QED Sum Rules in B Physics

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(He/Him)

With Roman Zwicky

Gauge variant operators

- 'Create' meson states with operators, e.g. $J_B = m_b \bar{b} i\gamma_5 q$.

$$\langle \bar{B} | J_B | 0 \rangle = m_B^2 f_B, \qquad \Gamma(\bar{B} \to l^- \bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_l^2 m_B \left(1 - \frac{m_l^2}{m_B^2}\right)^2 \times f_B^2$$

- Bound states in QED can carry charge. Interpolating operators become gauge variant.

$$J_B \to e^{-ie(Q_b - Q_q)\alpha} J_B = e^{-ieQ_{\bar{B}}\alpha} J_B$$

- Introduce a new charged scalar field Φ to compensate.

$$\Phi J_B \to e^{ie(Q_\Phi - Q_{\bar{B}})\alpha} \Phi J_B = \Phi J_B$$

S. Nabeebaccus and R. Zwicky, 'Resolving charged hadrons in QED – gauge invariant interpolating operators', JHEP 11 (2022) 101.

$\bar{B} \to l\nu(\gamma)$ from sum rules

Define the correlator

$$\Pi^{(\gamma)}(p_B^2) = i \int \mathrm{d}^D x \, e^{ix\cdot(p_\Phi - p_B)} \langle l^- \bar{\nu}(\gamma) | T i\mathcal{H}_w(x) (\Phi_B J_B)(0) | \Phi_B(p_\Phi) \rangle$$

Insert an identity $I = |0\rangle\langle 0| + \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{2E_q} |\bar{B}(q)\rangle\langle \bar{B}(q)| +$multiparticle continuum.

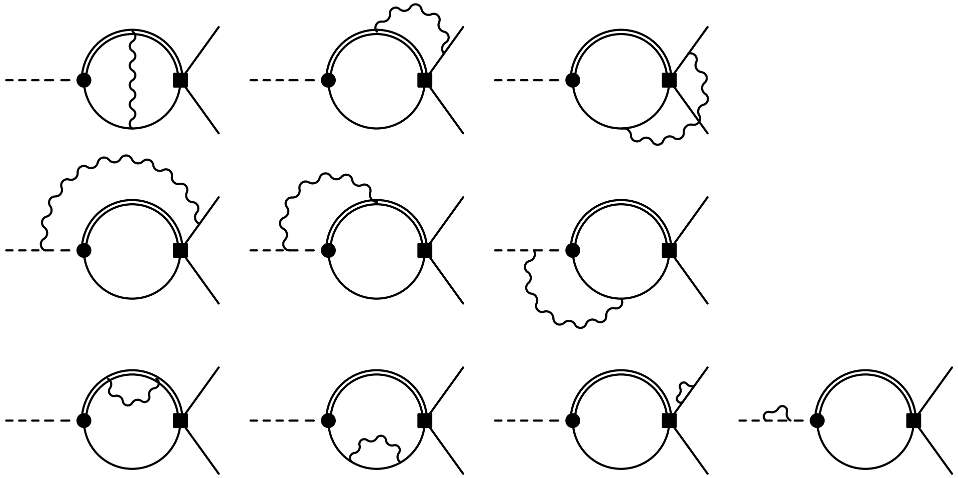
$$\langle l^- \bar{\nu}(\gamma) | i\mathcal{H}_w | \bar{B}\rangle \, \langle \bar{B} | \Phi_B J_B | \Phi_B \rangle \;\longrightarrow\; i\mathcal{A}^{(\gamma)} \, g_\Phi$$

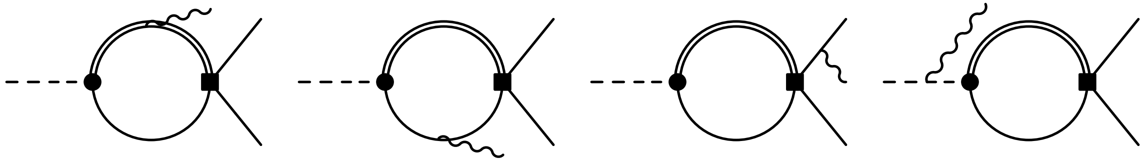
After integration / dispersion

$$\Pi^{(\gamma)}(p_B^2) = \int \mathrm{d}s \, \frac{\frac{1}{\pi} \operatorname{Im} \Pi(s)}{s - p_B^2} = \frac{i\mathcal{A}^{(\gamma)} \, g_\Phi}{m_B^2 - p_B^2} + \text{continuum.}$$

Partonic computation

- Calculate $\mathrm{Im}\,\Pi^{(\gamma)}$ in perturbative QCD/QED via cutting rules.

