

Prospects for lattice field theory beyond the standard model



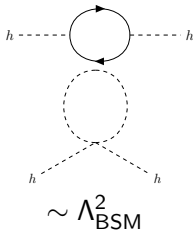
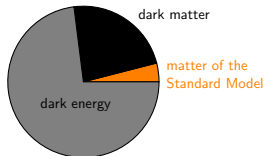
Georg Bergner
Friedrich Schiller University Jena



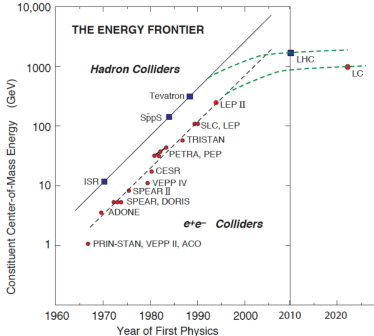
Lattice 2024: Aug 3, 2024

Shortcomings of the standard model

- missing pieces of the standard model
 - explanation for dark matter/dark energy
 - gravitational interactions
 - neutrino masses . . .
- aesthetic imperfection of the standard model
 - parametrization or explanation
 - hierarchy problem
 - missing unification or symmetry
 - flavor hierarchy
- missing understanding
 - confinement mechanism
 - quantum gravity
 - gauge/gravity duality



What can be expected from colliders?



[J. Nash, talk at 2010 IOP meeting]



- speed is slowing down and will stop eventually
- but there might be an oasis just a few miles away
- ... or it just means a free way to quantum gravity

Where is new physics hiding?

Our current BSM collider discoveries are constraints.

Directions for new observations:

- energy
- precision
- cosmology
- from on-shell to off-shell discoveries

Solution to SM problems without constraint violation:

- small coupling to the Standard Model
- large suppression of $\frac{1}{\Lambda_{BSM}}$
- large gap to new physics

Important contributions required from the lattice

- precision input for QCD contributions:
 - new routes towards detection
 - QCD-corrections to observed anomalies

→ not covered in this talk
- test of BSM strong interactions, goals at different levels
 - test specific models to add constraints – like an experiment
 - fundamental concepts behind the suggested models
 - theory space of strong interactions
 - relevant for model building and our general understanding

interplay with conceptual questions about strong interactions

BSM concept: Supersymmetry

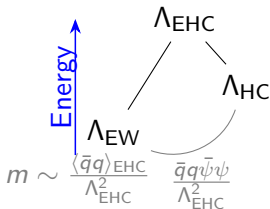
- original motivation for SUSY BSM: hierarchy problem
solution by symmetry: boson-fermion cancellation
- needs unknown (non-perturbative) breaking mechanism
- heavily investigated regarding BSM and collider signatures (MSSM etc.)
- dark matter: standard WIMP scenario
- general concepts with many applications: gauge/gravity duality etc.

Shifted perspective of SUSY applications:

supersymmetric gauge theories \leftrightarrow analytic predictions, dualities

BSM concept: Composite Higgs/ technicolor

- solve hierarchy problem by new strong interactions [S. Weinberg (1976)]
- naturally leads bound states as possible dark matter
- generate fermion masses from effective operators of extended hyper color theory
- constraints:
 - need bound state Higgs much lighter than other bound states
 - FCNC need to be suppressed
 - electroweak precision data



The diagram shows an energy scale on the vertical axis. At the top is the scale Λ_{EHC} . Below it are two scales: Λ_{EW} and Λ_{HC} . Λ_{EW} is connected to Λ_{EHC} by a straight line, and Λ_{HC} is connected to Λ_{EHC} by a curved line. Below Λ_{EW} is the operator $\frac{(\bar{q}q)_{\text{EHC}}}{\Lambda_{\text{EHC}}^2}$. Below Λ_{HC} is the operator $\frac{\bar{q}q\bar{\psi}\psi}{\Lambda_{\text{EHC}}^2}$. The mass m is indicated to be proportional to these operators.

