Hadron Structure via Parton Distribution Functions

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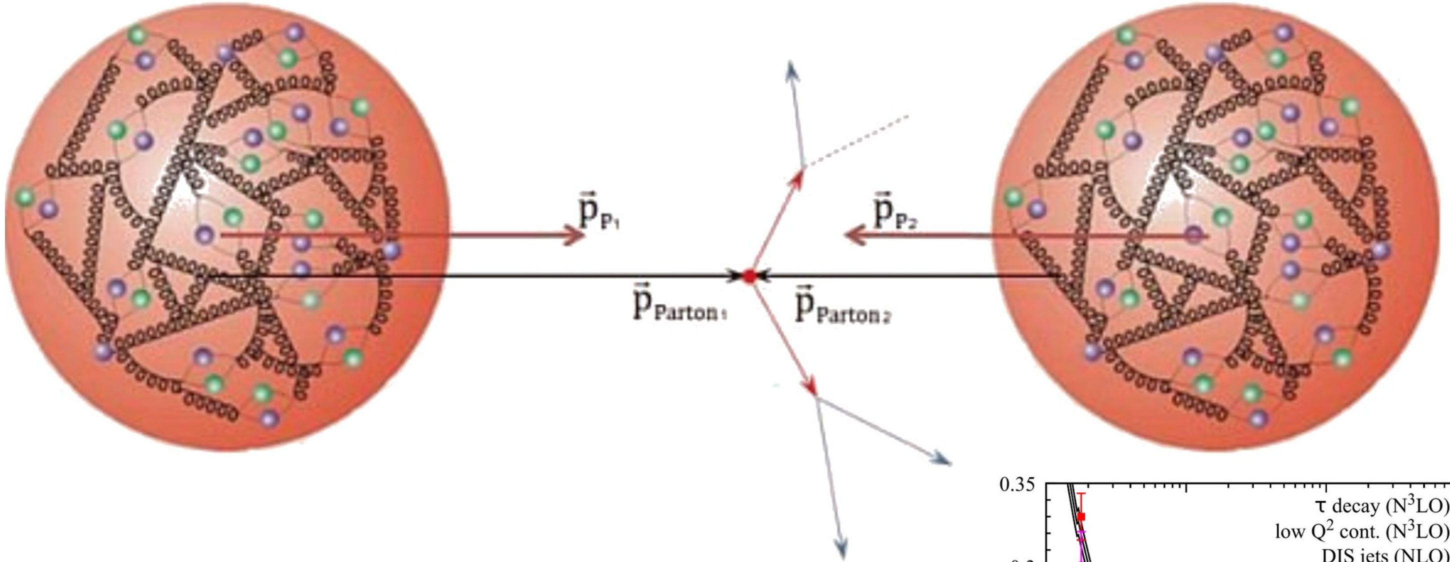
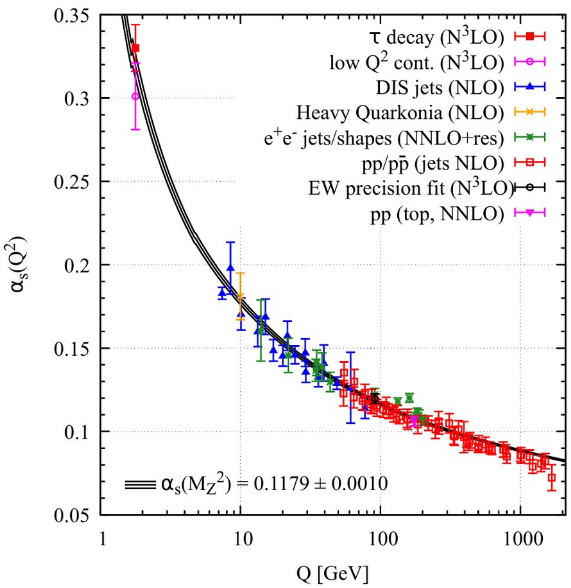
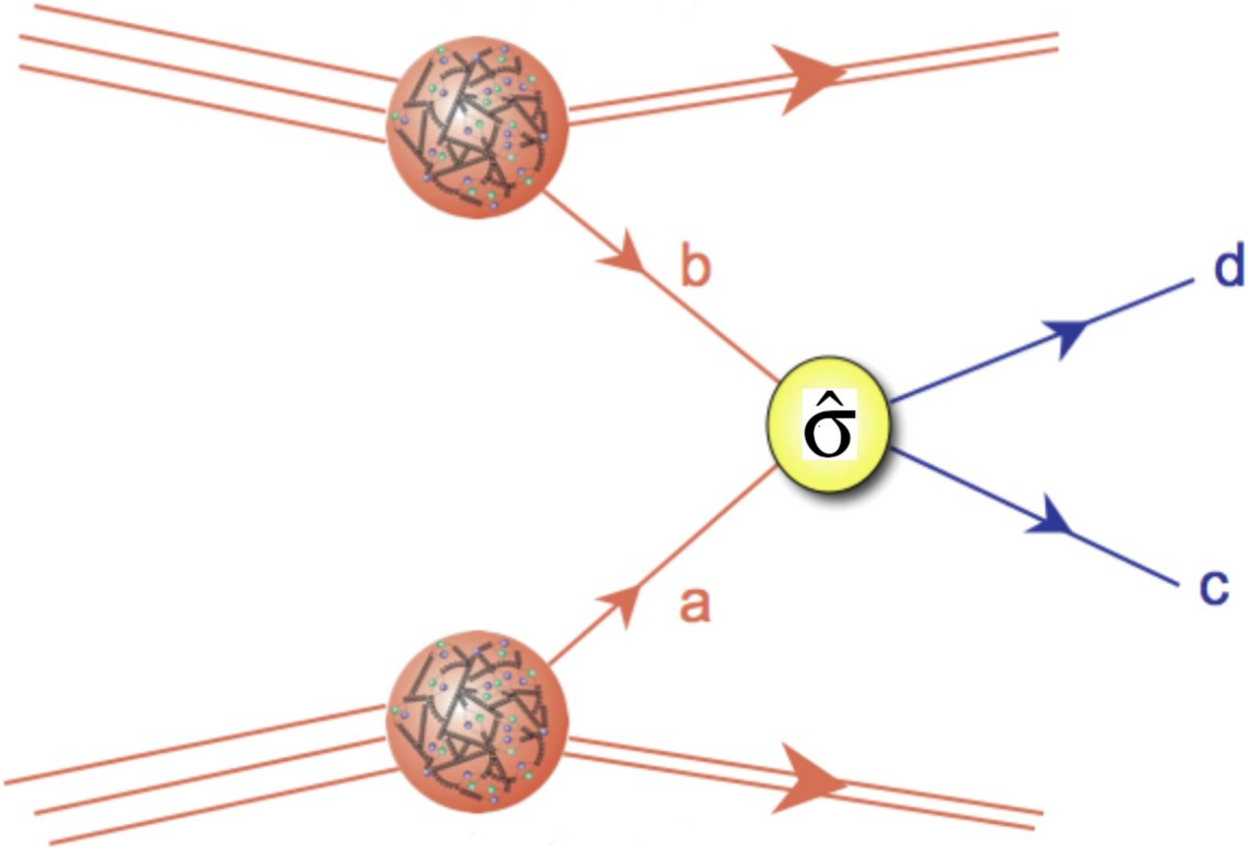
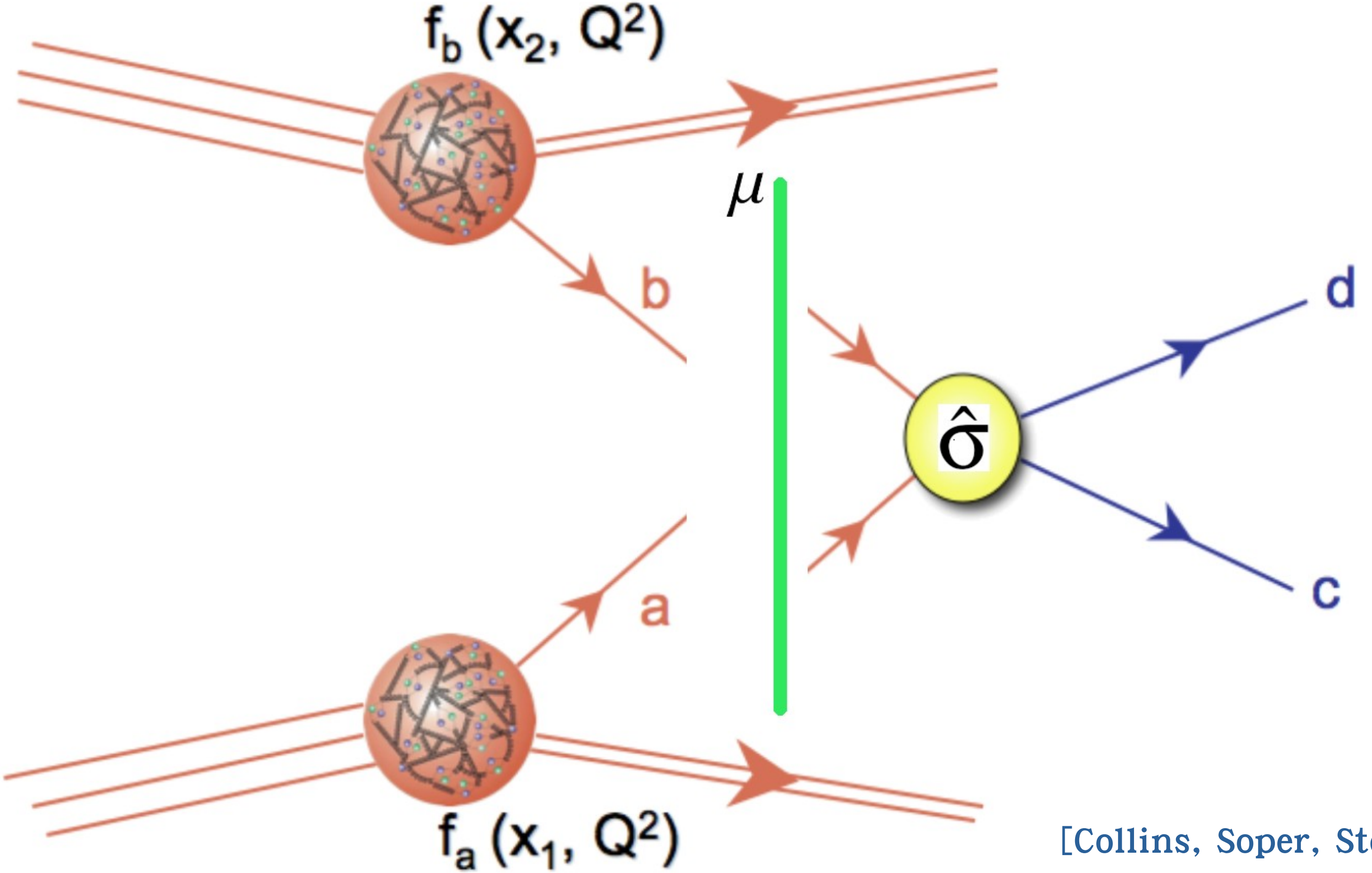


The strong interaction, which governs the collision, has the feature of asymptotic freedom.





The collision is characterized by low-energy **Parton Distribution Functions (PDFs)** and high-energy **hard-core interactions**.



[Collins, Soper, Sterman, 1989]

$$\sigma = f_a(x_1,\mu^2) \otimes f_b(x_2,\mu^2) \otimes \hat{\sigma}_{ab}(\mu^2)$$

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$$\frac{d\sigma}{d\ln\mu^2} = 0$$

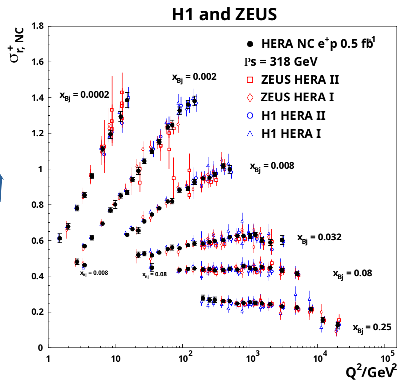
$$\frac{\partial f_a(x,\mu^2)}{\partial \ln\mu^2} = \sum_{j=g,q,\bar{q}} P_{a/j}(x,\mu^2) \otimes f_{j/p}(x,\mu^2)$$

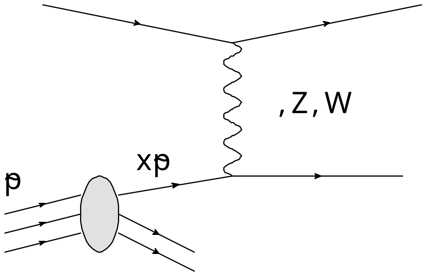
$$x f_a(x, Q_0, \{a_1, a_2, \ldots\}) = x^{a_1}(1-x)^{a_2} P_a(x)$$

- PDFs evolve with energy scale.
- With a given parametrized PDFs at Q_0 scale, the PDFs $f(x,\mu^2)$ is determined

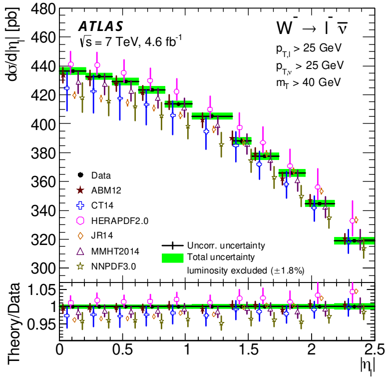
$$\sigma = f_a(x_1,\mu^2) \otimes f_b(x_2,\mu^2) \otimes \hat{\sigma}_{ab}(\mu^2)$$

