Universidad de Granada

High Energy Theory

On-Shell matching in effective field theories

Javier López Miras (he/him)

M. Chala, J. Santiago and F. Vilches

YTF|2023

Junta de Andalucía

Why do we need effective field theories?

EFT's are perturbative (Taylor) expansions of a full theory

$$F = G\,\frac{m_1 m_2}{r^2} \longrightarrow G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Why do we need effective field theories?

EFT's are perturbative (Taylor) expansions of a full theory

$$F = G\,\frac{m_1 m_2}{r^2} \quad \longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

In QFT's : Operators of mass dimension $d > 4$ $\quad \mathcal{O}_i^{(d)}$

Useful for finite precission computations

Search of new physics

Green's basis and redundant operators

EFT Lagrangian : $$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{d>4} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

Green's basis and redundant operators

EFT Lagrangian : $$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{d>4} \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

Valid operators

Local operators

Preserve the symmetries of the Lagrangian

Finite number of operators

Integration by parts and other identities (Fierz)

Green's basis

Green's basis and redundant operators

Green's basis of the bosonic sector of the SMEFT

X^3	
$\mathcal{O}_{3G}$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{3W}$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

X^2D^2	
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$

X^2H^2	
$\mathcal{O}_{HG}$	$G^A_{\mu\nu} G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu} G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu} W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu} W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HB}$	$B_{\mu\nu} B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu} B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu} B^{\mu\nu}(H^\dagger \sigma^I H)$
$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu} B^{\mu\nu}(H^\dagger \sigma^I H)$

H^2XD^2	
$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i \overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i \overleftrightarrow{D}_\mu H)$

H^2D^4	
$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger (D_\nu D^\nu H)$

H^4D^2	
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger (H^\dagger D_\mu H)$
$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger (D^\mu H)$
$\mathcal{O}''_{HD}$	$(H^\dagger H) D_\mu (H^\dagger i \overleftrightarrow{D}^\mu H)$

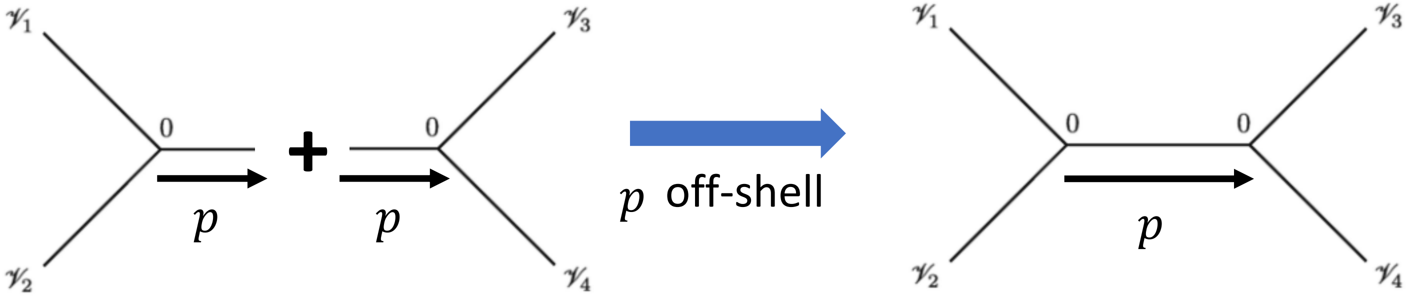
H^6	
$\mathcal{O}_H$	$(H^\dagger H)^3$

V. Gherardi, D. Marzocca y E. Venturini (2021) [2003.12525v5]

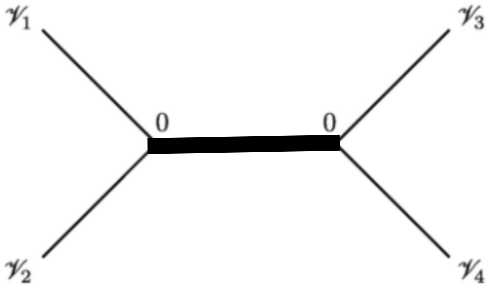
Matching: Off-Shell vs On-shell

Off-Shell matching

- Small number of diagrams (1 lPI)



- Hard region contribution directly local



$$p^2 \ll M^2$$

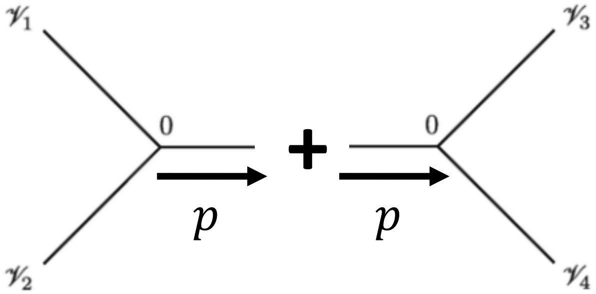
$$\sim \frac{1}{p^2 - M^2} = -\frac{1}{M^2}\left(1 + \frac{p^2}{M^2} + \frac{p^4}{M^4} + \cdots\right)$$

