Manifestly Causal Quantum Field Theory

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Manifestly Causal QFT

- For a quantum system evolving under the unitary time evolution operator, $U_t = T\left\{\exp\left(\frac{1}{i}\int_{t_0}^{t} dt'\, H_{\text{int}}(t')\right)\right\}$, with initial state, $|i\rangle$, and final state, $|f\rangle$,

$$\langle f|\, U_t\, |i\rangle$$

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$$\text{Define } E := |f\rangle\langle f| \quad \Rightarrow \quad \mathbb{P} = \langle i|\, U_t^\dagger\, E\, U_t\, |i\rangle$$

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- By an application of the Baker-Campbell-Hausdorff lemma,

$$\mathbb{P} = \sum_{j=0}^{\infty} \langle i|\underbrace{[H_j, \ldots [H_2, [H_1, E]] \ldots]}_{\text{j times}}|i\rangle$$

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Manifestly Causal QFT - Why is this useful?

1. **Retarded propagators** appear in results, highlighting the **causality** of interactions,

$$\Delta_{ij}^{(\mathrm{R})} := \Theta_{ij} \langle 0| \, \frac{1}{i} \left[\phi_i, \phi_j\right] \, |0\rangle$$

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2. **All final states** are inherently summed over, via the completeness of states,

$$\sum_{\kappa} |f\rangle_{\kappa} \langle f| = \mathbb{I}$$

This may mean that all infra-red divergences are intrinsically summed and cancel

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Result 1 - Fermi Two-Level Atom

- Consider a two-level source atom, S, and a two-level detector atom, D, each coupled to a scalar field, ϕ,

$$\mathcal{H}_{\rm int}(t') = M^S(t')\,\phi(\mathbf{x}^S, t') + M^D(t')\,\phi(\mathbf{x}^D, t')$$

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- We want to calculate something which should be **zero for space-like separations**, and insensitive to vacuum fluctuations,

$$\sigma_{pg} := \mathbb{P}_p - \mathbb{P}_g$$

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Result 1 - Fermi Two-Level Atom

$$\sigma_{pg} \supset \; 2\sum_n |\mu^S_{pn}|^2 \, |\mu^D_{qg}|^2 \Big\{$$

$$\cos\omega^D_{qg} t'_{12} \Big(\sin\omega^S_{pn} t'_{34} \, \Delta^{DS(\mathrm{H})}_{24} + \cos\omega^S_{pn} t'_{34} \, \Delta^{DS(\mathrm{R})}_{24} \Big) \Delta^{DS(\mathrm{R})}_{13}$$

$$+ \cos\omega^D_{qg} t'_{12} \Big(\sin\omega^S_{pn} t'_{34} \, \Delta^{DS(\mathrm{H})}_{14} + \cos\omega^S_{pn} t'_{34} \, \Delta^{DS(\mathrm{R})}_{14} \Big) \Delta^{DS(\mathrm{R})}_{23}$$

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Result 2 - Particle Scattering with Three Scalar Fields

- Consider a massive scalar particle, χ, decaying to any number of scalar particles, ϕ and h,

$$\mathcal{H}_{\text{int}}(t') = \int \mathrm{d}^3\mathbf{x} \, \left(g_\chi \, \phi^2 \, \chi + g_h \, \phi^2 \, h \right)$$

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- $E^\phi = \mathbb{I} \implies$ Interesting. **Looks different**.

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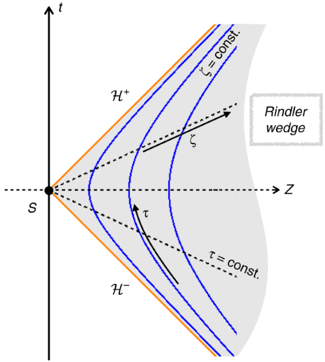
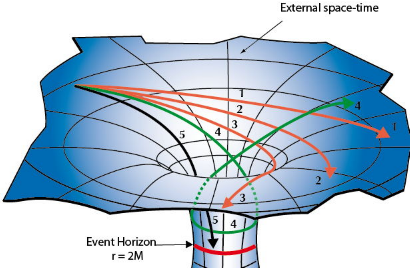
Future Results

- Quantify the response of an accelerated Unruh-DeWitt detector in a vacuum (**Unruh effect**)
- Quantify the response of a detector on different trajectories in the **Schwarzschild metric**





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Thank you!