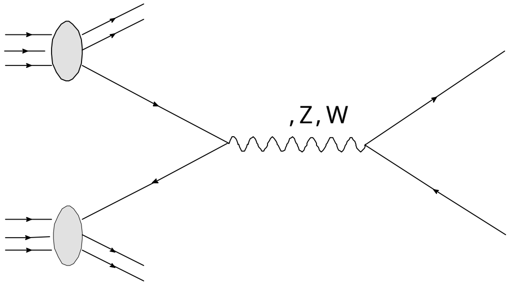
DIS



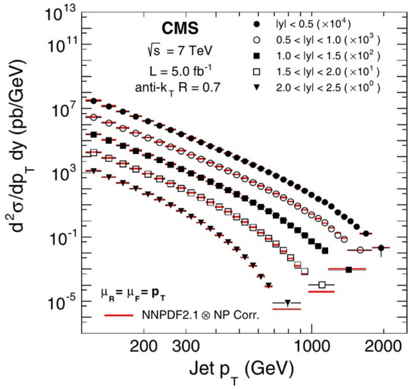


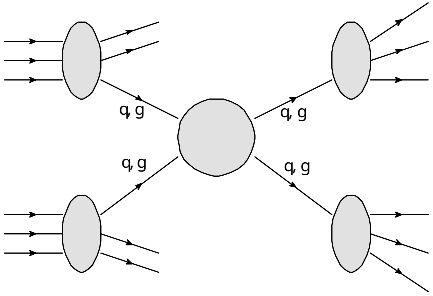
Drell-Yan



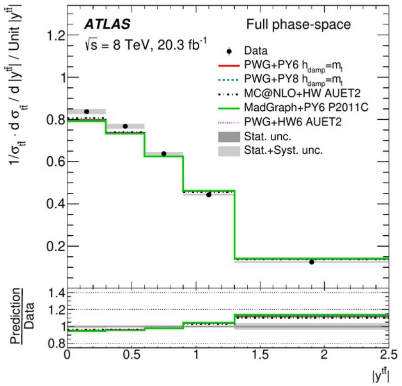


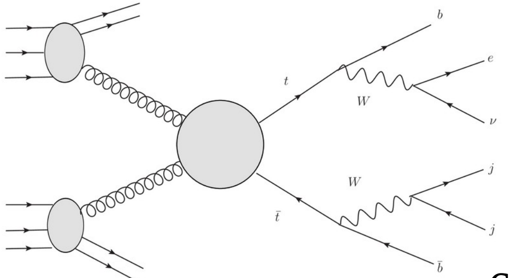
Jet





$t\bar{t}$





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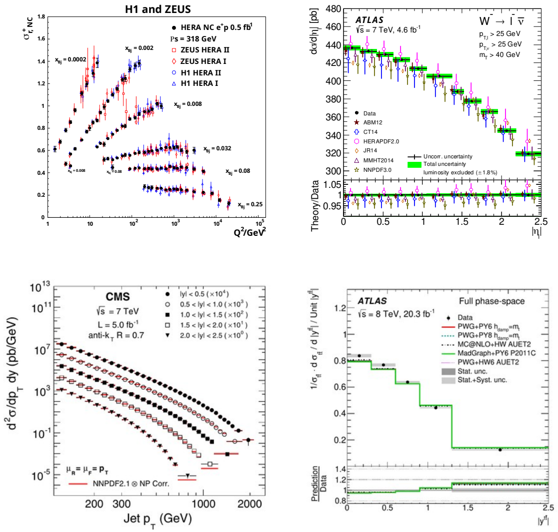
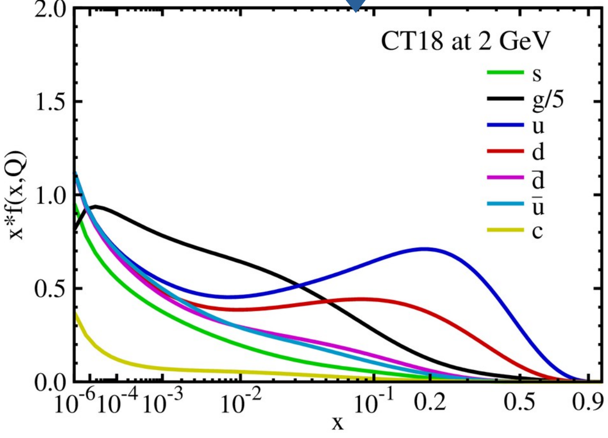
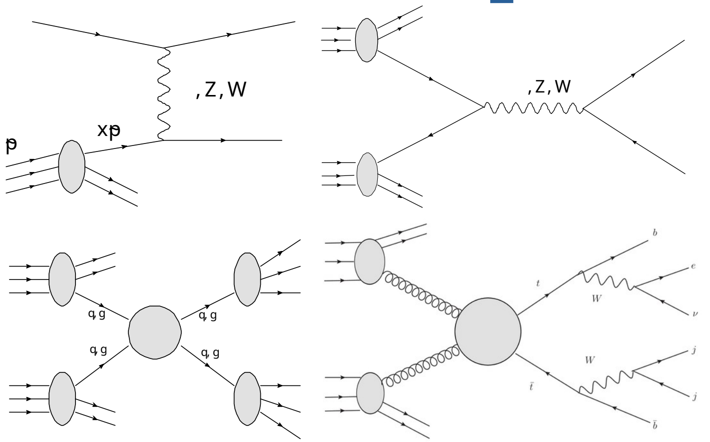
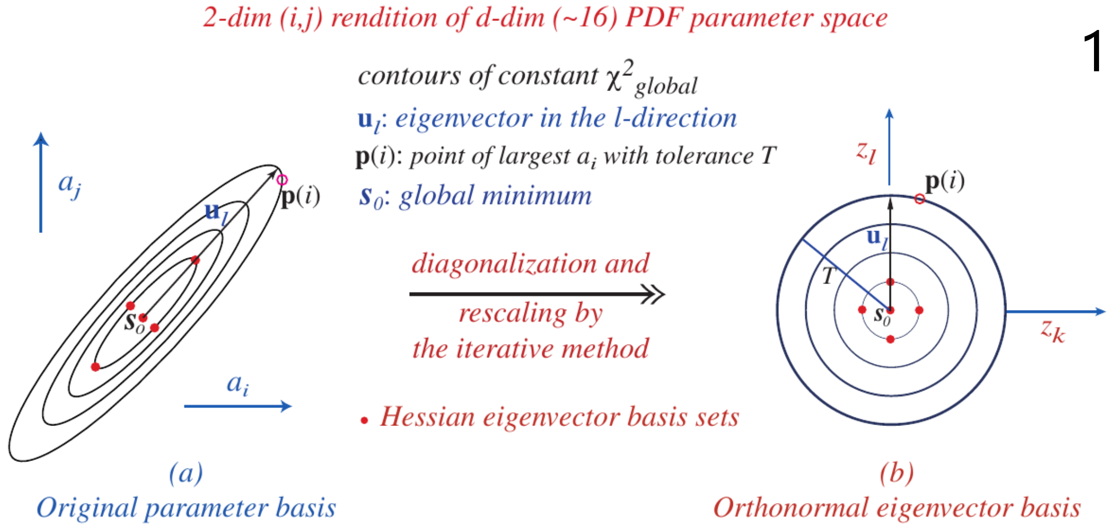






Uncertainty Estimation of PDFs



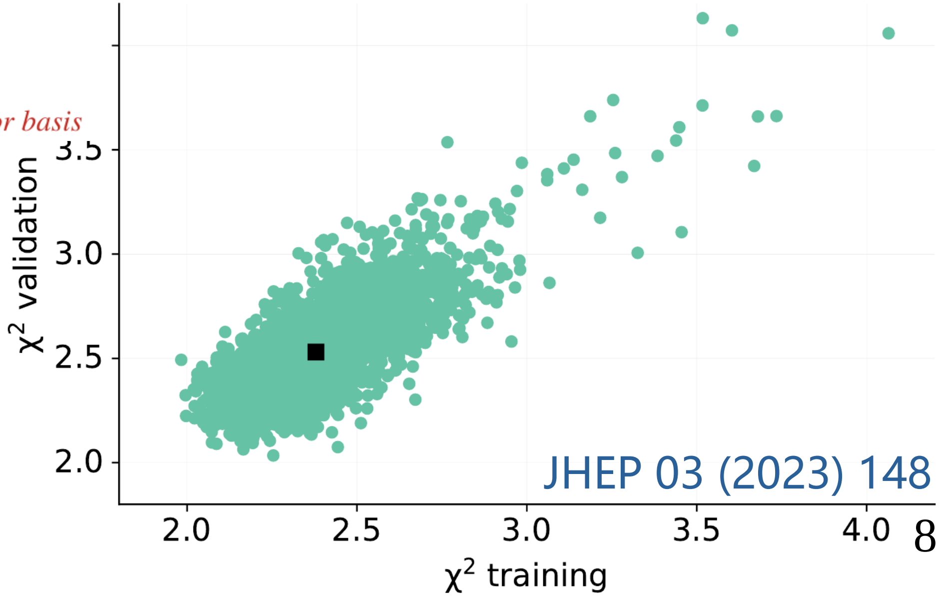
JHEP 07 (2002) 012

1. Hessian Method

Two replicas in each diagonalized parameter coordinate

Pro: less replicas, fast

Con: quadratic approximation



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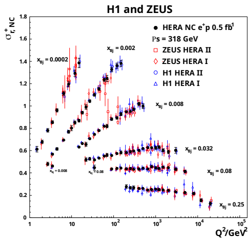
2. Monte Carlo Method

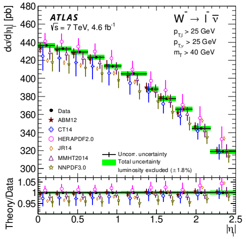
Replicas spread around the minimum

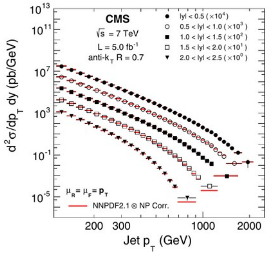
Pro: No assumption around minimum

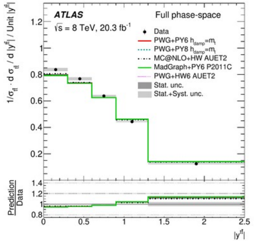
Con: Need more replicas, slow

$$\sigma = f_a(x_1,\mu^2) \otimes f_b(x_2,\mu^2) \otimes \hat{\sigma}_{ab}(\mu^2)$$





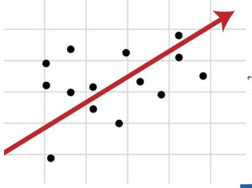


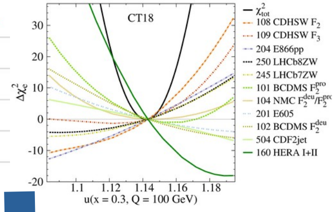


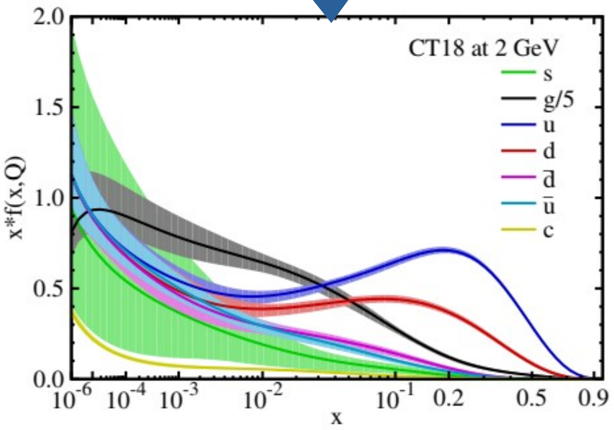
Statistic

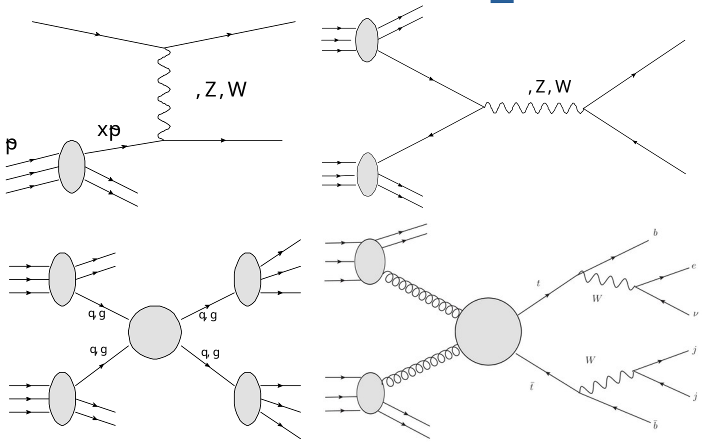
Regression analysis

Uncertainty Estimation

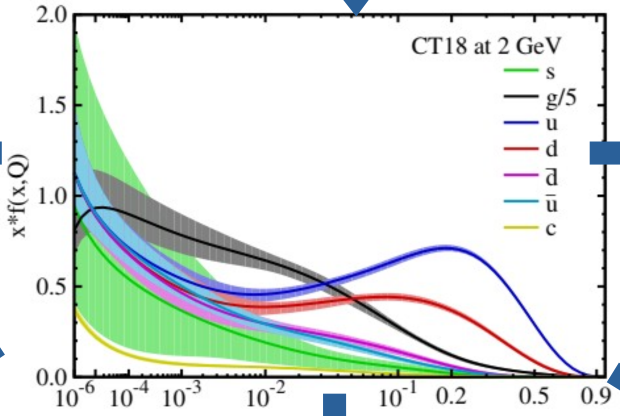




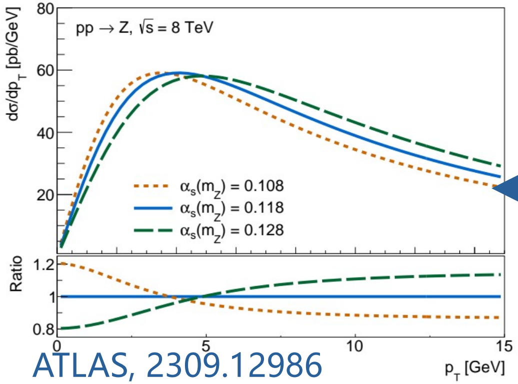




$$\sigma = f_a(x_1,\mu^2) \otimes f_b(x_2,\mu^2) \otimes \hat{\sigma}_{ab}(\mu^2)$$

