Froggatt-Nielsen models meet the SMEFT

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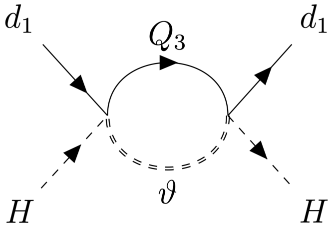


Based on work with Jim Talbert, to appear

Motivation

The flavour puzzle: What explains the dramatic hierarchies in fermion masses and mixings?

Patterns especially clear in the quark sector.

Quark masses:

$$\frac{m_u}{m_t} \sim 10^{-5}$$

CKM elements:

$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.009 & 0.04 & 1 \end{pmatrix}$$

$$\Rightarrow V_{11} \gg V_{21} \gg V_{31}$$

Yukawa sector of the SM

$$\mathcal{L} \supset y_{ij}\,\overline{\psi}_i\,H\,\psi_j \longrightarrow \frac{y_{ij}\,v_H}{\sqrt{2}}\,\overline{\psi}_i\,\psi_j$$

Two ingredients:

1. The Higgs vev v_H
2. Dimensionless Yukawa couplings y_{ij}

The mass hierarchies arise from the Yukawa couplings

Hierarchies in Yukawas could be generated anywhere between $\mathcal{O}(\text{TeV})$ and M_{Planck}

Potential solutions: introduce new symmetries, fields, extra dimensions, string theory etc.

No clear winner has emerged after decades of work.

Problems

Too many models available

They predict fermion masses by design. How to falsify or distinguish between them?

Too much work to go put bounds on all the different models.

$\longrightarrow$ Ideal situation to use the SMEFT.

Our goals

1. Take a simple model of fermion masses and mixings $\rightarrow$ Froggatt-Nielsen models
2. Match to the SMEFT
3. Study resulting operator and flavour structure

Froggatt-Nielsen Models[1]

One of the oldest and simplest models of flavour.

Setup:

SM fields & $\mathcal{G}_{\mathrm{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$+$ new $U(1)$ symmetry (global or gauged)
$+$ heavy flavon field θ to break the symmetry

$+$ unknown UV dynamics: vector-like fermions? We remain agnostic about the details.

[1] Froggatt and Nielsen, 1979

Toy model charge assignments

An example model producing down-quark masses:

Field	$\overline{Q}_1$	$\overline{Q}_2$	$\overline{Q}_3$	d_1	d_2	d_3	H	θ
FN charge	6	4	0	5	3	3	-3	-2

Which Yukawa-like terms are allowed?

dim-4: $y^d_{33}\overline{Q}_3 H d_3 + y^d_{32}\overline{Q}_3 H d_2$

dim-5: $c^d_{31}\,\overline{Q}_3 H d_1 \left(\dfrac{\theta}{\Lambda_{\rm UV}}\right)$

dim-6: $c^d_{23}\,\overline{Q}_2 H d_3 \left(\dfrac{\theta}{\Lambda_{\rm UV}}\right)^2 + c^d_{22}\,\overline{Q}_2 H d_2 \left(\dfrac{\theta}{\Lambda_{\rm UV}}\right)^2$

Yukawa sector

$$\mathcal{L} \supset y^d_{ij}\,\overline{Q}_i\,H\,d_j \longrightarrow \mathcal{L} \supset c^d_{ij}\,\overline{Q}_i\,H\,d_j\left(\frac{\theta}{\Lambda_{\text{UV}}}\right)^{x_{ij}}$$

Lower generations come with more powers of $\theta/\Lambda_{\text{UV}}$

Flavon takes a vev:

$$\theta = \frac{v_\theta + \vartheta}{\sqrt{2}}$$

Define $\lambda \equiv \frac{v_\theta}{\sqrt{2}\Lambda_{\text{UV}}} \sim 0.1$

$\longrightarrow$ Yukawa matrices populated hierarchically.

