Qubit Regularization: Asymptotic Freedom via New Renormalization Group Flows

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Phys.Rev.Lett. 126 (2021) 17, 172001
Lattice Gauge Theory Work in Progress

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Quantum Field Theory with Quantum Computers

Growing field of research within our community!

Plenary Talk 2018, Preskill

Two types of opportunities:

→ How to reformulate the QFT

→ How to use the quantum hardware

This talk is about showing you a glimpse of how quantum computing is already helping us learn

to formulate “old” problems with “new” variables and understand the field deeper.

Lagrangian → Hamiltonian

"Local" lattice Hilbert spaces play an important role

Traditional: $\mathcal{H}_{Trad}$ $\dim(\mathcal{H}_{Trad}) = \infty$

"Digital" quantum computer: $\mathcal{H}_{Q}$ $\dim(\mathcal{H}_{Q}) = \text{finite}$

qubit regularization

Hersh Singh, SC *Phys.Rev.D* 100 (2019) 5, 054505
Hanqing Liu, SC *Symmetry* 14 (2022) 2, 305

QFT is reformulated as a finite matrix model!

Continuum QFT emerges through the usual limiting process

continuum limit $a \to 0$

Thermodynamic limit $L \to \infty$

What about the traditional Hilbert space limit?

$$\mathcal{H}_{\mathrm{Q}} \longrightarrow \mathcal{H}_{\mathrm{Trad}}$$

Many papers seem to assume this is necessary!

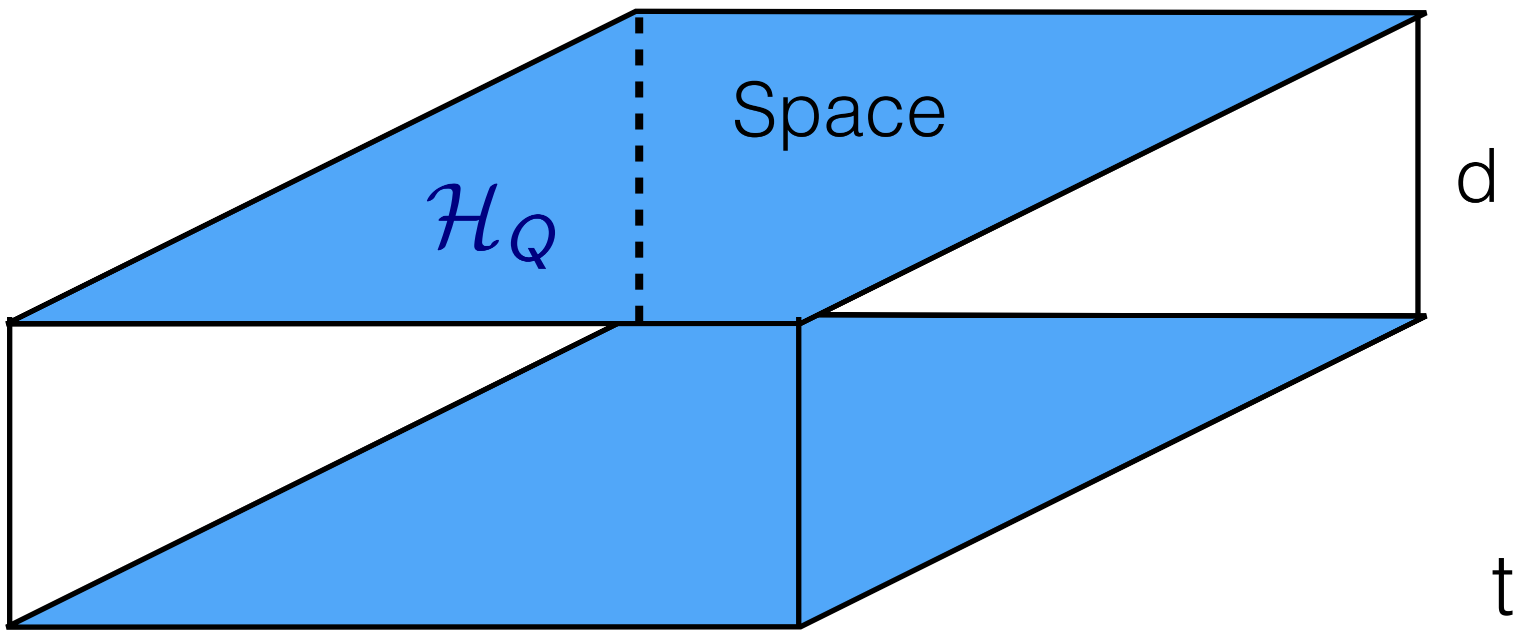
Hilbert space Truncation: $\mathcal{H}_{\mathrm{Trad}} \xrightarrow{\mathbb{P}_Q} \mathcal{H}_{\mathrm{Q}} \xrightarrow{\mathbb{P}_Q \to \mathbb{1}} \mathcal{H}_{\mathrm{Trad}}$

This is not required at least in some cases!

An interesting alternate approach is the D-theory

Lattice 1998, Plenary talk by Wiese.

Brower, SC, Riederer, Wiese, NPB 693 (2004),149



d is allowed to grow so the local Hilbert space can grow!

RG plays an important role!

Lot of work has been done over the past decade!

Talks in the conference

12 Parallel Talks on Monday

6 Parallel Talks on Tuesday

Various types of qubit regularizations.

How to prepare initial states.

How to evolve systems in real time on a real hardware!

Apologies: I do not plan on reviewing these results!

I want to focus on an interesting question!

Can we recover asymptotic freedom of traditional lattice models by reformulating it as lattice models with a finite dimensional “local” Hilbert space!

Yes!
(Two examples, Euclidean space)

We will discover a new a type of RG flow!

Collaborators

Siew

Maiti

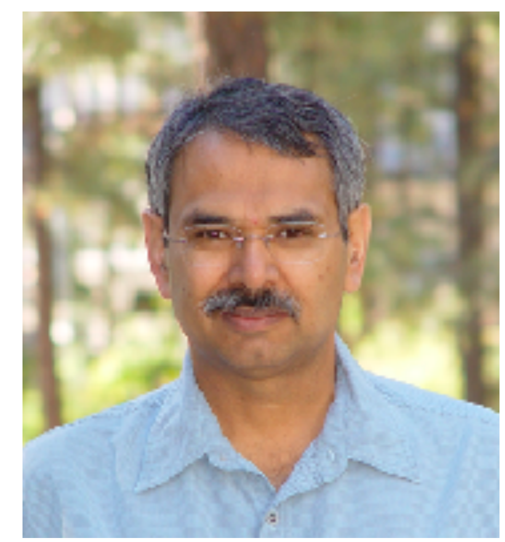
Liu

Singh

Marinkovic

Banerjee

Bhattacharya



Gupta

Outline

Asymptotic Freedom as an RG flow

Recovering Asymptotic Freedom with finite Hilbert space

- Example 1: O(3) Non-linear sigma model
- Example 2: BKT Transition

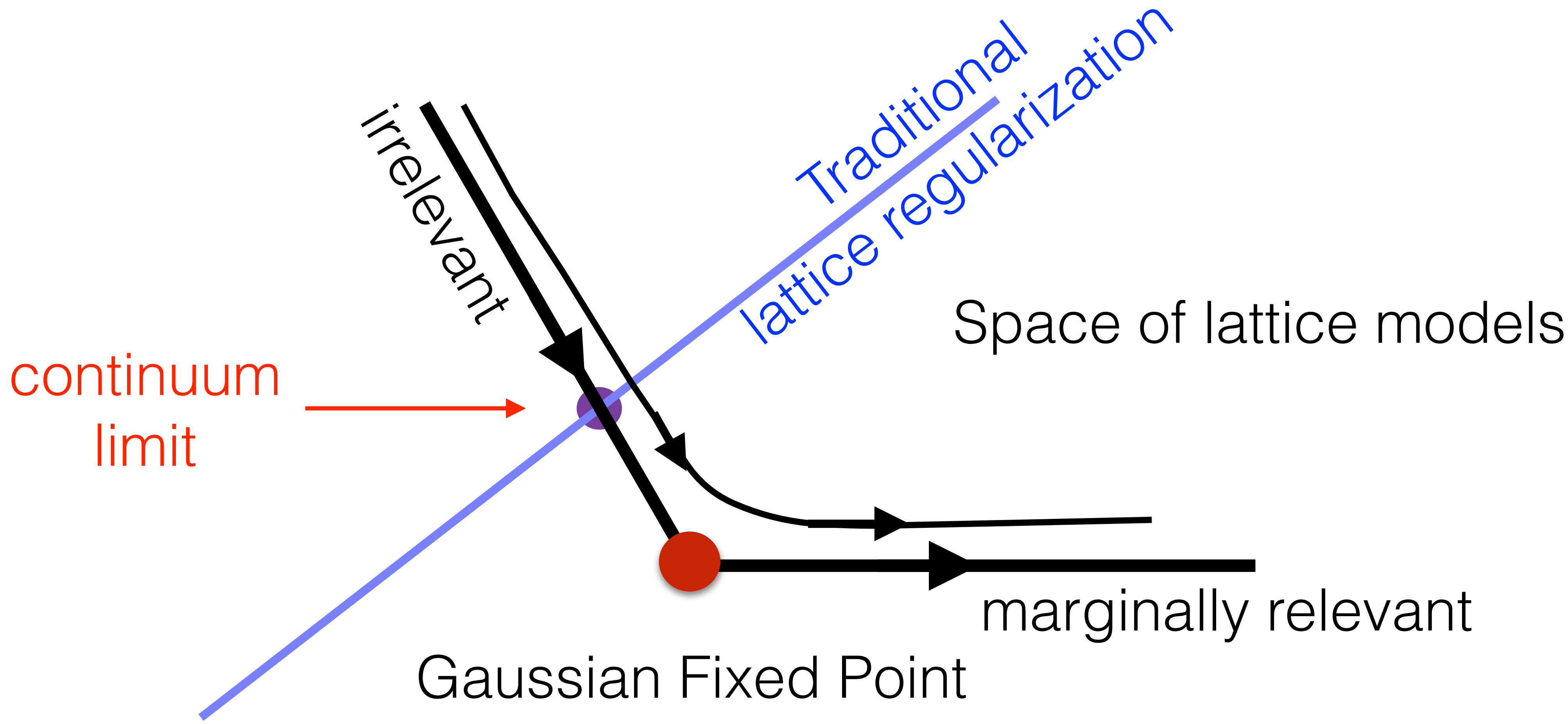
A Novel RG-Flow for asymptotic freedom

Qubit Regularization of Asymptotic Freedom in Gauge Theories

Conclusions

Asymptotic Freedom as an RG Flow

Traditional Lattice Regularization

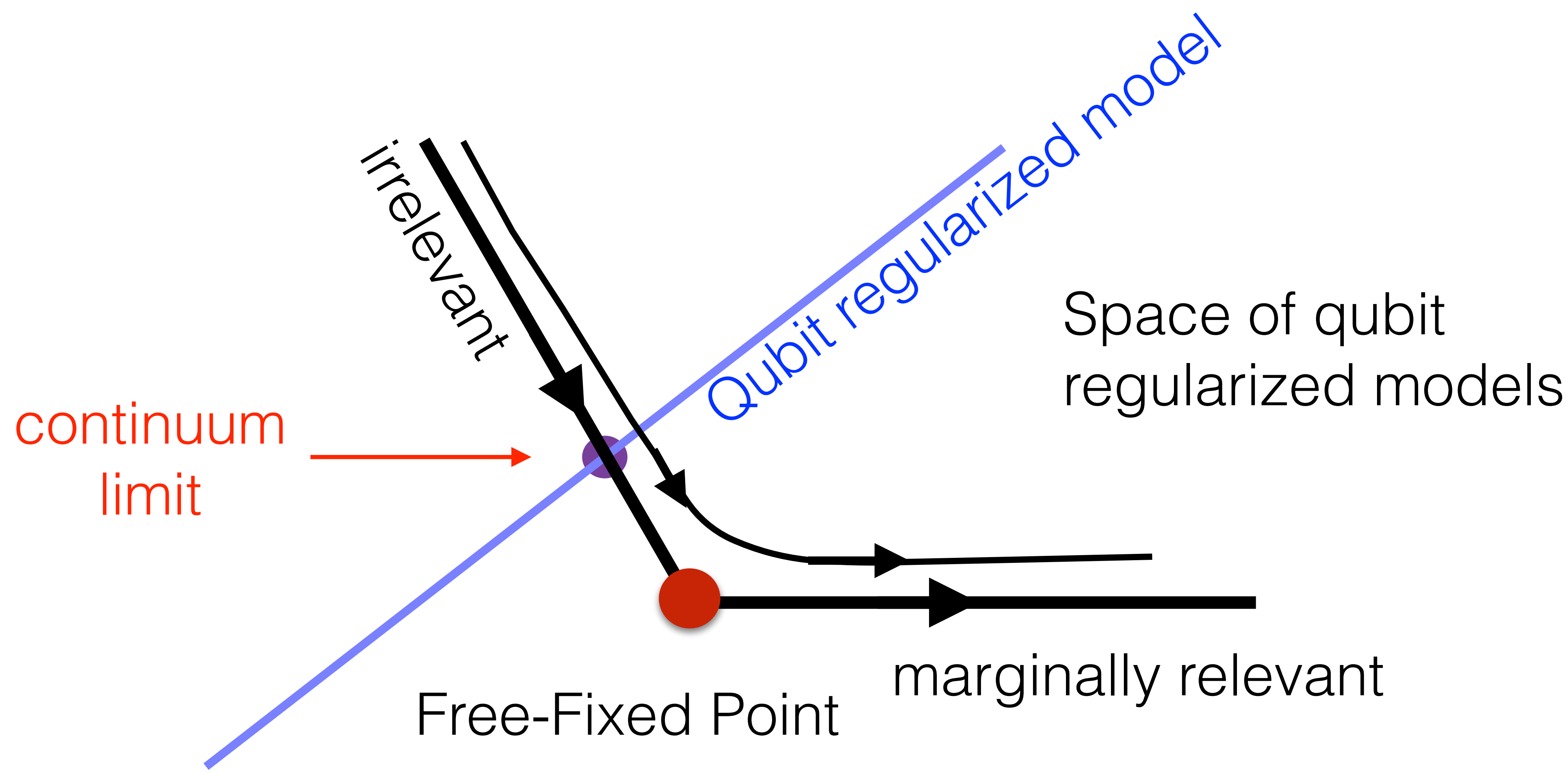


From an RG perspective, the limit

$$\mathcal{H}_{\mathsf{Q}} \longrightarrow \mathcal{H}_{\mathsf{Trad}}$$

may not be necessary!