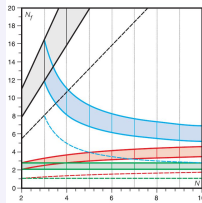
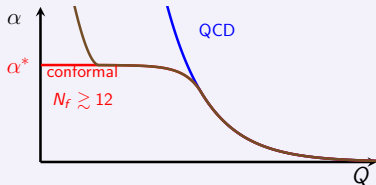
$$m \sim \frac{(\bar{q}q)_{\text{EHC}}}{\Lambda_{\text{EHC}}^2} \quad \frac{\bar{q}q\bar{\psi}\psi}{\Lambda_{\text{EHC}}^2}$$

BSM concept: Composite Higgs/ technicolor scenarios

Walking scenario

$$\beta(g) = -\frac{1}{(4\pi)^2} \left(\frac{11N_c}{3} - \frac{4}{3} T_R N_f \right) g^2 - \beta_1 g^5 + \dots$$

- conformal window: $N_{f,L} < N_f < N_{f,U}$ [T. Banks, A. Zaks (1981)]
- near conformal running enhances condensate over FCNC
 $\langle \bar{q}q \rangle_{\text{EHC}} \sim \left(\frac{\Lambda_{\text{EHC}}}{\Lambda_{\text{HC}}} \right)^{\gamma_m} \langle \bar{q}q \rangle_{\text{HC}}$, γ_m large [T. Appelquist et al. (1987), Yamawaki et al. (1986)]
- light scalar, possibly dilaton-like



[Dietrich, Sannino (2007)]

BSM concept: Composite Higgs scenarios

Walking scenario

Higgs from scalar pNGb

- spontaneous symmetry breaking generates light Higgs
[D. B. Kaplan, H. Georgi (1984)]
- requirements for chiral symmetry group,
e. g. by $SU(2N_f) \rightarrow Sp(2N_f)$ (pseudoreal)
- HC chiral symmetry breaking preserves $G_{SM} = SU(2)_L \times U(1)$
- scalar field generated at Λ_{HC} gets lower scale vev. v (EW breaking scale)

BSM concept: Composite Higgs scenarios

Walking scenario

Higgs from scalar pNGb

Partial compositeness [Kaplan (1991)]

- need to generate vastly different fermion masses
- large top mass generated by coupling with fermion bound state $\lambda\bar{\psi}B$
- UV theories classified according to symmetries [G. Ferretti, D. Karateev (2013)]
- 3 fermion operator (B) needs enhancement: combine with walking? [V. Ayyar et al. (2018), A. Hasenfratz et al. (2023)]

BSM concept: Strongly interacting dark matter

bound state of strong interactions: glueball, meson, baryon, gluino-gluon...

- dark matter needs to be stable, but can cover large mass range
- not visible by current direct detection
- astronomical observations limit self interaction, hints for small non-vanishing self interactions
- relic abundance, $\rho_{\text{DM}} = 5\rho_{\text{baryon}}$:
 - thermal freeze-out dark matter, e. g. WIMP and SIMP
 - SIMP: $3 \rightarrow 2$ process by Wess-Zumino-Witten term
[Y. Hochberg et al. (2014)]
 - asymmetric dark matter
 - ...
- first order transitions lead to gravitational wave signatures in early universe

