# Do we live in the matrix?

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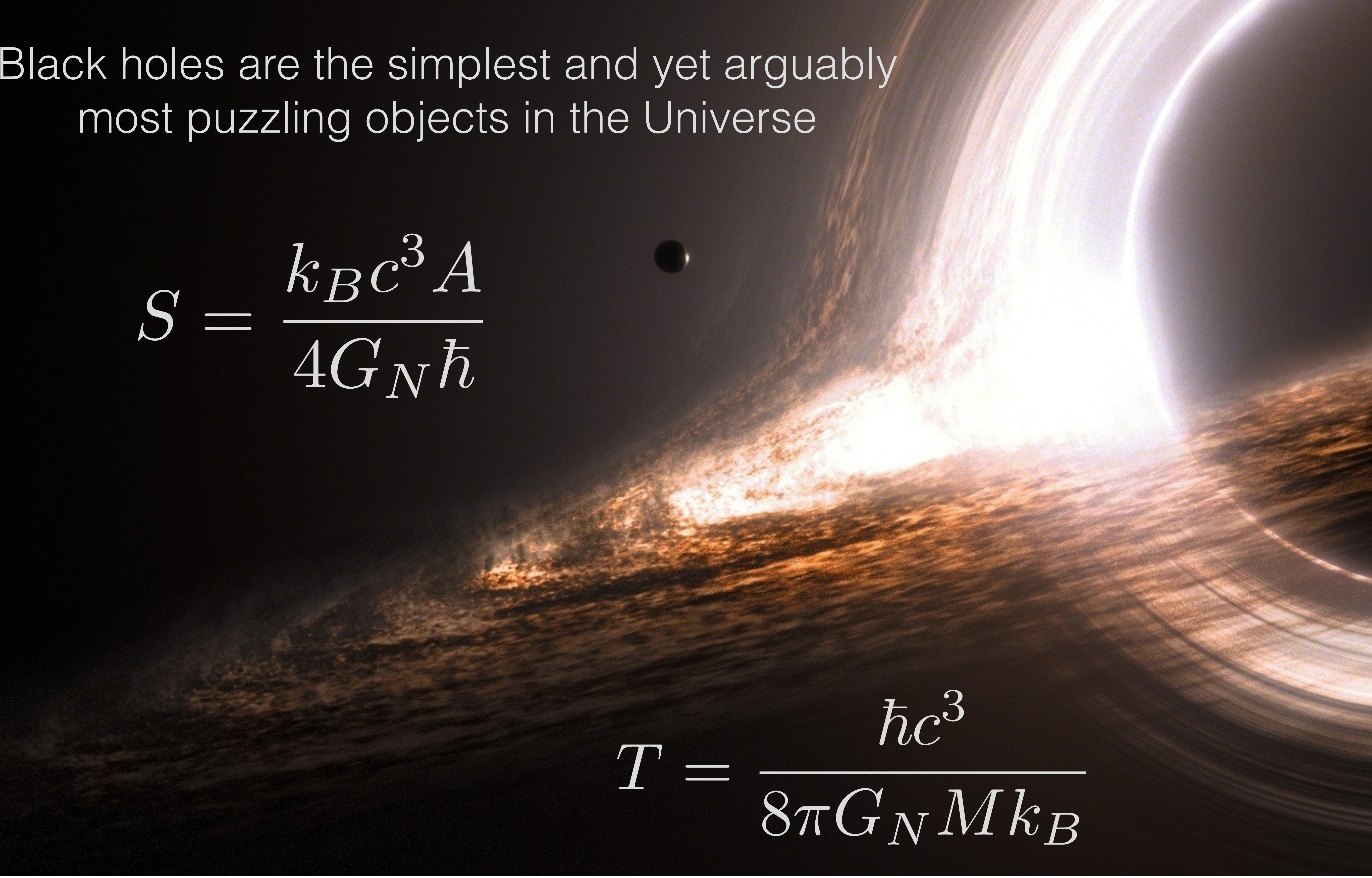
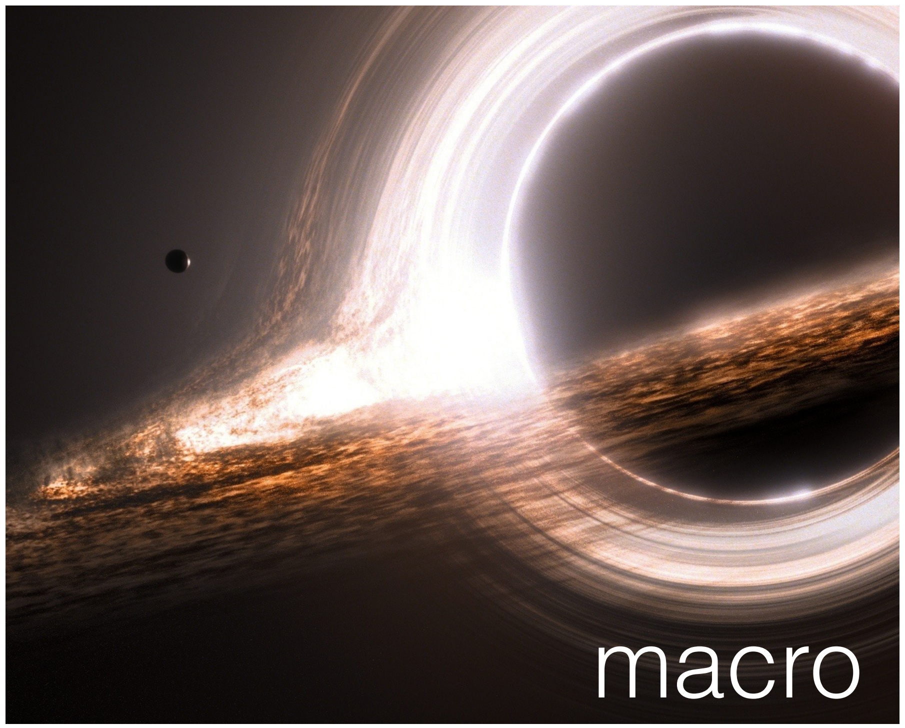
**A (-n incomplete) review on recent developments in the theory of random matrices and their ramifications in quantum gravity**