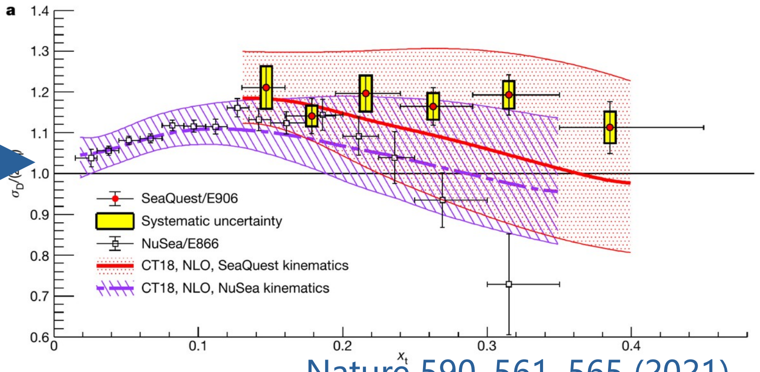


α_s



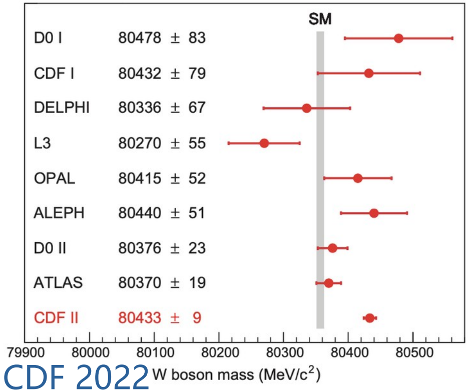
ATLAS, 2309.12986

$\sigma_D/(2\sigma_H)$



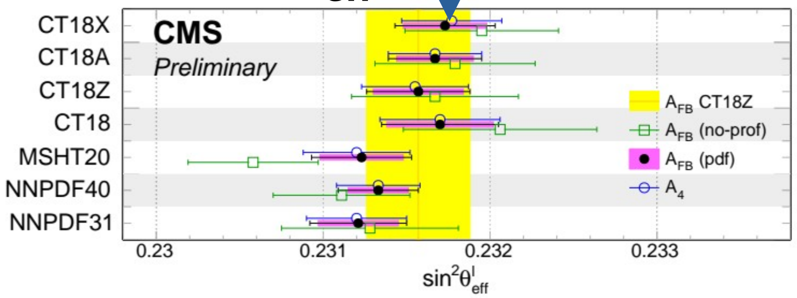
Nature 590, 561–565 (2021)

W Boson mass



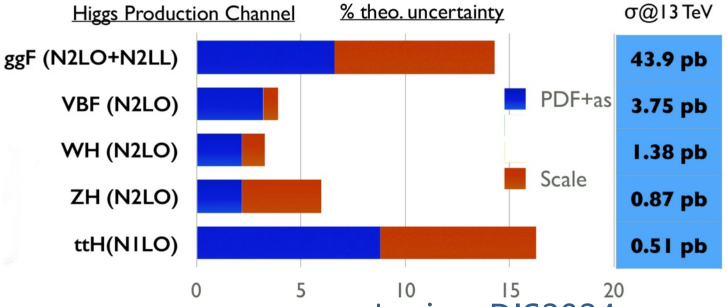
CDF 2022

$\sin^2\theta^l_{eff}$

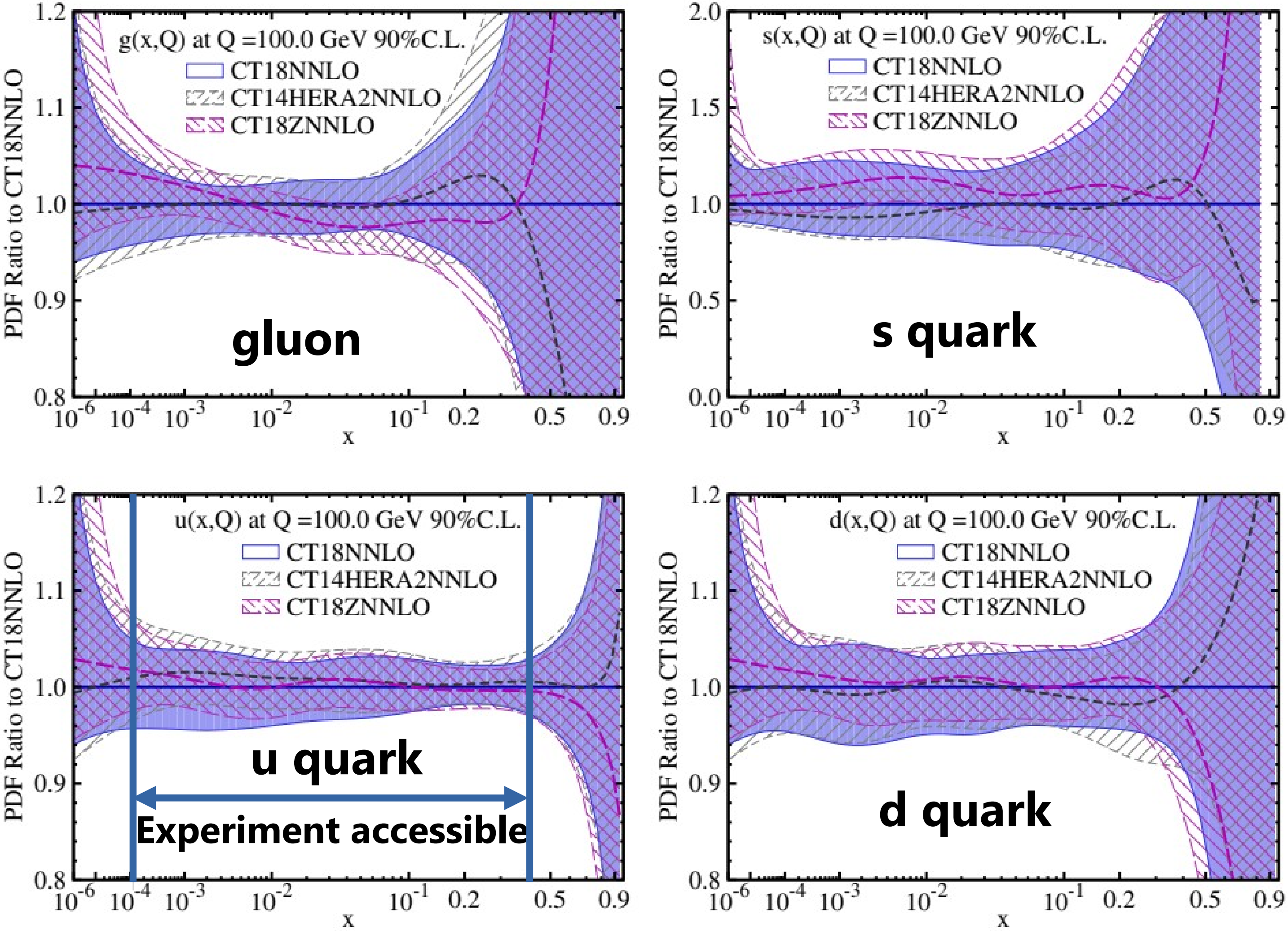


CMS PAS SMP-22-010

Higgs Production



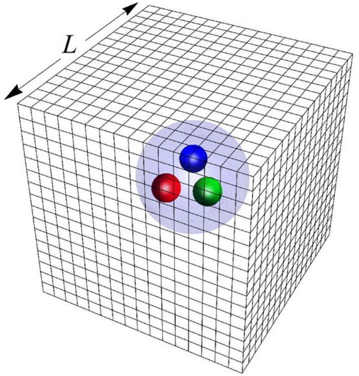
Lucian, DIS2024

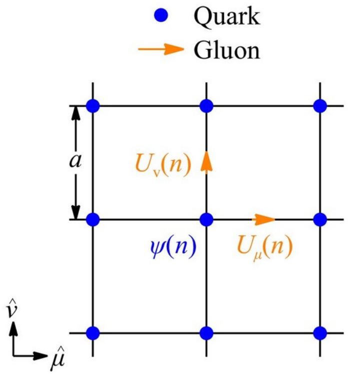


- PDFs are well determined in "middle-x" region: $10^{-4} \lesssim x \lesssim 0.4$
- Region of $x \to 1$ and $x \to 0$ are not experimental accessible.
- Rather large uncertainty for strangeness PDF, especially in large x region.

[Hou et al, 1912.10053]

- Earlier lattice calculations rely on operator product expansion, only provide moments $\langle x^n \rangle$, where $\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$
- Ideally, x-dependent information of PDFs can be obtained by full moments.
- However, for higher moments, the operators mix with lower-dimension operators, only lower moments are reliable.





- The common feature of all the approaches to the lattice PDF calculations is that they rely to some extent on the factorization framework:

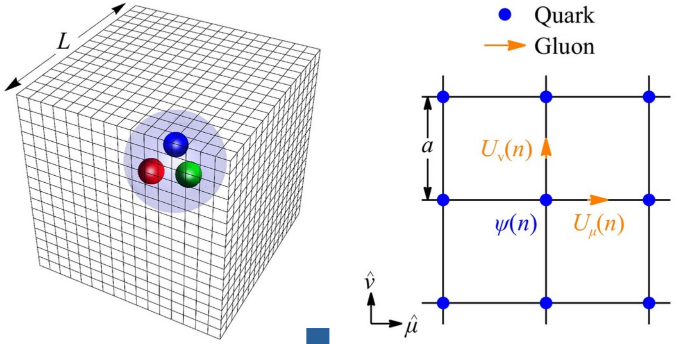
$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_R,\mu_F\right) \underbrace{q(y,\mu_F)}_{\text{PDF we want to extract}},$$

- Two classes of approaches:
  - ⁜ generalizations of light-cone functions; direct x-dependence,
  - ⁜ hadronic tensor; structure functions.

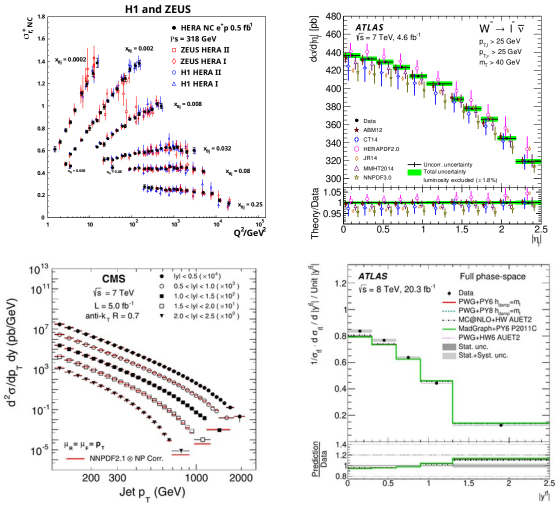
- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ, Γ' Dirac structure and objects $F(z)$.
  - ⁜ **hadronic tensor** - K.-F. Liu, S.-J. Dong, 1993
  - ⁜ **auxiliary scalar quark** - U. Aglietti et al., 1998
  - ⁜ **auxiliary heavy quark** - W. Detmold, C.-J. D. Lin, 2005
  - ⁜ **auxiliary light quark** - V. Braun, D. Müller, 2007

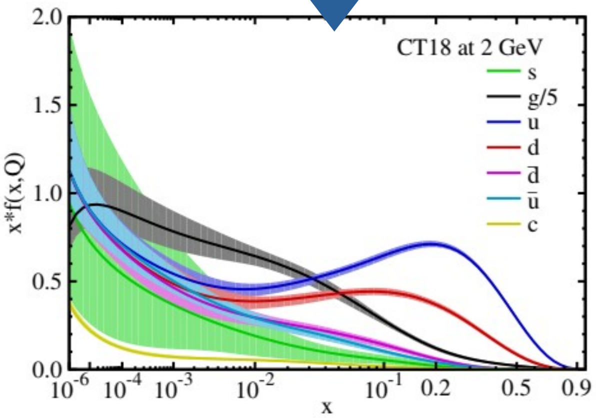
  (the four above) $\Rightarrow$ structure functions
  - ⁜ **quasi-distributions** - X. Ji, 2013
  - ⁜ ”**good lattice cross sections**” - Y.-Q. Ma, J.-W. Qiu, 2014, 2017
  - ⁜ **pseudo-distributions** - A. Radyushkin, 2017
  - ⁜ ”**OPE without OPE**” - QCDSF, 2017