We may be able to fine tune to the free critical surface within $\mathcal{H}_Q$



Lesson from RG:
Explore the space of qubit models,
don’t be stuck with the traditional Hamiltonian.

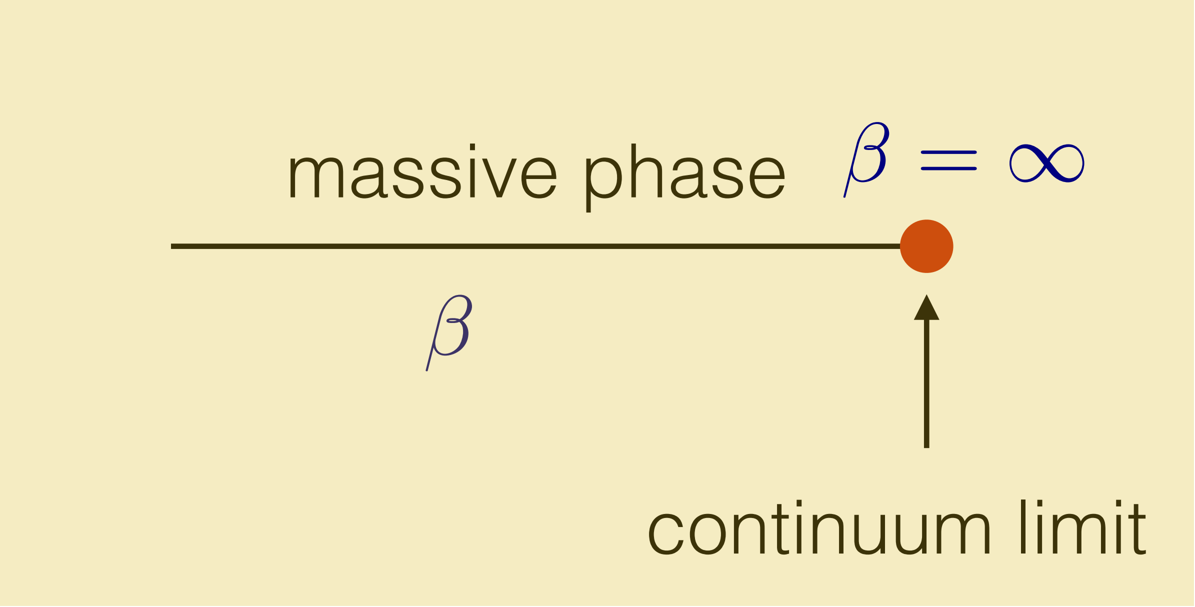
Recovering Asymptotic Freedom with a finite Hilbert space

Example: O(3) Non-linear sigma model

Traditional Lattice Action:

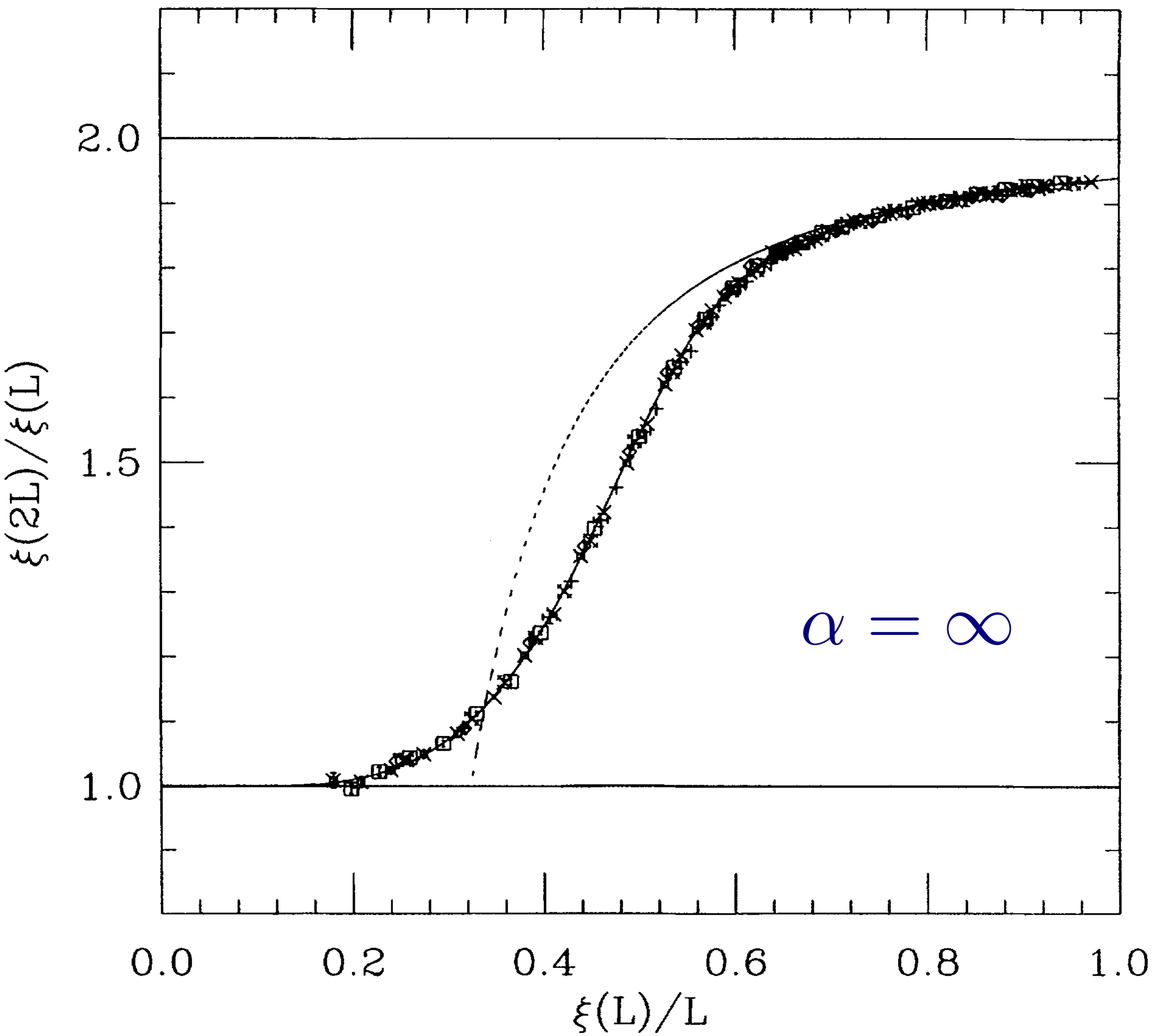
$$S_L = -\beta \sum_{\langle (x,\tau),(y,\tau')\rangle} \vec{\phi}_{x,\tau} \cdot \vec{\phi}_{y,\tau'}$$

$\vec{\phi}_{x,\tau}$ = three component unit vector $\qquad \vec{\phi}_{x,\tau} \cdot \vec{\phi}_{x,\tau} = 1$



At the critical point we get the desired free theory!

Continuum Physics: Universal step-scaling function

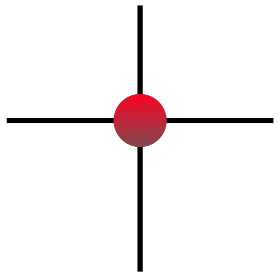


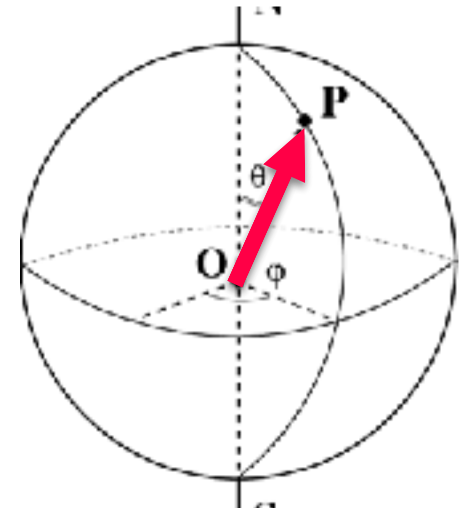
Can we reproduce this continuum physics in a lattice model with a finite Hilbert space?

Caracciolo, et.al., PRL 75, 1891 (1995)

Qubit Regularization of SO(3) fields

Local site Hilbert space describes a quantum particle on a surface of a unit sphere





$$\vec{\phi}_x = (\phi^1_x, \phi^2_x, \phi^3_x)$$

$$\vec{\phi}_x \cdot \vec{\phi}_x = 1$$

Basis of the traditional Hilbert space $\mathcal{H}_{\mathrm{Trad}}$:

$$\int d\Omega \; |\theta, \varphi\rangle\langle\theta, \varphi| \; = \; I$$

"position basis"

$$\sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \; | \; \ell, m \; \rangle\langle \; \ell, m \; | \; = \; I$$

"momentum basis"

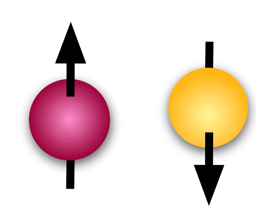
View the local Hilbert space is a direct sum of SO(3) representations:

Traditional Hilbert Space: $$\mathcal{H}_{\text{Trad}} = \bigoplus_{\ell=0,1,2,\ldots} \mathcal{H}_\ell$$

Qubit Regularized Hilbert Space $$\mathcal{H}_Q = \bigoplus_{\ell \in Q} \mathcal{H}_\ell$$

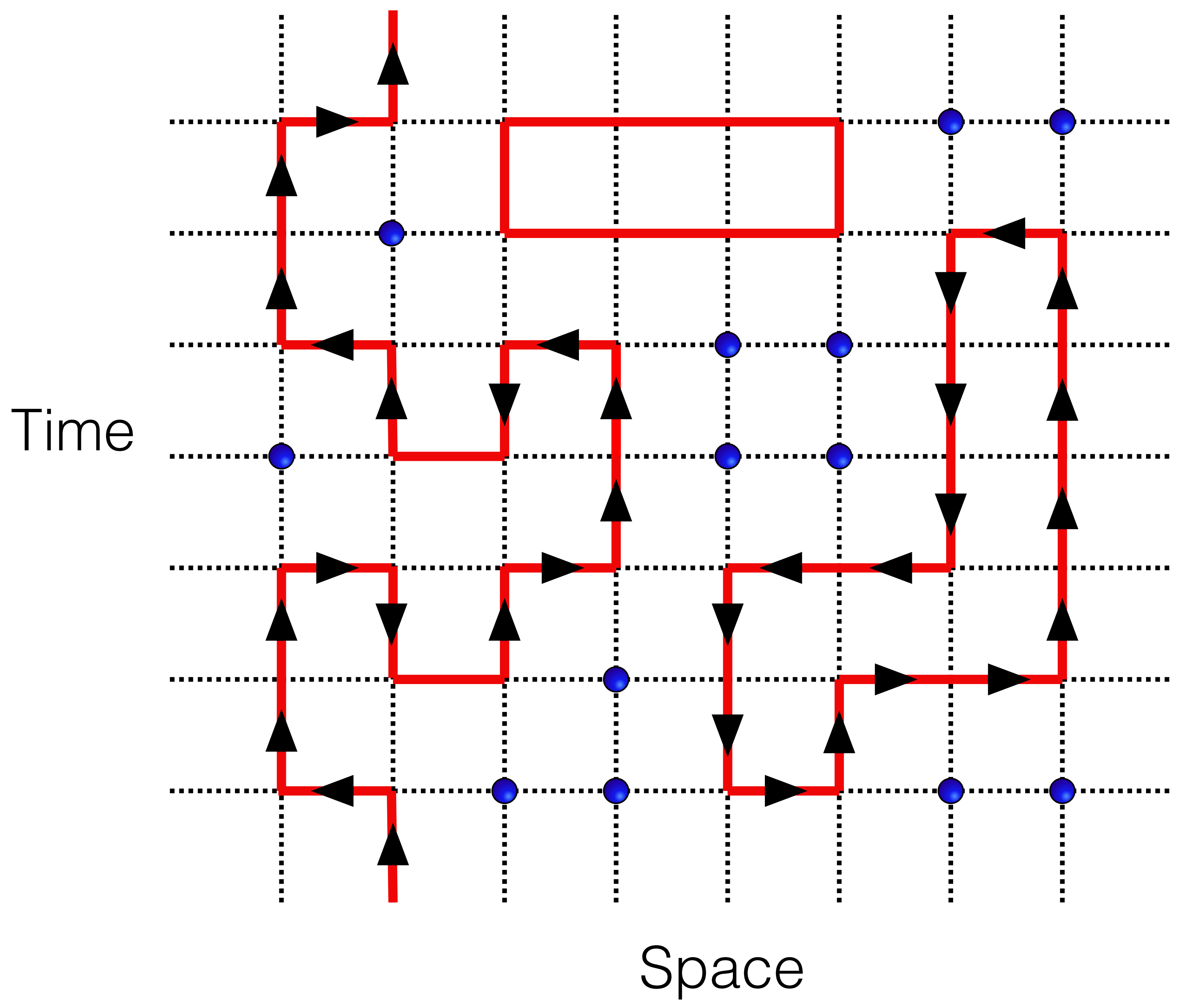
where $Q = \{\ell_1, \ell_2, \ell_3, \ldots\}$