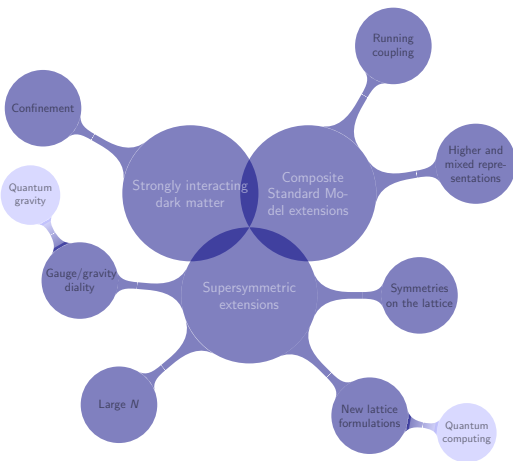
Other BSM concepts

Not covered in detail, though investigated on the lattice

- symmetric mass generation
(see talk by David Tong (Wed), S. Catterall (Fri))
- scalar gauge theories
(talk by G. Catumba (Fri) on 2HDM, poster by E. Carstensen)
- QCD Axions, closely related to QCD-vacuum structure
(talk by J. J. Hernández (Mo), many related talks)
- Hosotani mechanism
- extra dimensions
- quantum gravity
- ...

Beyond model building: BSM to extend our understanding

- standard model extension introduce new concepts
- challenge our knowledge of strong interactions
- provide new analytic approaches due to symmetries/properties of BSM strong interactions



Topics considered in this talk

- ① composite dark matter
SU(4) (hyper) stealth dark matter
SU(2), Sp(4) $N_f = 2$ as SIMP candidate
- ② composite Higgs and walking/conformal behavior
SU(2) $N_f = 2$
Sp(4) $N_f = 2 + N_{f,AS} = 3$ antisymmetric: partial compositeness
SU(3) $N_f = 8$ near conformal and SMG
SU(2) $N_f = 1$ adjoint QCD
- ③ progress of supersymmetric theories
supersymmetric gauge theories in 4d
gauge-gravity duality in lower dimensions

Hiding dark matter from direct detection

Some basic considerations:

- stable dark matter bound state:
⇒ baryonic bound state
- neutrality by confinement: larger interactions at higher energies in early universe
- avoid leading electromagnetic interactions:
⇒ scalar baryons (even $N_c > 2$)
- meson decay, but no conflict with ew precision data:
⇒ small Higgs coupling
- coupling to the standard model only by polarizability and Higgs boson exchange highly suppressed

Hidden, but still close to detection.



Stealth dark matter models

Stealth dark matter: $SU(4)$, $N_f = 4$ [T. Appelquist et al. [LSD] (2015)]

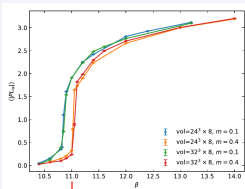
- direct detection limits: em polarizability [T. Appelquist et al. (2015)] and Higgs coupling [T. Appelquist et al. (2014)]
- $M_{DM} > 0.2$ TeV
- bound state masses [T. Appelquist et al. (2014)] and towards baryon scattering [R. C. Brower et al.2023]
- gravitational wave signatures from phase transition [R. C. Brower et al.2021]
- important difficult challenge: self interactions

Hyper stealth dark matter [V. Ayyar [LSD] arXiv:2402.07362]

- $SU(4)$ $N_f = 1$ light (plus heavy flavors)
- no light pNGbs, less detection constraints
- light flavor with possible tiny electroweak coupling (η' decay)
- lowest possible mass in this framework: $M_{DM} \sim$ few GeV

Gravitational wave signals from first order transition

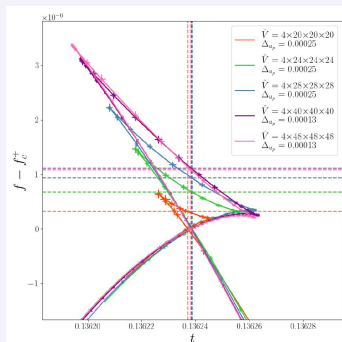
Phase transitions of hyper stealth dark matter [Talk by S. Park (Fri)]



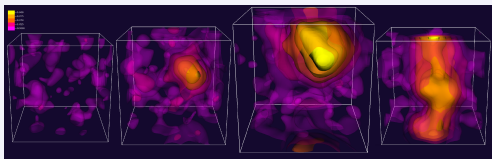
Domain-Wall fermions, signature of 1st order confinement trans.

