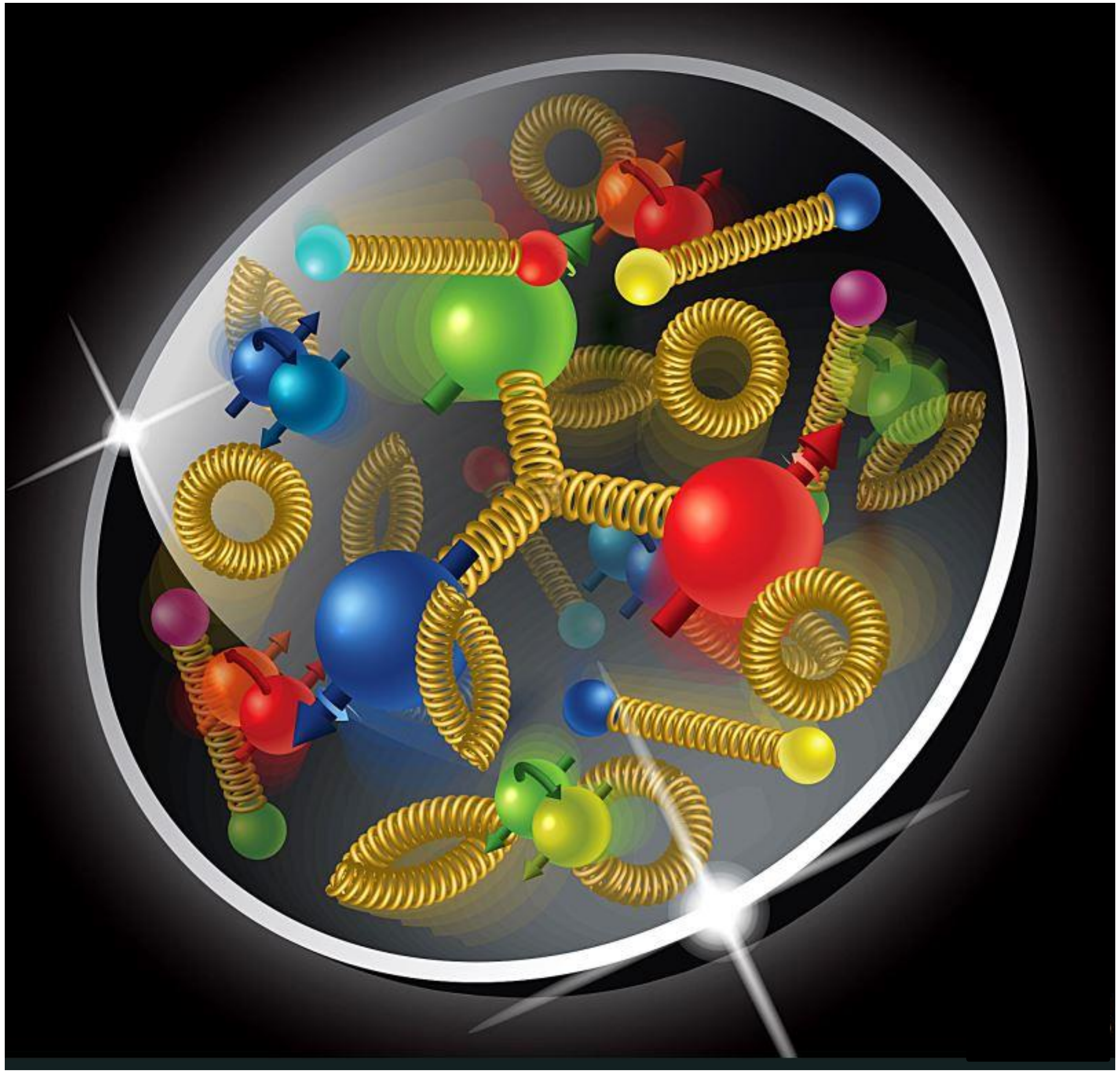
Hadron structure via Generalized Parton Distributions

Shohini Bhattacharya
Los Alamos National Laboratory
2 August 2024

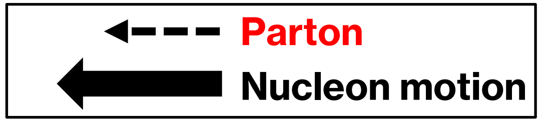
Introduction

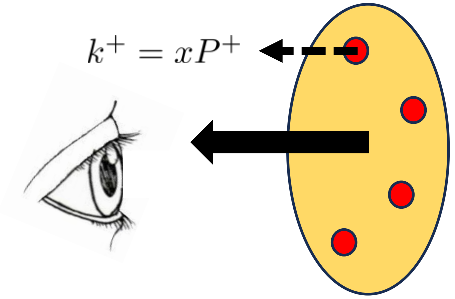


Unveil "cosmic" interior of nucleons

Nucleons are complicated dynamical systems of quarks & gluons (partons)

Non-perturbative functions in QCD

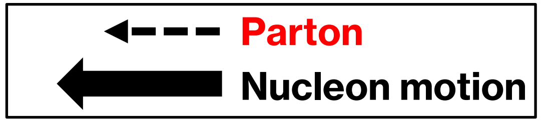


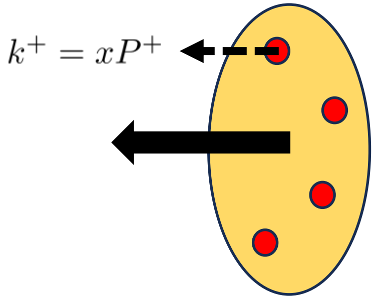


Parton Distribution Functions

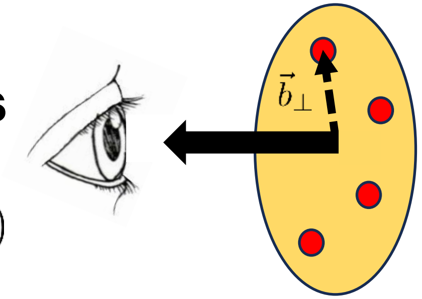
PDFs (x)

Non-perturbative functions in QCD





Form Factors



PDFs (x)

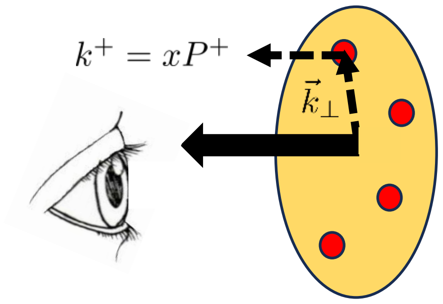
FFs (Δ)

Non-perturbative functions in QCD

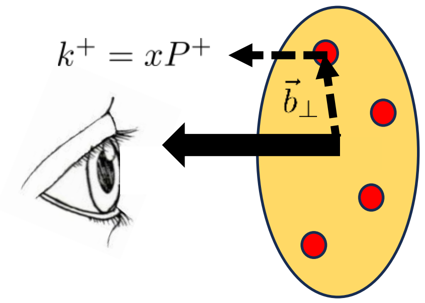
Parton

Nucleon motion

Transverse Momentum-dependent Distributions



Generalized Parton Distributions



TMDs $(x, \vec{k}_\perp)$

GPDs (x, Δ)

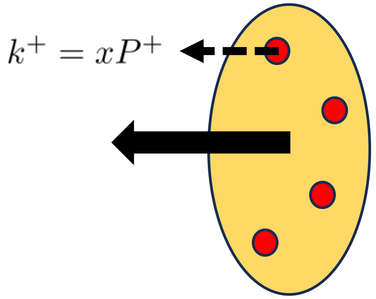
$\int d^2\vec{k}_\perp$

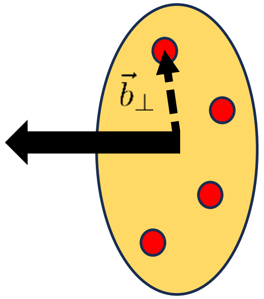
$\Delta = 0$

$\int dx$

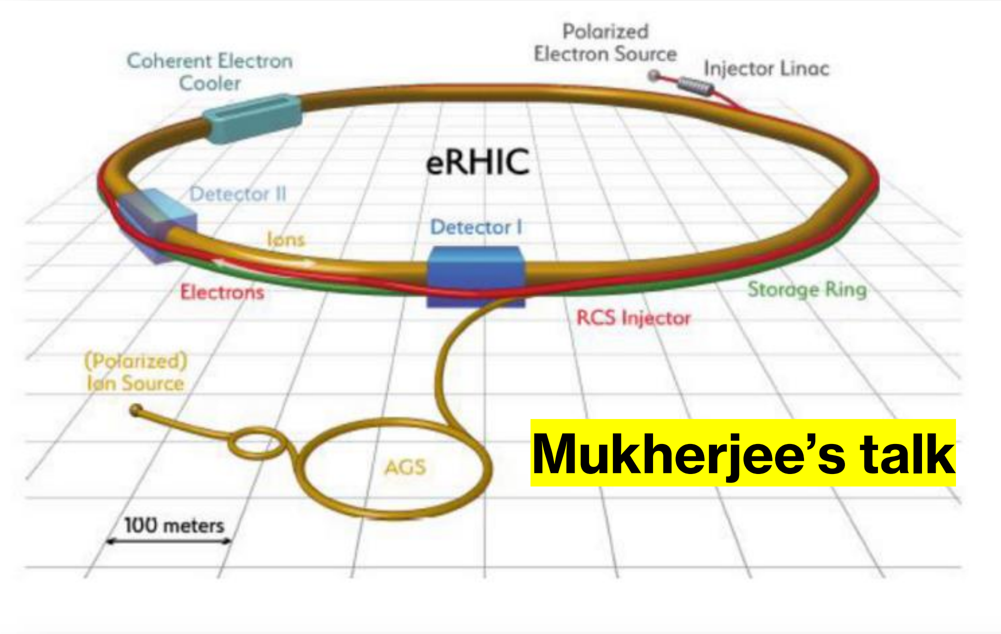
PDFs (x)

FFs (Δ)





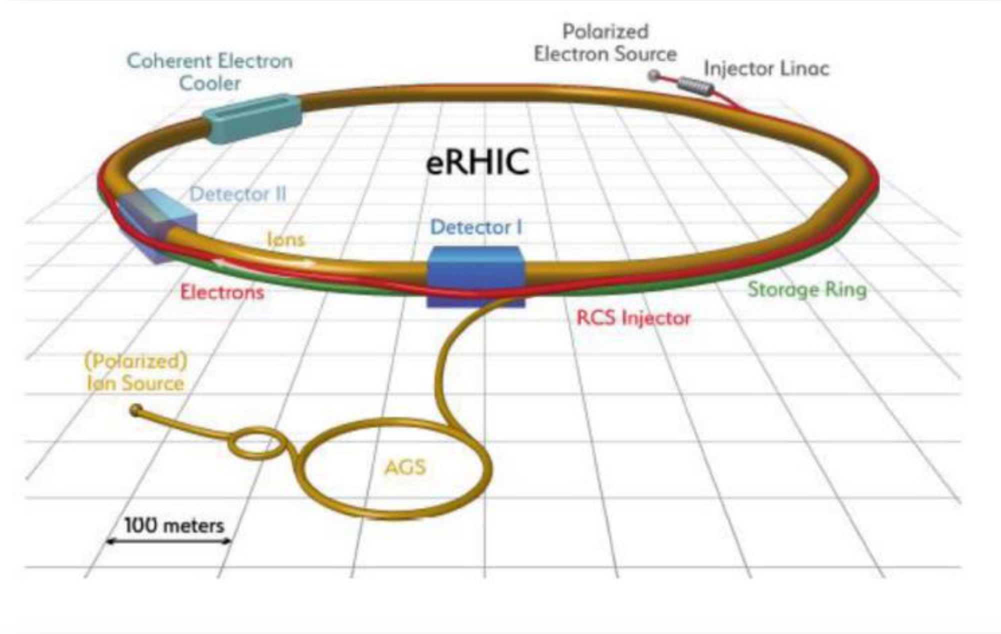
Electron-Ion Collider (EIC)



Efforts detailed in a decade worth of reports:

Nucleon tomography (mapping partonic distributions) is one of the major goals of the EIC

Electron-Ion Collider (EIC)



Efforts detailed in a decade worth of reports:

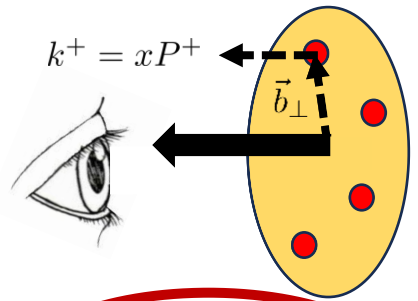
Nucleon tomography (mapping partonic distributions) is one of the major goals of the EIC

Lattice calculations can serve as a valuable complement to the ongoing efforts at the EIC

Outline

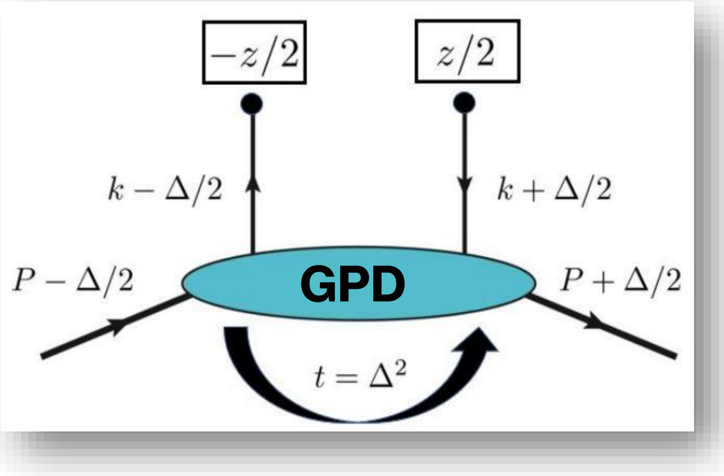
- **What are GPDs?**
- **Lattice results of GPDs**
- **Summary**

Generalized Parton Distributions



GPDs (x, Δ)

What are Generalized Parton Distributions?



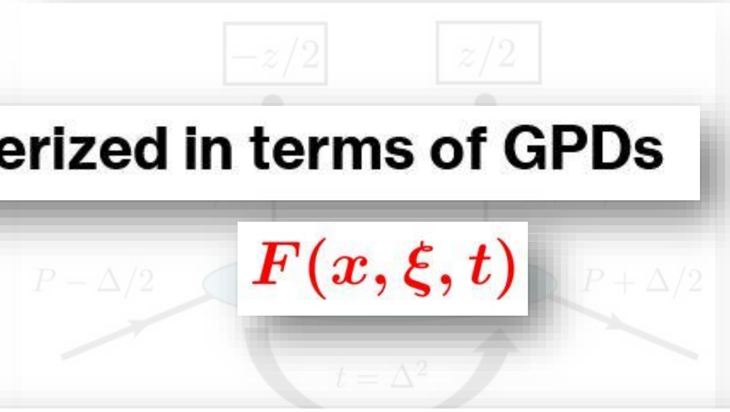
GPD correlator for quarks: Graphical representation

Definition of GPD correlator for quarks:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

What are Generalized Parton Distributions?

Correlator parameterized in terms of GPDs



$$F(x,\xi,t)$$

x : "average" longitudinal momentum fraction carried by parton

ξ : skewness parameter; longitudinal momentum transfer to nucleon

t : momentum transfer squared

Definition of (

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

What are Generalized Parton Distributions?

Example:

At twist 2 there are 8 GPDs

$F(x,\xi,t)$

Twist-2 GPDs

Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$i\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

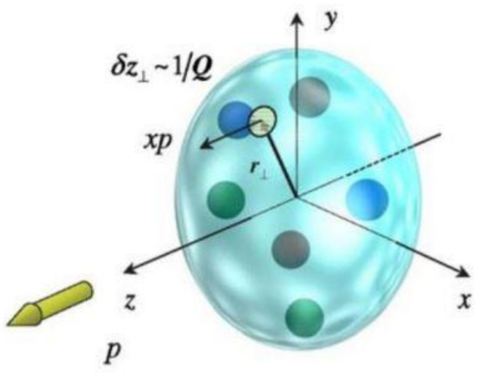
Motivation for studying GPDs

1) **3D imaging** (Burkardt, 0005108 ...)