**Before delving right in to the subject matter, let me give some motivating thoughts**

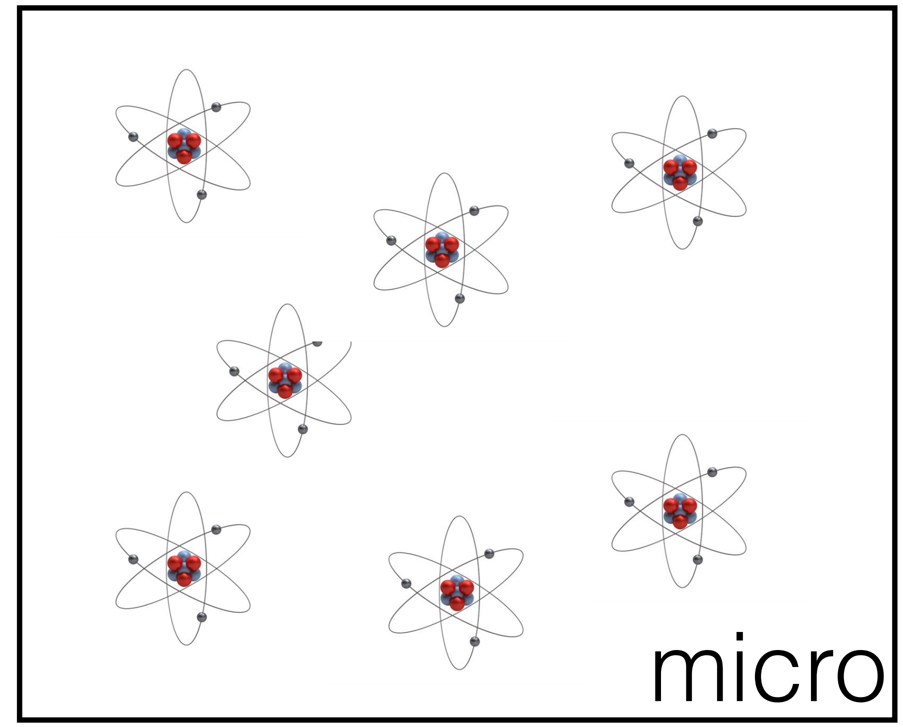



How do we understand this thermodynamic behaviour?

# WwBd*?




vs.



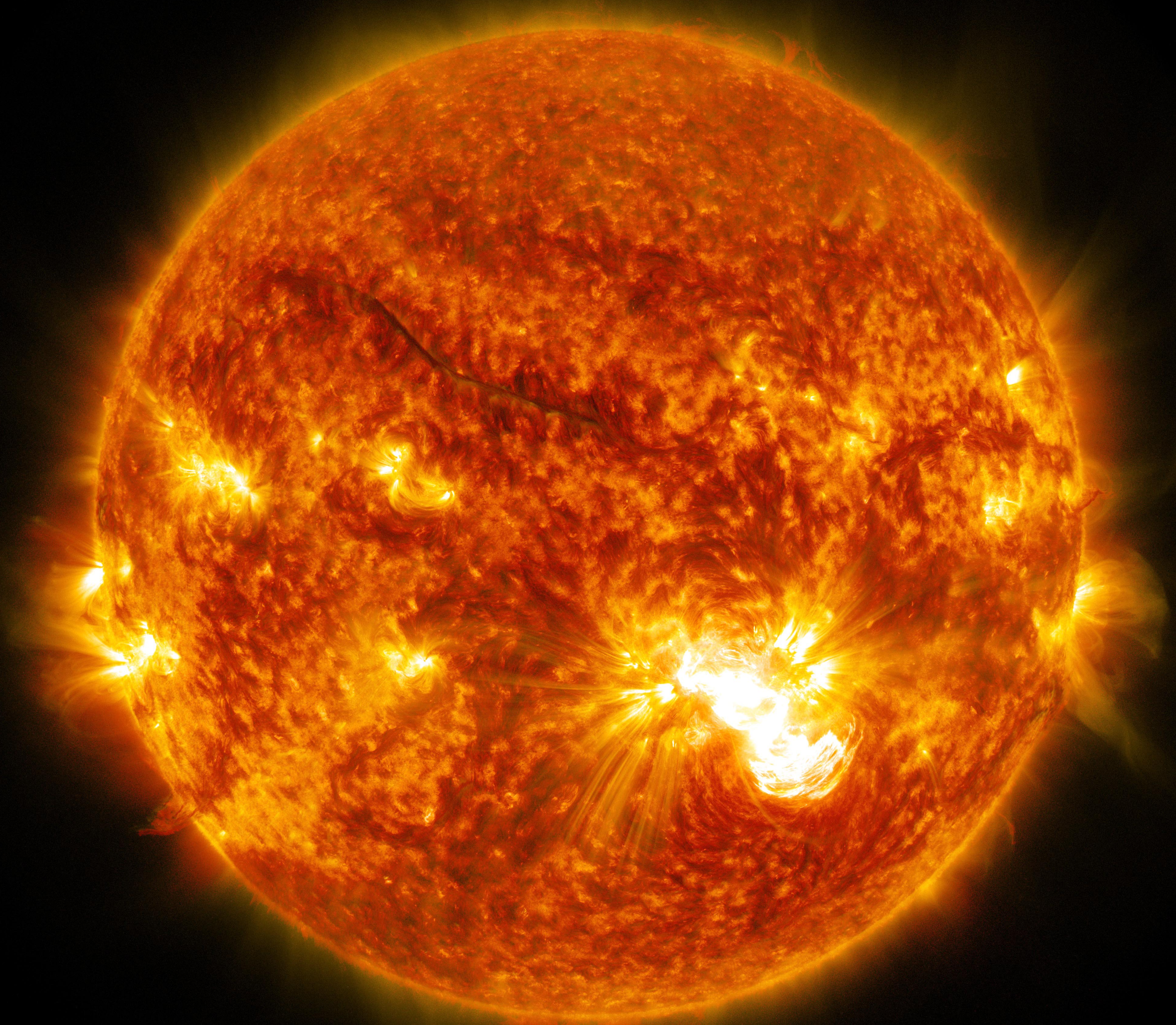

$$S_{\mathrm{BH}} = \frac{A}{4G_N} < \infty$$

$$S_{\mathrm{micro}} = \log \Omega$$

What is the microstructure of $\mathcal{H}_{\mathrm{BH}}$?

