Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

Fermilab

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3 August 2024

Outline

Dirac or Majorana?

- Oscillation experiments $\to m_\nu > 0$

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- Could also include Majorana mass term
$$-M \nu_R^T \nu_R$$
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$$-m_D \bar{\nu}_L \nu_R$$

- Could also include Majorana mass term

$$-M \nu_R^T \nu_R$$

  - Violates lepton number by 2 units
- Seesaw mechanism: m_ν naturally small (if $M \sim M_{\mathrm{Pl}}$)

$$m_\nu \propto \frac{(Yv)^2}{M} < 1 \text{ eV}$$

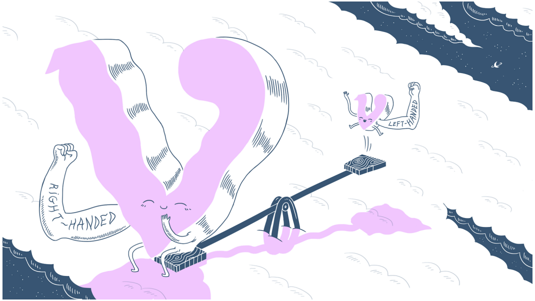
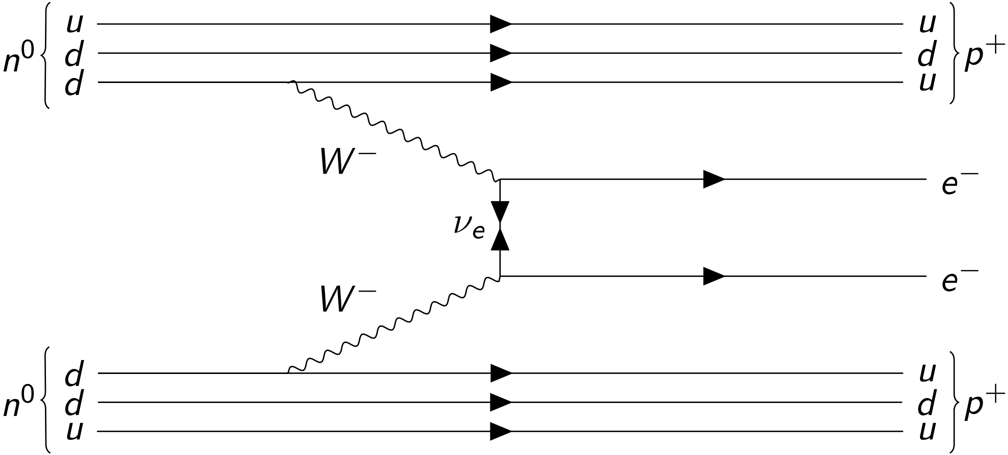


Image credit: Kova (Symmetry Magazine, Sandbox Studio)

$0\nu\beta\beta$ Diagram



Furry, PR 56, 1184 (1939); Figure credit: Detmold and Murphy, 2004.07404

Double-Beta Decay

$$\text{nuclear mass} \approx \left(Z - \frac{A}{2}\right)^2$$

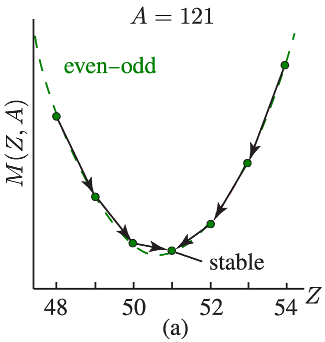
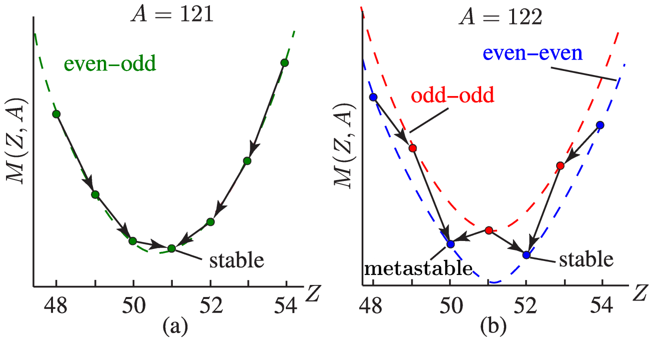


Figure credit: Adapted from Jaffe and Taylor (2018), after J. Lilley (2001)

Double-Beta Decay

$$\text{nuclear mass} \approx \left(Z - \frac{A}{2}\right)^2 + C \begin{cases} +1 & Z, N \text{ both odd} \\ -1 & Z, N \text{ both even} \\ 0 & \text{otherwise} \end{cases}$$



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Extraction of $m_{\beta\beta}$

$$\left(T_{1/2}^{0\nu}\right)^{-1} =$$

$0\nu\beta\beta$ half-life (measured experimentally)

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Effective double-beta neutrino mass

$$m_{\beta\beta} = \left| \sum_k U_{ek}^2 m_k \right|$$

Extraction of $m_{\beta\beta}$

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Kinematical factor (known functional form)

Extraction of $m_{\beta\beta}$

$$\left(T_{1/2}^{0\nu}\right)^{-1} = |\, m_{\beta\beta} \,|^2 \, G^{0\nu} \, |\, \langle A, Z+2 | JJ | A, Z \rangle \,|^2$$

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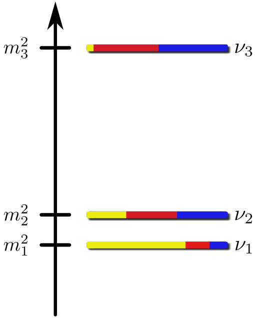
Kinematical factor (known functional form)

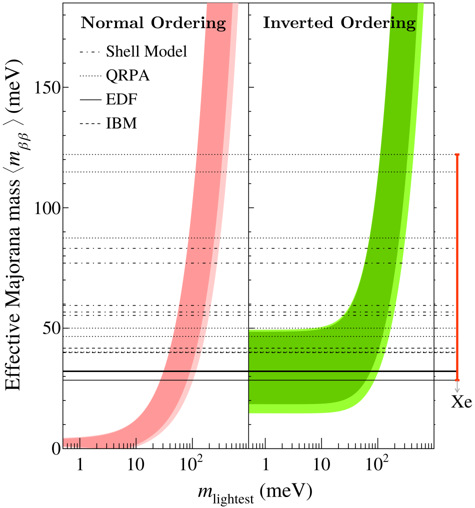
Nuclear matrix element

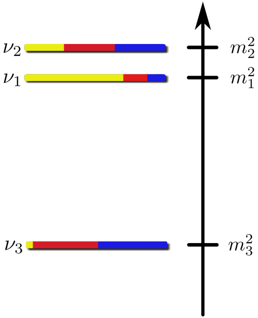
Note: Additional short-distance contributions in some BSM theories

KamLAND-Zen Results



Normal





Inverted

Figure credits: Adapted from KamLAND-Zen (2406.11438); Kismalac, Wikimedia Commons

