Lattice fermions, topological phases and Floquet insulators

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Work with

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Ties between lattice and Floquet:

Phys.Rev.Lett. 132 (2024) 13, 136601

Phys.Rev.Res. 6 (2024) 1, 013098

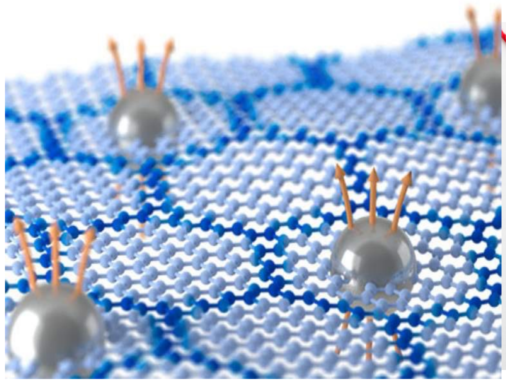
Lattice 2024

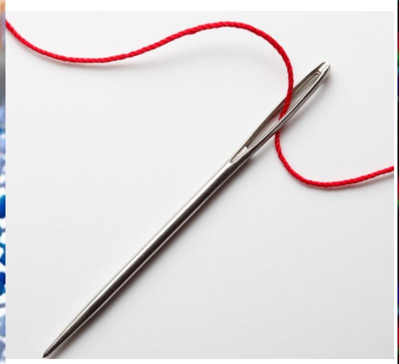
Big picture

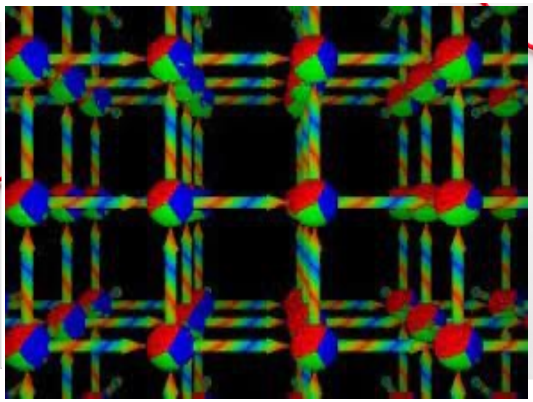
Thinking about ideas at the intersection of nuclear and particle physics, lattice QFT and condensed matter physics (topological phases), may be profitable for all the fields involved.

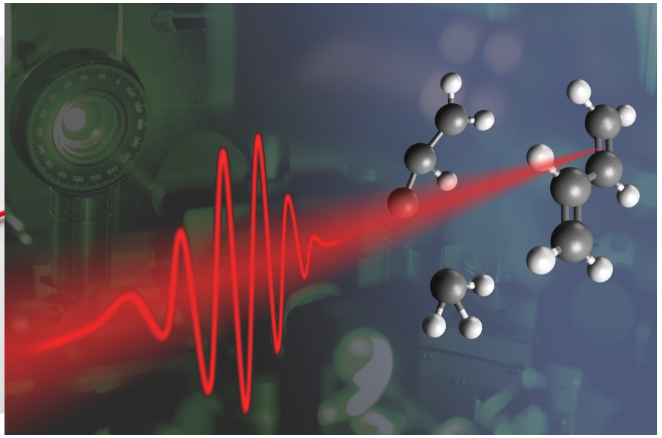
Can offer a new point of view in understanding a certain theory/system?

Possibility of using machinery of one field in the other.