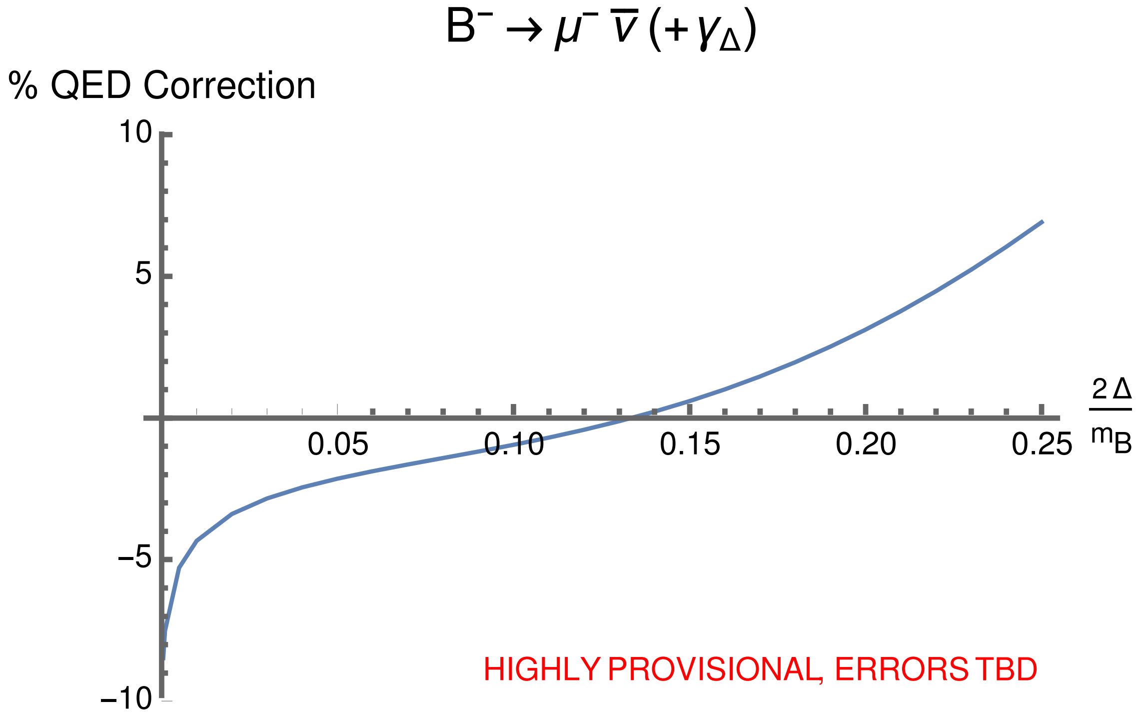




- Include condensates and Borel transform to get $\mathcal{A}^{(\gamma)}$.

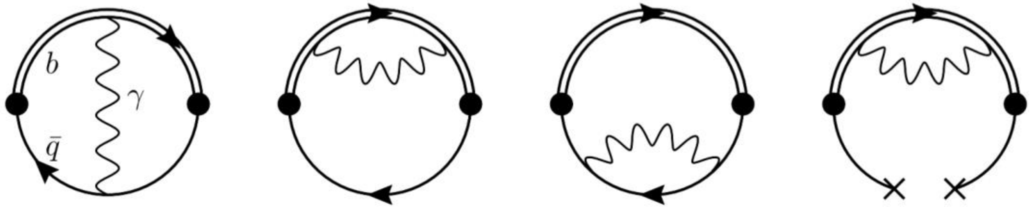
Results

- Correct IR + gauge behaviour
- No factorisation in QED $\to$ large hard-collinear logs?
  - Cancel for S−P
  - Exist in V−A
- Real dominated for $m_l << \Delta E$.