Nuclear Matrix Element Estimates

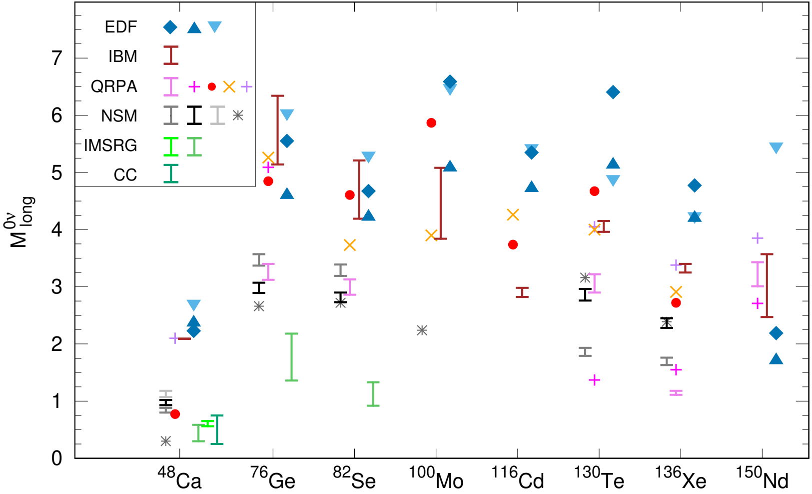


Figure credit: Agostini et al. (RMP 95, 025002 (2202.01787))

Nuclear Effective Field Theory

- Effective field theory (EFT): Approximate low-energy description of problem
- Quark-gluon interactions $\to$ effective hadronic couplings
- Inputs: NN scattering and ^{2}H, ^{3}H binding energies (Bansal et al., PRC 98, 054301 (1712.10246))
  - For χEFT, also need interactions of $N\pi$, $\pi\pi$, $NN\pi$, etc.
  - For weak decays, also need axial and vector nucleon charges
- Successful phenomonologically – can compute binding energies up to ^{132}Sn to within 10–20% (Binder et al., PRC 93, 044332 (1512.03802))

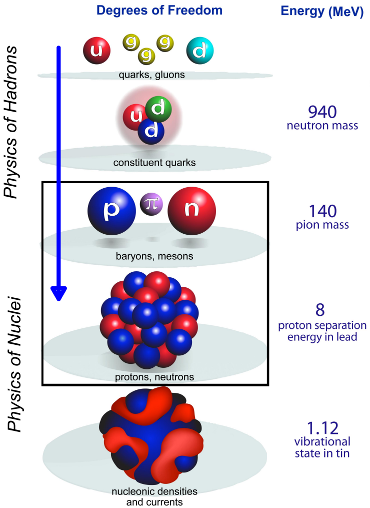
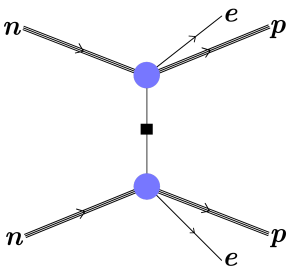
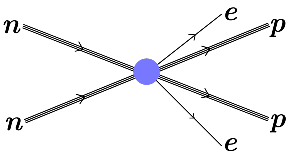


Figure credit: DOE/NSF NSAC (0809.3137)

Nuclear EFT for $0\nu\beta\beta$





- Neutrino energy can be hard or soft
- Low-energy contribution factorises into two SM weak currents
  - Can be computed from existing experimental data
- High-energy intermediate ν outside of EFT validity
- Need contact term g^{ν}_{NN} to absorb high-energy behavior (Cirigliano et al., PRC 97, 065501 (1710.01729), PRL 120, 202001 (1802.10097))
- Contact term promoted to leading order in EFT

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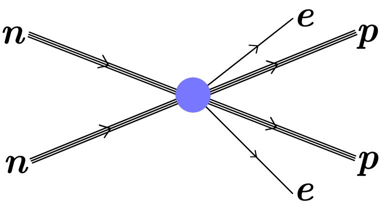

- EFT contact term g_{NN}^{ν} unique to $0\nu\beta\beta$
  - No experimental data!
  - Cannot be computed from $2\nu\beta\beta$
- Can be estimated using dispersive relations (generalized Cottingham formula) (Cottingham, AP 25, 424 (1963); Cirigliano et al., JHEP 05, 289 (2102.03371))
  - Likely correct to within 40% but requires model assumptions
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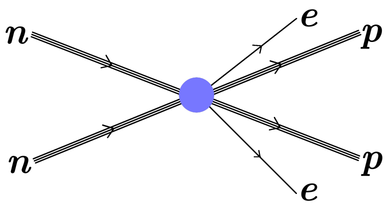

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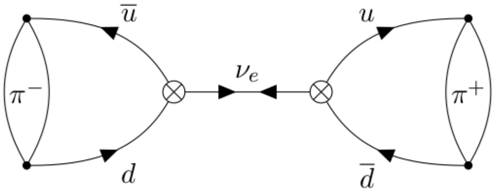
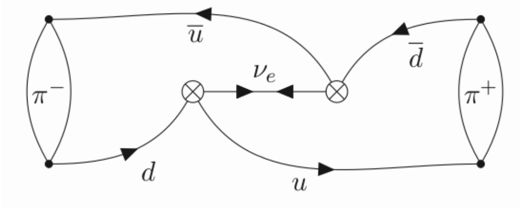
  - Likely correct to within 40% but requires model assumptions
  - Ongoing work to refine calculation (Van Goffrier, PhD thesis (2023))
- Calculate simple system with lattice QCD, match to EFT

$0\nu\beta\beta$ for $\pi^- \to \pi^+$





$$C_{\pi^- \to \pi^+} = \sum_{\mathbf{x},\mathbf{y}} \int \frac{d^4q}{(2\pi)^4} \frac{e^{iq\cdot(x-y)}}{q^2} \langle \mathcal{O}_{\pi^+}(t_+) J_\mu(x) J_\mu(y) \mathcal{O}^\dagger_{\pi^-}(t_-) \rangle$$

- Compute quark propagators from wall source and sink, contract at operators

Figure credit: 2004.07404

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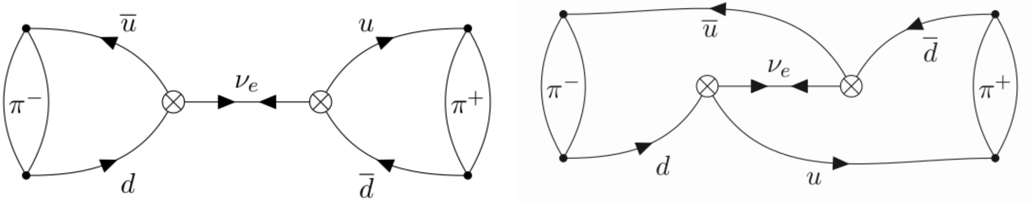

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- Double sum over both operator spatial positions
    - Naïve cost: L^6 (expensive)
    - FFT convolution theorem reduces cost to $O(L^3 \log L)$

