Qubit Regularization: Asymptotic Freedom via New Renormalization Group Flows

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(Duke University)

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Lattice Gauge Theory Work in Progress

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Quantum Field Theory with Quantum Computers

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Growing field of research within our community!

Plenary Talk 2018, Preskill

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Two types of opportunities:

→ How to reformulate the QFT

→ How to use the quantum hardware

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Two types of opportunities: → How to reformulate the QFT

→ How to use the quantum hardware

This talk is about showing you a glimpse of how quantum computing is already helping us learn

to formulate "old" problems with "new" variables and understand the field deeper.

Lagrangian → Hamiltonian

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“Local” lattice Hilbert spaces play an important role

Lagrangian → Hamiltonian

"Local" lattice Hilbert spaces play an important role

Traditional: $\mathcal{H}_{Trad}$ $\mathsf{dim}(\mathcal{H}_{Trad}) = \infty$

"Digital" quantum computer: $\mathcal{H}_Q$ $\mathsf{dim}(\mathcal{H}_Q) = \mathsf{finite}$

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↓

qubit regularization

Hersh Singh, SC *Phys.Rev.D* 100 (2019) 5, 054505
Hanqing Liu, SC *Symmetry* 14 (2022) 2, 305

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Continuum QFT emerges through the usual limiting process

continuum limit	Thermodynamic limit
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Many papers seem to assume this is necessary!

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This is not required at least in some cases!

An interesting alternate approach is the D-theory

Lattice 1998, Plenary talk by Wiese.

Brower, SC, Riederer, Wiese, NPB 693 (2004),149

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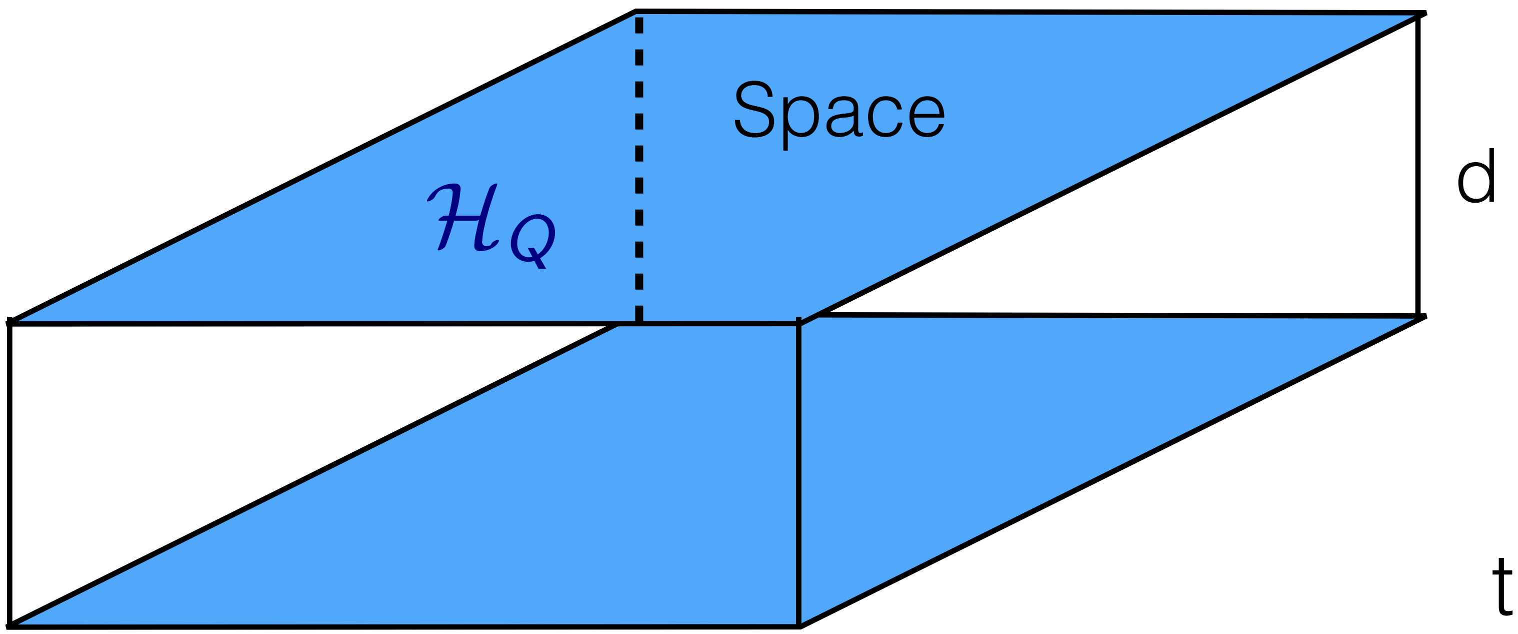

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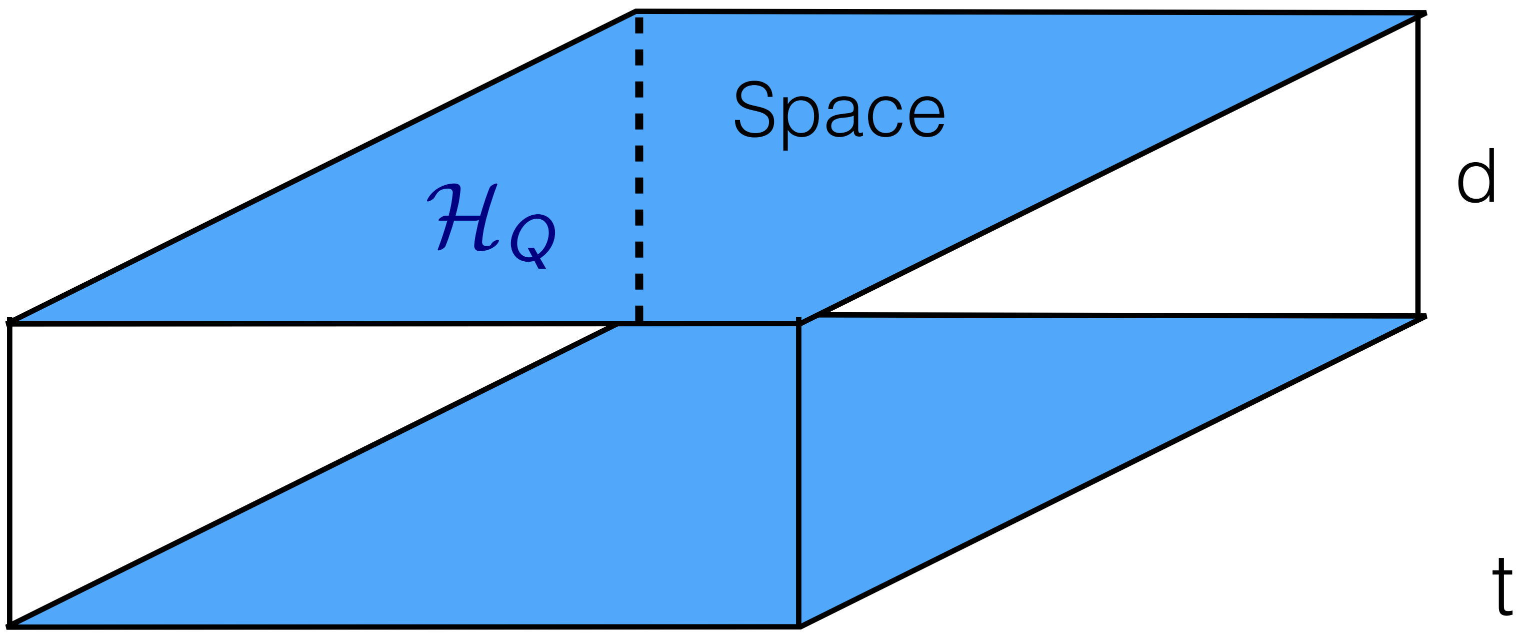

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RG plays an important role!

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Talks in the conference

12 Parallel Talks on Monday

6 Parallel Talks on Tuesday

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How to prepare initial states.

How to evolve systems in real time on a real hardware!

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Apologies: I do not plan on reviewing these results!

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(Two examples, Euclidean space)

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Can we recover asymptotic freedom of traditional lattice models by reformulating it as lattice models with a finite dimensional "local" Hilbert space!

Yes!
(Two examples, Euclidean space)

We will discover a new a type of RG flow!

Collaborators

Siew

Maiti

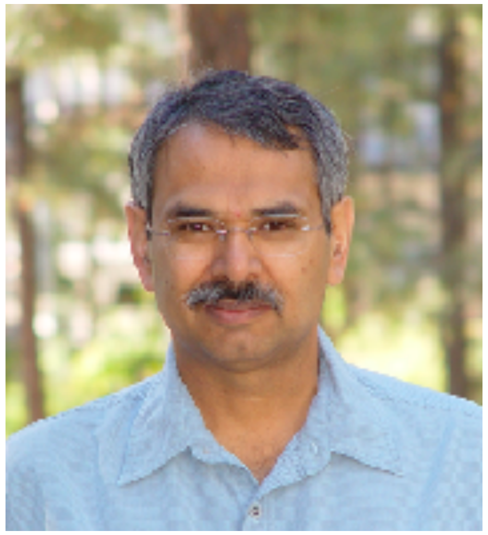
Liu

Singh

Marinkovic

Banerjee

Bhattacharya



Gupta

Outline

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Asymptotic Freedom as an RG flow

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Recovering Asymptotic Freedom with finite Hilbert space

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Example 1: O(3) Non-linear sigma model

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Conclusions

Asymptotic Freedom as an RG Flow

Traditional Lattice Regularization

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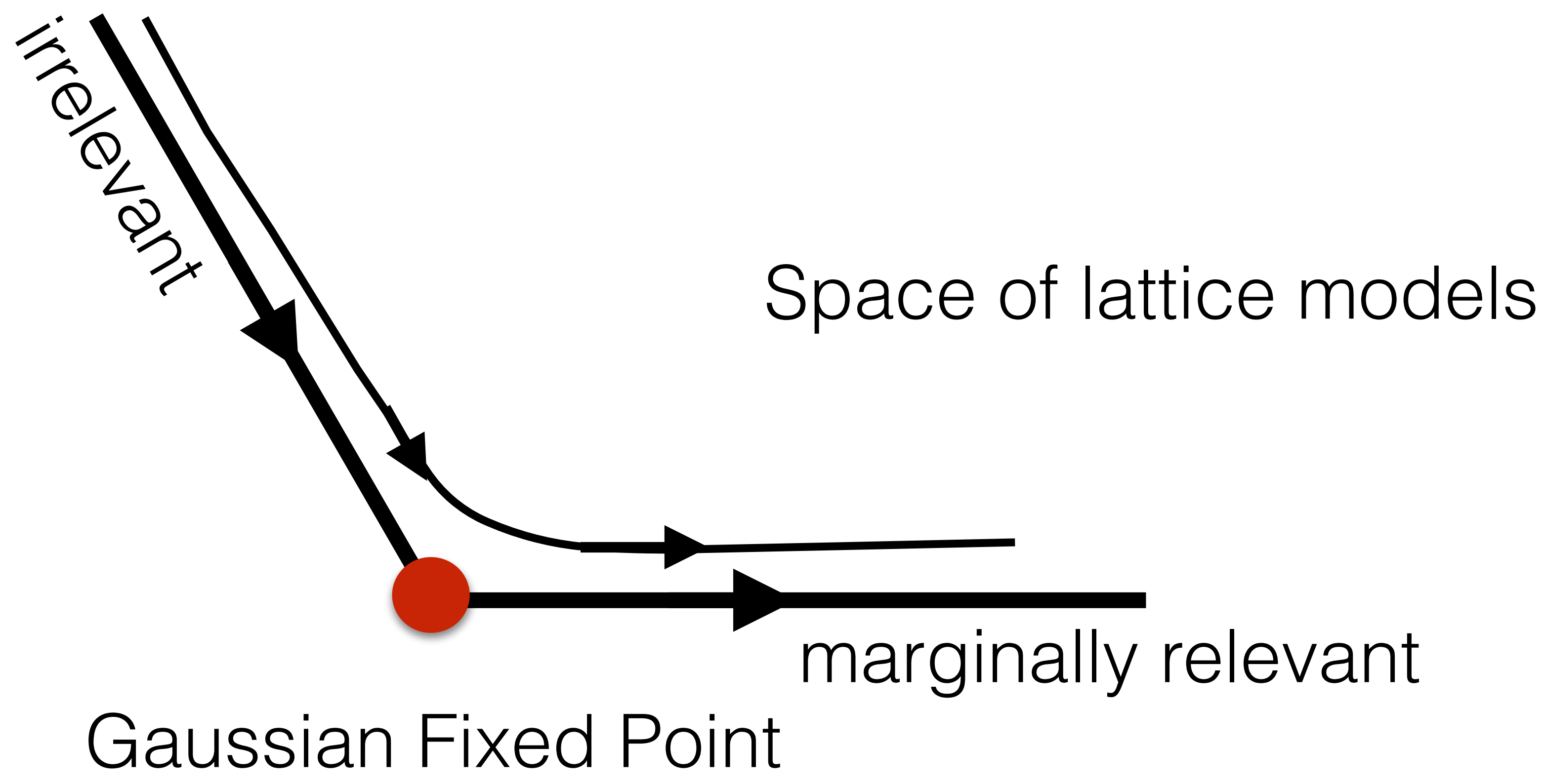

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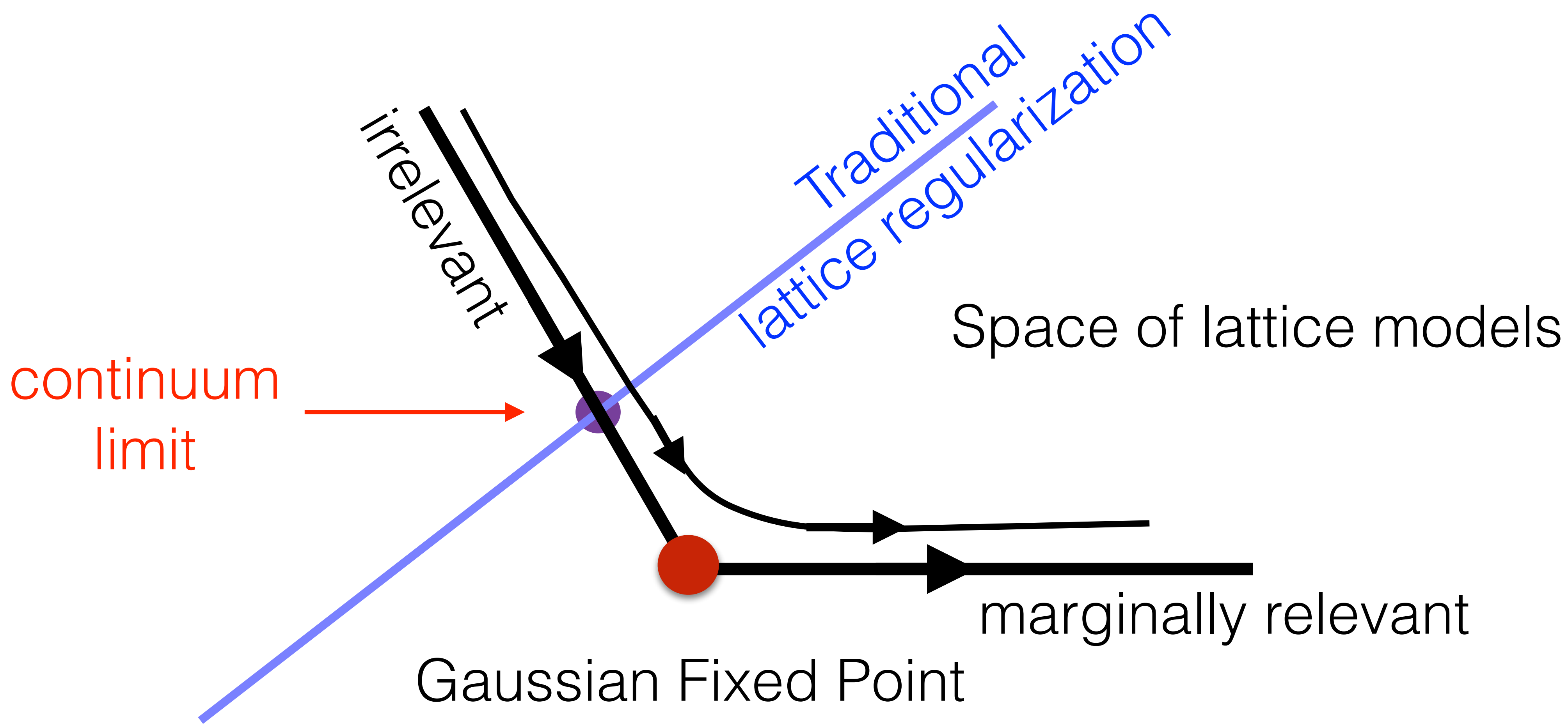

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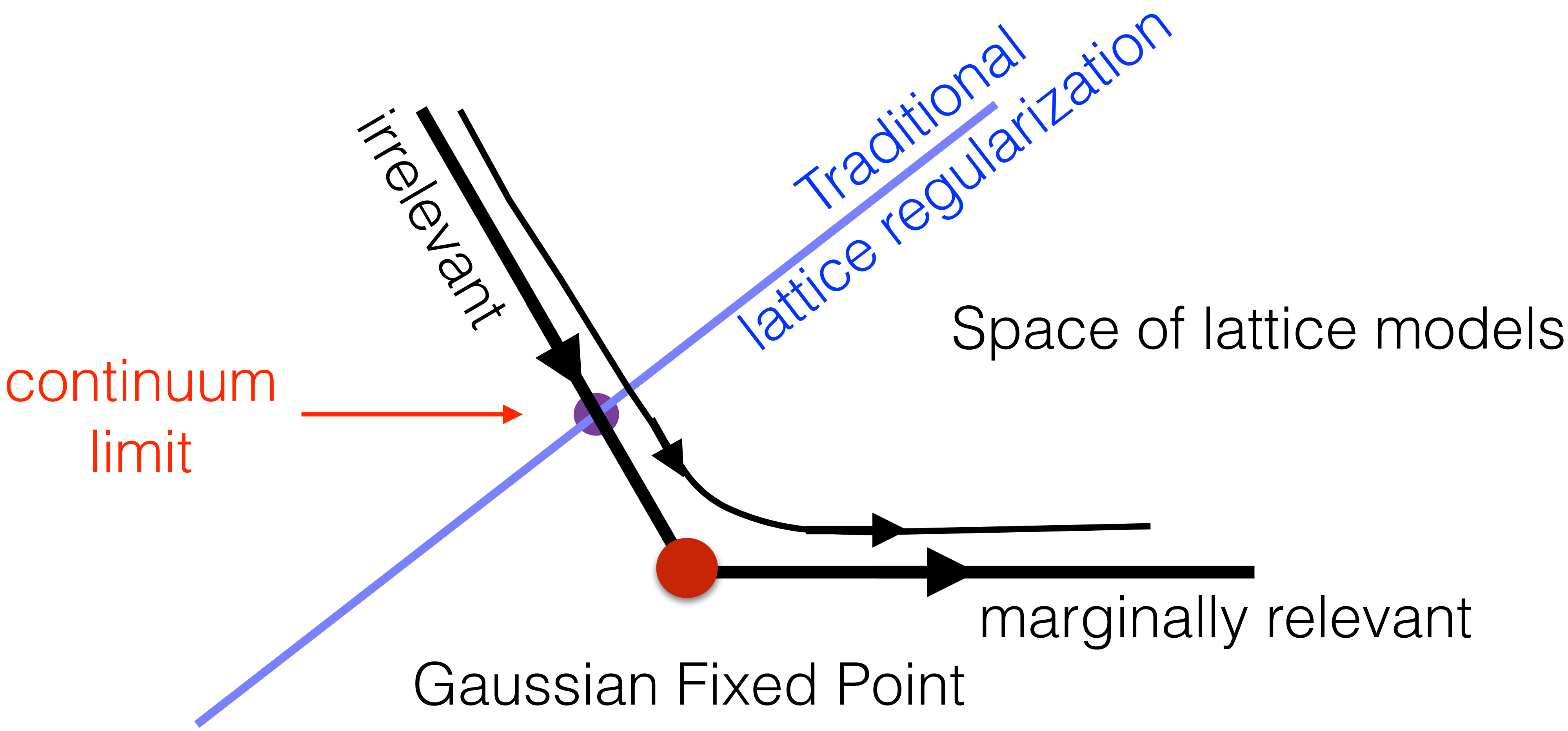

From an RG perspective, the limit

$$\mathcal{H}_{\mathrm{Q}} \longrightarrow \mathcal{H}_{\mathrm{Trad}}$$

may not be necessary!