IMPACT PARAMETER SPACE INTERPRETATION FOR GENERALIZED PARTON DISTRIBUTIONS

MATTHIAS BURKARDT*

Department of Physics, New Mexico State University
Las Cruces, New Mexico 88011, U.S.A. †



3D quark/gluon dist.

$$F(x, \xi = 0, \Delta_\perp) \xrightarrow{\mathcal{FT}} f(x, r_\perp)$$

Motivation for studying GPDs

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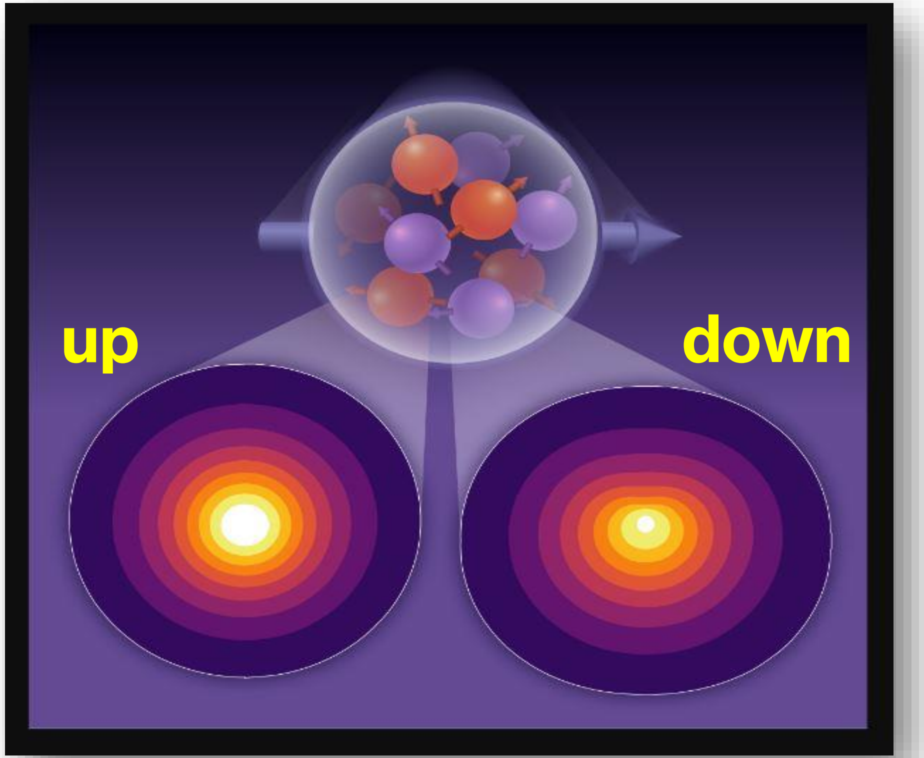
Lattice QCD results of impact-parameter distributions:



Differential distribution of up versus down quarks inside protons

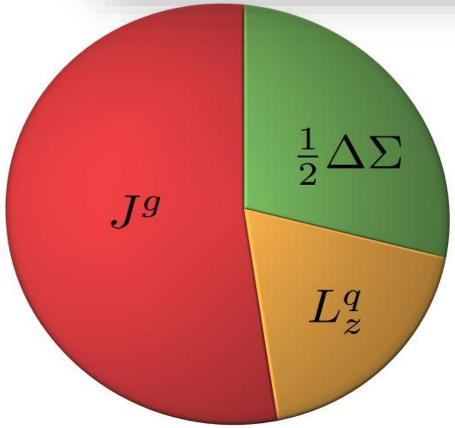
(Temple/BNL/ANL)

Motivation for studying GPDs

2) Spin sum rule & orbital angular momentum (Ji, 9603249)

GAUGE-INVARIANT DECOMPOSITION OF NUCLEON SPIN AND ITS SPIN-OFF *

Xiangdong Ji



$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma(\mu) + L_z^q(\mu)}_{J^q} + J^g(\mu)$$

Example:

$$\boxed{J^q = \int_{-1}^{1} dx\, x\big(H^q + E^q\big)\big|_{t=0}}$$

Motivation for studying GPDs

3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)

On "dual" parametrizations of generalized parton distributions

M.V. Polyakov[a,b], A.G. Shuvaev[a]

Energy Momentum Tensor (EMT) carries information about mechanical properties

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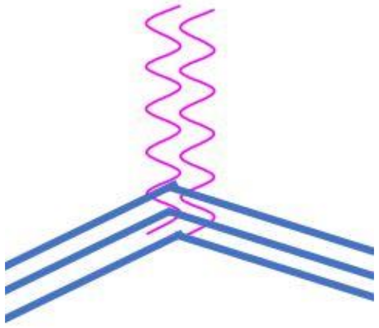

Gravitational Form Factors

Gravitational Form Factors:

$$\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)$$

Gravitational Form Factors characterize the EMT in the context of proton scattering with a graviton

Motivation for studying GPDs

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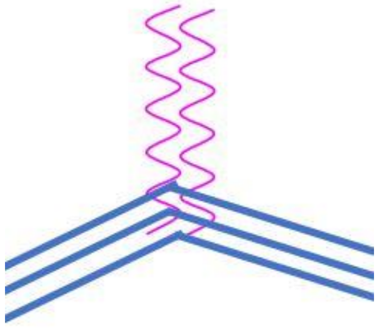
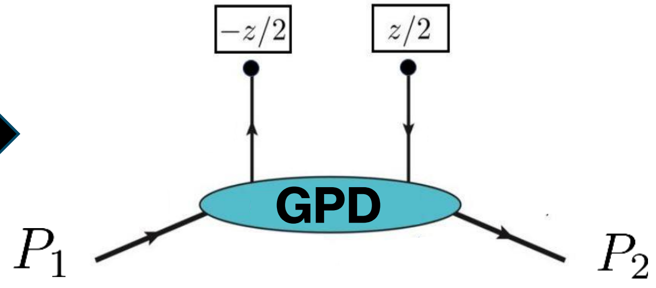
On "dual" parametrizations of generalized parton distributions

M.V. Polyakov[a,b], A.G. Shuvaev[a]

Explore mechanical properties of nucleons through connections between Gravitational Form Factors and GPDs



Gravitational Form Factors



$$A + \xi^2 D = \int_{-1}^{1} dx\, xH$$

$$B - \xi^2 D = \int_{-1}^{1} dx\, xE$$

Motivation for studying GPDs

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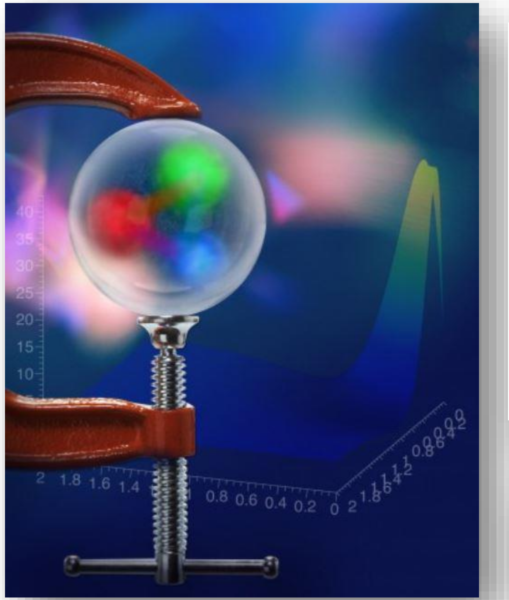


Courtesy: JLab media

LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

QUARKS FEEL THE PRESSURE IN THE PROTON

Motivation for studying GPDs

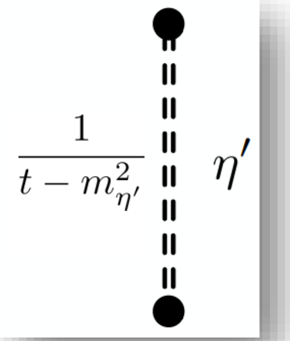
4) **Mass generations & chiral symmetry breaking**

(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)

Chiral and trace anomalies in Deeply Virtual Compton Scattering: QCD factorization and beyond

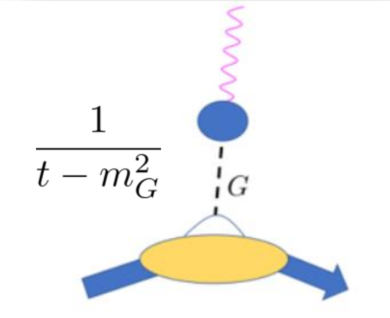
Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

Unraveled profound & previously undiscovered connections between chiral/trace anomalies & GPDs



Eta meson mass generation:

$$\boxed{\tilde{E}(x) \sim \frac{1}{t-m_{\eta'}^2}}$$



Glueball mass generation:

$$\boxed{H(x), E(x) \sim \frac{1}{t-m_G^2}}$$

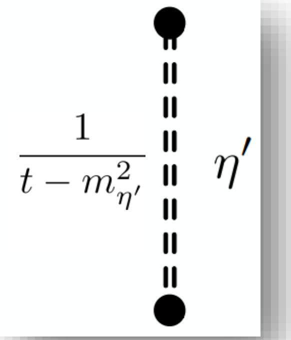
Motivation for studying GPDs

4) Mass generations & chiral symmetry breaking

(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)

Novel avenue of GPD research

Profound physical implication of anomaly poles:
Touches questions on mass generations, Chiral symmetry breaking, ...



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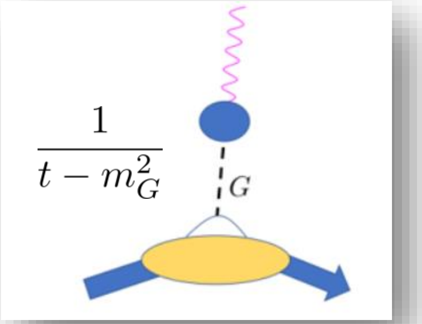

Glueball mass generation:

$$\boxed{H(x), E(x) \sim \frac{1}{t - m_G^2}}$$

Motivation for studying GPDs

4) Ma

Trace anomaly spatial distribution of the pion

B. Wang, et al, Phys.Rev.D 109 (2024) 9, 094504

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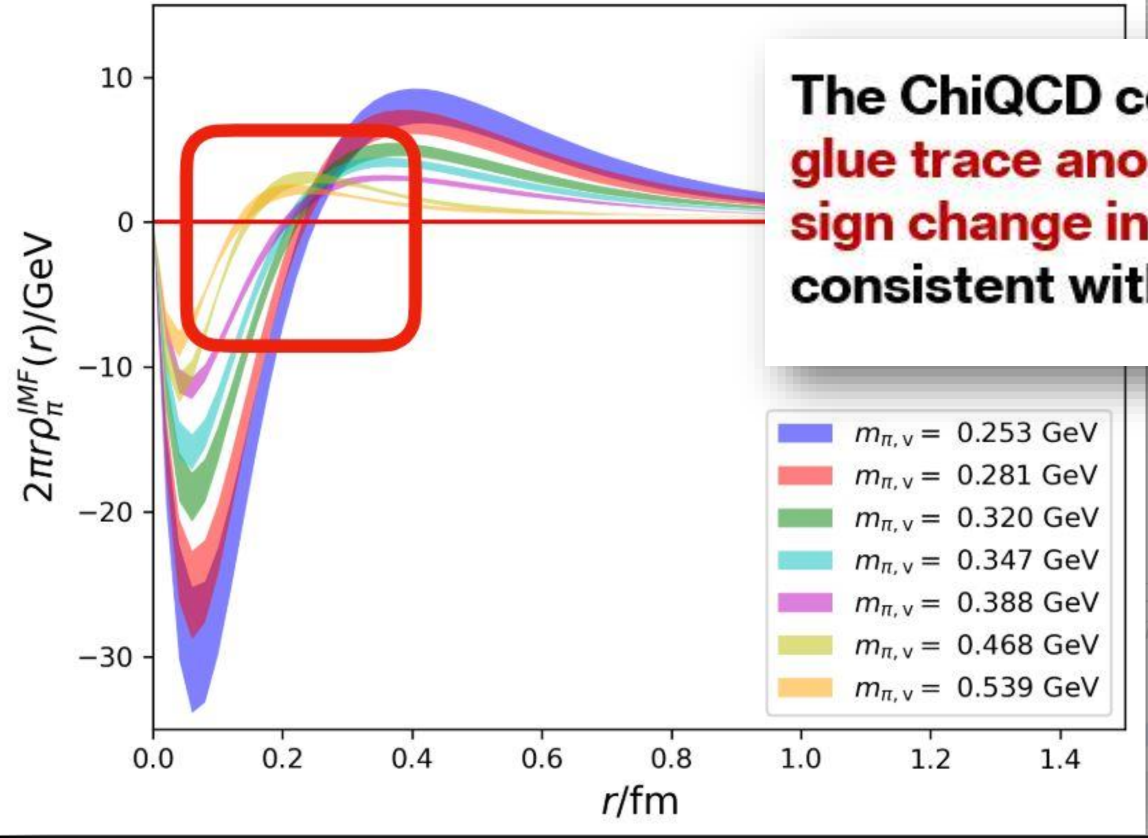


The ChiQCD collaboration finds that for the pion the **glue trace anomaly form factor** shows a **sign change in the small- r region**, consistent with predictions from chiral perturbation theory

Glueball mass generation:

$$H(x), E(x) \sim \frac{1}{t - m_G^2}$$

Motivation for studying GPDs

1) **3D imaging** (Burkardt, 0005108 ...)

2) **Spin sum rule & orbital angular momentum** (Ji, 9603249)

Example:

$$\boxed{J_q = \int_{-1}^{1} dx\, x(H_q + E_q)\big|_{t=0}}$$

We have numerous compelling reasons to engage in GPD studies!

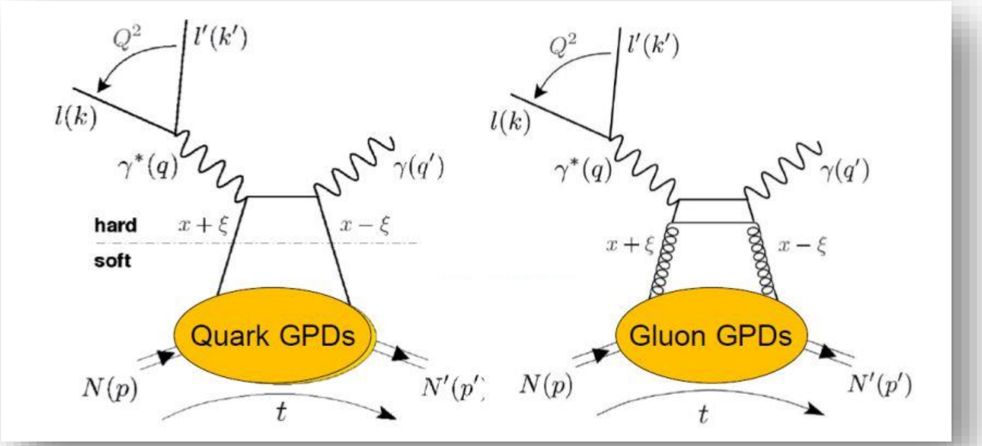
3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)

4) **Mass generations & chiral symmetry breaking**

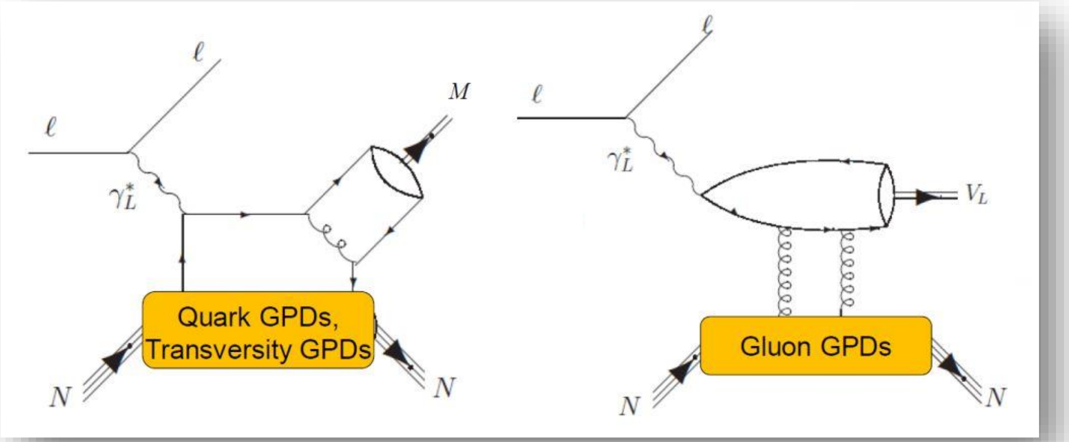
(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)

Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering



Deeply Virtual Meson Production

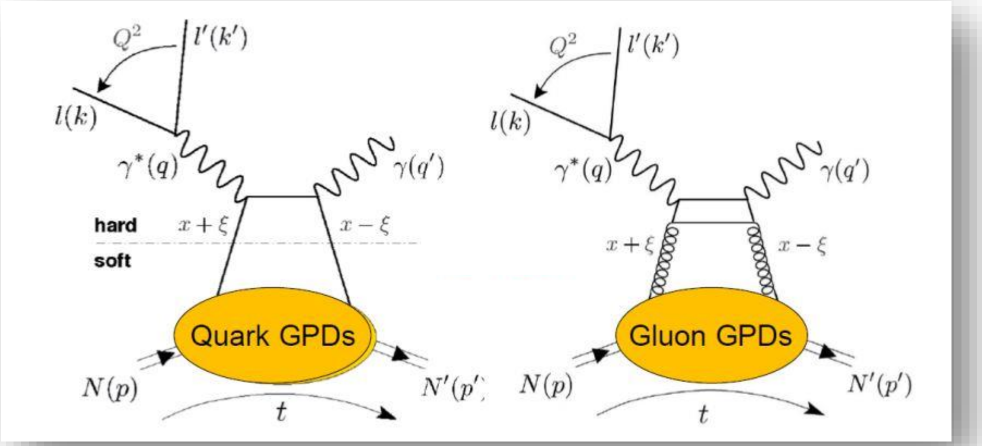


Courtesy: Hyon-Suk Jo, KPS Meeting

No access to x-dependence of GPDs

Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering



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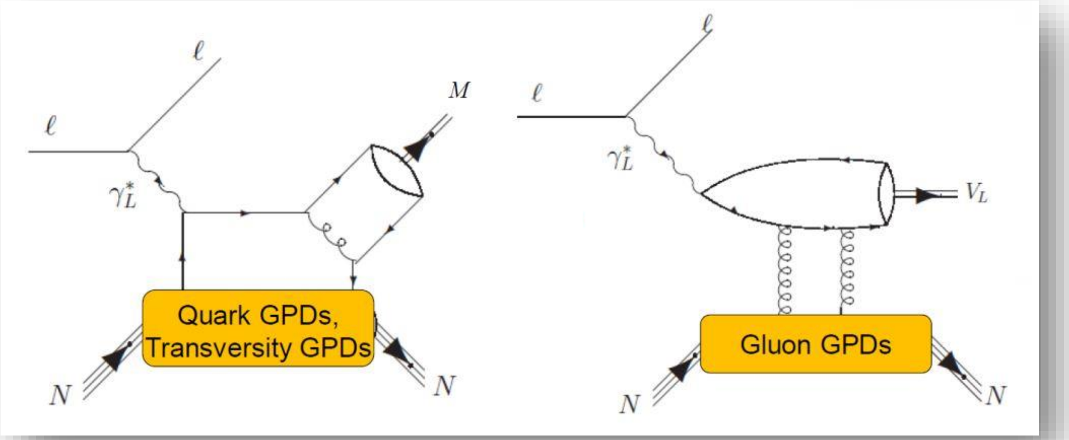

Courtesy: Hyon-Suk Jo, KPS Meeting

No access to x-dependence of GPDs

Complementarity: Lattice results can be integrated into global analysis of experimental data

Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering

Deeply Virtual Meson Production

KPS Meeting

Exclusive production of a pair of high transverse momentum photons in pion-nucleon collisions for extracting generalized parton distributions

Jian-Wei Qiu[a,b] Zhite Yu[c]

Hard photoproduction of a diphoton with a large invariant mass

A. Pedrak,[1] B. Pire,[2] L. Szymanowski,[1] and J. Wagner[1]

Access to x-dependence of GPDs

Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering

Deeply Virtual Meson Production

We require complementary measurements of the GPDs using Lattice QCD

In recent years, significant breakthroughs have been made in our ability to access the x-dependence of GPDs

Ex...
m...
extracting generalized parton distributions

Jian-Wei Qiu[a,b] Zhite Yu[c]

Hard photoproduction of a diphoton with a large invariant mass

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Access to x-dependence of GPDs

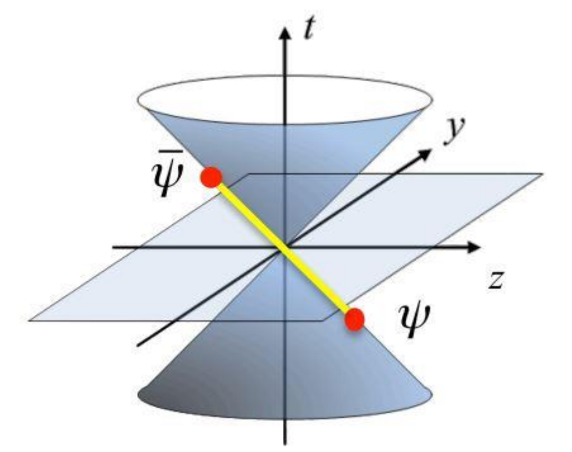
Calculating Parton Distributions in Lattice QCD

"Physical" distributions

Light-cone (standard) correlator $-1 \leq x \leq 1$

$$F^{[\Gamma]}(x, \Delta; \lambda, \lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p'; \lambda' | \bar{\psi}(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \psi(\tfrac{z}{2}) | p; \lambda \rangle \Big|_{z^+ = \vec{z}_\perp = 0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **Cannot be computed on Euclidean lattice**



Calculating Parton Distributions in Lattice QCD

"Physical" distributions

Parton Physics on Euclidean Lattice

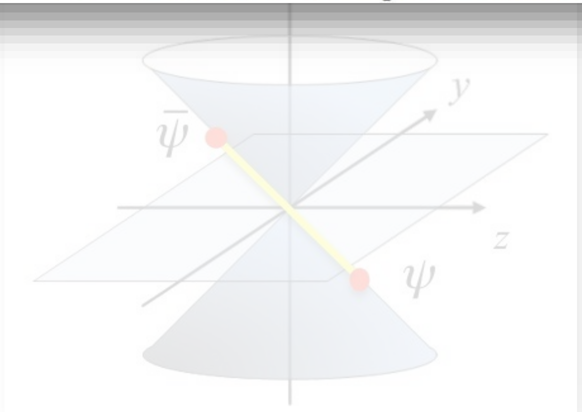
Xiangdong Ji[1,2]

[1] *INPAC, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, 200240, P. R. China*
[2] *Maryland Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

(Dated: May 8, 2013)

Abstract

I show that the parton physics related to correlations of quarks and gluons on the light-cone can be studied through the matrix elements of frame-dependent, equal-time correlators in the large momentum limit. This observation allows practical calculations of parton properties on an

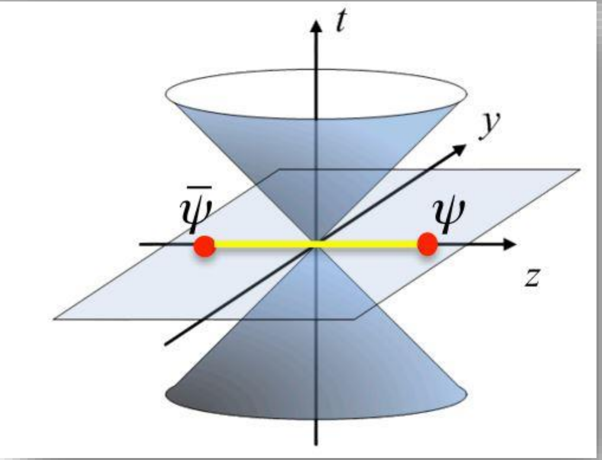


"Auxiliary" distributions

Correlator for quasi-GPDs (Ji, 2013) $\quad -\infty \le x \le \infty$

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=z_\perp=0}$$

- **Non-local correlator depending on position z^3**
- **Can be computed on Euclidean lattice**



Calculating Parton Distributions in Lattice QCD

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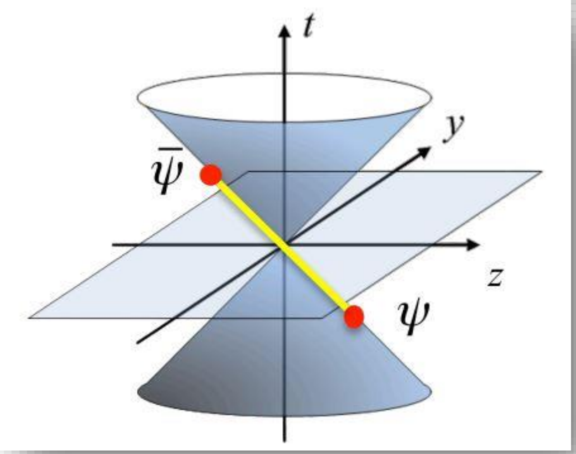

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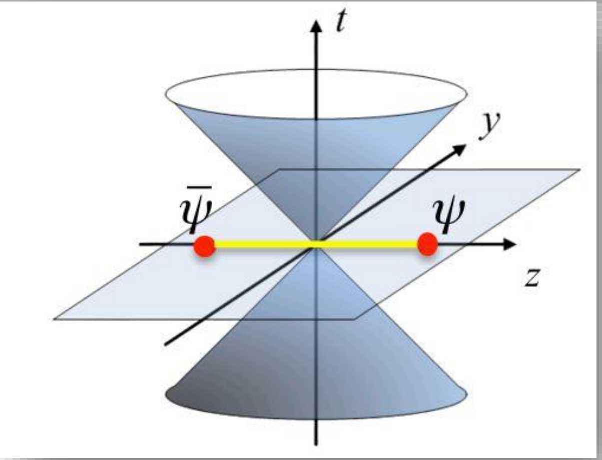

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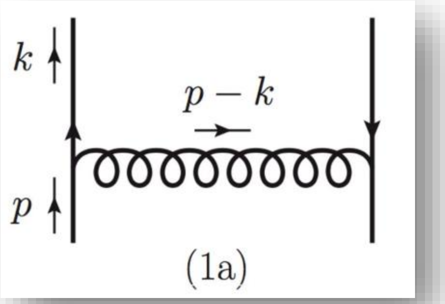
Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



Light-cone PDF:

$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2}{x\boldsymbol{m_g^2}} - 2\right)}$$

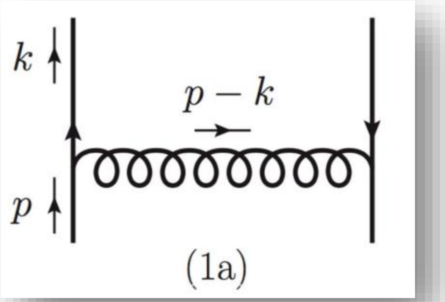
$$\int_{\mathbf{0}}^{\boldsymbol{\infty}} dk_\perp$$

$0 < x < 1$

$$\mathcal{P}_{\mathrm{UV}} = \frac{1}{\epsilon_{\mathrm{UV}}} + \ln 4\pi - \gamma_E$$

Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



Light-cone PDF:

$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\rm UV} + \ln\frac{\mu^2}{x m_g^2} - 2\right)}$$

$0 < x < 1$

$$\mathcal{P}_{\rm UV} = \frac{1}{\epsilon_{\rm UV}} + \ln 4\pi - \gamma_E$$

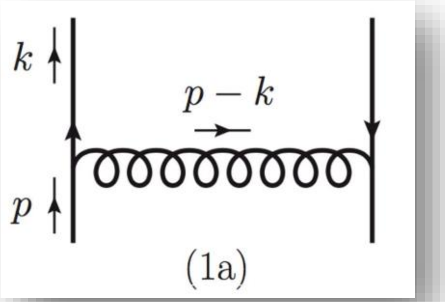
$$\int_0^\infty dk_\perp$$

Quasi PDF:

$$f_1(x,p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases} (1-x)\ \ln\dfrac{x}{x-1} + 1 & x > 1 \\ (1-x)\ \ln\dfrac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ \ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases}$$

Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



(1a)

Light-cone PDF:

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$0 < x < 1$

$$\mathcal{P}_{\mathrm{UV}} = \frac{1}{\epsilon_{\mathrm{UV}}} + \ln 4\pi - \gamma_E$$

$\int_0^\infty dk_\perp$

Quasi PDF:

Support outside physical region $0 < x < 1$

$$f_1(x, p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases} (1-x)\ln\dfrac{x}{x-1} + 1 & x > 1 \\ (1-x)\ln\dfrac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases}$$

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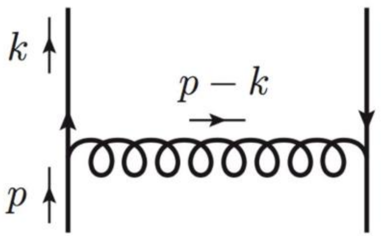

(1a)

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$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2}{x m_g^2} - 2\right)}$$

$0 < x < 1$

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$$\int_0^\infty dk_\perp$$

Absence of UV divergence: They manifest only after $\int dx$

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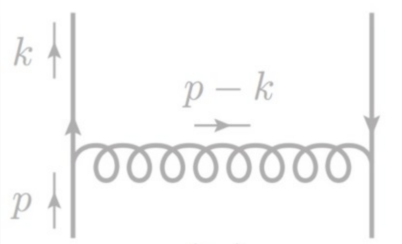
Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



(1a)

Light-cone PDF:

$$f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\rm UV} + \ln\frac{\mu^2}{\cdots} - 2\right) \qquad 0<x<1$$

By construction, if one boosts the quasi-observable to infinite-momentum frame, then it reduces to the light-cone observable

$\int_0^\infty dk_\perp$

Absence of UV divergence: They manifest only after $\int dx$

Quasi PDF:

Support outside physical region $0<x<1$

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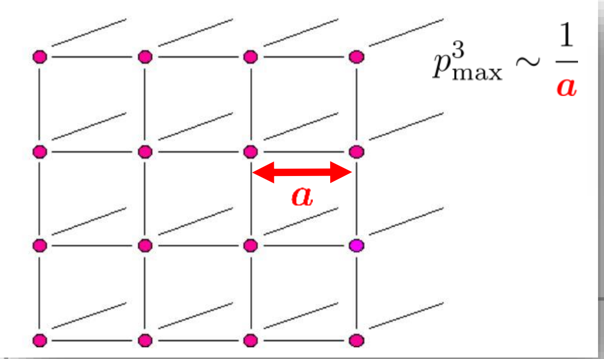
Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



Light-cone PDF:

$0 < x < 1$

By construction, if one boosts the quasi-observable to infinite-momentum frame, then it reduces to the light-cone observable

$\int_0^\infty dk_\perp$

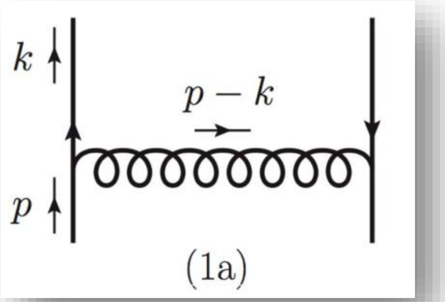
Absence of UV divergence: They manifest only after $\int dx$

In lattice computations, UV cut-offs (Λ) are given by the finite lattice spacing a ($\Lambda \sim a^{-1}$), and one (naturally) deals with UV renormalization before taking the limit $P^3 \to \infty$. The limits $\Lambda \to \infty$ and $P^3 \to \infty$ do not commute, which leads to non-trivial differences in the UV behavior of the quasi-PDFs and light-cone PDFs.

$$f_1(x, p^3) = \frac{\alpha_s C_F}{2\pi} \begin{cases} (1-x)\ln\frac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ln\frac{x-1}{x} - 1 & x < 0 \end{cases}$$

Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



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IR pole structure of light-cone & quasi-PDFs are same

Calculating Parton Distributions in Lattice QCD

Matching formula: **(PDF)**

Matching coefficient

$$\tilde{q}(x,\mu,P^3) = \int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\mu}{P^3}\right)q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{(P^3)^2},\frac{M_N^2}{(P^3)^2}\right)$$

Xiong, Ji, Zhang, Zhao/ Stewart, Zhao/ Izubuchi, Ji, Jin, Stewart, Zhao ...

Essence of the quasi-PDF approach

IR pole structure of light-cone & quasi-PDFs are same

Quasi PDF:

Absence of UV divergence: They manifest only after $\int dx$

Support outside physical region $0 < x < 1$

$$f_1(x,p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases}(1-x)\ln\frac{x}{x-1}+1 & x>1\\ (1-x)\ln\frac{4(1-x)p_3^2}{m_g^2}+x & 0<x<1\\ (1-x)\ln\frac{x-1}{x}- & \end{cases}$$

IR pole structure of light-cone & quasi-PDFs are same

Calculating Parton Distributions in Lattice QCD

Matching formula: **(GPD)**

Matching coefficient

$$\tilde{q}(x,\xi,t,\mu,P^3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{P^3}\right) q(y,\xi,t,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(P^3)^2},\frac{M_N^2}{(P^3)^2},\frac{t}{(P^3)^2}\right)$$

GPD matching known up to one-loop order (non-singlet & singlet)

References: (not exhaustive)

Connecting Euclidean to light-cone correlations: From flavor nonsinglet in forward kinematics to flavor singlet in non-forward kinematics

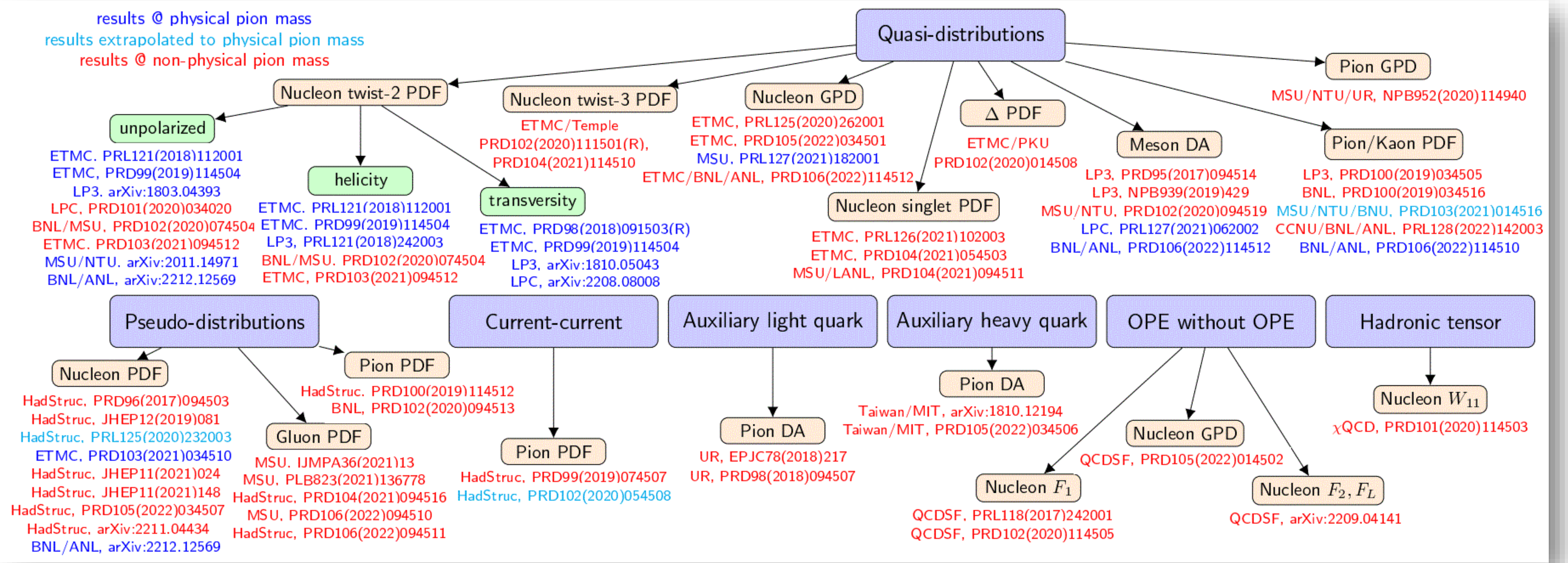
Yao Ji,[a] Fei Yao[b] and Jian-Hui Zhang[c,b]