- But requires the construction and reduction of the Green's basis

Matching: Off-Shell vs On-shell

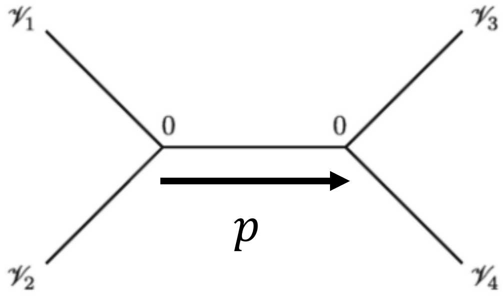
On-Shell matching

- Huge number of diagrams (1 lPI)



NO!!

p on-shell



- There is delicate cancellation of non-local contributions between between UV and EFT

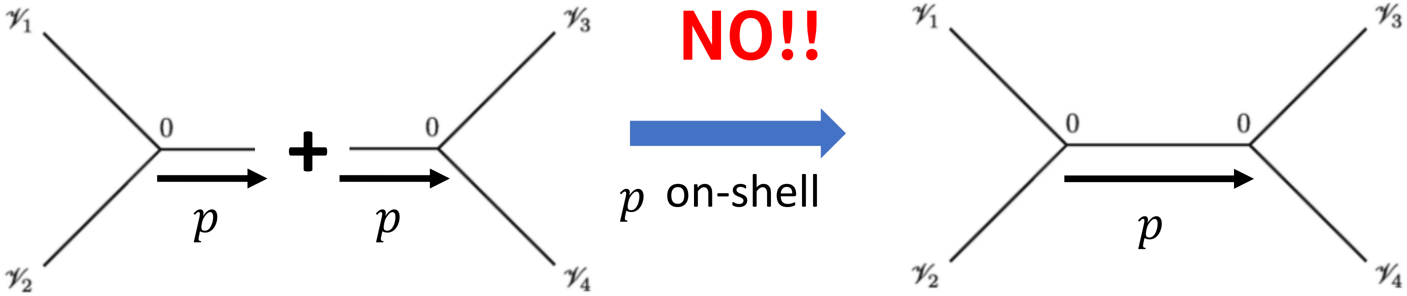
$$\left.\frac{1}{p^2-m^2}\right|_{\text{UV}} - \left.\frac{1}{p^2-m^2}\right|_{\text{EFT}} = Polynomial(p^2)$$

- But gives directly the matching in the physical minimal basis

Matching: Off-Shell vs On-shell

On-Shell matching

- Huge number of diagrams (1 lPI)



- There is delicate cancellation of non-local contributions between between UV and EFT

$$\left.\frac{1}{p^2-m^2}\right|_{\text{UV}} - \left.\frac{1}{p^2-m^2}\right|_{\text{EFT}} = Polynomial(p^2)$$

Substitution of randomly generated physical momenta

- But gives directly the matching in the physical minimal basis

Reduction to the physical basis in a toy model

Identification of redundant operators

- Field redefinitions
- EOMs (only valid up to linear order)

Reduction to the physical basis in a toy model

Identification of redundant operators

- Field redefinitions
- EOMs (only valid up to linear order)

TOY MODEL

Real scalar ϕ

Symmetry $\mathbb{Z}_2$

$$\mathcal{L} = \mathcal{L}_4 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^4}\mathcal{L}_8$$

$$\mathcal{L}_4 = -\frac{1}{2}\phi(\partial^2 + m^2)\phi - \lambda\phi^4,$$

$$\mathcal{L}_6 = \alpha_{61}\phi^6 + \beta_{61}\partial^2\phi\partial^2\phi + \beta_{62}\phi^3\partial^2\phi,$$

$$\mathcal{L}_8 = \alpha_{81}\phi^8 + \alpha_{82}\phi^2\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi + \beta_{81}\phi\partial^6\phi + \beta_{82}\phi^3\partial^4\phi + \beta_{83}\phi^2\partial^2\phi\partial^2\phi + \beta_{84}\phi^5\partial^2\phi.$$

Reduction to the physical basis in a toy model

Identification of redundant operators

- Field redefinitions
- EOMs (only valid up to linear order)