We may be able to fine tune to the free critical surface within $\mathcal{H}_{\mathrm{Q}}$

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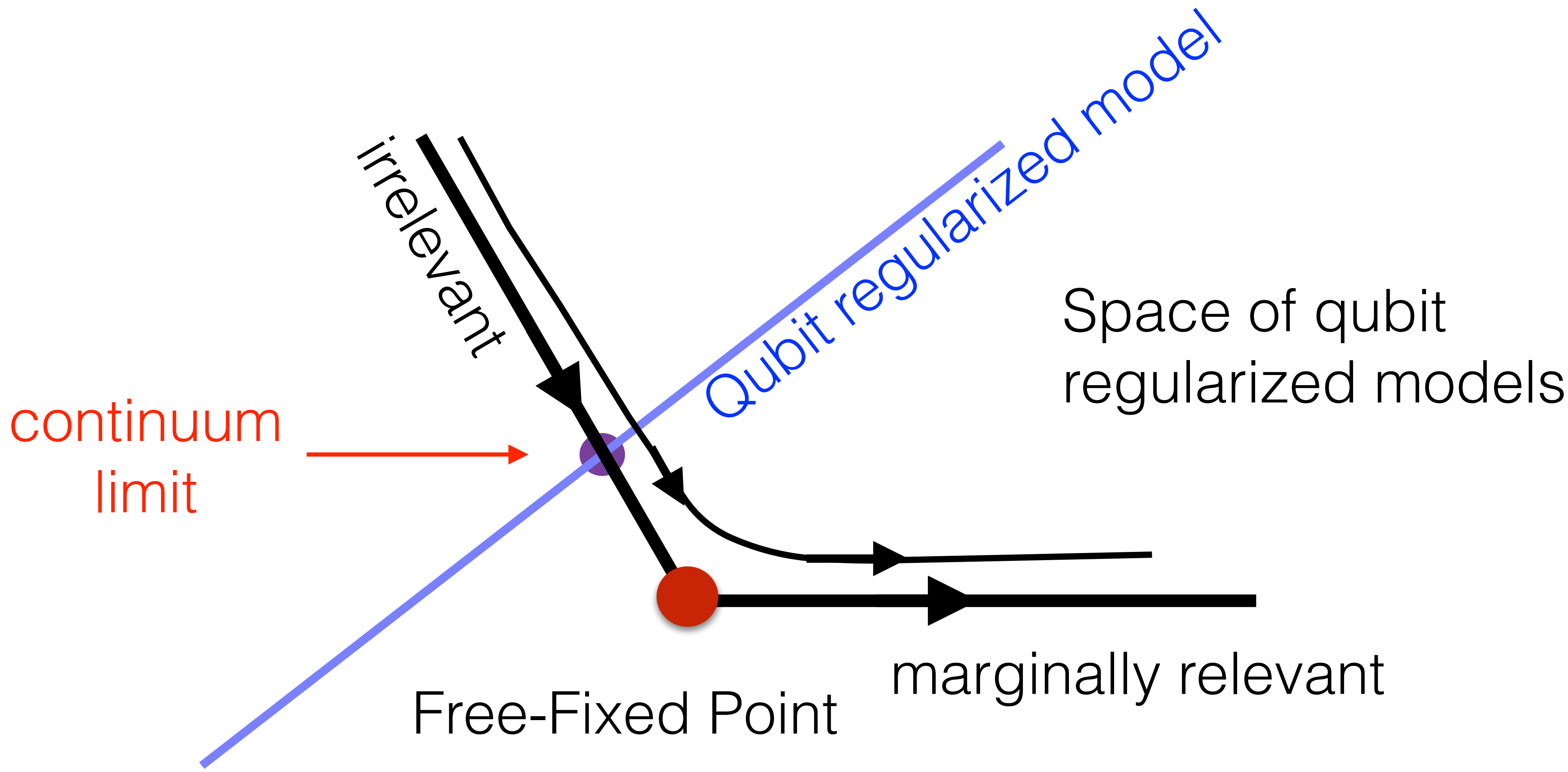

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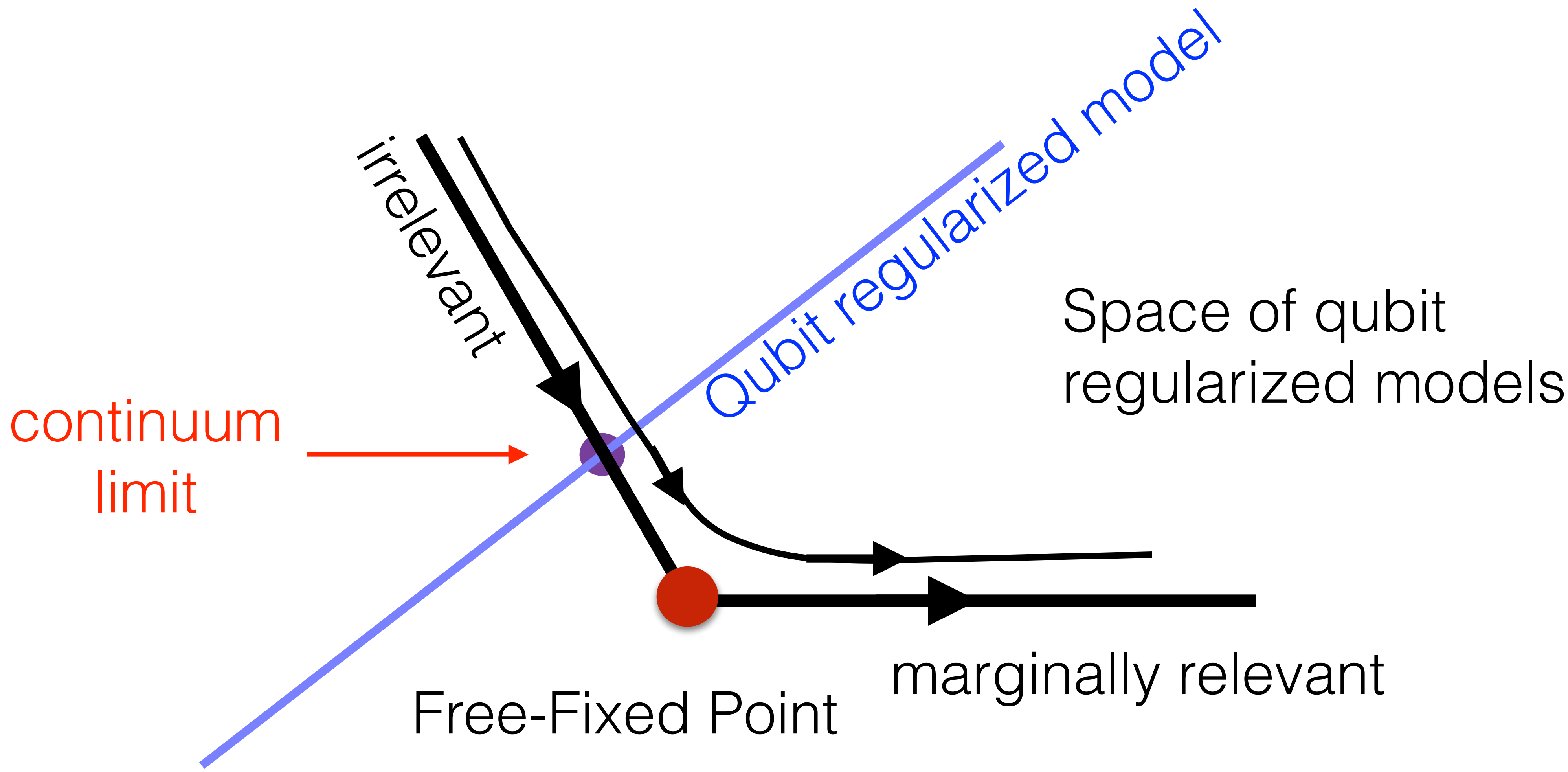

Lesson from RG:
Explore the space of qubit models,
don’t be stuck with the traditional Hamiltonian.

Recovering Asymptotic Freedom with a finite Hilbert space

Example: O(3) Non-linear sigma model

Traditional Lattice Action:

$$S_L = -\beta \sum_{\langle (x,\tau),(y,\tau')\rangle} \vec{\phi}_{x,\tau} \cdot \vec{\phi}_{y,\tau'}$$

$\vec{\phi}_{x,\tau}$ = three component unit vector $\quad \vec{\phi}_{x,\tau} \cdot \vec{\phi}_{x,\tau} = 1$

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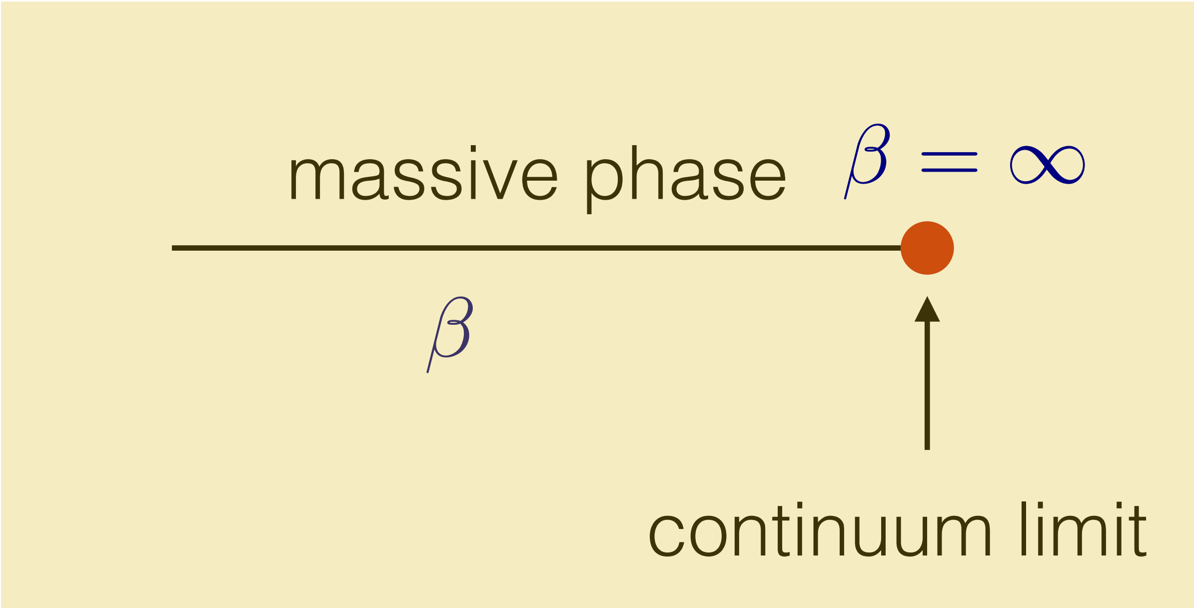

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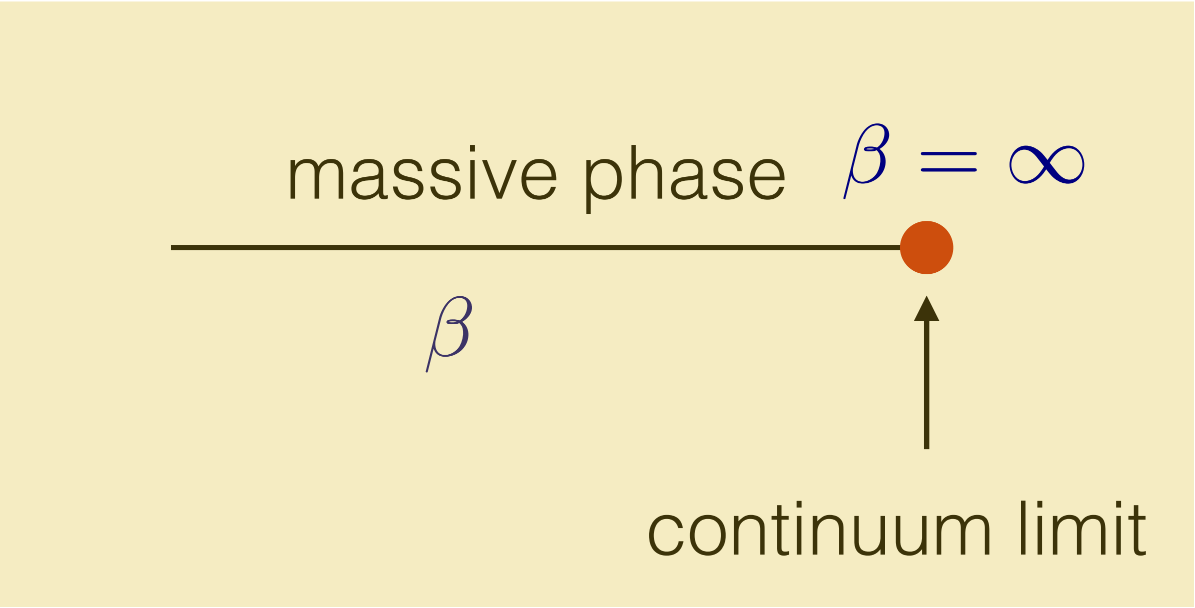

At the critical point we get the desired free theory!

Continuum Physics: Universal step-scaling function

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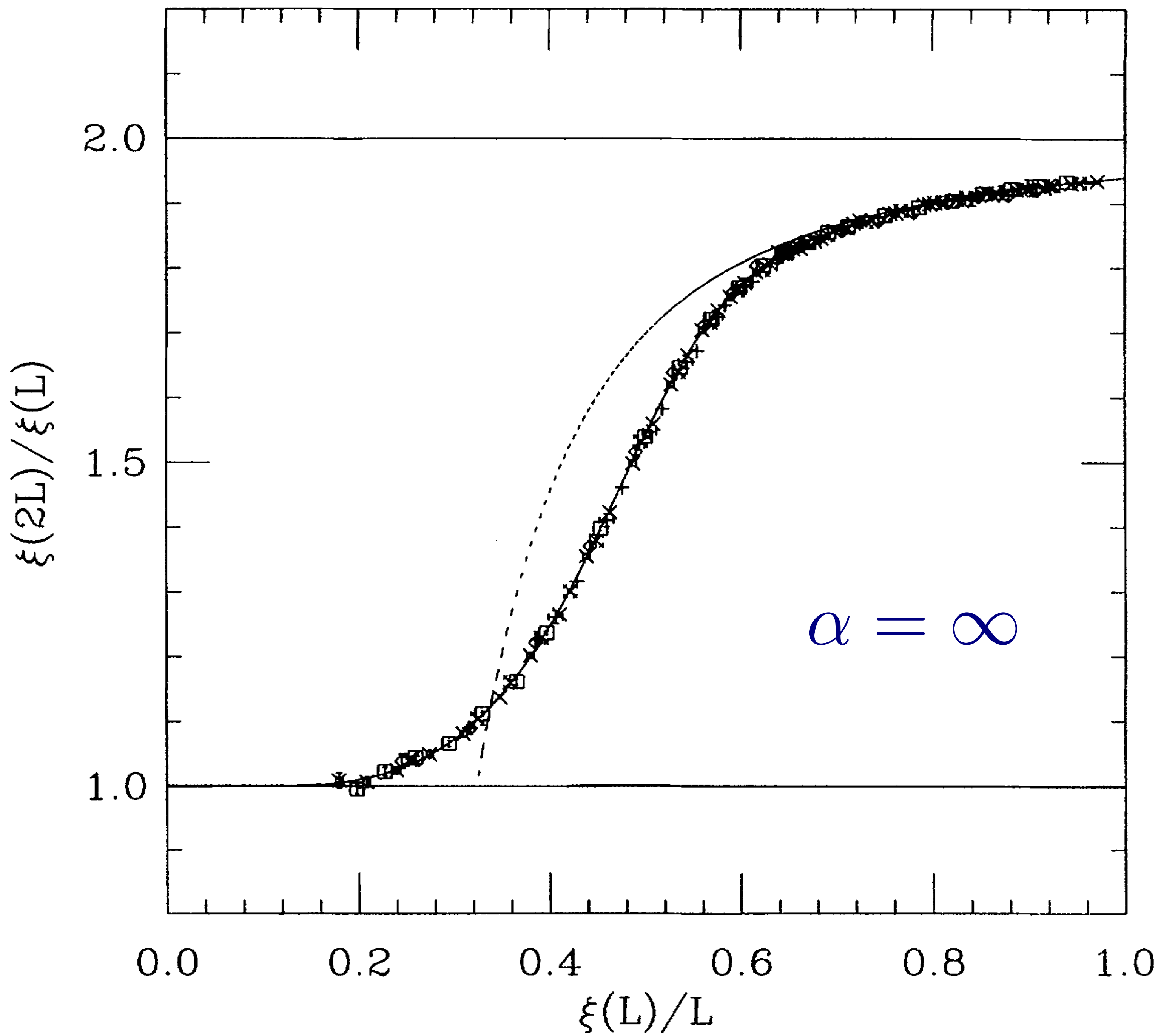
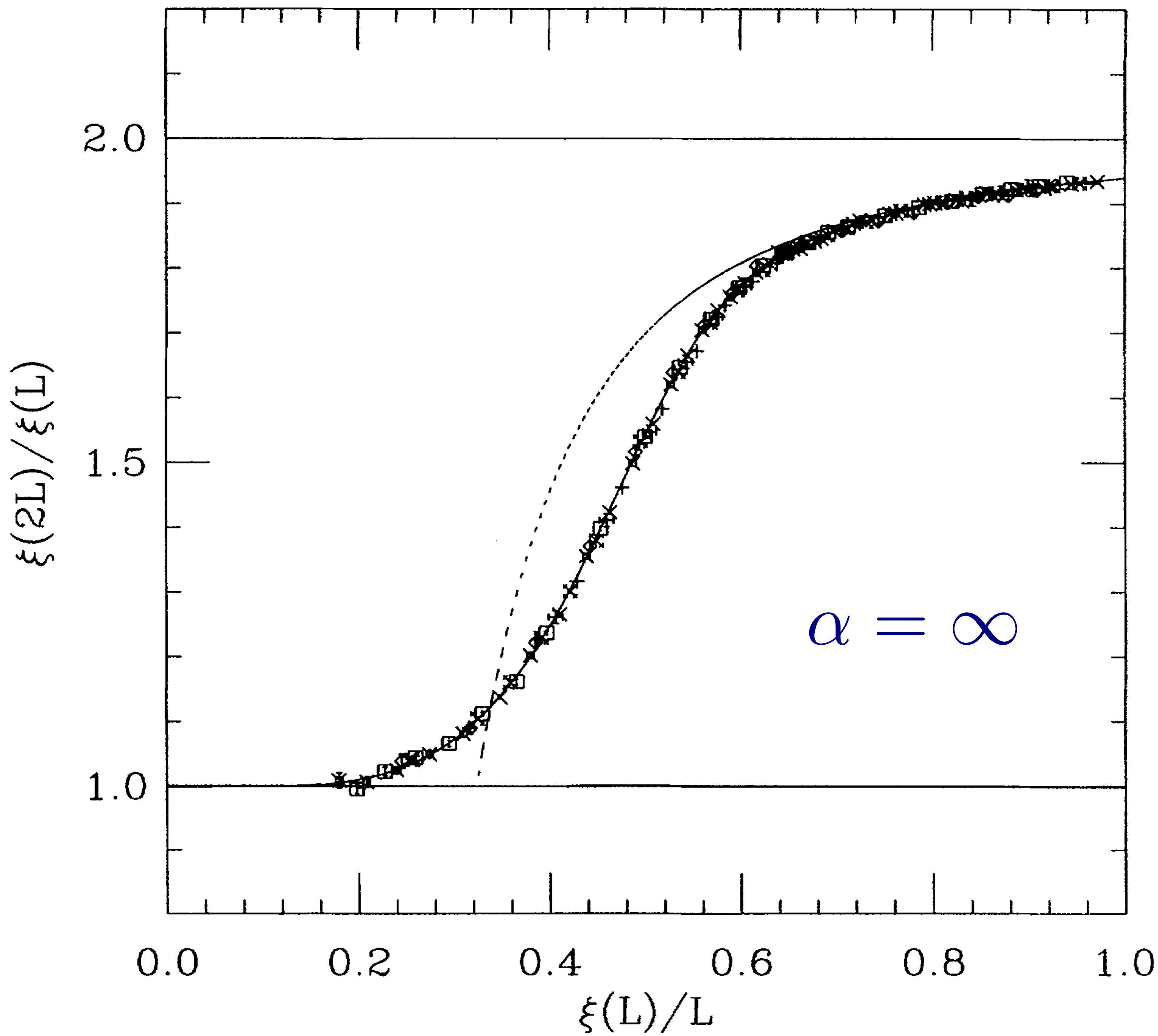


Caracciolo, et.al., PRL 75, 1891 (1995)

Continuum Physics: Universal step-scaling function



Can we reproduce this continuum physics in a lattice model with a finite Hilbert space?

