



Maximally supersymmetric Yang–Mills in three dimensions

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Phase diagram from lattice calculations

System: Naive dimensional reduction of maximal super-Yang–Mills (4d $\mathcal{N} = 4$ SYM) [arXiv:2010.00026]
Richer dynamics than 2d [arXiv:2312.04980], computationally cheaper than 4d [arXiv:2304.04655]

Goal: Map ‘spatial deconfinement’ transition [dual to black brane / black hole]
from high-temperature 2d bosonic theory to low-temperature holographic regime

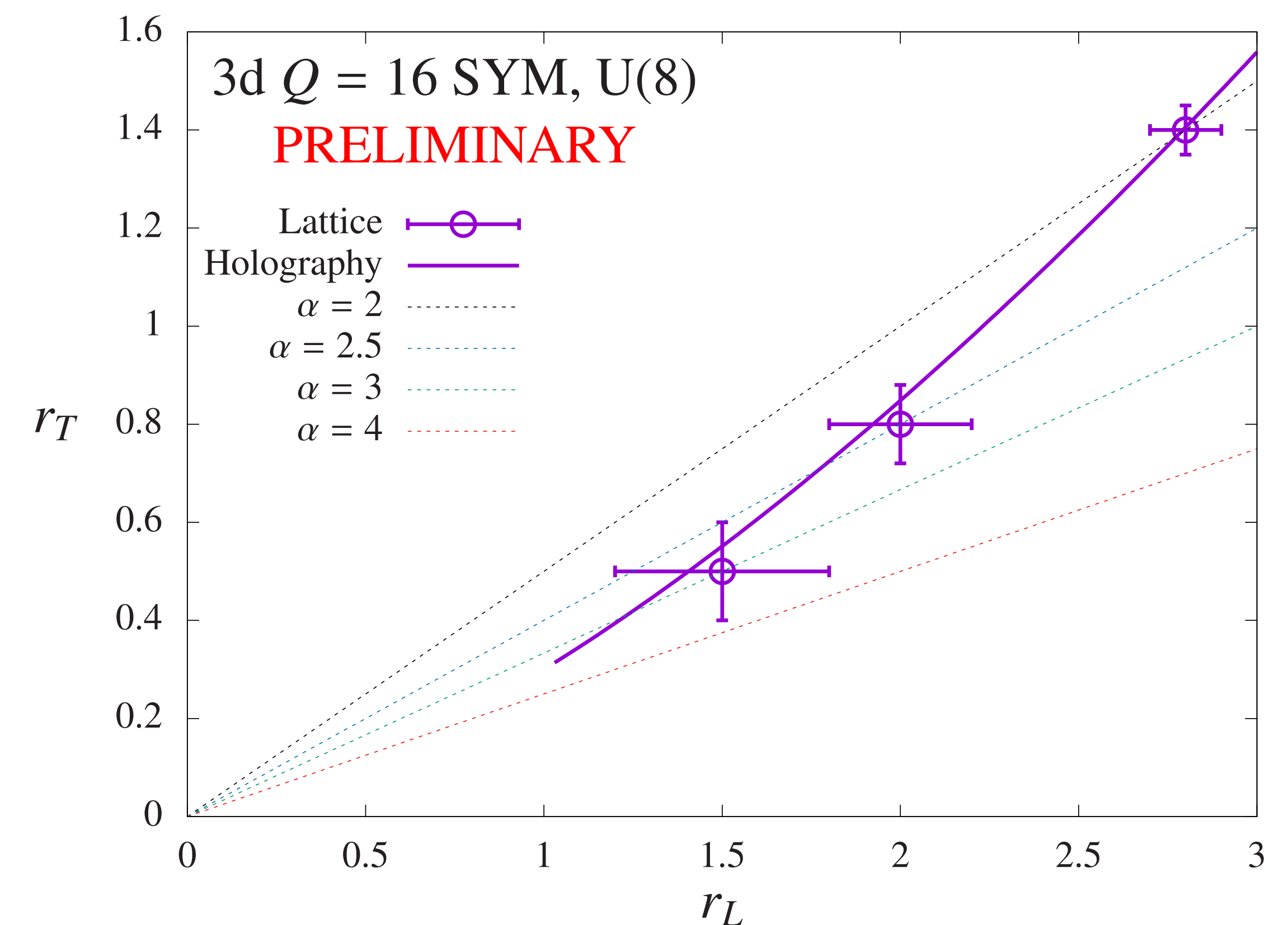
Dim’less torus size $r_L \times r_L \times r_T \rightarrow$ **aspect ratio** $\alpha = r_L/r_T$

