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FTAE High Energy Theory

Aspects of dimensional reduction in thermal QFT

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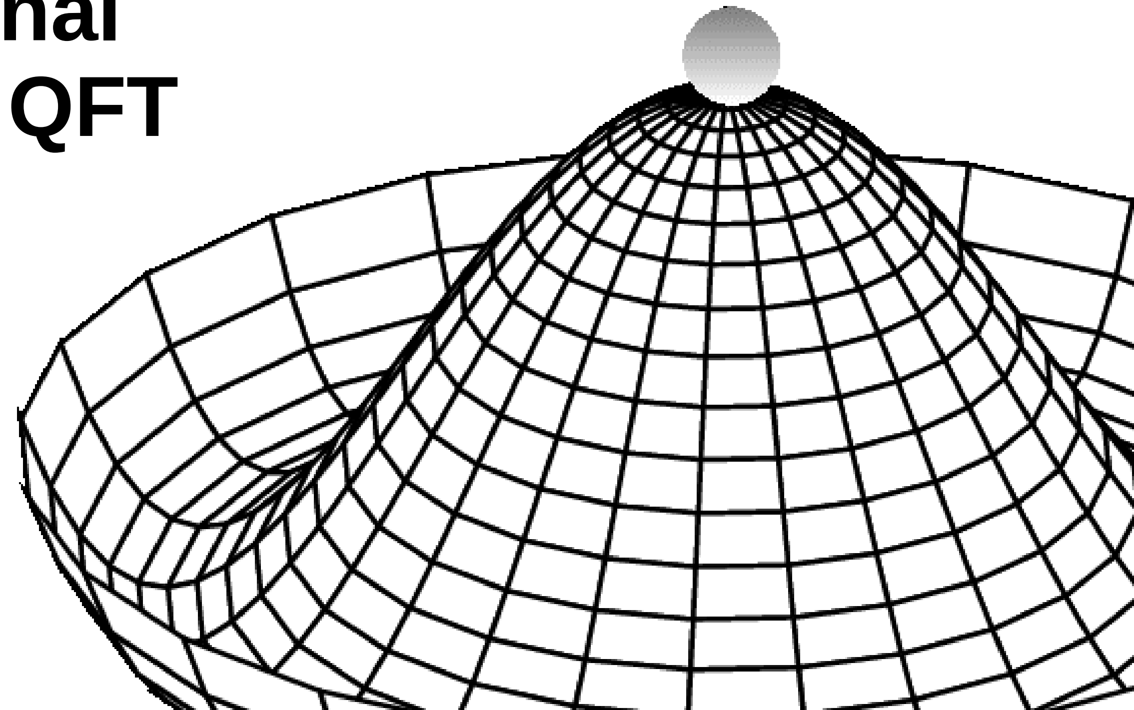


Outline

Need for thermal QFT

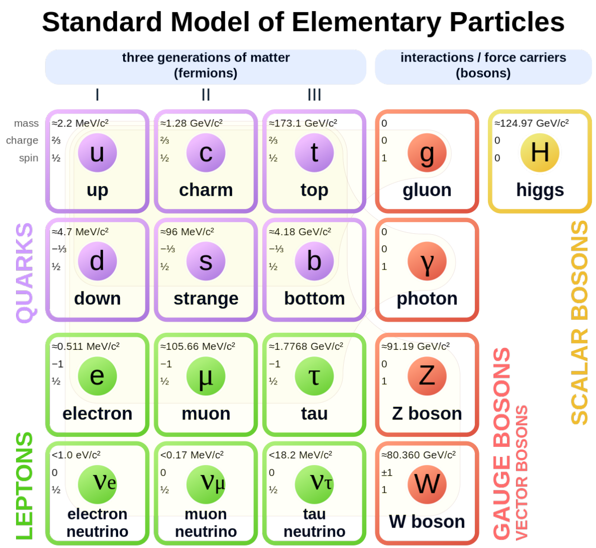
Formal aspects

3dEFT approach

Future applications

Need for thermal QFT

- The Standard Model is usually studied at **zero temperature** in a **non-zero temperature Universe**
- At finite temperature, collective statistical effects appear