What do we learn about semi-classical gravity?

*What would Boltzmann do?



Studying **black holes** means studying the non-equilibrium process of **thermalisation**

**Quantum thermalisation** and quantum **ergodicity**
lead inevitably to random **matrices**

# The plan

I Background: matrix models for quantum chaos and gravity

II Statistical mechanics, chaos and thermalization

III Chaotic (AdS)/CFT

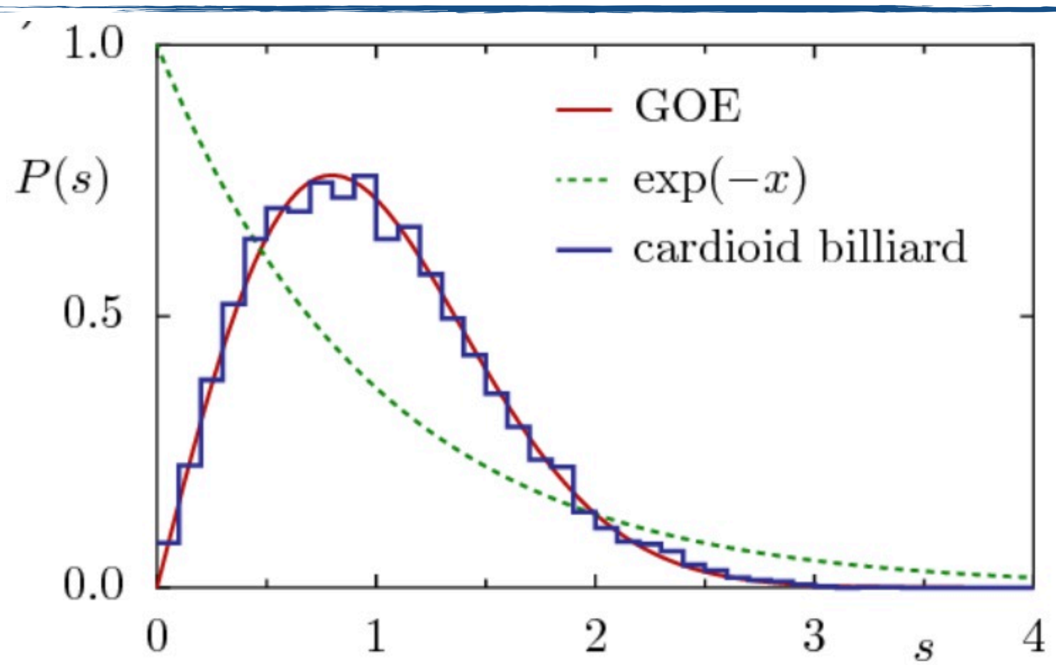
IV chaos ➥ quantum geometry

# Matrix Models and Quantum Chaos

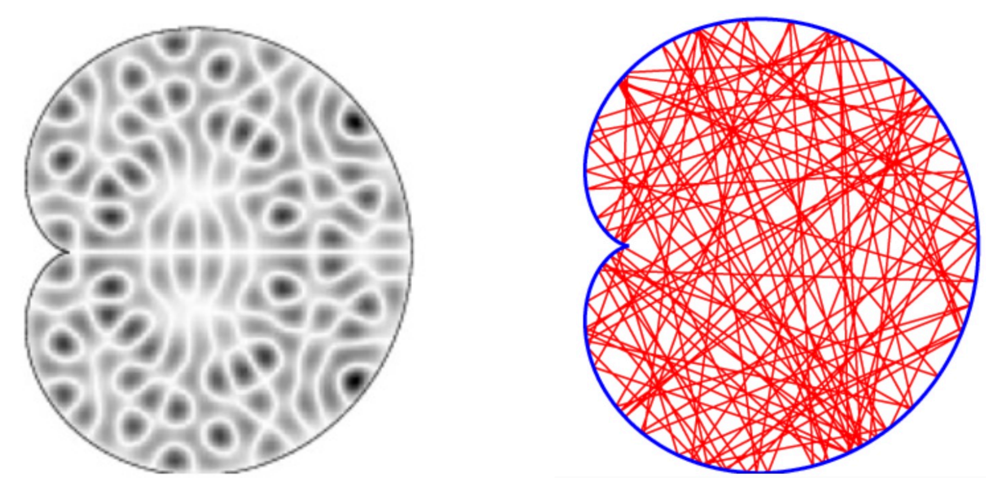
Matrix models = random matrix theory (RMT) = matrix integrals
[Wigner, Dyson,…]

Quantum chaotic H(amiltonian) $\cong$ RMT

Exemplified by spectral probes:




level statistics



wave function

trajectories

# A primer on random matrices

RMT = probability distribution for matrices P(H), generalising classical probability theory [Wigner, Dyson,…Voiculescu]