One-Loop Matching for Generalized Parton Distributions

Xiangdong Ji,[1,2,3] Andreas Schäfer,[4] Xiaonu Xiong,[5,6] and Jian-Hui Zhang[1,4]

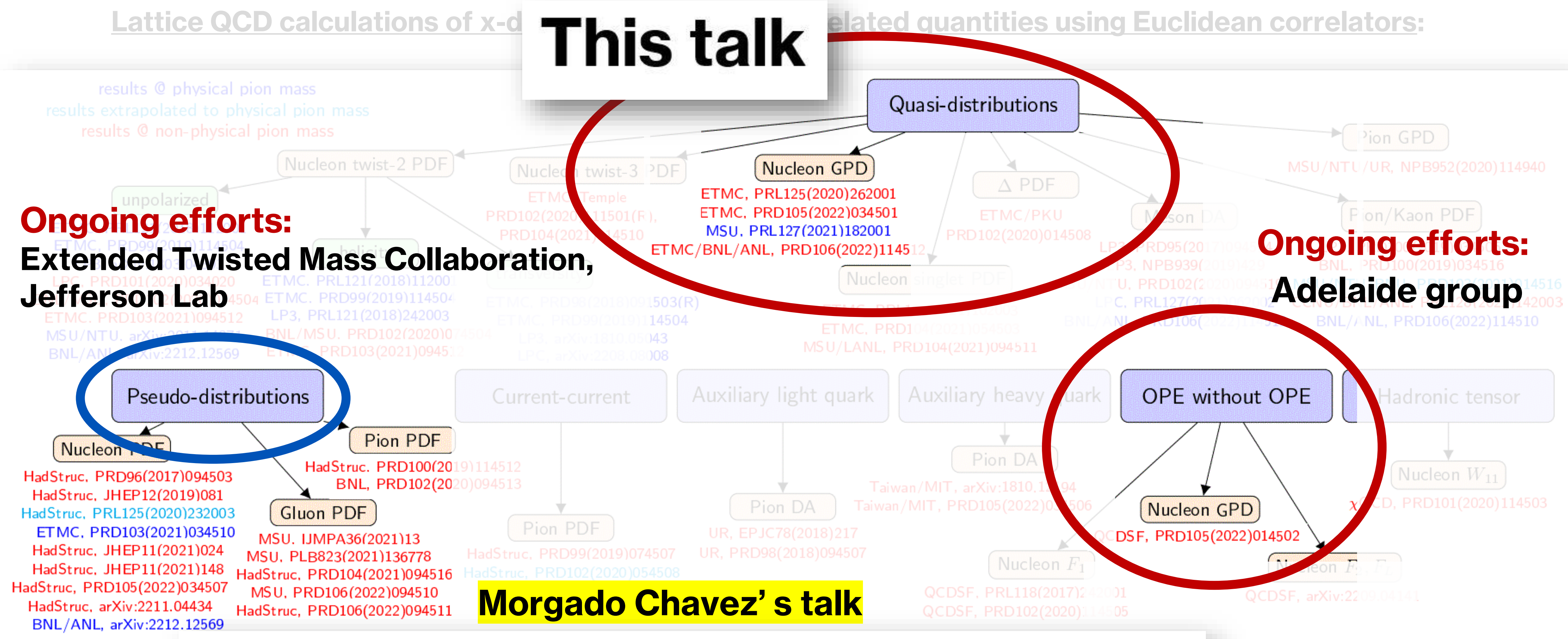
Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



Compilation by Cichy, 2110.07440

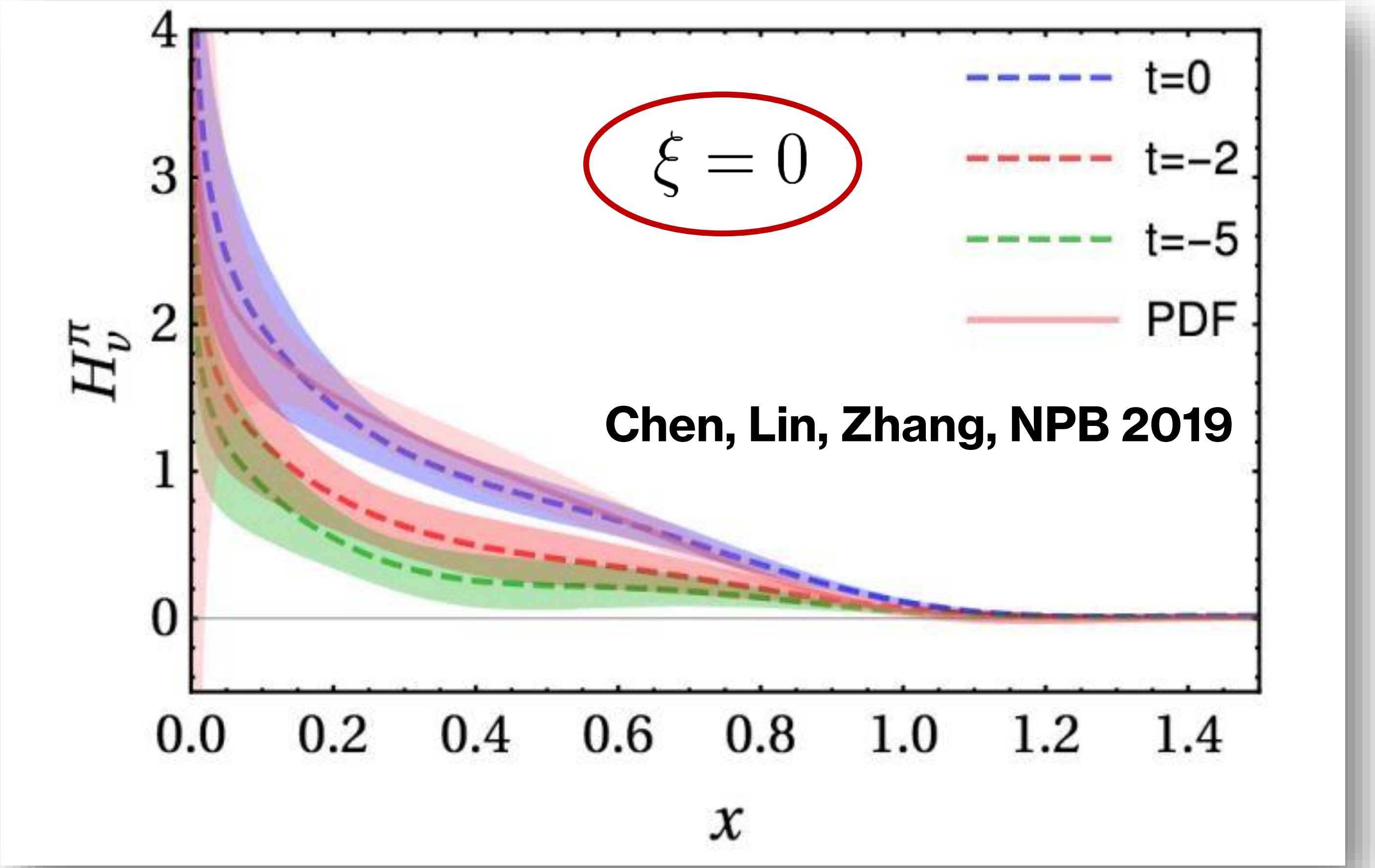
Dynamical Progress of Lattice QCD calculations of PDFs/GPDs



Forward-limit generalized parton distributions of the η_c-meson

First Lattice QCD results of the x-dependent GPDs

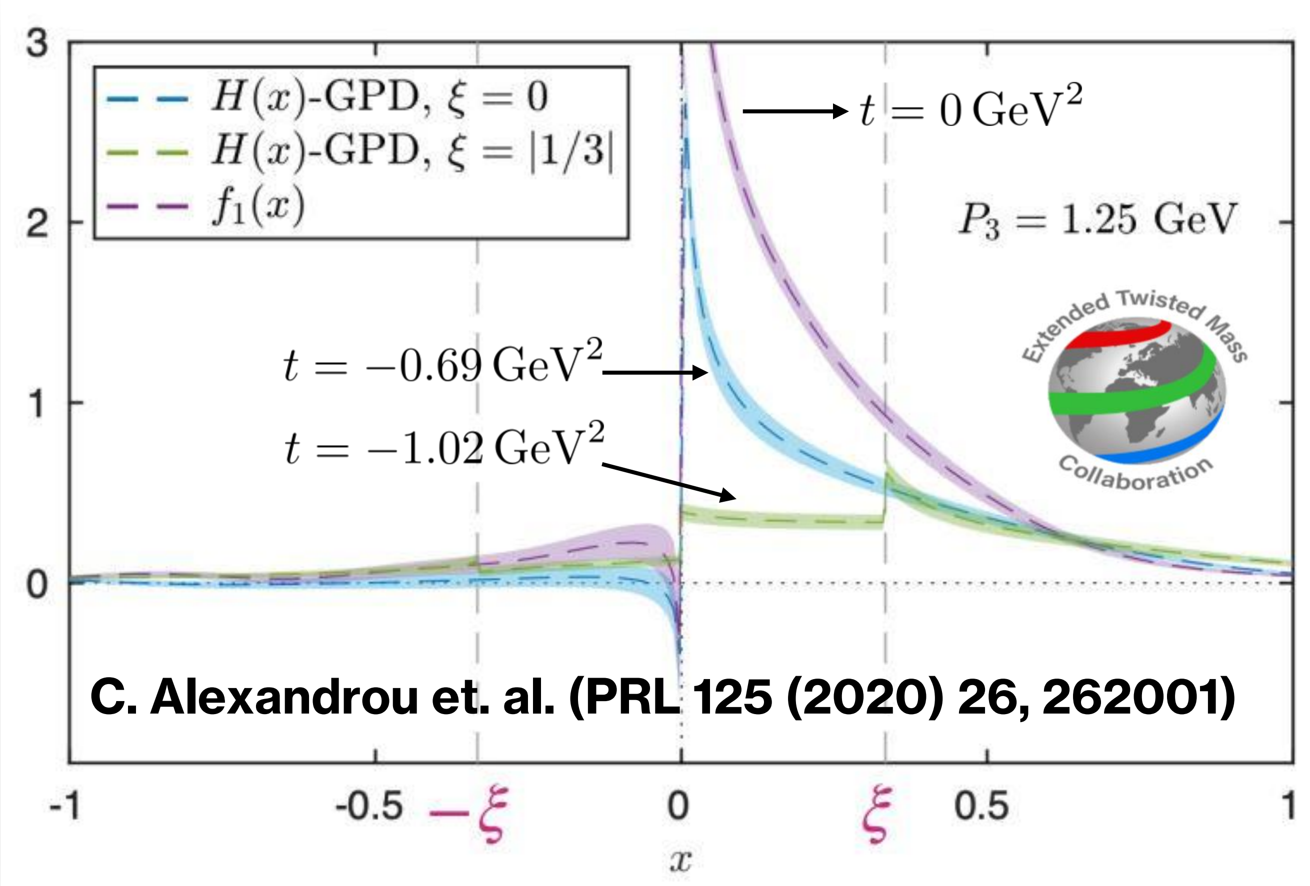
Pion:



As t increases, the distribution flattens

First Lattice QCD results of the x-dependent GPDs

Proton:



ERBL/DGLAP: Qualitative differences

As $x \to 1$, qualitative behavior in agreement with power counting analysis

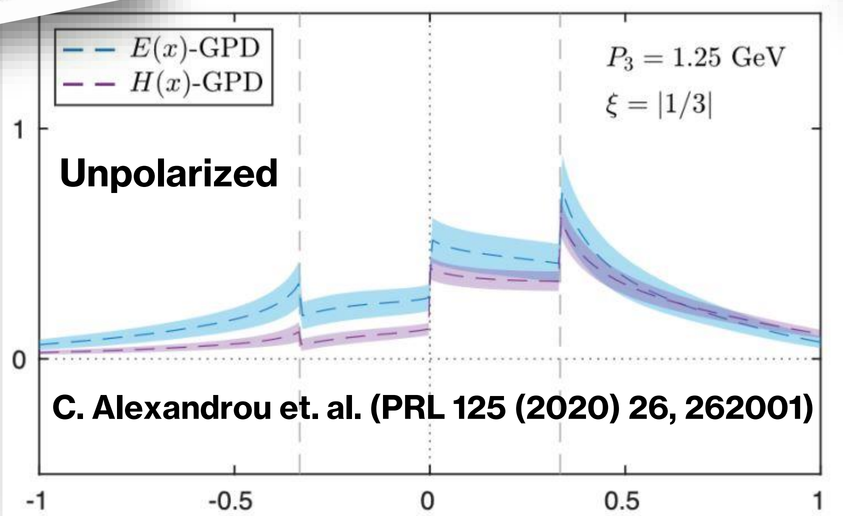
(F. Yuan, 0311288)

First Lattice QCD he x-dependent GPDs

Twist-2 GPDs

Pol \ Γ	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

Proton:



C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

First Lattice QCD he x-dependent GPDs

Twist-2 GPDs

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U	H		E_T
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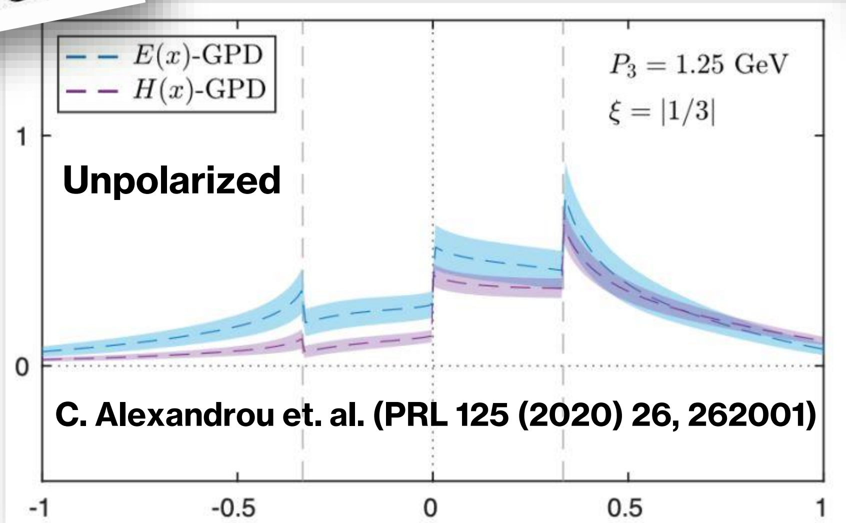