Density of states in finite temperature $Sp(4)$

[Talk D. Mason (Wed)]



Resolving the critical bubble in Yang-Mills theory [poster R. Seppä, talk D. Weir (Mon)]



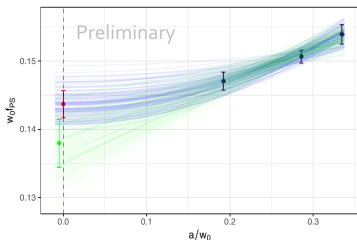
SU(2) gauge theory, dark matter and composite Higgs

SU(2) gauge theory with $N_f = 2$ fundamental flavors:

- dark matter with SIMP scenario
- composite Higgs theory: scalar pNGBs
- can be extended to walking technicolor by adding adjoint fermions [T. A. Ryttov, F. Sannino (2008)] [GB, S. Piemonte (2021)]
- lattice studies:
 - particle spectrum
 - scattering and self interactions
 - . . .

Talk by L. S. Bowes and S. Martins (Fri):

- exp. clover Fermion action
- target smaller fermion masses for singlet meson decay: $m_\sigma < 2m_{ps}$



Summary: [R. Arthur et al. (2016)]

Sp(4) gauge theories as composite Higgs and dark matter

- $SU(2N_f) \rightarrow Sp(2N_f)$ chiral symmetry breaking for $Sp(2N_c)$ theory: scalar pNGbs
- $SU(2) = Sp(2)$, $N_f = 2$ simplest example with Wess-Zumino-Witten term (SIMP dark matter)
- $Sp(4)$, $N_f = 2$ next complicated choice
- $N_f = 2 + N_{f,AS} = 3$ allows for partial compositeness
- larger N_c might provide a larger parameter range for SIMP

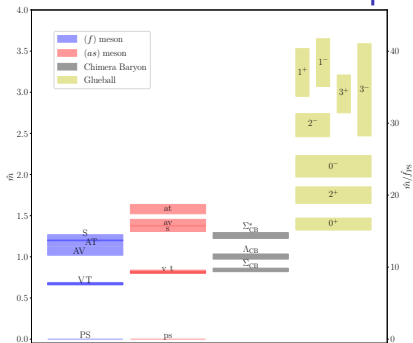
[M. Hansen et al. (2016)]

Lattice investigations of $Sp(4)$

Recent investigations

- scan of theory space: quenched meson and glueball spectroscopy in the large N_c limit: $Sp(2N_c)$ for $N_c = 2, 3, 4$
[E. Bennett et al. (2023)]
- analysis of the top partner (chimera baryon)
- dynamical $N_f = 2 + N_{f,AS} = 3$ spectroscopy
- $N_f = 2$ dark matter self interaction (meson scattering) provides lower bound of dark pion mass: 100 MeV
[Y. Dengler et al. (2024)]
- important missing information for partial compositeness: anomalous dimension of chimera baryon (see $SU(4)$ partial compositeness model [V. Ayyar et al. (2018), A. Hasenfratz et al. (2023)])

Particle spectrum of Sp(4)

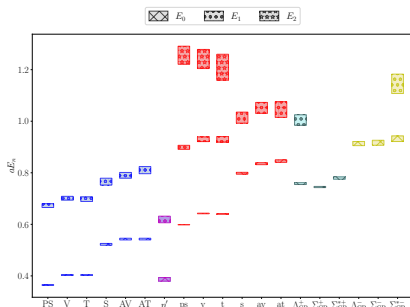


[E. Bennett et al. Phys.Rev.D 109 (2024)]

massless continuum limit from
quenched simulations
gradient flow units

Spectroscopy using spectral densities (see Talk by H. Hsiao Fri)