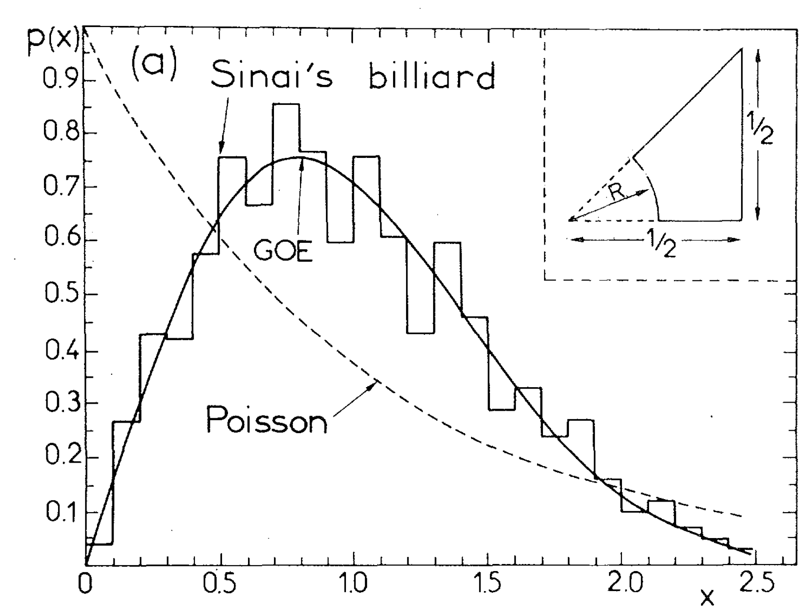
Simplest example: $$H = \begin{pmatrix} h_1 & \frac{V}{\sqrt{2}} \\ \frac{V^*}{\sqrt{2}} & h_2 \end{pmatrix}$$ i.i.d. from a Gaussian distribution

Direct calculation gives the gap distribution

$$P(s) = \int dh_1 dh_2 dV \, \delta\left(E_{12} - s\right) \exp\left(-\frac{h_1^2 + h_2^2 + V^2}{2\sigma^2}\right)$$

$$P(s) = \frac{\omega}{2\sigma^2} e^{-\frac{s^2}{4\sigma^2}}$$

GOE Wigner surmise



# The unreasonable effectiveness of RMT

(why was Wigner's guess so good?)

Chaos-RMT as a Goldstone **effective theory**
[Efetov, Wegner, …. Altland, JS]

$$\int dQ\, e^{-S[Q;\omega]} \quad \text{where} \quad Q \in \frac{U(2|2)}{U(1|1)\times U(1|1)} := \mathcal{M}(Q)$$

10 Altland-Zirnbauer symmetry classes (includes Wigner/Dyson)

Principle of **maximal ignorance** [Shannon, Balian]

The unique P(H) constrained only by fixing the norm of H, and which minimises the Shannon information $I = \sum_m P_m \log P_m$ is Gaussian RMT

$$P(H) \propto e^{-\mathrm{Tr} H^\dagger H}$$

Further constraints may add non-Gaussianity

# The quantum microscope

Two-level correlation ➛ analytically continued partition function

$$\boxed{F_\beta(t) = \sum_{i,j} e^{-\beta(E_i+E_j)\underbrace{+it(E_i-E_j)}_{\text{quantum microscope}}}}$$

Laplace transform relates this to spectral information

$$Z(\beta+it)Z^*(\beta-it) \quad \longleftrightarrow \quad \rho(E+\omega/2)\rho(E-\omega/2)$$

Probes differences of levels at finer & finer resolution!

In gravity, this is non-perturbative information about BH micro states

# Semiclassical gravity

We can calculate $F_\beta(t)$ semiclassically (including 1-loop dets):

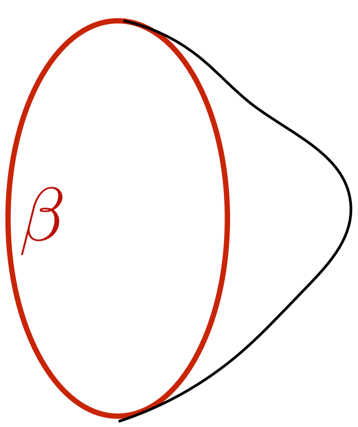
$$Z(\beta)_{\text{grav}} = e^{-I_{\text{grav}}[\bar{g}]} \det(I''[\bar{g}])$$

