Update on Glueballs

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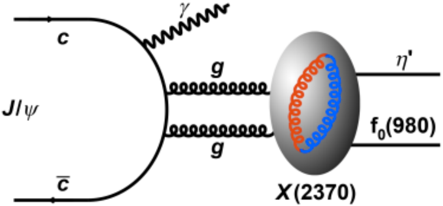
Why an update on glueballs?

- BESIII first-time determination of 0^{-+} quantum numbers of $X(2370)$ in PRL **132**, 181901 (2024)

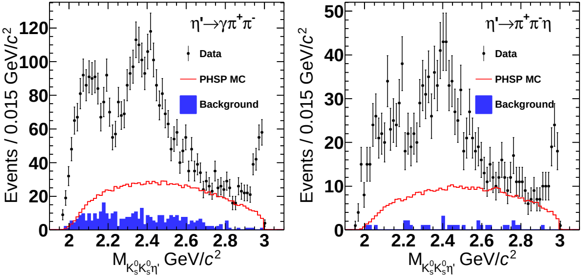


- BESIII earlier observation of $X(2370)$ in $J/\psi \to \gamma\pi^+\pi^-\eta'$ [PRL **106**, 072002 (2011)]
- mass consistent with lightest 0^{-+} glueball from lattice QCD
- gluon rich environment

$X(2370)$ from BESIII

- partial wave analysis of $J/\psi \to \gamma K_S^0 K_S^0 \eta'$ gives

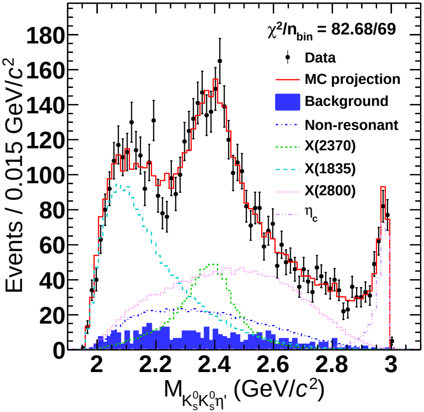
$$m = 2395 \pm 11(\text{stat})^{+26}_{-94}(\text{syst})\ \text{MeV}/c^2$$
$$\Gamma = 188^{+18}_{-17}(\text{stat})^{+124}_{-33}(\text{syst})\ \text{MeV}$$



- unfortunate name $X(2370)$ from earlier paper

$X(2370)$ from BESIII

- optimal PWA fit: $X(1835), X(2370), \eta_c$ and broad 0^{-+} $X(2800)$ Breit-Wigner decays through $f_0(980)\eta'$ to $(K_S^0 K_S^0)_S\eta'$ and $(K_S^0 K_S^0)_D\eta'$ with nonresonant components
- statistical significance of $X(2370)$ is $> 11.7\sigma$



Identifying glueballs in experiments

- difficult to identify glueballs
- mass ratios only really known in pure gluon theory
- expect flavor symmetric decays, but differing quark masses leads to differing phase spaces
- no rigorous predictions on decay patterns and their branching ratios
- glueball decays could be similar to that of charmonium
- states could be admixtures with quark-antiquark states

Some past glueball candidates

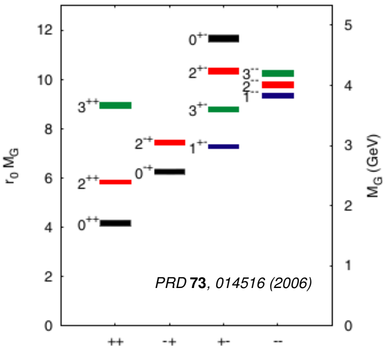
- light scalar candidates $f_0(1370), f_0(1500), f_0(1710)$ (MarkII in 1980s, Crystal Barrel in 1990s)
- narrow $\xi(2230)$ tensor glueball candidate due to good flavor symmetric decay property from MarkIII in 1980s/BESI in 1990s $\longrightarrow$ not confirmed by BESII nor BESIII with much higher statistics
- odderon (odd C-parity) from D0 and TOTEM [PRL **127**, 062003 (2021)]