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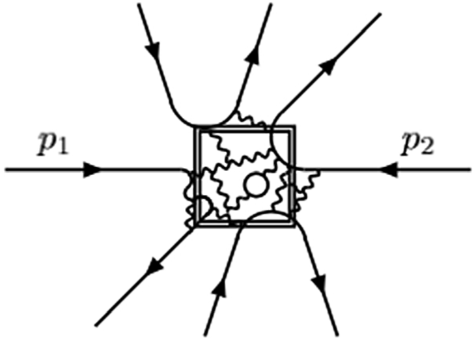
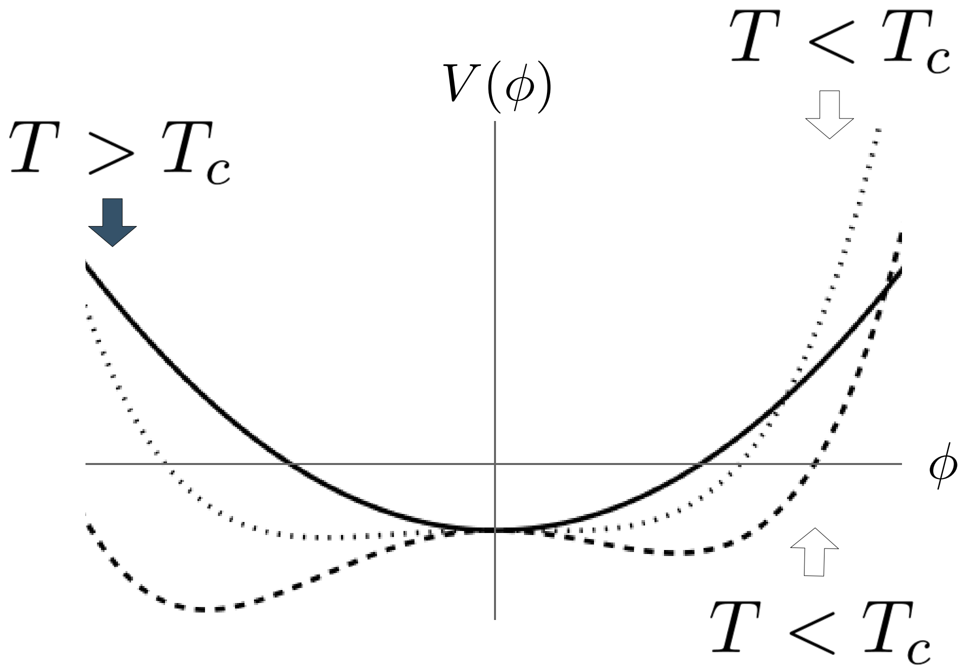


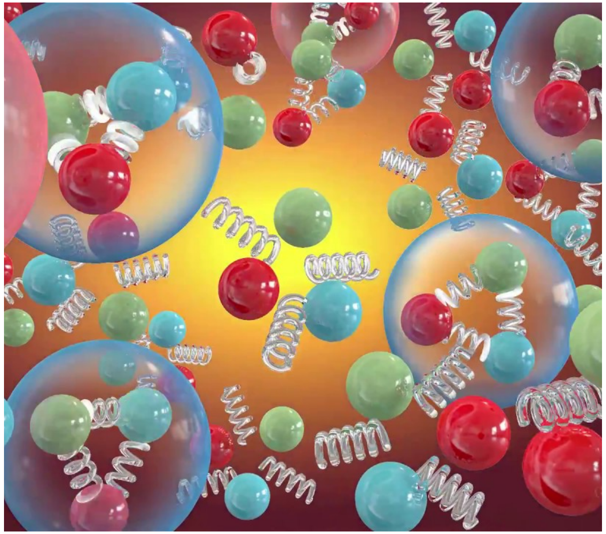
$$\mathcal{Z}^{-1}\mathrm{Tr}\left(e^{-\beta\mathcal{H}}\langle p_1\ldots|p_2\ldots\rangle\right)$$