Gibbons-Hawking

Euclidean saddles

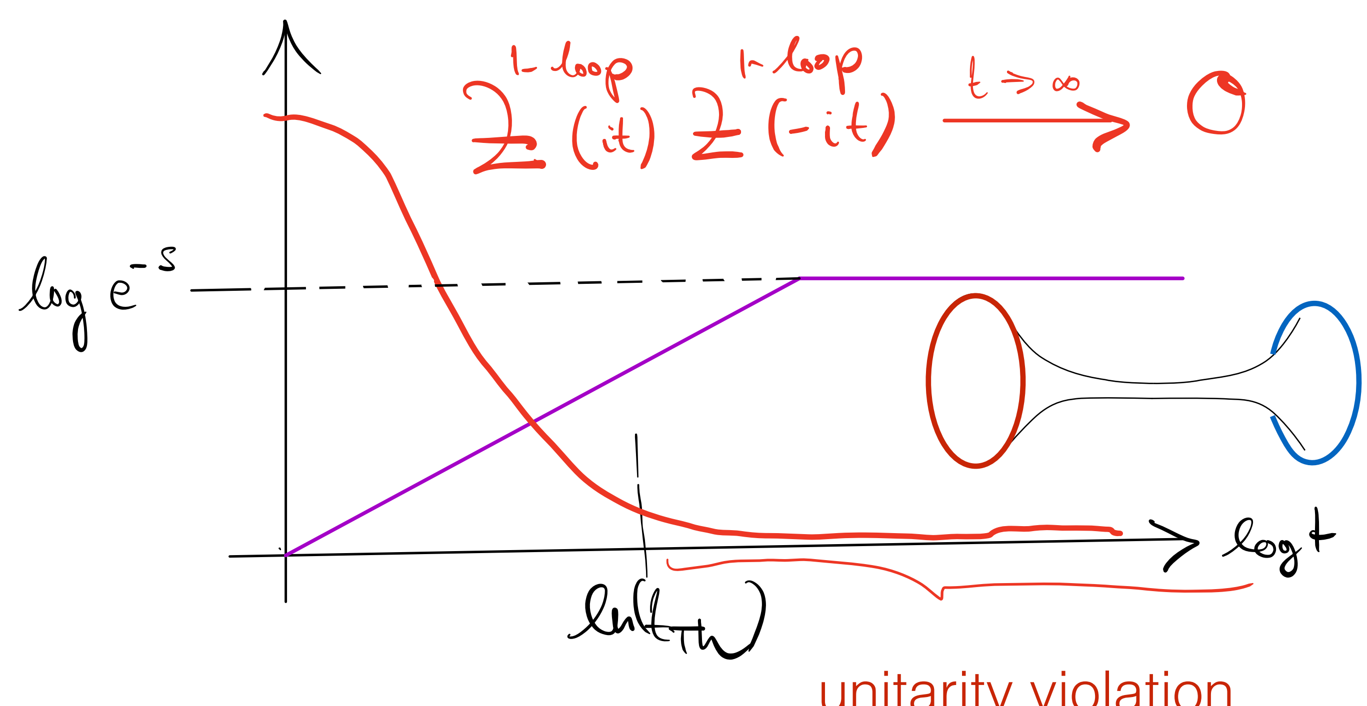
Graviton fluctuations

The relevant saddle:




Euclidean BH: $\tau \sim \tau + \beta$

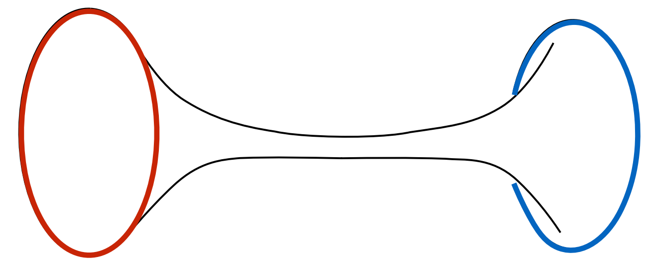
$\Rightarrow$



# Chaos and gravity

$t_{\mathrm{H}}$

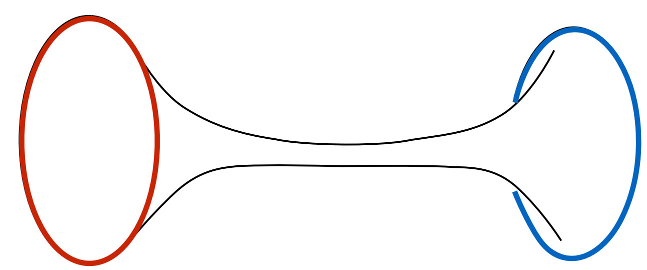
Heisenberg
$\sim \Delta^{-1} \sim e^S$

$e^{ie^S}$ ( Bdy 1  Bdy 2 ) genus - ∞

[Altland, Post, JS, vd Heijden, E.Verlinde; …]

$t_{\mathrm{Th}}$

Thouless
$\sim N^{1/2} \log N$

Bdy 1  Bdy 2 topology

[Saad, Shenker, Stanford; Mertens, Turiaci; Altland, Bagrets, Nayak, JS; Stanford, Witten…]

$t_s$

scrambling
$\sim \hbar^{-1} \log S$

$\langle A_t B A_t B \rangle$  $\sim$ $e^{2\lambda t}$ Gravity shocks

[Larkin, Ovchinnikov; Kitaev; Shenker, Stanford,….]

$t_0$

# Operators: Eigenstate Thermalization (ETH)

Attempts to answer: How do unitary quantum systems thermalize?

$$\langle i|\mathcal{O}|j\rangle = \overline{\mathcal{O}}(\overline{E})\delta_{ij} + e^{-S/2}f(E_i,E_j)R_{ij}$$

[Deutsch; Srednicki,…]

Smooth function of average energy

smooth function
theory dept.

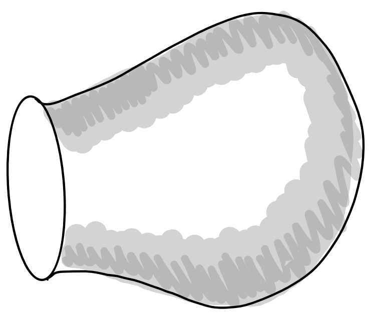
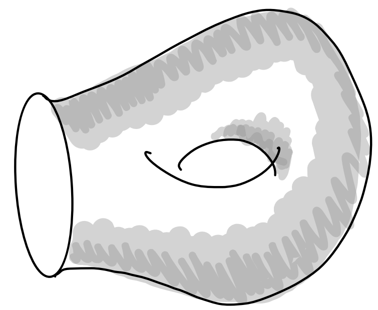
random matrix
(Gaussian, for now)

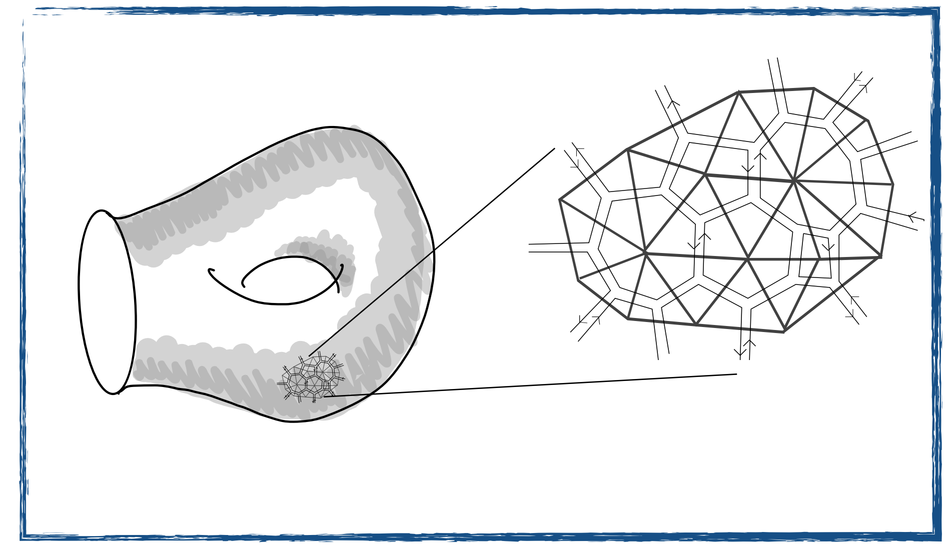
**Goal**: fit ETH and RMT into a joint framework, the "ETH matrix model"