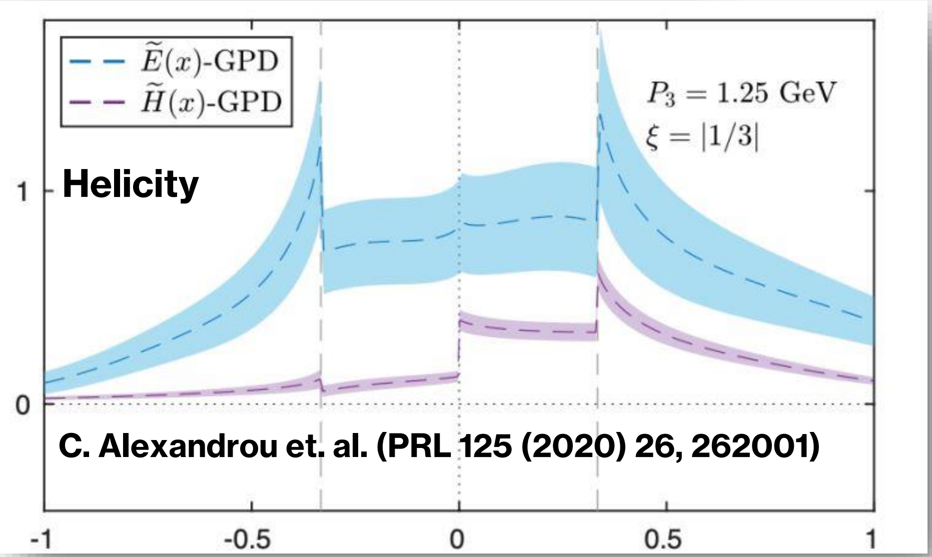

First Lattice QCD ... he x-dependent GPDs

Twist-2 GPDs

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U	H		E_T
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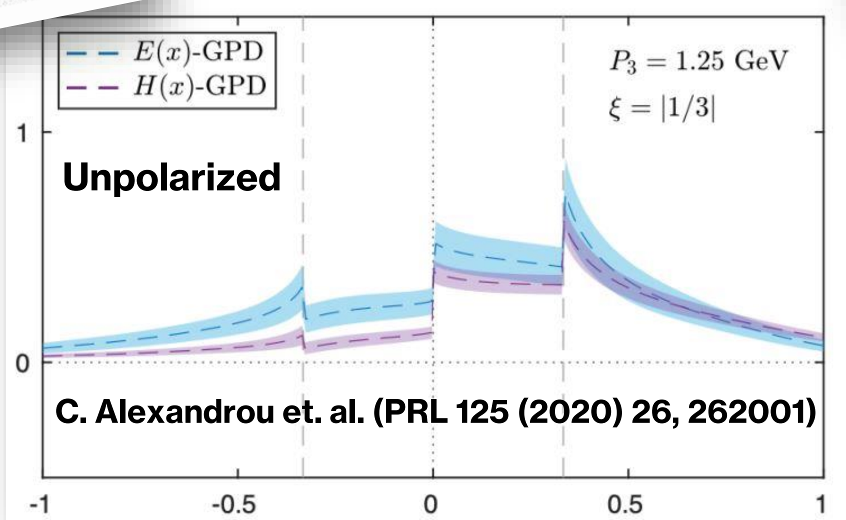
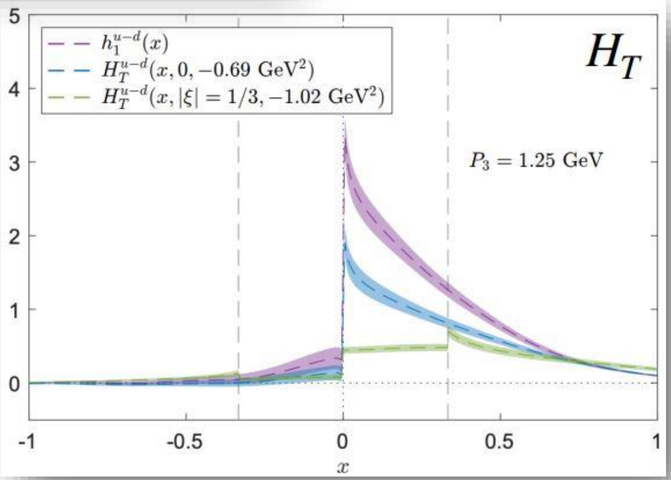
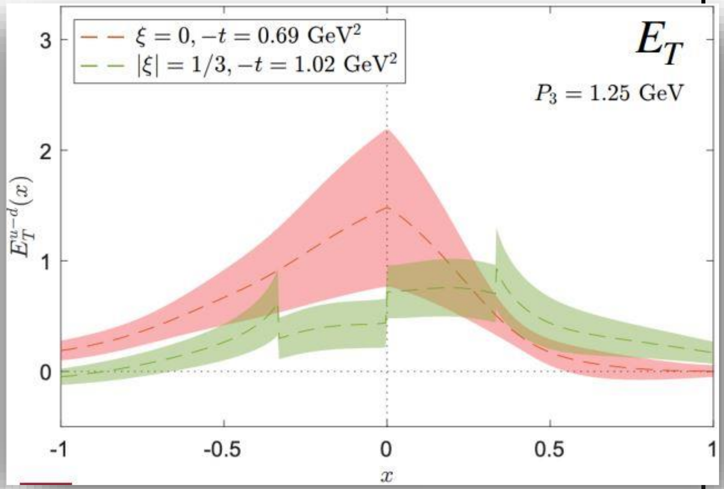
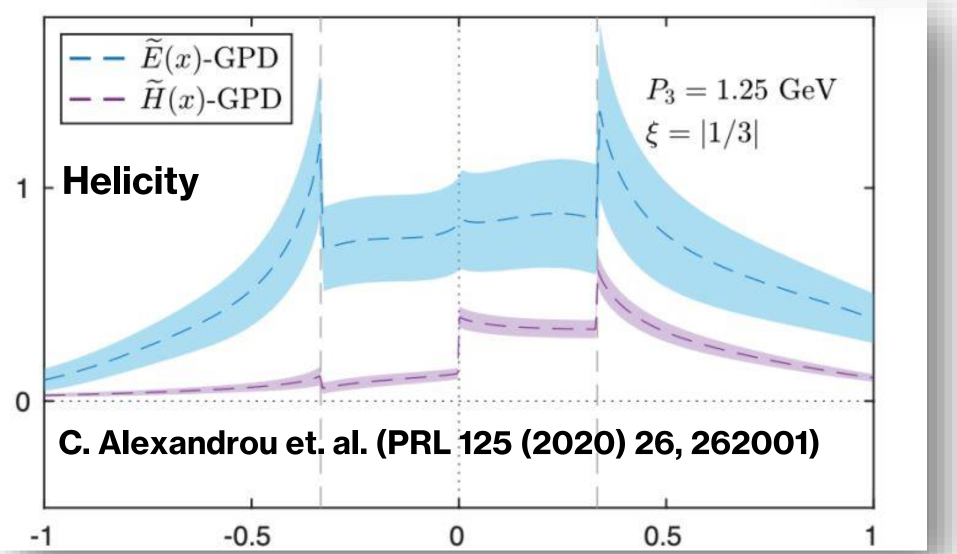
Proton:

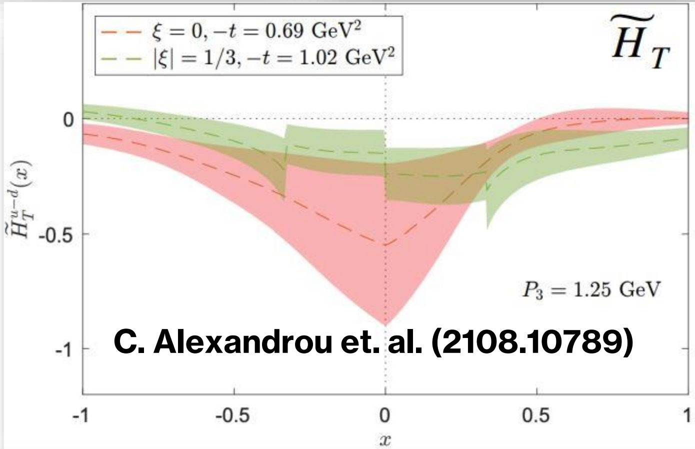


Transversity









GPD $\widetilde{E}_T$ is small/zero within uncertainties (not shown)

First exploration of twist-3 GPDs

Why twist 3?

- As sizeable as twist 2
- Contain information about quark-gluon-quark correlations inside hadrons ...

First exploration of twist-3 GPDs

Definition:

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

First exploration of twist-3 GPDs

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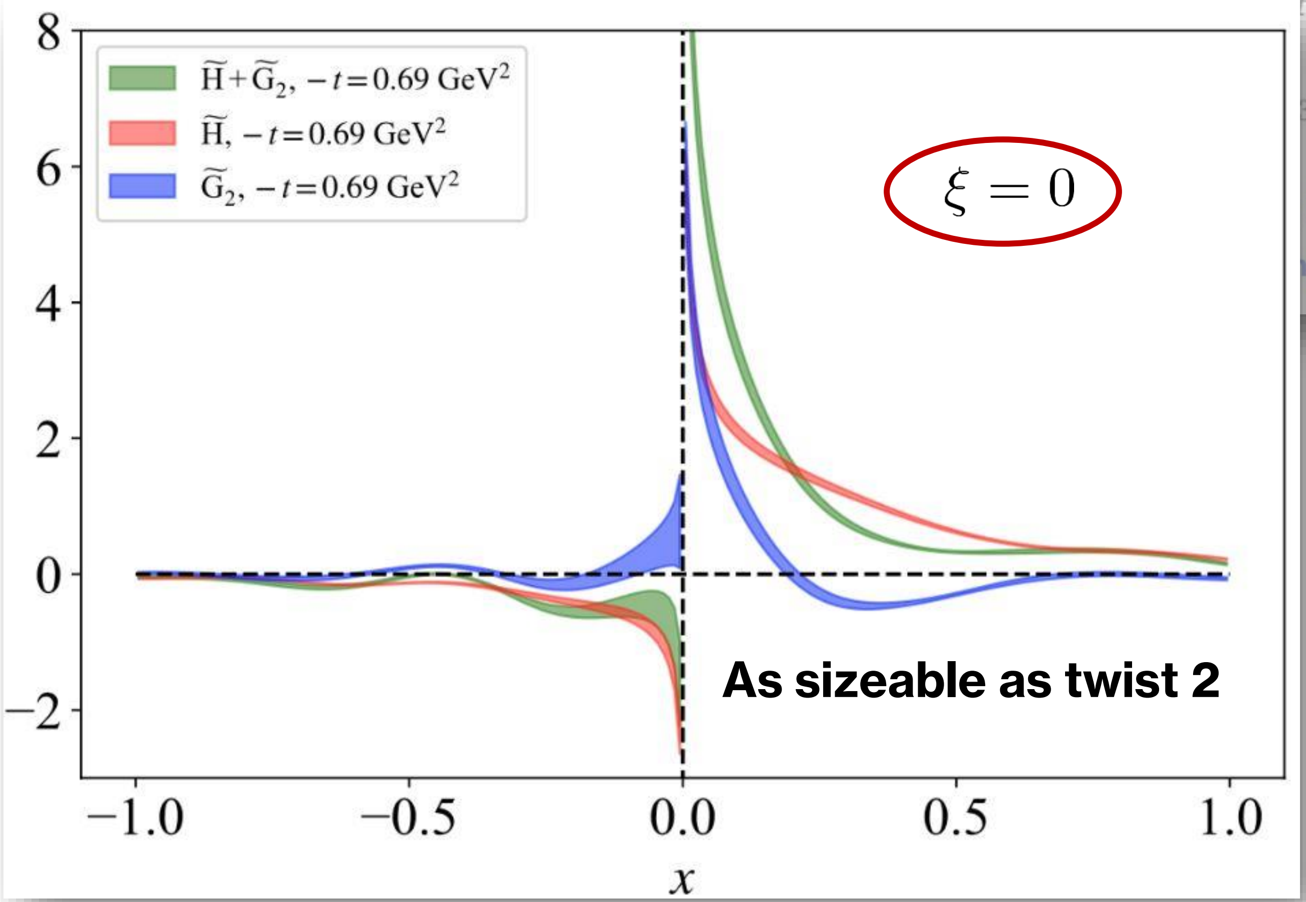
$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$

$$+\gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$

$$) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

ys. Rev. D 98, 014038 (2018), arXiv:1802.06243]



Bhattacharya et al, 2306.05533

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$$\cdots;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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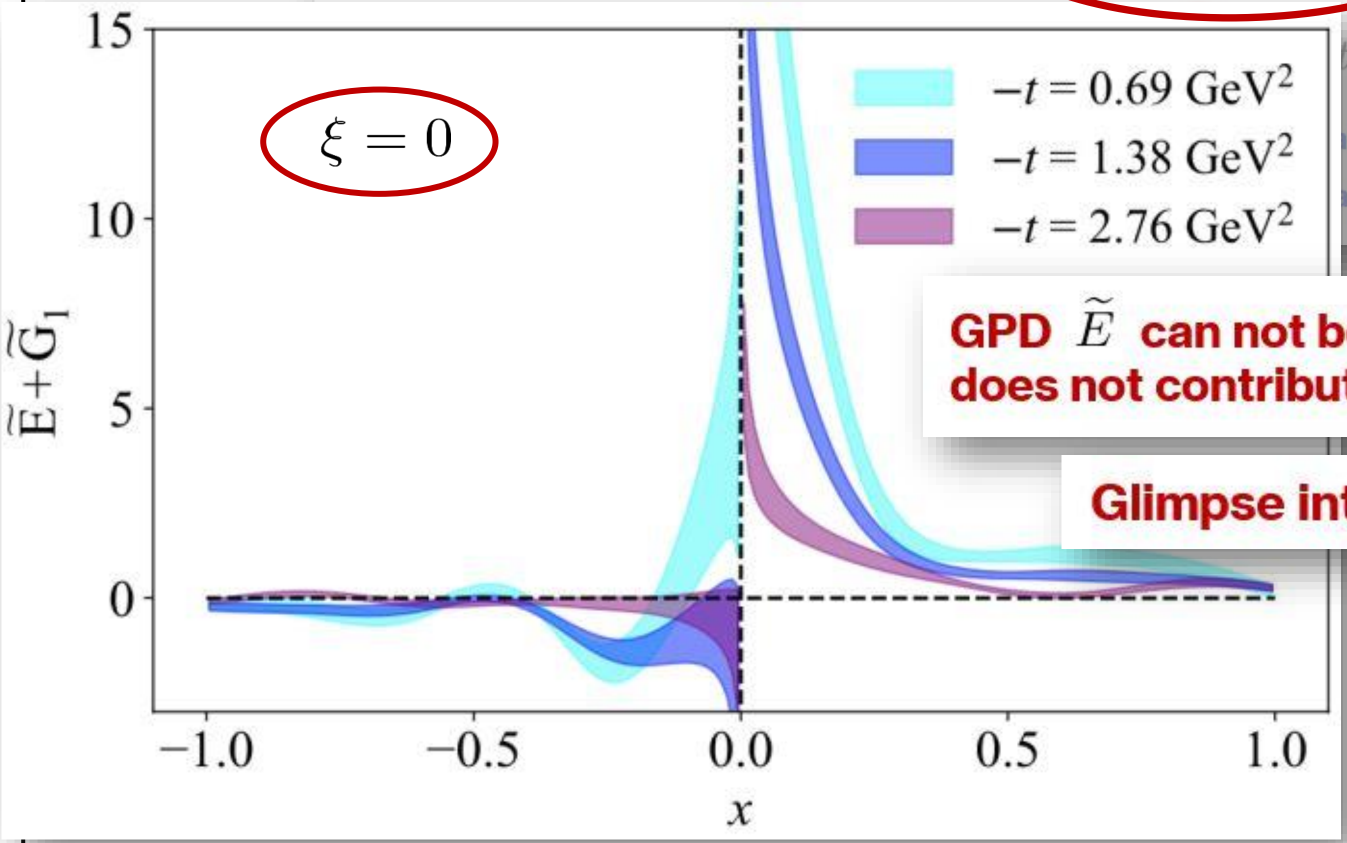

GPD $\widetilde{E}$ can not be accessed at zero skewness because it simply does not contribute to the matrix element at this point

Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness

Bhattacharya et al, 2306.05533

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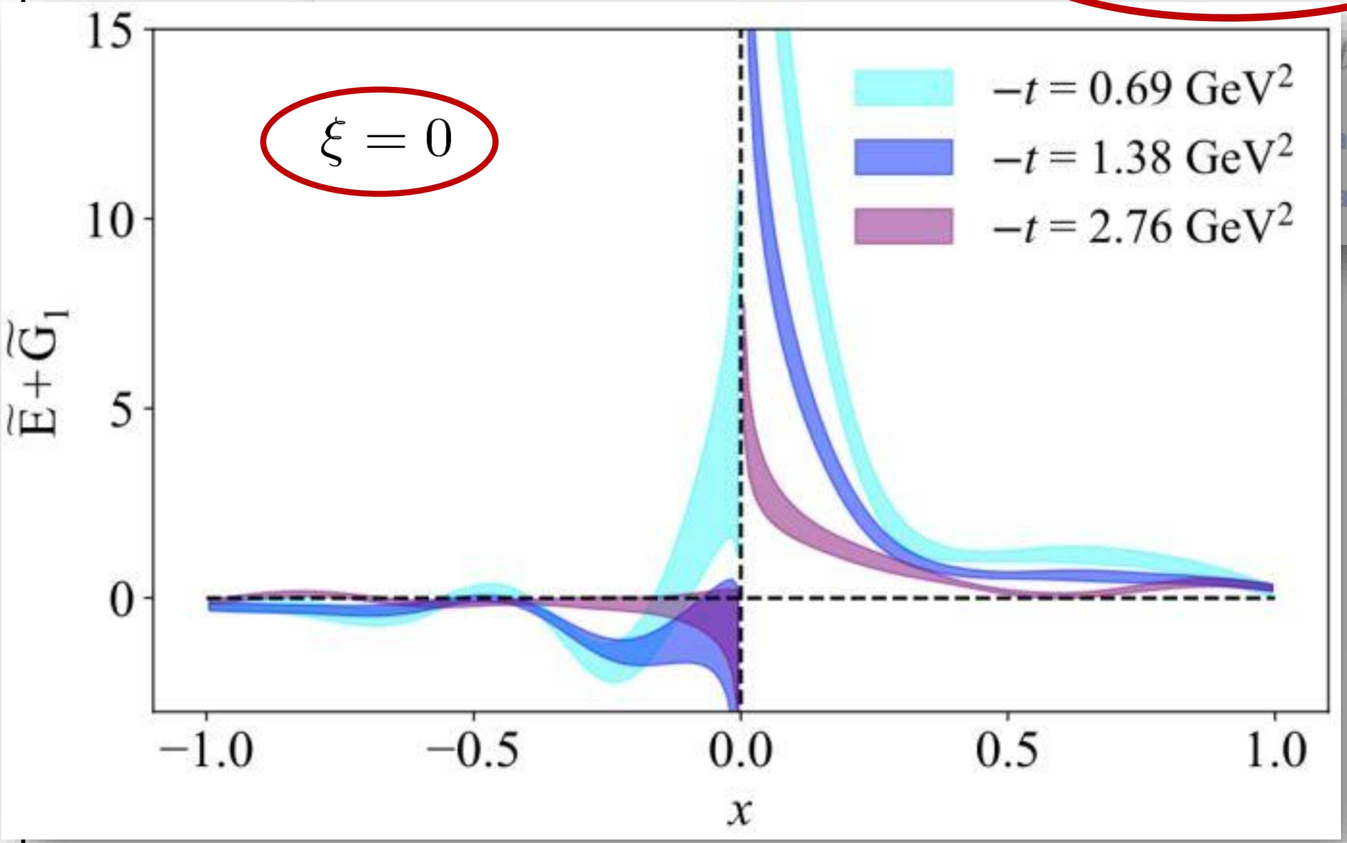

First indication of pion pole from Lattice QCD!

$$\tilde{E}_u - \tilde{E}_d \sim \frac{1}{l^2 - m_\pi^2}$$

(Penttinen, Polyakov, Goeke)

Bhattacharya et al, 2306.05533

But little hiccup ...