Dim’less temperature $T = 4/(r_T\sqrt{3})$ [skewed 3-torus]

Fix gauge group $U(8)$ and $N_T = 8$
 $\rightarrow \alpha = 2, 2.5, 3, 4$ from $N_L = 16, 20, 24, 32$

Preliminary results consistent with holographic expectation

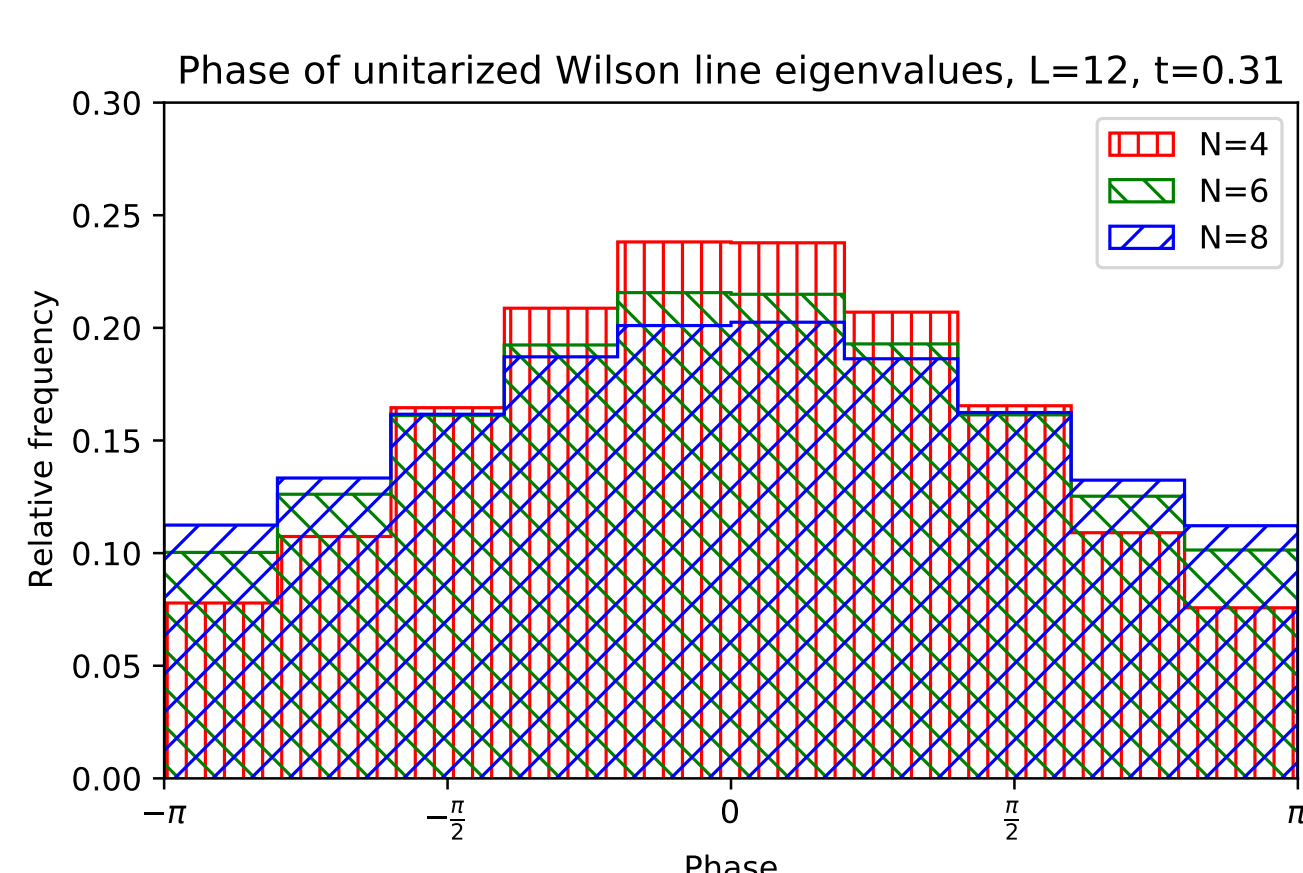
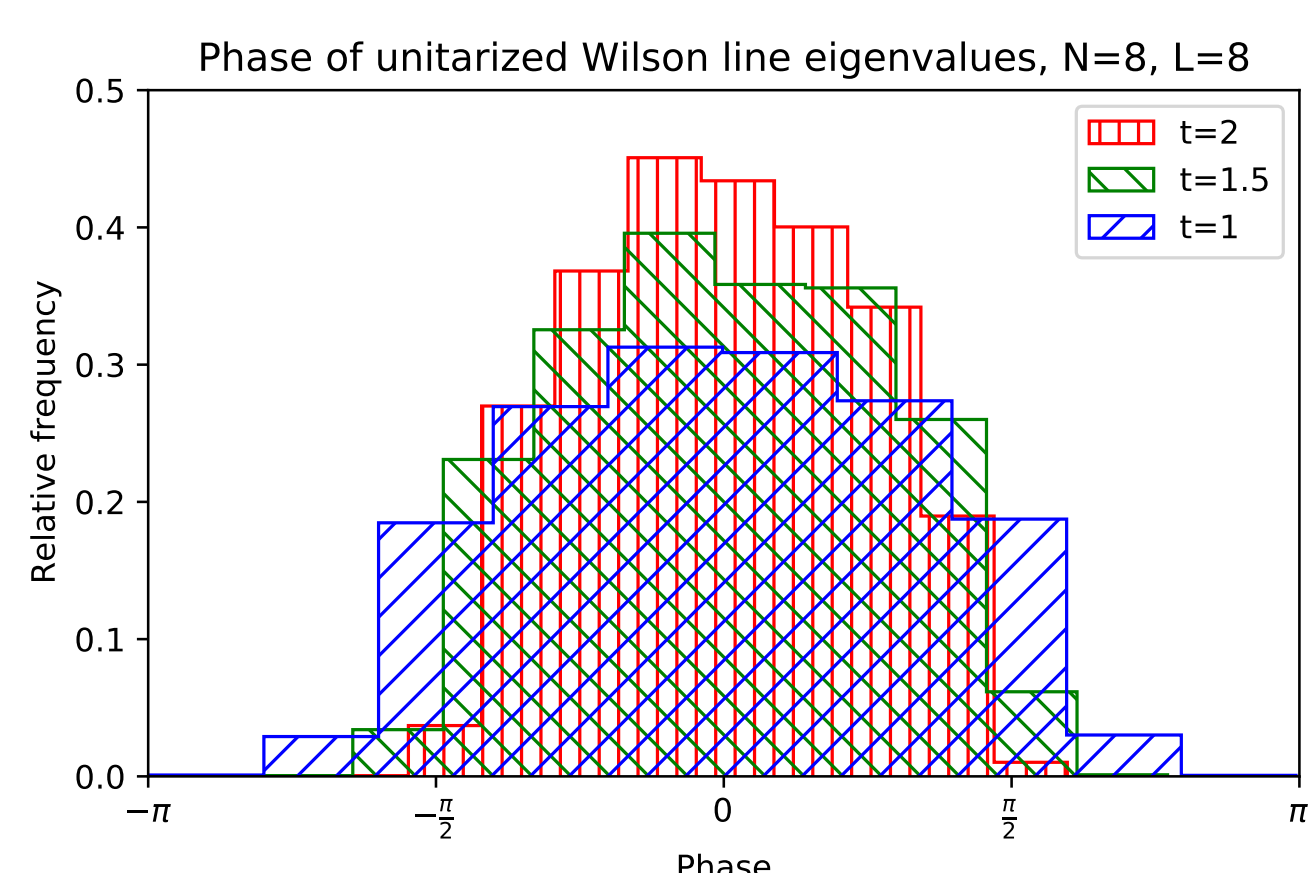
$$r_L^{3/2} \propto r_T \rightarrow T_c \propto \alpha^3$$