Caracciolo, et.al., PRL 75, 1891 (1995)

Qubit Regularization of SO(3) fields

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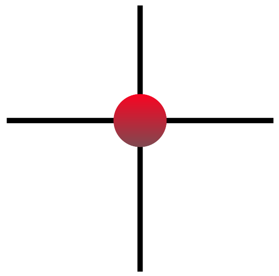
Local site Hilbert space describes a quantum particle on a surface of a unit sphere



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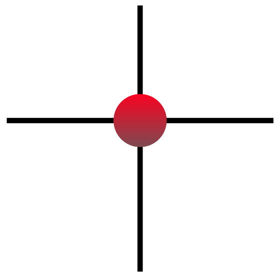

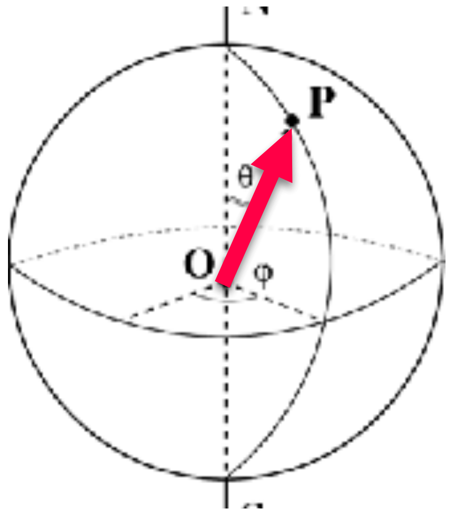

$$\vec{\phi}_x = (\phi_x^1, \phi_x^2, \phi_x^3)$$

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Qubit Regularization of SO(3) fields

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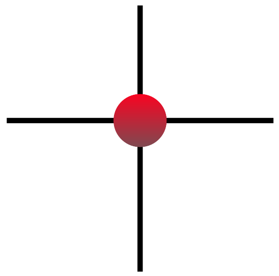

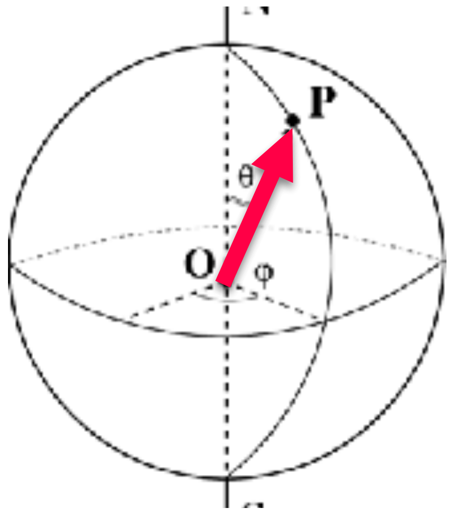

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Basis of the traditional Hilbert space $\mathcal{H}_{\mathrm{Trad}}$:

$$\int d\Omega \; |\theta, \varphi\rangle\langle\theta, \varphi| \;=\; I$$

"position basis"

$$\sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} |\,\ell, m\,\rangle\langle\,\ell, m\,| \;=\; I$$

"momentum basis"

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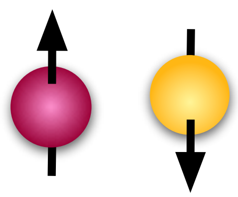
A simple qubit regularization scheme is $Q = \{0, 1\}$

($dim(\mathcal{H}_Q) = 4$) Two qubits per lattice site



Space-time Euclidean configurations in the qubit regularization scheme $Q = \{0, 1\}$

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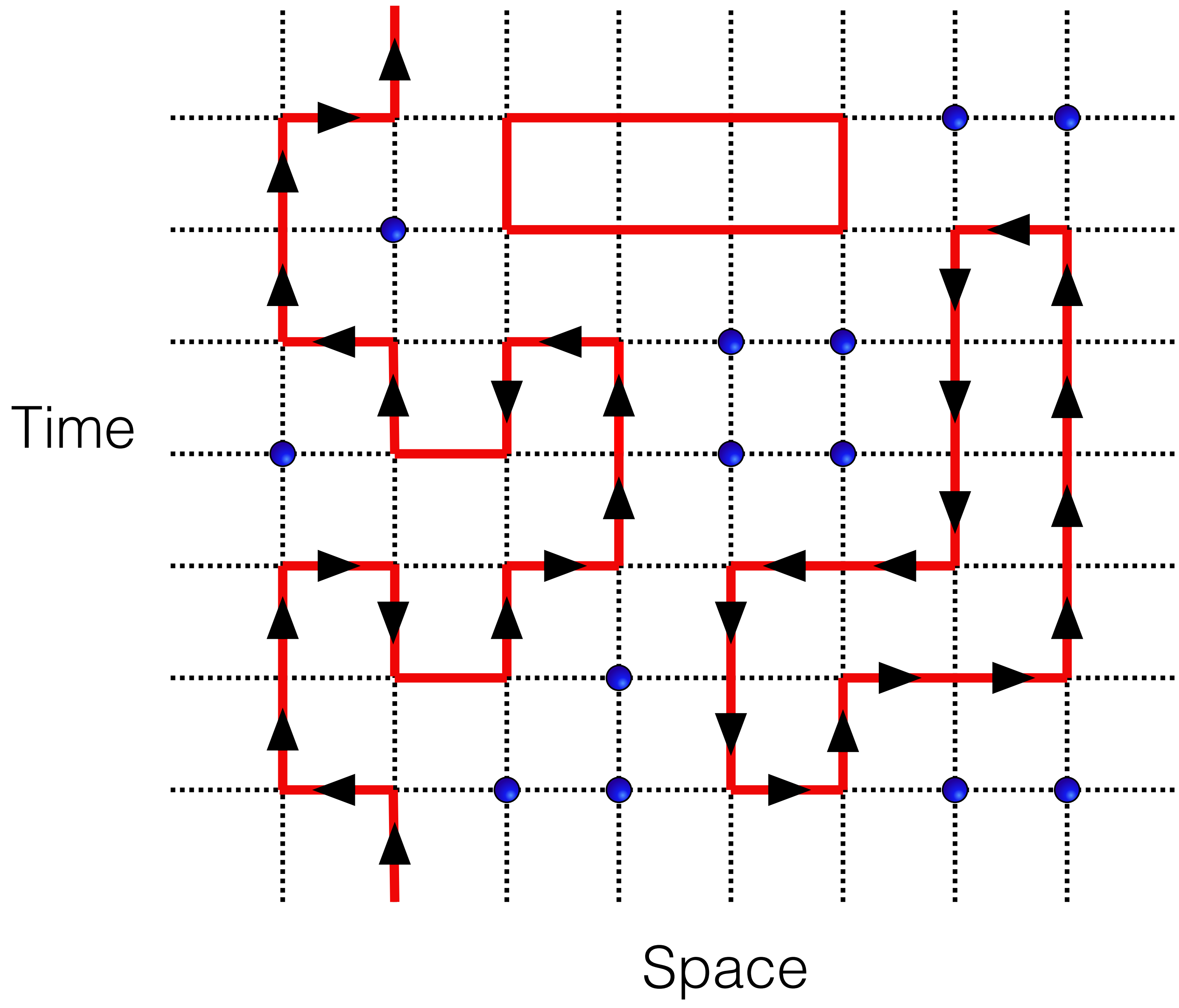

Qubit regularized model

Bhattacharya, Buser, SC, Gupta, Singh PRL 126 (2021), 172001

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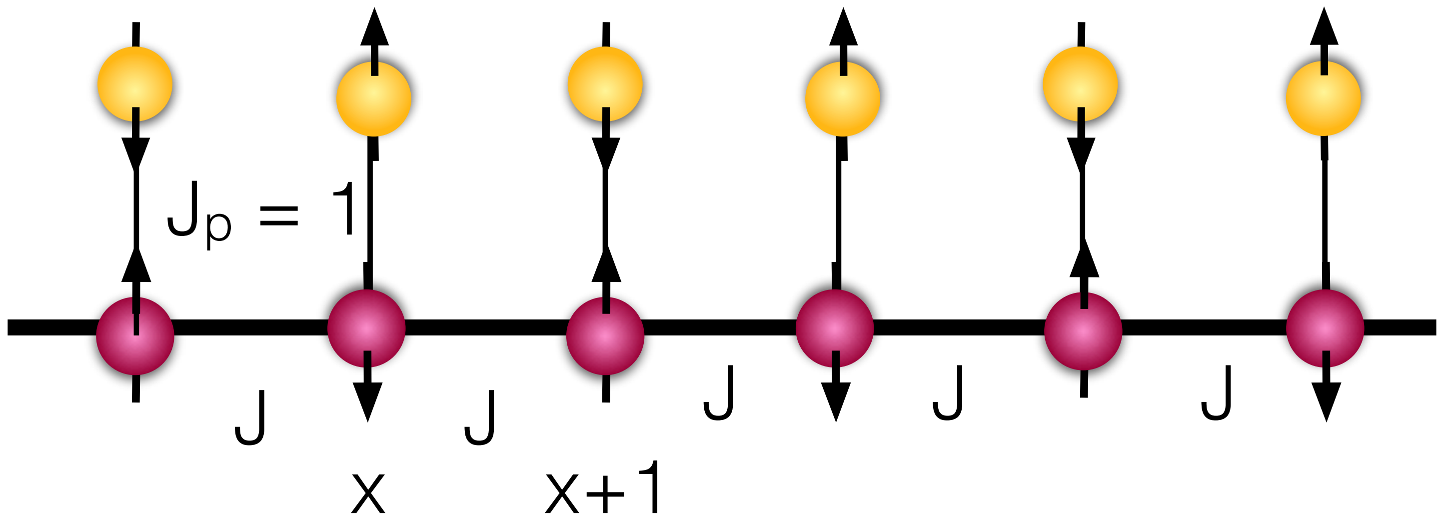
Heisenberg-Comb



$$H = \sum_x J\, \mathbf{S}_{x,1} \cdot \mathbf{S}_{x+1,1} + \mathbf{S}_{x,1} \cdot \mathbf{S}_{x,2}$$

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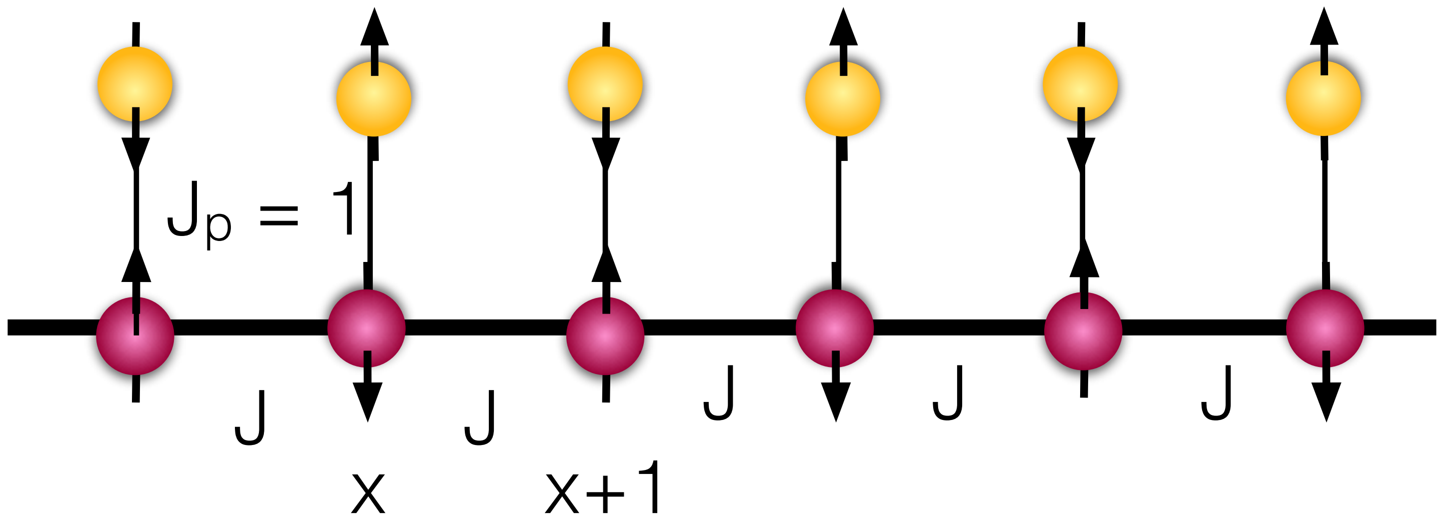

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Quantum Critical Point: $J \rightarrow \infty$ (Spin-1/2 Chain)

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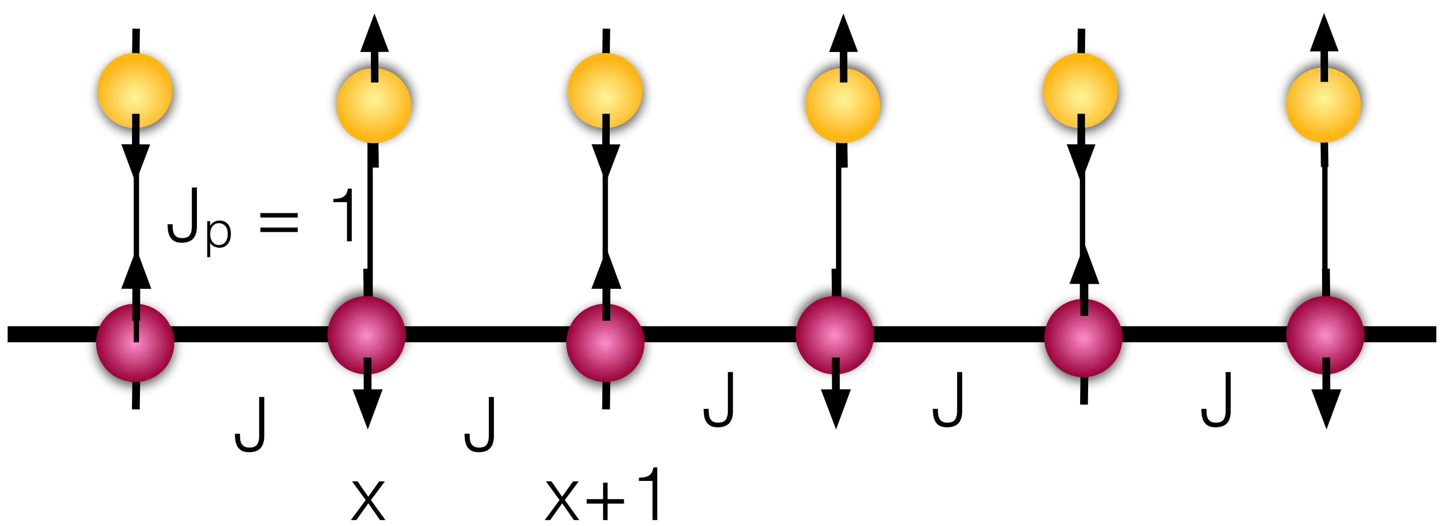

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At the critical point we get a decoupled critical theory that is not the desired free theory!

Universal Step Scaling Function

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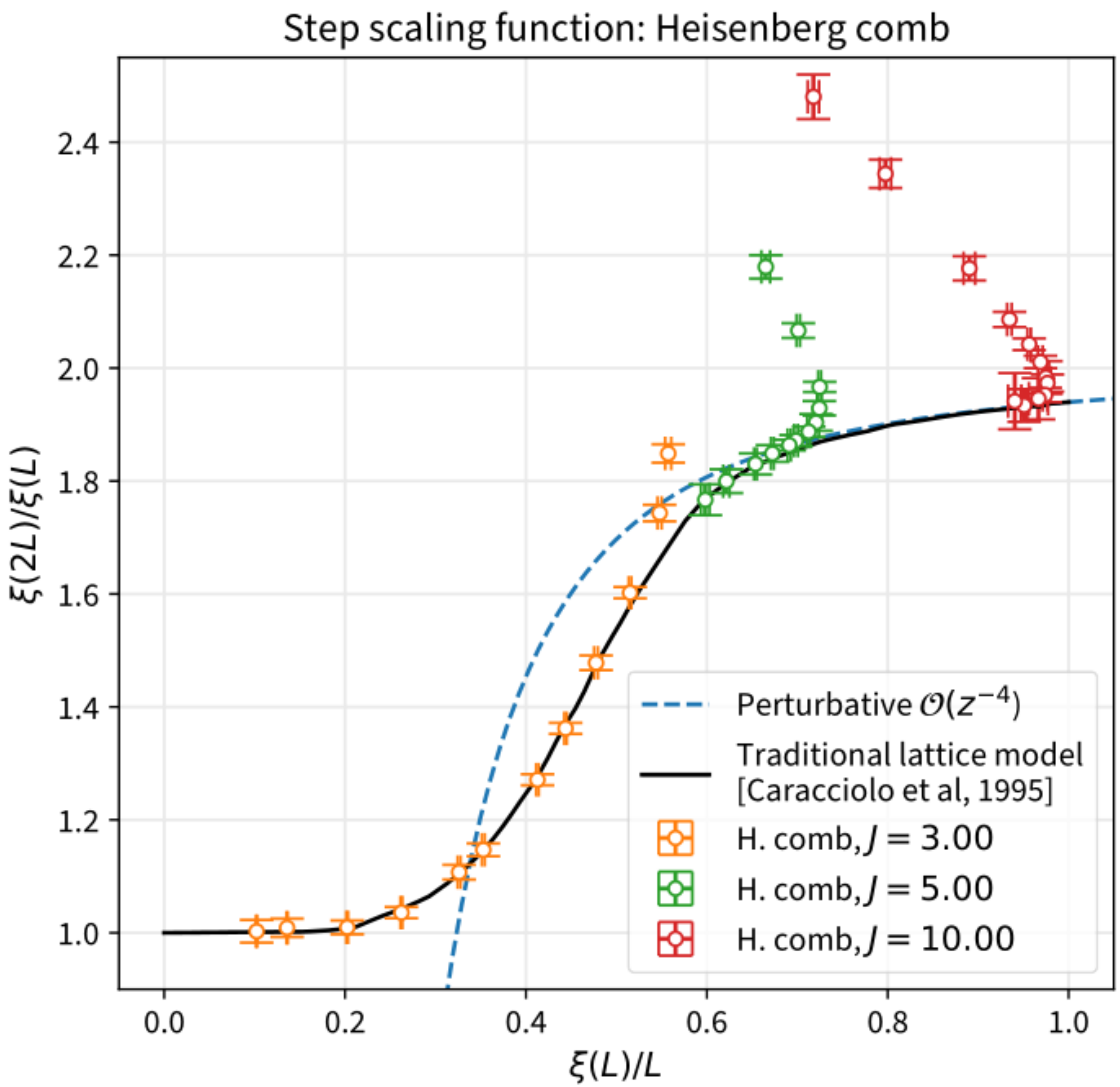

Traditional Model at

$$\beta \to \infty$$

$$\mathcal{H}_{\mathsf{Trad}} = \textit{infty}$$

$$\equiv$$

Heisenberg Comb at

$$J \to \infty$$

$$\mathcal{H}_{\mathsf{Q}} = 4$$

Example: BKT Transition

Lattice Action:

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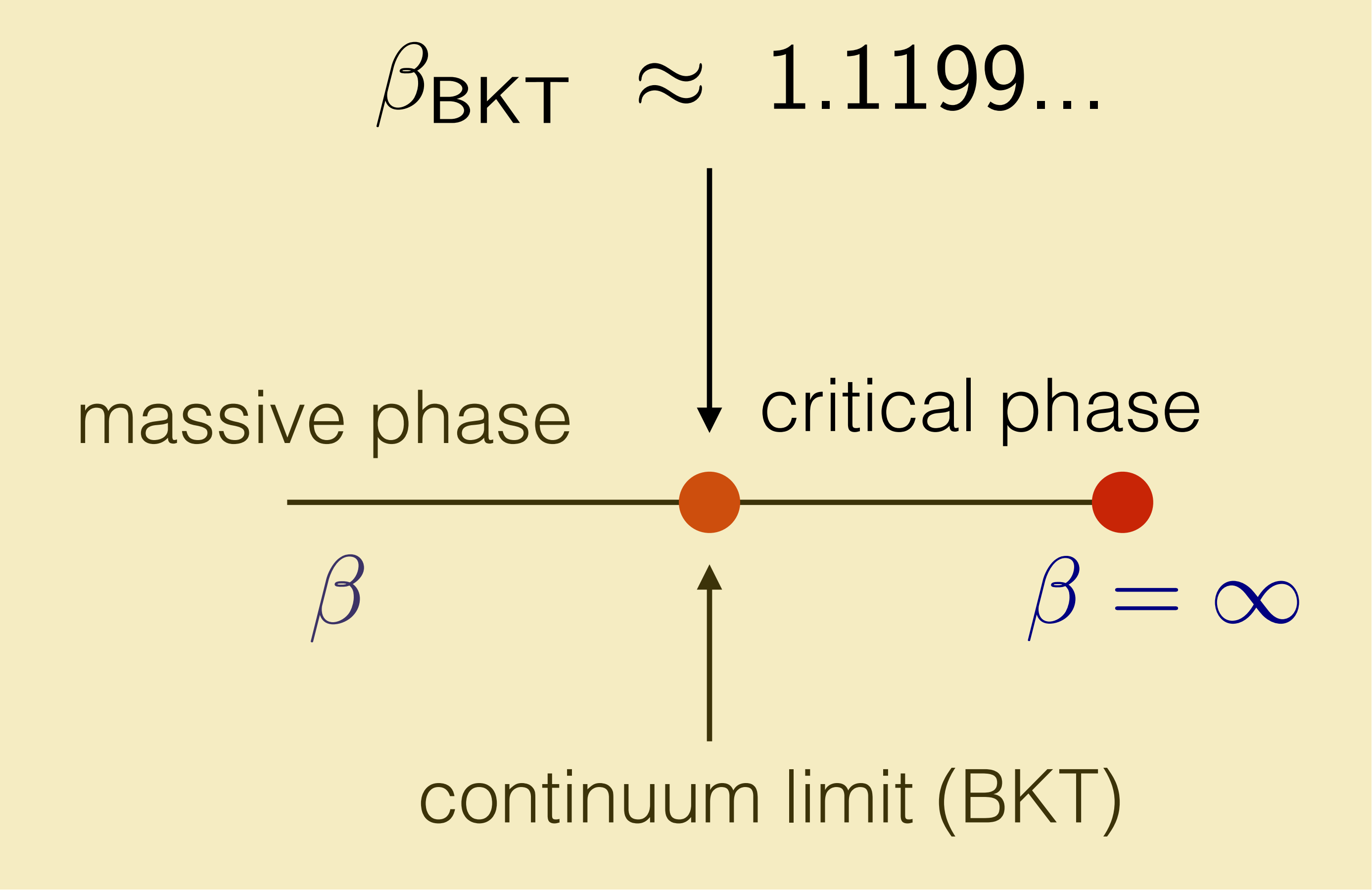