Thermal average

Need for thermal QFT

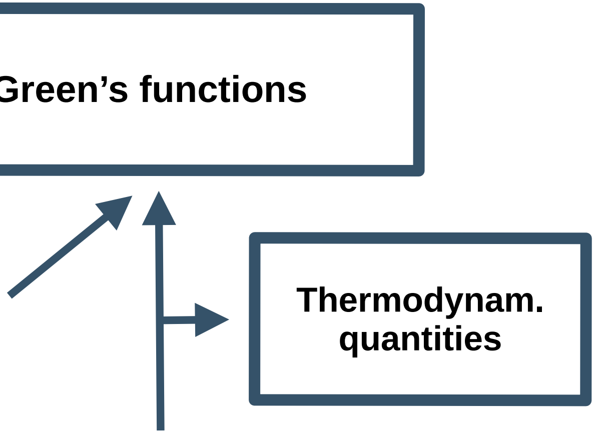


Phase transition



Quark-gluon plasma

Formal aspects



- **Generating functional** (J=0) in QFT:

$$\mathcal{Z}[0] = \langle q' \ t | q \ 0 \rangle = \langle q' \ 0 | e^{-i\mathcal{H}t} | q \ 0 \rangle = \mathcal{N} \int \mathcal{D}q \exp\left(iS\right)$$

- **Partition function** in quantum statistical mechanics:

$$\mathcal{Z}_{\text{th}} = \text{Tr}\left(e^{-\beta\mathcal{H}}\right) = \sum_q \langle q \ 0 | e^{-\beta\mathcal{H}} | q \ 0 \rangle = \mathcal{N} \int_{q(0)=q(-i\beta)} \mathcal{D}q \exp\left(-S_E\right)$$

where

$$S = \int d^4x \ \mathcal{L}(q, \dot{q}) \qquad \text{and} \qquad S_E = iS = \int_0^\beta d\tau \int d^3x \ \mathcal{L}_E(q_E, \dot{q}_E)$$

Formal aspects

Green's functions

Thermodynam. quantities

- **Generating functional** (J=0) in QFT:

$$\mathcal{Z}[0] = \langle q'\ t | q\ 0 \rangle = \langle q'\ 0 | e^{-i\mathcal{H}t} | q\ 0 \rangle = \mathcal{N} \int \mathcal{D}q \exp\left(iS\right)$$

- **Partition function** in quantum statistical mechanics:

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QFT at finite temperature **=** Euclidean QFT at zero temperature with periodic time

Formal aspects

Check out this review by M. Laine and A. Vuorinen [1701:01554]

Summary

	$\mathbf{T = 0}$	$\mathbf{T \neq 0}$
Space-time	Minkowski $\mathbb{R}^3 \times \mathbb{R}^1$ $(-\infty < t < \infty)$	Euclidean $\mathbb{R}^3 \times \mathcal{S}^1$ $(0 < (\tau = it) \leq \beta)$
Bosonic fields	$\phi(x)$	$\sum_{n=-\infty}^{\infty} \phi_n(\mathbf{x}) \exp(i2\pi nT\tau)$
Bosonic momentum	p^μ	$P = (2\pi nT, \mathbf{p})$
Fermionic fields	$\psi(x)$	$\sum_{n=-\infty}^{\infty} \psi_n(\mathbf{x}) \exp(i\pi(2n+1)T\tau)$
Fermionic momentum	p^μ	$P = (\pi(2n+1)T, \mathbf{p})$

Formal aspects

Feynman rules

	$\mathbf{T = 0}$	$\mathbf{T \neq 0}$
Vertex	Same	Same
Product of momenta	$p^{\mu} q_{\mu}$	$-P \cdot Q$
Propagator	$\dfrac{i}{p^2 - m^2}$	$\dfrac{1}{P^2 + m^2}$
Loop integral	$\displaystyle\int \frac{d^4 q}{(2\pi)^4}$	$\displaystyle\sum\!\!\!\!\!\!\!\int_Q \equiv T \sum_{n=-\infty}^{\infty} \int \frac{d^3 q}{(2\pi)^3}$