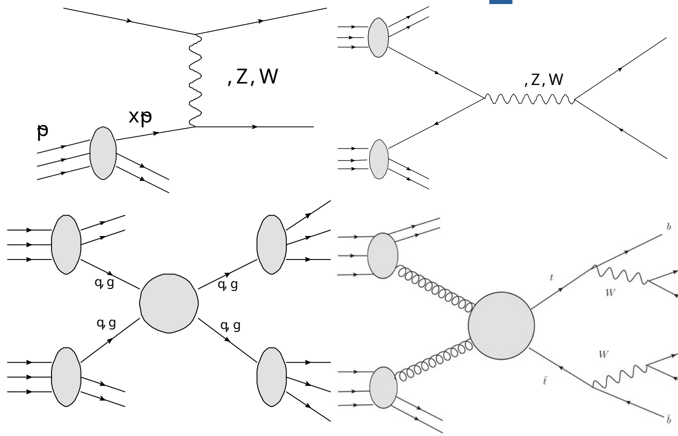
[Cichy and Sato, 2020]



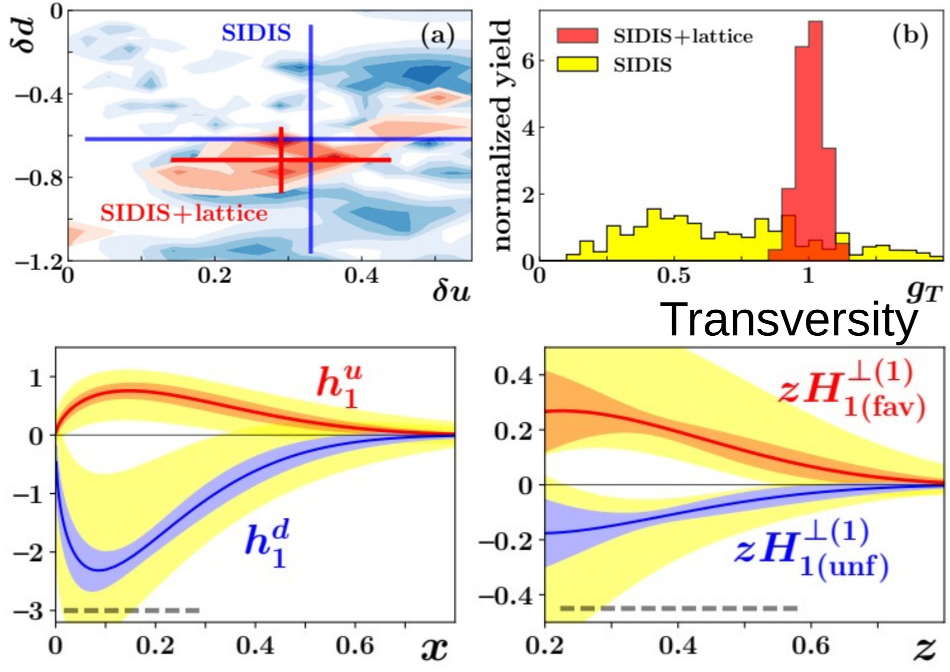
$$\sigma = f_a(x_1,\mu^2) \otimes f_b(x_2,\mu^2) \otimes \hat{\sigma}_{ab}(\mu^2)$$



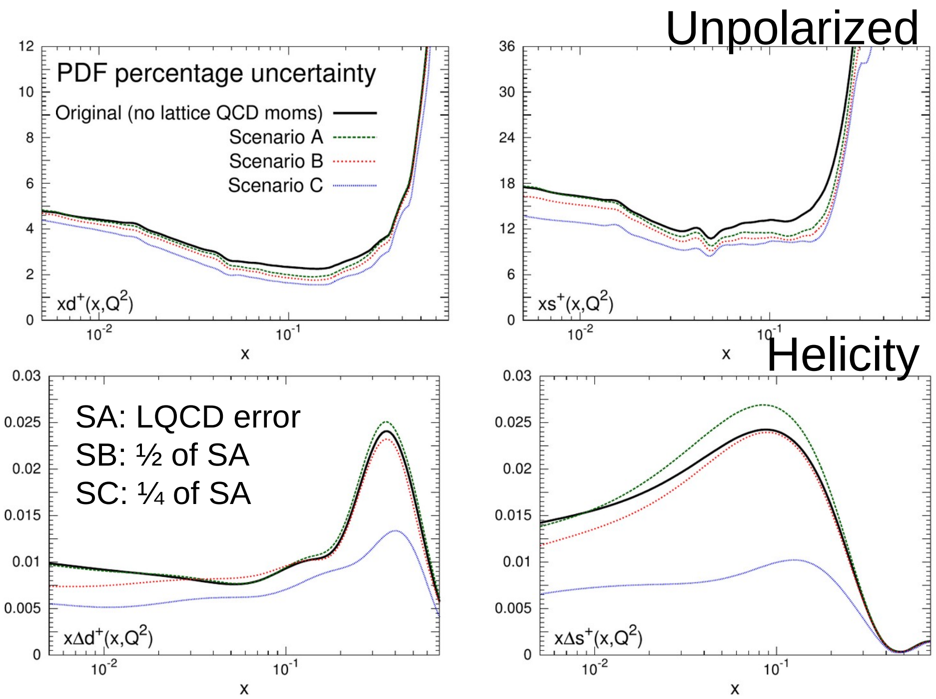




Global analysis with moments from LQCD

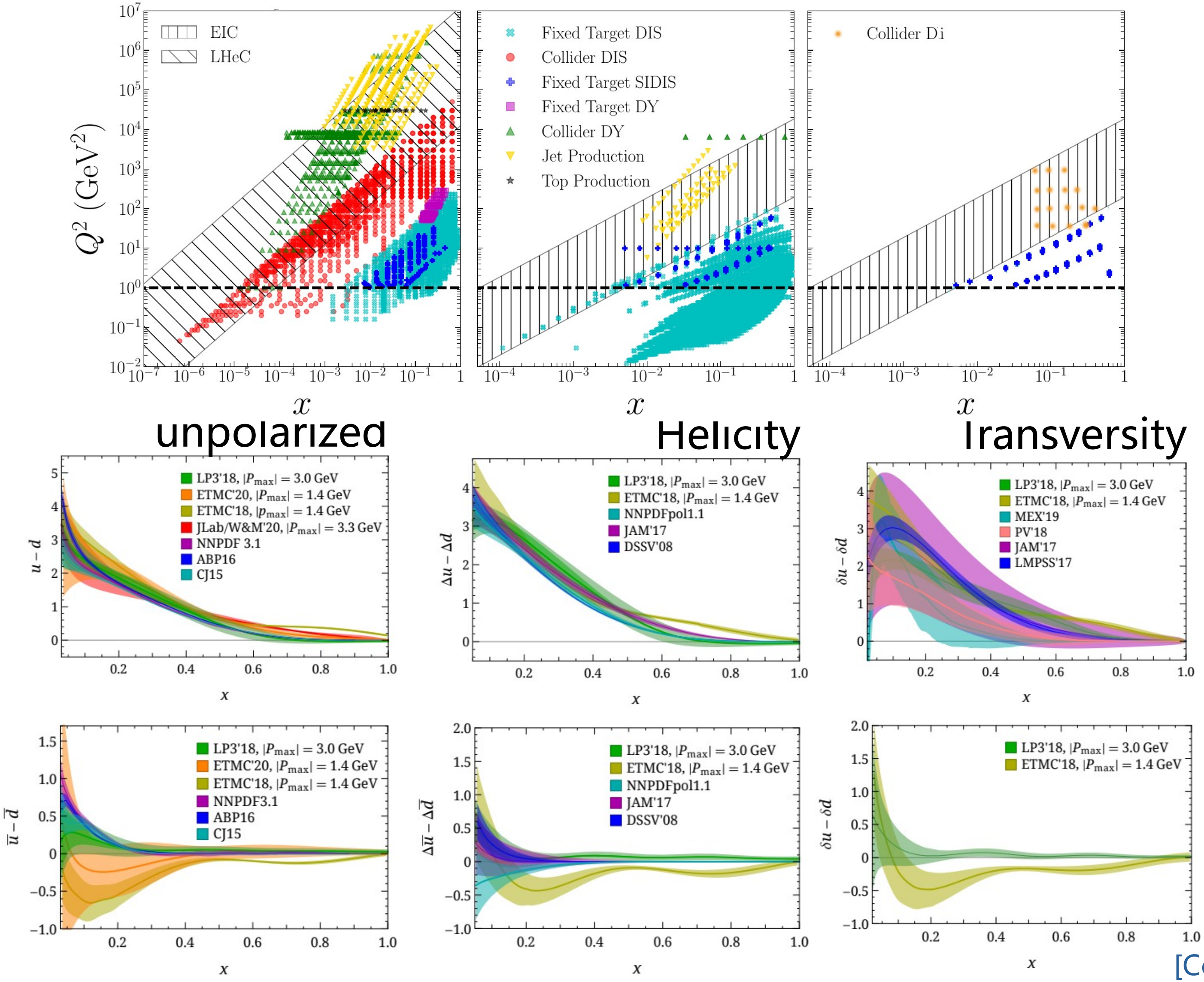


Global QCD analysis of the quark transversity distributions receiving constraint from the average lattice value of g_T [Lin et al, 1710.09858].



Potential impact of future lattice-QCD calculations in global unpolarized and polarized PDF fits [Lin et al, 1711.07916]

With the Lattice input in the format of moments, it is helpful on reducing PDFs uncertainty



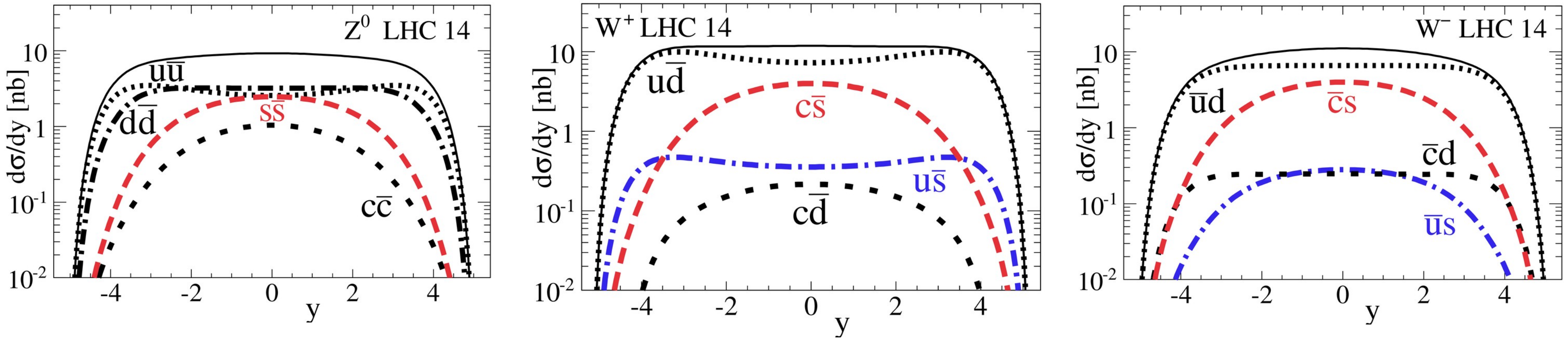
- Lots of efforts on including x-dependent lattice calculation in global analysis have been made. [1907.06037, 2009.05522, 2010.00548, 2010.03996, 2204.00543]
- Unpolarized collinear PDFs receive more experimental measurements in global analysis; while the helicity and transversity PDFs receive rather less experiments.

[Constantinou et al, 2006.08636]

Despite constraints on *moments*, can Lattice input directly assist in unpolarized PDF determination through global analysis?

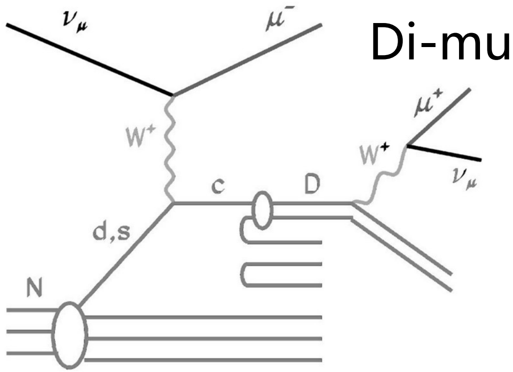
Flavor separation is one of the most challenging tasks in QCD global analysis, specially in the strangeness sector which plays an important role in precision electroweak physics, such as the determination of the W mass.