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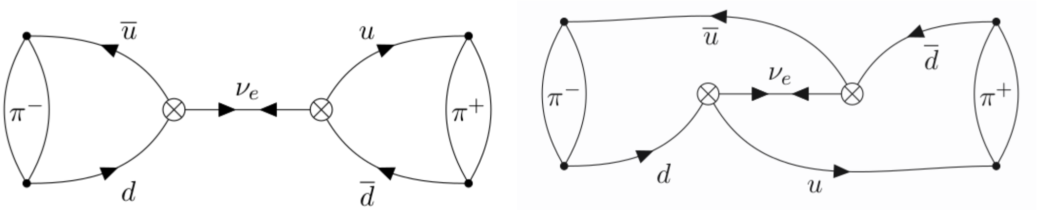

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- Final integration over $t = x_4 - y_4$ required for matrix element

Figure credit: 2004.07404

$0\nu\beta\beta$ for $\pi^- \to \pi^+$

$$\langle \pi^+ | J^\mu J_\mu | \pi^- \rangle \propto 1 + \frac{m_\pi^2}{8\pi^2 f_\pi^2}\left(3\log\left(\frac{\mu^2}{m_\pi^2}\right) + \frac{7}{2} + \frac{\pi^2}{4} + \frac{5}{6}\boldsymbol{g_\nu^{\pi\pi}(\mu)}\right)$$

- Matrix element completely determined up to $g_\nu^{\pi\pi}$

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- Matrix element completely determined up to $g_\nu^{\pi\pi}$
- $g_\nu^{\pi\pi}(\mu = m_\rho)$ measured by two groups with domain-wall fermions, extrapolated to physical point
  - $-10.9(8)$ (Tuo, Feng, Jin, PRD 100, 094511 (1909.13525))
  - $-10.8(5)$ (Detmold, Murphy, 2004.07404)

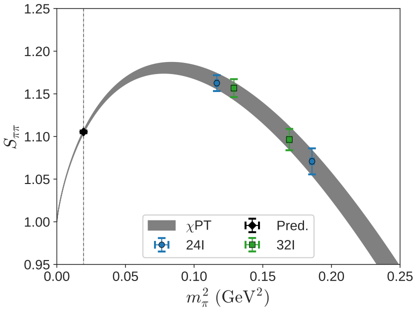
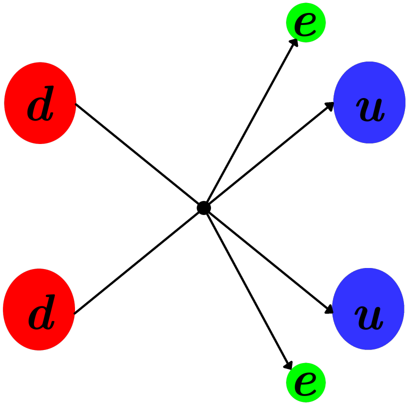


Figure credit: 2004.07404

Short-Distance Mechanism

$$\mathcal{O} = \left(\bar{d}\Gamma_i u\right)\left(\bar{d}\Gamma_j u\right)$$

- Contact interactions at scale of QCD
- Basis of 9 operators
  - 5 scalar operators ($\Gamma^i\Gamma^j = s$): $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}'_1, \mathcal{O}'_2$
  - 4 vector operators ($\Gamma^i\Gamma^j = v^\mu$): $\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4$
- Coefficients determined by BSM theories
  - Compute matrix elements of 9 operators separately
- Scalar operator matrix elements calculated for $\pi^- \to \pi^+$ by CalLat (Nicholson et al., PRL 121, 172501 (1805.02634)) and NPLQCD (Detmold et al., PRD 107, 094501 (2208.05322))



Dimension-9 $0\nu\beta\beta$ Coefficients

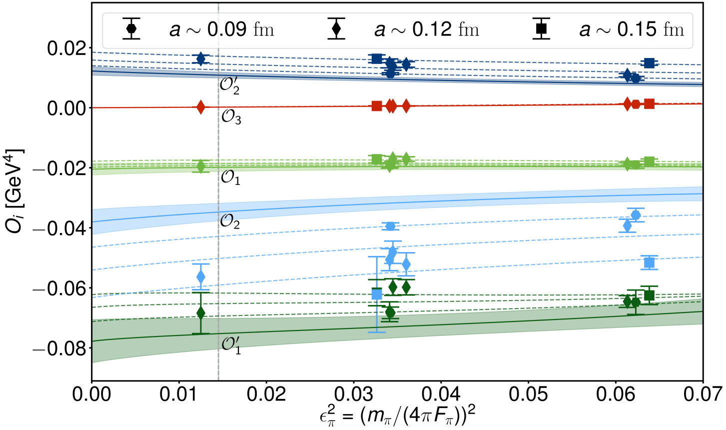


Figure credit: Nicholson et al., PRL 121, 172501 (1805.02634)

Neutrinoful Double-Beta Decay ($2\nu\beta\beta$)

- Rarest observed Standard Model process
- Experimental data used as inputs or tests of nuclear models of $0\nu\beta\beta$ (Engel, Menéndez, RPP 80, 046301 (1610.06548))

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  - Single lattice spacing and $m_\pi = 800$ MeV
  - Computed matrix element to $\sim 2\%$ uncertainty (stat.) and extracted $2\nu\beta\beta$ counterterm

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- No intermediate ν prop – weak currents decouple
  - Background field method – quark propagators computed in presence of uniform weak field (Fucito et al., PLB 115, 148; Martinelli et al., PLB 116, 434; Bernard et al., PRL 49, 1076)

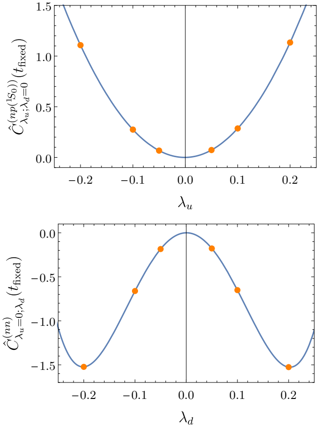

Figure credit: 1702.02929

Challenges for $0\nu\beta\beta$ in $nn \to pp$

$$C_{nn\to pp} = \sum_{\mathbf{x},\mathbf{y}} \int \frac{d^4q}{(2\pi)^4} \frac{e^{iq\cdot(x-y)}}{q^2} \langle \mathcal{O}_{pp}(t_+) J_\mu(x) J_\mu(y) \mathcal{O}^\dagger_{nn}(t_-)\rangle$$

- Current insertions coupled by ν propagator
  - Cannot use background field method

Davoudi, Detmold, Fu, **AVG**, Jay, Murphy, Oare, Shanahan, Wagman (NPLQCD), PRD 109, 114514 (2402.09362)

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- Complexity of contractions $\propto N_q!$
  - $N_c!^4 N_u! N_d! = 6^4 24^2 \approx 10^6$ contractions needed

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Dinucleon Interpolating Operators

- Dibaryon (bi-local) operators – good signal quality but computationally expensive
  - Require cost reduction techniques, e.g. sparsening (Detmold et al., PRD 104, 034502 (1908.07050), Amarasinghe et al., PRD 107, 094508 (2108.10835)), distillation (Peardon et al., PRD 80, 054506 (0905.2160); Hörz et al., PRC 103, 014003 (2009.11825))