A simple qubit regularization scheme is $Q = \{0, 1\}$

($dim(\mathcal{H}_Q) = 4$) Two qubits per lattice site 

Space-time Euclidean configurations in the qubit regularization scheme $Q = \{0, 1\}$

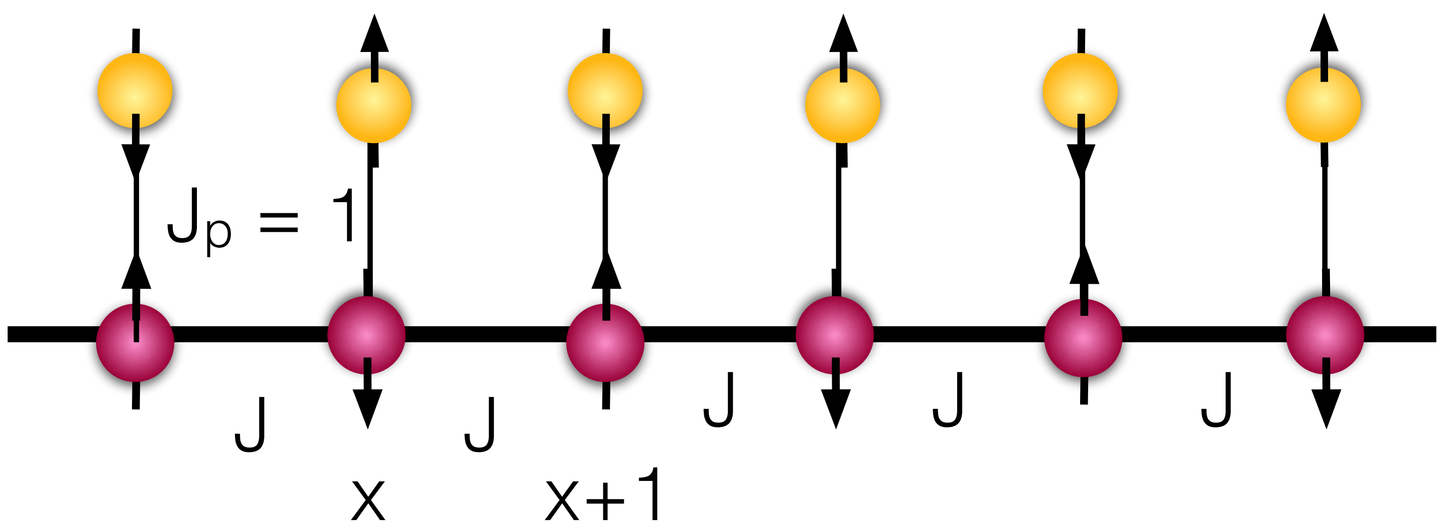
($dim(\mathcal{H}_Q) = 4$)



Qubit regularized model

Bhattacharya, Buser, SC, Gupta, Singh PRL 126 (2021), 172001

Heisenberg-Comb

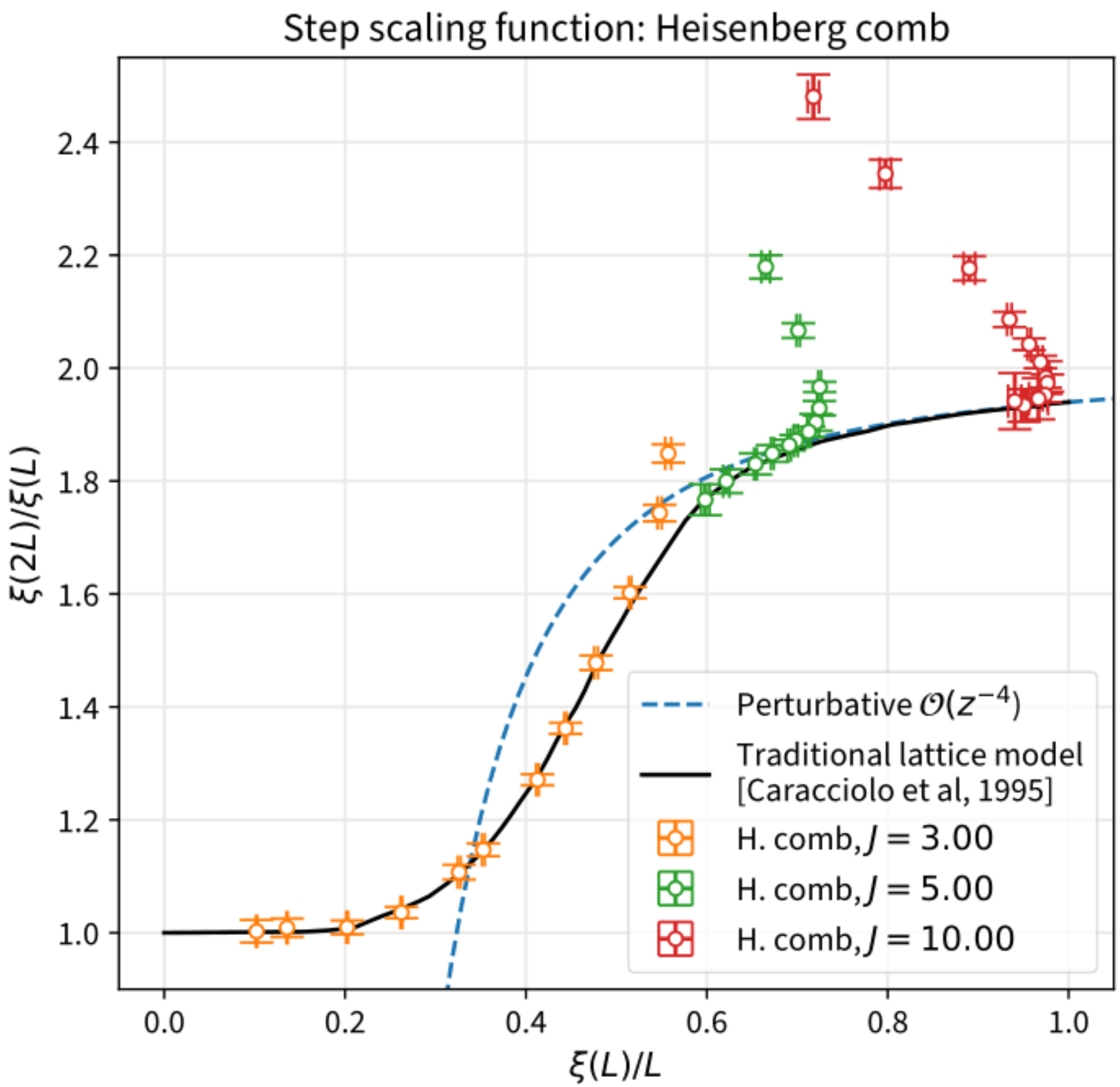


$$H = \sum_x J\, \mathbf{S}_{x,1} \cdot \mathbf{S}_{x+1,1} + \mathbf{S}_{x,1} \cdot \mathbf{S}_{x,2}$$

Quantum Critical Point: $J \to \infty$ (Spin-1/2 Chain)

At the critical point we get a decoupled critical theory that is not the desired free theory!

Universal Step Scaling Function



Traditional Model at

$\beta \to \infty$

$\mathcal{H}_{\mathsf{Trad}} = infty$

$\equiv$

Heisenberg Comb at

$J \to \infty$

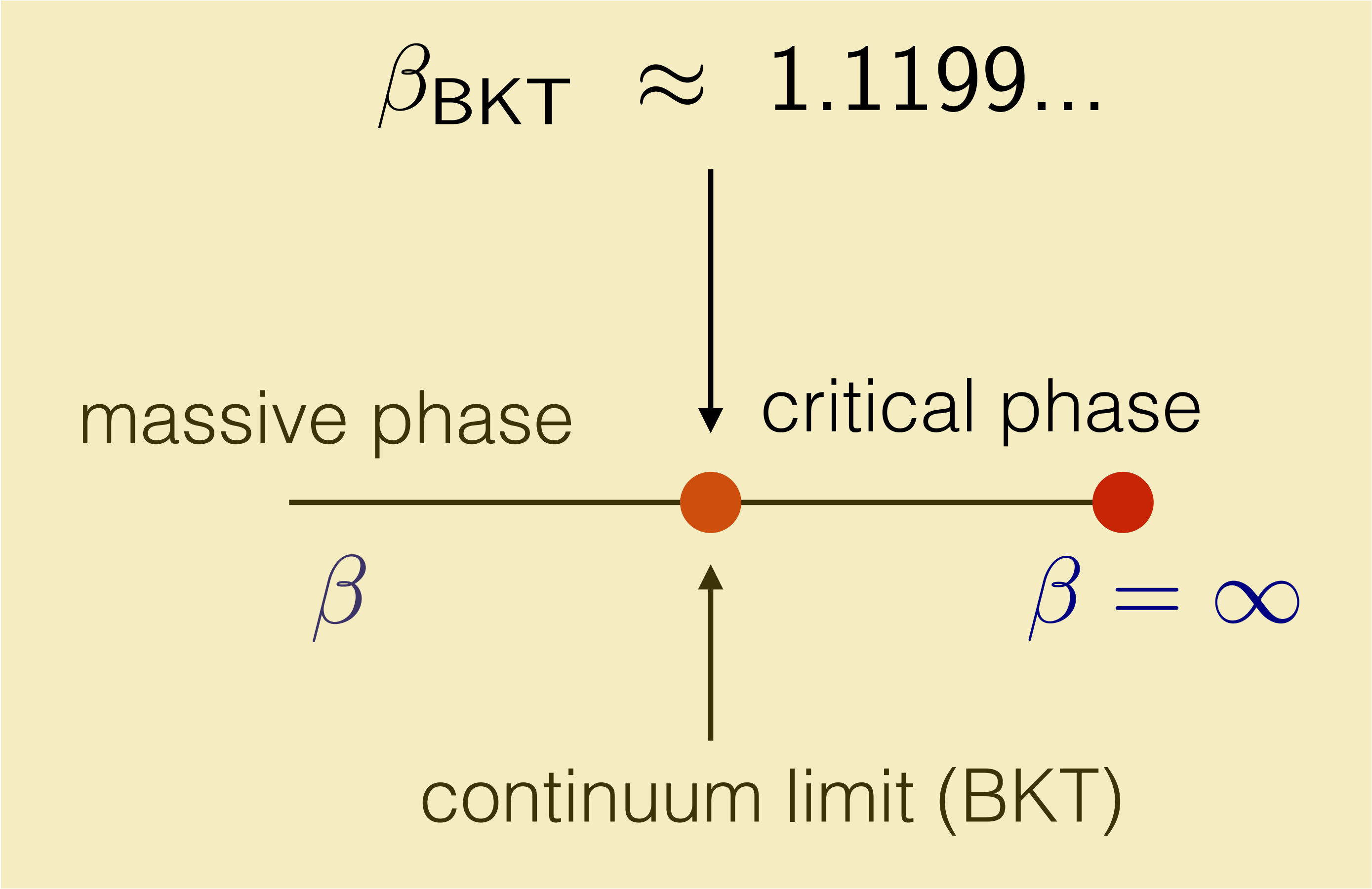
$\mathcal{H}_{\mathsf{Q}} = 4$

Example: BKT Transition

Lattice Action:

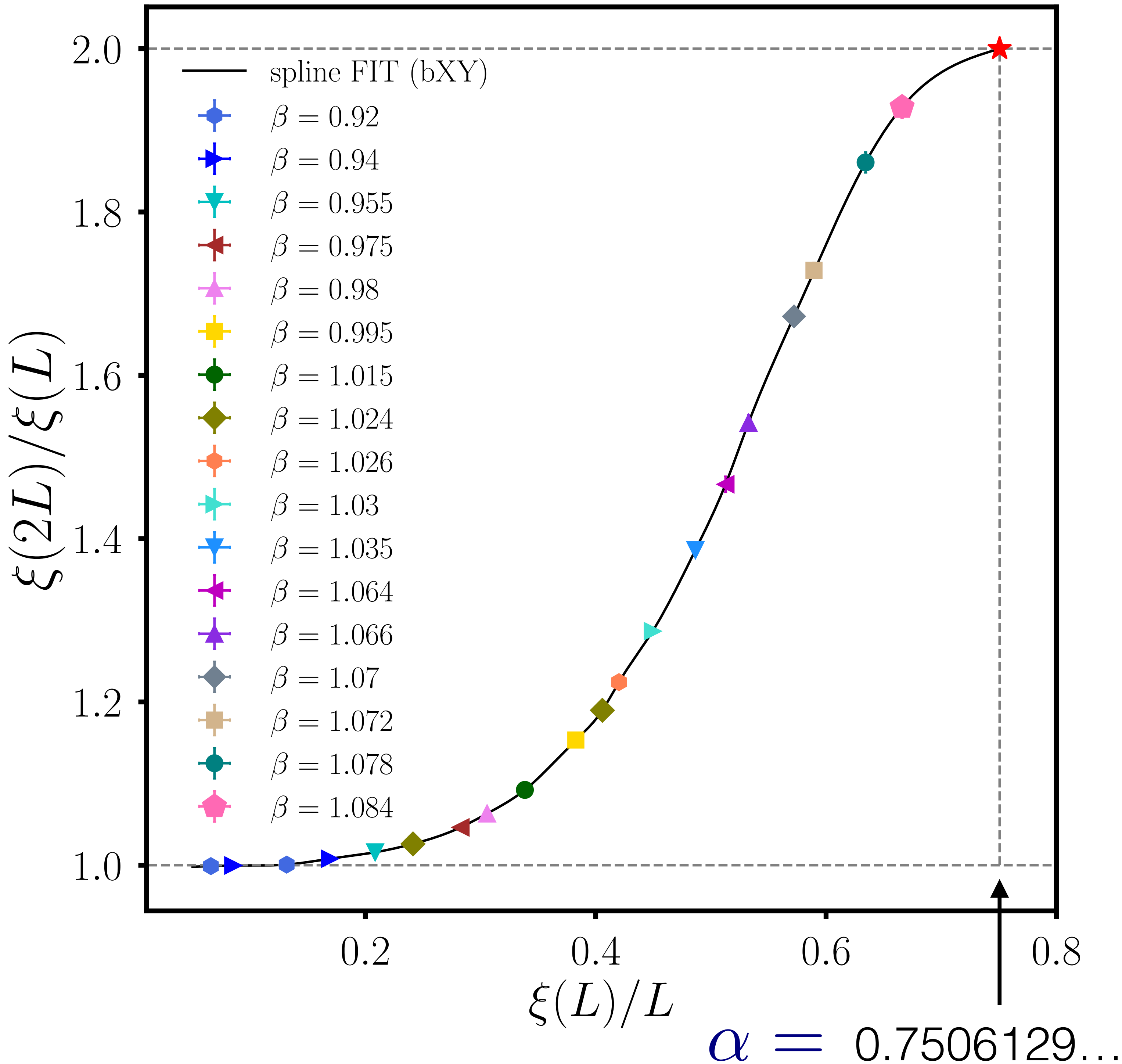
$$S_L = -\beta \sum_{\langle (x,\tau),(y,\tau') \rangle} \vec{\phi}_{x,\tau} \cdot \vec{\phi}_{y,\tau'}$$

$\vec{\phi}_{x,\tau}$ = two component unit vector $\vec{\phi}_{x,\tau} \cdot \vec{\phi}_{x,\tau} = 1$



At the critical point we get a Gaussian theory!

The Step-Scaling function at the BKT transition using the traditional lattice model

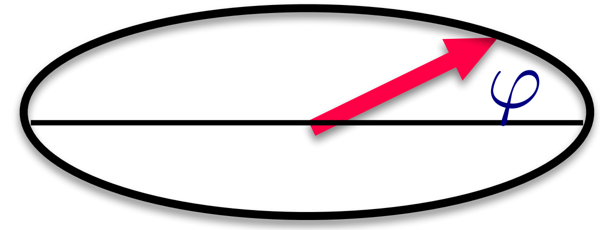


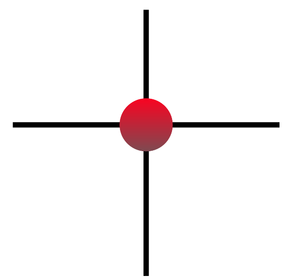
Can we reproduce this continuum physics in a lattice model with a finite Hilbert space?

Hasenbusch, cond-mat/0506552v2 (2008)

Qubit Regularization of SO(2) fields

Local site Hilbert space describes a quantum particle on a circle





$$\vec{\phi}_x = (\cos(\varphi_x), \sin(\varphi_x))$$

Basis of the traditional Hilbert space $\mathcal{H}_{\mathrm{Trad}}$:

$$\int d\varphi \; |\varphi\rangle\langle\varphi| = I$$

"position basis"

$$\sum_{m=-\infty}^{\infty} |m\rangle\langle m| = I$$

"momentum basis"

View the local Hilbert space is a direct sum of SO(2) representations:

Traditional Hilbert Space: $\mathcal{H}_{\text{Trad}} = \bigoplus_{m=0,\pm1,\pm2,\ldots} \mathcal{H}_m$

Qubit Regularized Hilbert Space $\mathcal{H}_Q = \bigoplus_{m\in Q} \mathcal{H}_m$

where $Q = \{m_1, m_2, m_3, \ldots\}$

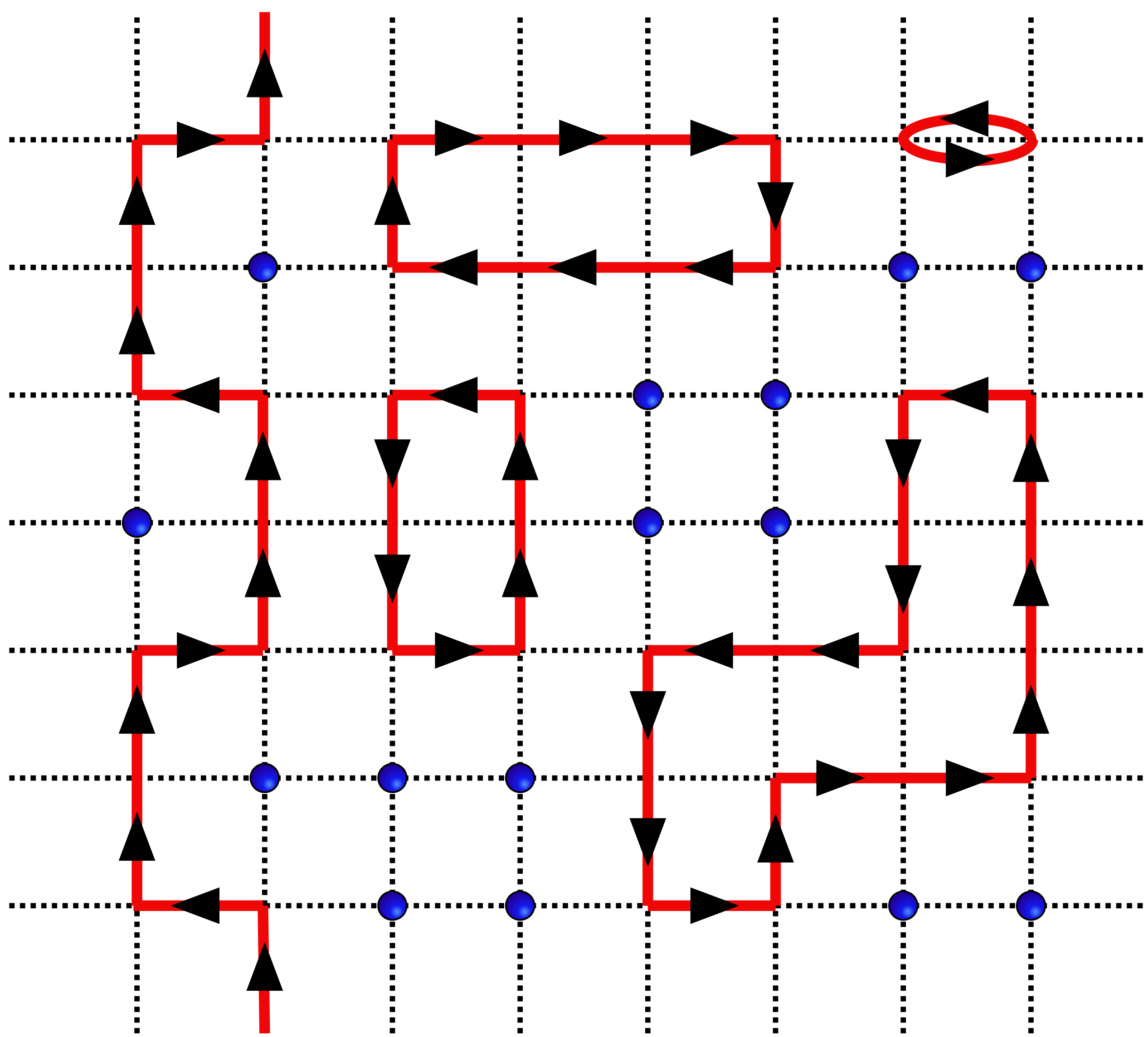
A simple qubit regularization scheme is $Q = \{0, 0, 1, -1\}$

($dim(\mathcal{H}_Q) = 4$)

Space-time Euclidean configurations in the qubit regularization scheme $Q = \{0, 0, 1, 1\}$

($dim(\mathcal{H}_Q) = 4$)