**Remark**: (universal) emergence of RMT and operator statistics can be understood in terms of a Goldstone EFT: [Wegner; Efetov; Altland, JS]

# Random matrices and quantum gravity

2D gravity can be defined from random triangulations of surfaces:

 $+ \; e^{-S_0}$  $+ \cdots$



[Kazakov; Gross, Migdal; Moore, Shenker;… SSS]

$$\sum_{\text{top}} \int \mathcal{D}h e^{-S[h]} \quad \leftrightarrow \quad \int d\mu[H] e^{-\mathrm{tr}V(H;\lambda,N)}$$

RMT = discrete triangulation of 2D universes, continuum gravity path integral emerges from double-scaling limit

Recent developments suggest unity of points I & II

[SSS; Altland, Bagrets, JS, Nayak; Johnson,….]

# Some milestones

- perturbative equivalence of RMT and 2D gravity (v2.0?)

  - Euclidean wormholes encode chaotic eigenstatistics [SSS; Altland, Bagrets, JS, Nayak; Johnson,….]
  - Ensemble vs. factorized duality
  - Baby universes and $\boldsymbol{\alpha}$-states

  [SSS; Altland, JS, Nayak; Marolf, Mayfield; Blommaert, Mertens; Iliesu,…..]

- non-perturbative 2D gravity

  - Construction of universe field theory for JT-gravity
  - Chaotic sigma model = universe field theory
  - late-time plateau in spectral form factor and correlation functions

  [Altland, JS; Post, vdH, Verlinde; Saad, Shenker, Stanford; Blommaert, Kruthoff, Yao,…]

- quantum chaos in quantum gravity

  - ETH in CFT as statistics of OPE coefficients [Dymarsky, Lashkari, Liu; Belin, de Boer; Ross,..]
  - statistical mechanics interpretation of apparent averaging in gravity [Pollack, Rozali, Sully, Wakeham; …]
  - 3D gravity as ensemble of chaotic 2D CFT
  - Virasoro/Teichmüller TQFT and random tensor models

  [Cotler, Jensen; Chandra, Collier, Hartman, Maloney; Belin, de Boer, Jafferis, Nayak, JS;]

  [Eberhard, Collier, Zhang]

# A few questions:

I What is the Wigner ensemble for a (holographic) CFT?

II Can we establish (in controlled examples) a connection to gravity?

III More insight about gravity as an ensemble of quantum theories?

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# Packaging as a (two) matrix model

[Jafferis, Kolchmeyer, Mukhametzhanov, JS, PRX '23]

We have two inputs:

- Energy-level statistics: leads to random matrix $H_{ij}$
- Operator matrix-element statistics: leads to random matrix $R_{ij}$

$$\mathcal{Z} = \int d\mu[H] d\mu[\mathcal{O}] e^{-\mathrm{tr} V(H,O)}$$

In energy eigenbasis we have

$$V = -\sum_i V(E_i) - e^{S_0} \sum_{i,j} \tfrac{1}{2} F_{ij} \mathcal{O}_{ij} \mathcal{O}_{ji} + e^{S_0} \sum_{i_1,\ldots i_n} G^{(n)}_{i_1 \ldots i_n} \mathcal{O}_{i_1 i_2} \cdots \mathcal{O}_{i_n i_1}$$

"Gaussian ETH"

Determined by matching $\boldsymbol{\rho}(E)$

Determined by matching hierarchy of $g^{(n)}$

# The ETH matrix model

Comments:

- $V(H)$ determines $\boldsymbol{\rho}(E)$, $\boldsymbol{\rho}^{(2)}(E, E')$,… gives connected energy correlations

- $F_{ij}$, $G^{(n)}{}_{ij}$…. non-Gaussianities of operator statistics

Like in standard ETH, these are free functions: how to determine them?

Input more knowledge by imposing constraints:

- EFT of quantum chaos (quantum-chaos universality)

- Determine non-Gaussianities by imposing additional constraints (e.g. modular crossing, s/t crossing,…). First example: JT + matter

[Jafferis, Kolchmeyer, Mukhametzhanov, JS, PRX '23]

⋮

# Approximate CFTs

[de Boer, Belin, Jafferis, Nayak, JS;]

# Random ensembles for chaotic CFT

[Cotler, Jensen; de Boer, Belin, Nayak, JS; Chandra, Collier, Maloney, Hartman]

Let us think about the idea of “ETH matrix models” for (chaotic) CFT:

Do not expect straight-forward Wigner (“Altland-Zirnbauer”)-type RMTs
BUT: RMT universality can hold ‘spin-by-spin’

[very recent: Ubaldi, Perlmutter; Haehl, Reeves, Rozali,…]

Most natural approach, average over CFT data

$$\{\Delta_i \, , \, J_i \, , \, C_{ijk}\}$$

These data define a CFT only if they satisfy consistency conditions

→ so what are the elements of such ensembles?

# Definition of approximate CFT

[de Boer, Belin, Jafferis, Nayak, JS;]

An approximate CFT is the set of data $\{\Delta_i\ ,\ J_i\ ,\ C_{ijk}\}$, which *approximately satisfy* CFT constraints, in the sense that

- CFT constraints are only imposed for a subset $\{\mathbb{O}_{\mathrm{restr}}\}$ of "light operators"
- These constraints are only imposed up to a tolerance parameter $\mathbb{T}$ where $\mathbb{T} \sim e^{-S}$
- This can equivalently be seen as imposing restrictions $(n_{\max}, \Delta_{\max}, z^L_{\min}\ \ldots)$ on number of insertions, maximal dimension, minimal cross ratio,…
- Allows in principle large violations of CFT constraints (!), but in a way that is carefully correlated across the spectrum