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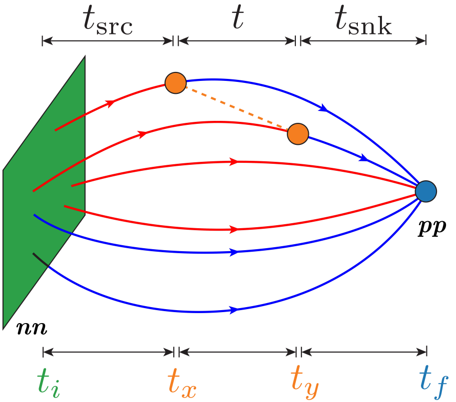
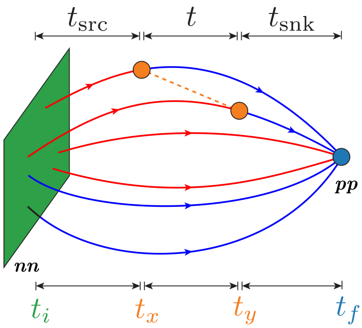
- Variational analysis – expensive
- Compromise: Wall source, point sink
  - Improve signal with sparse (4^3) grid at sink



Reducing Computational Cost

- 4-point function requires nuclear contractions ($O(10^6)$) and convolution over operator positions ($O(V^2)$): $10^6 V^2 \sim 10^{15}$



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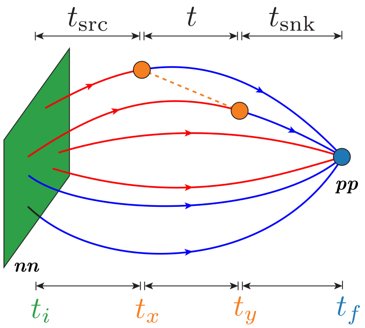

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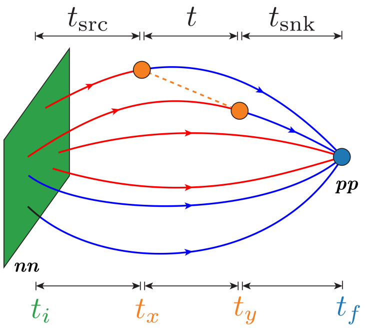

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  - Reduces work to

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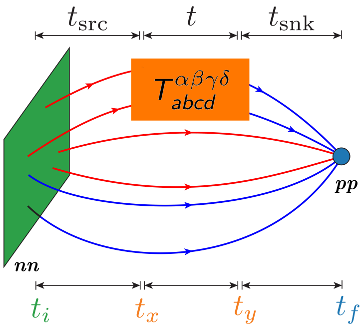

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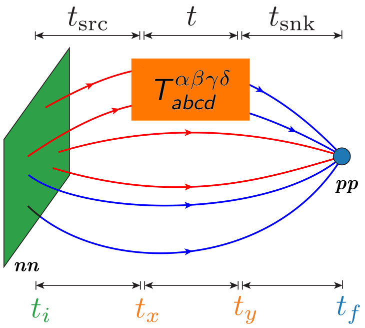
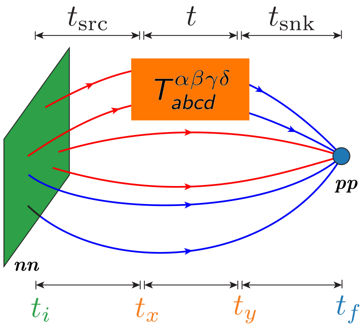
- Project quarks to positive parity: $N_s \to 2$



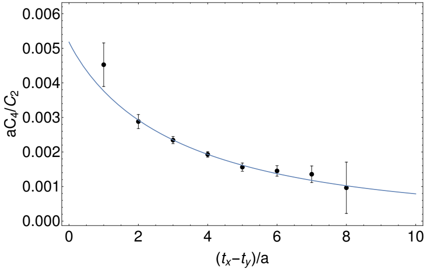
Reducing Computational Cost

- 4-point function requires nuclear contractions ($O(10^6)$) and convolution over operator positions ($O(V^2)$): $10^6 V^2 \sim 10^{15}$
- Fast Fourier transform $V^2 \to V \log V$ ($\sim 10^{12}$)
- ~~Sparsening at operator $\to$ wrong answer~~
- Decouple operator position sum from nuclear contractions
  - Sum 4-quark tensor $T^{\alpha\beta\gamma\delta}_{abcd}$ over x, y
  - Reduces work to
    $$(N_c N_s)^4 V \log V + 10^6 \sim 10^{10}$$
- Project quarks to positive parity: $N_s \to 2$
- Total cost of $O(10^9)$ prop multiplications/sink location/(t_x, t_y, T)
  - ~ 200 CPU core-hours/config

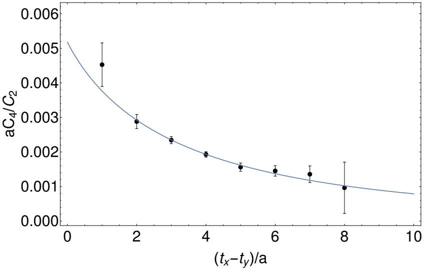


Neutrino Propagator

- Long-distance amplitude contains significant contribution from low-E_ν tail
  - Contribution from separation $t = t_y - t_x$ falls off as t^{-2}
  - Corresponds to large temporal separation between operators
  - Difficult to control (signal-to-noise problem)



Neutrino Propagator



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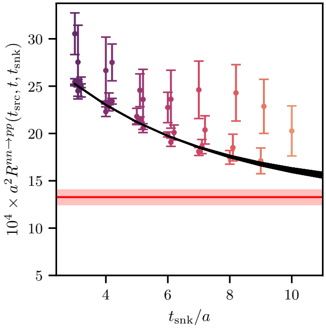
  - Contribution from separation $t = t_y - t_x$ falls off as t^{-2}
  - Corresponds to large temporal separation between operators
  - Difficult to control (signal-to-noise problem)
- Solution: Use zero-mode subtracted propagator (Davoudi and Kadam, PRL 126, 152003 (2012.02083))

$$S_\nu(\tau, \mathbf{z}) = \frac{m_{\beta\beta}}{2L^3} \sum_{\mathbf{q} \in \frac{2\pi}{L}\mathbb{Z}^3 \setminus \{\mathbf{0}\}} \frac{e^{i\mathbf{q}\cdot\mathbf{z}}}{|\mathbf{q}|} e^{-|\mathbf{q}||\tau|}$$

  - Contribution falls off exponentially in t
  - Match to zero-mode removed EFT amplitude

Fitting Procedure

- Asymmetric excited state contamination from source and sink
  - More severe from point sink than wall source
- Extrapolate $t_{\text{src}}, t_{\text{snk}} \to \infty$ at given operator separation t



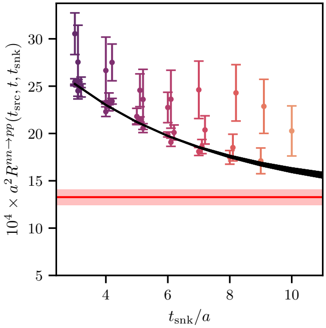
Fitting Procedure

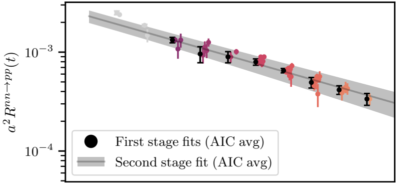
- Asymmetric excited state contamination from source and sink
  - More severe from point sink than wall source
- Extrapolate $t_{src}, t_{snk} \to \infty$ at given operator separation t
- Fit t dependence to exponential and integrate:

$$\langle pp|JJ|nn\rangle \propto 2m_{nn} \int_{-\infty}^{\infty} dt \, \frac{C_4(t,\tau)}{C_2(\tau)}$$
$$= 0.14(3) \text{ GeV}^2 \text{ (stat.)}$$

- Need high stats (5M total sources) to resolve dependence on t, t_{src}, t_{snk}

Thanks to XSEDE/ACCESS, TACC, and RCAC for compute time!