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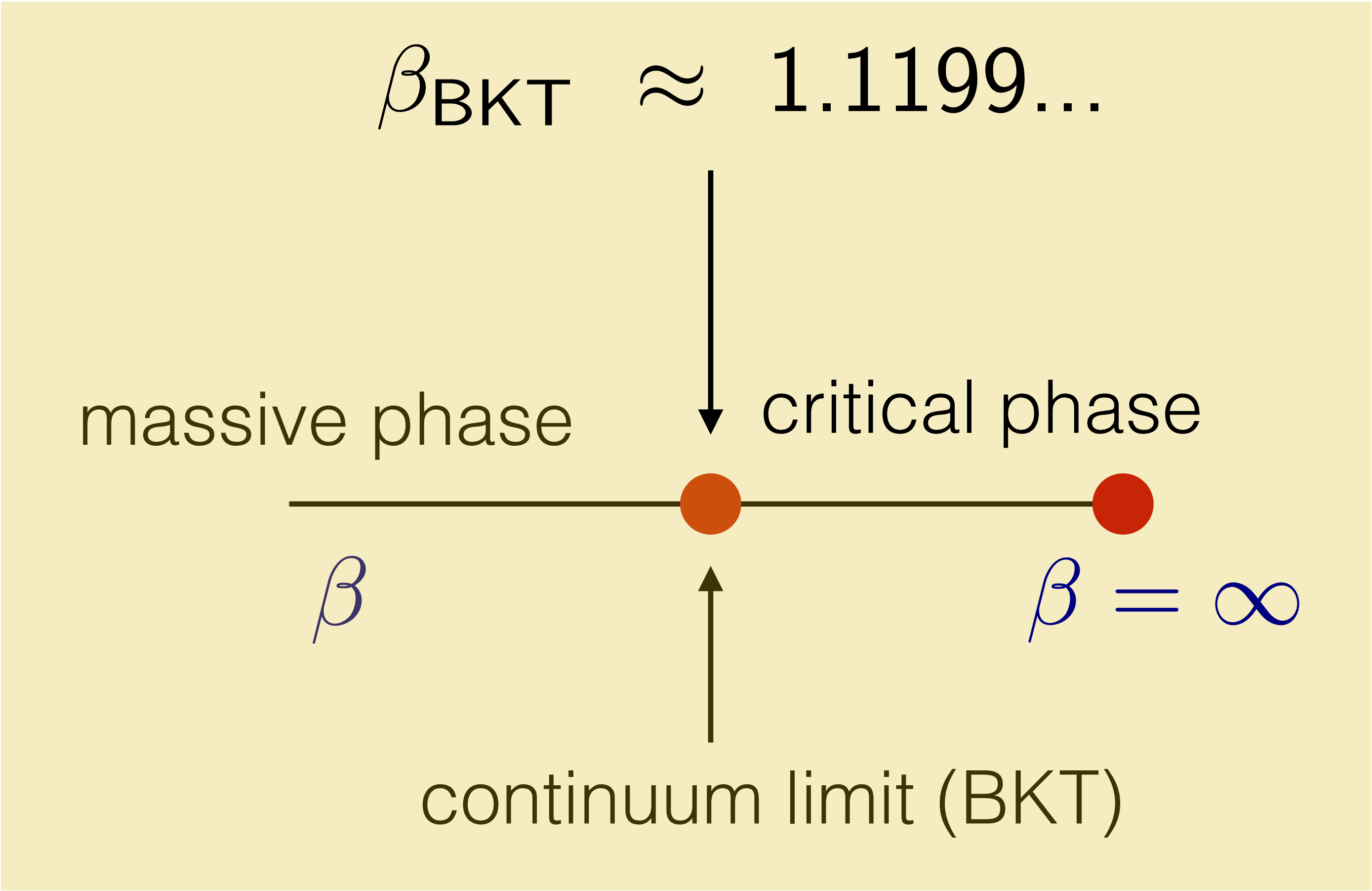
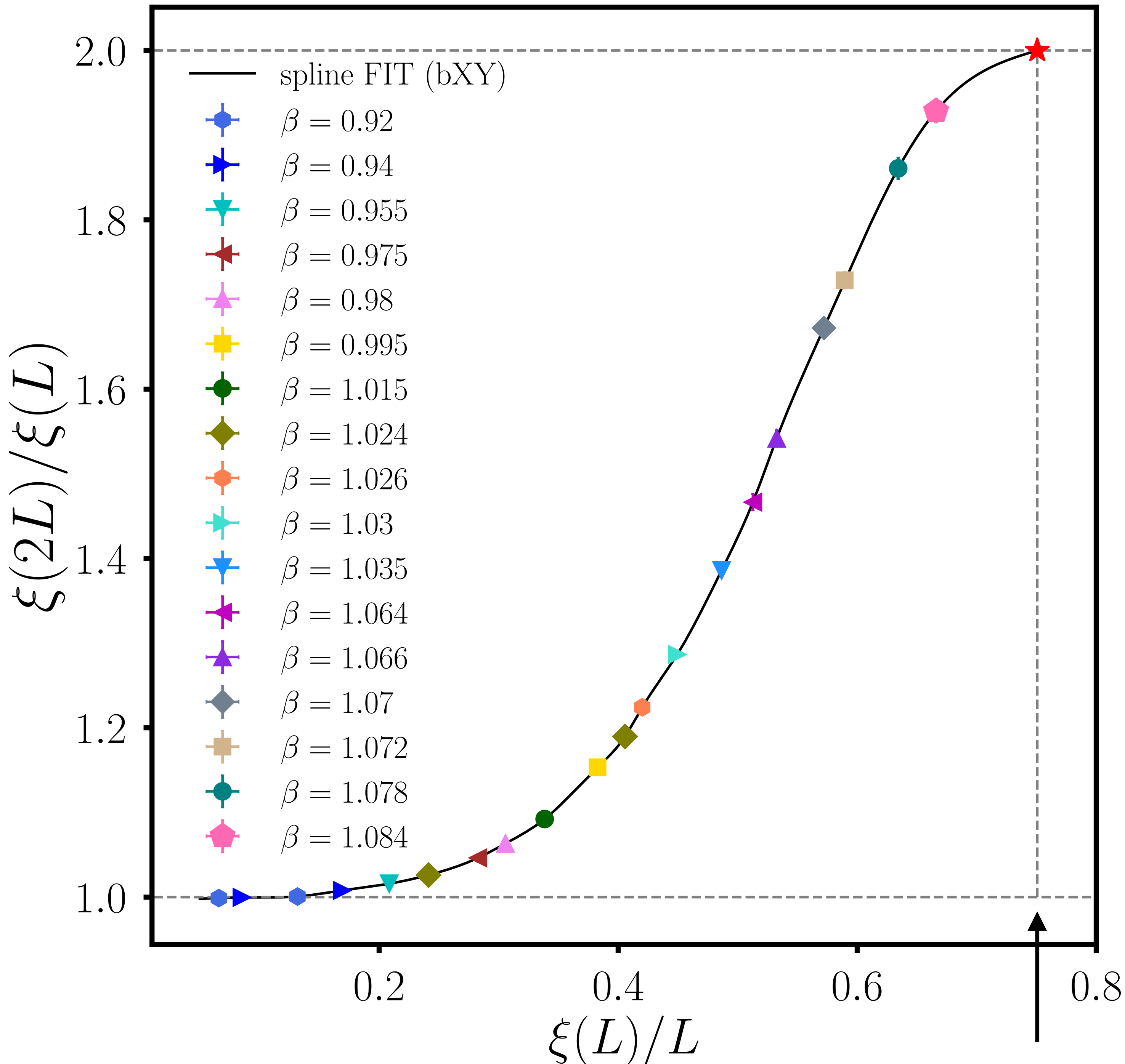


At the critical point we get a Gaussian theory!

The Step-Scaling function at the BKT transition using the traditional lattice model



$\alpha =$ 0.7506129...

Hasenbusch, cond-mat/0506552v2 (2008)

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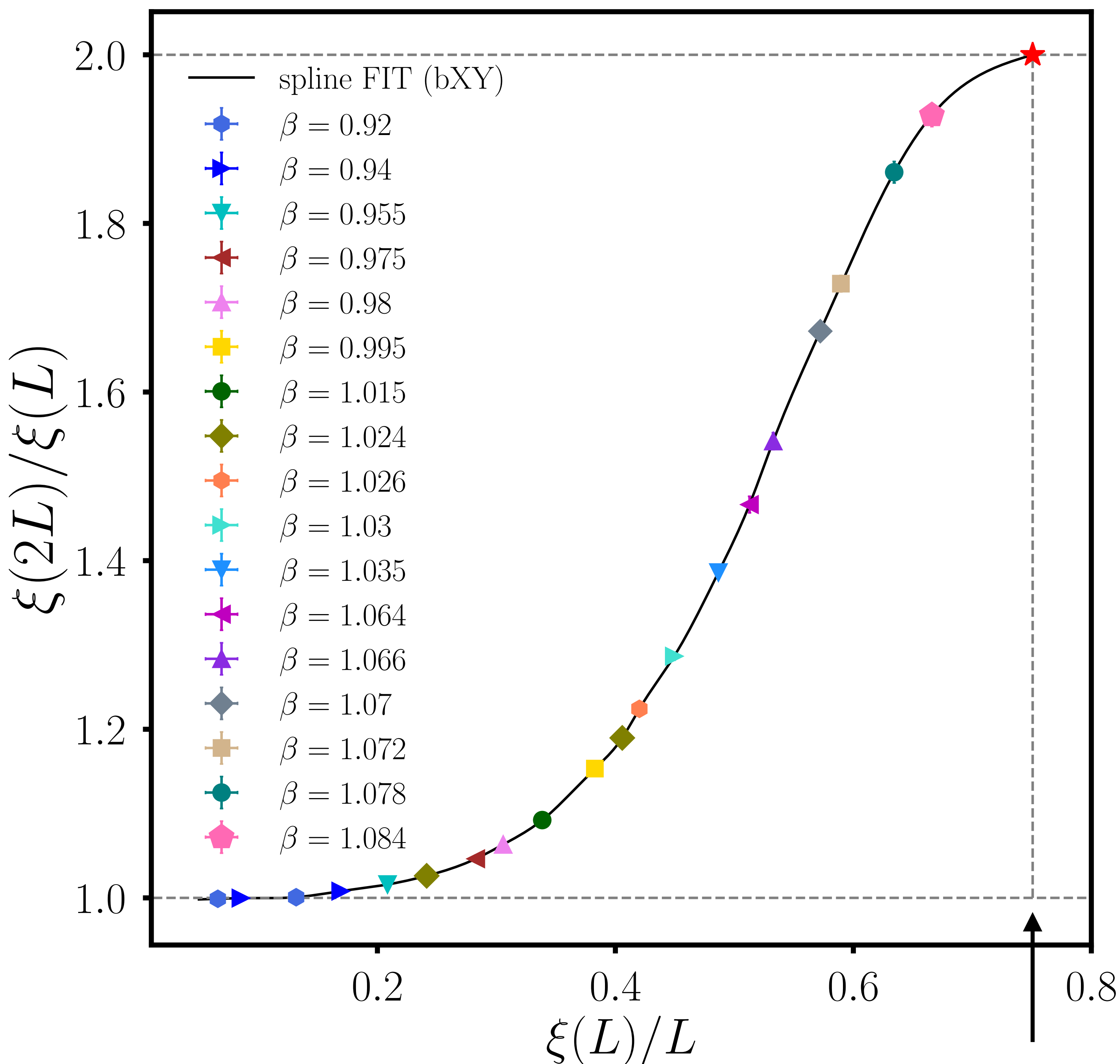

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Can we reproduce this continuum physics in a lattice model with a finite Hilbert space?

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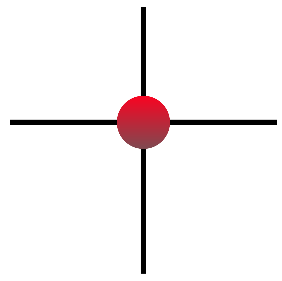
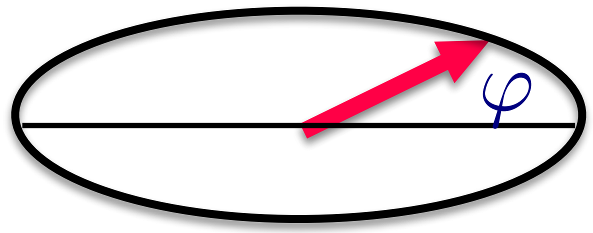
Qubit Regularization of SO(2) fields

Local site Hilbert space describes a quantum particle on a circle





$$\vec{\phi}_x = (\cos(\varphi_x), \sin(\varphi_x))$$

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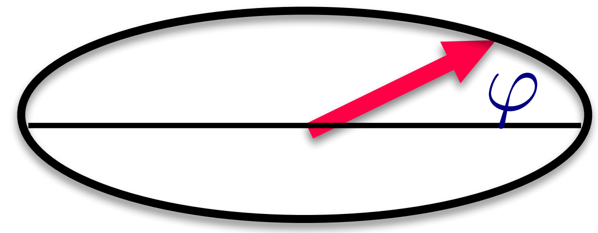

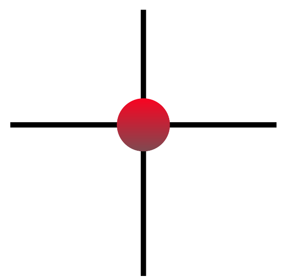

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"position basis"

$$\sum_{m=-\infty}^{\infty} |m\rangle\langle m| = I$$

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A simple qubit regularization scheme is $Q = \{0, 0, 1, -1\}$

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Space-time Euclidean configurations in the qubit regularization scheme $Q = \{0, 0, 1, 1\}$

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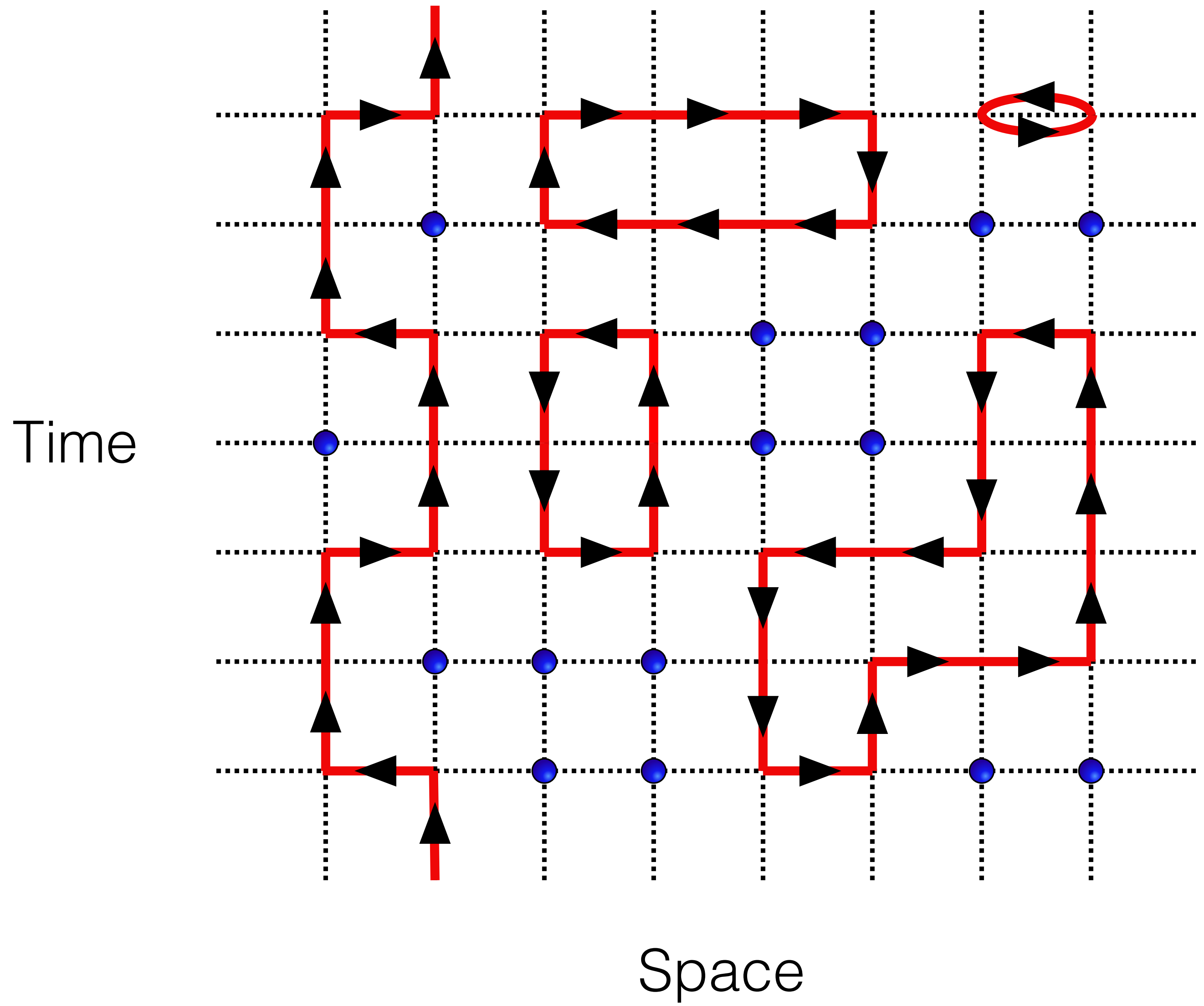

Mapping to two layers of closed pack dimer model

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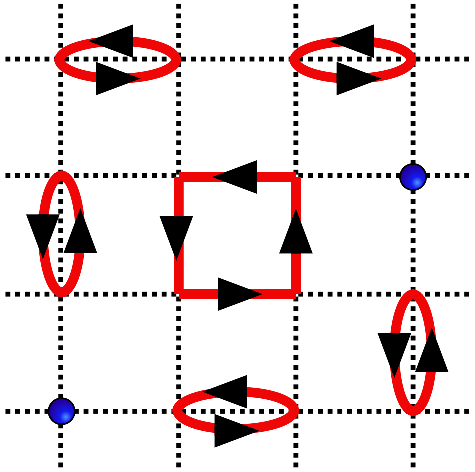
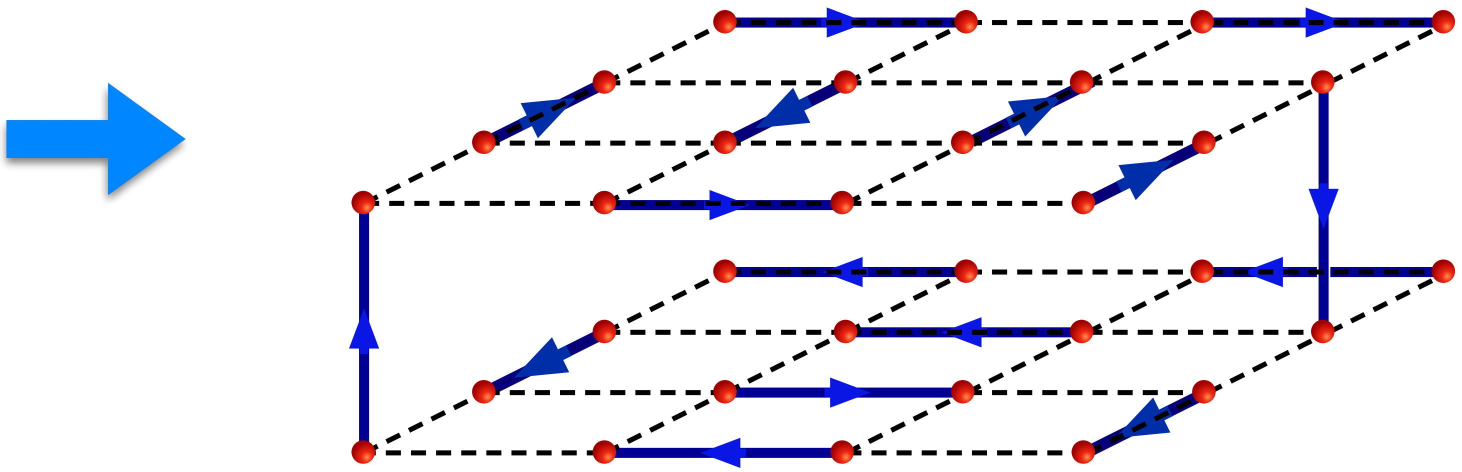
worldline configuration



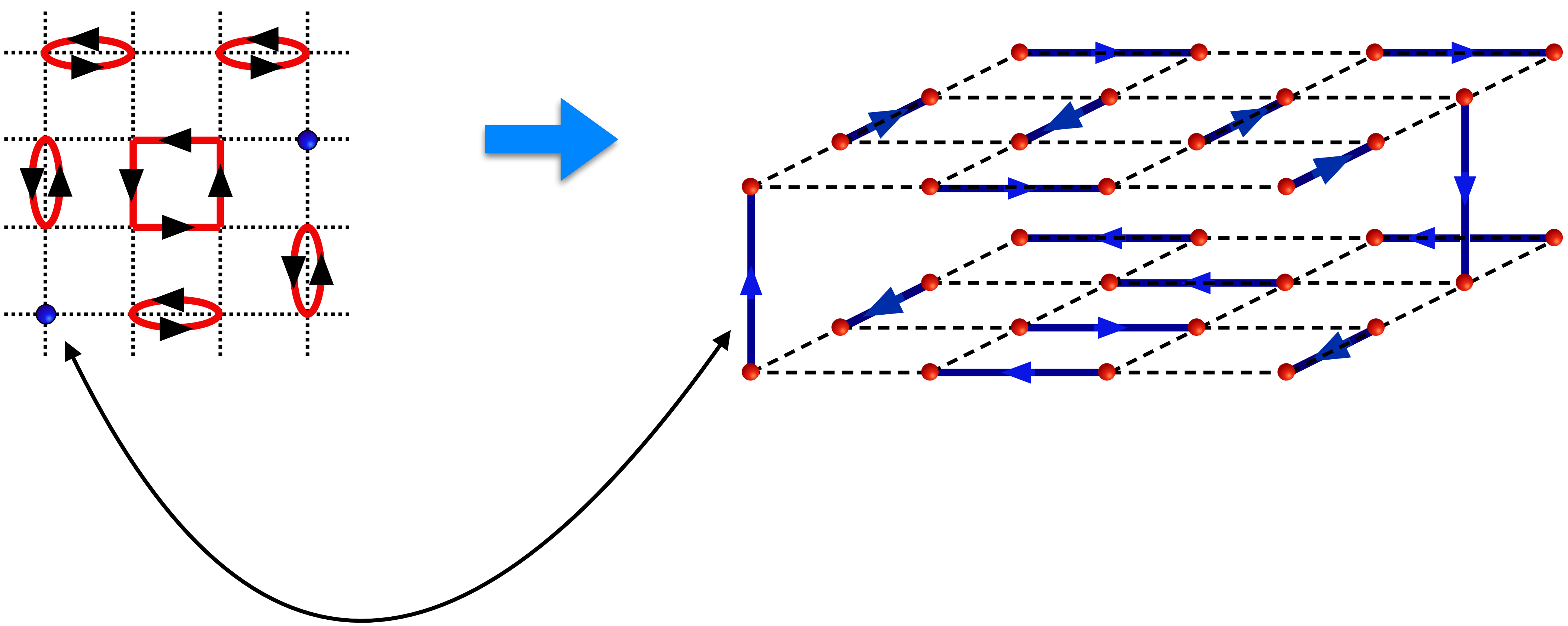
closed packed dimer configurations



Mapping to two layers of closed pack dimer model

worldline configuration

closed packed dimer configurations



Singlets are mapped to inter-layer dimers

Qubit regularized model

Maiti, Banerjee, SC, Marinkovic PRL 132 (2024), 041601

Qubit regularized model

Maiti, Banerjee, SC, Marinkovic PRL 132 (2024), 041601

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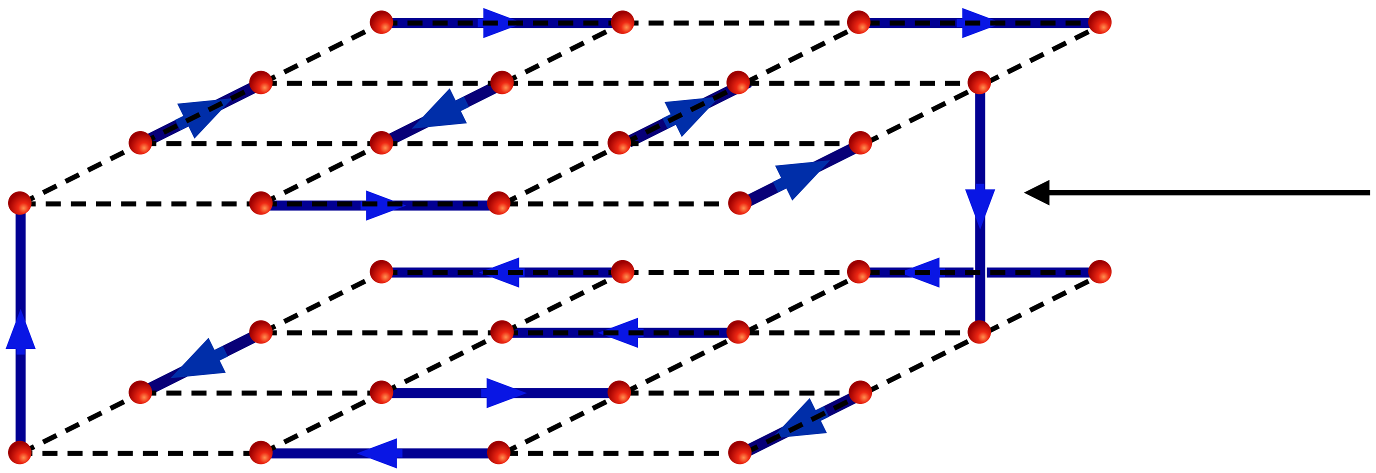