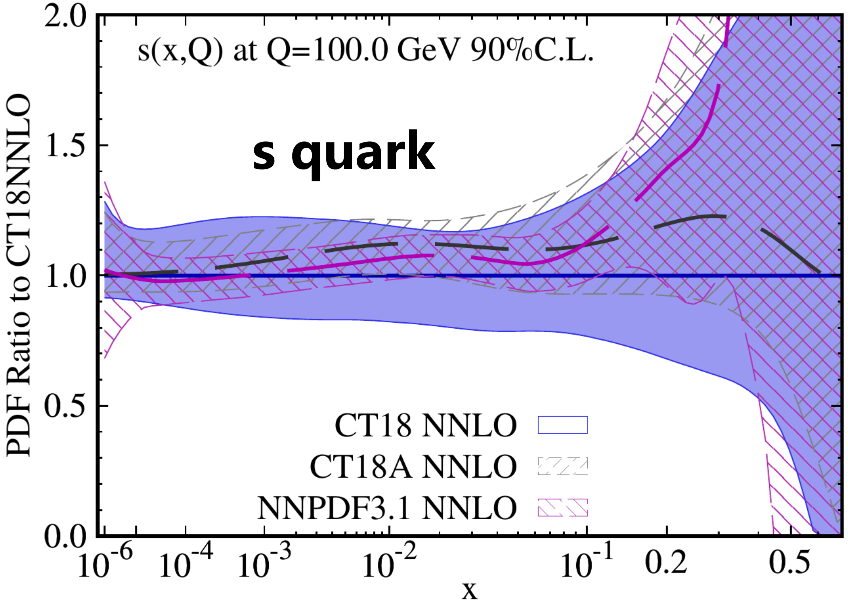
[Kusina et al, 1203.1290]



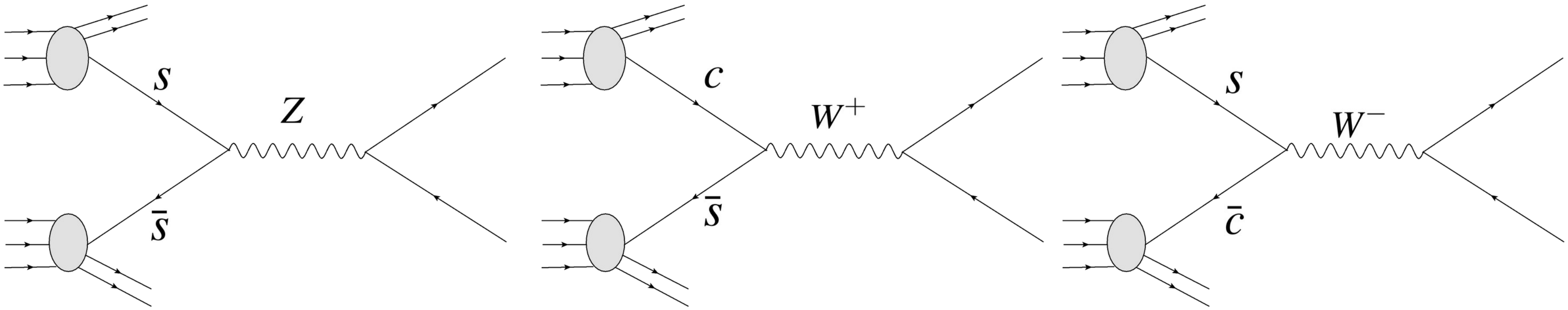
In global analysis, only the SIDIS di-muon[Goncharov et al, hep-ex/0102049] data probe the strangeness directly: the neutrino process probe the strangeness PDF, while the anti-neutrino process probe the antistrangeness PDF.

Di-muon

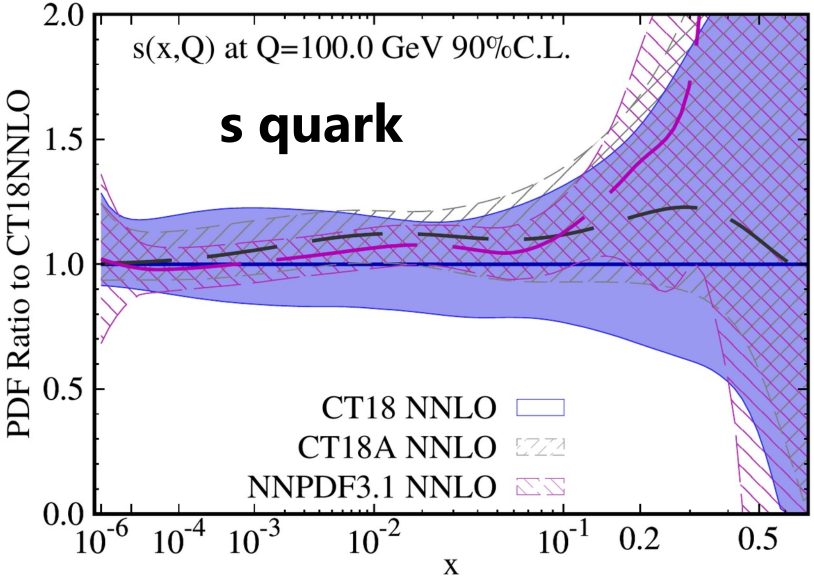


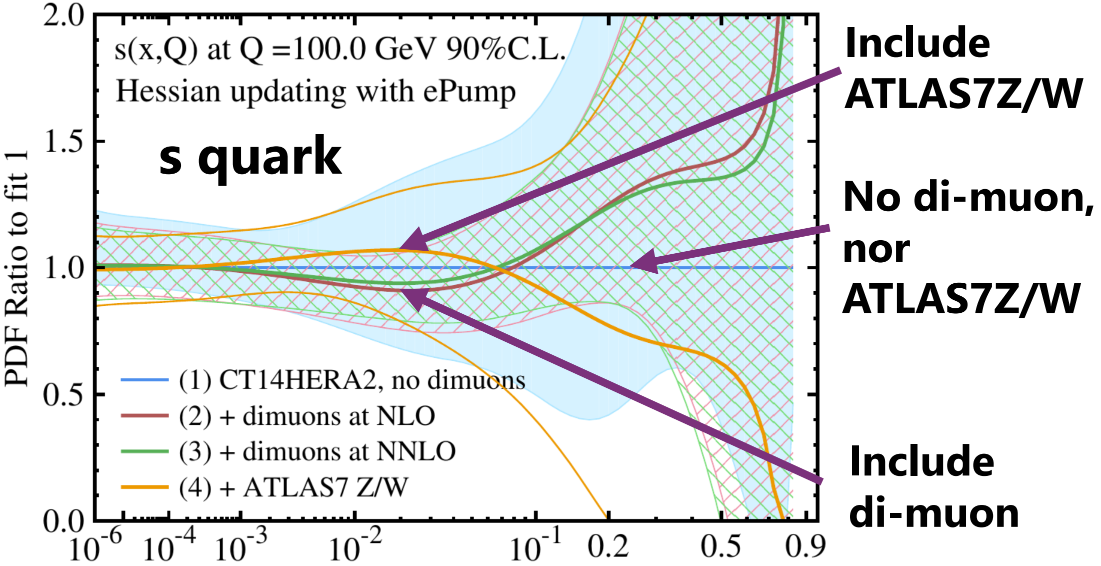


In the CT18A global analysis with **ATLAS 7 TeV Z/W**[ATLAS, 1612.03016] data included, we observe significant enhancement of strangeness PDF as compared to CT18. This is also observed in MSHT[Bailey, 2012.04684] and NNPDF[Ball, 2109.02653].



Strangeness decomposition

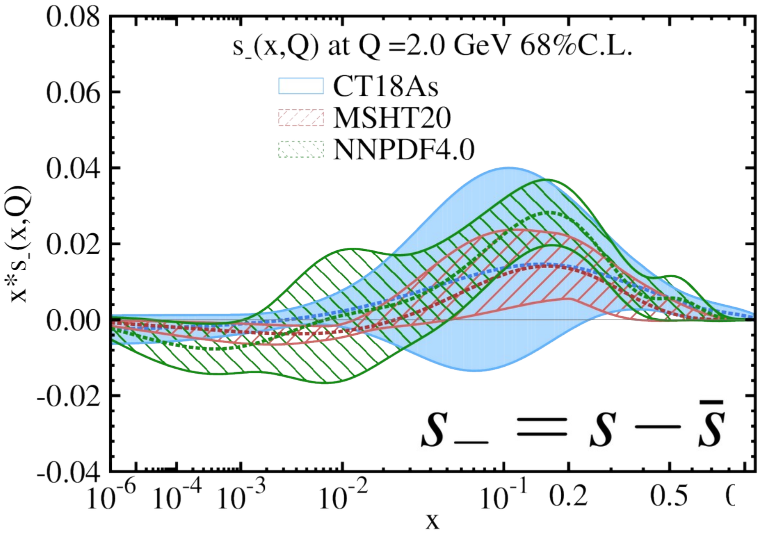




[Hou et al, 1912.10053]

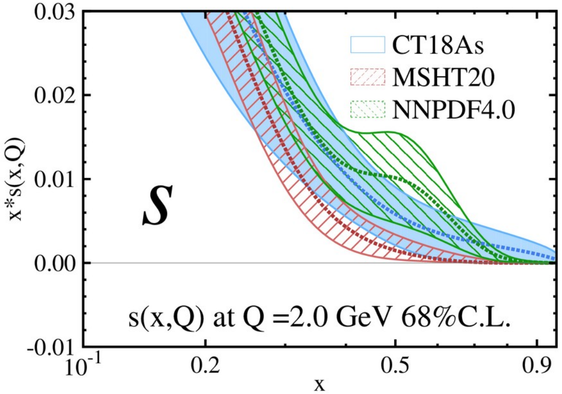
- Noticeable tensions between the SIDIS di-muon data and the precision ATLAS 7 TeV Z/W data were found in global analysis.
- In MSHT20[Bailey, 2012.04684], it was concluded that allowing $s \neq \bar{s}$ at the Q_0 scale can release some of these tensions.

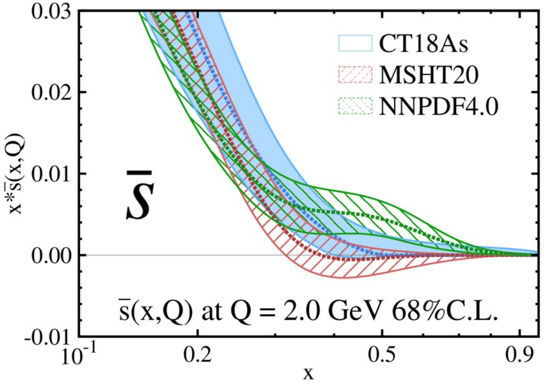
CT18As: CT18A allowing $s \neq \bar{s}$ at $Q_0 = 1.3$ GeV



Allowing strangeness not equal to antistrangeness, the CT18As, which is the CT18A with strangeness asymmetry, presents similar strangeness asymmetry as MSHT20 and NNPDF4.0.

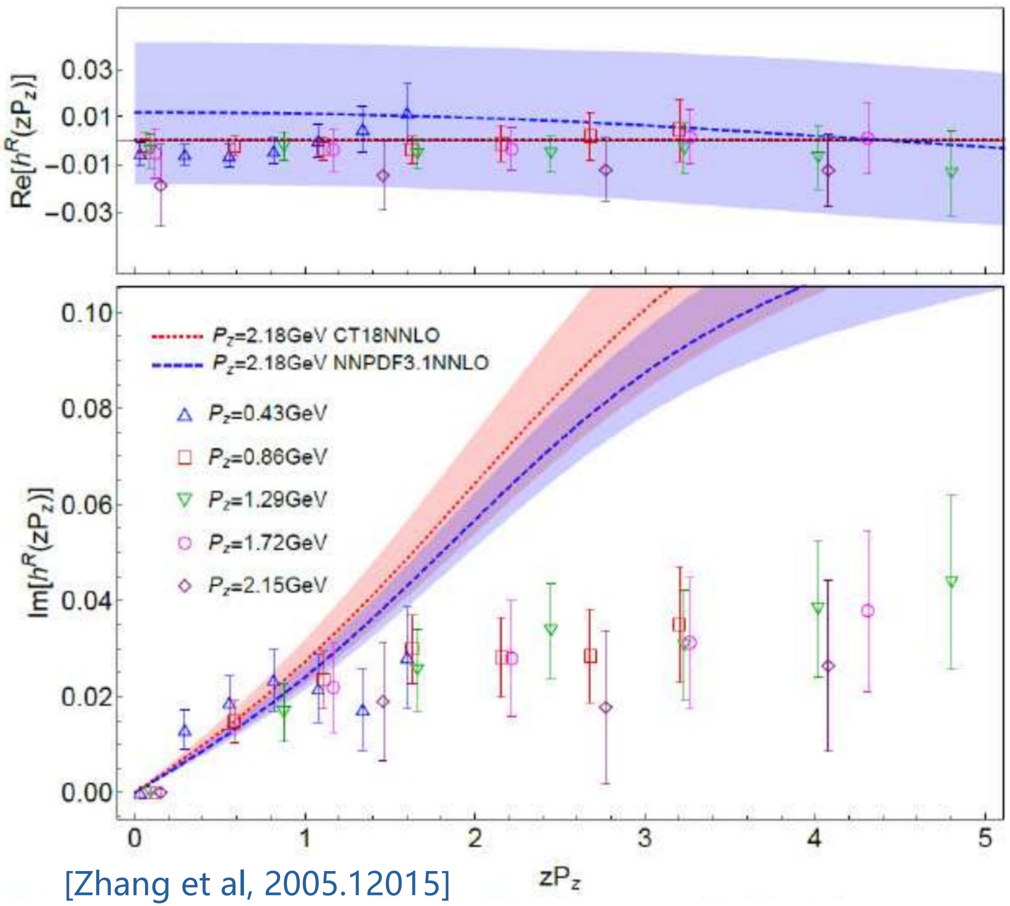
- Both SIDIS di-muon and ATLAS 7 TeV Z/W data can constraint strangeness PDF.
- **Are there any other data for determining the strangeness asymmetry?**





[Tie-Jiun Hou, Huey-Wen Lin, Mengshi Yan, C.-P. Yuan, 2211.11064]