TOY MODEL

Real scalar ϕ

Symmetry $\mathbb{Z}_2$

$$\mathcal{L} = \mathcal{L}_4 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^4}\mathcal{L}_8$$

$$\mathcal{L}_4 = -\frac{1}{2}\phi(\partial^2 + m^2)\phi - \lambda\phi^4,$$

$$\mathcal{L}_6 = \alpha_{61}\phi^6 + \beta_{61}\cancel{\partial^2\phi\partial^2\phi} + \beta_{62}\cancel{\phi^3\partial^2\phi},$$

$$\mathcal{L}_8 = \alpha_{81}\phi^8 + \alpha_{82}\phi^2\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi + \beta_{81}\phi\partial^6\phi + \beta_{82}\phi^3$$

Field redefinition :

$$\phi \to \phi + \frac{\gamma_{61}}{\Lambda^2}\partial^2\phi + \frac{\gamma_{62}}{\Lambda^2}\phi^3$$

$$\gamma_{61} = \beta_{61} - \frac{m^2\beta_{61}^2}{2\Lambda^2},$$

$$\gamma_{62} = \beta_{62} - 4\lambda\beta_{61} + \frac{m^2}{\Lambda^2}\beta_{61}(6\lambda\beta_{61} - \beta_{62}).$$

Reduction to the physical basis in a toy model

Identification of redundant operators

- Field redefinitions
- EOMs (only valid up to linear order)

TOY MODEL

Real scalar ϕ

Symmetry $\mathbb{Z}_2$

$$\mathcal{L} = \mathcal{L}_4 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^4}\mathcal{L}_8$$

$$\mathcal{L}_4 = -\frac{1}{2}\phi(\partial^2 + m^2)\phi - \lambda\phi^4,$$

$$\mathcal{L}_6 = \alpha_{61}\phi^6 + \cancel{\beta_{61}\partial^2\phi\partial^2\phi} + \cancel{\beta_{62}\phi^3\partial^2\phi},$$

$$\mathcal{L}_8 = \alpha_{81}\phi^8 + \alpha_{82}\phi^2\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi + \cancel{\beta_{81}\phi\partial^6\phi} + \cancel{\beta_{82}\phi^3\partial^4\phi} + \cancel{\beta_{83}\phi^2\partial^2\phi\partial^2\phi} + \cancel{\beta_{84}\phi^5\partial^2\phi}.$$

EOM : $\partial^2\phi = -m^2\phi - 4\lambda\phi^3$

Reduction to the physical basis in a toy model

$$m^2 \to m^2 \left(1 - \frac{2m^2\beta_{61}}{\Lambda^2} + \frac{1}{\Lambda^4}(8m^4\beta_{61}^2 + 2\beta_{81})\right),$$

$$\lambda \to \lambda + \frac{m^2}{\Lambda^2}(\beta_{62} - 8\lambda\beta_{61}) + \frac{m^4}{\Lambda^4}(64\lambda\beta_{61}^2 - 10\beta_{61}\beta_{62} + 12\lambda\beta_{81} - \beta_{82} - \beta_{83}),$$

$$\begin{aligned}\alpha_{61} \to \alpha_{61} &+ 16\lambda^2\beta_{61} - 4\lambda\beta_{62} - \\ &- \frac{m^2}{\Lambda^2}\left(\frac{1728}{5}\lambda^2\beta_{61}^2 + \frac{22}{5}\beta_{62}^2 - \frac{512}{5}\lambda\beta_{61}\beta_{62} + 12\alpha_{61}\beta_{61} + \frac{304}{5}\lambda^2\beta_{81} - \frac{56}{5}\lambda\beta_{82} - 8\lambda\beta_{83} + \beta_{84}\right),\end{aligned}$$

$$\begin{aligned}\alpha_{81} \to \alpha_{81} &- \frac{3072}{5}\lambda^3\beta_{61}^2 - \frac{108}{5}\lambda\beta_{62}^2 + \frac{1248}{5}\lambda^2\beta_{61}\beta_{62} - 48\alpha_{61}\beta_{61} + 6\alpha_{61}\beta_{62} - \frac{576}{5}\lambda^3\beta_{81} + \\ &+ \frac{144}{5}\lambda^2\beta_{82} + 16\lambda^2\beta_{83} - 4\lambda\beta_{84},\end{aligned}$$

$$\alpha_{82} \to \alpha_{82}.$$

On-Shell matching approach

- Find the Green's basis up to dimension d → $\mathcal{L}_{Green}$

- Find the physical basis → $\mathcal{L}_{phys}$

  R. Fonseca [1907.12584]
  J.C. Criado [1901.03501]

- Compute n-points amplitudes with n $\leq d$ **on-shell** → **By the substitution of randomly generated physical momenta**