Difficulties in Extracting g_{NN}

$$\frac{\langle pp|JJ|nn\rangle}{2m_{nn}}\frac{1}{\mathcal{R}(E)\mathcal{M}(E)^2} = (1+3g_A^2)(J^\infty + \delta J^V) - \frac{m_n^2}{8\pi^2}\tilde{g}_\nu^{NN}$$

- $\langle pp|JJ|nn\rangle = 0\nu\beta\beta$ amplitude from LQCD
- $\tilde{g}_\nu^{NN} \propto g_\nu^{NN}$ = EFT counterterm of interest
- Known functions of NN interactions:
  - $\mathcal{M}$ = NN scattering (from effective-range expansion)
  - $\mathcal{R}$ = Lellouch-Lüscher residue
  - J^∞ = contribution from soft ν exchange
  - δJ^V = FV correction

Kaplan et al., PLB 424, 390 (nucl-th/9801034); Lellouch and Lüscher, CMP 219, 31 (hep-lat/0003023); Davoudi and Kadam, PRD 102, 114521 (2007.15542), PRL 126, 152003 (2012.02083), PRD 105, 094502 (2111.11599)

Difficulties in Extracting g_{NN}

$$\mathcal{M}(E) = -\frac{4\pi}{m_N}\frac{1}{1/a - rp^2/2 + ip}$$

- Inputs required:
  - a = scattering length
  - r = effective range
  - $E = p^2/2m_N$ = FV energy shift
- Difficult to determine at $m_\pi = 800$ MeV
  - Values for $\mathcal{M}$, $\mathcal{R}$ very different for bound vs. scattering states
- Well determined from experiment ($a = 23.5$ fm, $r = 2.75$ fm)

Physical Point

- EFT matching is more straightforward
- Lattice calculation more difficult
  - More expensive propagators
  - Worse signal-to-noise problem

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  (Perry, 31 Jul, 11:35; Dhindsa, 2 Aug, 11:15; Green, 2 Aug, 11:35)
  - *NN* correlators being computed at $m_\pi = 170$ MeV $\approx m_\pi^{\text{phys}}$

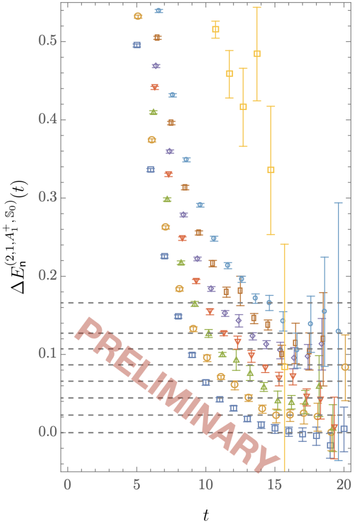

Figure credit: Davoudi et al. (NPLQCD), unpublished

Physical Point

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  - NN correlators being computed at $m_\pi = 170$ MeV $\approx m_\pi^{\text{phys}}$
- Goal: Find good interpolating operator(s) at physical point, use these for $0\nu\beta\beta$
  - $t > 2$ fm difficult to resolve $\to$ need to reduce excited states

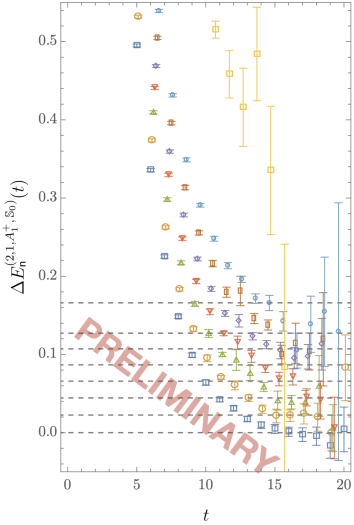
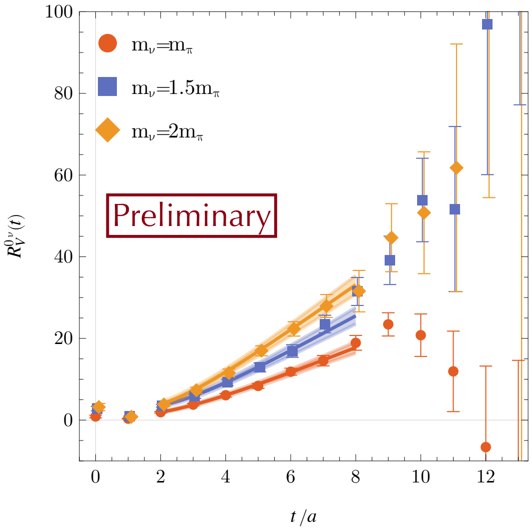


Figure credit: Davoudi et al. (NPLQCD), unpublished

Progress Toward Physical Point

- Progress toward $0\nu\beta\beta$ at $m_\pi = 432$ MeV
- Use bi-local interpolators at source and sink to suppress excited states
- Use unphysical $m_\nu \sim m_\pi$ to suppress large-t tail and FV corrections
- Stochastic noise vectors to represent neutrino propagator
- Summation method – read contribution to $0\nu\beta\beta$ from slope versus t

Wang, 31 Jul, 12:35



Conclusion

- $0\nu\beta\beta$ experiments need theory input from hadronic physics
- Simulate light nuclear systems on lattice, extract EFT coefficients
- Can use EFT coefficients as input to *ab initio* nuclear many-body methods
  - Ongoing work to refine these methods, push to larger A
  - Also progress using lattice EFT for β-decay (Wang, 2 Aug, 12:55)
- Important whether we observe $0\nu\beta\beta$ or place improved bounds
  - Next-gen experiments could rule out inverted ordering of ν masses