Time

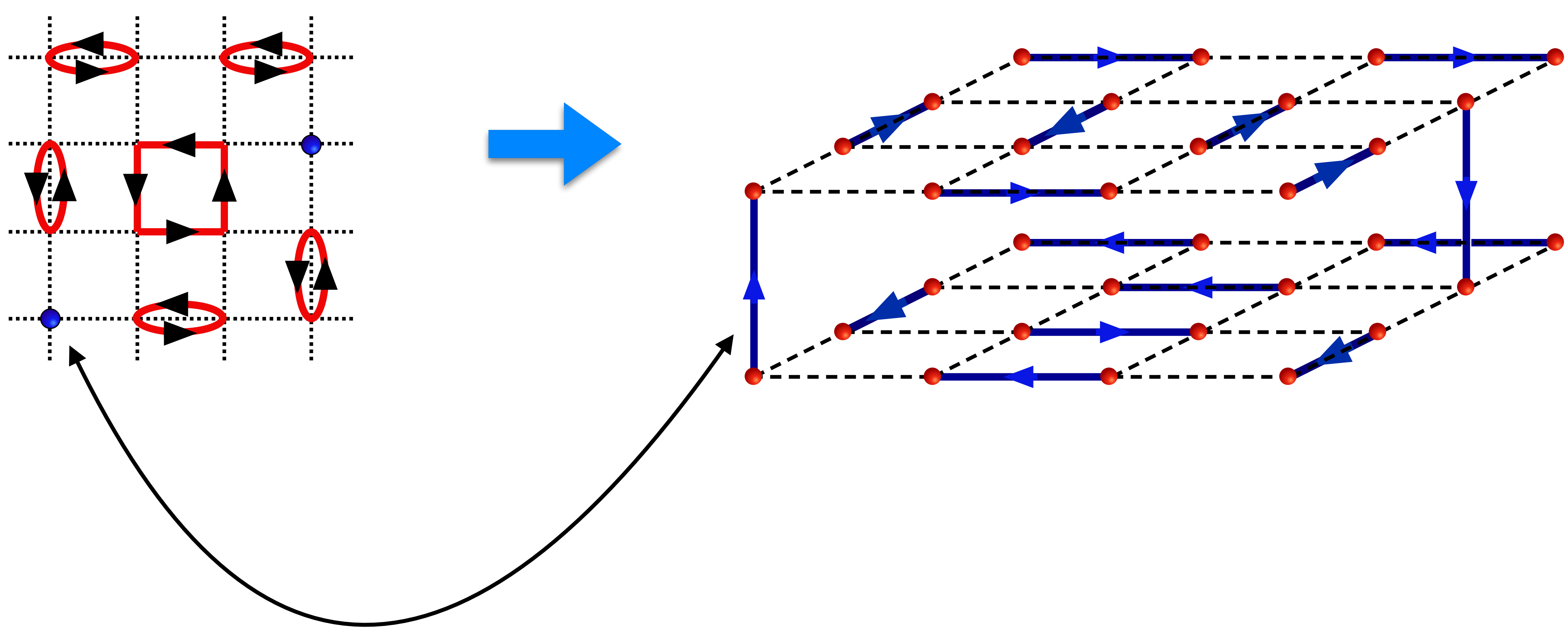


Space

Mapping to two layers of closed pack dimer model

worldline configuration

closed packed dimer configurations

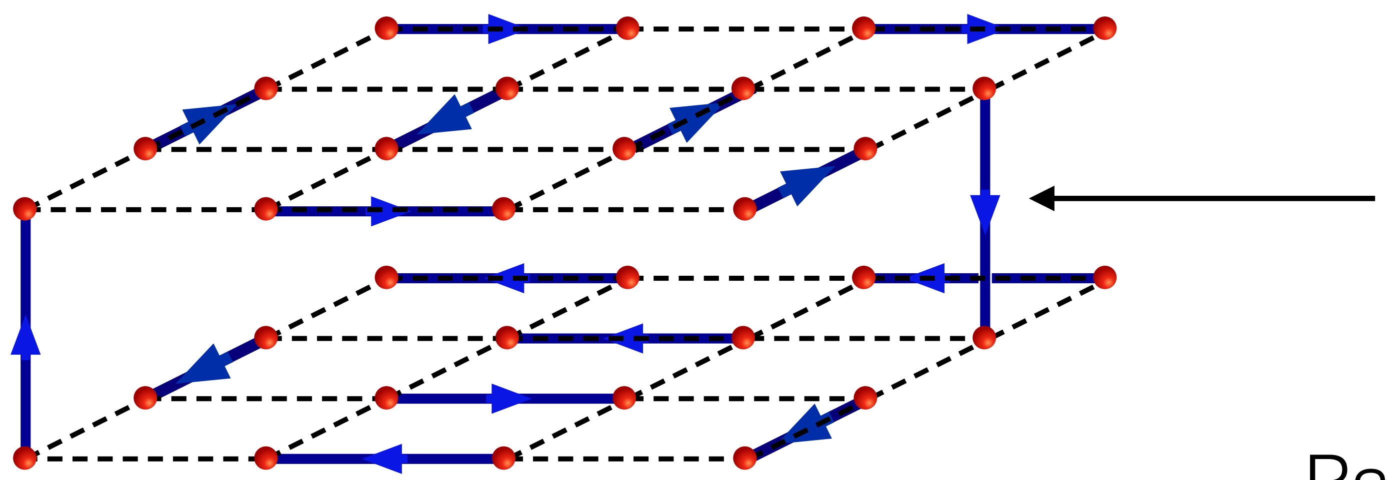


Singlets are mapped to inter-layer dimers

Qubit regularized model

Maiti, Banerjee, SC, Marinkovic PRL 132 (2024), 041601

closed packed dimer configurations



λ

Fugacity of interlayer dimers

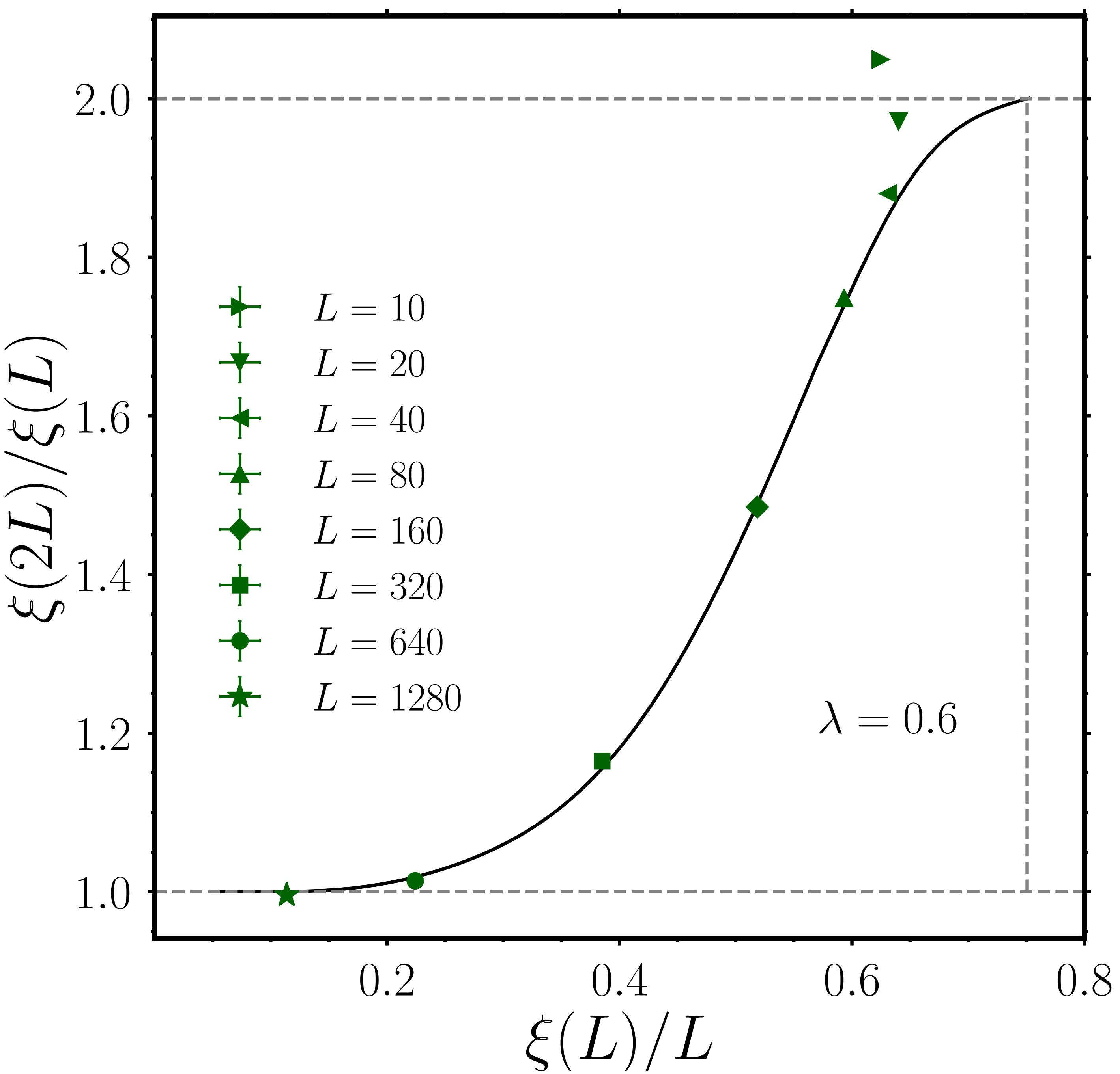
Partition function

$$Z = \sum_{C} \lambda^{N_I}$$

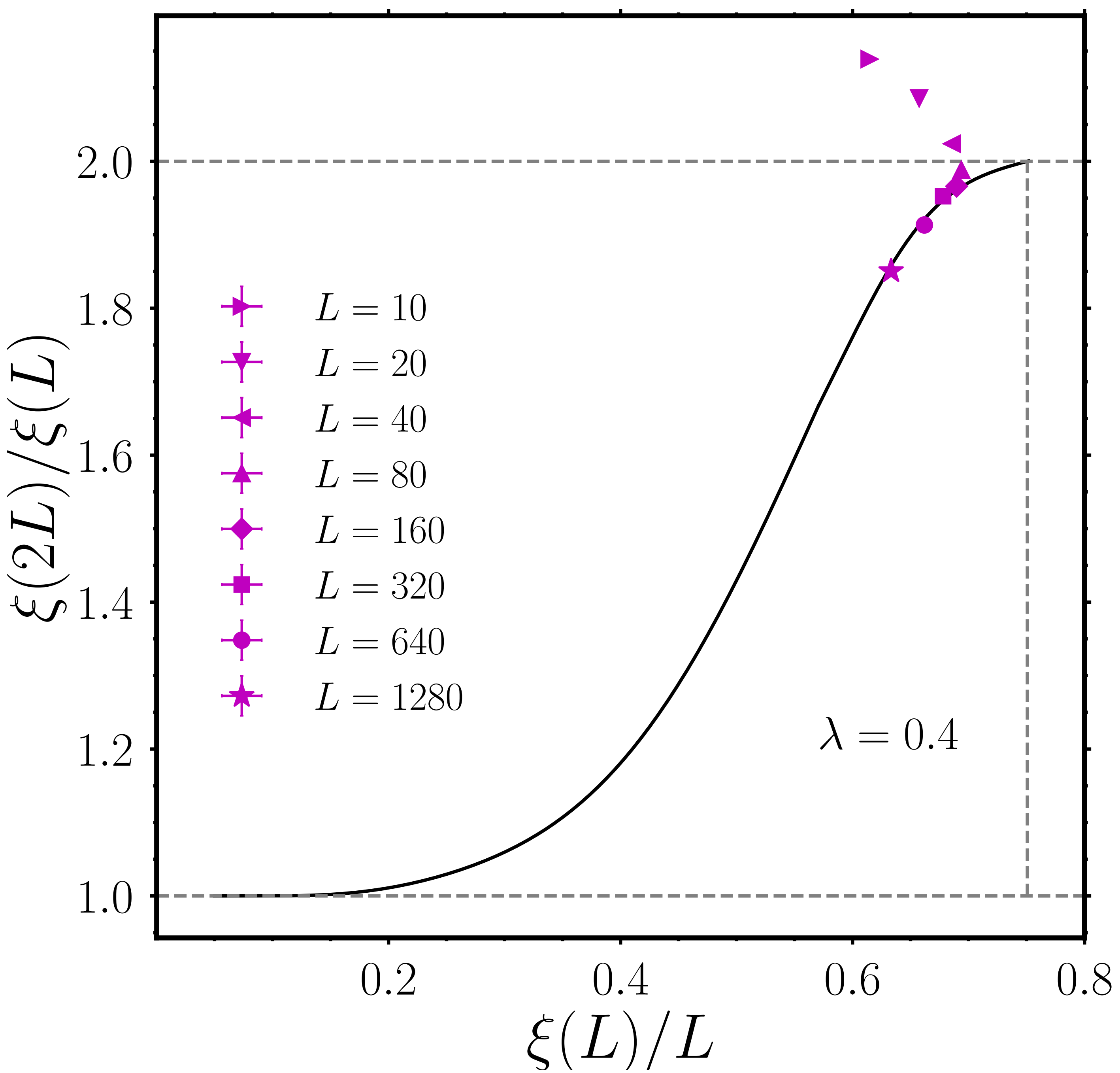
Continuum Limit: $\lambda \to 0$

At the critical point we get a decoupled critical theory that is again not the desired free theory!

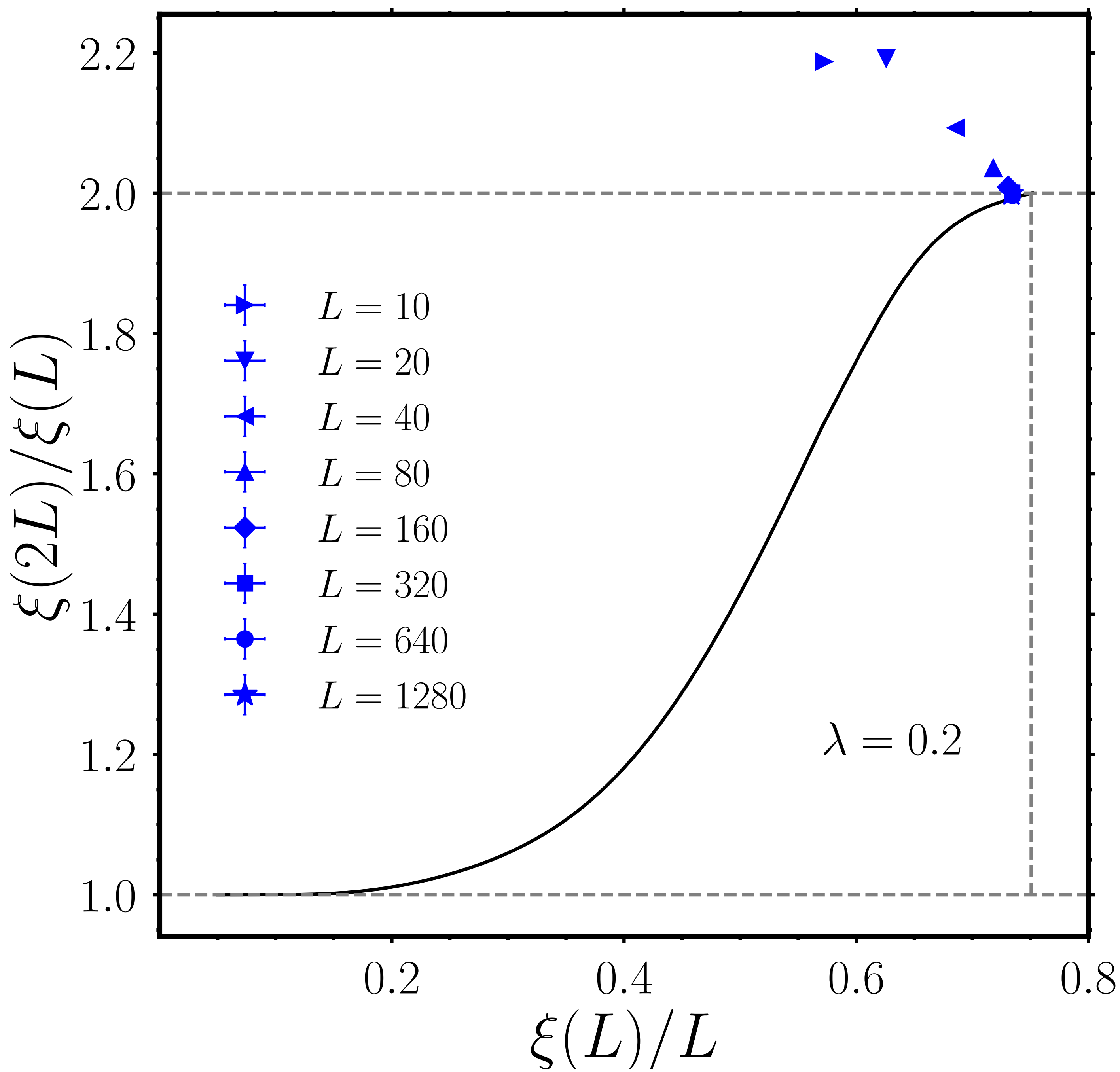
Step-scaling function of the qubit model