- Solve the system $\mathcal{M}_{i,Green} = \mathcal{M}_{i,phys}$

Some results in the SMEFT

HIGG'S SECTOR

$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green} \qquad \mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

Some results in the SMEFT

HIGG'S SECTOR

$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green} \qquad \mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

$$\mathcal{L}^{(4)} = \frac{1}{2}(D_\mu H)^\dagger (D^\mu H) - m_0^2 H^\dagger H - \lambda \left(H^\dagger H\right)^2$$

$$\begin{aligned}\mathcal{L}^{(6)}_{Green} = {} & c_H \left(H^\dagger H\right)^3 + c_{H\Box} \left(H^\dagger H\right) \Box \left(H^\dagger H\right) + c_{HD} \left(H^\dagger D^\mu H\right)^\dagger \left(H^\dagger D_\mu H\right) + \\ & r'_{HD} \left(H^\dagger H\right) (D_\mu H)^\dagger (D^\mu H) + r''_{HD} \left(H^\dagger H\right) D_\mu \left(H^\dagger i \overleftrightarrow{D}^\mu H\right) + r_{DH} \left(D^2 H\right)^\dagger \left(D^2 H\right)\end{aligned}$$

Some results in the SMEFT

HIGG'S SECTOR

$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green} \qquad \mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

$$\mathcal{L}^{(4)} = \frac{1}{2}\left(D_\mu H\right)^\dagger\left(D^\mu H\right) - m_0^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$\mathcal{L}^{(6)}_{phys}$$

$$\mathcal{L}^{(6)}_{Green} = c_H\left(H^\dagger H\right)^3 + c_{H\Box}\left(H^\dagger H\right)\Box\left(H^\dagger H\right) + c_{HD}\left(H^\dagger D^\mu H\right)^\dagger\left(H^\dagger D_\mu H\right) +$$
$$r'_{HD}\left(H^\dagger H\right)\left(D_\mu H\right)^\dagger\left(D^\mu H\right) + r''_{HD}\left(H^\dagger H\right)D_\mu\left(H^\dagger i\overleftrightarrow{D}^\mu H\right) + r_{DH}\left(D^2 H\right)^\dagger\left(D^2 H\right)$$

Some results in the SMEFT

HIGG'S SECTOR

$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green} \qquad \mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

$$\mathcal{L}^{(4)} = \frac{1}{2}\left(D_\mu H\right)^\dagger\left(D^\mu H\right) - m_0^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

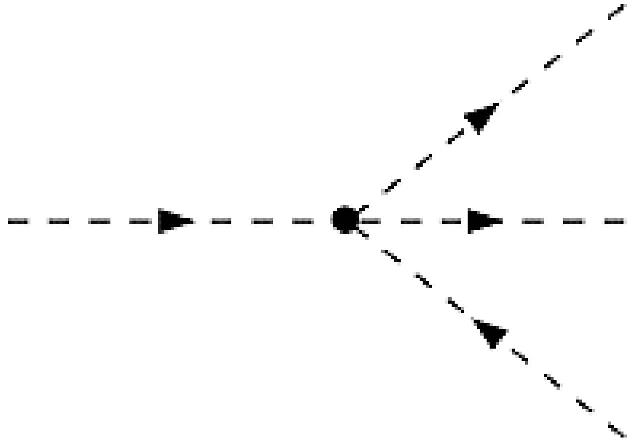
$$\mathcal{L}^{(6)}_{phys}$$

$$\mathcal{L}^{(6)}_{Green} = c_H\left(H^\dagger H\right)^3 + c_{H\Box}\left(H^\dagger H\right)\Box\left(H^\dagger H\right) + c_{HD}\left(H^\dagger D^\mu H\right)^\dagger\left(H^\dagger D_\mu H\right) +$$
$$r'_{HD}\left(H^\dagger H\right)\left(D_\mu H\right)^\dagger\left(D^\mu H\right) + r''_{HD}\left(H^\dagger H\right) D_\mu\left(H^\dagger i \overleftrightarrow{D}^\mu H\right) + r_{DH}\left(D^2 H\right)^\dagger\left(D^2 H\right)$$

$$\mathcal{L}^{(8)}_{phys} = c_{H^8}\left(H^\dagger H\right)^4 + c^{(1)}_{H^6D^2}\left(H^\dagger H\right)^2\left(D_\mu H^\dagger D^\mu H\right) + c^{(2)}_{H^6D^2}\left(H^\dagger H\right)\left(H^\dagger\sigma^I H\right)\left(D_\mu H^\dagger\sigma^I D^\mu H\right) +$$
$$c^{(1)}_{H^4D^4}\left(D_\mu H^\dagger D_\nu H\right)\left(D^\nu H^\dagger D^\mu H\right) + c^{(2)}_{H^4D^4}\left(D_\mu H^\dagger D_\nu H\right)\left(D^\mu H^\dagger D^\nu H\right) +$$
$$c^{(3)}_{H^4D^4}\left(D^\mu H^\dagger D_\mu H\right)\left(D^\nu H^\dagger D_\nu H\right)$$