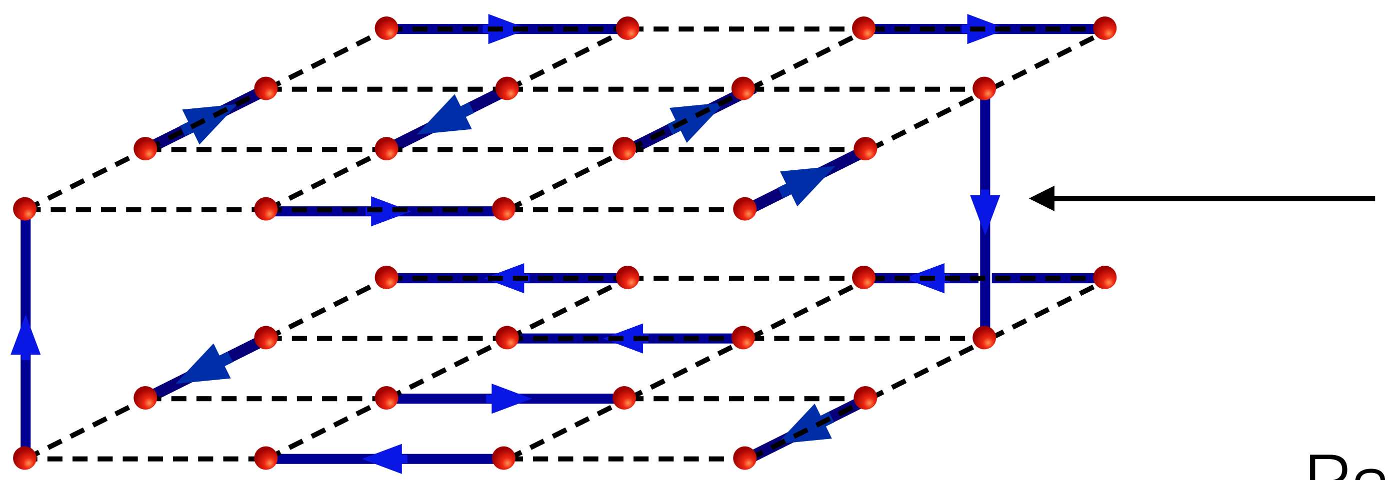
λ

Fugacity of interlayer dimers

Qubit regularized model

Maiti, Banerjee, SC, Marinkovic PRL 132 (2024), 041601

closed packed dimer configurations



λ

Fugacity of interlayer dimers

Partition function

$$Z = \sum_{C} \lambda^{N_I}$$

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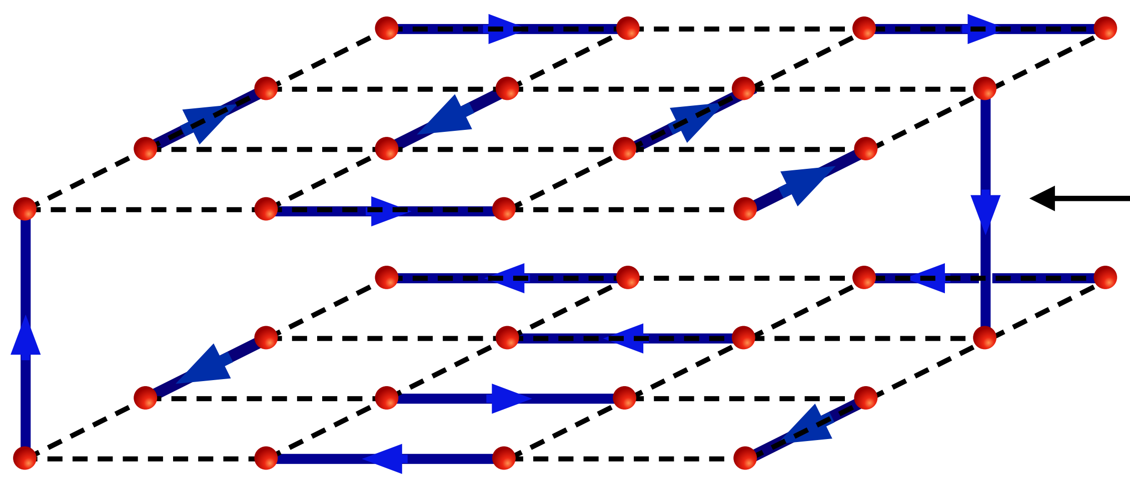
Maiti, Banerjee, SC, Marinkovic PRL 132 (2024), 041601

closed packed dimer configurations



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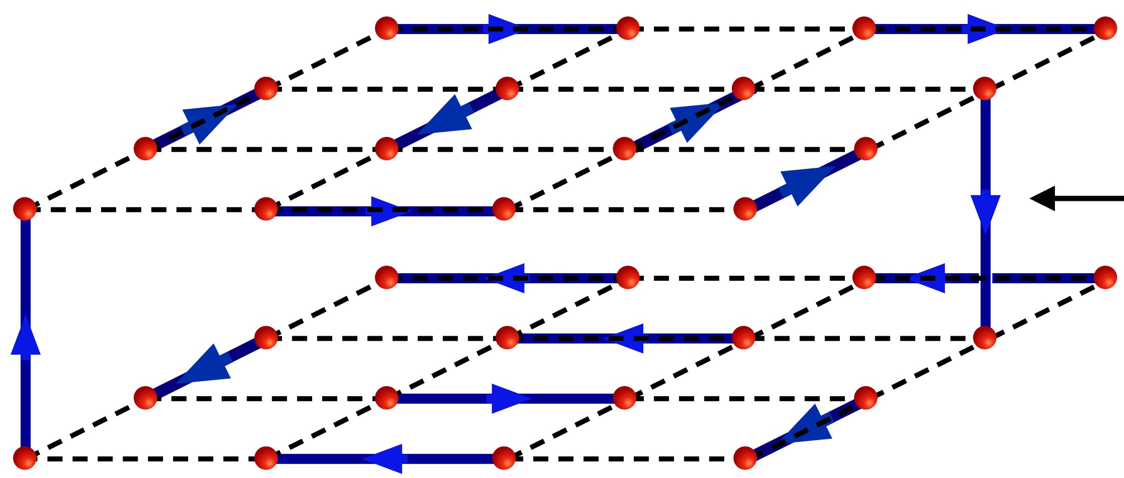

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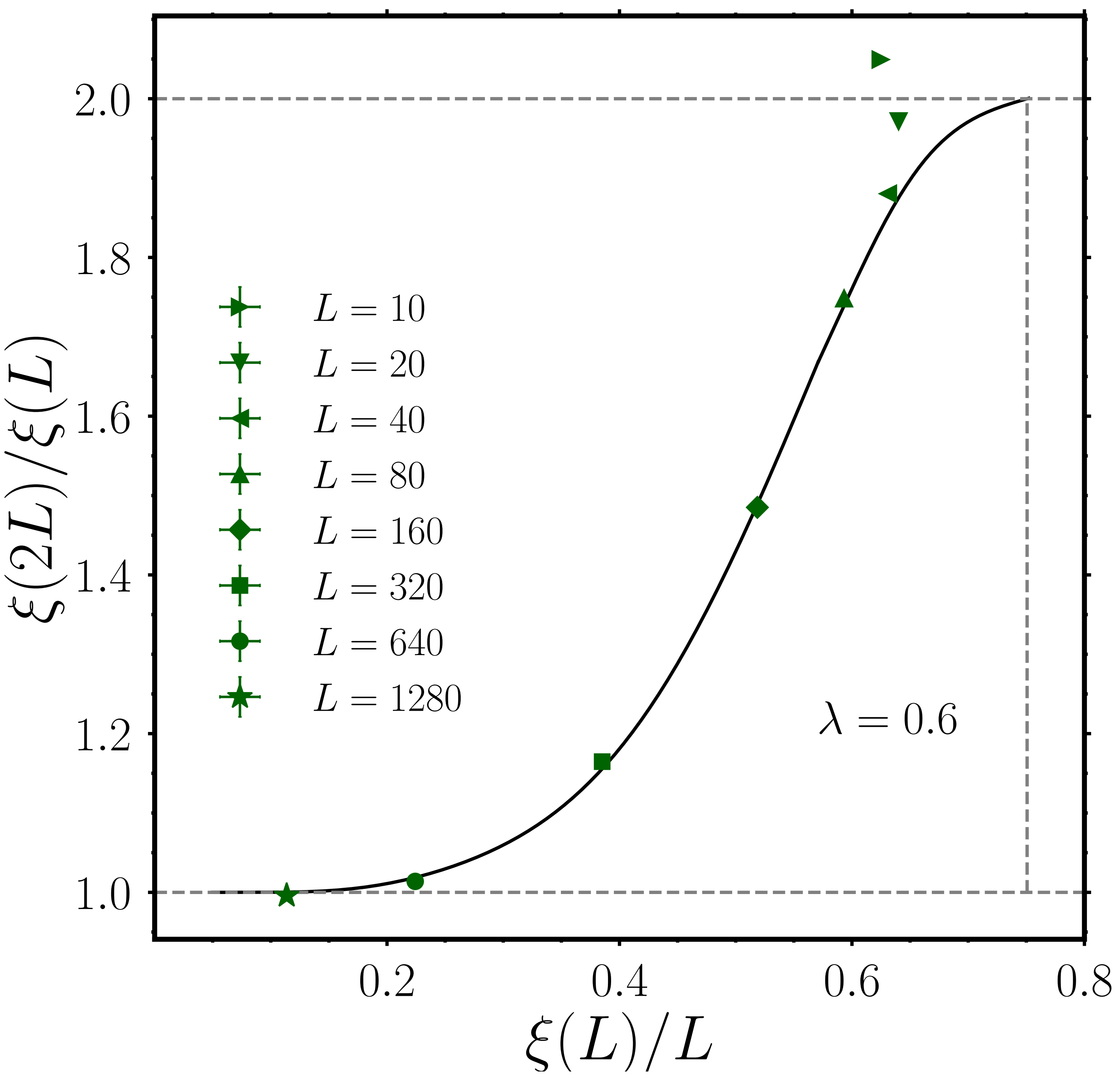
Continuum Limit: $\lambda \to 0$

At the critical point we get a decoupled critical theory that is again not the desired free theory!

Step-scaling function of the qubit model



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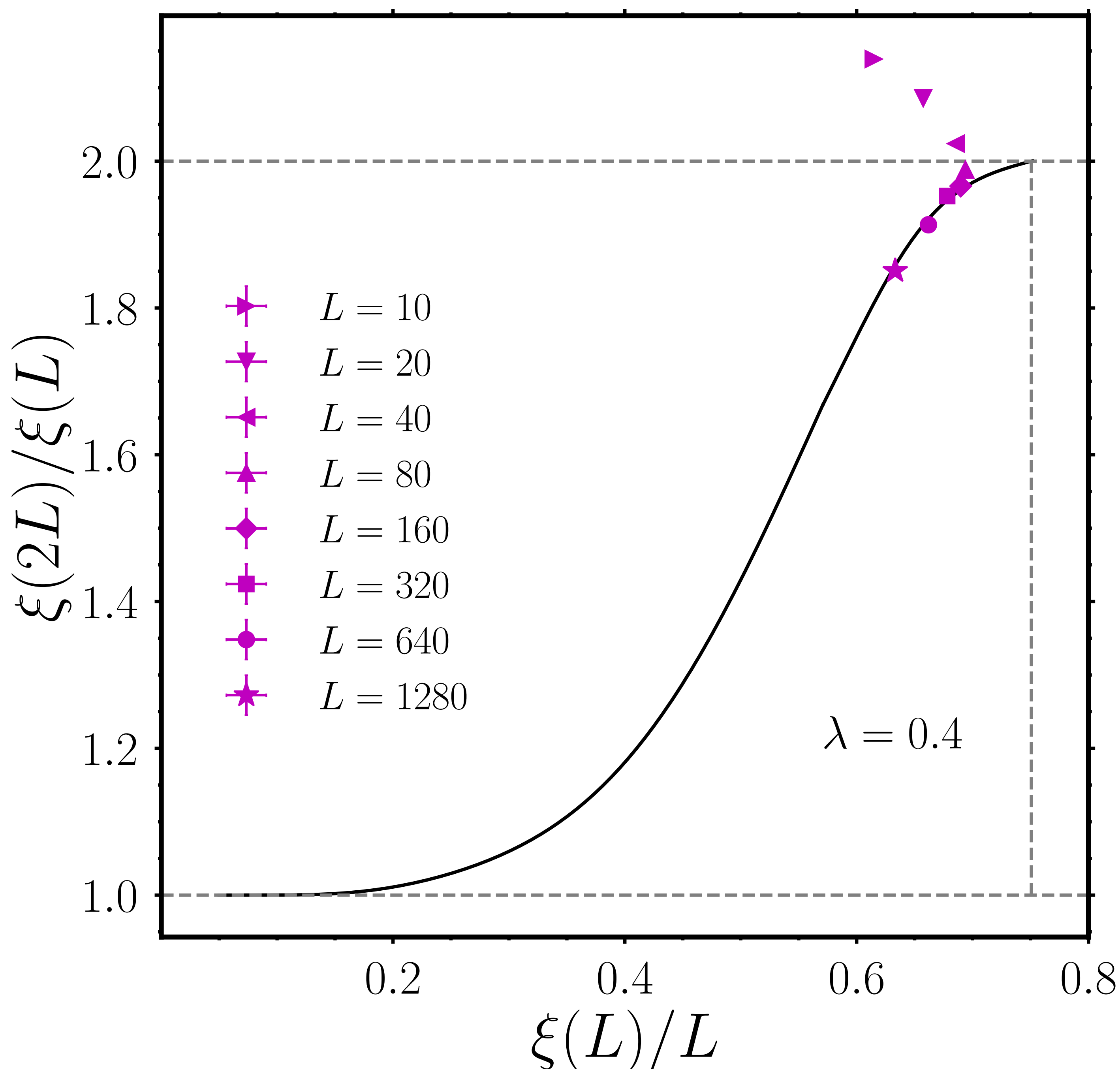

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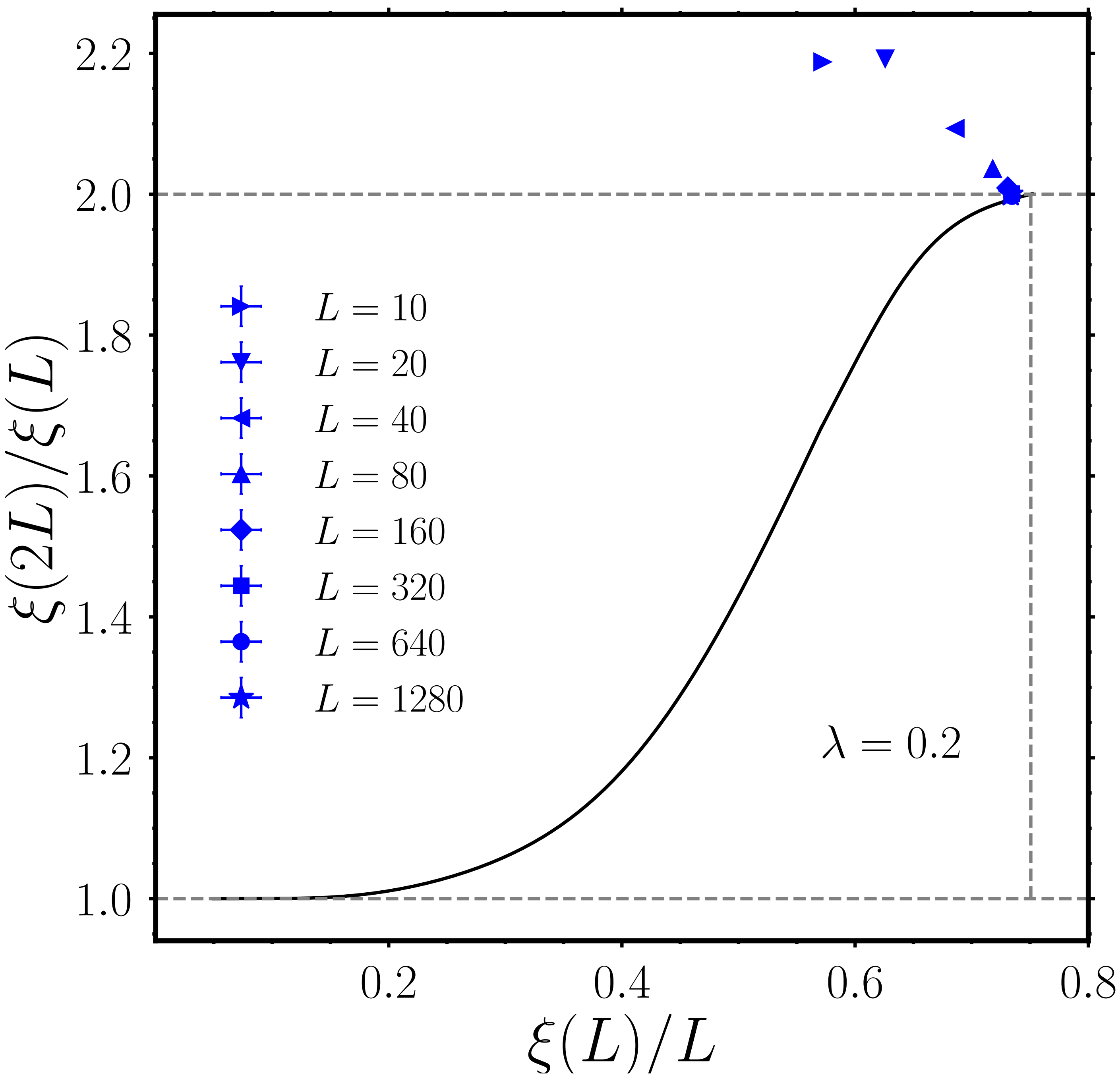