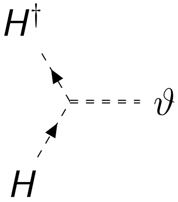
Scalar potential

$$V(H,\theta) = -\mu_H^2 H^\dagger H - \mu_\theta^2 \theta^* \theta$$
$$+ \lambda_{20}(H^\dagger H)^2 + \lambda_{02}(\theta^*\theta)^2 + \lambda_{11}\theta^*\theta H^\dagger H$$

After symmetry breaking:

$$\theta = \frac{v_\theta + \vartheta}{\sqrt{2}}$$

$$V(H,\theta) \supset -\lambda_{11} v_\theta \vartheta H^\dagger H \longrightarrow$$



Matching strategy

1) Write down a Froggatt-Nielsen EFT up to a given operator dimension. At dimension-4:

$$\mathcal{L}_{\mathsf{FN}} \supset y^d_{33}\overline{Q}_3 H d_3 + y^d_{32}\overline{Q}_3 H d_2 - \lambda_{11}(\theta^*\theta)\big(H^\dagger H\big).$$

At dimension-5:

$$\mathcal{L}_{\mathsf{FN}} \supset y^d_{33}\overline{Q}_3 H d_3 + y^d_{32}\overline{Q}_3 H d_2 - \lambda_{11}(\theta^*\theta)\big(H^\dagger H\big)$$

$$+ c^d_{31}\,\overline{Q}_3 H d_1 \left(\frac{\theta}{\Lambda_{\mathsf{UV}}}\right)$$

and so on.

2) Break the $U(1)_{\mathsf{FN}}$ symmetry:

$$\theta = \frac{v_\theta + \vartheta}{\sqrt{2}}$$

3) Integrate out ϑ and match to the SMEFT up to a given operator dimension.

Technical details

We have obtained our tree-level results manually and loop-level results using `Matchete`[2] which uses the functional method.

Have manually cross-checked loop-level results using diagrammatic matching.

[2] Fuentes-Martin et al., 2212.04510

Organisation

We need to approach the matching systematically.
We can:

1. Go to higher operator dimensions in $\mathcal{L}_{\text{FN}}$

2. Go to higher operator dimensions in the SMEFT

3. Match at tree-level, one-loop, two-loop...?

Organisation

We need to approach the matching systematically.
We can:

1. Go to higher operator dimensions in $\mathcal{L}_{\mathsf{FN}}$
   $d_{\mathsf{FN}} = 4, 5$
2. Go to higher operator dimensions in the SMEFT
   $d_{\mathsf{SMEFT}} = 6$
3. Match at tree-level, one-loop, two-loop...?
   Tree- and one-loop-level

$d_{\mathsf{FN}} = 4$; $d_{\mathsf{SMEFT}} = 6$; tree-level

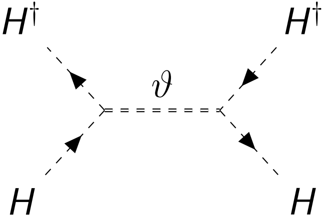
The only non-trivial Lagrangian term comes from the scalar potential:

$$\mathcal{L}_{\mathsf{FN}}^{d=4} \supset y_{33}^{d} \overline{Q}_3 H d_3 + y_{32}^{d} \overline{Q}_3 H d_2 - \lambda_{11} \theta^* \theta H^\dagger H$$

After SSB:

$$\mathcal{L}_{\mathsf{FN}}^{d=4} \supset y_{33}^{d} \overline{Q}_3 H d_3 + y_{32}^{d} \overline{Q}_3 H d_2 - \lambda_{11} v_\theta \vartheta \left(H^\dagger H\right) - \frac{\lambda_{11}}{2} \vartheta^2 \left(H^\dagger H\right)$$

Integrate out ϑ:

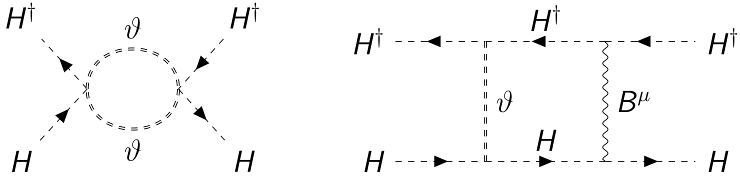


$$\longrightarrow (H^\dagger H)\Box(H^\dagger H)$$

$$\mathcal{L}_{\mathsf{SMEFT}} \supset -\frac{\lambda_{11}^2 v_\theta^2}{2m_\theta^4}(H^\dagger H)\Box(H^\dagger H)$$

$d_{\text{FN}} = 4$; $d_{\text{SMEFT}} = 6$; loop-level

Many more diagrams. E.g.