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
para cada      longitud

      For[i = 1, i ≤ Length[rules12], i++,
      para cada      longitud

        final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
                             aplana

        final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
                  suma                 longitud            expande factores

        final = final /. Sust;
        final = final /. {x^3 → 0, x^4 → 0,  x^5 → 0, x^6 → 0} /. {x → 1};
        ampIR = final /. propEFT /. limitIR;
        ampUV = Z^2 final /. propEFT /. limitUV;
        ampsUV[[i]] = ampsUV[[i]] + ampUV;
        ampsIR[[i]] = ampsIR[[i]] + ampIR;

   ];
   AppendTo[equations, Table[ampsUV[[i]] == ampsIR[[i]], {i, 1, Length[rules12]}]];
   añade al final      tabla                               longitud
  ];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            resuelve aplana                                 simplifica
```

H0 → H0 H0 H0



```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
para cada       longitud

    For[i = 1, i ≤ Length[rules12], i++,
    para cada       longitud

     final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
                          aplana

     final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
             ·· suma                  longitud            expande factores

     final = final /. Sust;
     final = final /. {x^3 → 0, x^4 → 0,  x^5 →  0,  x^6 → 0} /. {x → 1};
     ampIR = final /. propEFT /. limitIR;
     ampUV = Z^2 final /. propEFT /. limitUV;
     ampsUV[[i]] = ampsUV[[i]] + ampUV;
     ampsIR[[i]] = ampsIR[[i]] + ampIR;

  ];
  AppendTo[equations, Table[ampsUV[[i]] == ampsIR[[i]], {i, 1, Length[rules12]}]];
  añade al final      tabla                                  longitud
 ];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            resuelve aplana                                simplifica
```

→ Replacing momenta by randomly generated values

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
para cada     longitud

    For[i = 1, i ≤ Length[rules12], i ++,
    para cada     longitud

     final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
                          aplana

     final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
               ·· suma                 longitud            expande factores

     final = final /. Sust;
     final = final /. {x^3 → 0, x^4 → 0, x^5 → 0, x^6 → 0} /. {x → 1};
     ampIR = final /. propEFT /. limitIR;
     ampUV = Z^2 final /. propEFT /. limitUV;
     ampsUV[[i]] = ampsUV[[i]] + ampUV;
     ampsIR[[i]] = ampsIR[[i]] + ampIR;

  ];
  AppendTo[equations, Table[ampsUV[[i]] == ampsIR[[i]], {i, 1, Length[rules12]}]];
  añade al final      tabla                              longitud
 ];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            resuelve aplana                                 simplifica
```

Replacing momenta by randomly generated values

Running through every amplitude in the process

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
    For[i = 1, i ≤ Length[rules12], i ++,
      final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
      final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
      final = final /. Sust;
      final = final /. {x^3 → 0, x^4 → 0, x^5 → 0, x^6 → 0} /. {x → 1};
      ampIR = final /. propEFT /. limitIR;
      ampUV = Z^2 final /. propEFT /. limitUV;
      ampsUV[[i]] = ampsUV[[i]] + ampUV;
      ampsIR[[i]] = ampsIR[[i]] + ampIR;

  ];
  AppendTo[equations, Table[ampsUV[[i]] == ampsIR[[i]], {i, 1, Length[rules12]}]];
];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
```

Replacing momenta by randomly generated values

Running through every amplitude in the process

Setting both theories amplitudes with their propagators and wavefunction renormalizations

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
para cada        longitud

      For[i = 1, i ≤ Length[rules12], i++,
      para cada        longitud

       final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
                           aplana

       final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
                 ·· suma                 longitud          expande factores

       final = final /. Sust;
       final = final /. {x^3 → 0, x^4 → 0,  x^5 → 0, x^6 → 0} /. {x → 1};
       ampIR = final /. propEFT /. limitIR;
       ampUV = Z^2 final /. propEFT /. limitUV;
       ampsUV[[i]] = ampsUV[[i]] + ampUV;
       ampsIR[[i]] = ampsIR[[i]] + ampIR;

  ];
  AppendTo[equations, Table[ampsUV[[i]] == ampsIR[[i]], {i, 1, Length[rules12]}]];
  añade al final          tabla                                longitud
];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            resuelve aplana                                  simplifica
```