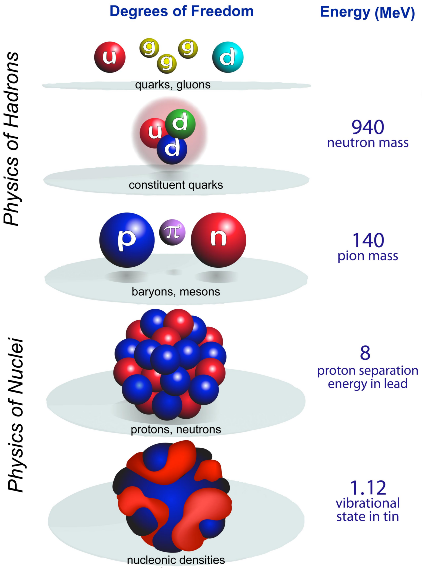


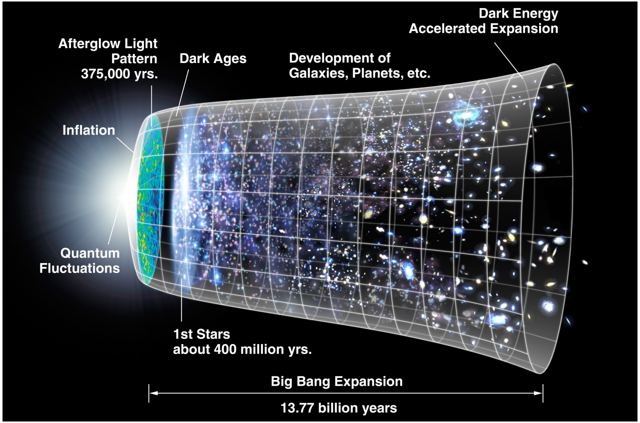
Figure credit: DOE/NSF NSAC (0809.3137)

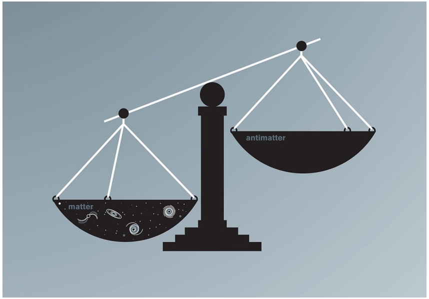
Neutrino Masses

- Original formulation of Standard Model had $m_\nu = 0$
- Homestake experiment $\rightarrow m_\nu \neq 0$
- Exact values unknown but $m_\nu < 1$ eV for all generations

Image credit: Wikimedia Commons

Origin of Matter





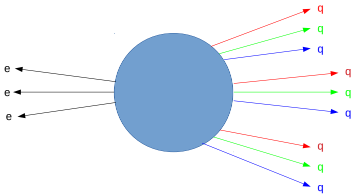


Image credits: Wikimedia Commons; Symmetry Magazine, Sandbox Studio

Neutrino Mass Scales

- Only mass gaps accessible by (most) experiments
- Known that $\Delta^2_{12} \equiv m_2^2 - m_1^2 \ll \Delta^2_{13}, \Delta^2_{23}$
- Two possible orderings: normal and inverted
- More precise measurements needed to resolve ordering

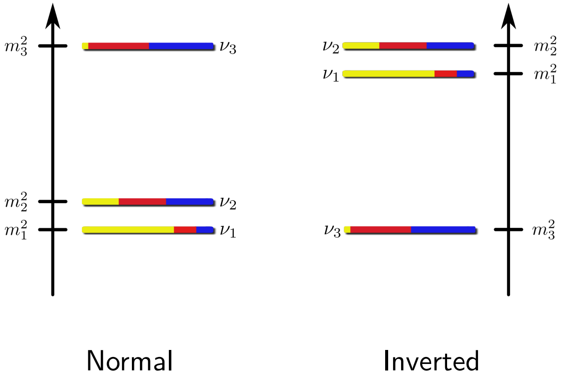
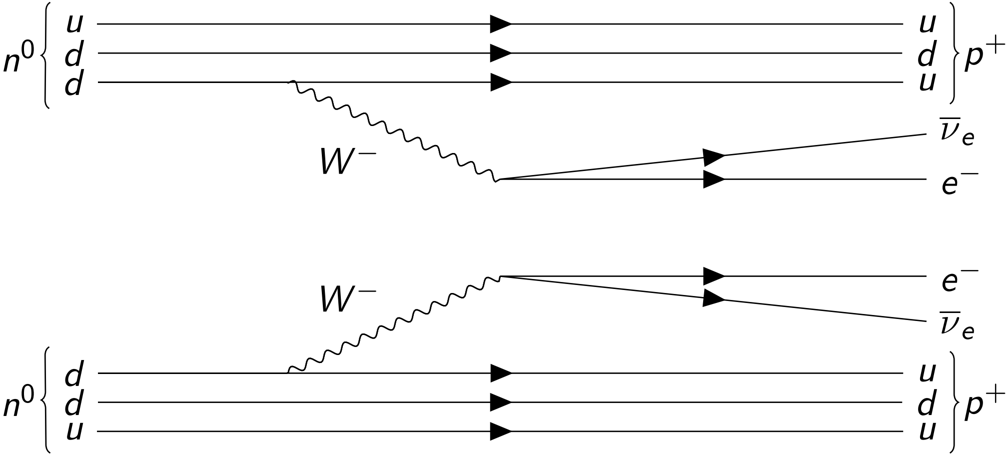


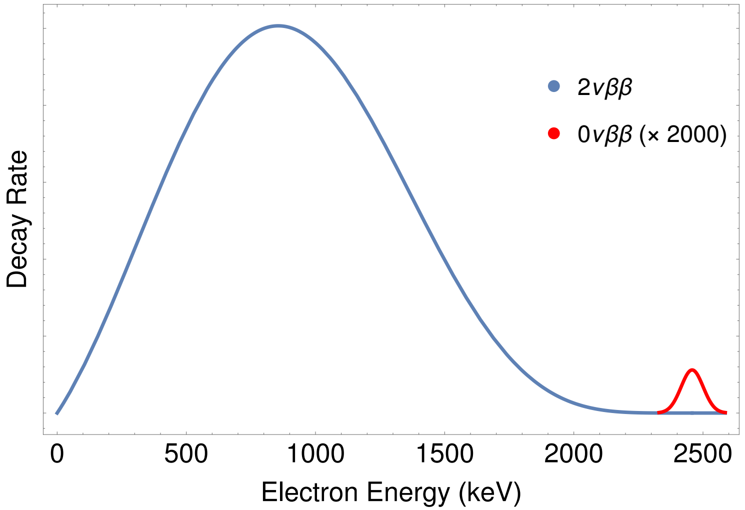
Figure credit: Adapted from Kismalac, Wikimedia Commons

$2\nu\beta\beta$ Diagram



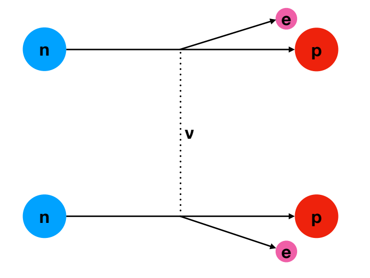
Goeppart-Mayer, PR 48, 512 (1935); Figure credit: Detmold and Murphy, 2004.07404

Experimental $0\nu\beta\beta$ Signature



Short- and Long-Distance Mechanisms

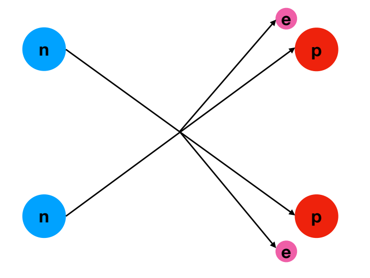
Long-Distance



$$(\bar{d}P_L\gamma_\mu u)(x)S_\nu(x-y)(\bar{d}P_L\gamma^\mu u)(y)$$

- Present in any theory of $0\nu\beta\beta$
- Minimal extension to original SM
- Only parameter $= m_{\beta\beta}$