Matching done by Jiang et al.,1811.08878
and Haisch et al., 2003.05936

$d_{\mathsf{FN}} = 5$; $d_{\mathsf{SMEFT}} = 6$; tree-level

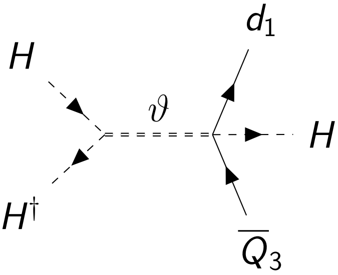
$$\mathcal{L}_{\mathsf{FN}}^{d=5} = \mathcal{L}_{\mathsf{FN}}^{d=4} + c_{31}^{d}\, \overline{Q}_3 H d_1 \left(\frac{\theta}{\Lambda_{\mathsf{UV}}}\right)$$

After SSB:

$$\mathcal{L}_{\mathsf{FN}}^{d=5} = \mathcal{L}_{\mathsf{FN}}^{d=4} + c_{31}^{d} \lambda\, \overline{Q}_3 H d_1 + c_{31}^{d}\, \overline{Q}_3 H d_1 \left(\frac{\vartheta}{\Lambda_{\mathsf{UV}}}\right)$$

(Recall $\lambda \sim v_\theta / \Lambda_{\mathsf{UV}} \sim 0.1$)

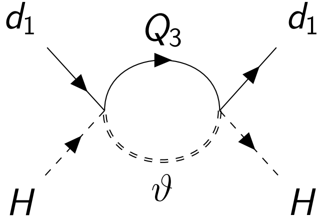
Matching



$$\longrightarrow \left(H^\dagger H\right)\left(\overline{Q}_3 H d_1\right)$$

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{\lambda\lambda_{11} c^d_{31}}{m^2_\theta}\left(H^\dagger H\right)\left(\overline{Q}_3 H d_1\right) + \text{H.c.}$$

$d_{\text{FN}} = 5$; $d_{\text{SMEFT}} = 6$; loop-level



$$\longrightarrow C_{Hd}^{11}(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{d}_1 \gamma^\mu d_1)$$

where

$$C_{Hd}^{11} = \frac{|c_{31}^d|^2}{128\pi^2 \Lambda_{\text{UV}}^2}(1 + 2\mathbb{L}).$$

(Have defined $\mathbb{L} = \log \mu^2/m_\theta^2$)

Key findings at 1-loop

Main operator types:

Higgs-enhanced Yukawas: $(H^\dagger H)\,\overline{\psi}_i H \psi_j$

Higgs kinetic operators: $(H^\dagger i \overleftrightarrow{D}_\mu H)\big(\overline{\psi}_i \gamma^\mu \psi_j\big)$

4-fermion operators: $(\overline{\psi}_i \psi_j)(\overline{\psi}_k \psi_l)$

Higgs kinetic operators

$$\frac{1}{128\pi^2 m_\theta^2}\left[4\lambda\lambda_{11}\left(c^{d\dagger}y^d + y^{d\dagger}c^d\right)_{ij} + \frac{m_\theta^2|c_{31}^d|^2}{\Lambda_{\rm UV}^2}\delta_{i1}\delta_{j1}(1+2\mathbb{L})\right]\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)(\overline{d}_i\gamma^\mu d_j)$$

where

$$\left(c^{d\dagger}y^d + y^{d\dagger}c^d\right)_{ij} = \begin{pmatrix} 0 & c_{31}^{d*}y_{32}^d & c_{31}^{d*}y_{33}^d \\ c_{31}^d y_{32}^{d*} & 0 & 0 \\ c_{31}^d y_{33}^{d*} & 0 & 0 \end{pmatrix}$$

Flavour hierarchies appear in SMEFT Wilson coefficients too!

Conclusions

Goal: Understand the infrared imprint of Froggatt-Nielsen models.

Method: Systematically match a Froggatt-Nielsen EFT to the SMEFT.

Findings: Rich flavour structure especially in $(H^\dagger H)\overline{\psi}_i H\psi_j$, $(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{\psi}_i\gamma^\mu\psi_j)$ and $(\overline{\psi}_i\gamma^\mu\psi_j)(\overline{\psi}_k\gamma^\mu\psi_l)$ operators.

Wilson coefficients show hierarchies too.

The End

Thank you for listening!