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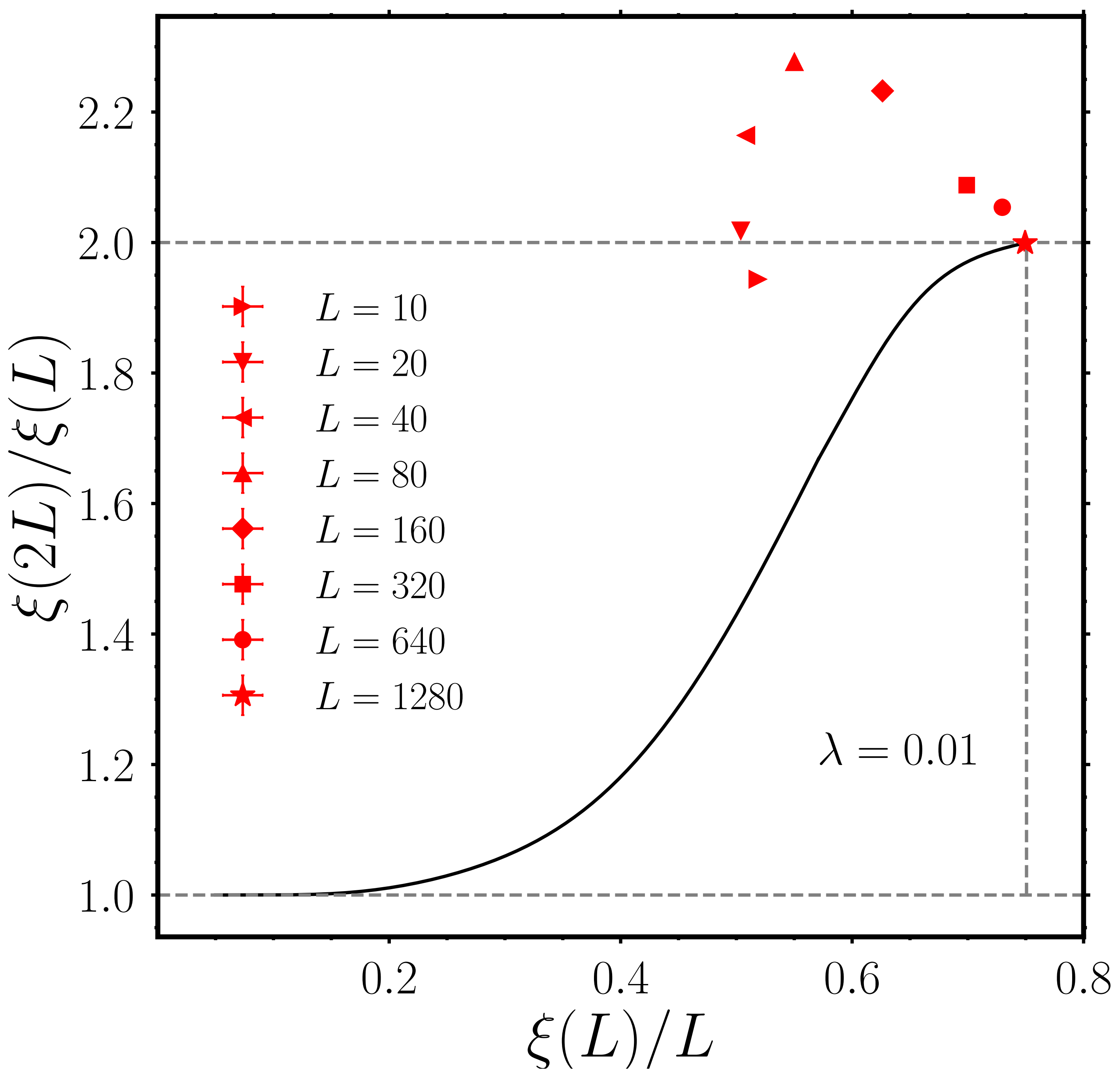
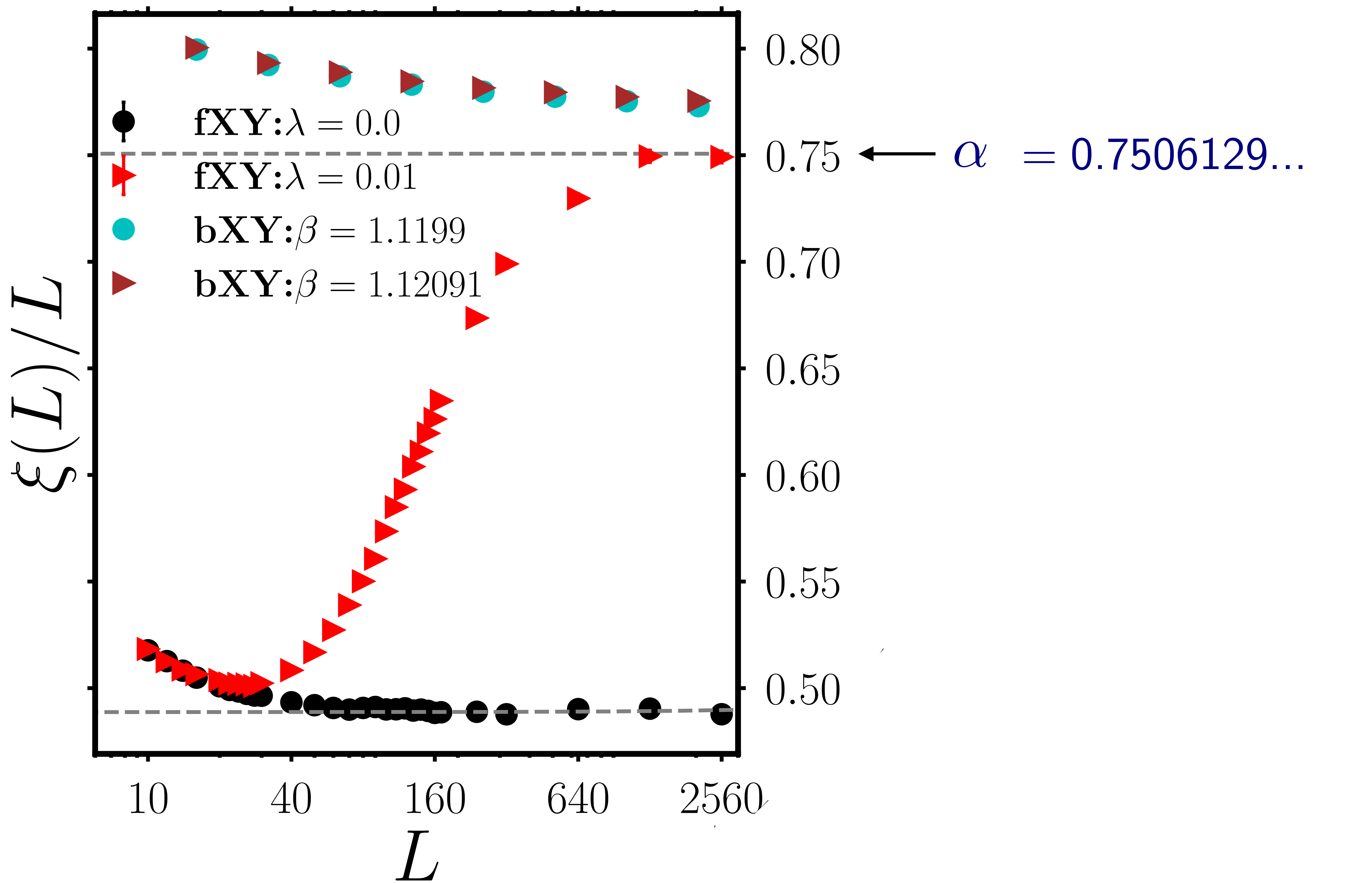


Decoupled vs. Coupled Theory



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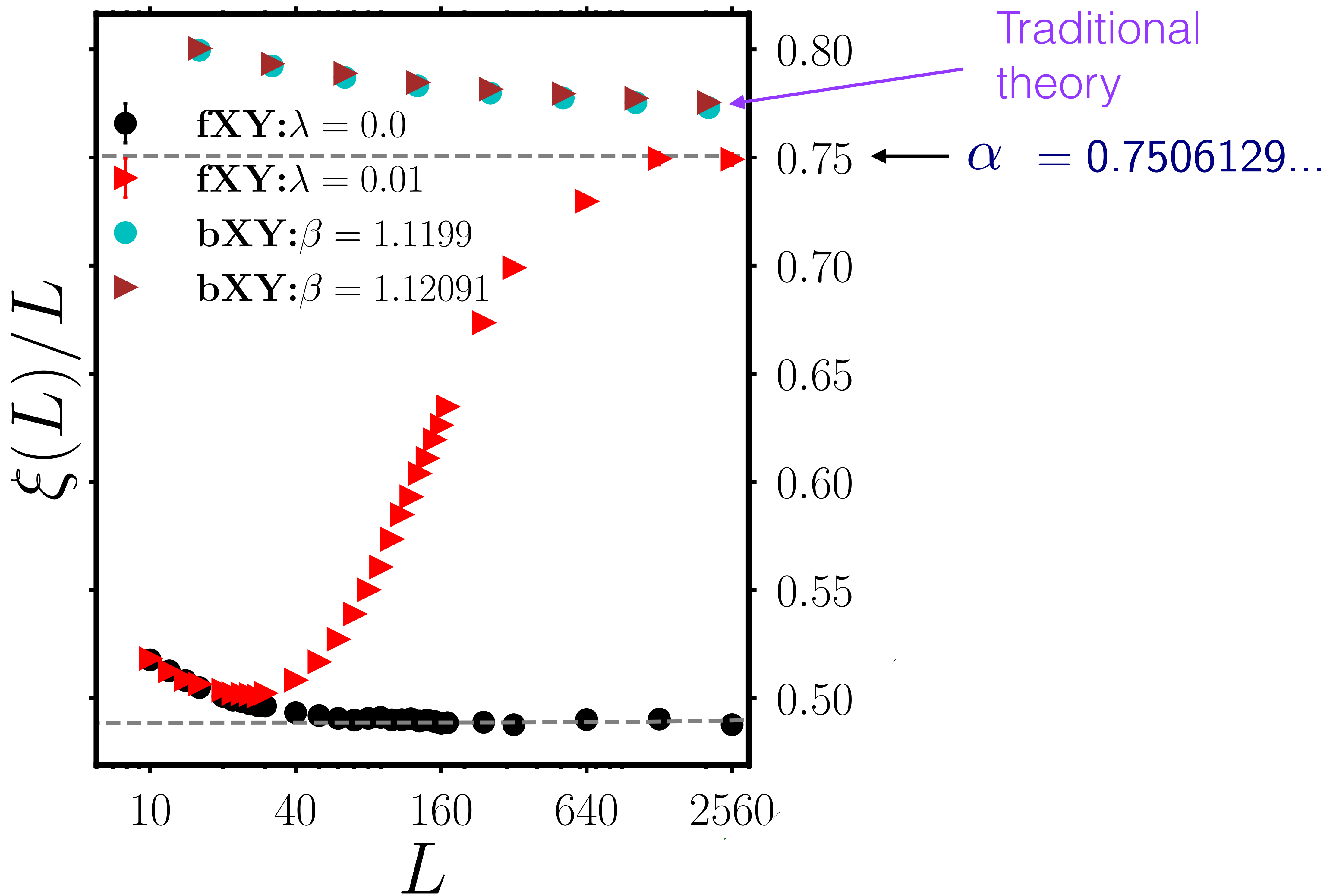

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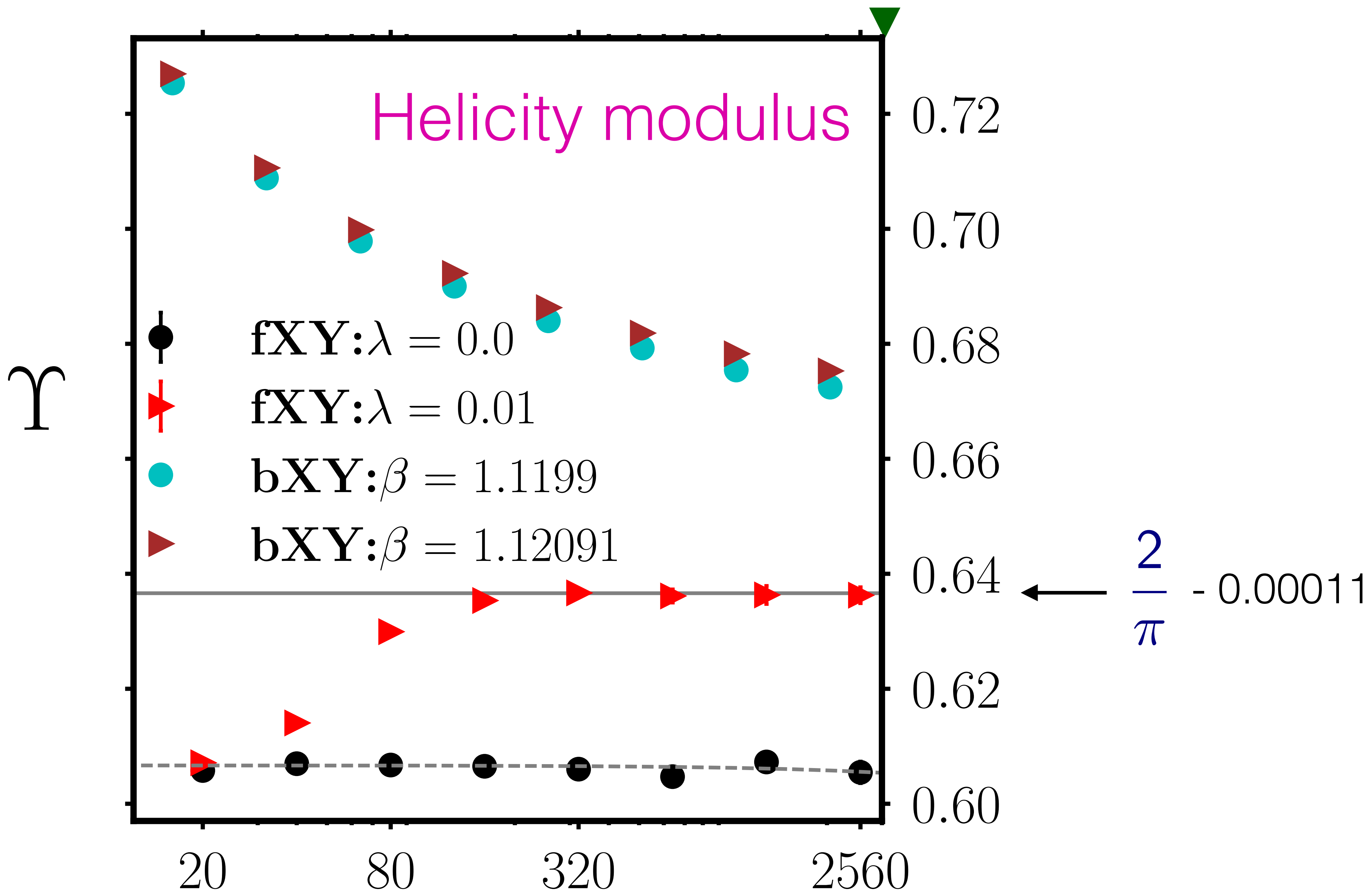

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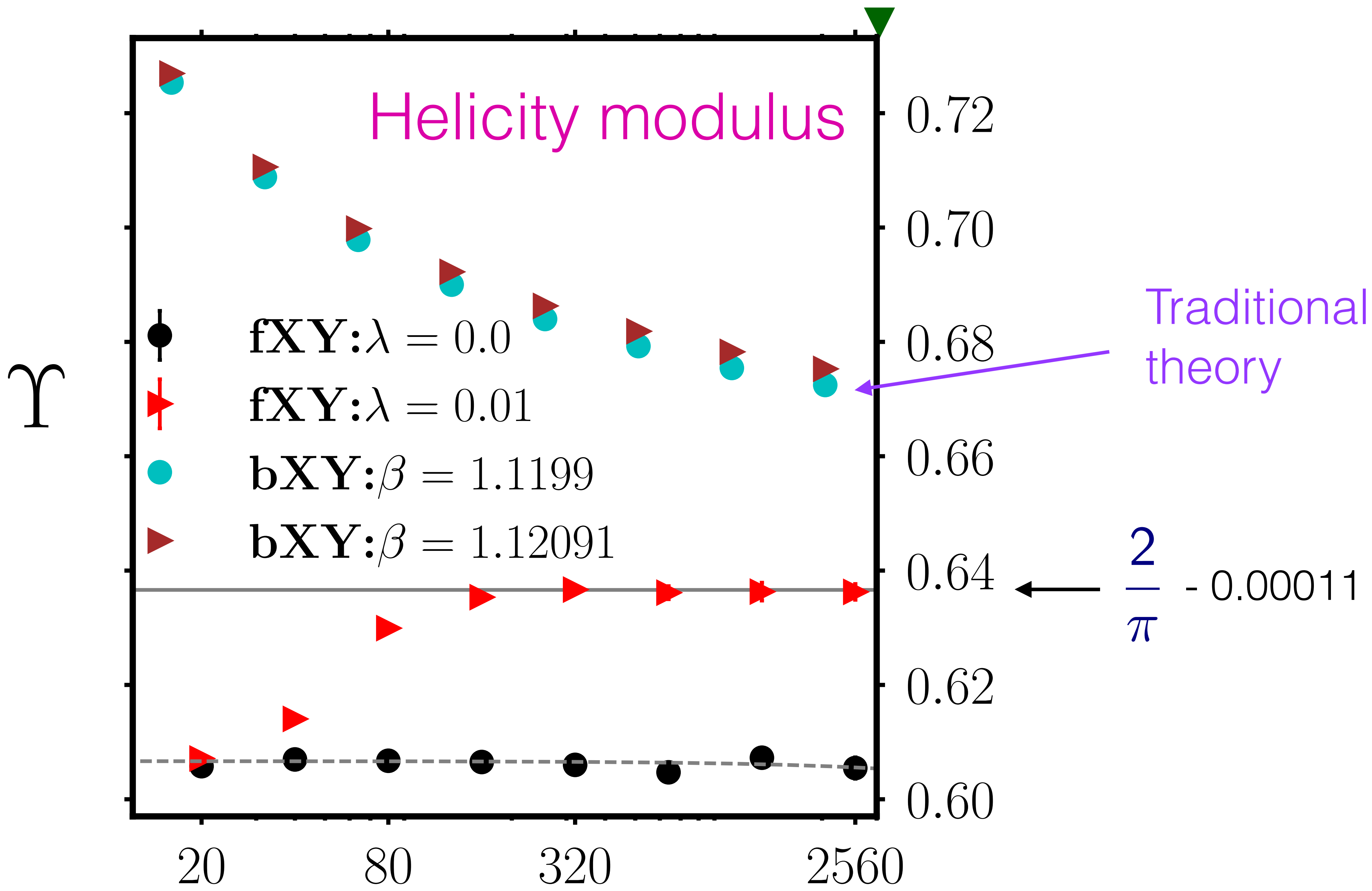

Massive Phase of the XY model
at the BKT transition

$\mathcal{H}_{\mathsf{Trad}} = \textit{infty}$

$\equiv$

Two layers of coupled
closed pack dimers

$\lambda \to 0$

$\mathcal{H}_{\mathsf{Q}} = 4$

A Novel-RG flow for Asymptotic Freedom

In both these examples, asymptotic freedom is recovered via new type of RG flow

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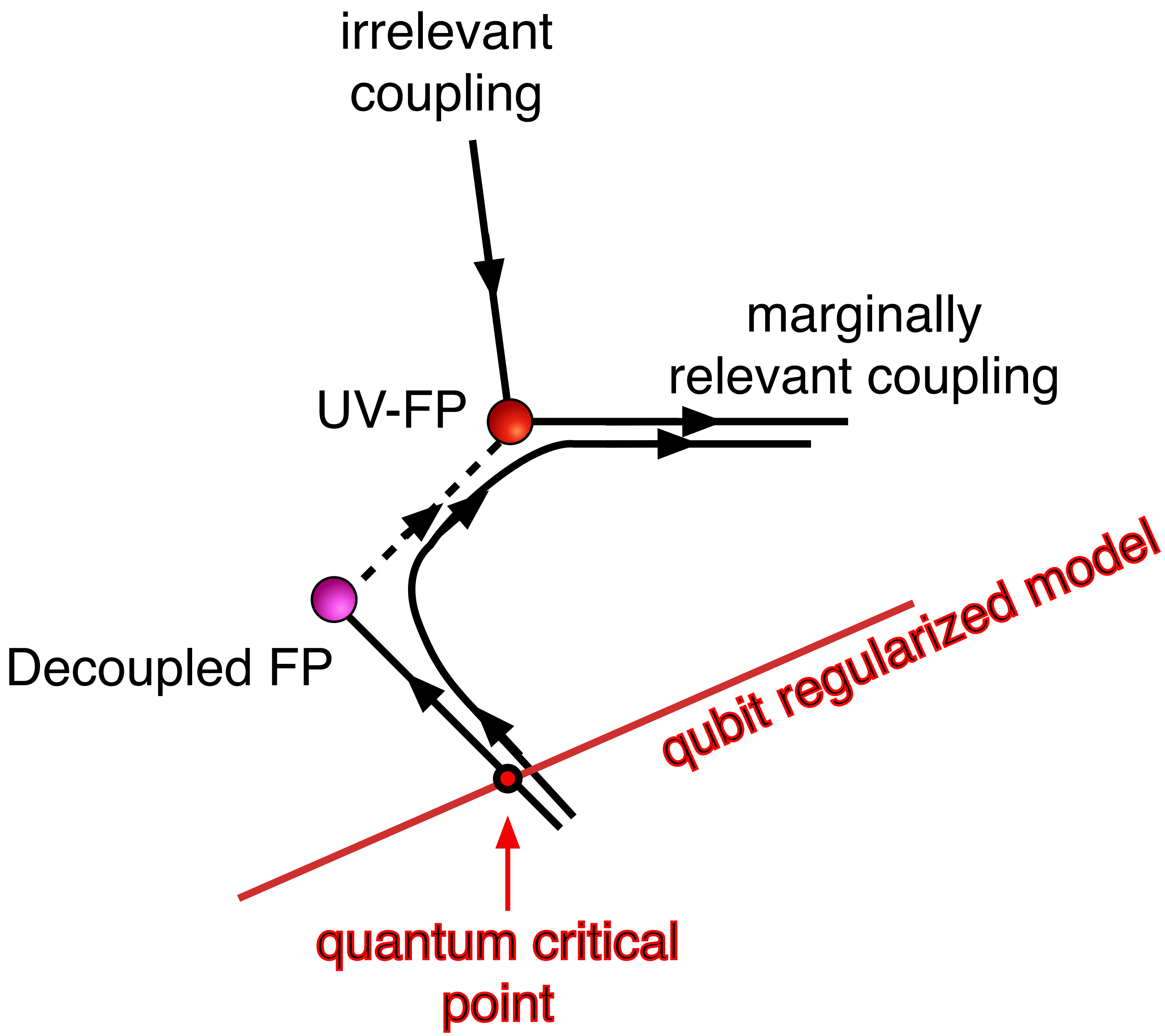

Qubit Regularization of Asymptotic Freedom in Gauge Theories

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Asymptotic Freedom of Yang Mills theory

Deconfined phase at high temperatures

Confined massive phase at zero temperatures

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Classical qubit models must already show this!