The approximate bootstrap constraints open up islands to average over

# Examples of approximate CFT

Take a true CFT and to shift the dimension of a single operator $\mathcal{O}_0$

$$\Delta_0 \to \Delta_0 + \epsilon$$

If $\mathcal{O}_0 \in \{\mathbb{O}_{\mathrm{rest}}\}$ the correlation functions of $\mathcal{O}_0$ violate crossing arbitrarily strongly, in other words $\mathcal{O}_0 \notin \{\mathbb{O}_{\mathrm{rest}}\}$

However, even light correlation functions will violate crossing, due to heavy operators in intermediate channels

In fact, we show that these kinds of violations can be counterbalanced by shifting other heavy operators in correlated fashion

Crucially this produces violations of crossing of operators $\in \{\mathbb{O}_{\mathrm{rest}}\}$ uniformly bounded by $\epsilon$ in cross-ratio space

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# Ensembles for random (2D)CFT

We will now construct a CFT ensemble for 2D CFT concretely

This will be a joint statistical model of the data for 2D CFT:

$$\{h_i, \bar{h}_i, C_{ijk}\}$$

random matrix
$h_i = \mathrm{eig}(L_{ij})$

random tensor

We can construct the resulting tensor model by imposing approximate bootstrap constraints (→ notion of approximate CFT)

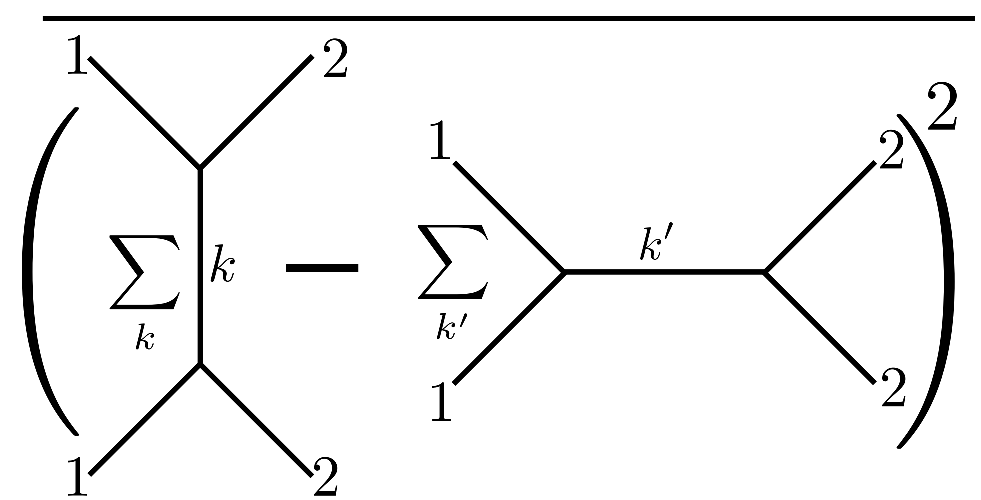
In order to define the measure can a) consider moments or b) use CFT constraints more directly

# a) moments: crossing implies non-Gaussianity

Suppose crossing in the ensemble is satisfied on average, then

imposing the variance of crossing on average to be ≈ 0

$$\overline{\left(\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_2\mathcal{O}_1\rangle_s - \langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_2\mathcal{O}_1\rangle_t\right)^2} =$$




$$\approx 0$$

This implies the existence of large non-Gaussianities among OPE coefficients

Virasoro 6j (crossing kernel)

$$\overline{C_{ijk}C_{iml}C_{njl}C_{nmk}}\Big|_c = \begin{Bmatrix} \mathcal{O}_k & \mathcal{O}_j & \mathcal{O}_i \\ \mathcal{O}_l & \mathcal{O}_m & \mathcal{O}_n \end{Bmatrix}$$

This fixes the second moment. What about higher moments?

# b) fixing all moments by statistics of crossing

In fact, by approximately enforcing crossing, we can construct the full potential. For example:

$$\sum_q \left( C_{i_1 i_2 q} C_{i_3 i_4 q} \delta^{(2)} \left(P_s - P_q\right) - C_{i_1 i_4 q} C_{i_2 i_3 q} \left| \mathbb{F}_{P_q P_s} \begin{bmatrix} P_3 & P_4 \\ P_2 & P_1 \end{bmatrix} \right|^2 \right) = 0$$

The 4-point crossing on the sphere thus is the constraint equation

$$M^{i_1 i_2}_{i_3 i_4}(P_s, \bar{P}_s) = 0$$

Modular crossing of torus 1-point function gives a similar (simpler) condition

The total potential is then defined as the maximum ignorance ensemble of CFT data that imposes (on average) these constraints

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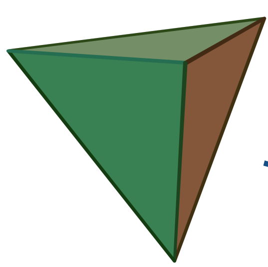
# A tensor model for random CFT

Builds again a type of "ETH tensor model", with potential given by the square of the crossing equation

$$\mathcal{Z} = \int d\mu[H] d\mu[C] \exp\left(-V(H,C)\right)$$

with quartic non-linearity implementing the square of crossing

$$V_4 = 2 \sum_{i_1 \cdots i_4}{}' \sum_{p,q} \left( \frac{C_{i_1 i_2 p} C_{i_3 i_4 p} C_{i_1 i_2 q} C_{i_3 i_4 q}}{|\rho_0(p) C_0(12p) C_0(34p)|^2} \delta^{(2)}\left(P_p - P_q\right) - \right.$$



$$\left. \frac{C_{i_1 i_2 p} C_{i_3 i_4 p} C_{i_1 i_4 q} C_{i_2 i_3 q}}{|C_0(12p) C_0(34p) C_0(23q) C_0(14q)|^2} \begin{Bmatrix} \mathcal{O}_q & \mathcal{O}_4 & \mathcal{O}_1 \\ \mathcal{O}_p & \mathcal{O}_2 & \mathcal{O}_3 \end{Bmatrix} \right)$$

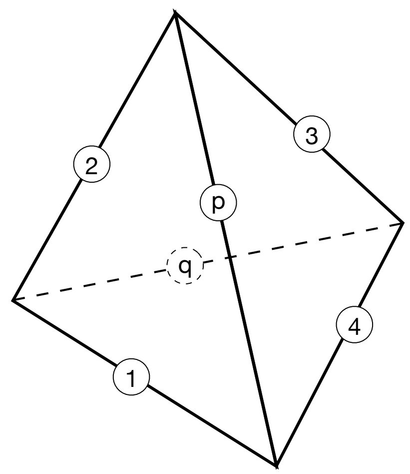
More terms in the potential come from modular crossing to fix $\rho(\Delta)$

[= maxRMT of Ubaldi and Perlmutter?]