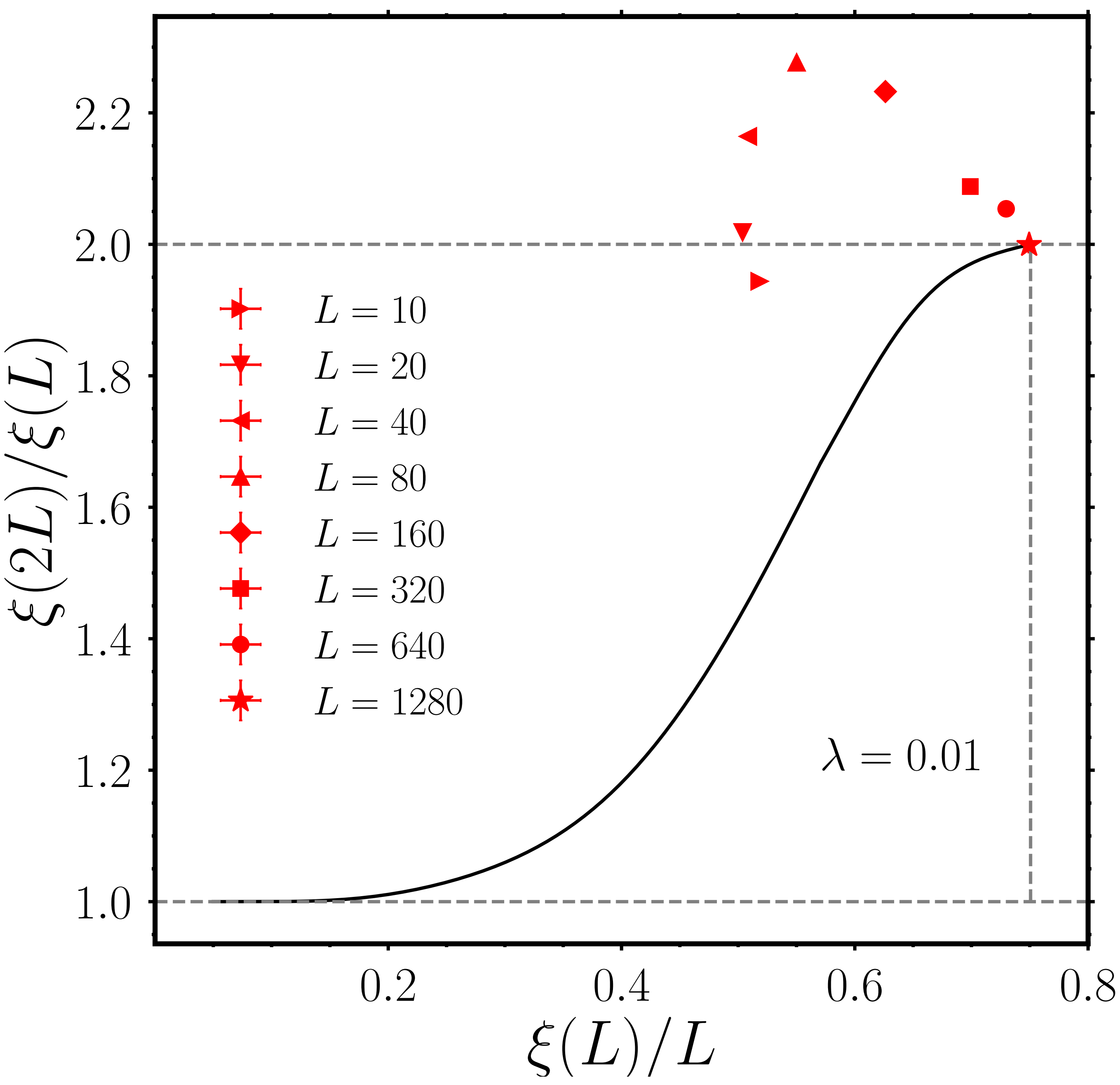
Step-scaling function of the qubit model



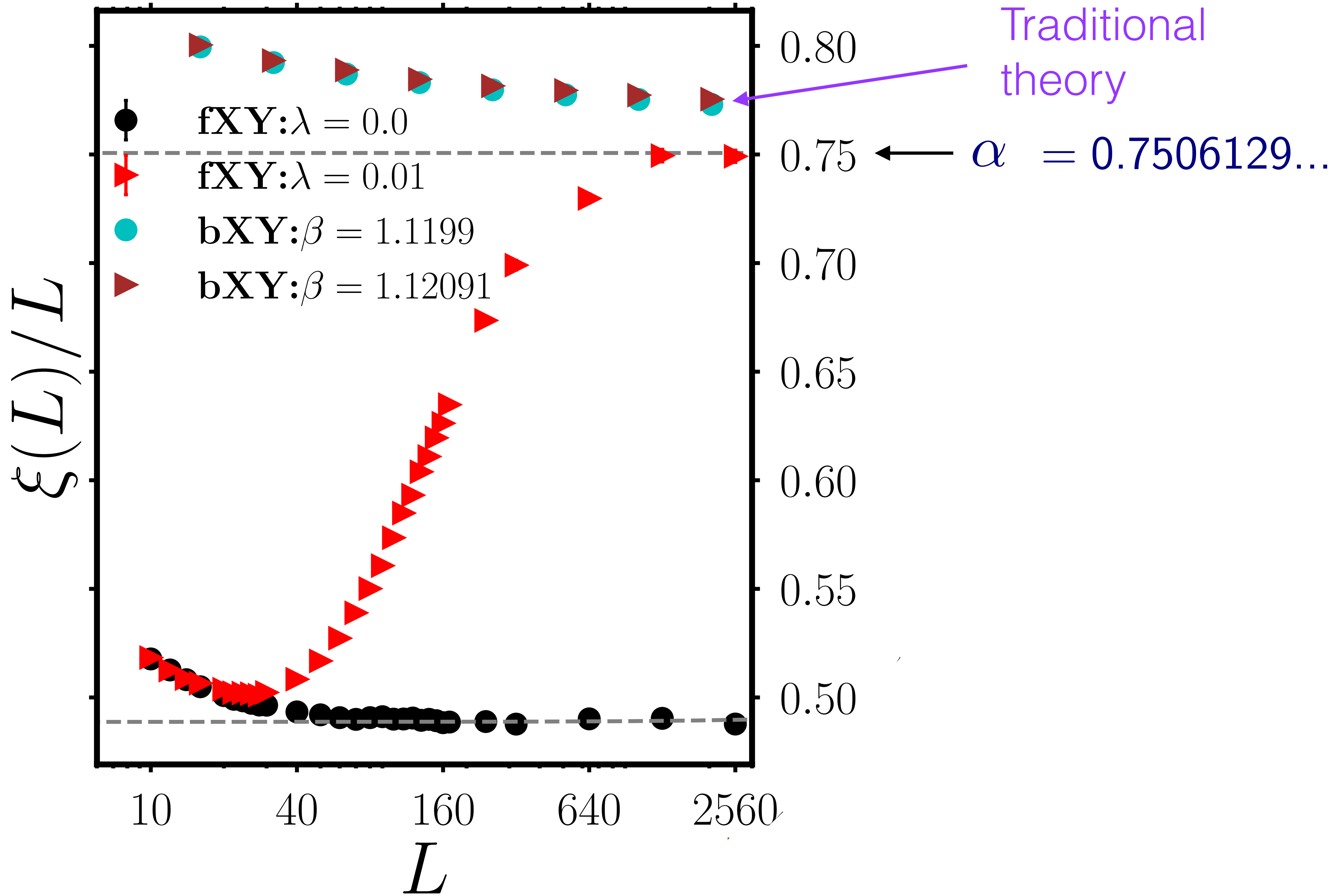
Step-scaling function of the qubit model



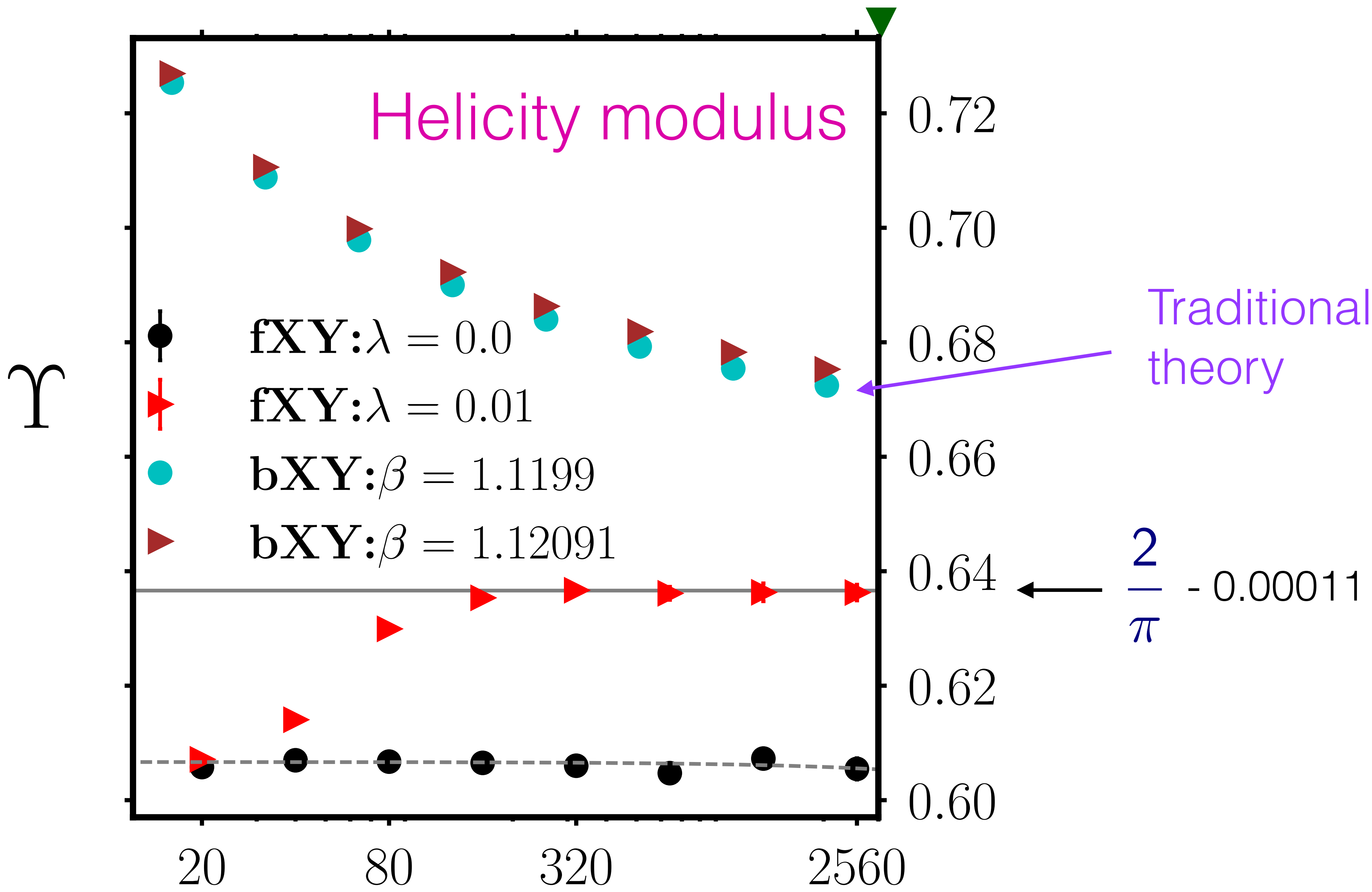
Step-scaling function of the qubit model



Decoupled vs. Coupled Theory



Decoupled vs. Coupled Theory



Massive Phase of the XY model
at the BKT transition

$$\mathcal{H}_{\mathsf{Trad}} = \mathsf{\textit{infty}}$$

$$\equiv$$

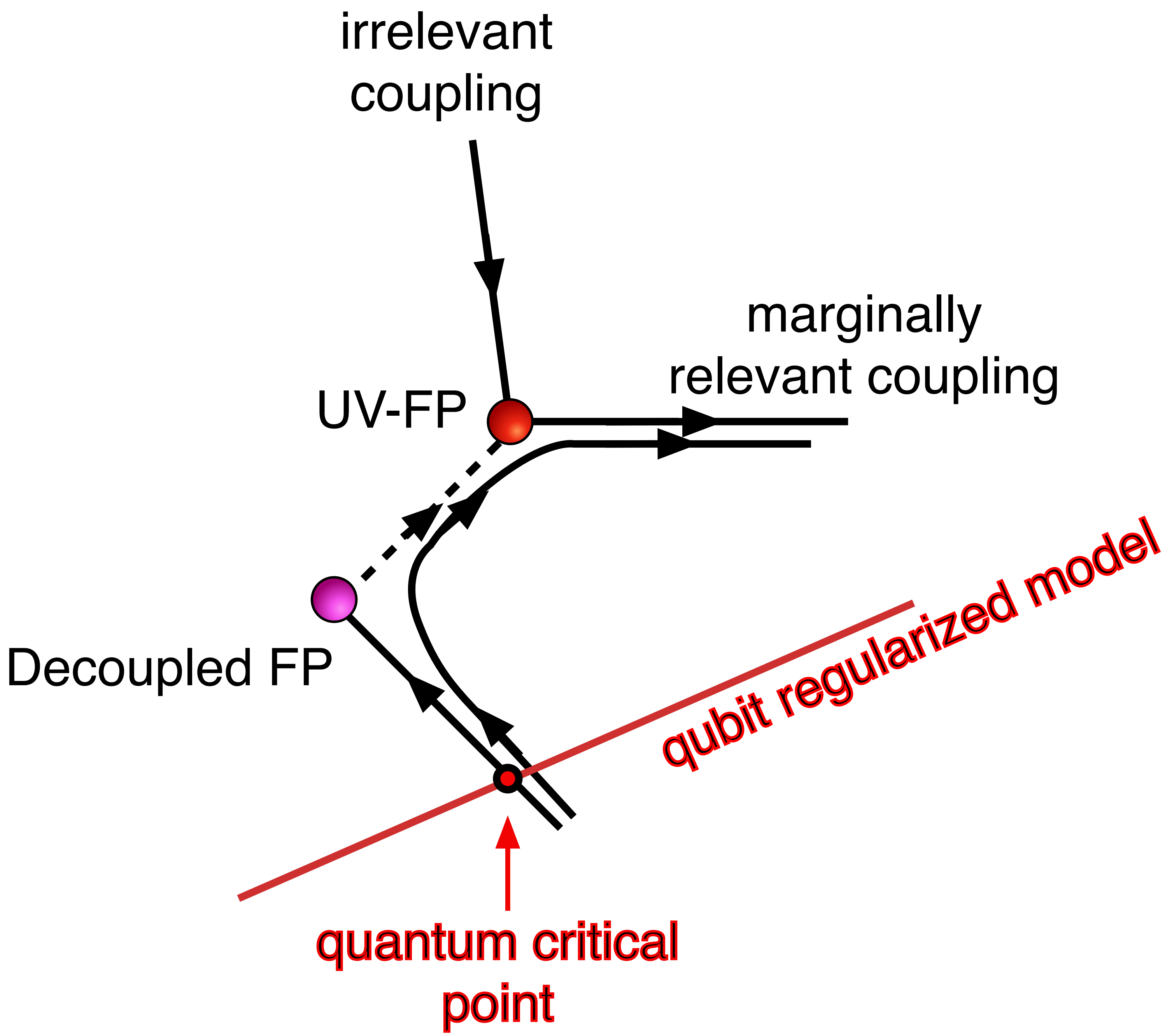
Two layers of coupled
closed pack dimers