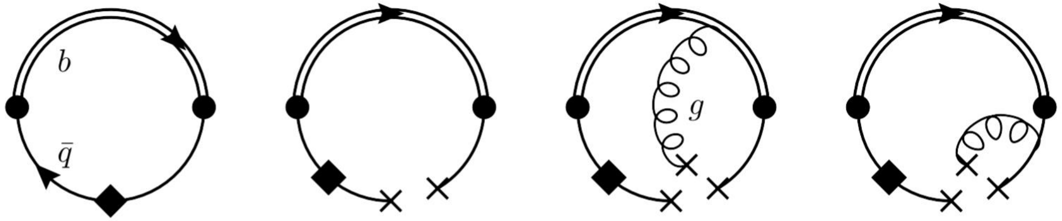


Meson mass splittings

- Charged and neutral mesons do not have the same mass, $m_{B^-} \neq m_{B^0}$.
  - QED effects $\langle B | T j_\mu(x) j_\nu(0) | B \rangle$
  - $m_u \neq m_d$.



- Consider $\Pi \sim \int \langle 0 | T J_B^\dagger j_\mu j_\nu J_B | 0 \rangle$
  - Insert $|\bar{B}\rangle\langle\bar{B}|$ twice (double dispersion)
  - Gives $m_B^4 f_B^2 \times \langle B | T j_\mu(x) j_\nu(0) | B \rangle$.



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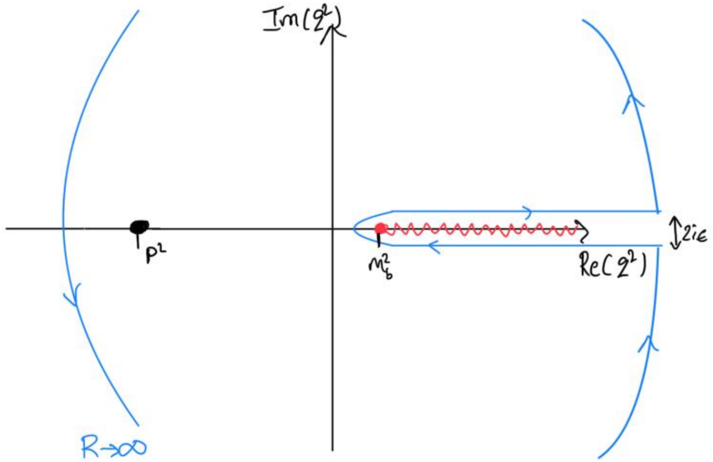
Thank you for listening

[EXTRA] QCD SR: The main idea

- Define a correlator with the right quantum numbers $\Pi(q^2)$
- At high energies calculate it in PT
- At low energies it can be represented in terms of non-perturbative hadronic quantities
- Link the two via complex analysis (a dispersion relation)
- Extract hadronic stuff from the PT calculation.



[EXTRA] f_B from QCD SR

Define

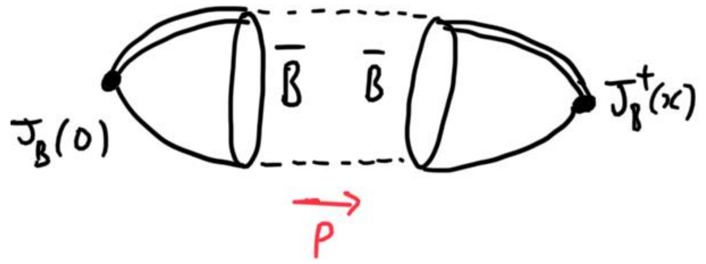
$$\Pi(p^2) = i\int \mathrm{d}^d x\, e^{ip\cdot x}\langle 0|T J_B^\dagger(x) J_B(0)|0\rangle$$