From quasi-PDF to PDF



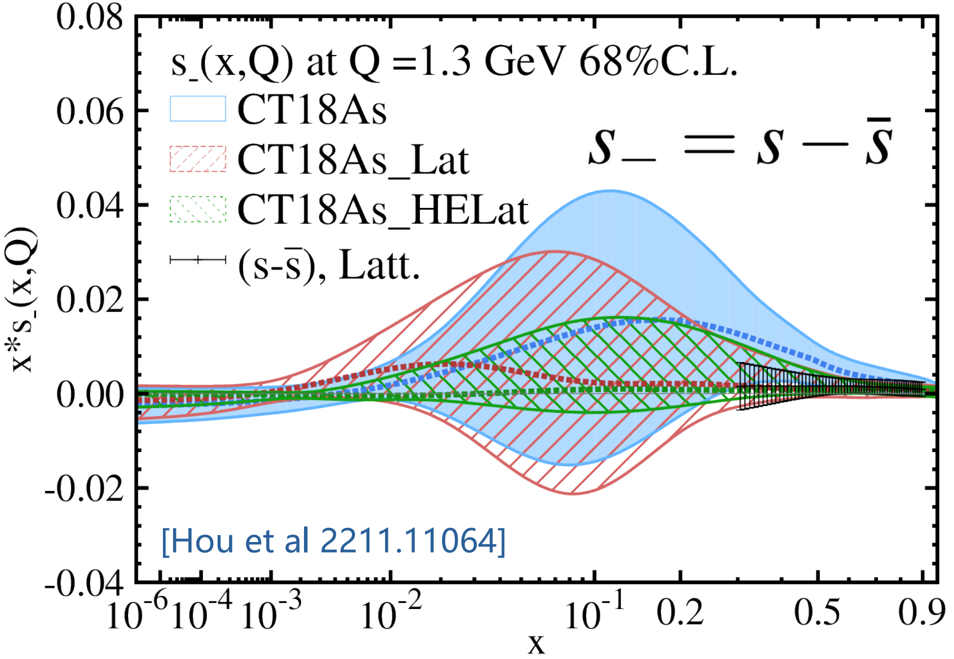
[Zhang et al, 2005.12015]

- Due to the large uncertainty in strangeness PDF from global analysis, lattice QCD calculation is able to provide more information.

$$\mathrm{Re}[h(z)] \propto \int dx\, (s(x) - \bar{s}(x)) \cos(xzP_z)$$

$$\mathrm{Im}[h(z)] \propto \int dx\, (s(x) + \bar{s}(x)) \sin(xzP_z)$$

- MSULat/quasi-PDF method
- Clover on 2+1+1 HISQ
- 0.12 fm, 310 MeV QCD vacuum
- RI/MOM renormalization
- Extropolartion to M_π = 140 MeV

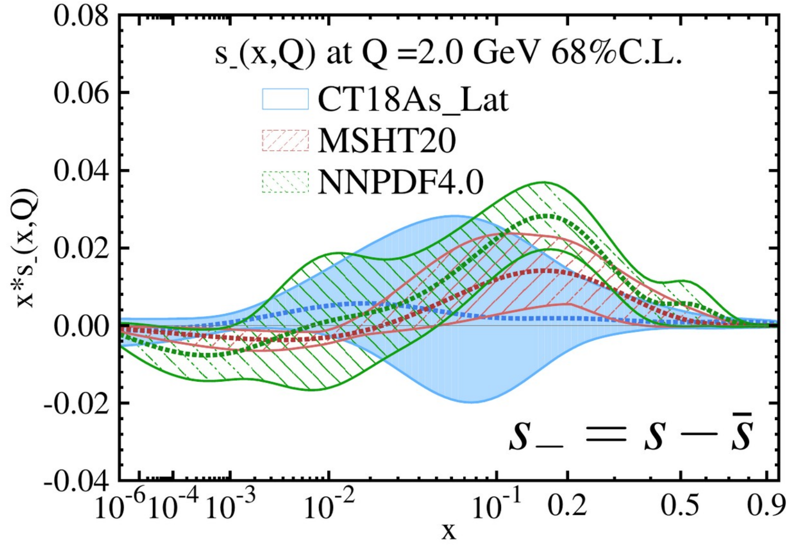


CT18As: CT18A with strangeness asymmetry at $Q_0 = 1.3$ GeV.

CT18As_Lat: PDFs with lattice input.

CT18As_HELat: PDFs with the lattice errors reduced by half.

- Lattice QCD calculation provide prediction at $0.3 < x < 0.8$, while the di-muon data constraint strangeness at $0.015 < x < 0.336$.
- Lattice input improves the determination of strangeness asymmetry.
- LQCD can improve heavy flavor decomposition.



Beyond heavy flavor decomposition,
can Lattice input directly assist in
reducing uncertainties of unpolarized PDFs
through global analysis?

Uncertainty of Gluon PDF

Jet data sets included in each fits

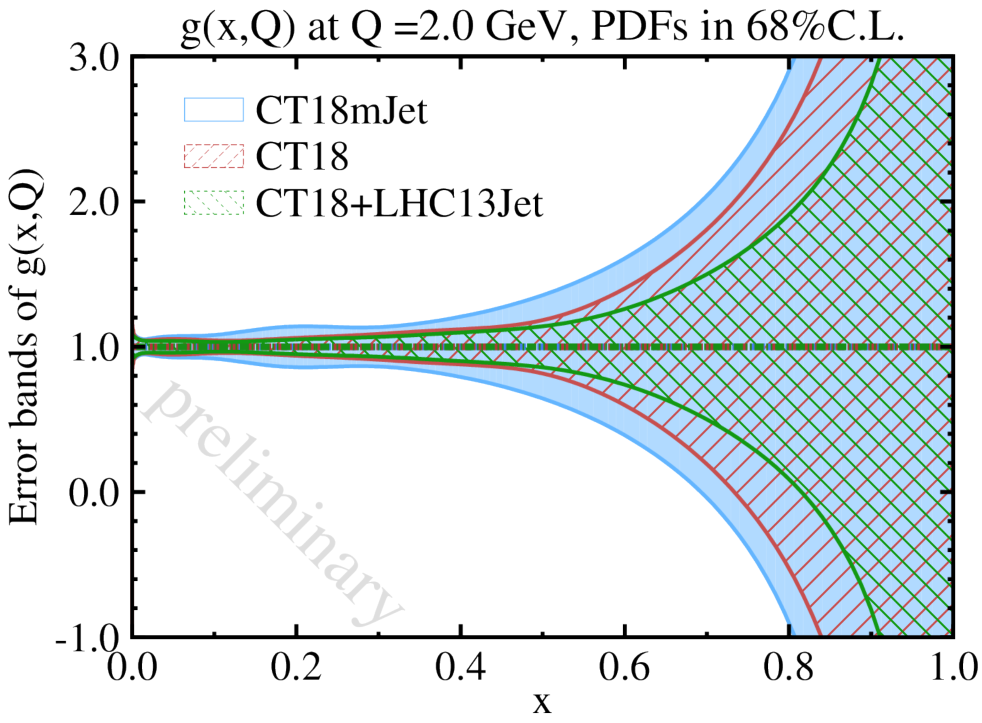
CT18mJet: no jet data

CT18:

CDF	0807.2204	1.13 fb^{-1}
D0	0802.2400	0.7 fb^{-1}
CMS7	1406.0324	5.0 fb^{-1}
ATL7	1410.8857	4.5 fb^{-1}
CMS8	1609.05331	19.7 fb^{-1}

CT18+LHC13jet:

ATL8	1706.03192	20.2 fb^{-1}
ATL13	1711.02692	3.2 fb^{-1}
CMS13	2111.10431	36.3 fb^{-1}

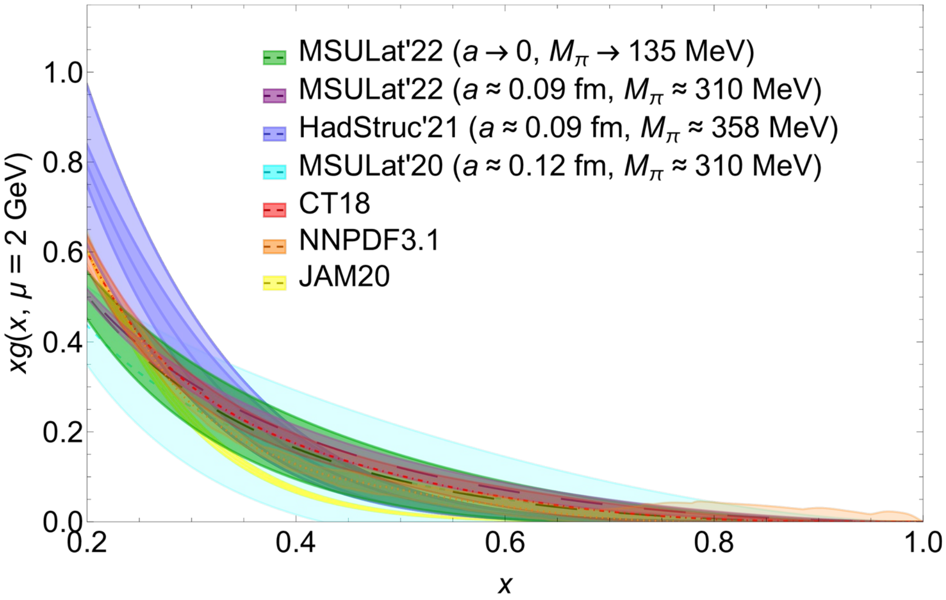


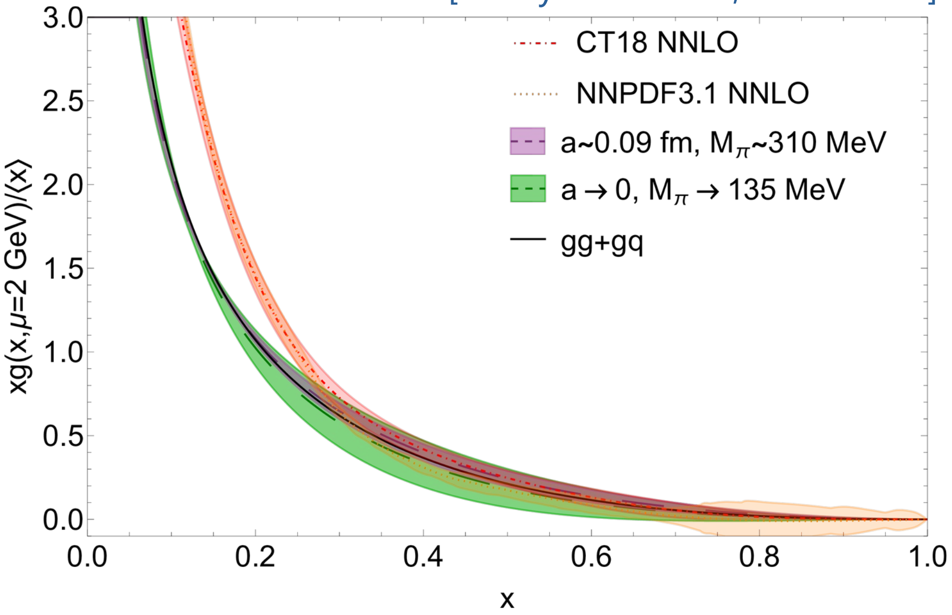
The uncertainty of the gluon PDF in the large-x region receives constraints from jet data.

Gluon PDF from Lattice QCD

Result from MSULat/quasi-PDF method

[Zhouyou Fan et al, 2210.09985]





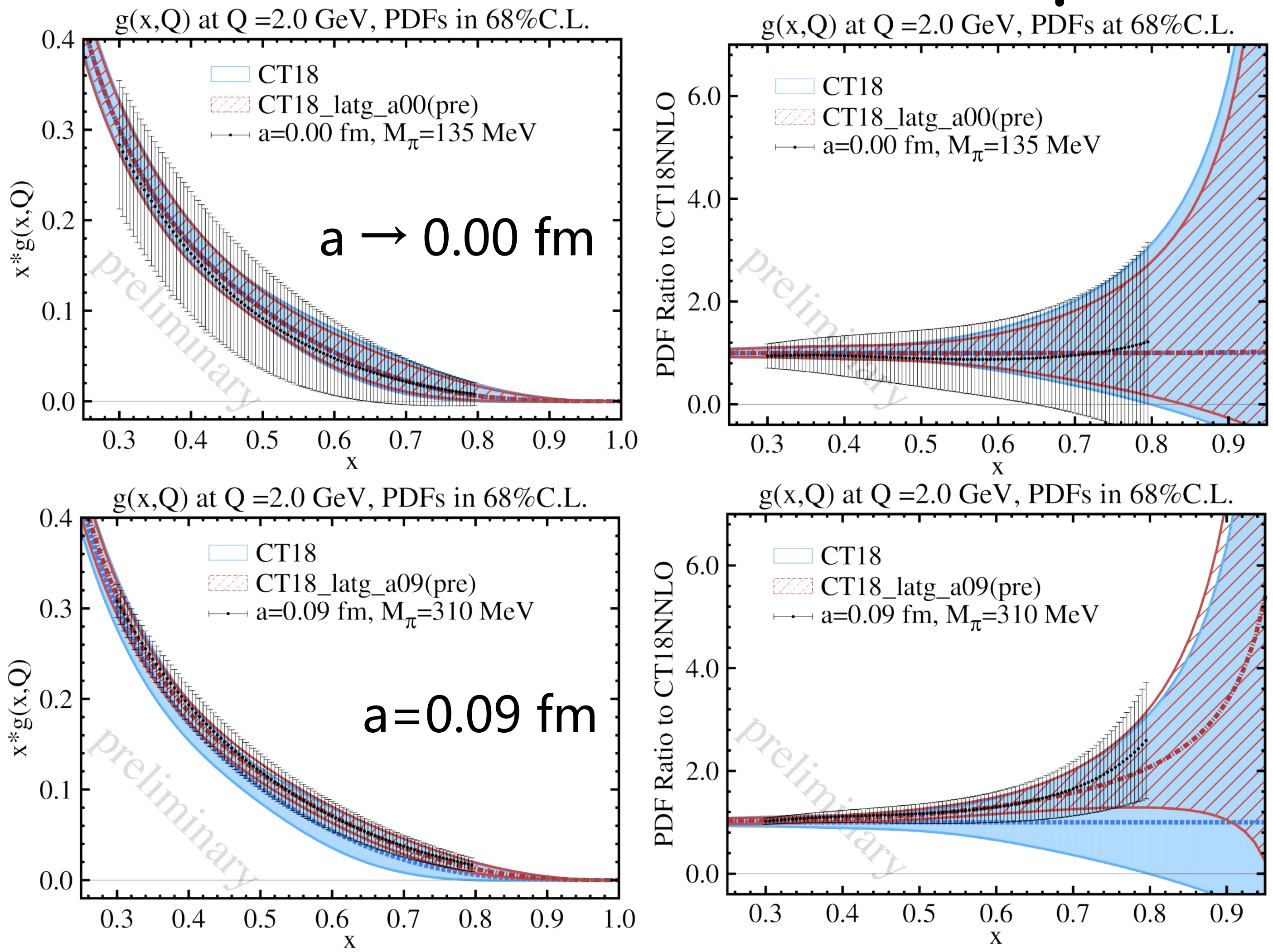
- Clover on 2+1+1 HISQ
- $a \to 0$ fm, $M_\pi \to 135$ MeV and $a \approx 0.09$ fm, $M_\pi \approx 310$ MeV which represent the current and future levels of uncertainty.