Qubit Regularization of SU(N) gauge fields

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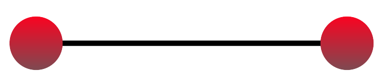
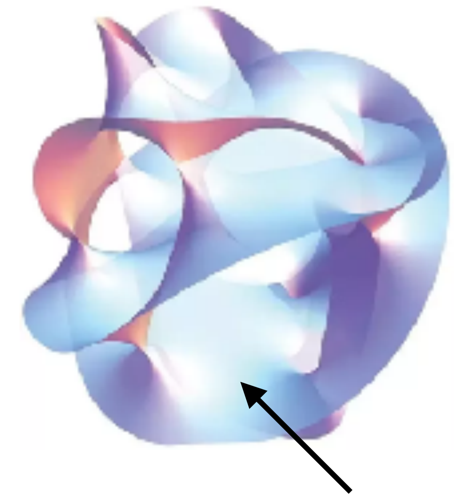
Local link Hilbert space describes a quantum particle on the surface of the SU(N) group manifold





$g \in SU(N)$

Qubit Regularization of SU(N) gauge fields

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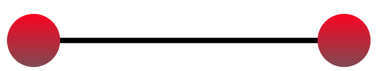

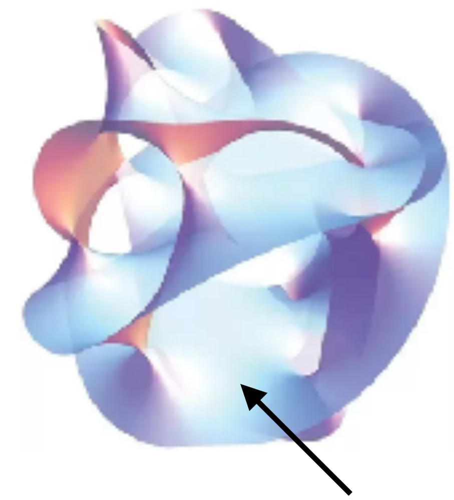

$g \in SU(N)$

Basis of the full Hilbert space $\mathcal{H}_{\text{Trad}}$:

$$\int [dg]\ |g\rangle\langle g| = I$$

"position basis"

$$\sum_{\lambda} \sum_{i,j} |D^{\lambda}_{ij}\rangle\langle D^{\lambda}_{ij}| = \mathbb{I}$$

"momentum basis"

λ labels distinct irreps of SU(N)

This means the traditional link Hilbert space is given by

$$\mathcal{H}_{\mathsf{Trad}} = \bigoplus_{\lambda} \mathcal{H}_\lambda$$ ⟵ Peter-Weyl Theorem

where $\mathcal{H}_\lambda = V_\lambda \otimes V_\lambda^*$ Is spanned by $\{|D_{ij}^{\lambda}\rangle\}, i, j = 1, 2, ..., d_\lambda$

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Qubit Regularized Hilbert Space $\mathcal{H}_{Q} = \bigoplus_{\lambda \in Q} V_{\lambda} \otimes V_{\lambda}^{*}$

$$dim(\mathcal{H}_{Q}) = \sum_{\lambda \in Q} (d_{\lambda})^{2}$$

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A simple qubit regularization involves $Q = \{\underbrace{1, \square, \begin{smallmatrix}\square\\ \square\end{smallmatrix}, \ldots}\}$

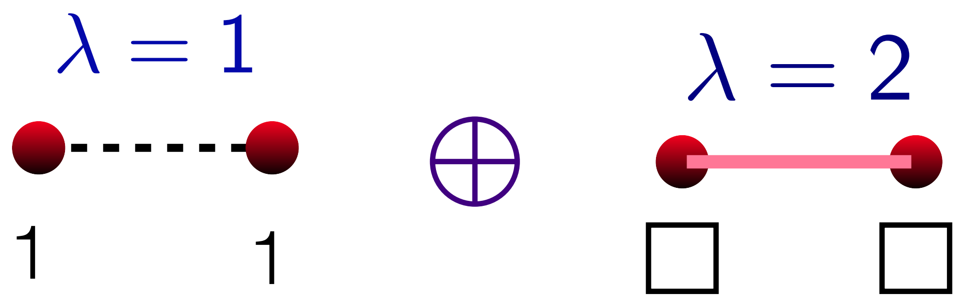
Hanqing Liu, SC Symmetry 14 (2022) 2 305,

↑ All anti-symmetric irreps

SU(2):

$$Q = \{1, 2\}$$

$$\mathcal{H}_Q =$$



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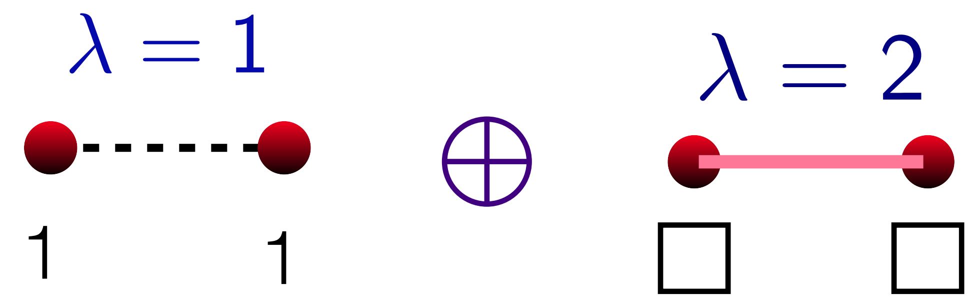

The physical Hilbert Space $\mathcal{H}_{\text{phys}}$ only involves gauge invariant states

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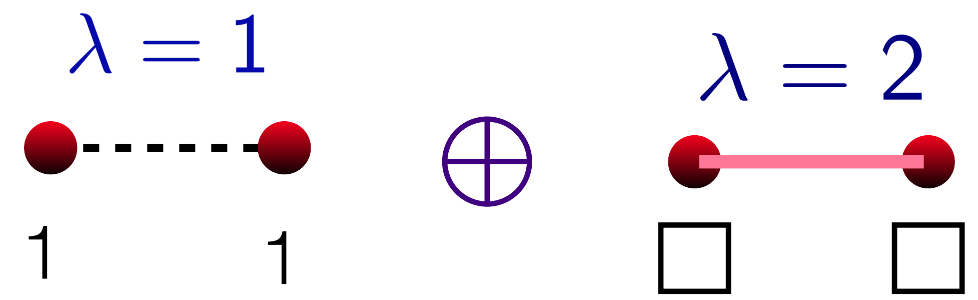
$\mathcal{H}_Q =$ 

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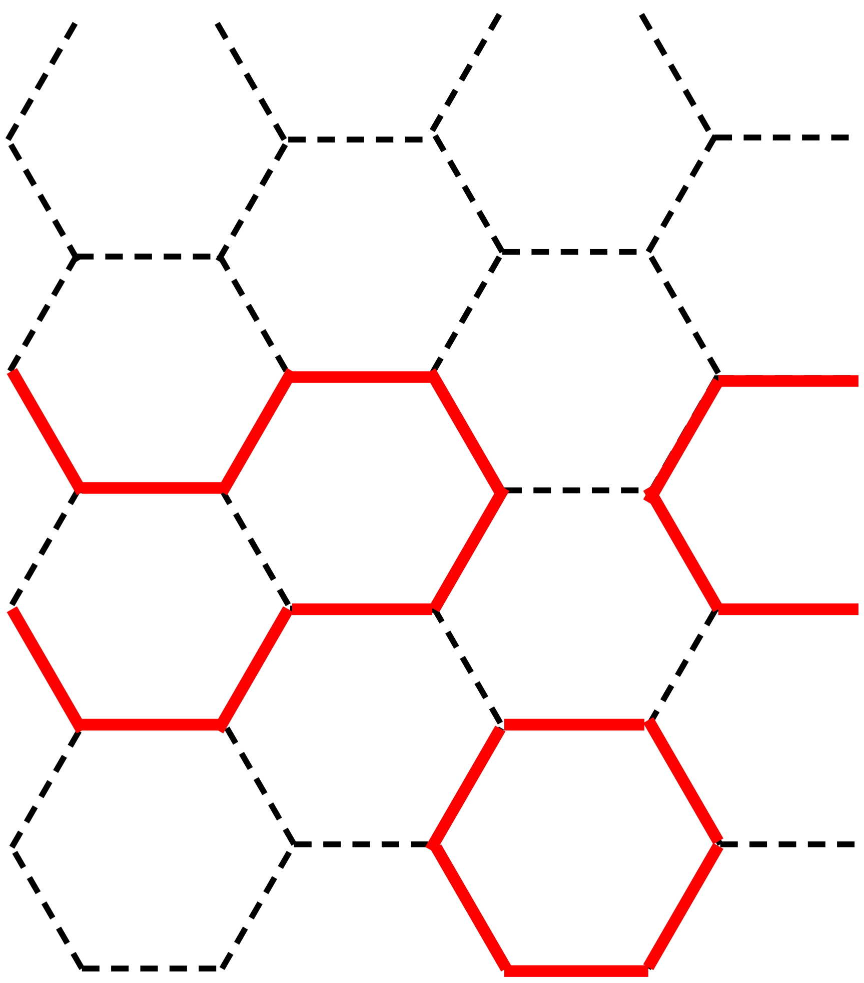

Gauge invariant
pure gauge state

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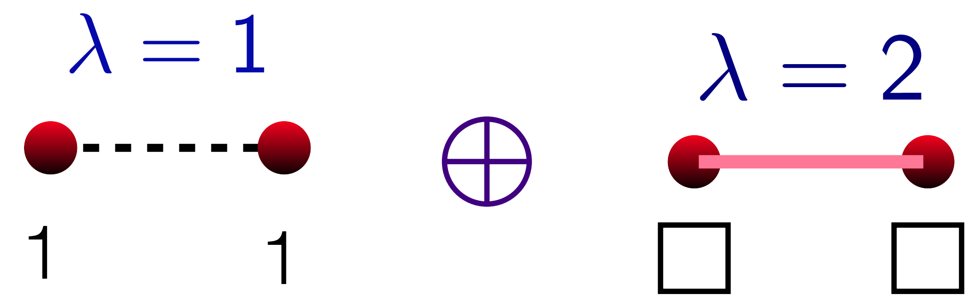
$\mathcal{H}_Q =$ 

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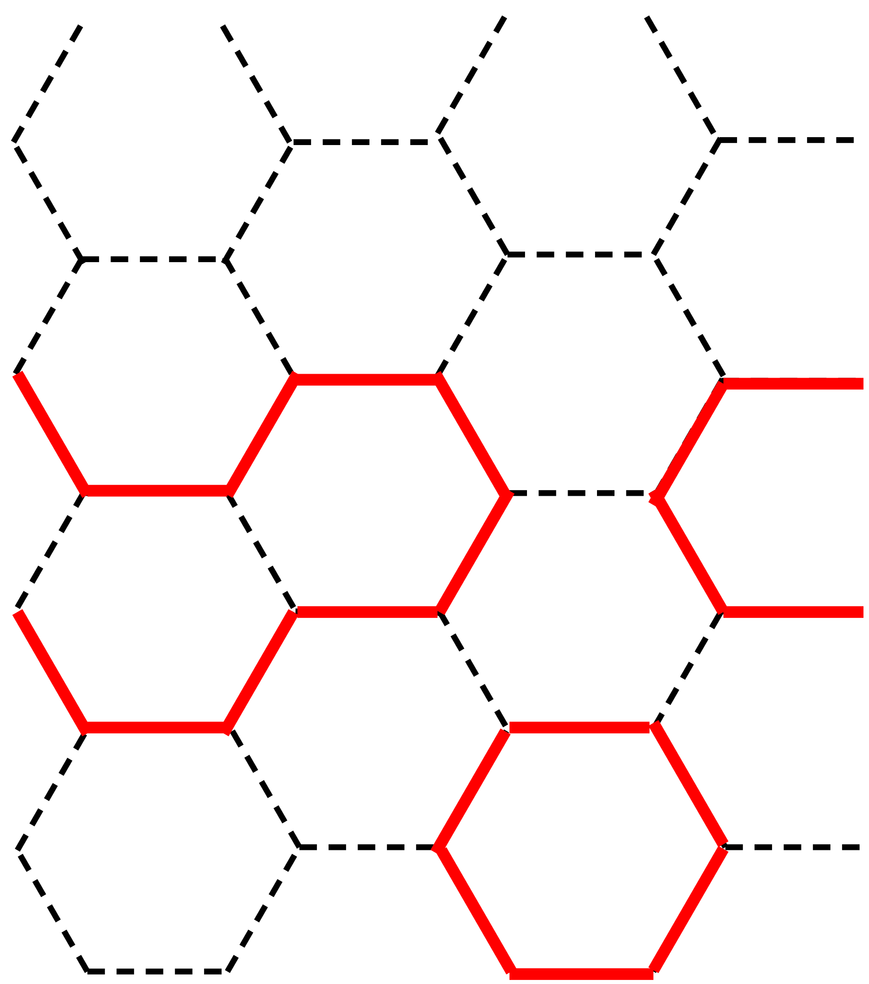

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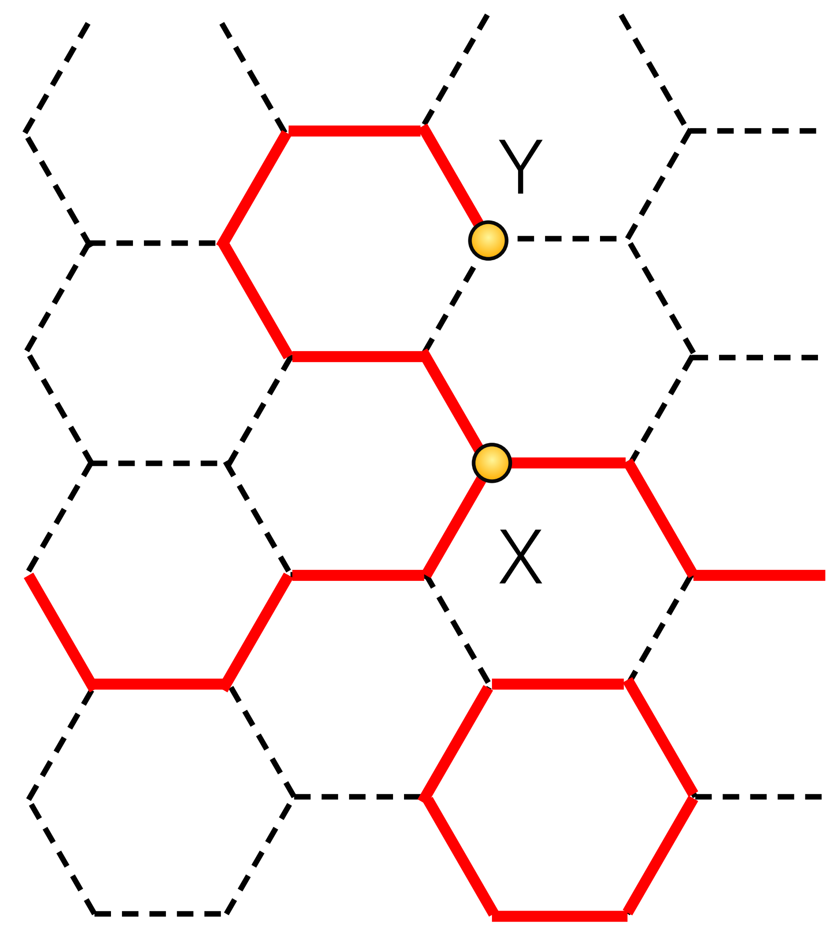
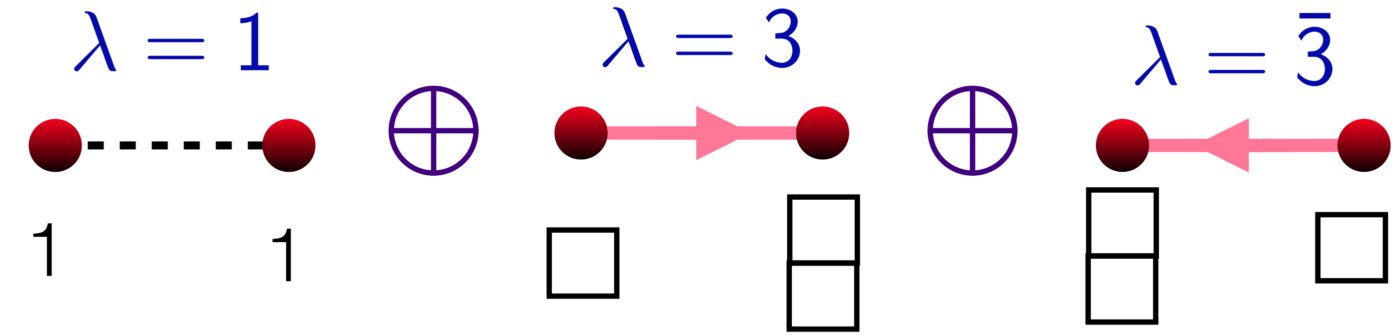


Gauge invariant state
with two quarks

SU(3):

$Q = \{1, 3, \bar{3}\}$

$\mathcal{H}_Q =$



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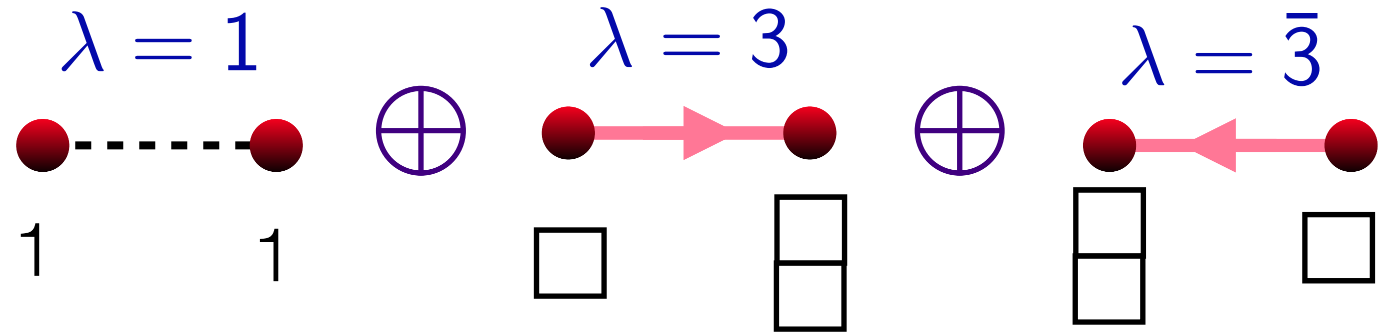

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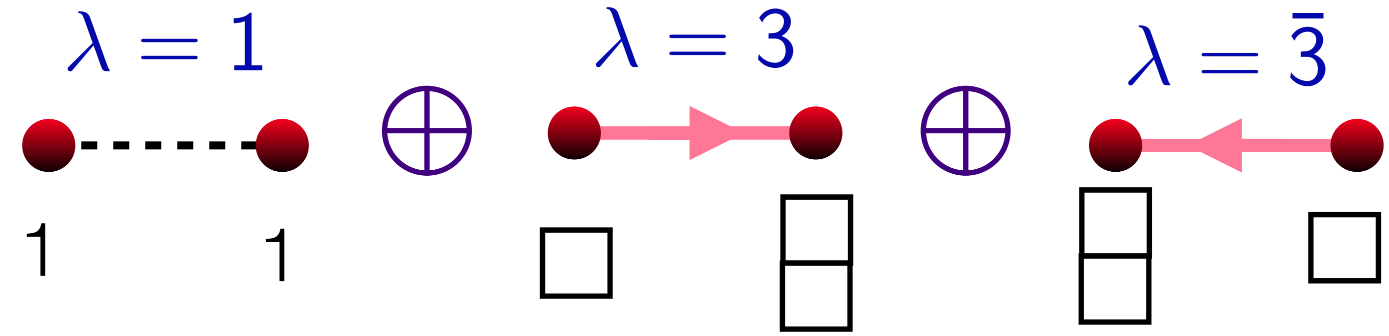

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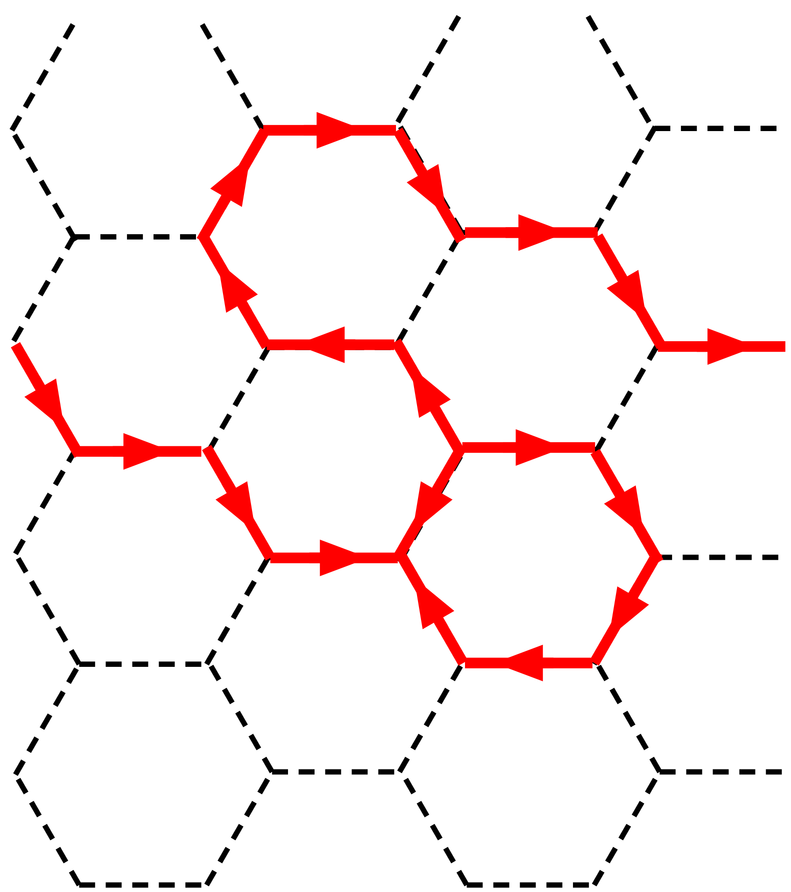