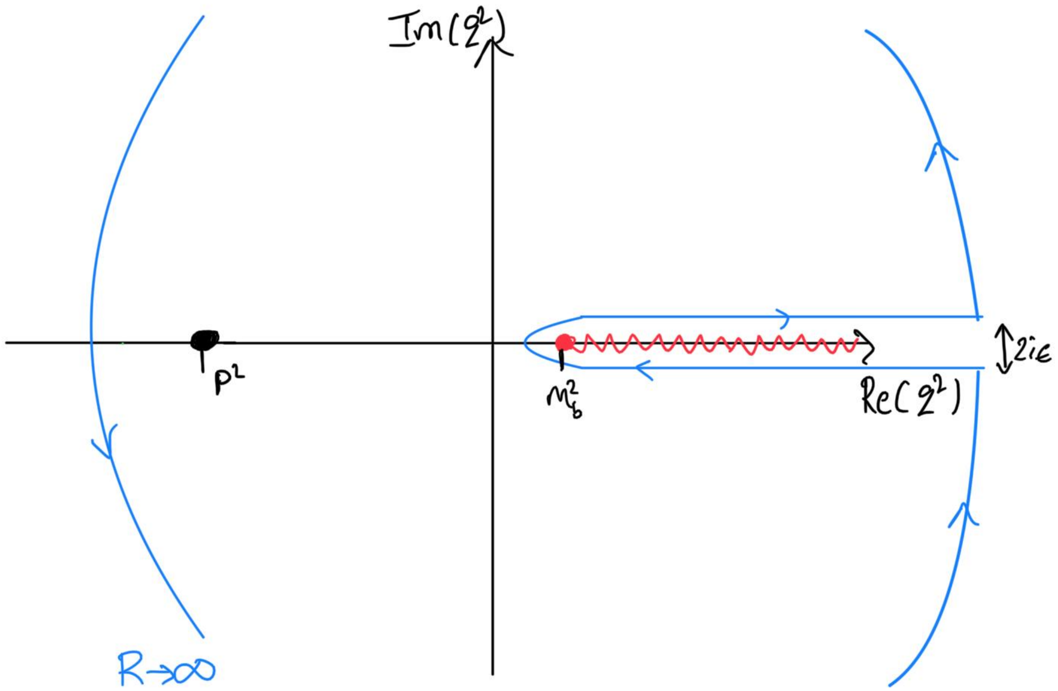
By Cauchy

$$\Pi(p^2) = \frac{1}{2\pi i}\oint \mathrm{d}z\, \frac{\Pi(z)}{z-p^2}$$

Provided the circle vanishes at ∞

$$\Pi(p^2) = \int_{m_b^2}^{\infty} \frac{\mathrm{d}s}{2\pi i}\frac{\mathrm{Disc}\Pi(s)}{s-p^2}$$





[EXTRA] f_B from QCD SR - 2

On the hadronic side, insert an identity

$$I = |0\rangle\langle 0| + \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{2E_q} |\bar{B}(q)\rangle\langle\bar{B}(q)| + \text{multiparticle}$$

into the correlator

$$\Pi(p^2) = i \int \mathrm{d}^d x \, \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{e^{ip\cdot x}}{2E_q} \underbrace{\langle 0|J_B^\dagger(x)|\bar{B}(q)\rangle}_{m_B^2 f_B e^{-iq\cdot x}} \underbrace{\langle\bar{B}(q)|J_B(0)|0\rangle}_{m_B^2 f_B}$$

Doing the integrations and taking the discontinuity

$$\mathrm{Disc}\Pi(p^2) = 2\pi i \, m_B^4 f_B^2 \, \delta(p^2 - m_B^2)$$

$$\implies \Pi(p^2) = \frac{m_B^4 f_B^2}{m_B^2 - p^2} + \text{continuum}$$

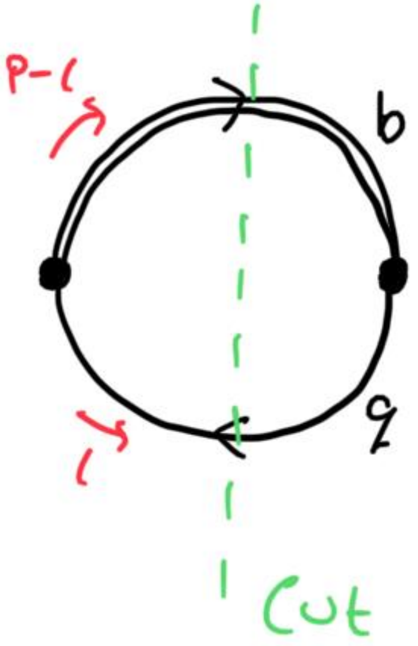
$$\mathrm{Disc} \frac{1}{x - a + i\epsilon} = -2\pi i \, \delta(x - a)$$