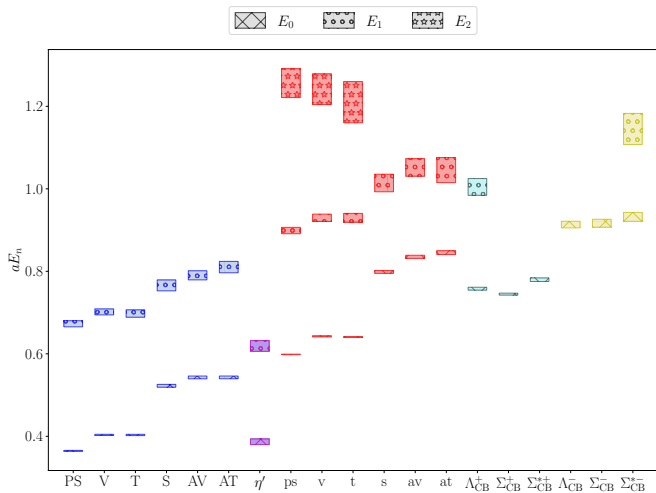
Further results: flavor singlet mesons (Talk by F. Zierler, Tue)



[E. Bennett et al. (2024)]

single ensemble, in lattice units

Particle spectrum of $Sp(4)$



Considerations of walking scenario and conformal window

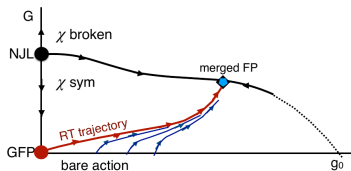
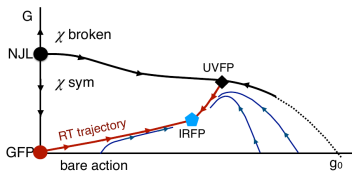
- landscape of gauge theories with gauge group G , N_f fermions possible in multiple representations
- conjectured IR behavior has to be confirmed with lattice data
- methods:
 - scaling of particle spectrum
 - Schrödinger functional
 - gradient flow
 - mode number
 - ...
- conformal: scaling of masses $M_H \sim m^{1/\gamma_m}$
- requires also control of phase transition, topology, etc.
- recent new methods: gradient flow [A.Carosso et al. (2018), C. T. Peterson et al. (2021)], Pauli-Villars fields

SU(3) with $N_f = 8$ possible edge of conformal window

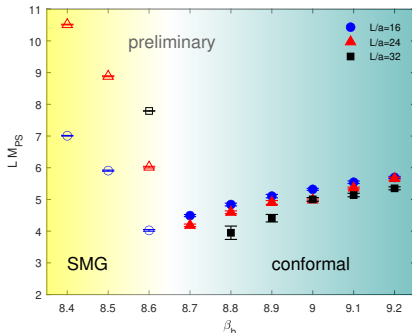
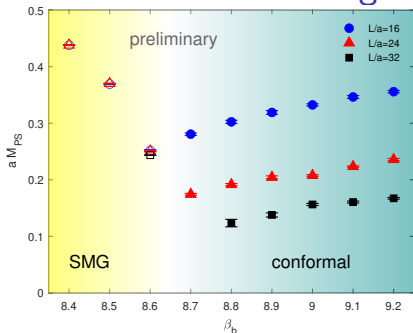
Indications for a rich phase space with staggered fermions [A. Hasenfratz

PRD 106 (2022)]

- no clear signal from running coupling, but observation of two phases, separated by continuous phase transition
- weak coupling phase: looks conformal
- strong coupling: SMG chiral symmetric, confining
- Pauli-Villars fields to avoid bulk transitions



SU(3) with $N_f = 8$ walking theory and symmetric mass generation



- SMG phase: mass gap independent of volume
- conformal phase: mass scaling given by the volume
- more details need to be explored

see talks by A. Hasenfratz, O. Witzel

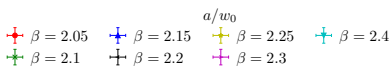
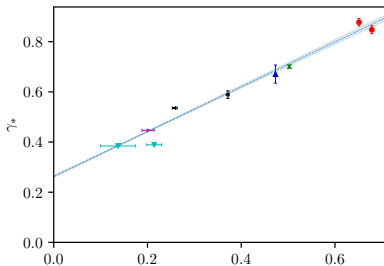
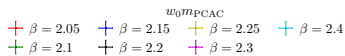
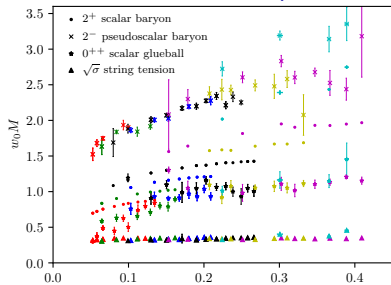
Conformal window for fermions in the adjoint representation