Short-Distance



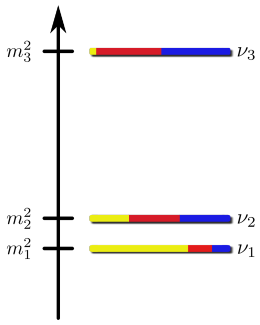
$$(\bar{u}\Gamma^i d)\,(\bar{u}\Gamma^j d)$$

- Present in some BSM theories
- High-energy intermediate states
- Parameters $=$ 9 operator coefficients

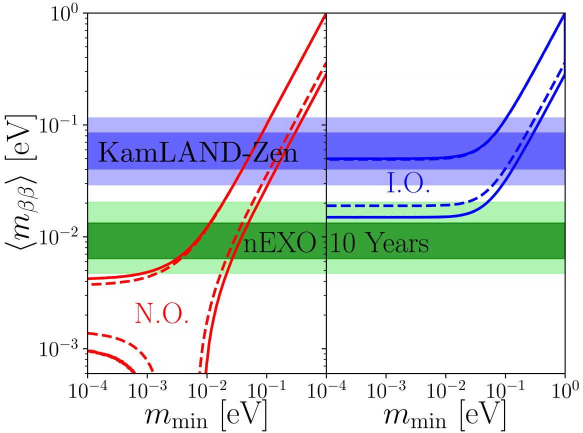
Nuclear Models

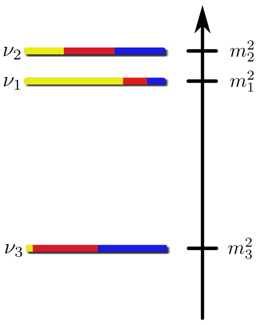
- **Shell Model (SM)**: Nucleons arranged in shells, outer shell(s) studied most closely
- **Quasiparticle random phase approximation (QRPA)**: Hartree-Fock approximation plus collective excitations
- **Energy density functional (EDF)**: Mean field approach (like QRPA) but with additional support for large corrections away from mean field behavior
- **Interacting boson model (IBM)**: Groups nucleons into bosonic pairs to lower effective degrees of freedom
- Subvariants of each model (e.g. density functional used in EDF)

nEXO Planned Sensitivity



Normal

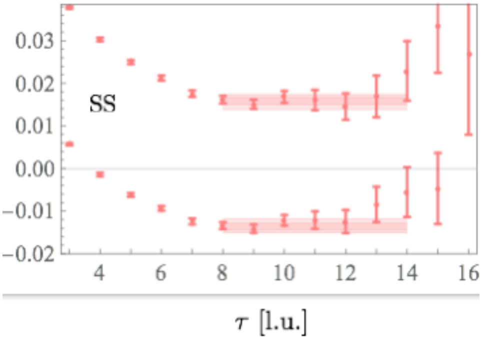


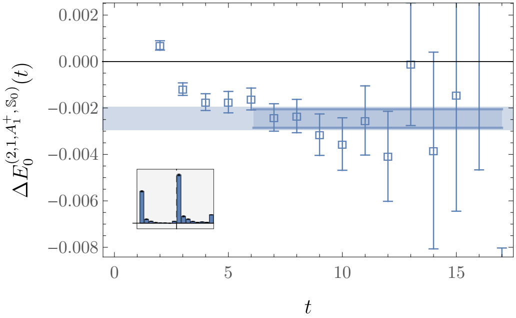


Inverted

Figure credits: Adapted from nEXO (J. Phys. G 49, 015104 (2106.16243)); KamLAND-Zen (2406.11438); Kismalac, Wikimedia Commons

NN Controversy ($m_\pi = 800$ MeV)





NPLQCD, asymmetric (1706.06550)

NPLQCD, variational (2108.10835)

- Energy shift of *NN* state → *a*, *r*
- (At least) one of these is false plateau from excited states

EFT Matching

- NN scattering approximated by effective range expansion (ERE)

$$\mathcal{M} = \frac{4\pi}{m_N} \frac{1}{p\cot\delta - ip}$$

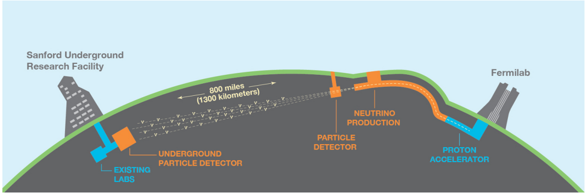
$$p\cot\delta = -\frac{1}{a} + \frac{1}{2} r p^2 + \cdots$$

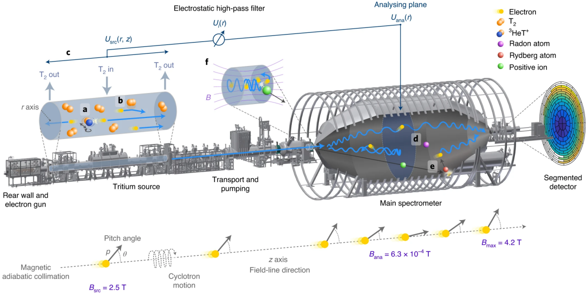
- Relates $\mathcal{A}^{0\nu}$ for $nn \to pp$ to EFT coefficient

$$\frac{\mathcal{A}^{0\nu}}{2m_{nn}} \frac{1}{\mathcal{R}(E)\mathcal{M}(E)^2} = (1 + 3g_A^2)(J^\infty + \delta J^V) - \frac{m_n^2}{8\pi^2} \tilde{g}_\nu^{NN}$$

  - $\mathcal{R}(E)$ = Lellouch-Lüscher residue (known function)
  - δJ^V = FV correction

Complementary Experiments





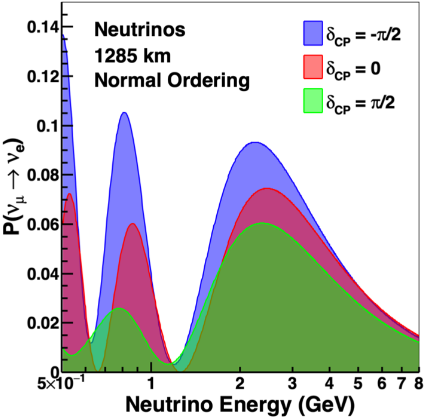


Image credits: Fermilab; DUNE (EPJC 80, 978 (2020), 2006.16043); KATRIN, Wikimedia Commons

Impact on NMEs

- g_{NN}^{ν} induces short-range contribution to nuclear $0\nu\beta\beta$ potential
- Resultant contribution to nuclear matrix elements
- Can be estimated with many-body methods (e.g. quantum Monte Carlo)
- With $g_{NN} \approx -1$ fm^2, increases $|\mathcal{M}^{0\nu}|$ by 25–40% (Weiss et al., PRC 106, 065501 (2112.08146))