[EXTRA] f_B from QCD SR - 3

On the partonic side

$$\Pi(p^2) = -im_b^2 \int \mathrm{d}^d x\, e^{ip\cdot x} \langle 0|\, \bar{q}(x)\gamma_5\, b(x)\bar{b}(0)\gamma_5 q(0)|0\rangle$$

$$= -im_b^2 N_c \int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{\mathrm{Tr}[\gamma_5(-\not{l})\gamma_5(\not{p}-\not{l}+m_b)]}{[(p-l)^2 - m_b^2]\, l^2}$$



Via Cutkosky cutting rules

$$\mathrm{Disc}\Pi(p^2) = -im_b^2 N_c\, (-2\pi i)^2 \int \frac{\mathrm{d}^d l}{(2\pi)^d}\, \delta^+(l^2)\, \delta^+((p-l)^2 - m_b^2) \times \mathrm{Tr}[\cdots]$$

[EXTRA] f_B from QCD SR - 4

Evaluating the first delta function

$$\frac{\mathrm{Disc}\Pi(p^2)}{2\pi i} = \frac{m_b^2 N_c}{2\pi^3} \int \frac{\mathrm{d}^3\vec{l}}{2|\vec{l}|} \delta^+(p^2 - m_b^2 - 2p\cdot l)\, p\cdot l$$

Choose the $p-$RF for convenience

$$\begin{aligned}\frac{\mathrm{Disc}\Pi(p^2)}{2\pi i} &= \frac{m_b^2 N_c}{8\pi^3}(p^2 - m_b^2) \int 4\pi|\vec{l}|\,\mathrm{d}|\vec{l}|\,\delta^+(p^2 - m_b^2 - 2\sqrt{p^2}|\vec{l}|) \\ &= \frac{m_b^2 N_c}{8\pi^2}\frac{(p^2 - m_b^2)^2}{p^2}\,, \qquad p^2 \geq m_b^2\,.\end{aligned}$$

[EXTRA] f_B from QCD SR - 5

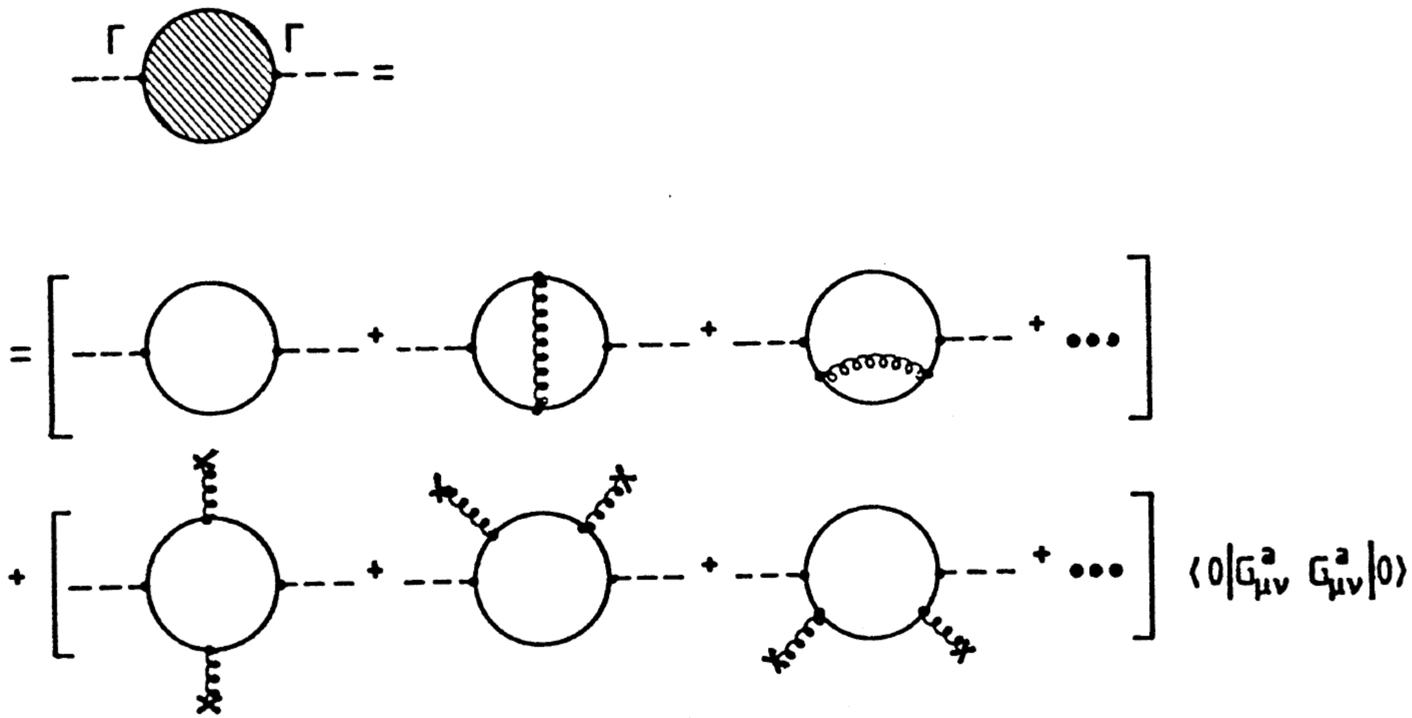
Equating the hadronic and partonic representations then:

$$\frac{m_B^4 f_B^2}{m_B^2 - p^2} + \text{continuum} = \frac{m_b^2 N_c}{8\pi^2} \int_{m_b^2}^{s_0} \frac{\mathrm{d}s}{s - p^2} \frac{(s - m_b^2)^2}{s} + \text{subtractions}$$

Thus we can calculate f_B. In practise we Borel transform $\Pi(p^2) \to \tilde{\Pi}(M^2)$.

[EXTRA] It's not that simple!

- ❑ $O(\alpha_s^n)$ corrections
- ❑ Condensates
- ❑ UV behaviour (subtractions)
- ❑ Daughter SR
- ❑ Quark-hadron duality
- ❑ Borel transformations
- ❑ Instantons



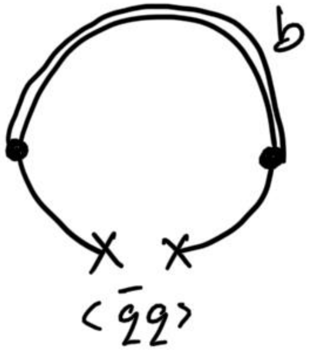
L.J. Reinders, QCD Sum Rules: An Introduction and some Applications. CERN Library, Ref.TH.3701-CERN,1983.

[EXTRA] It's not that simple! - 2

Leave a pair of q quark fields uncontracted

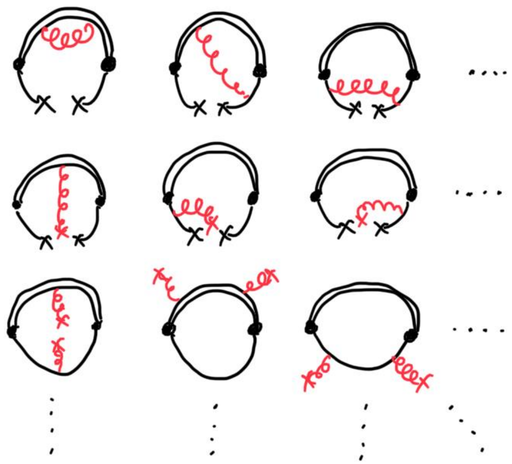
$$\Pi(p^2) = -im_b^2 \int \mathrm{d}^d x\, e^{ip\cdot x} \langle 0|\bar{q}(x)\gamma_5\, b(x)\bar{b}(0)\gamma_5 q(0)|0\rangle$$

$$= -im_b^2 N_c \int \mathrm{d}^d x\, e^{ip\cdot x} \left(\gamma_5 S_b(x)\gamma_5\right)_{\alpha\beta} \langle 0|\bar{q}_\alpha(x) q_\beta(0)|0\rangle$$



This matrix element would vanish in PT. However it is non-zero due to non-PT effects

$$\langle 0|\bar{q}_\alpha(x) q_\beta(0)|0\rangle = \frac{\langle \bar{q}q\rangle}{4N_c}\delta_{\alpha\beta} + \mathcal{O}(x)\,, \qquad \langle \bar{q}q\rangle_{2\mathrm{GeV}} = (-267\mathrm{MeV})^3$$