Formal aspects: λϕ⁴ model

$$\mathcal{L}_M = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \lambda\phi^4$$

Thermal bath

$$\mathcal{L}_E = \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}(\partial_i \phi_n)^2 + \frac{1}{2}\left(m^2 + (2\pi n T)^2\right)\phi_n^2\right] + \text{coupling between modes}$$

- At energies below the temperature scale, heavy thermal modes can be **integrated out**
- We are left with an **effective 3D field theory (3dEFT)** for the **zero mode** only

3dEFT approach

Analogous calculation without effective operators [2101:05528]

Full theory:

$$\mathcal{L}_E = \frac{1}{2}\left(\partial_\tau \phi\right)^2 + \frac{1}{2}\left(\partial_i \phi\right)^2 + \frac{m^2}{2}\phi^2 + \lambda\phi^4$$

Off-shell matching
$\mathcal{O}\left(\lambda^2, P^4\right)$ $m^2 \sim \lambda T^2$ $P_0 = 0$

$$\mathcal{L}_{(4)} = \frac{K}{2}(\partial_i\varphi)^2 + \frac{m_3^2}{2}\varphi^2 + \lambda_3\varphi^4,$$

$$\mathcal{L}_{(6)} = \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi,$$

$$\mathcal{L}_{(8)} = \alpha_{82}\varphi^2\partial_i\partial_j\varphi\partial^i\partial^j\varphi + \beta_{82}\varphi^3\partial^4\varphi + \beta_{83}\varphi^2\partial^2\varphi\partial^2\varphi.$$

3d EFT:
(up to 4 deriv. and 4 field insertions)

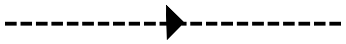
$$\mathcal{L}_3 = \mathcal{L}_{(4)} + \mathcal{L}_{(6)} + \mathcal{L}_{(8)}$$

See talk by J. López!

3dEFT approach

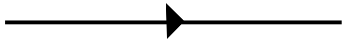
- We first compute the **2-point 1PI diagrams** in both theories:

$\varphi \to \varphi$ (tree-level)



$$= \quad -K\mathbf{p}^2 - m_3^2 + 2\beta_{61}\mathbf{p}^4$$

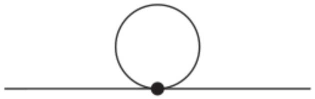
$\phi \to \phi$ (tree-level)



$$= \quad -\left(P^2 + m^2\right)$$

3dEFT approach

$\phi \to \phi$ (1-loop)



$$= -\frac{4!\lambda}{2}\sum\!\!\!\!\!\!\!\int' \frac{1}{Q^2+m^2} = -\frac{4!\lambda}{2}\sum\!\!\!\!\!\!\!\int' \left[\frac{1}{Q^2}\left(1-\frac{m^2}{Q^2}+\mathcal{O}\left(\frac{m^4}{Q^4}\right)\right)\right]$$

$$= -\frac{4!\lambda}{2}\left(\sum\!\!\!\!\!\!\!\int' \frac{1}{Q^2} - m^2 \sum\!\!\!\!\!\!\!\int' \frac{1}{Q^4}\right) + \mathcal{O}\left(\lambda^3\right)$$

1) Hard region expansion in $\frac{m^2}{(\pi T)^2}$. Double hard region expansion at 2-loop level [2311.13630]

scaleless vanish in dim. reg.