Transition observables

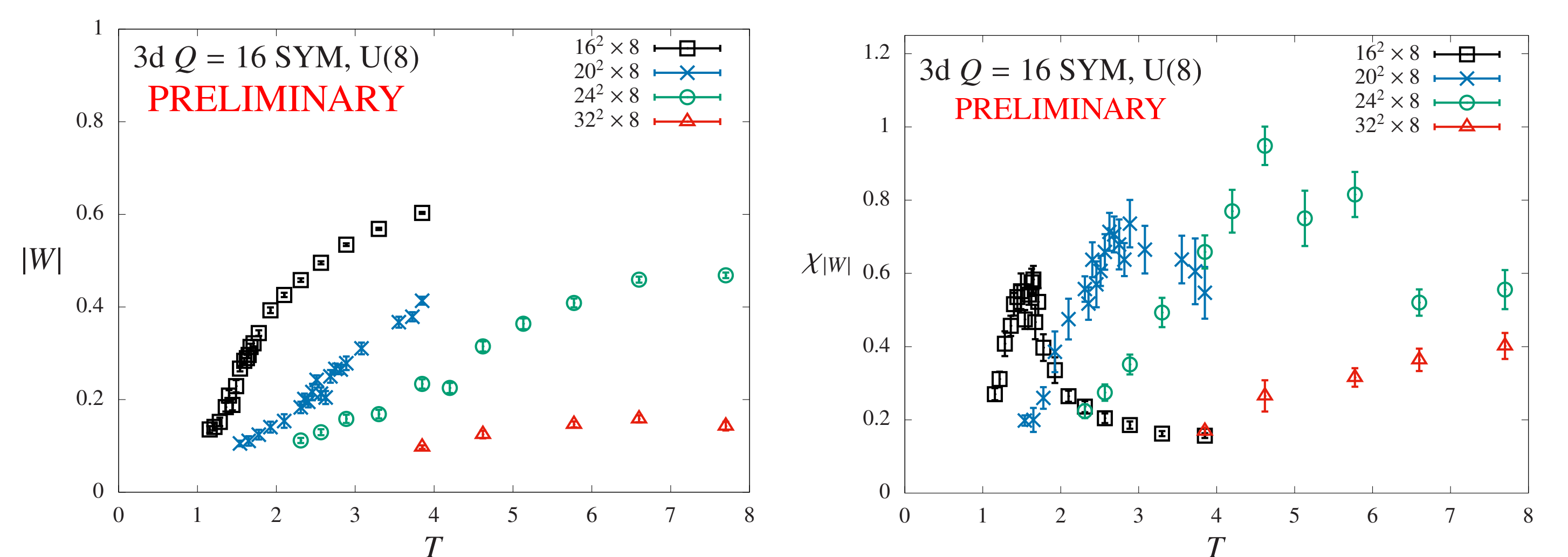
Magnitude of spatial Wilson line (WL)
Large for high- T deconfined phase
Vanishes for low- T confined (in large- N limit)

WL eigenvalue distribution
Localized for high- T deconfined phase
Homogeneous for low- T confined (large- N)



Transition signals

T_c from peaks in WL magnitude **susceptibility**



Larger aspect ratio (larger spatial volume)
 \rightarrow higher T_c
 \rightarrow harder to stabilize lattice system
Critical scaling needs $U(N)$ with larger $N > 8$

More technical details of the lattice system

Twisted formulation preserves two (of 16) supersymmetries at non-zero lattice spacing
Requires A_3^* (body-centered cubic) lattice \rightarrow skewed 3-torus [arXiv:2010.00026]

16 fermions \rightarrow 2 site $\{\eta, \psi_5\}$ + 8 link $\{\psi_a, \chi_{5a}\}$ + 6 plaquette $\{\chi_{ab}\}$

Gauge field A_μ + 7 scalars \rightarrow complexified $\{\mathcal{U}_5, \bar{\mathcal{U}}_5\}$ + $\{\mathcal{U}_a, \bar{\mathcal{U}}_a\}$ \rightarrow $U(N)$ gauge invariance

Use 't Hooft coupling λ to define dim’less $r_T = \beta\lambda$ and $r_L = \alpha\beta\lambda$

Fixed torus size \rightarrow larger lattice volumes produce smaller lattice spacing a and smaller $\lambda_{\text{lat}} = a\lambda$

Soft-supersymmetry-breaking scalar potential $\propto \lambda_{\text{lat}}$ needed to stabilize RHMC lattice calculations

Future work to adapt gradient flow to naive dimensional reduction [$1 \times N_L \times N_L \times N_T$ lattices]