- hard to determine conformal window: limited range of scales on the lattice, mass deformations
- SU(3) N_f fundamental: large number of investigations
- motivation for considering adjoint rep.: lower $N_{f,L}$ might be better for pheno.
- adjoint representation: $N_f = 2$ investigated as MWT seems to be inside conformal window [S. Catterall, F. Sannino (2007), L. Del Debbio et al. (2010)...] like also $N_f = 3/2$ [GB et al. (2018)]

$N_f = 1$ very challenging, new data [A. Athenodorou et al. (2024)]:

- large volumes with Wilson fermions
- large coupling range
- smallest possible masses

$N_f = 1$ adjoint QCD



- conformal-like: light scalar
- neither agreement with conformal scaling, nor with chiral perturbation theory
- γ^* from mode number shows β dependence

poster by E. Bennett: improved strategies for further investigations

Tensions and systematics

EFT powerful framework, how to extend to walking case?

- recent new investigations of relations between different approaches [M. Golterman, Y. Shamir (2024)]
- dilaton EFT and soft theorems: R. Zwicky (Tue)

Other general questions:

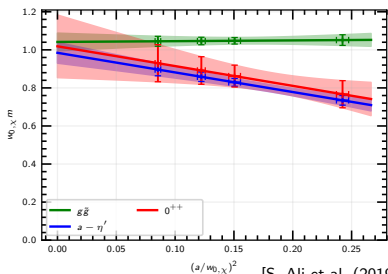
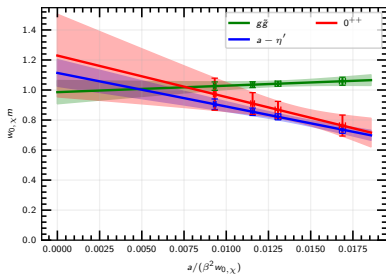
- How to think about lattice artefacts and scaling corrections?
- What kind of additional fixed points might be induced?
- Why is there such a light scalar in the theories with adjoint fermions?
- How large can anomalous dimensions be in practice?

Supersymmetry on the lattice

Supersymmetry on the lattice can only be fine-tuned or accidental

- short explanation: $\{Q, \bar{Q}\} \sim \gamma^\mu P_\mu$, but space-time symmetries are broken
- long explanation: conflict between locality and symmetry (like chiral symmetry), but no Ginsparg-Wilson solution
- accidental:
 - $\mathcal{N} = 1$ supersymmetric Yang-Mills theory:
chiral symmetry \leftrightarrow SUSY
 - $\mathcal{N} = 4$ supersymmetric Yang-Mills theory:
subset of SUSY \leftrightarrow full SUSY
- lower dimensional theories allow alternative solutions

Supersymmetric gauge theories in four dimensions



[S. Ali et al. (2019)]

- $\mathcal{N} = 1$ supersymmetric Yang-Mills theory: SUSY recovered on the lattice
- non-perturbative properties like phase transitions and condensate determined on the lattice
- towards SQCD (tuning of $O(10)$ counterterms): talk by H. Herodotou (Tue)

New twists to $\mathcal{N} = 1$ supersymmetric Yang-Mills theory

Gluino condensate in SYM:

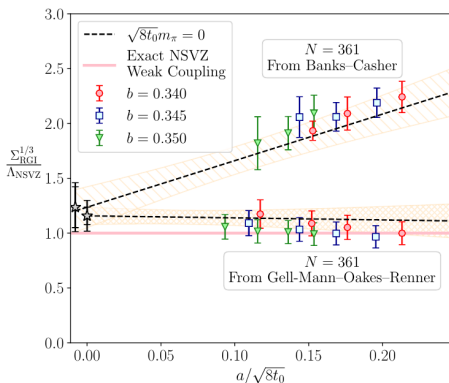
$$\Sigma = \begin{cases} 2e\Lambda^3/N & \text{strong coupling [G. C. Rossi, G. Veneziano (1984)]} \\ \Lambda^3 & \text{weak coupling [V. A. Novikov et al. (1984)]} \\ N\Lambda^3 & \text{twisted bc [M. Anber, E. Poppitz (2023)]} \end{cases}$$

- analytically calculable, but three different results are obtained
- previous studies of the condensate: Domain-Wall fermions: [J. Giedt et al. (2009)]; overlap and gradient flow [GB et al.(2019)][S. Piemonte, GB (2020)]

$\mathcal{N} = 1$ SYM on the lattice: the chiral condensate

- new simulations based on twisted Eguchi-Kawai:
lattice formulation of large N SYM with Wilson fermions

[A. Gonzalez-Arroyo, M. Okawa (2013)]



Methods:

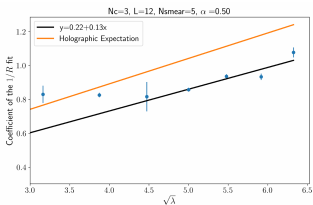
- 1) mode number from lowest eigenmodes of Wilson-Dirac
- 2) GMOR relation

N_c dependence as predicted by weak coupling result [C. Bonanno et al. (2024)]

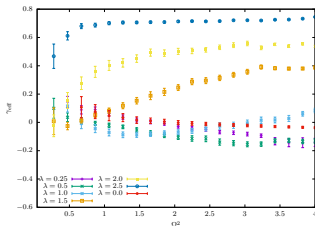
Talk by C. Bonanno (Mon)

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory

- combining the space-time and R symmetries to preserve one SUSY on the lattice, recover 15 SUSY in continuum limit
- main obstacle has been overcome: simulations have reached larger couplings [S. Catterall et al. (2023)]
- promising first data indicates correctness of the method, waiting for coming investigations
- interesting: RG flow with conformal theory



[S. Catterall et al. (2023)]

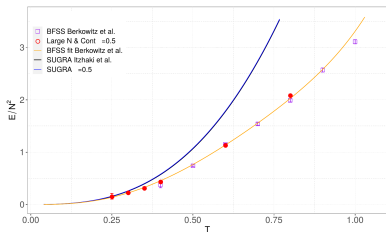


[GB, D. Schaich (2021)]

Review: [S. Catterall et al. (2016), D. Schaich (2018)]

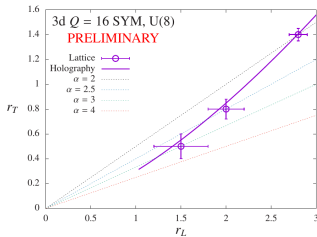
Supersymmetry and gauge/gravity duality

- towards quantum gravity on the lattice
- evidence for gauge/gravity duality conjectures
- extend beyond large N_c strong coupling limit



[S. Pateloudis et al. [MCSMC] (2023)]

Poster by D. Schaich:
Maximal SUSY in 3d



$r_L^2 \times r_T$ torus, critical T_c

Conclusions

- long standing problem, how to fix the standard model might still require new solutions
- like collider experiment: quite mature developments for BSM strong interactions can be further constrained by lattice simulations
- after years of investigations: still surprised and puzzles in the landscape of strongly interacting theories
- BSM leads to new perspectives on strong interactions
- interesting interplay to theoretical developments like gauge/gravity duality

*Thanks for all the contributions, discussions, and input material.
Unfortunately not all could be covered in this talk.*