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x,\mu^2)}{\langle x \rangle_g} R_{gg}(x\nu, z^2\mu^2)$$

- Good agreement with CT18 and NNPDF3.1
- Gluon PDF xg(x) extra at 2 GeV

Work in progress

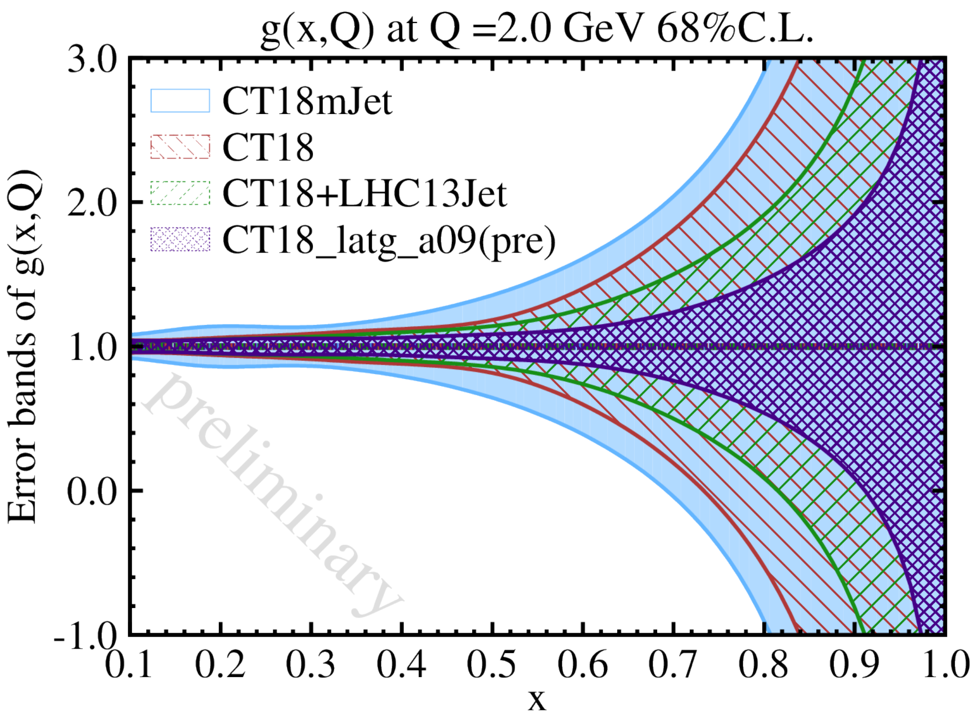
CT18 with Lattice input on gluon

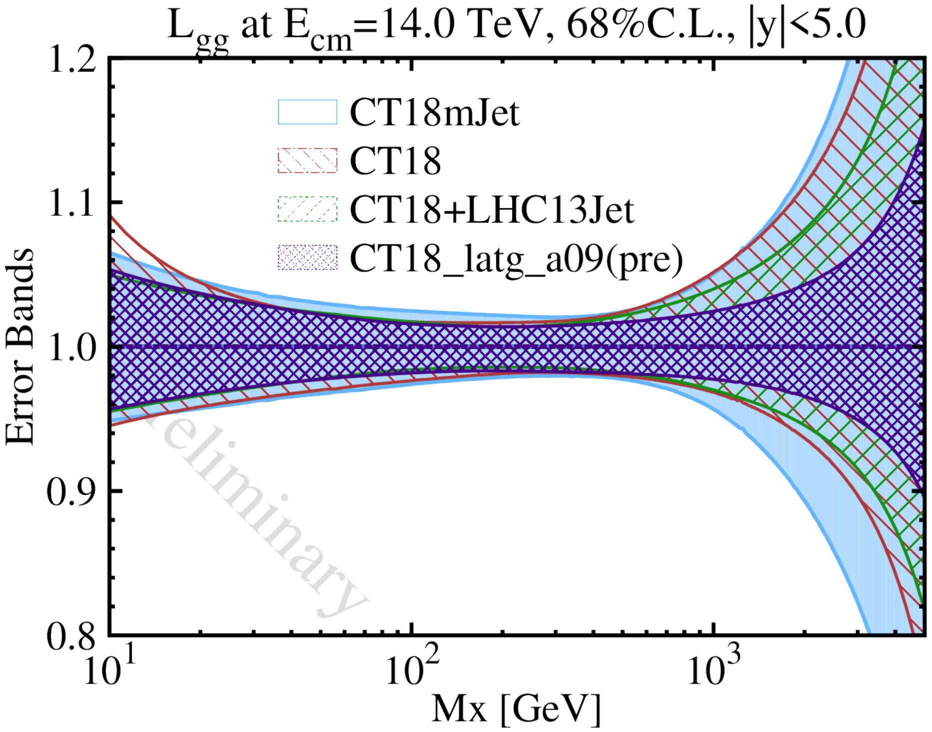


- Inclusion of lattice input in the CT18 global analysis has the potential to reduce its uncertainty.
- Here, a=0.00 fm case represents the current status, while a=0.09 fm case represents future potential.

Work in progress

CT18 with Lattice input on gluon





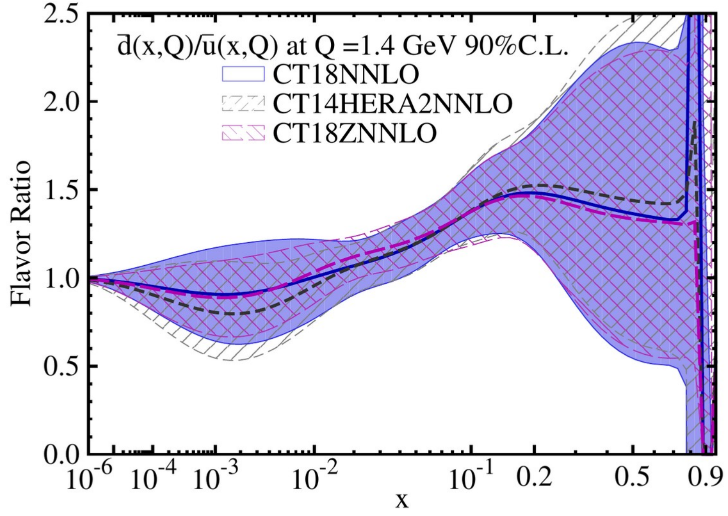
- The precision of the gluon PDF in the large-x region could potentially be improved by both incorporating jet data and lattice QCD input.

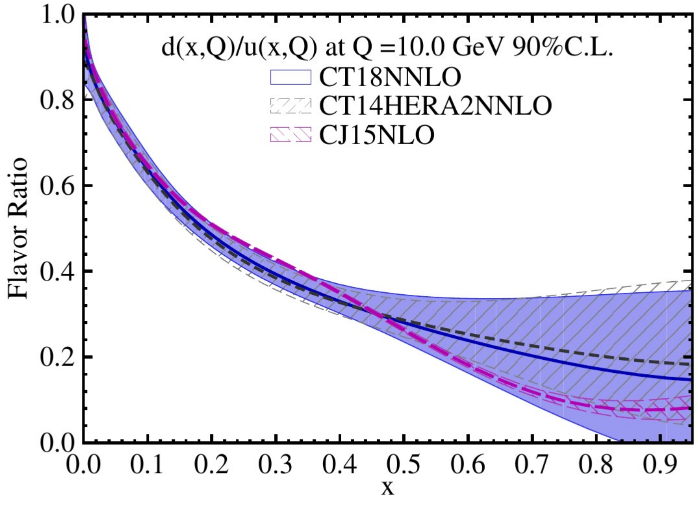
Besides reducing uncertainties of PDFs and improving heavy flavor decomposition, what else can we gain from Lattice QCD in assisting PDFs global analysis?

$\bar{d}/\bar{u}$?

d/u ?

$c-\bar{c}$?





Besides reducing uncertainties of PDFs and
improving heavy flavor decomposition,
what else can we gain
for **better understanding of hadron structure**
from the collaboration between
global analysis of PDFs and
lattice calculations?

Gottfried sum rule

New Muon Collaboration[NMC PRL 66, 2712 (1991), PRD 50, R1 (1994)] first discover $\bar{u} \neq \bar{d}$, which violates the Gottfried sum rule.

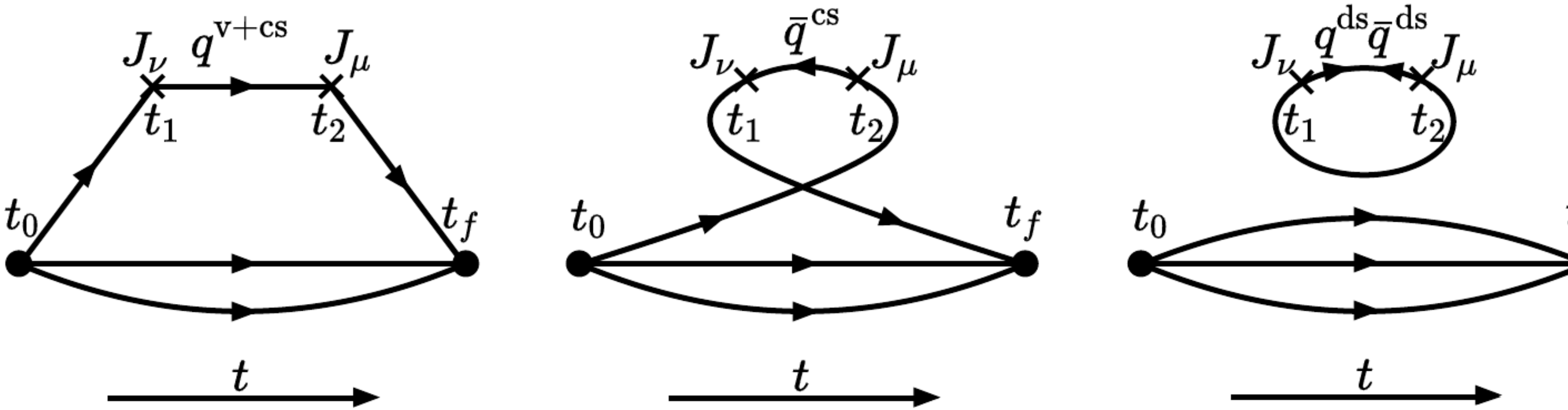
$$S_G = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\left(\bar{d}(x) - \bar{u}(x)\right) + O(\alpha_s^2)$$

The following experiments like HERMES[PLB387, 419 (1996)] and E866[PRD64, 052002 (2001)] also show preference of u - d flavor asymmetry.

Experiment	$\langle Q^2 \rangle$ (GeV^2)	$\int_0^1 [\bar{d}(x) - \bar{u}(x)]dx$
NMC/DIS	4.0	0.147 ± 0.039
HERMES/SIDIS	2.3	0.16 ± 0.03
FNAL E866/DY	54.0	0.118 ± 0.012

What is the origin of $\bar{u} \neq \bar{d}$?

- Euclidean path-integral formulation of the hadronic tensor predicts two kinds of sea partons: connected and disconnected



[Liu, 2007.15075]

$$
\begin{aligned}
u &= u^{v+cs} + u^{ds}, \quad d = d^{v+cs} + d^{ds} \\
\bar{u} &= \bar{u}^{cs} + \bar{u}^{ds}, \quad \bar{d} = \bar{d}^{cs} + \bar{d}^{ds}
\end{aligned}
$$

Define $u^v \equiv u^{v+cs} - \bar{u}^{cs}$, which is equivalent to defining $u^{cs} \equiv \bar{u}^{cs}$.