Replacing momenta by randomly generated values

Running through every amplitude in the process

Setting both theories amplitudes with their propagators and wavefunction renormalizations

Matching both theories

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
para cada    longitud

      For[i = 1, i ≤ Length[rules12], i++,
      para cada    longitud

       final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
                            aplana

       final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
                 suma                  longitud             expande factores

       final = final /. Sust;
       final = final /. {x^3 → 0, x^4 → 0,  x^5 → 0,  x^6 → 0} /. {x → 1};
       ampIR = final /. propEFT /. limitIR;
       ampUV = Z^2 final /. propEFT /. limitUV;
       ampsUV[[i]] = ampsUV[[i]] + ampUV;
       ampsIR[[i]] = ampsIR[[i]] + ampIR;

  ];
  AppendTo[equations, Table[ampsUV[[i]] == ampsIR[[i]], {i, 1, Length[rules12]}]];
  añade al final        tabla                                longitud
 ];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            resuelve aplana                                  simplifica
```

Replacing momenta by randomly generated values

Running through every amplitude in the process

Setting both theories amplitudes with their propagators and wavefunction renormalizations

Matching both theories

Solving the system

Some results in the SMEFT

Final solution: redefinition of coefficients

$$\text{cH} \to \text{aH} - \text{lmbd}^2\,\text{rDH} + \text{lmbd}\,\text{rHDp} + \text{m0}^2\left(6\,\text{aH}\,\text{rDH} + \text{aHD}\,\text{lmbd}\,\text{rDH} - 8\,\text{aHDD}\,\text{lmbd}\,\text{rDH} - 11\,\text{lmbd}^2\,\text{rDH}^2 - \frac{\text{aHD}\,\text{rHDp}}{2} + 4\,\text{aHDD}\,\text{rHDp} + 9\,\text{lmbd}\,\text{rDH}\,\text{rHDp} - \frac{\text{rHDp}^2}{4} - \text{rHDpp}^2\right)$$

$$\text{cH41} \to 0$$

$$\text{cH42} \to 0$$

$$\text{cH43} \to 0$$

$$\text{cH61} \to \text{aHD}\,\text{lmbd}\,\text{rDH} + 8\,\text{aHDD}\,\text{lmbd}\,\text{rDH} + \text{lmbd}^2\,\text{rDH}^2 - \frac{\text{aHD}\,\text{rHDp}}{2} - 4\,\text{aHDD}\,\text{rHDp} + 3\,\text{lmbd}\,\text{rDH}\,\text{rHDp} - \frac{7\,\text{rHDp}^2}{4} + \text{rHDpp}^2$$

$$\text{cH62} \to 2\,\text{aHD}\,\text{lmbd}\,\text{rDH} - \text{aHD}\,\text{rHDp}$$

$$\text{cHD} \to \text{aHD} + 4\,\text{aHD}\,\text{m0}^2\,\text{rDH}$$

$$\text{cHDD} \to \frac{1}{2}\left(2\,\text{aHDD} + \text{rHDp}\right) + \text{m0}^2\left(4\,\text{aHDD}\,\text{rDH} + 2\,\text{rDH}\,\text{rHDp}\right)$$

$$\text{mH}^2 \to \text{m0}^2 + \text{m0}^4\,\text{rDH} + 2\,\text{m0}^6\,\text{rDH}^2$$

$$\lambda \to \text{lmbd} + \text{m0}^2\left(4\,\text{lmbd}\,\text{rDH} - 2\,\text{rHDp}\right) + \text{m0}^4\left(16\,\text{lmbd}\,\text{rDH}^2 - 10\,\text{rDH}\,\text{rHDp}\right)$$

J. Aebischer, M. Fael and J. Fuentes-Martín|2023
[2307.08745v1]

V. Gherardi, D. Marzocca and E. Venturini |2021
[2003.12525v5]

Some results in the SMEFT

Final solution: redefinition of coefficients