2) Write dashed sum-integrals as un-dashed $\sum\!\!\!\!\!\!\!\int_{Q'} f(\mathbf{q},Q_0) = \sum\!\!\!\!\!\!\!\int_{Q} f(\mathbf{q},Q_0) - \int_q f(\mathbf{q},0)$

3dEFT approach

3) Use (scheme-dependent) master bosonic 1-loop sum-integral formula in d=3 limit:

$$I_\alpha^{\beta\gamma}(d) \equiv \sum\!\!\!\!\!\!\!\int_Q \frac{\left(Q_0^2\right)^\beta \left(\mathbf{q}^2\right)^\gamma}{\left(Q^2\right)^\alpha} = \left(\frac{e^{\gamma_E}\mu^2}{4\pi}\right)^\epsilon \frac{2T(2\pi T)^{d-2\alpha+2\beta+2\gamma}}{(4\pi)^{d/2}} \frac{\Gamma(d/2+\gamma)\Gamma(-d/2+\alpha-\gamma)}{\Gamma(d/2)\Gamma(\alpha)} \zeta(-d+2\alpha-2\beta-2\gamma)$$

[2101.05528]

We need:

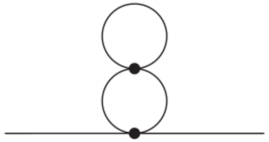
$$I_1^{00}(3) = \frac{T^2}{12}$$

$$I_2^{00}(3) = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{T^2}\right) + \psi^{(0)}\left(\frac{1}{2}\right) + \text{constants}\right)$$

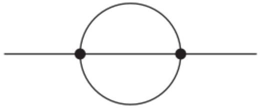
Requires renormalization

3dEFT approach

$\phi \to \phi$ (2-loop)



$$= \frac{(-4!\lambda)^2}{4} \sum\!\!\!\!\!\!\!\int_{Q\,K} \frac{1}{(Q^2+m^2)^2(K^2+m^2)} = \frac{(-4!\lambda)^2}{4} I_2^{00}(d) I_1^{00}(d) + \mathcal{O}(\lambda^3)$$



$$= \frac{(-4!\lambda)^2}{2} \sum\!\!\!\!\!\!\!\int_{Q\,K} \frac{1}{(Q^2+m^2)(K^2+m^2)((P-Q-K)^2+m^2)}$$

3dEFT approach

Sum-integrals like these appear in the 2-loop sunset diagram:

$$L^{s_4 s_5}_{s_1 s_2 s_3}(d) \equiv \SumInt_{QK} \frac{(Q_0)^{s_4}\,(K_0)^{s_5}}{(Q^2)^{s_1}\,(K^2)^{s_2}\,((Q+K)^2)^{s_3}}$$

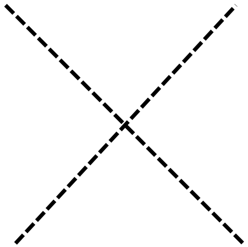
We do **not** have an **easy and systematic** way to compute these. We find some specific examples in the literature using IBP relations and tensor reduction formulae. [1207.4042, 1208.0284]

Would be useful to have codes to solve some families of loop sum-integrals for generic indices.

3dEFT approach

- Now, we compute the **4-point 1PI diagrams** in both theories:

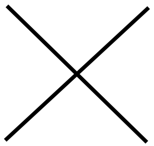
$\varphi\varphi \to \varphi\varphi$ (tree-level)



$$= -4!\lambda_3 + 4\alpha_{82}\left[(p_1\cdot p_2)^2 + (p_1\cdot p_3)^2 + (p_1\cdot p_4)^2 + (p_2\cdot p_3)^2 + (p_2\cdot p_4)^2 + (p_3\cdot p_4)^2\right] + 6\beta_{62}\left[p_1^2+p_2^2+p_3^2+p_4^2\right] + 6\beta_{82}\left[p_1^4+p_2^4+p_3^4+p_4^4\right] + 4\beta_{83}\left[p_2^2p_1^2+p_3^2p_1^2+p_4^2p_1^2+p_2^2p_3^2+p_2^2p_4^2+p_3^2p_4^2\right]\Big|_{p_1+p_2+p_3+p_4=0}$$