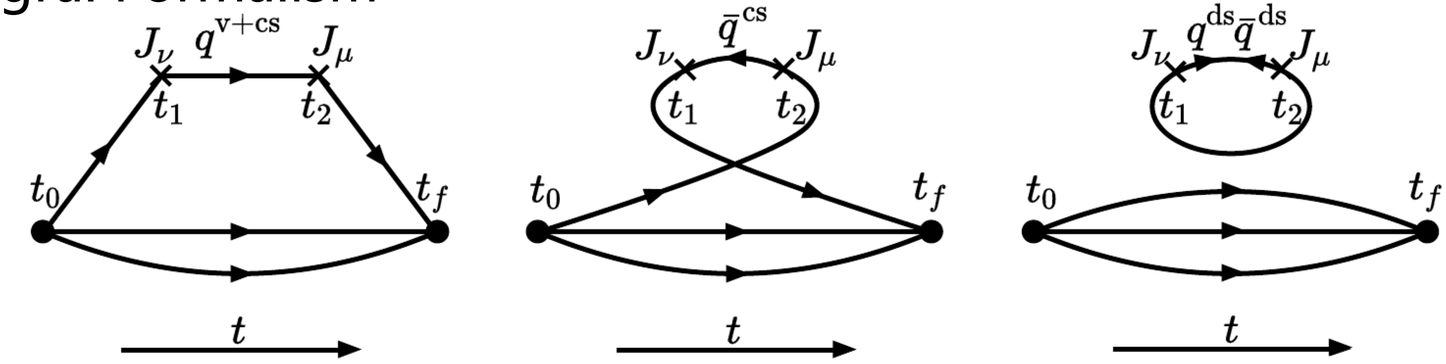
$$
\begin{aligned}
u - \bar{u} &\equiv (u^{v+cs} + u^{ds}) - (\bar{u}^{cs} + \bar{u}^{ds}) = u^v + (u^{ds} - \bar{u}^{ds}) \\
&\neq u^v, \quad \text{unless } u^{ds} = \bar{u}^{ds}
\end{aligned}
$$

Hadronic tensor in Euclidean path-integral formalism

versus

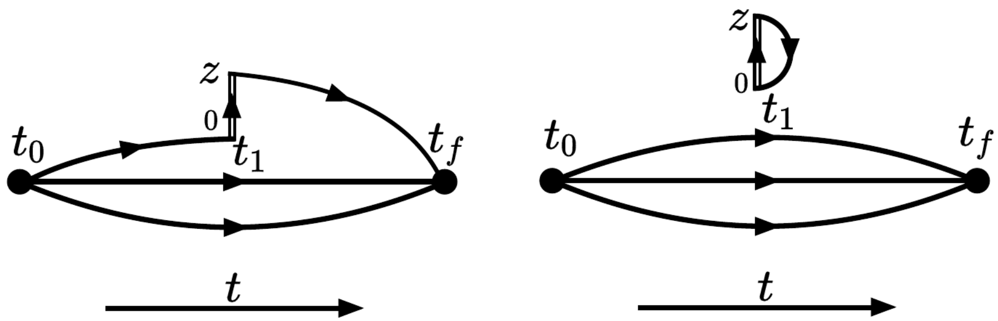
Quasi-PDF from Lattice QCD

Path-Integral Formalism



[Liu, 2007.15075]

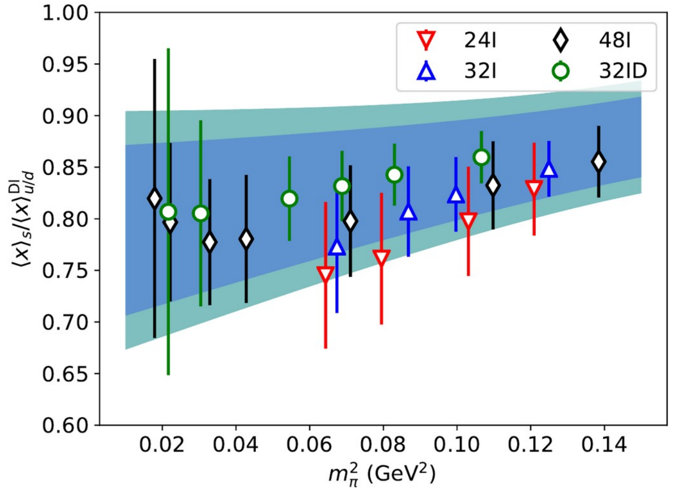
Quasi-PDF from Lattice QCD

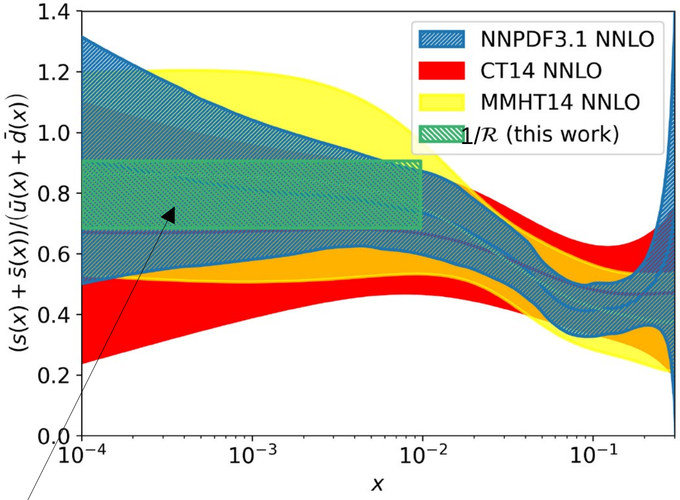


Connected Insertion(CI) Disconnected Insertion(DI)

Lattice input to global fitting of PDFs

With only one input from Lattice QCD, the ratio of moments between u, d and s in disconnected insertion(DI), the connected and disconnected sea are distinguishable in global analysis.





Lattice result from overlap on Nf = 2 + 1 DWF on 4 lattices, with one at physical pion mass [Liang et al, χQCD, 1901.07526]

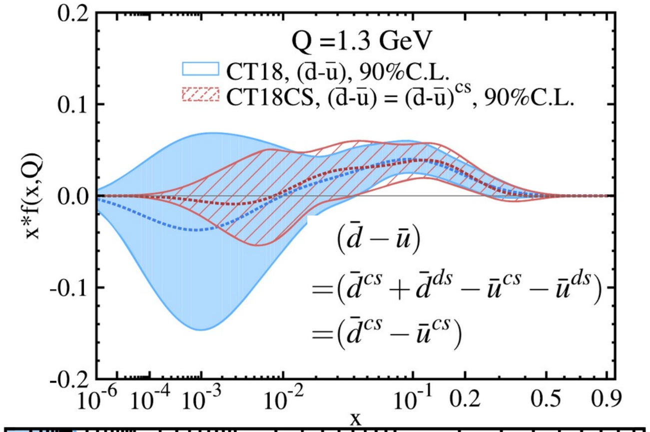
$$\frac{1}{R} = \frac{\langle x\rangle_{s+\bar{s}}}{\langle x\rangle_{\bar{u}+\bar{d}}(DI)}(\text{at } 1.3\text{ GeV}) = 0.822(69)(78)$$

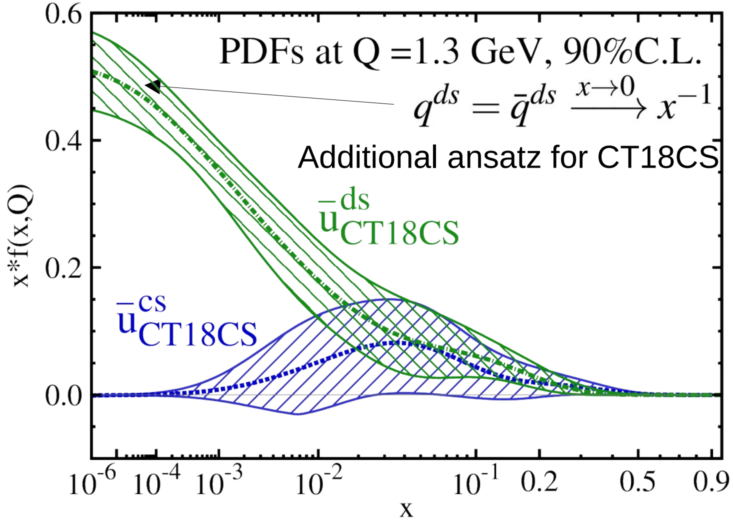
Connected and disconnected sea d.o.f. can be distinguished by assuming

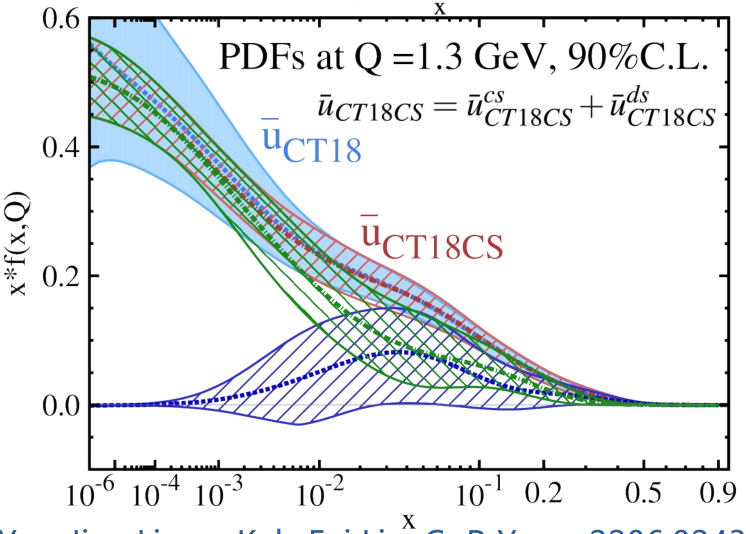
$$u^{ds} = \bar{u}^{ds} = d^{ds} = \bar{d}^{ds} = Rs = R\bar{s},$$

- Distinguish connected and disconnected flavor d.o.f. at Q_0=1.3 GeV in global analysis.
- The difference between $\bar{u}$ and $\bar{d}$ come from the connected sea contribution.

CT18		CT18CS
g	$=$	g
u^v	$=$	u^v
d^v	$=$	d^v
$\bar{u}$	$=$	$\bar{u}^{cs} + \bar{u}^{ds} = \bar{u}^{cs} + Rs^{ds}$
$\bar{d}$	$=$	$\bar{d}^{cs} + \bar{d}^{ds} = \bar{d}^{cs} + Rs^{ds}$
s	$=$	$\bar{s} = s^{ds}$







[Tie-Jiun Hou, Mengshi Yan, Jian Liang, Keh-Fei Liu, C.-P. Yuan, 2206.02431]

- Direct comparison of all connected and disconnected parton moments between global analysis and lattice calculation instead of being limited to only u - d and s.

$$u^+ - d^+ = (u+\bar{u}) - (d+\bar{d}) = (u^{v+cs} + u^{ds} + \bar{u}^{cs} + \bar{u}^{ds}) - (d^{v+cs} + d^{ds} + \bar{d}^{cs} + \bar{d}^{ds})$$

$$\xrightarrow{CT18CS} (u^{v+cs} - d^{v+cs}) + (\bar{u}^{cs} - \bar{d}^{cs})$$

$$s^+ = s + \bar{s} = s^{ds} + \bar{s}^{ds} \xrightarrow{CT18CS} 2s^{ds}$$

	$Q = 2.0$ GeV CT18	Lattice	$Q = 1.3$ GeV CT18CS	CT18
$\langle x \rangle_{u^+-d^+}$	0.156(7)	$0.111-0.209^{N_f=2+1}$ $0.153-0.194^{N_f=2+1+1\dagger}$ $0.166-0.212^{N_f=2}$	0.173(7)	0.175(8)
$\langle x \rangle_{s^+}$	0.033(9)	$0.051(26)(5)^{\ddagger}$	0.027(8)	0.027(10)

† Prog. Part. Nucl. Phys., 121:103908, 2021. ‡ Phys. Rev. Lett., 121(21):212001, 2018

CT18 CT18CS

$$\begin{aligned} g &= g \\ u^v &= u^v \\ d^v &= d^v \\ \bar{u} &= \bar{u}^{cs} + \bar{u}^{ds} = \bar{u}^{cs} + Rs^{ds} \\ \bar{d} &= \bar{d}^{cs} + \bar{d}^{ds} = \bar{d}^{cs} + Rs^{ds} \\ s &= \bar{s} = s^{ds} \end{aligned}$$

PDF	$\langle x \rangle_{u^v}$	$\langle x \rangle_{d^v}$	$\langle x \rangle_g$	$\langle x \rangle_{\bar{u}}$	$\langle x \rangle_{\bar{d}}$	$\langle x \rangle_s$
CT18	0.325(5)	0.134(4)	0.385(10)	0.0284(22)	0.0361(27)	0.0134(52)
CT18CS	0.323(4)	0.136(3)	0.384(12)	0.0287(25)	0.0364(34)	0.0137(39)

PDF	$\langle x \rangle_{u^{v+cs}}$	$\langle x \rangle_{d^{v+cs}}$	$\langle x \rangle^*_{\bar{u}^{cs}}$	$\langle x \rangle^*_{\bar{d}^{cs}}$	$\langle x \rangle^\dagger_{u^{ds}}$
CT18CS	0.335(7)	0.155(8)	0.0120(64)	0.0197(70)	0.0167(49)

1.3 GeV

To be tested by lattice calculation

Hadron Structure via PDFs
Complementarity between Perturbative and Lattice QCD

- PDF global analysis requires a large amount of data to access hadron structure, but the resulting PDFs are inherently limited by the constraints of available experimental data.
- Lattice QCD provides constraints on hadron structures that are not experimentally accessible.
- Incorporating lattice QCD calculations into PDF global analysis has the potential to aid in heavy flavor decomposition.
- Utilizing lattice QCD results in global analysis could potentially reduce gluon uncertainty at large x.
- Incorporating connected and disconnected sea degrees of freedom within global analysis would help in better understanding the non-perturbative nature of hadron structure. Additionally, this would allow for direct comparison between lattice calculations and all separated connected and disconnected sea moments.

Thank you for your attention!