Pure-gauge glueballs in lattice QCD

- pure-gauge glueball spectrum
  $\longrightarrow$ long history in lattice QCD
- calculations date back to early days of lattice QCD (1970s)
- Monte Carlo computations very noisy, rapid correlator falloffs
  $\longrightarrow$ need for large amount of statistics
- progress from extended smeared operators in 1980s/1990s
  $\longrightarrow$ M. Teper, C. Michael, D. Weingarten, among others
- anisotropic lattice: better temporal resolution of correlators
  $\longrightarrow$ CM + M. Peardon late 1990s

Pure-gauge glueball spectrum

- pure-gauge glueball masses (no quarks)
- mass ratios well determined
- setting scale ambiguous since no quarks: string tension from Cornell potential
- lightest scalar $\sim 1600 - 1700$ MeV



- states are not fixed numbers of gluons
- scalar mass gap has $1 million bounty (Clay Mathematics Institute)

Excited states from correlation matrices

- energies from temporal correlations $C_{ij}(t) = \langle 0|\overline{O}_i(t) O_j(0)|0\rangle$
- in finite volume, energies are discrete (neglect wrap-around)

$$C_{ij}(t) = \sum_n Z_i^{(n)} Z_j^{(n)*}\, e^{-E_n t}, \qquad Z_j^{(n)} = \langle 0|\, O_j\, |n\rangle$$

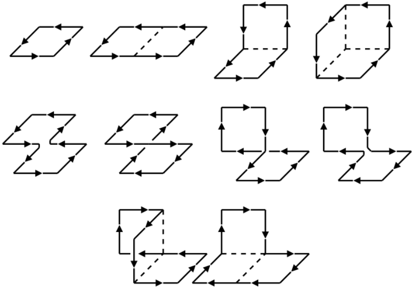
- not practical to do fits using above form
- define new correlation matrix $\widetilde{C}(t)$ using a single rotation

$$\widetilde{C}(t) = U^\dagger\, C(\tau_0)^{-1/2}\, C(t)\, C(\tau_0)^{-1/2}\, U$$

- columns of U are eigenvectors of $C(\tau_0)^{-1/2}\, C(\tau_D)\, C(\tau_0)^{-1/2}$
- choose τ_0 and τ_D large enough so $\widetilde{C}(t)$ diagonal for $t > \tau_D$
- 2-exponential fits to $\widetilde{C}_{\alpha\alpha}(t)$ yield energies E_α and overlaps $Z_j^{(n)}$

Glueball operators

- important: use good operators for signal before noise growth
- glueball operators: gauge-invariant loops of link variables $U_\mu(x)$
- Teper fuzzing: large links from links+staples
- multiple sizes for radial structure
- different shapes for orbital structure