3dEFT approach

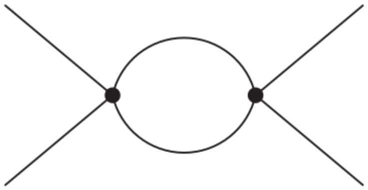
$\phi\phi \to \phi\phi$ (tree-level)



$$= \quad -4!\lambda$$

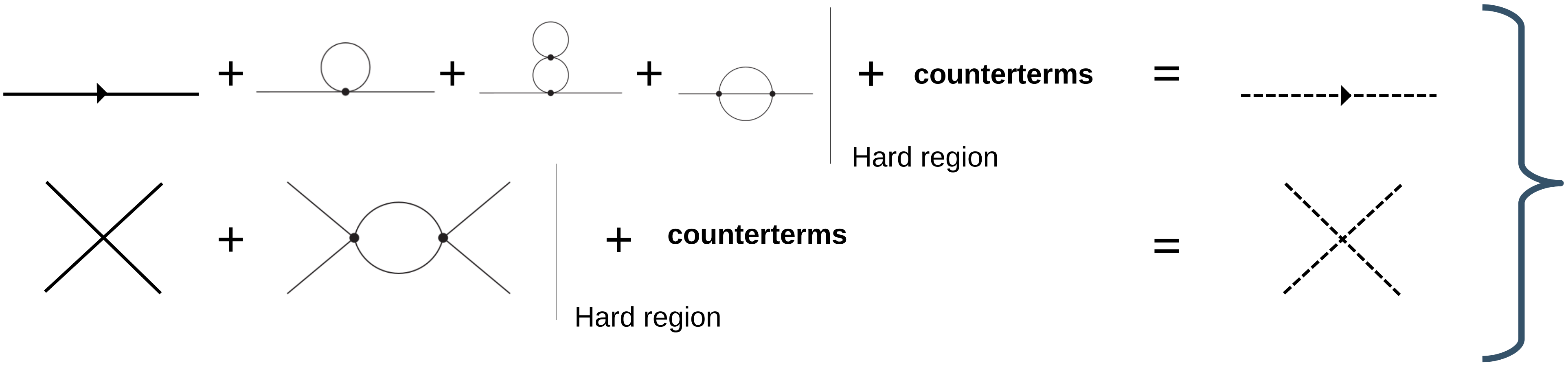
$\phi\phi \to \phi\phi$ (1-loop)

Can be written in terms of $I_\alpha^{\beta\gamma}(d)$



$$= \frac{(-4!\lambda)^2}{2} \sum_{j=\{s,t,u\}} \sum\!\!\!\!\!\!\!\int' \frac{1}{(Q^2+m^2)((Q+P_j)^2+m^2)}$$

3dEFT approach

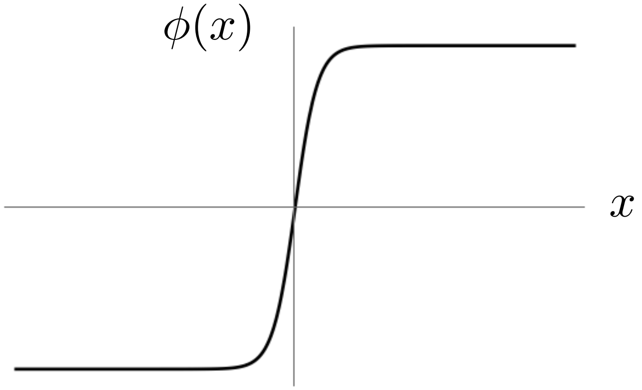


- Wilson coefficients are matched at scale $\mu \sim T$
- 3dEFT coefficients are then run (RGE) to a lower energy scale $\mu \sim m \sim \sqrt{\lambda} T$

Future applications

Why such a simple model?

- We want to test our EFT-building procedures and **gain some intuition** about 3D Lagrangians
- What are the consequences of adding effective operators with **higher order derivatives**?
  - In 4D theories with asymmetric potentials we find **non-constant classical field configurations** called *bounces.*
  - What kind of classical solutions can we find in a 3dEFT?



Future applications

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- What are the consequences of adding effective operators with **higher order derivatives**?

What are our future goals?

- Cosmological phase transitions and gravitational waves production pre-CMB (?) [2305.02357]
- Apply our knowledge to the Standard Model to study the EW phase transition

Still A LOT for me to learn :)

Thank you for your attention!

¡Gracias por vuestra atención!