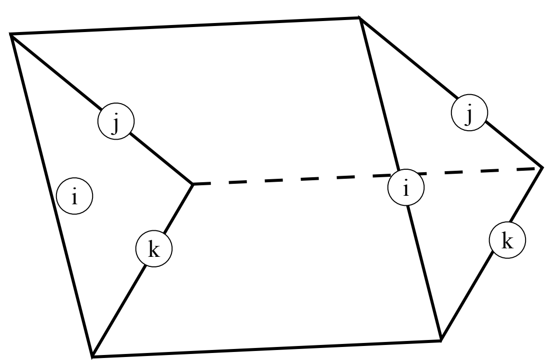
Propagator of the model comes from identity running in the sum

# 2D random CFT = 3d gravity?

Tensor diagrams are simplicial decompositions weighted by:




$$= \begin{Bmatrix} \mathcal{O}_q & \mathcal{O}_4 & \mathcal{O}_1 \\ \mathcal{O}_p & \mathcal{O}_2 & \mathcal{O}_3 \end{Bmatrix} |C_0(12p)C_0(34p)C_0(23q)C_0(14q)|^{-2}$$



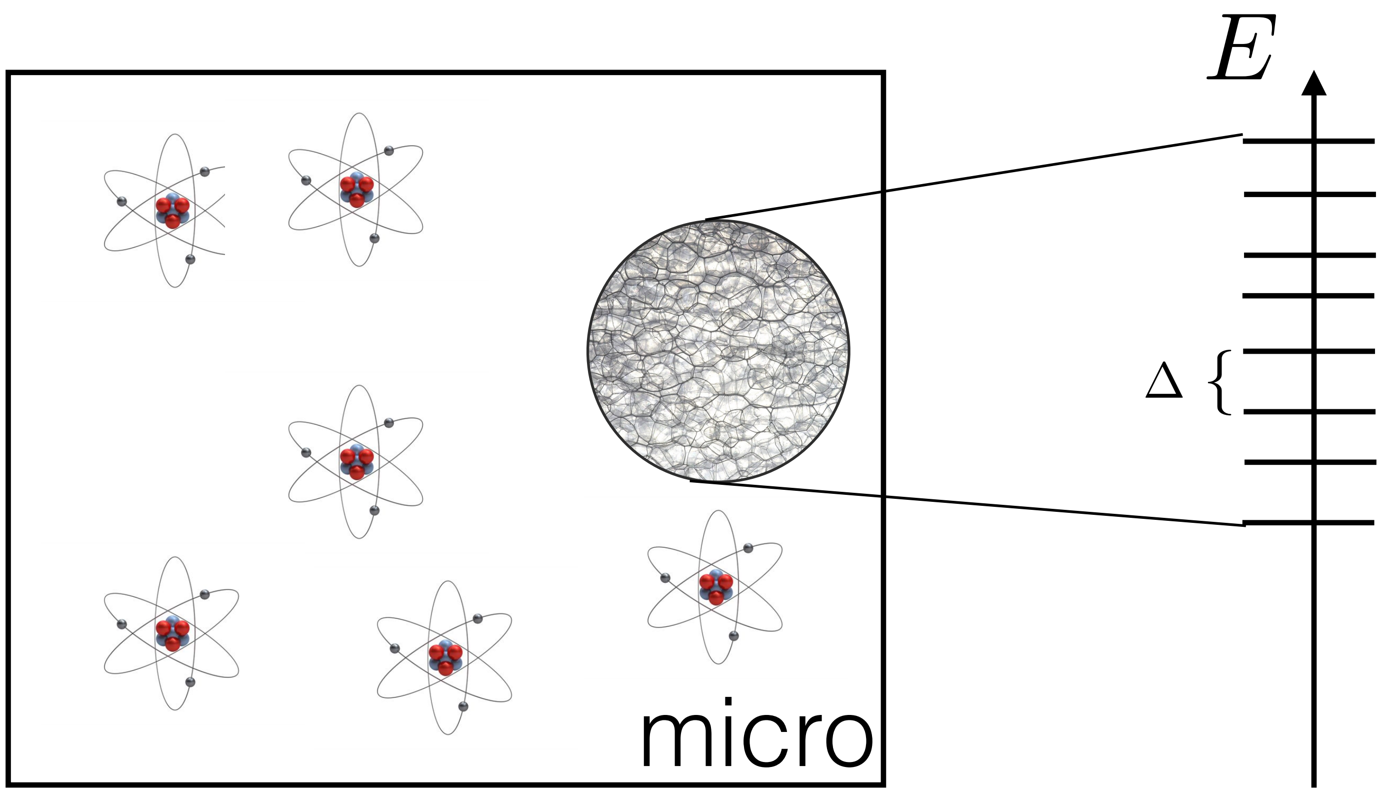

$$= \quad C_0\,(P_i, P_j, P_k)\, C_0\left(\bar{P}_i, \bar{P}_j, \bar{P}_k\right)$$

Non-perturbative Schwinger-Dyson equations generate moments of approximate CFTs studied earlier

A new role for “simplicial-gravity” in 3D? [Regge, Ponzano…] [Boulatov; M. Gross, Ambjørn,…]

# Summary

Matrix description of quantum spacetime = chaotic correlations




Is averaging an approximation or fundamental?


BULK
FICTION
Thank you !