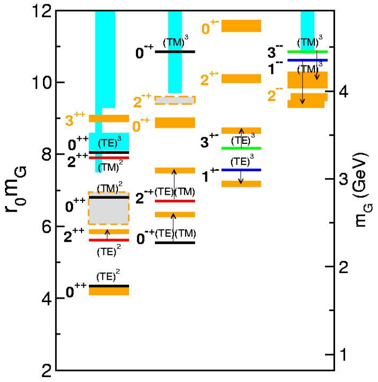


Qualitative features of glueball spectrum

- spectrum qualitatively understood in terms of interpolating operators of minimal dimension (Jaffe,Johnson,Ryzak, Ann. Phys. 168, 344 (1986))
- dimension 4:
$$\mathrm{Tr}F_{\mu\nu}F_{\alpha\beta} \Rightarrow 0^{++}, 0^{-+}, 2^{++}, 2^{-+}$$
- dimension 5:
$$\mathrm{Tr}F_{\mu\nu}D_\rho F_{\alpha\beta} \Rightarrow 1^{++}, 3^{++}$$
- dimension 6:
$$\mathrm{Tr}F_{\mu\nu}F_{\rho\omega}F_{\alpha\beta} \Rightarrow 0^{\pm+}, 1^{\pm\pm}, 2^{\pm\pm}, 3^{\pm-}$$
$$\mathrm{Tr}F_{\mu\nu}\{D_\rho, D_\omega\}F_{\alpha\beta} \Rightarrow 1^{-+}, 3^{-+}, 4^{\pm+}$$
- of lightest 6 states, 4 have the J^{PC} of the dimension 4 operators
- absence of low-lying $0^{\pm-},\ 1^{-+}$ glueballs explained

Glueballs from MIT bag model

- qualitative agreement with bag model
- constituent gluons are TE or TM modes in spherical cavity
- Hartree modes with residual perturbative interactions
- center-of-mass correction
- parameter modifications 1983$\rightarrow$ 1993
  $\alpha_s : 1.0 \rightarrow 0.5$
  bag parameter $B^{1/4}$:
  $230\ \mathrm{MeV} \rightarrow 280\ \mathrm{Mev}$

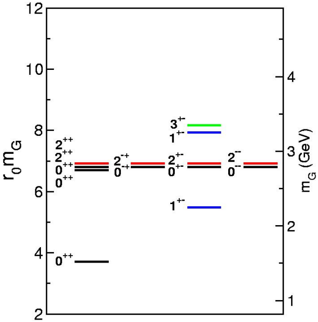


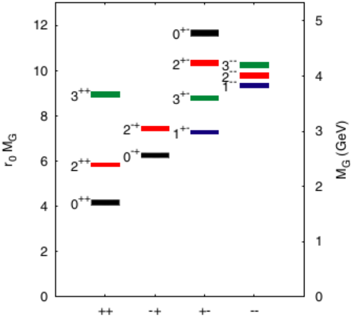
Carlson, Hansson, Peterson, PRD27, 1556 (1983);

J. Kuti (private communication)

Glueballs from Isgur-Paton flux tube model

- disagreement with one particular string model
- Isgur, Paton, PRD31, 2910 (1985)



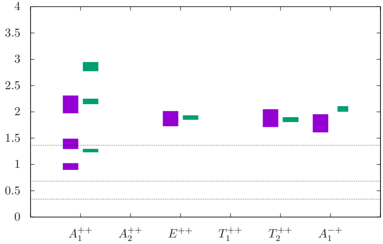


Why are glueballs with quarks so hard in lattice QCD?

- must extract all levels lying below glueballs of interest
- many 2-meson, 3-meson, 4-meson levels expected below
- 2-meson correlators require timeslice-to-timeslice propagators
- glueballs expected to be resonances
- glueballs require high statistics: difficult with quarks
- scalar sector requires large VEV subtraction

Glueballs with $N_f = 4$ light quarks

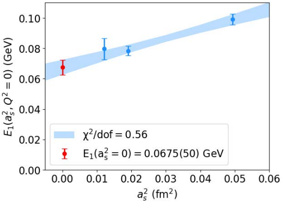
- Athenodorou et al. arXiv:2308.10054 [hep-lat]
- examine effect of quark loops on glueball spectrum
- several ensembles used with $m_\pi \approx 250$ MeV
- GEVP used on correlation matrices with only glueball operators
- no meson-meson operators!! (really need these)

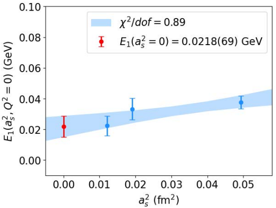


- conclusions: scalar channel lowers toward 2π, tensor and pseudoscalar spectrum only slightly affected