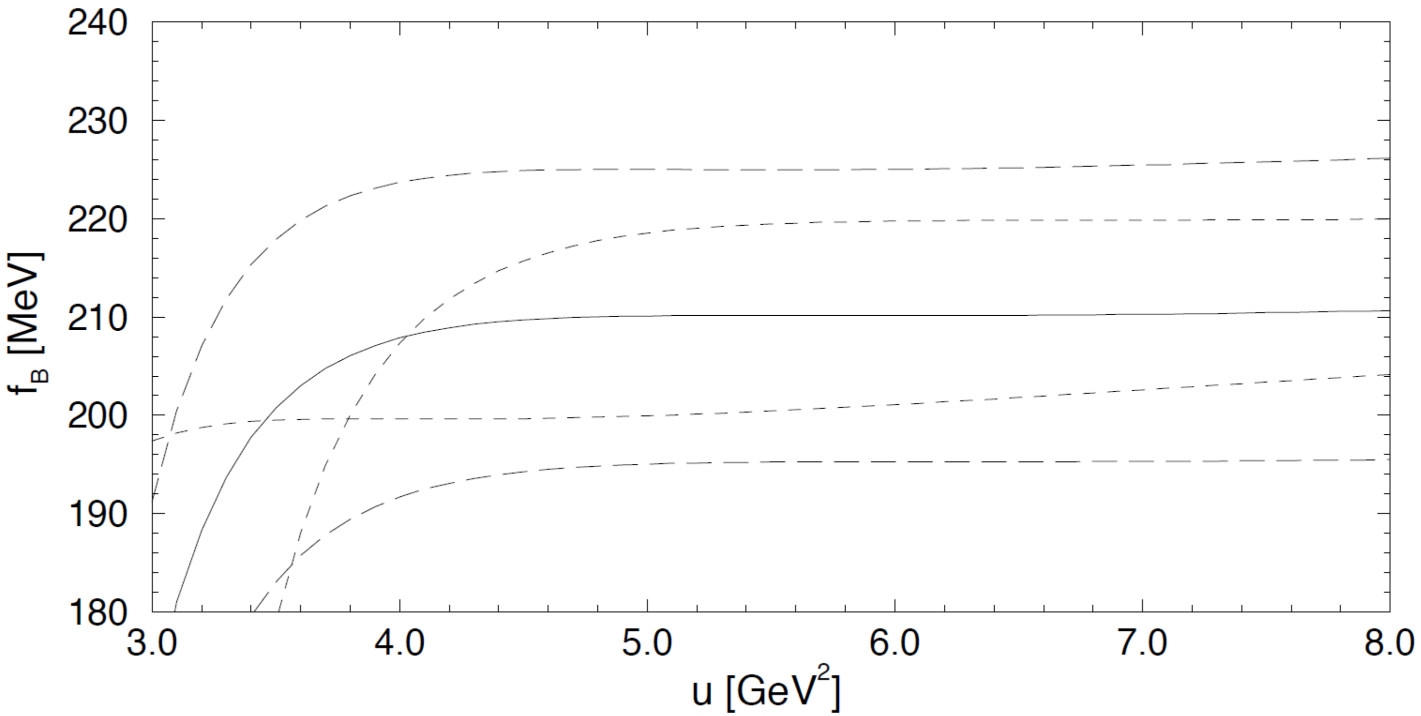


This is all formalised via Wilson's Operator Product Expansion (OPE)

$$\Pi(p^2) = \sum C_j(p^2)\langle 0|O_j|0\rangle\,, \qquad O_j = \{I, \bar{q}q, \bar{q}\sigma G q, G^2, \ldots\}$$

[EXTRA] Numerics

- Classic Jamin & Lange Result
  - 3-loop ($O(\alpha_s^2)$) MS
  - PT, $\langle \bar{q}q \rangle$, $\langle G^2 \rangle$,$\langle \bar{q}\sigma G q \rangle$
  - Borel window $M^2 \in [4,6]$ GeV
- $f_B = 210 \pm 19$ MeV
  - Errors estimated by varying parameters (dashed lines). Mainly $\overline{m}_b, \mu$.



M. Jamin and B. O. Lange. f_B and f_{B_s} from QCD sum rules. Phys. Rev. D, 65:056005, 2002.