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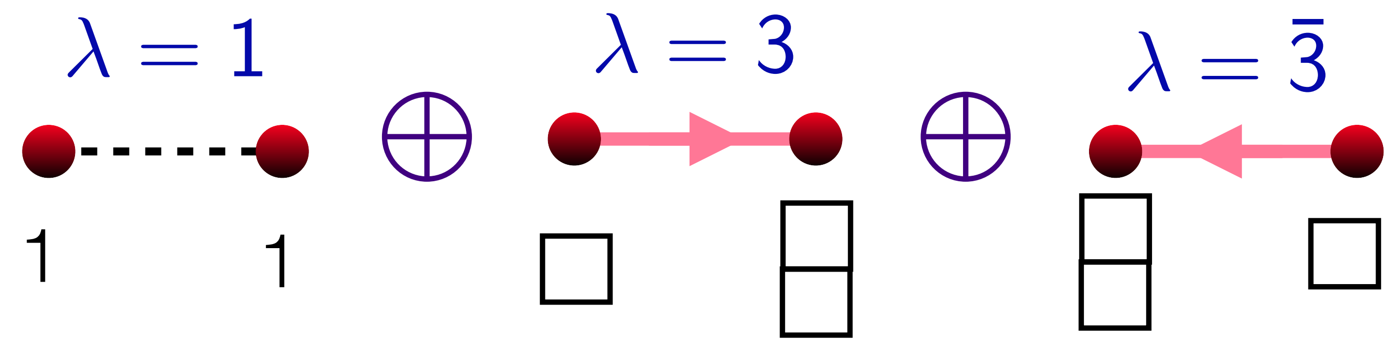

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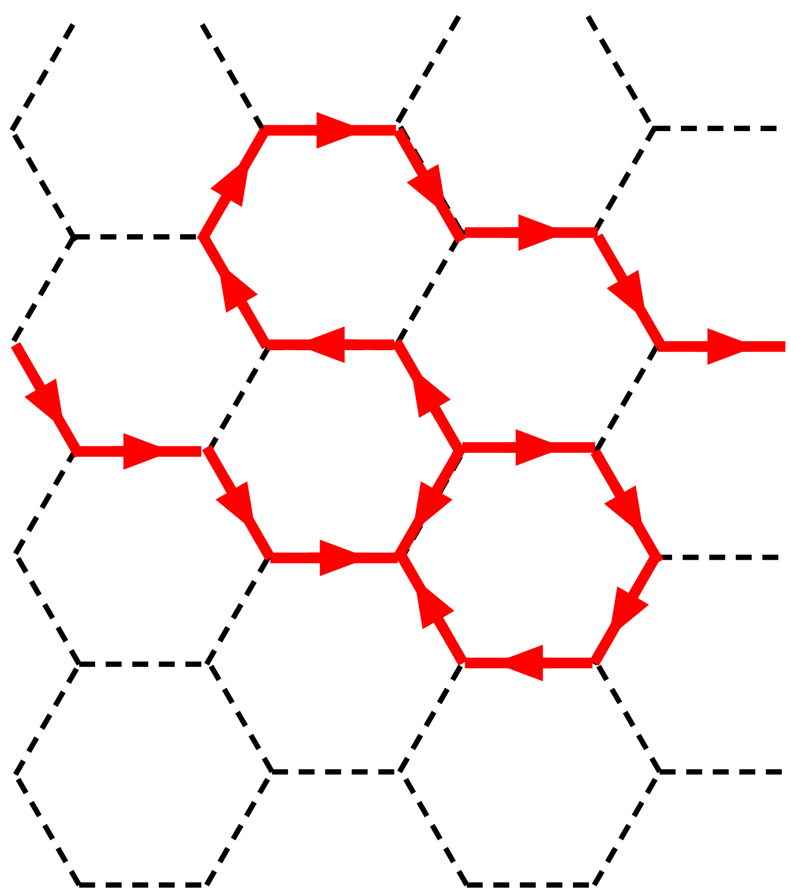
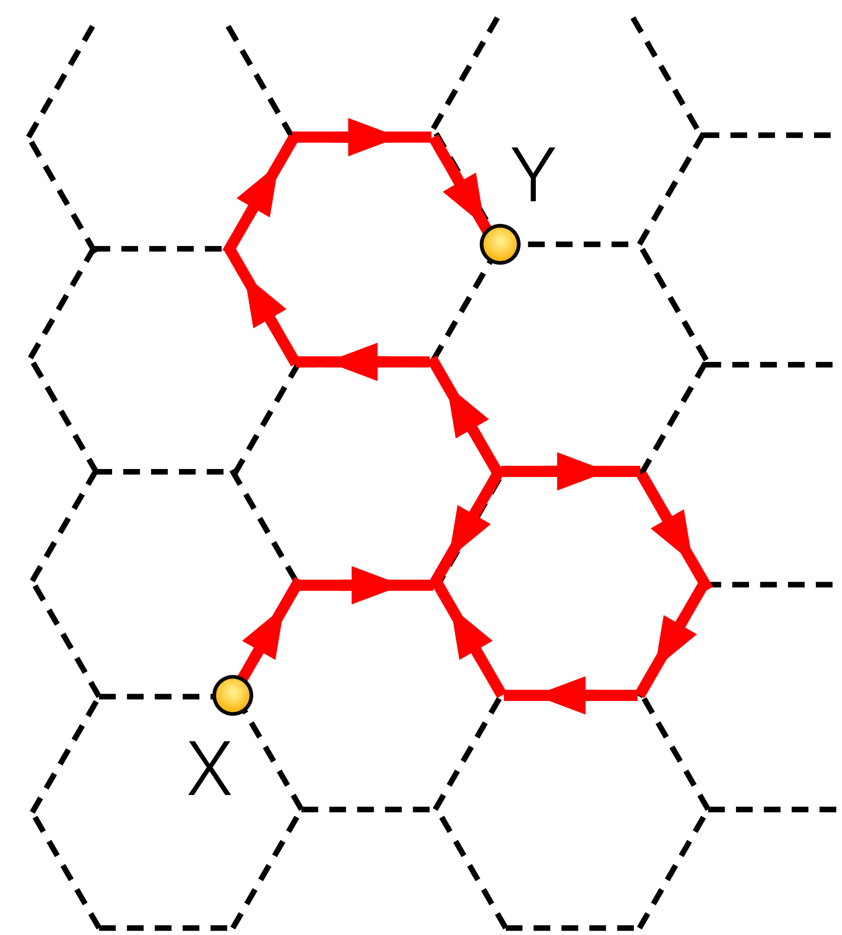


Gauge invariant pure gauge state



Gauge invariant state with a quarks and an anti-quark.

Non-Abelian Gauge Theories

“momentum basis” →

Generalized Quantum Dimer Models

Non-Abelian Gauge Theories → ("momentum basis") → Generalized Quantum Dimer Models

“momentum basis”

Easy to study classical dimer models with Monte Carlo methods!

Non-Abelian Gauge Theories → "momentum basis" → Generalized Quantum Dimer Models

Easy to study classical dimer models
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A "confinement" observable
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$$\chi = \frac{1}{V} \sum_{x,y} \frac{Z_{xy}}{Z}$$

$$Z \;=\; \mathrm{Tr}\left(e^{-\beta H_{\mathrm{cl}}}\right) \qquad Z_{xy} \;=\; \mathrm{Tr}_{xy}\left(e^{-\beta H_{cl}}\right)$$

Non-Abelian Gauge Theories $\xrightarrow{\text{“momentum basis”}}$ Generalized Quantum Dimer Models

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Deconfined phase: $\chi \sim L^3$ Confined phase: $\chi \sim \mathrm{Const}$

Confinement-Deconfinement Transition in (3+1)d

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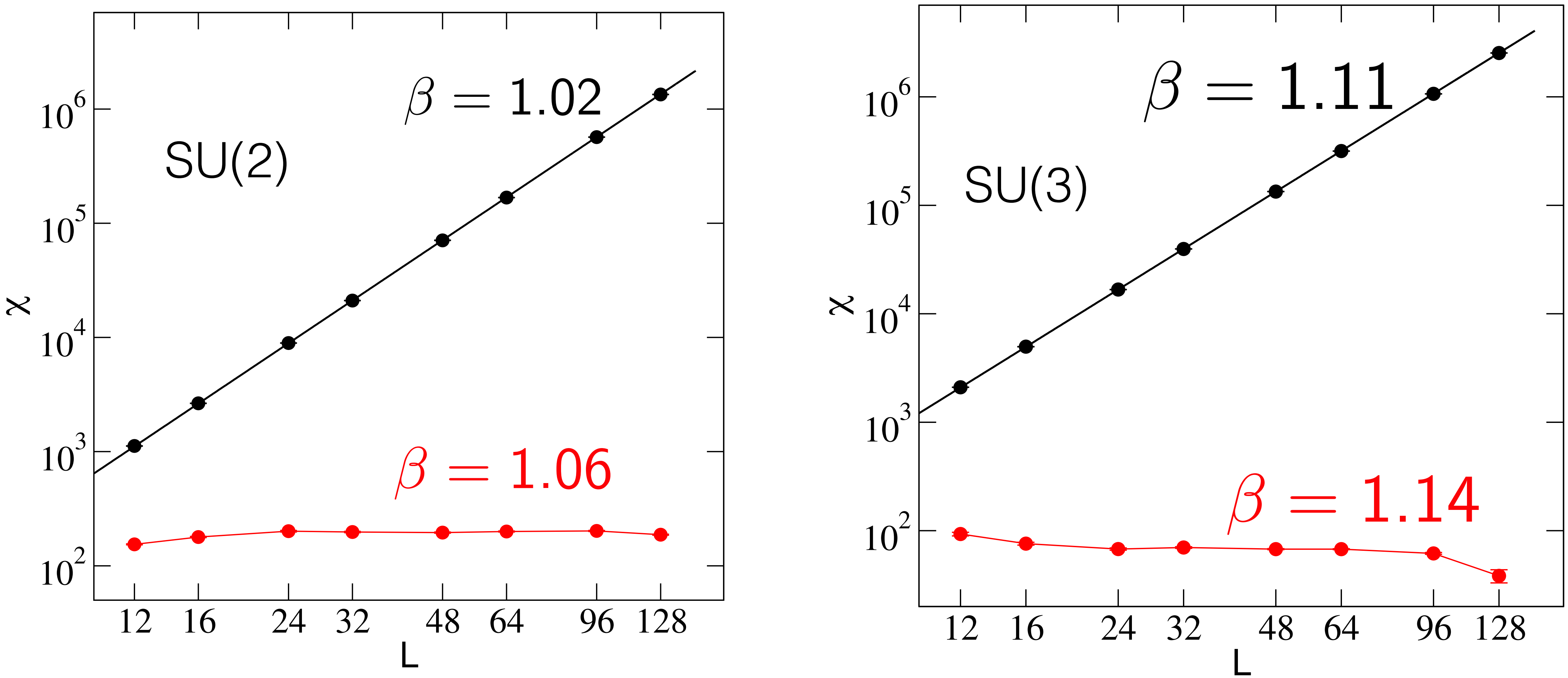
Preliminary Results



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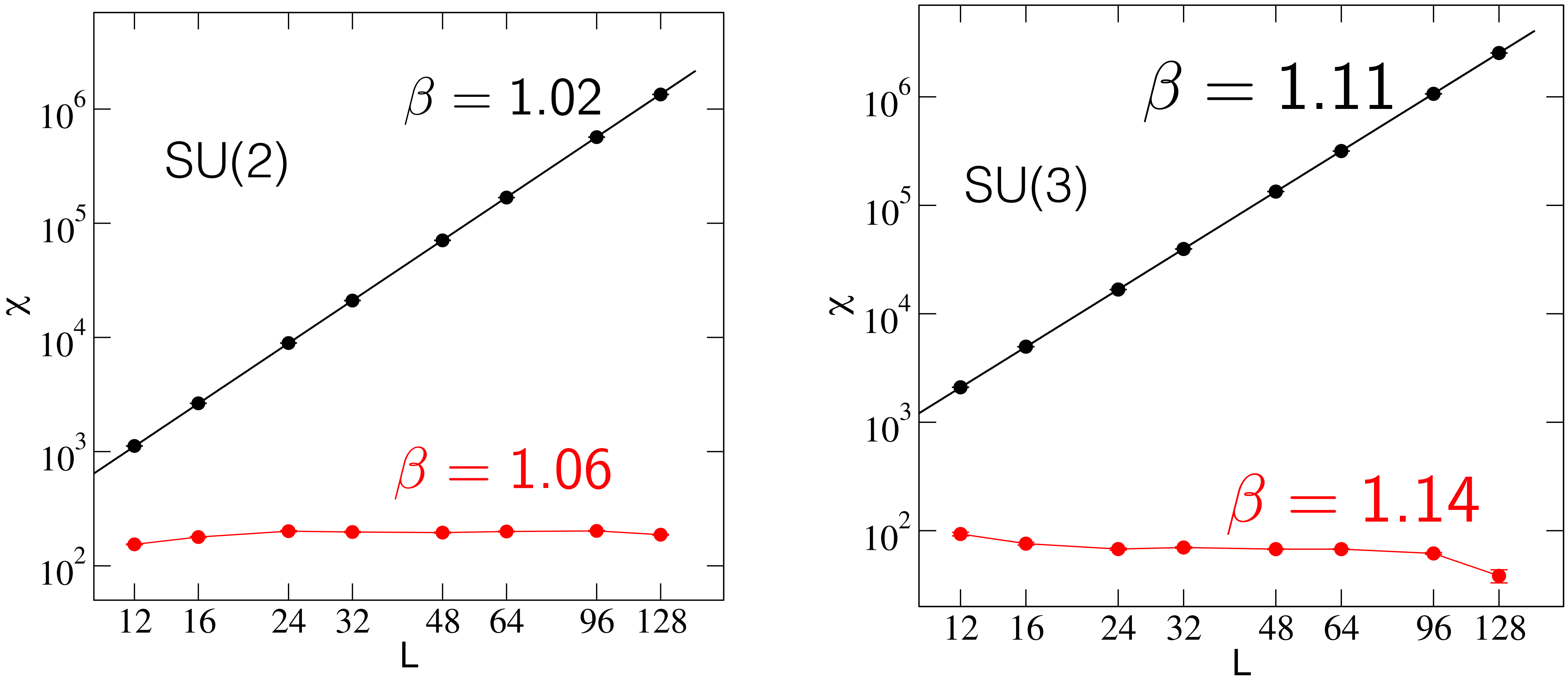

Both seem like first order transitions.

Easy to construct quantum dimer models
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Are there quantum critical points separating
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Easy to construct quantum dimer models
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Does asymptotic freedom of Yang-Mills theory
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SU(N) gauge theories with massless staggered matter in 1+1 dimensions is also very interesting.

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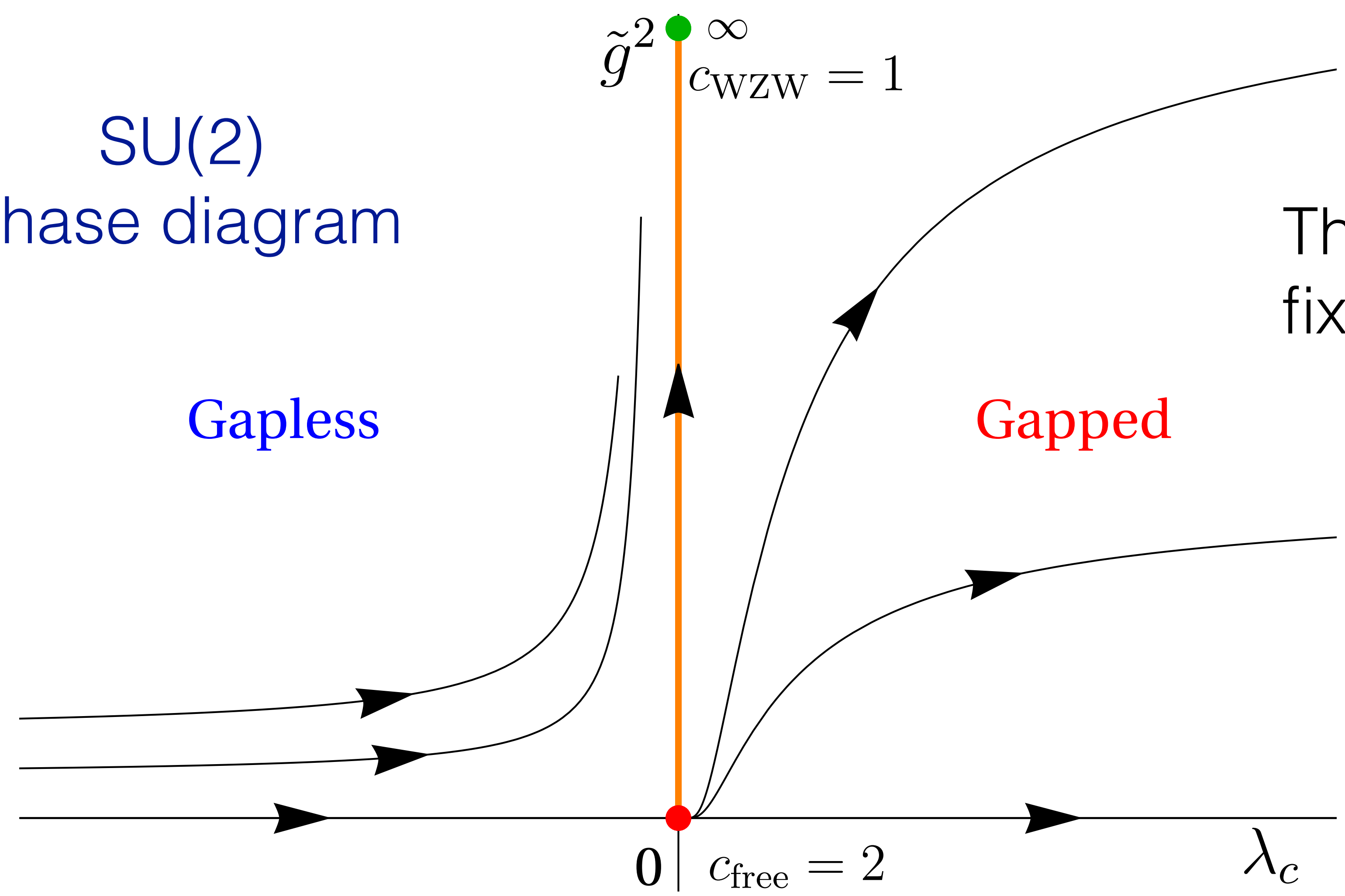
SU(2)
Phase diagram



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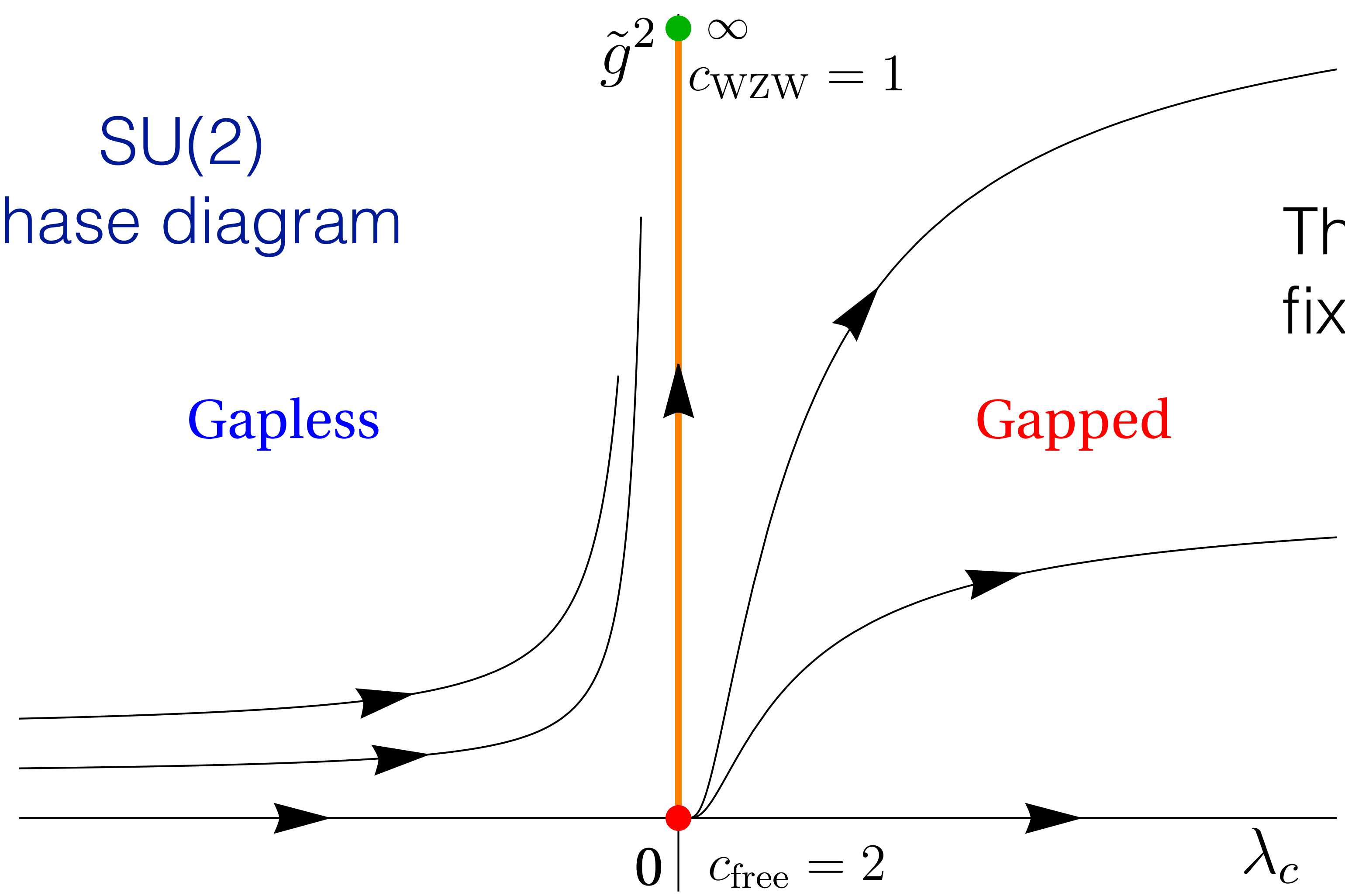

There are two fixed points

How can we recover this physics with qubit regularization?

Liu, Bhattacharya, SC, Gupta, arXiv:2312.17734

Conclusions

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Quantum Computation suggests new regularization schemes for QFTs with finite local Hilbert spaces.

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Can Yang-Mills theory arise at a quantum critical point of quantum dimer models?