$$\mathrm{cH} \to \mathrm{aH} - \mathrm{lmbd}^2\,\mathrm{rDH} + \mathrm{lmbd}\,\mathrm{rHDp} + \mathrm{m0}^2 \left(6\,\mathrm{aH}\,\mathrm{rDH} + \mathrm{aHD}\,\mathrm{lmbd}\,\mathrm{rDH} - 8\,\mathrm{aHDD}\,\mathrm{lmbd}\,\mathrm{rDH} - 11\,\mathrm{lmbd}^2\,\mathrm{rDH}^2 - \frac{\mathrm{aHD}\,\mathrm{rHDp}}{2} + 4\,\mathrm{aHDD}\,\mathrm{rHDp} + 9\,\mathrm{lmbd}\,\mathrm{rDH}\,\mathrm{rHDp} - \frac{\mathrm{rHDp}^2}{4} - \mathrm{rHDpp}^2\right)$$

$$\mathrm{cH41} \to 0$$

$$\mathrm{cH42} \to 0$$

$$\mathrm{cH43} \to 0$$

$$\mathrm{cH61} \to \mathrm{aHD}\,\mathrm{lmbd}\,\mathrm{rDH} + 8\,\mathrm{aHDD}\,\mathrm{lmbd}\,\mathrm{rDH} + \mathrm{lmbd}^2\,\mathrm{rDH}^2 - \frac{\mathrm{aHD}\,\mathrm{rHDp}}{2} - 4\,\mathrm{aHDD}\,\mathrm{rHDp} + 3\,\mathrm{lmbd}\,\mathrm{rDH}\,\mathrm{rHDp} - \frac{7\,\mathrm{rHDp}^2}{4} + \mathrm{rHDpp}^2$$

$$\mathrm{cH62} \to 2\,\mathrm{aHD}\,\mathrm{lmbd}\,\mathrm{rDH} - \mathrm{aHD}\,\mathrm{rHDp}$$

$$\mathrm{cHD} \to \mathrm{aHD} + 4\,\mathrm{aHD}\,\mathrm{m0}^2\,\mathrm{rDH}$$

$$\mathrm{cHDD} \to \frac{1}{2}\left(2\,\mathrm{aHDD} + \mathrm{rHDp}\right) + \mathrm{m0}^2\left(4\,\mathrm{aHDD}\,\mathrm{rDH} + 2\,\mathrm{rDH}\,\mathrm{rHDp}\right)$$

$$\mathrm{mH}^2 \to \mathrm{m0}^2 + \mathrm{m0}^4\,\mathrm{rDH} + 2\,\mathrm{m0}^6\,\mathrm{rDH}^2$$

$$\lambda \to \mathrm{lmbd} + \mathrm{m0}^2\left(4\,\mathrm{lmbd}\,\mathrm{rDH} - 2\,\mathrm{rHDp}\right) + \mathrm{m0}^4\left(16\,\mathrm{lmbd}\,\mathrm{rDH}^2 - 10\,\mathrm{rDH}\,\mathrm{rHDp}\right)$$

J. Aebischer, M. Fael and J. Fuentes-Martín|2023
[2307.08745v1]

V. Gherardi, D. Marzocca and E. Venturini |2021
[2003.12525v5]

Some results in the SMEFT

Final solution: redefinition of coefficients

$$\mathrm{cH} \to \mathrm{aH} - \mathrm{lmbd}^2\,\mathrm{rDH} + \mathrm{lmbd}\,\mathrm{rHDp} + \boxed{\mathrm{m0}^2 \left(6\,\mathrm{aH}\,\mathrm{rDH} + \mathrm{aHD}\,\mathrm{lmbd}\,\mathrm{rDH} - 8\,\mathrm{aHDD}\,\mathrm{lmbd}\,\mathrm{rDH} - 11\,\mathrm{lmbd}^2\,\mathrm{rDH}^2 - \frac{\mathrm{aHD}\,\mathrm{rHDp}}{2} + 4\,\mathrm{aHDD}\,\mathrm{rHDp} + 9\,\mathrm{lmbd}\,\mathrm{rDH}\,\mathrm{rHDp} - \frac{\mathrm{rHDp}^2}{4} - \mathrm{rHDpp}^2\right)}$$

$$\mathrm{cH41} \to 0$$

$$\mathrm{cH42} \to 0$$

$$\mathrm{cH43} \to 0$$

$$\mathrm{cH61} \to \boxed{\mathrm{aHD}\,\mathrm{lmbd}\,\mathrm{rDH} + 8\,\mathrm{aHDD}\,\mathrm{lmbd}\,\mathrm{rDH} + \mathrm{lmbd}^2\,\mathrm{rDH}^2} - \frac{\mathrm{aHD}\,\mathrm{rHDp}}{2} - 4\,\mathrm{aHDD}\,\mathrm{rHDp} + \boxed{3\,\mathrm{lmbd}\,\mathrm{rDH}\,\mathrm{rHDp}} - \frac{7\,\mathrm{rHDp}^2}{4} + \mathrm{rHDpp}^2$$

$$\mathrm{cH62} \to \boxed{2\,\mathrm{aHD}\,\mathrm{lmbd}\,\mathrm{rDH}} - \mathrm{aHD}\,\mathrm{rHDp}$$

$$\mathrm{cHD} \to \mathrm{aHD} + \boxed{4\,\mathrm{aHD}\,\mathrm{m0}^2\,\mathrm{rDH}}$$

$$\mathrm{cHDD} \to \frac{1}{2}\left(2\,\mathrm{aHDD} + \mathrm{rHDp}\right) + \boxed{\mathrm{m0}^2\left(4\,\mathrm{aHDD}\,\mathrm{rDH} + 2\,\mathrm{rDH}\,\mathrm{rHDp}\right)}$$

$$\mathrm{mH}^2 \to \mathrm{m0}^2 + \mathrm{m0}^4\,\mathrm{rDH} + \boxed{2\,\mathrm{m0}^6\,\mathrm{rDH}^2}$$

$$\lambda \to \mathrm{lmbd} + \mathrm{m0}^2\left(4\,\mathrm{lmbd}\,\mathrm{rDH} - 2\,\mathrm{rHDp}\right) + \boxed{\mathrm{m0}^4\left(16\,\mathrm{lmbd}\,\mathrm{rDH}^2 - 10\,\mathrm{rDH}\,\mathrm{rHDp}\right)}$$

J. Aebischer, M. Fael and J. Fuentes-Martín|2023
[2307.08745v1]

V. Gherardi, D. Marzocca and E. Venturini |2021