Radiative decay of the scalar glueball

- Zou et al. arXiv:2404.01564 [hep-lat]
- quenched approximation, 3 gauge ensembles
  $a_s \sim 0.11, 0.14, 0.22$ fm, continuum extrapolation
- evaluate EM transition matrix element $\langle S|J^{\mu}_{\rm em}|V\rangle$
- multipole expansion: two form factors $E_1(Q^2)$ and $C_1(Q^2)$
- get widths from $E_1(0)$: $Q^2 \to 0,\ a \to 0$ limits taken

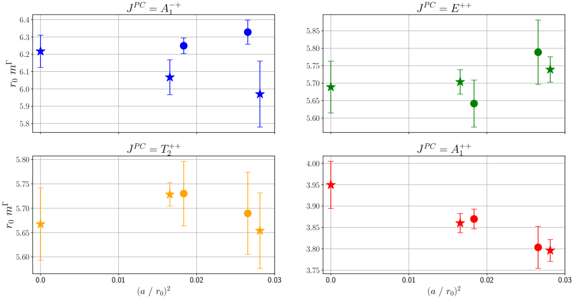




- find $\Gamma(J/\psi \to \gamma G) = 0.578(86)$ keV with
  ${\rm Br}(J/\psi \to \gamma G) = 6.2(9) \times 10^{-3}$ and $\Gamma(G \to \gamma\phi) = 0.074(47)$ keV
- conclude $J\psi \to \gamma G \to \gamma\gamma\phi$ not detectable by BESIII

Error reduction algorithm

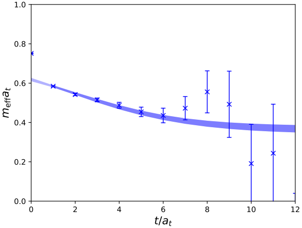
- Barca et al. arXiv:2406.12656 [hep-lat]
- new multi-level sampling procedure proposed for error reduction of glueball correlators in pure gauge theory

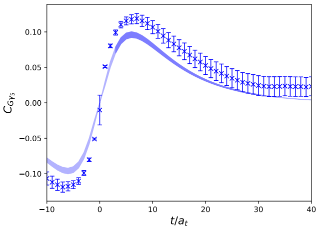


- state-of-the-art stars, new method circles
- no reduction in glueball mass errors, but significant error reduction in large$-t$ correlators to improve confidence in plateau estimates

Glueball-η mixing

- X. Jiang et al., PRD **107**, 094510 (2023)
- studied mixing of 0^{-+} glueball and pseudoscalar $\overline{q}q$ meson
- $16^3 \times 128$ anisotropic $N_f = 2$ lattice with $m_\pi \approx 350$ MeV
- used distillation, no GEVP, diagonal and cross correlators
- obtained very small $3.5°$ mixing angle glueball-η from cross correlator

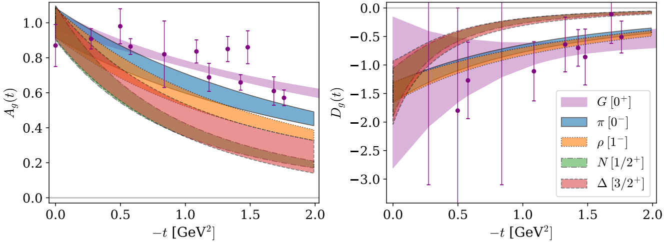




- (left) Effective mass 0^{-+} glueball (right) Cross correlator (shifted horizontally) with fit from temporal derivative

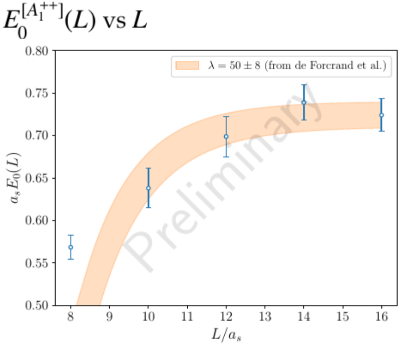
Gravitational form factors of glueballs

- D. Pefkou (poster), Abbott, Hackett, Romero-Lopez, Shanahan
- use gravitational form factors to probe structure of glueballs
- obtained from energy-momentum tensor matrix elements
- express matrix element of $T_{\mu\nu}$ in scalar glueball state in terms of form factors $A(Q^2)$ and $D(Q^2)$
- results also for π, ρ, N, Δ
- preliminary results in pure gauge theory $24^3 \times 48$ lattice $a = 0.1$ fm