First exploration of twist-3 GPDs

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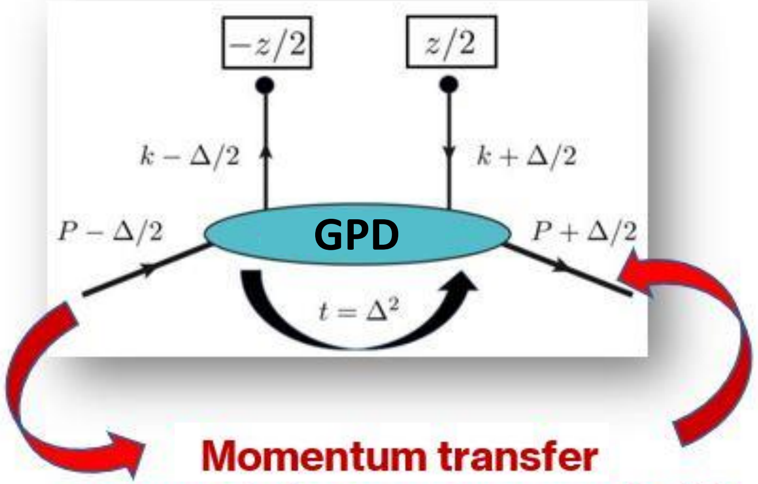
But little hiccup ...

First exploration of twist-3 GPDs

Traditionally, GPDs have been calculated from "symmetric frames"

Practical drawback



Momentum transfer symmetric between source & sink

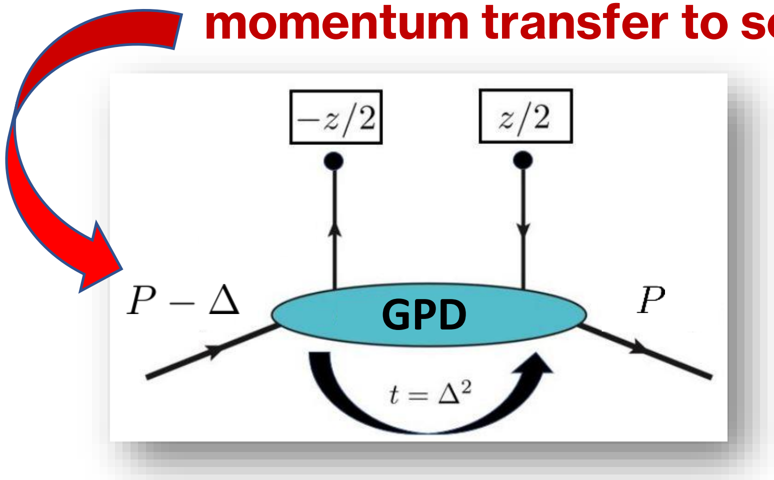
Lattice QCD calculations of GPDs in symmetric frames are expensive

In symmetric frame, full new calculation required for each momentum transfer (Δ)

GPDs from asymmetric frames

Resolution:

All momentum transfer to source



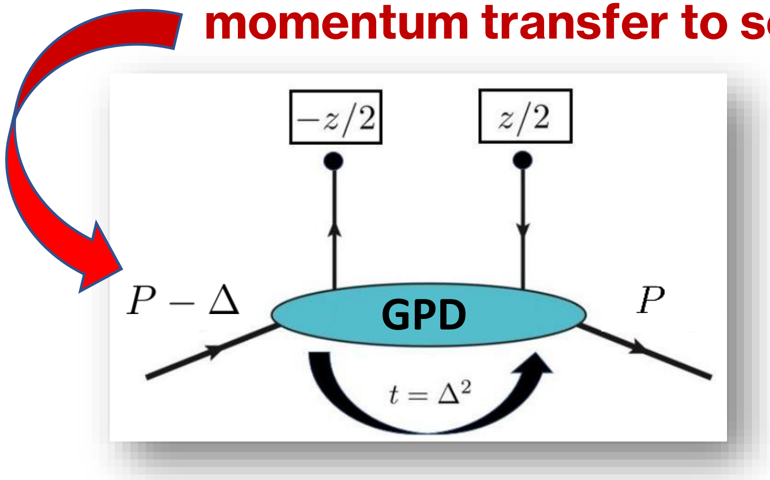
Perform Lattice QCD calculations of GPDs in asymmetric frames:

- **Reduction in computational cost**
- **Access to broad range of t (enabling creation of high-resolution partonic maps)**

GPDs from asymmetric frames

Resolution:

All momentum transfer to source



Major theoretical advances (Bhattacharya et al, 2209.05373, 2310.13114):

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source

Major theoretical advances:

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- Elimination of power corrections potentially allowing faster convergence to light-cone GPDs

GPDs from asymmetric frames

Example

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}\boldsymbol{A_1} + mz^\mu \boldsymbol{A_2} + \frac{\Delta^\mu}{m}\boldsymbol{A_3} + im\sigma^{\mu z}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_6} + mz^\mu i\sigma^{z\Delta}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Vector operator $F^\mu_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^\mu q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$

Features:

- **8** linearly-independent Dirac structures
- **8** Lorentz-invariant (frame-independent) amplitudes $\boldsymbol{A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)}$

GPDs from asymmetric frames

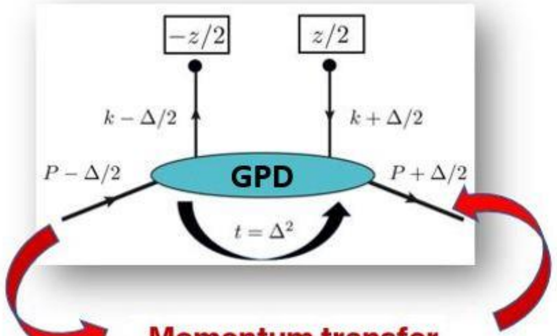
Example

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$F^{\mu}(z,P,\Delta)=\bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}\boldsymbol{A_1}+mz^{\mu}\boldsymbol{A_2}+\frac{\Delta^{\mu}}{m}\boldsymbol{A_3}+im\sigma^{\mu z}\boldsymbol{A_4}+\frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5}+\frac{P^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_6}+mz^{\mu}i\sigma^{z\Delta}\boldsymbol{A_7}+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Traditional definition (symmetric frame):



Momentum transfer symmetric between source & sink

$$F^{0}_{\lambda,\lambda'}\Big|_s=\langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$=\bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\Big|_s+\frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{Q(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

Quasi-GPDs are intrinsically frame-dependent

GPDs from asymmetric frames

Example

Lorentz covariant formalism

Nove

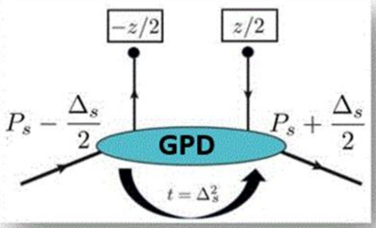
Main point:

$$H^{s}_{\mathrm{Q}(0)} = \sum_i c_i\, A_i$$

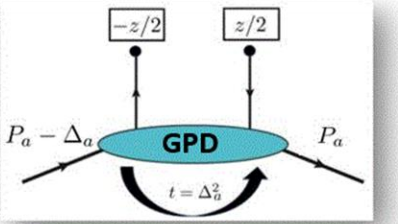
$$F^{\mu}(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}A_1 + m \ldots + m z^{\mu} i\sigma^{z\Delta} A_7 + \frac{\Delta^{\mu} i\sigma^{z\Delta}}{m} A_8\right] u(p_i,\lambda)$$

Main point:

Calculate quasi-GPD in symmetric frame through matrix elements of asymmetric frame



Symmetric frame



Asymmetric frame

- 8 Lorentz $A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source

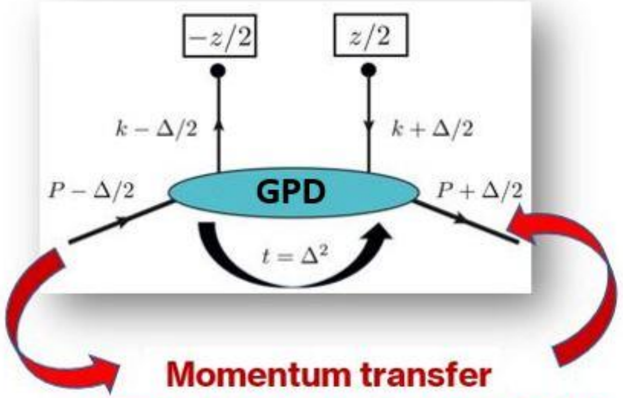
Major theoretical advances:

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Example: Symmetric frame



Quasi-GPD:

$$H^s_{\mathrm{Q}(0)}(z,P^s,\Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$
$$+\left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

Quasi-GPD: (Symmetric frame)

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$
$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

Contamination from additional amplitudes or explicit power corrections

Quasi-GPD: (Symmetric frame)

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$
$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

Main finding

Schematic structure of (operator-level) Lorentz-invariant definition of quasi-GPD:

$$\boxed{H_{\rm Q} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle}$$

Here, c's are frame-dependent kinematic factors that cancel additional amplitudes such that quasi-GPD has the same functional form as light-cone GPD (Lorentz invariant)

$$+\left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s} z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s} z^3 (\Delta_\perp^s)^2}{2P^{0,s}P^{3,s}}\right] A_8\,,$$

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

$$\boxed{H(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3}$$

$$A_i \equiv A_i(z^2 = 0)$$

Lorentz-invariant definition of quasi-GPD:

$$\boxed{\mathcal{H}(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3}$$

$$A_i \equiv A_i(z^2 \neq 0)$$

Same functional forms

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

$$\boxed{H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3}$$

$A_i \equiv A_i(z^2 = 0)$

Lorentz-invariant definition of quasi-GPD:

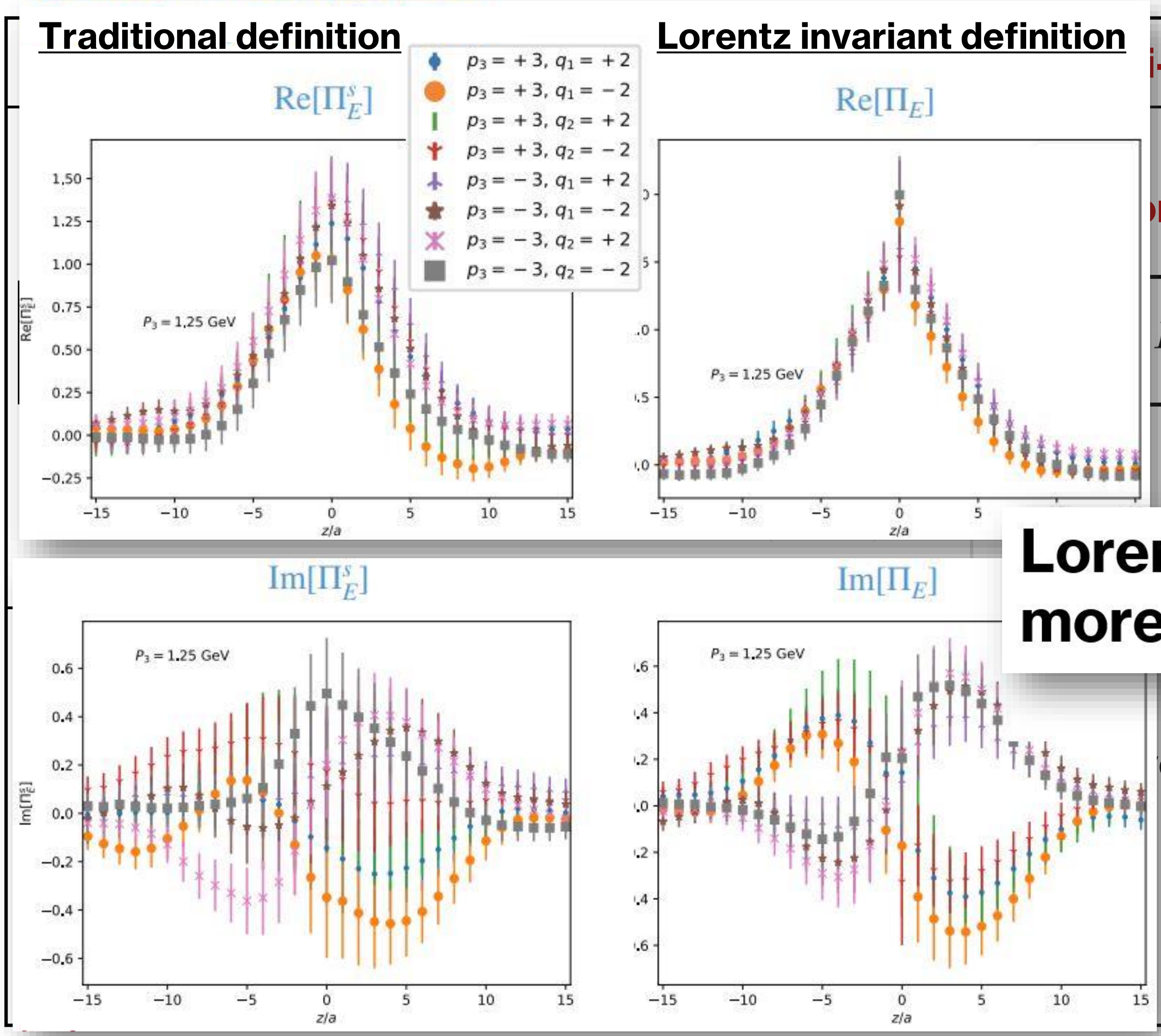
$$\boxed{\mathcal{H}(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3}$$

$A_i \equiv A_i(z^2 \neq 0)$

Feature:

- **Lorentz-invariant definition of quasi-GPDs may converge faster**

GPDs from asymmetric frames



i-GPDs

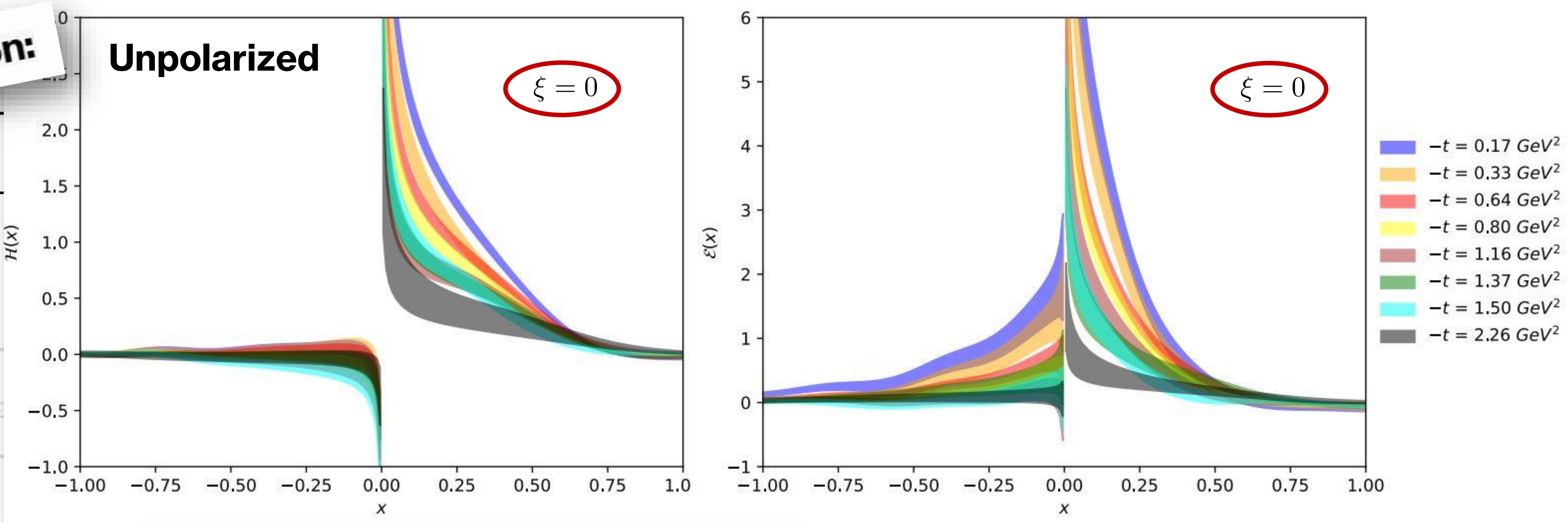
rentz-invariant definition of quasi-GPD:

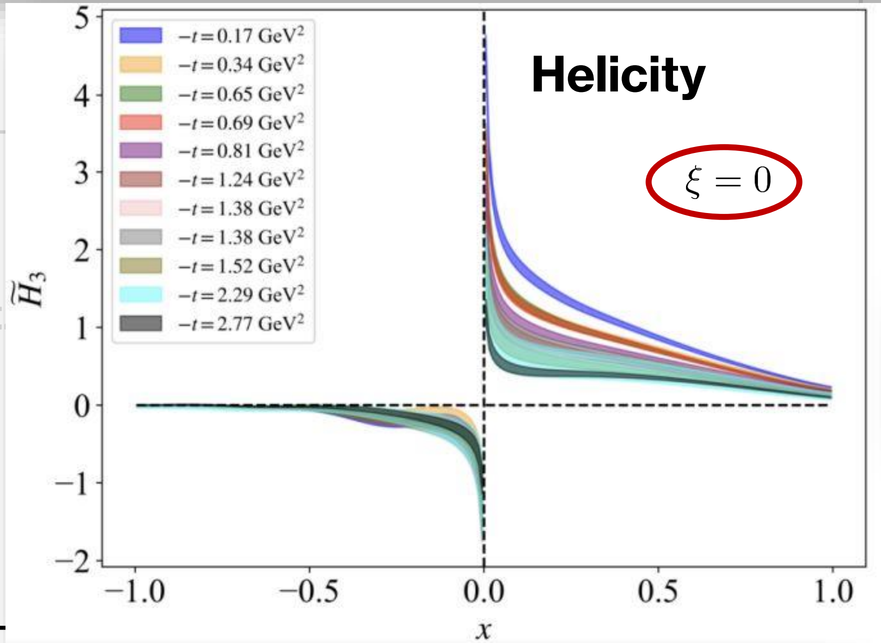
$$P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z} A_3$$

Lorentz invariant definition leads to more precise results for GPD E

erge faster

Proton:





GPDs derived from asymmetric frames within the amplitude formalism

Feature:

- Lorent

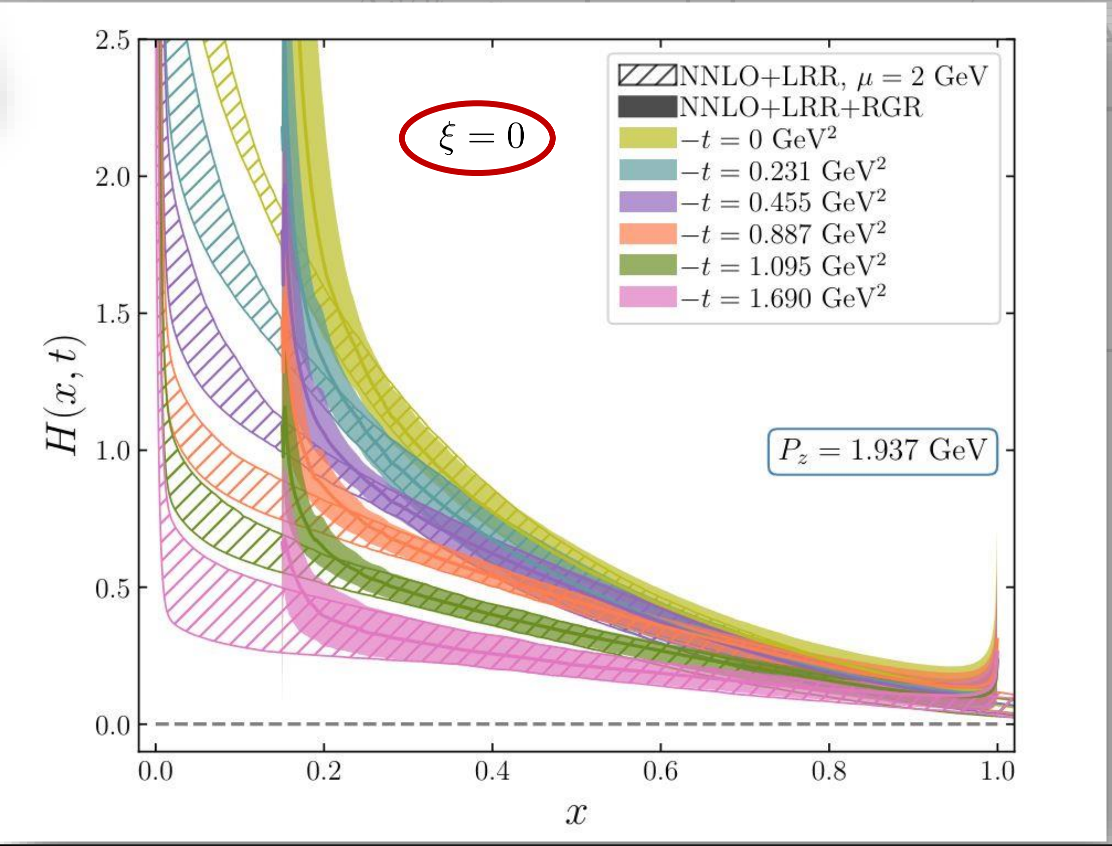
$A_i \equiv A_i(z^2 \neq 0)$

GPDs from asymmetric frames

New definition of quasi-GPDs

GPDs derived from asymmetric frames within the amplitude formalism

Pion:



Ding et al, 2407.03516

Feature:

- Lorentz-invaria

$A_i \equiv A_i(z^2 \neq 0)$

Summary

- **Tremendous recent activity in studying parton structure of hadrons in lattice QCD through Euclidean correlators**

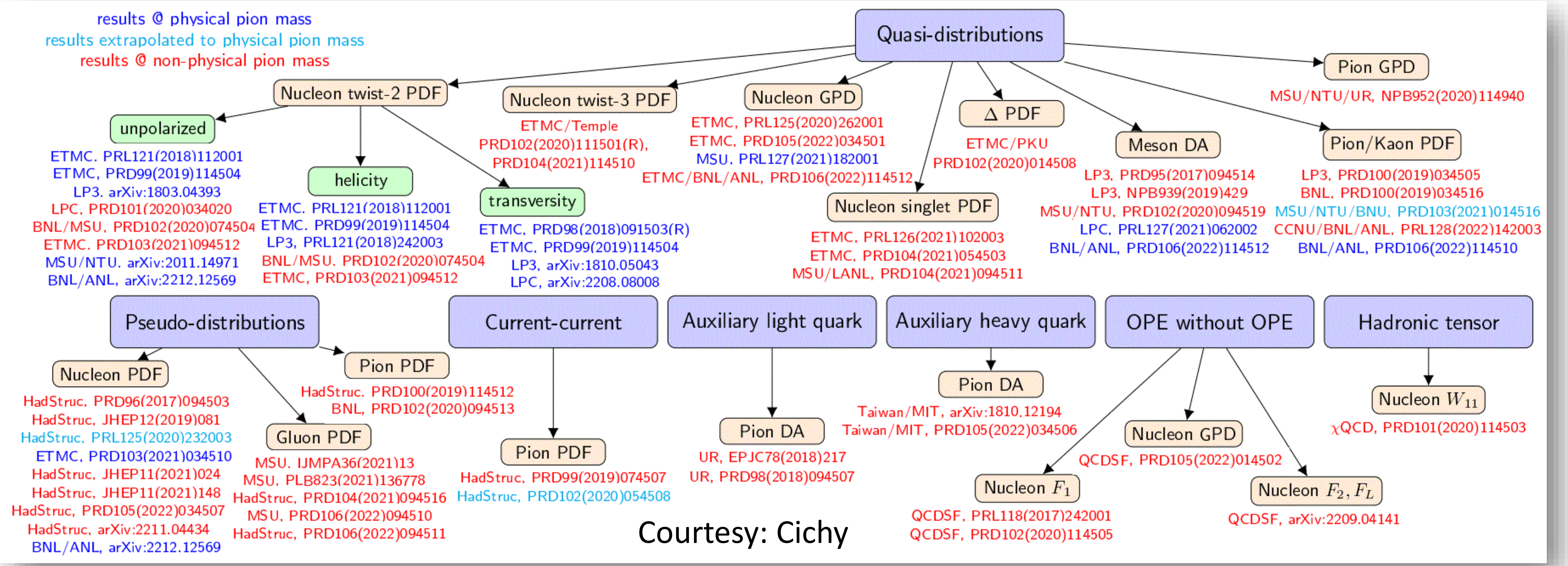
- **Impact of approach(es) largest where experiments are difficult → GPDs**

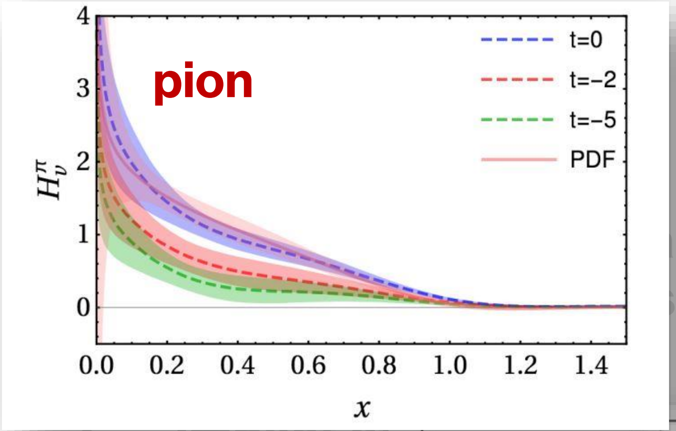
Summary

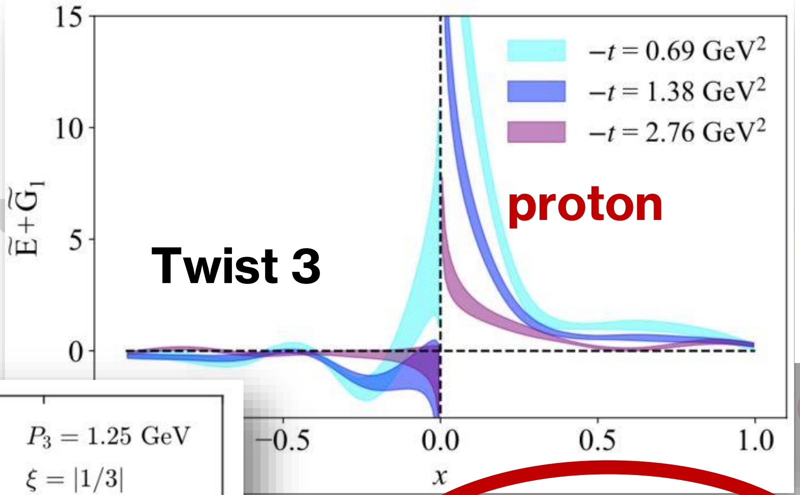
- Tremendous recent activity in studying parton structure of hadrons in lattice QCD through Euclidean correlators

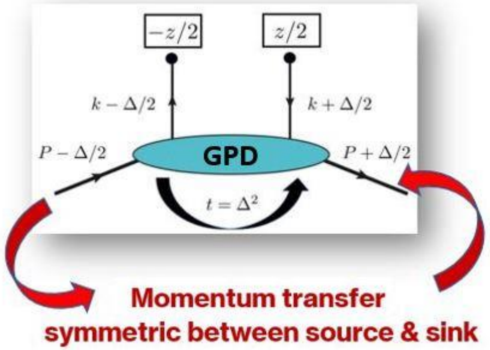
- Impact of approach(es) largest where experiments are difficult → GPDs

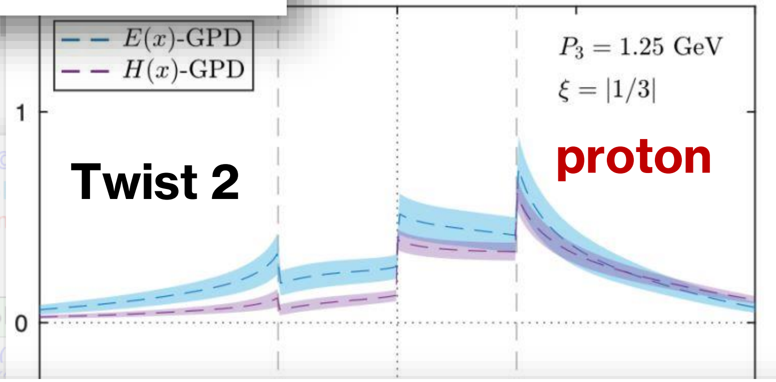
Overview of Euclidean-correlator approaches

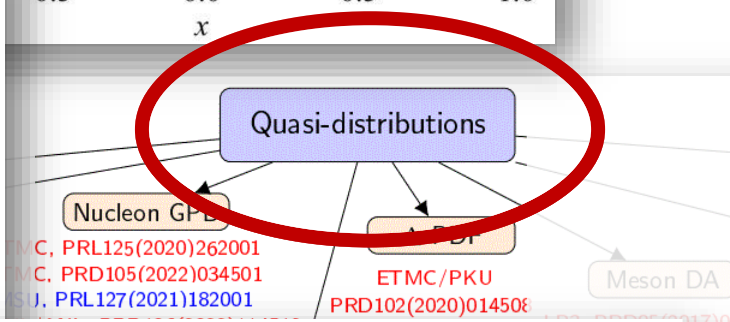


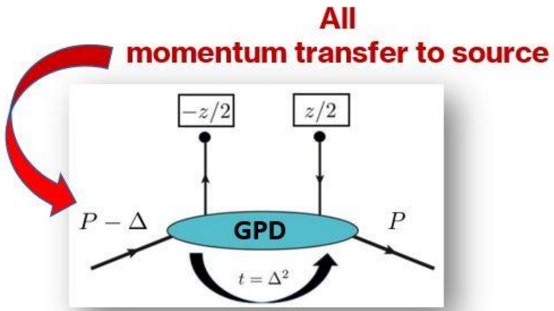




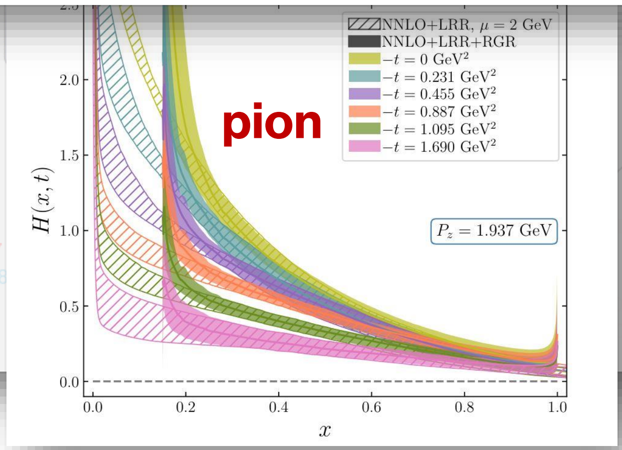


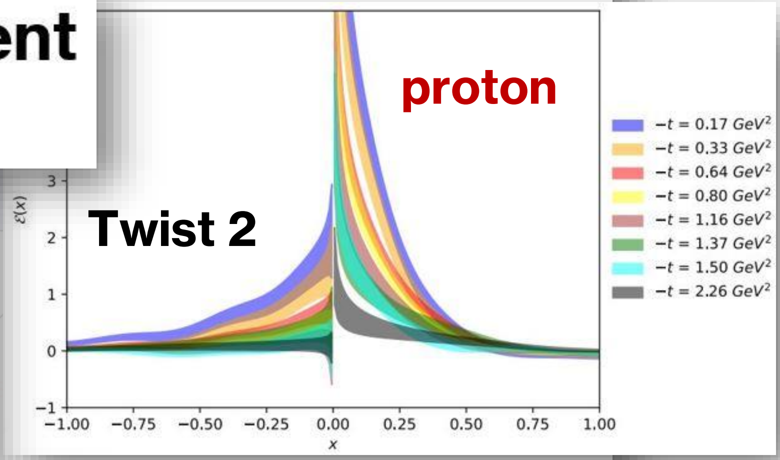






Lattice calculations can serve as a valuable complement to the ongoing efforts at the EIC





Significant progress!

Backup slides

Dynam[...]l Progress of Lattice QCD calculations of PDFs/GPDs

Check out!