[2003.12525v5]

Future work

BOSONIC SECTOR

$\boldsymbol{X^3}$		$\boldsymbol{X^2H^2}$		$\boldsymbol{H^2D^4}$	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	$\boldsymbol{H^4D^2}$	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
$\boldsymbol{X^2D^2}$		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$	$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\boldsymbol{H^6}$	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		$\boldsymbol{H^2XD^2}$			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

Future work

BOSONIC SECTOR

X^3	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
X^2D^2	
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$

X^2H^2	
$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$
$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$
H^2XD^2	
$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$

H^2D^4	
$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
H^4D^2	
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
H^6	
$\mathcal{O}_H$	$(H^\dagger H)^3$

Future work

BOSONIC SECTOR

X^3		X^2H^2		H^2D^4	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	H^4D^2	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$	$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	H^6	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

$$m_0^2 \to m_0^2$$

$$\lambda \to \lambda - 2m_0^2 r'_{HD}$$

$$c^{(1)}_{H^4D^4} \to 2r^2_{BDH}$$

$$c^{(2)}_{H^4D^4} \to -2r^2_{BDH}$$

$$c^{(3)}_{H^4D^4} \to 0$$

$$c_{H\Box} \to c_{H\Box} + \frac{1}{2}g' r_{BDH} + \frac{1}{2}r'_{HD}$$

$$c_{HD} \to c_{HD} + 2g' r_{BDH}$$

$$D_\mu = \partial_\mu - ig'B_\mu$$

$$g' \to g'$$

$$c_{HB} \to c_{HB}$$

Notice that ...

Reduction of the Green's basis

- Field redefinitions and EOMs
  - Non-systematic
  - Highly non-lineal
- On-Shell matching approach
  - Systematic
  - Algebraic system of linear equations

Notice that ...

Reduction of the Green's basis

- Field redefinitions and EOMs
  - Non-systematic
  - Highly non-trivial
- On-Shell matching approach
  - Systematic
  - Algebraic system of linear equations

The reduction of **ANY** theory to **ANY** physical basis will be completely **AUTOMATIC**

Universidad de Granada

High Energy Theory

THANKS FOR YOUR ATTENTION !

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \quad \begin{array}{l} \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \quad \begin{array}{l} \lambda^{\alpha} = \varepsilon^{\alpha\beta}\lambda_{\beta} \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

Massless momenta : $P_{\alpha\dot{\alpha}} = \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}}$ ➡ $P = p_{\mu}\sigma^{\mu} = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \quad \begin{array}{l} \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

Massless momenta : $P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ $\Rightarrow$ $P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$

Massive momenta : $P^\mu := q^\mu + \dfrac{m^2}{2q\cdot k} k^\mu$ $\quad \left| \begin{array}{rcl} q^2, k^2 &=& 0 \\ q_{\alpha\dot{\alpha}} &=& \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \\ k_{\alpha\dot{\alpha}} &=& \mu_\alpha \tilde{\mu}_{\dot{\alpha}} \end{array} \right.$