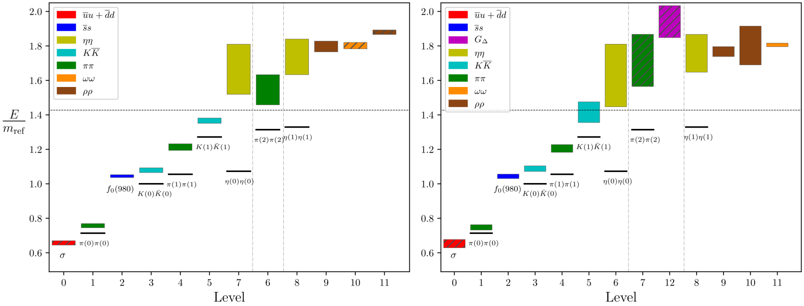
Scalar glueball scattering

- M. Hansen parallel talk (with M. Bruno and A. Rago)
- calculate finite-volume energies in Yang-Mills to extract $GG \to GG$ amplitudes
- anisotropic lattice, use of multi-level algorithm, scale set using t_0
- obtained volume dependence of A_1^{++} energy (single and 2 glueball operators)
- use Lüscher relation to get trilinear coupling λ from energies



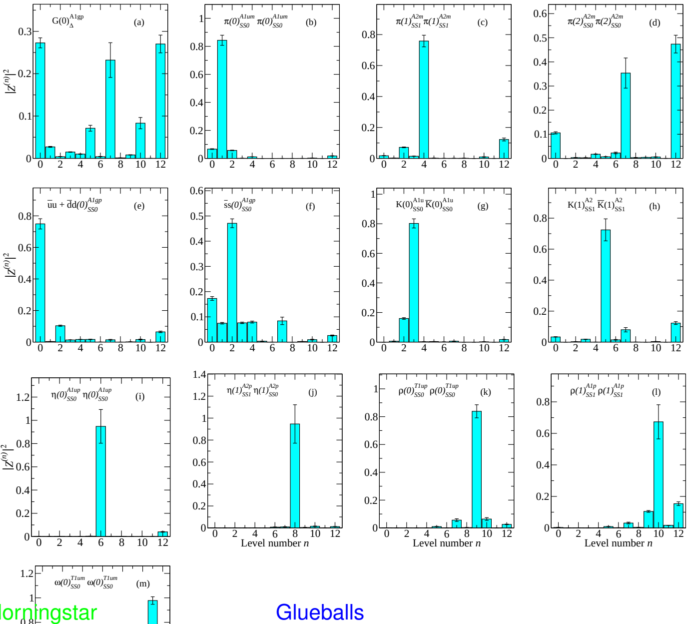
Scalar glueball in $N_f = 2+1$ QCD

- R. Brett et al. arXiv:1909.07306 [hep-lat]
- $24^3 \times 128$ anisotropic lattice, $m_\pi \sim 390$ MeV
- A_{1g}^{+} spectrum show below:
  - (left) 4 $\bar{q}q$ operators, 10 two-meson operators
  - (right) added glueball operator



- bad news for the scalar glueball?
- need for Lüscher analysis (ongoing)

A_{1g}^{+} overlaps



Conclusion

- glueballs very challenging in full QCD
- recent study suggests no scalar state below 2 GeV is pure or dominant glueball
- pseudoscalar glueball is new focus of attention
- identifying glueballs in experiments is challenging too
- long history in lattice QCD and still very active