$$\lambda \to 0$$

$$\mathcal{H}_{\mathsf{Q}} = 4$$

A Novel-RG flow for Asymptotic Freedom

In both these examples, asymptotic freedom is recovered via new type of RG flow



Qubit Regularization of Asymptotic Freedom in Gauge Theories

Asymptotic Freedom of Yang Mills theory

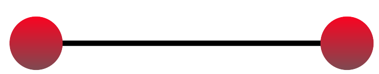
Deconfined phase at high temperatures

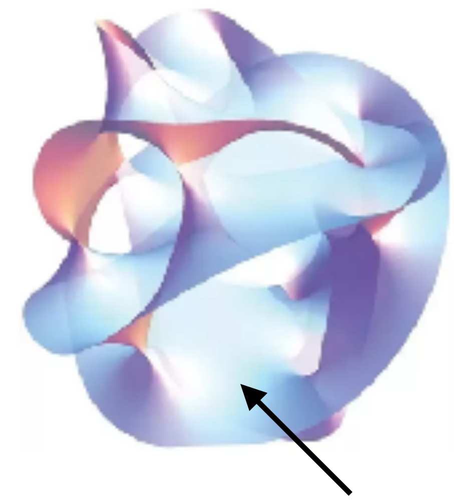
Confined massive phase at zero temperatures

Classical qubit models must already show this!

Qubit Regularization of SU(N) gauge fields

Local link Hilbert space describes a quantum particle on the surface of the SU(N) group manifold





$g \in SU(N)$

Basis of the full Hilbert space $\mathcal{H}_{\text{Trad}}$:

$$\int [dg]\ |g\rangle\langle g| = I$$

"position basis"

$$\sum_{\lambda} \sum_{i,j} |D^{\lambda}_{ij}\rangle\langle D^{\lambda}_{ij}| = \mathbb{I}$$

"momentum basis"

λ labels distinct irreps of SU(N)

This means the traditional link Hilbert space is given by

$$\mathcal{H}_{\text{Trad}} = \bigoplus_{\lambda} \mathcal{H}_{\lambda} \quad \longleftarrow \text{Peter-Weyl Theorem}$$

where $\mathcal{H}_{\lambda} = V_{\lambda} \otimes V_{\lambda}^{*}$ Is spanned by $\{|D_{ij}^{\lambda}\rangle\}, i, j = 1, 2, ..., d_{\lambda}$

Qubit Regularized Hilbert Space $\mathcal{H}_{Q} = \bigoplus_{\lambda \in Q} V_{\lambda} \otimes V_{\lambda}^{*}$

$$dim(\mathcal{H}_{Q}) = \sum_{\lambda \in Q} (d_{\lambda})^{2}$$

A simple qubit regularization involves $Q = \{1, \underbrace{\square, \begin{smallmatrix}\square\\ \square\end{smallmatrix}, \ldots}\}$

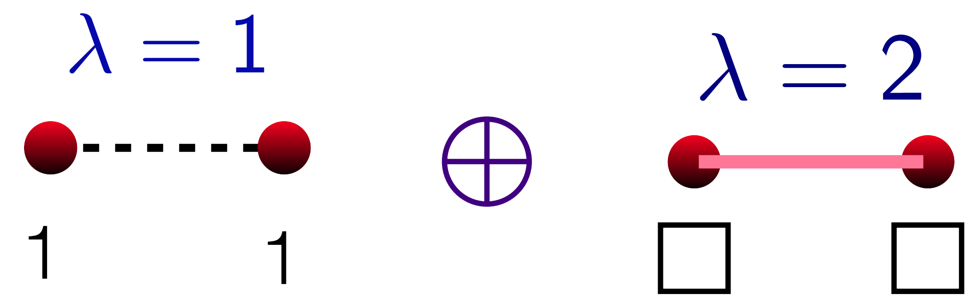
Hanqing Liu, SC Symmetry 14 (2022) 2 305,

↑ All anti-symmetric irreps

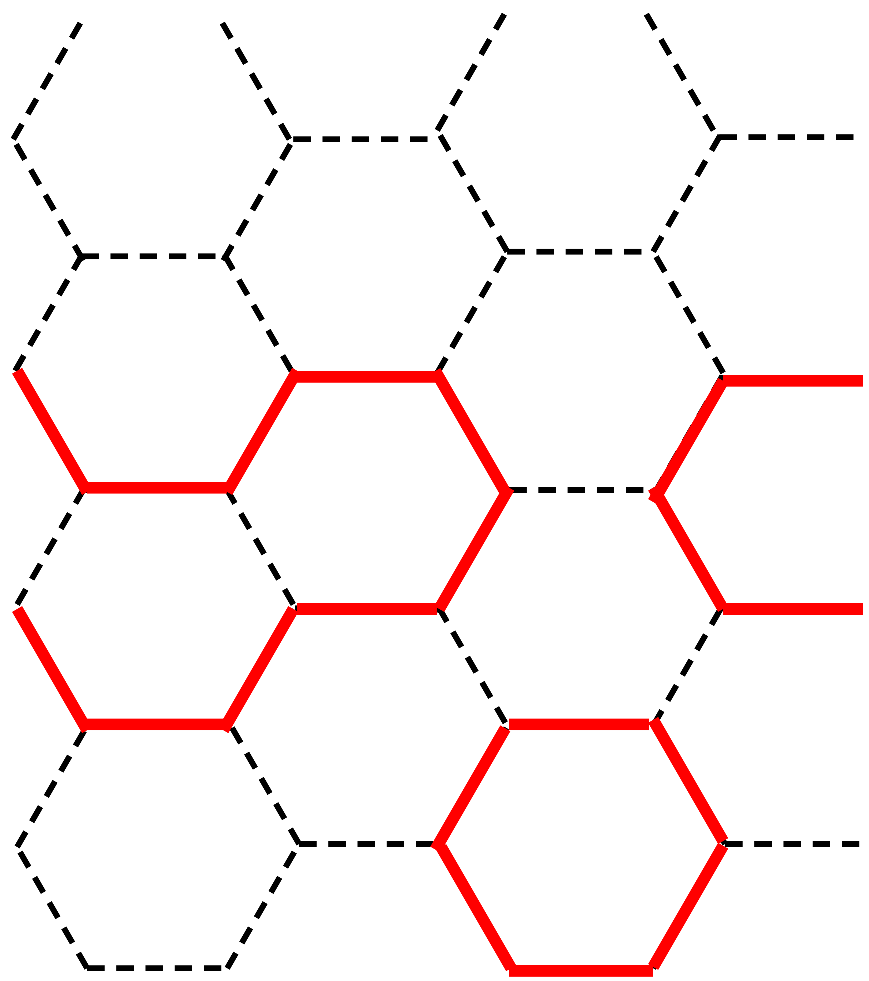
SU(2):

$Q = \{1, 2\}$

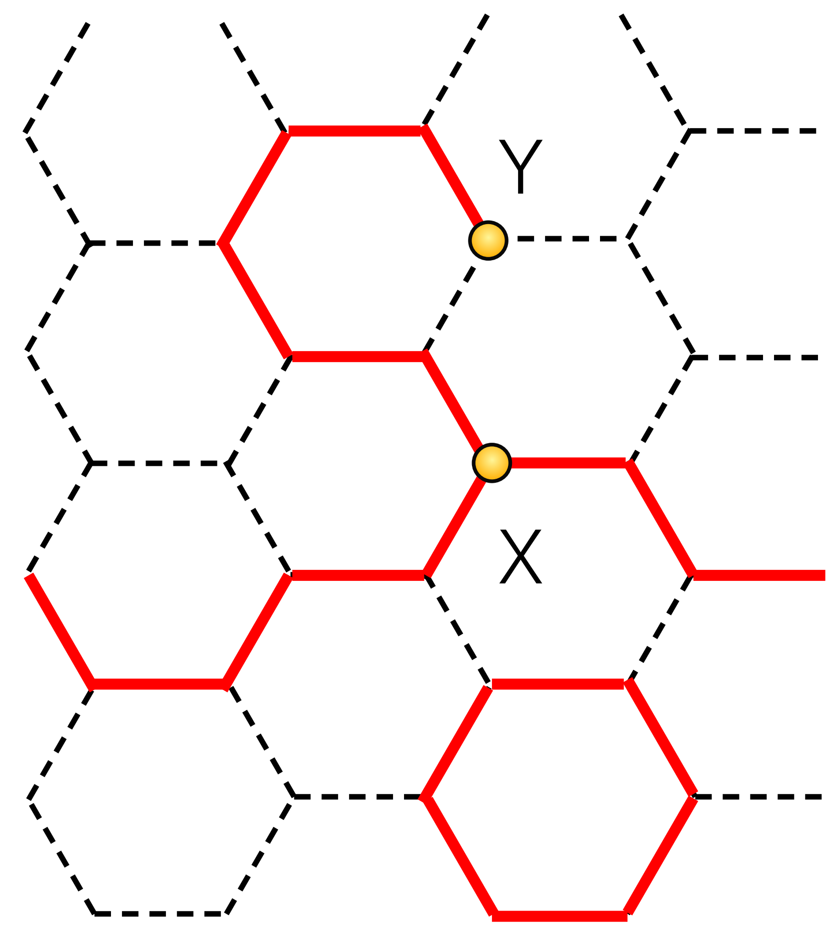
$\mathcal{H}_Q =$



The physical Hilbert Space $\mathcal{H}_{\text{phys}}$ only involves gauge invariant states



Gauge invariant
pure gauge state

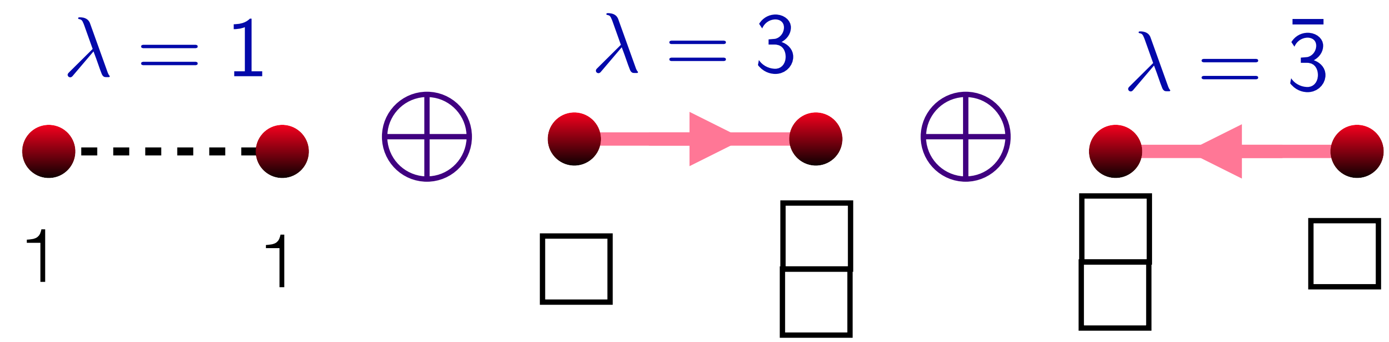


Gauge invariant state
with two quarks

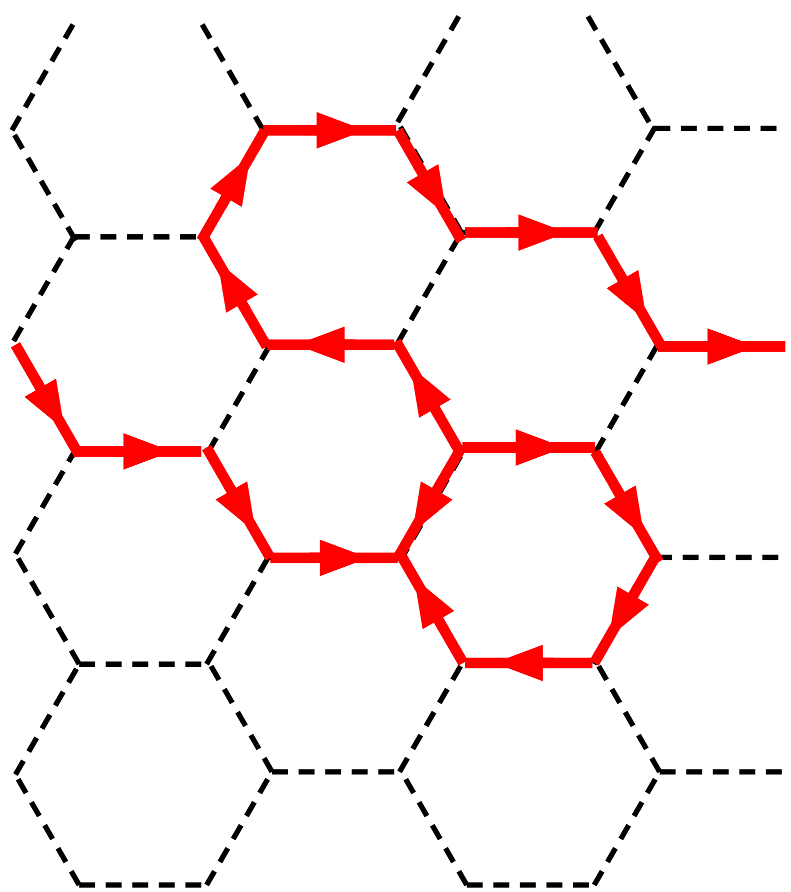
SU(3):