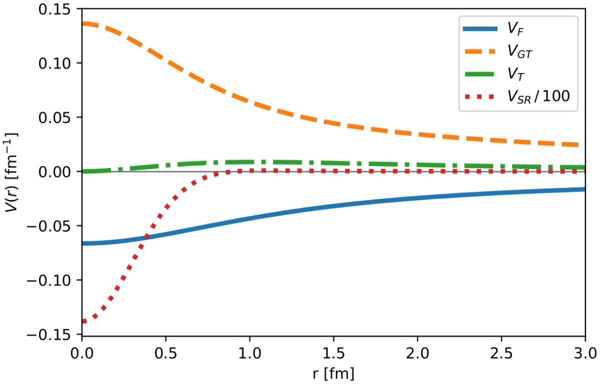


Figure credit: 2112.08146

Short- and Long-Distance Mechanisms

- Standard $0\nu\beta\beta$ paradigm: Two weak currents with light Majorana neutrino

$$(\bar{d}P_L\gamma_\mu u)(x)S_\nu(x-y)(\bar{d}P_L\gamma^\mu u)(y)$$

  - Intermediate neutrino propagates across nuclear scales
  - All operators fully determined by SM
- Some BSM theories predict additional high-energy interactions
- Effective dimension-9 contact interactions (Cirigliano et al., PPNP 112, 103771 (2003.08493))

$$\left(\bar{d}\Gamma_i u\right)\left(\bar{d}\Gamma_j u\right)$$

- Relative sizes of operators (for different i, j) model dependent
- NB: Contact interaction at scale of quarks/gluons
  - Distinct from short-distance effective operator in nuclear EFT

Dimension-9 $0\nu\beta\beta$ Operators in χEFT

- In Weinberg power counting, dominant effect of short-distance term is through $\pi\pi ee$ interaction
- Can extract coefficient from $\pi^- \to \pi^+ ee$
- Only scalar operators contribute
  - Vector operators suppressed by m_e/F_π
- NB: Inconsistencies with Weinberg power counting
- Calculated by CalLat (Nicholson et al., PRL 121, 172501 (1805.02634)) and NPLQCD (Detmold et al., PRD 107, 094501 (2208.05322))

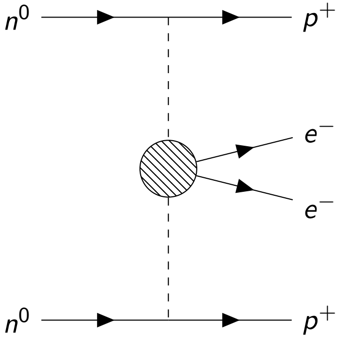


Figure credit: 2208.05322

Neutrinoful Double-Beta Decay ($2\nu\beta\beta$)

- Rarest observed Standard Model process
- Experimental data used as inputs or tests of nuclear models of $0\nu\beta\beta$ (Engel, Menéndez, RPP 80, 046301 (1610.06548))
- Computed for $nn \to pp$ transition from lattice QCD (Shanahan et al., PRL 119, 062003 (1701.03456); Tiburzi et al., PRD 96, 054505 (1702.02929))
  - Single lattice spacing and volume
- No intermediate ν prop – weak currents decouple
  - Background field method – quark propagators computed in presence of uniform weak field (Fucito et al., PLB 115, 148; Martinelli et al., PLB 116, 434; Bernard et al., PRL 49, 1076)

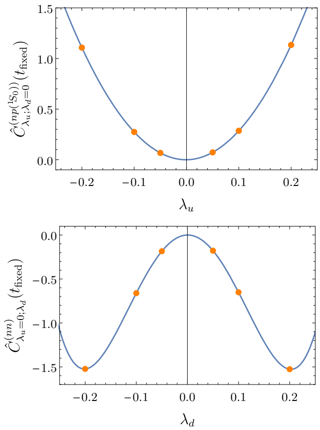


Figure credit: 1702.02929

Neutrinoful Double-Beta Decay ($2\nu\beta\beta$)

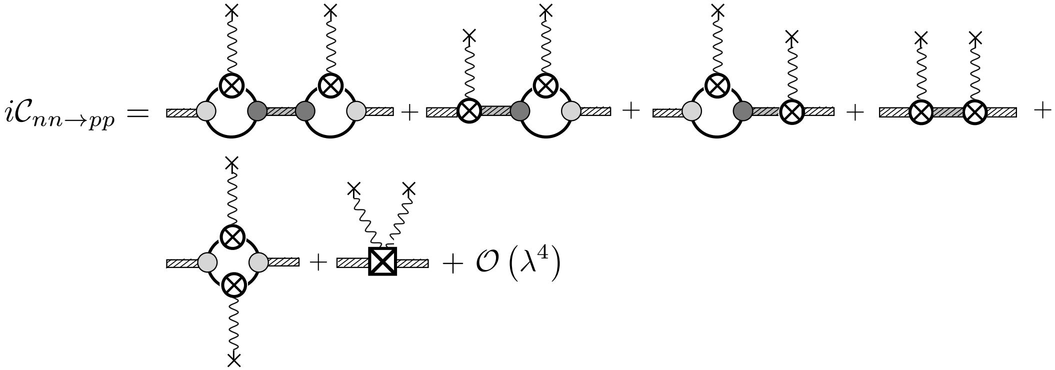


Figure credit: Tiburzi et al., PRD 96, 054505 (1702.02929)

Neutrinoful Double-Beta Decay ($2\nu\beta\beta$)

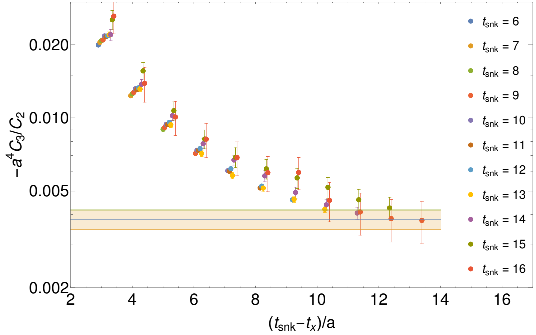
- Can write full decay amplitude as single-current pieces and two-current LEC $\mathbb{H}_{2,S}$

$$M_{nn\to pp} = \underbrace{\frac{M g_A^2}{4\gamma_S^2} - \frac{|M_{pp\to d}|^2}{\Delta}}_{\text{single-}\beta\text{ pieces}} - \underbrace{\mathbb{H}_{2,S}}_{\text{counterterm}}$$

- Computed as $\mathbb{H}_{2,S} = 4.7(2.2)$ fm
- $\mathbb{H}_{2,S}$ is about 5% correction to full amplitude
  - NLO contribution in $2\nu\beta\beta$
  - $0\nu\beta\beta$ equivalent is LO – $O(1)$ correction!

Dimension-9 $0\nu\beta\beta$ Coefficients in $nn \to pp$

- Power counting different in $nn \to pp$ versus $\pi^- \to \pi^+$
- Vector operators, $\mathcal{O}_3$ no longer suppressed
- Need to measure all nine operators to constrain BSM models
- Renormalization is still in progress



Preliminary (unrenormalized) extraction of $\mathcal{O}_3^{nn\to pp}$ (Davoudi, **AVG**, et al., unpublished)