Hindawi
Advances in High Energy Physics
Volume 2019, Article ID 3036904, 68 pages
https://doi.org/10.1155/2019/3036904

Hindawi

Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy [1] **and Martha Constantinou** [2]

[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

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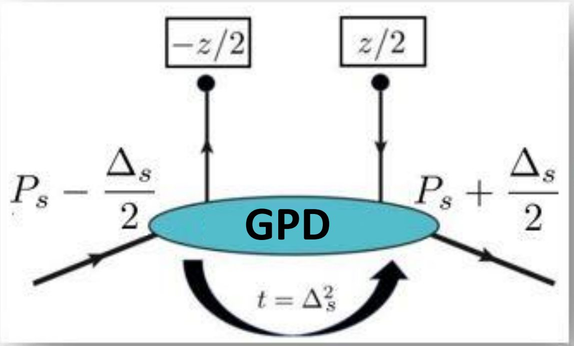
GPDs from asymmetric frames

Historic definitions of quasi-GPDs H & E are not manisfestly Lorentz invariant

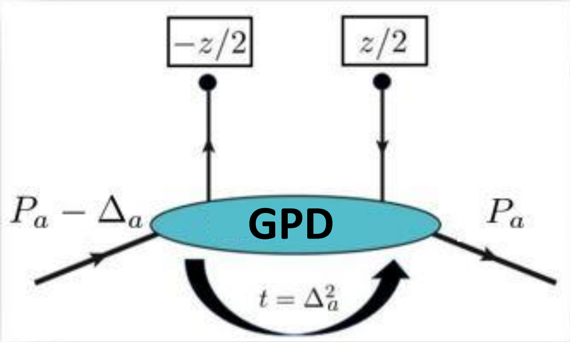
Think about how γ^0 transforms under Lorentz transformation

Tra
(s

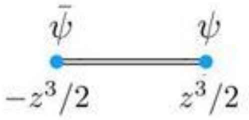


Symmetric frame

"Transverse" Lorentz transformation



Asymmetric frame



"Transverse" with respect to Wilson Line

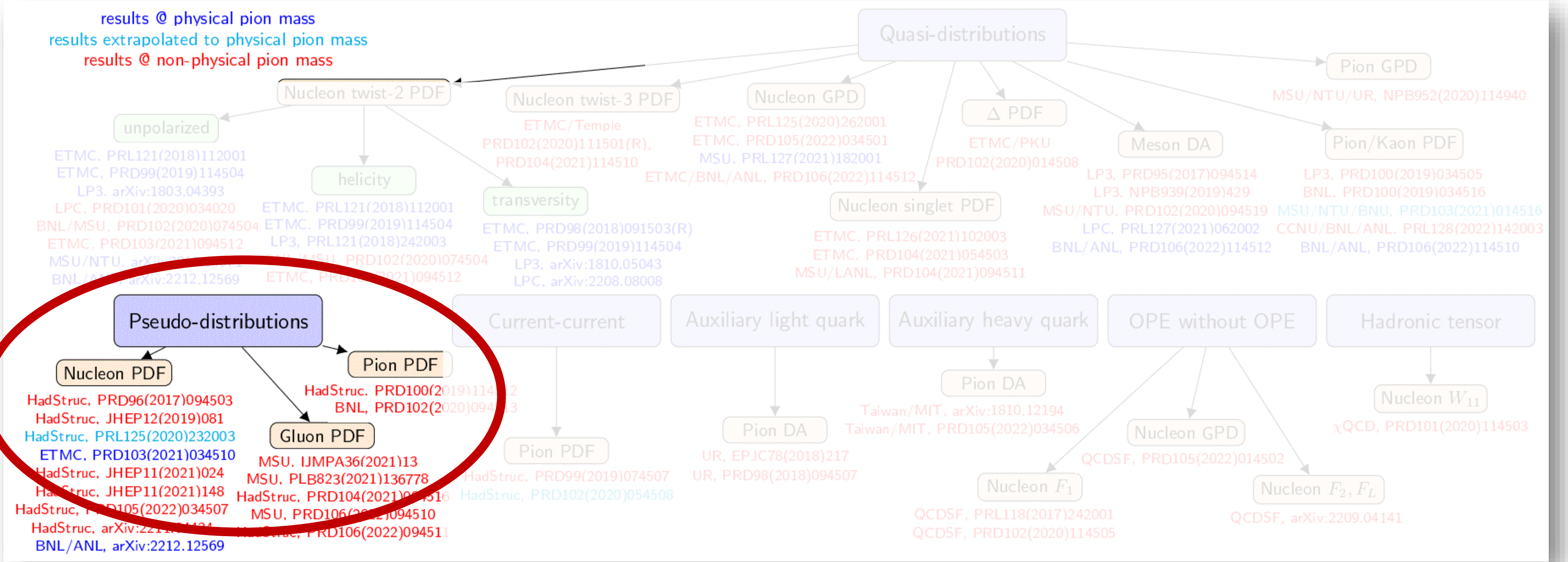
$$\boxed{F_s^0 = \gamma F_0^a - \gamma\beta F_\perp^a}$$

$$\beta = -\sqrt{\frac{E_i^a - E_f^a}{E_i^a + E_f^a}} < 0$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



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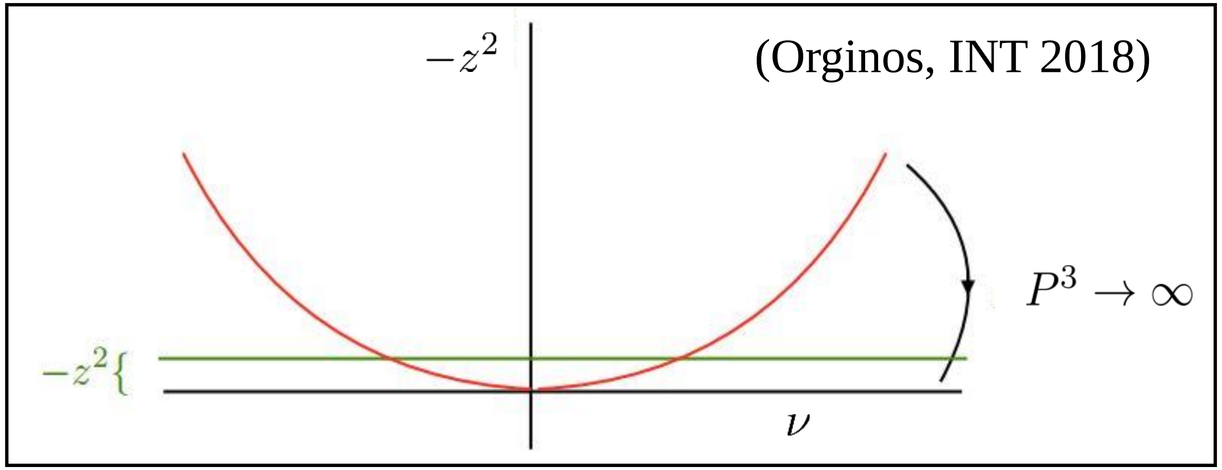
Pseudo-GPD approach

Generalized Parton Distributions and Pseudo-Distributions

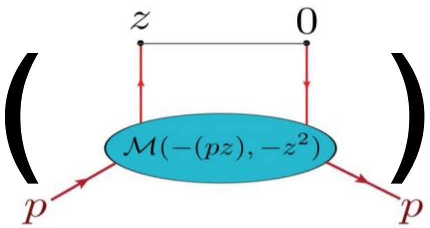
A. V. Radyushkin[1,2]

Sketch of the approach:



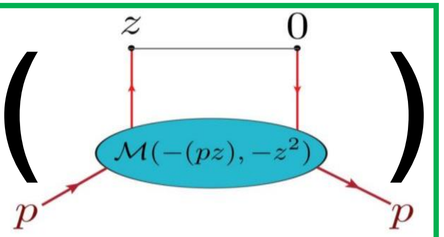
Quasi-PDF : Fixed P^3

$$Q(x, P^3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-ix\nu} \Bigg(\quad \Bigg)$$



Pseudo-PDF : Fixed z^2

$$P(x, -z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-ix\nu} \Bigg(\quad \Bigg)$$



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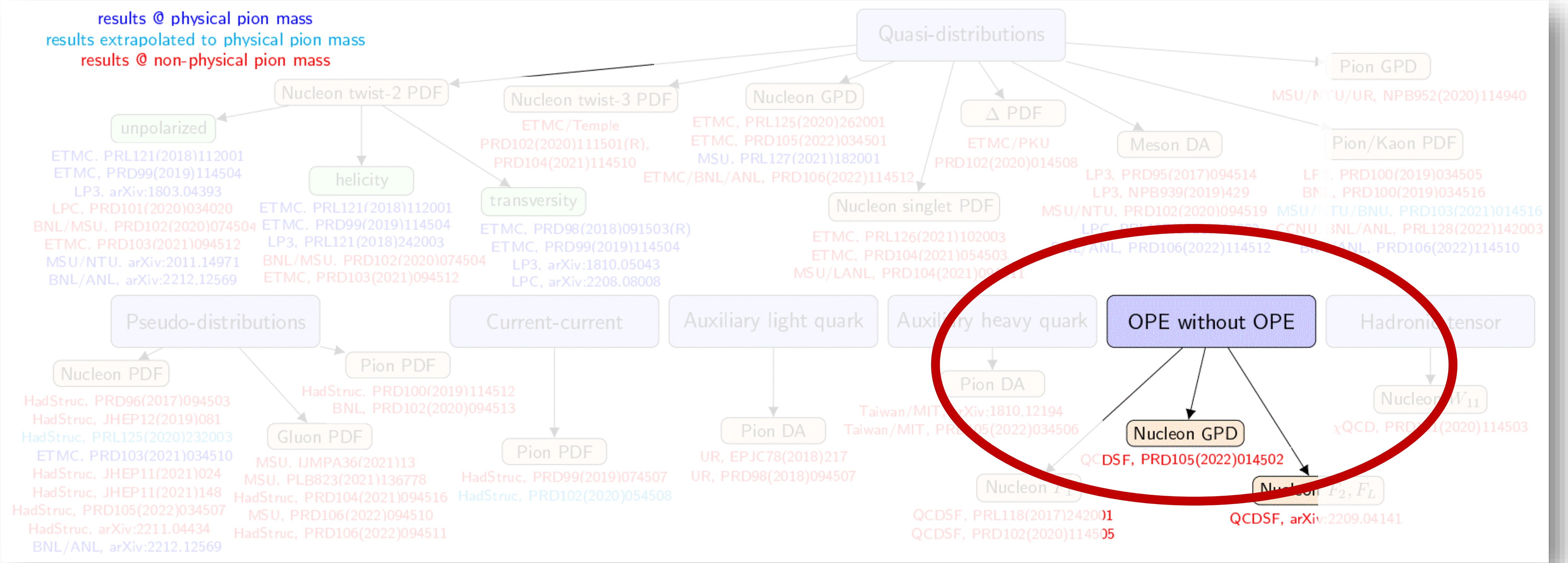
Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



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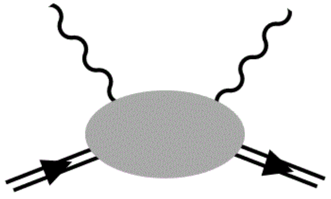
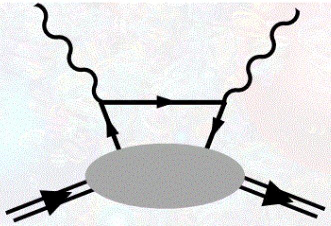
Compton amplitude in Lattices

Generalised parton distributions from the off-forward Compton amplitude in lattice QCD

A. Hannaford-Gunn,[1] K. U. Can,[1] R. Horsley,[2] Y. Nakamura,[3] H. Perlt,[4]
P. E. L. Rakow,[5] G. Schierholz,[6] H. Stüben,[7] R. D. Young,[1] and J. M. Zanotti[1]
(CSSM/QCDSF/UKQCD Collaborations)

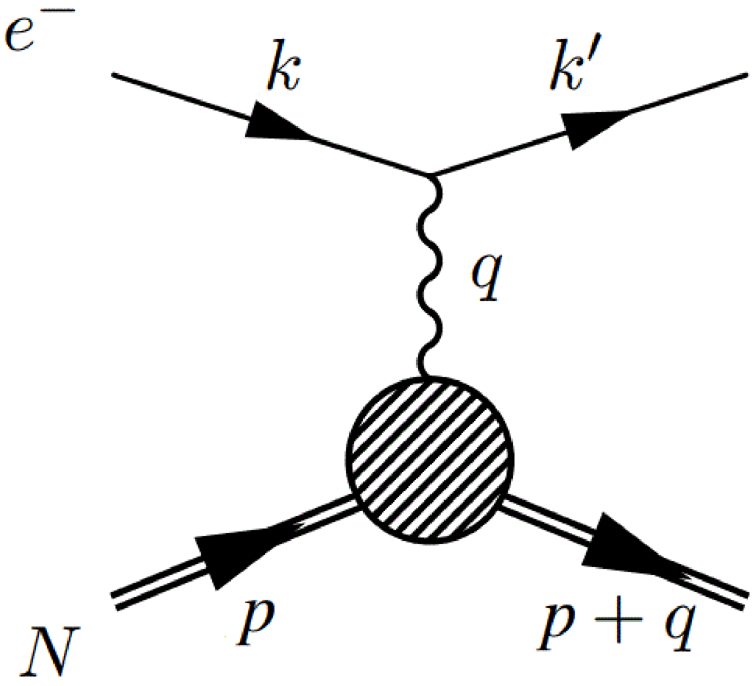
Example: **Forward Compton amplitude**

 $= $  $+ \ \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{1}{Q^2}\right)$

Courtesy: Utku Can

Compton amplitude in Lattices

Deep Inelastic Scattering (DIS)



DIS & Hadronic Tensor:

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \frac{F_2(x, Q^2)}{p \cdot q}$$

Structure Functions

Compton amplitude in Lattices

Forward Compton amplitude:

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$
$$= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2)+\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Same Lorentz decomposition as the Hadronic tensor

Compton amplitude in Lattices

Forward Compton amplitude:

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$
$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Dispersion relations connecting Compton SFs to DIS SFs:

$$\underbrace{\mathcal{F}_1(\omega,Q^2) - \mathcal{F}_1(0,Q^2)}_{\equiv \overline{\mathcal{F}}_1(\omega,Q^2)} = 2\omega^2\int_0^1 dx\frac{2x\,F_1(x,Q^2)}{1-x^2\omega^2-i\epsilon}$$

$$\mathcal{F}_2(\omega,Q^2) = 4\omega\int_0^1 dx\frac{F_2(x,Q^2)}{1-x^2\omega^2-i\epsilon}$$

Compton amplitude in Lattices

Forward Compton amplitude:

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathscr{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Compton amplitude approach gives access to moments of DIS SFs:

Example:

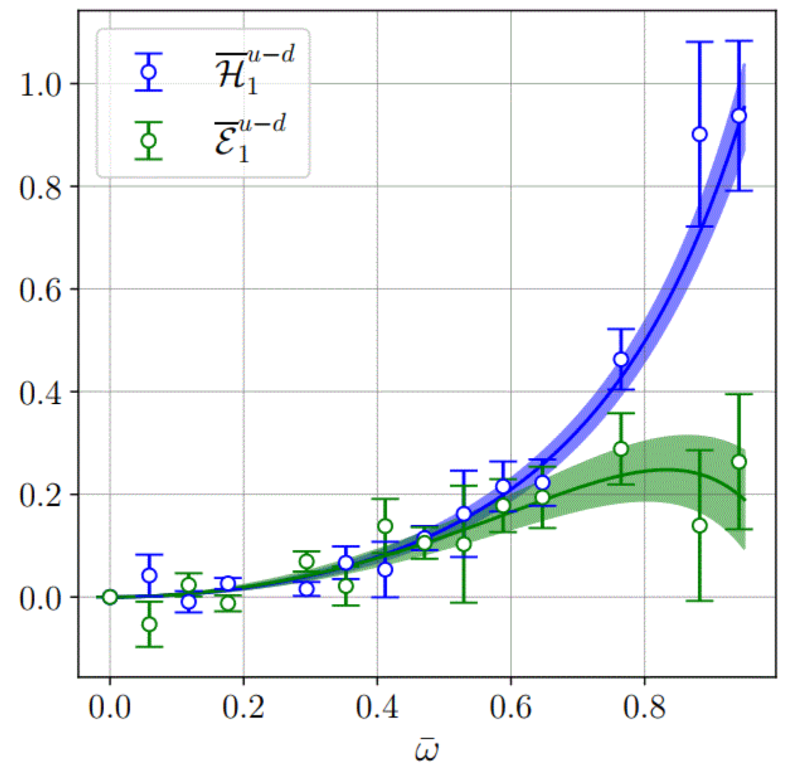
$$\mathscr{F}_2(\omega,Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1}M_{2n}^{(2)}(Q^2), \text{ where } M_{2n}^{(2,L)}(Q^2) = \int_0^1 dx\, x^{2n-2}F_{2,L}(x,Q^2)$$

Compton amplitude in Lattices

Off-forward is very similar

$$T_{\mu\nu} = \frac{1}{2\bar{P}\cdot\bar{q}}\left[-\left(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\right)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\left(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\right)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\right] + \ldots$$

This approach gives access to moments GPDs:



Isovector results for $t = -0.57$ GeV^2.

Outlook

- **Improving perturbative calculations**

- **Better understanding of power corrections**

- **Synergy with phenomenology ...**