$Q = \{1, 3, \bar{3}\}$

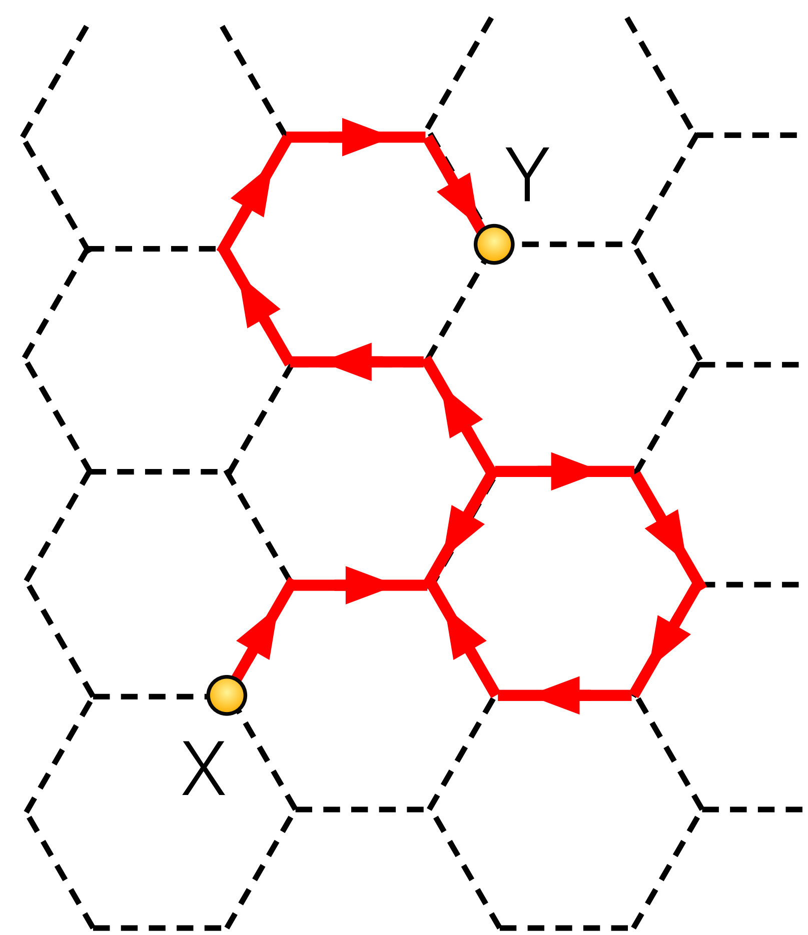
$\mathcal{H}_Q =$



The physical Hilbert Space $\mathcal{H}_{\text{phys}}$ only involves gauge invariant states



Gauge invariant
pure gauge state



Gauge invariant state
with a quarks and an
anti-quark.

Non-Abelian Gauge Theories —"momentum basis"→ Generalized Quantum Dimer Models

Easy to study classical dimer models with Monte Carlo methods!

A "confinement" observable accessible in dimer models!

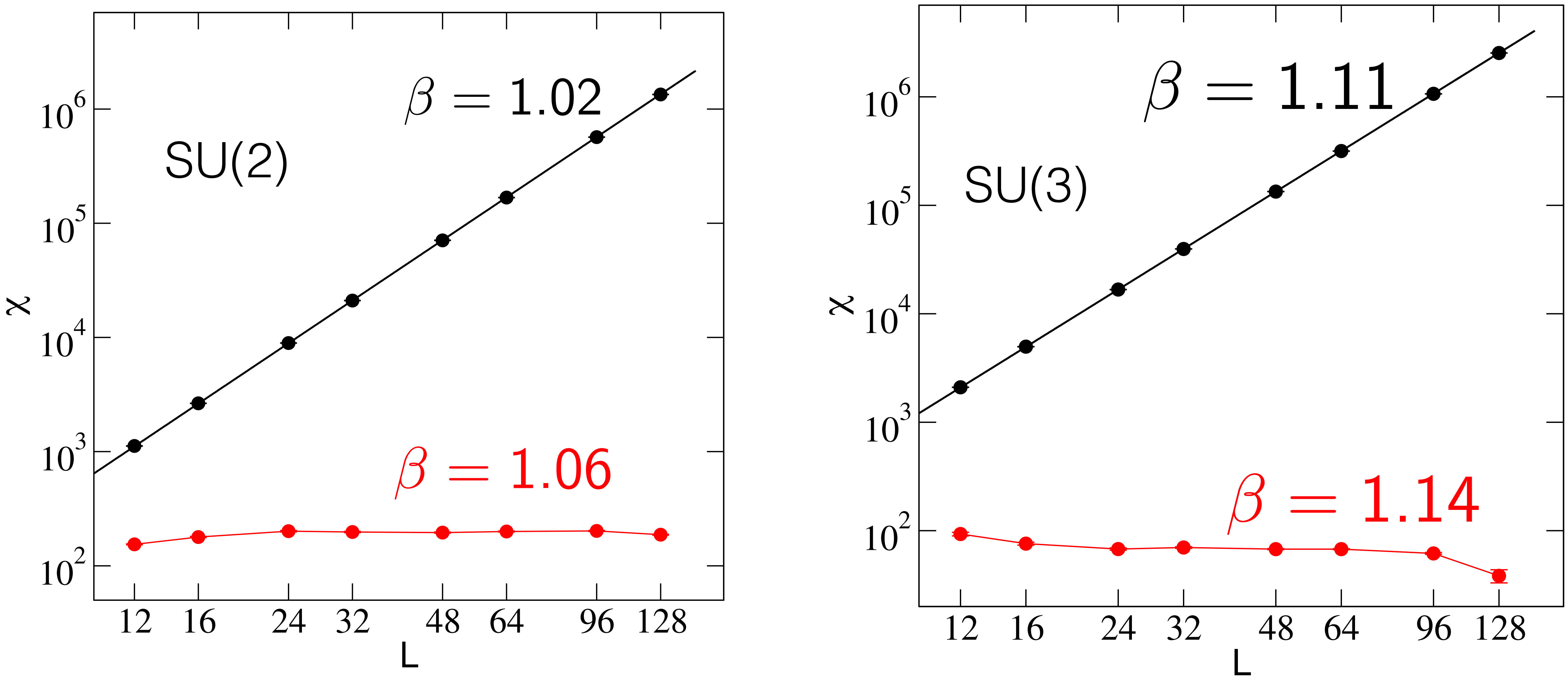
$$\chi = \frac{1}{V} \sum_{x,y} \frac{Z_{xy}}{Z}$$

$$Z = \mathrm{Tr}\left(e^{-\beta H_{\mathrm{cl}}}\right) \qquad Z_{xy} = \mathrm{Tr}_{xy}\left(e^{-\beta H_{cl}}\right)$$

Deconfined phase: $\chi \sim L^3$ Confined phase: $\chi \sim \mathrm{Const}$

Confinement-Deconfinement Transition in (3+1)d

Preliminary Results



Both seem like first order transitions.

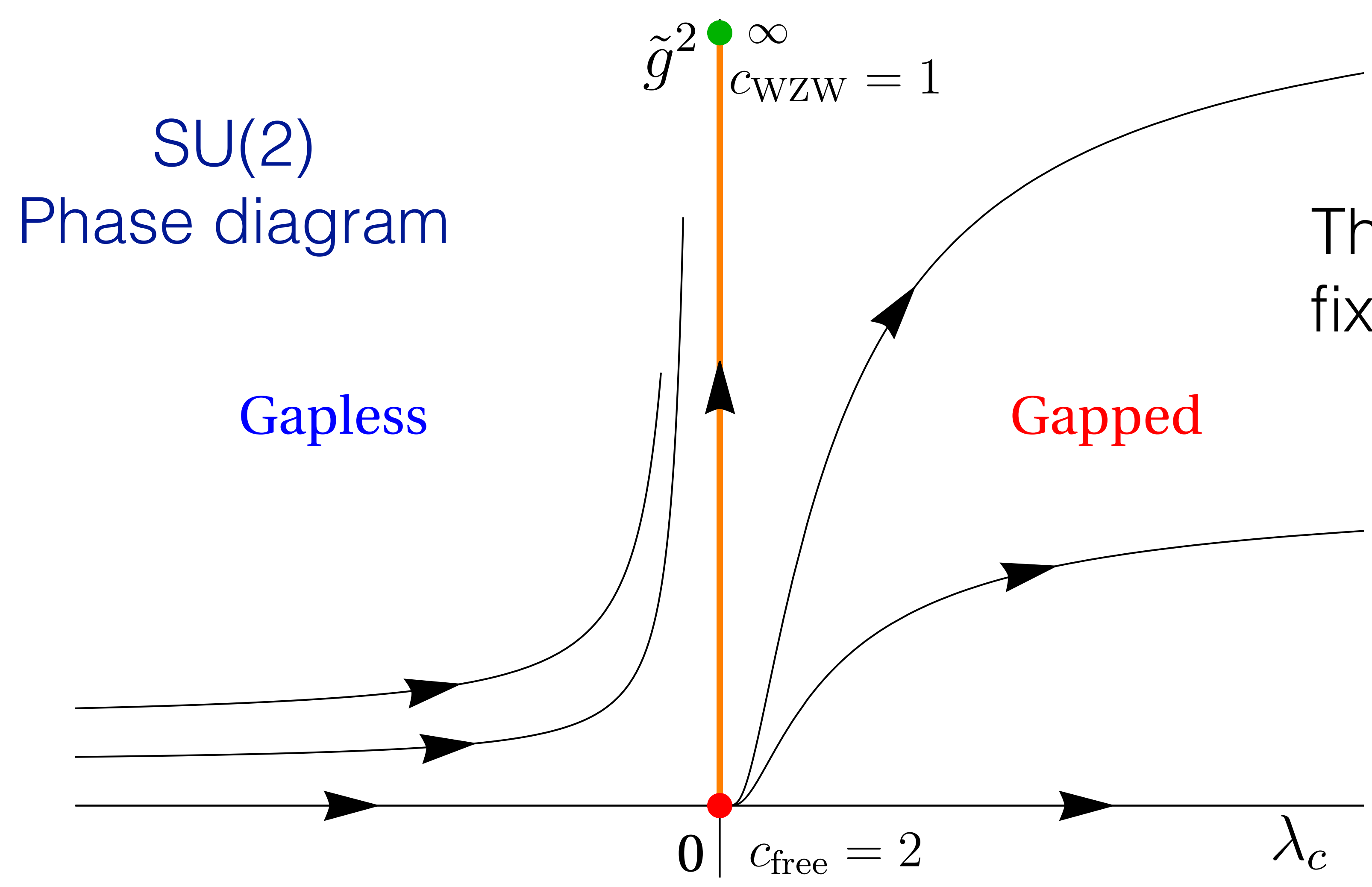
Easy to construct quantum dimer models
free of sign problems!

Are there quantum critical points separating
confined and deconfined phases?

Does asymptotic freedom of Yang-Mills theory
arise at one of these quantum critical points?

Adding SU(N) matter is straight forward

SU(N) gauge theories with massless staggered matter in 1+1 dimensions is also very interesting.



There are two fixed points

How can we recover this physics with qubit regularization?

Liu, Bhattacharya, SC, Gupta, arXiv:2312.17734

Conclusions

Quantum Computation suggests new regularization schemes for QFTs with finite local Hilbert spaces.

Qubit Regularization

Asymptotic freedom can emerge via a new type of RG flow in qubit regularized models

Think beyond traditional Hamiltonians!

Qubit regularization of gauge theories lead to quantum dimer models.

Both confined and deconfined phases exist!

Can Yang-Mills theory arise at a quantum critical point of quantum dimer models?