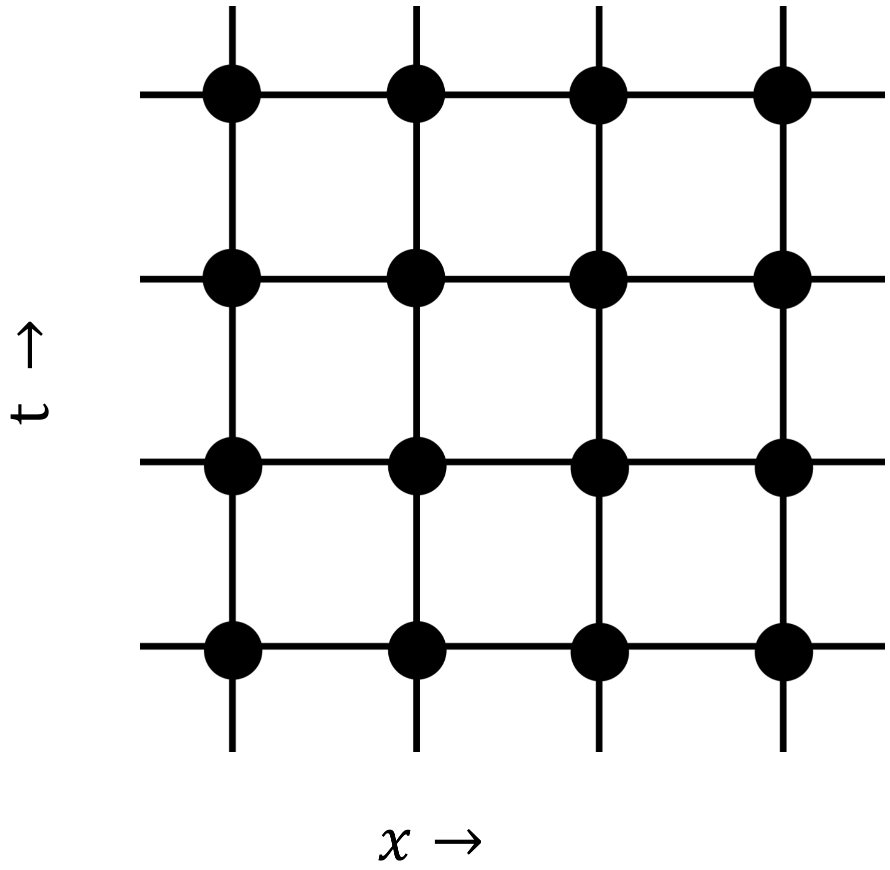
Old ties

Equilibrium topological insulators and lattice chiral fermions: the most celebrated example.
(Avron, Seiler, Kaplan, Jansen, Golterman, Schmaltz, Thouless, Kohmoto, Nightingale, Nijs, Haldane, Zhang, Aoki, Bernevig, Hughes)

New ties

Non-equilibrium quantum systems (Floquet) and lattice fermions?
(with T. Iadecola and L. Sivertsen)

Straight to the point



space-time lattice for lattice QFT

Fermion doubling on the lattice from discretization of space and time.

Fermion doubling can be found in materials, spatial discretization in crystals. But no time doubler in the real world?

Not the full story!!

The ties in continuous space-time, topological insulators and Dirac fermion

Insulators two types:

1. Uninteresting (Non-Topological) → Trivial, uninteresting bulk and boundary. Gapped bulk and boundary

2. Interesting (Topological) → Topological, sometimes boring bulk physics but interesting with a boundary. Gapped bulk, gapless boundary.

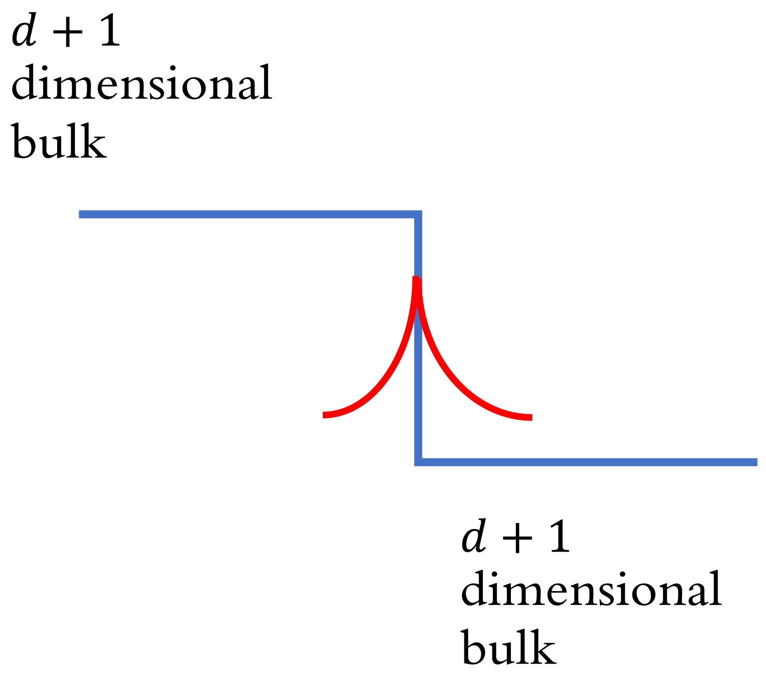
The ties in continuous space-time

Relativistic fermion with a domain wall in mass:

Massive fermion → Gapped bulk

Domain wall → Boundary

Domain wall is a boundary between boring and interesting



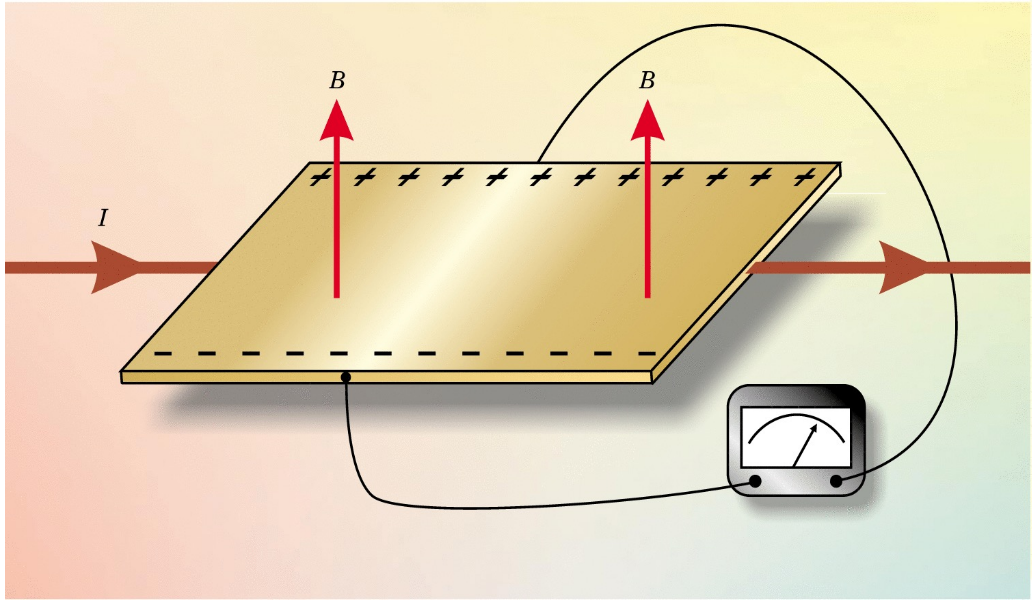
Relativistic fermion with open boundary condition:

Positive mass: Trivial, no boundary modes.

Negative mass: Topological, boundary modes etc.

Jackiw, Rebbi, Callan, Harvey

Dirac fermion and Quantum Hall Effect(QHE)



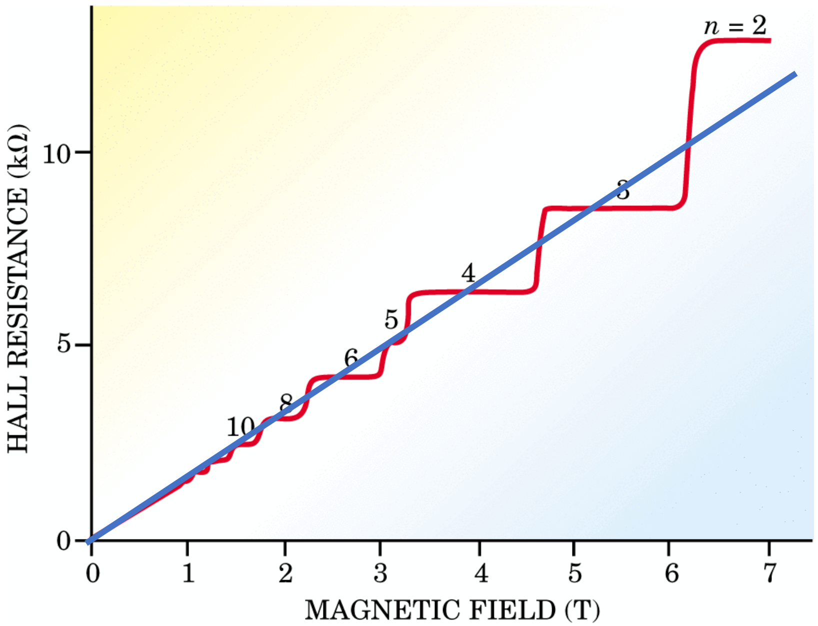
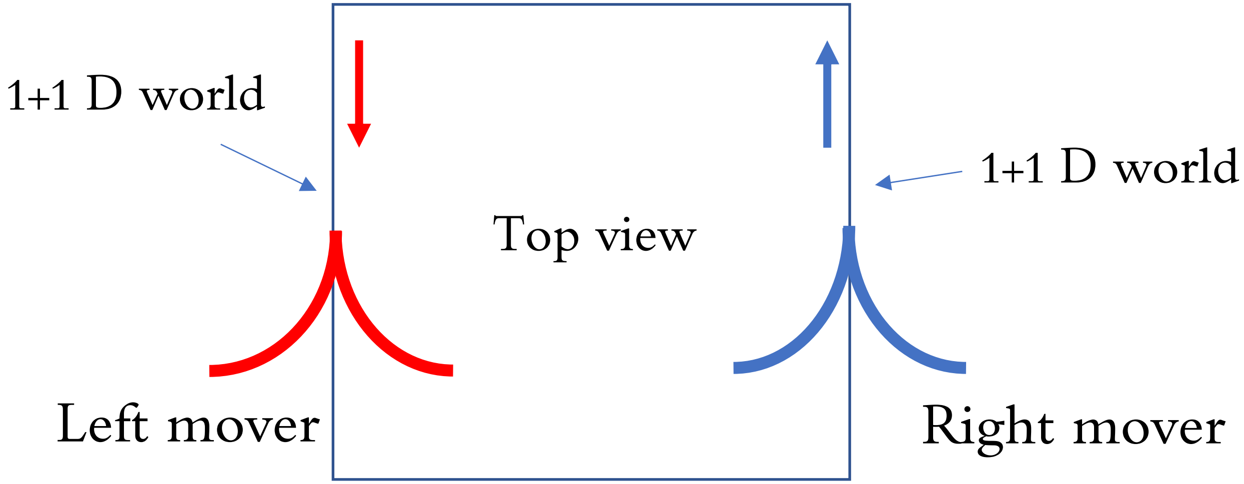
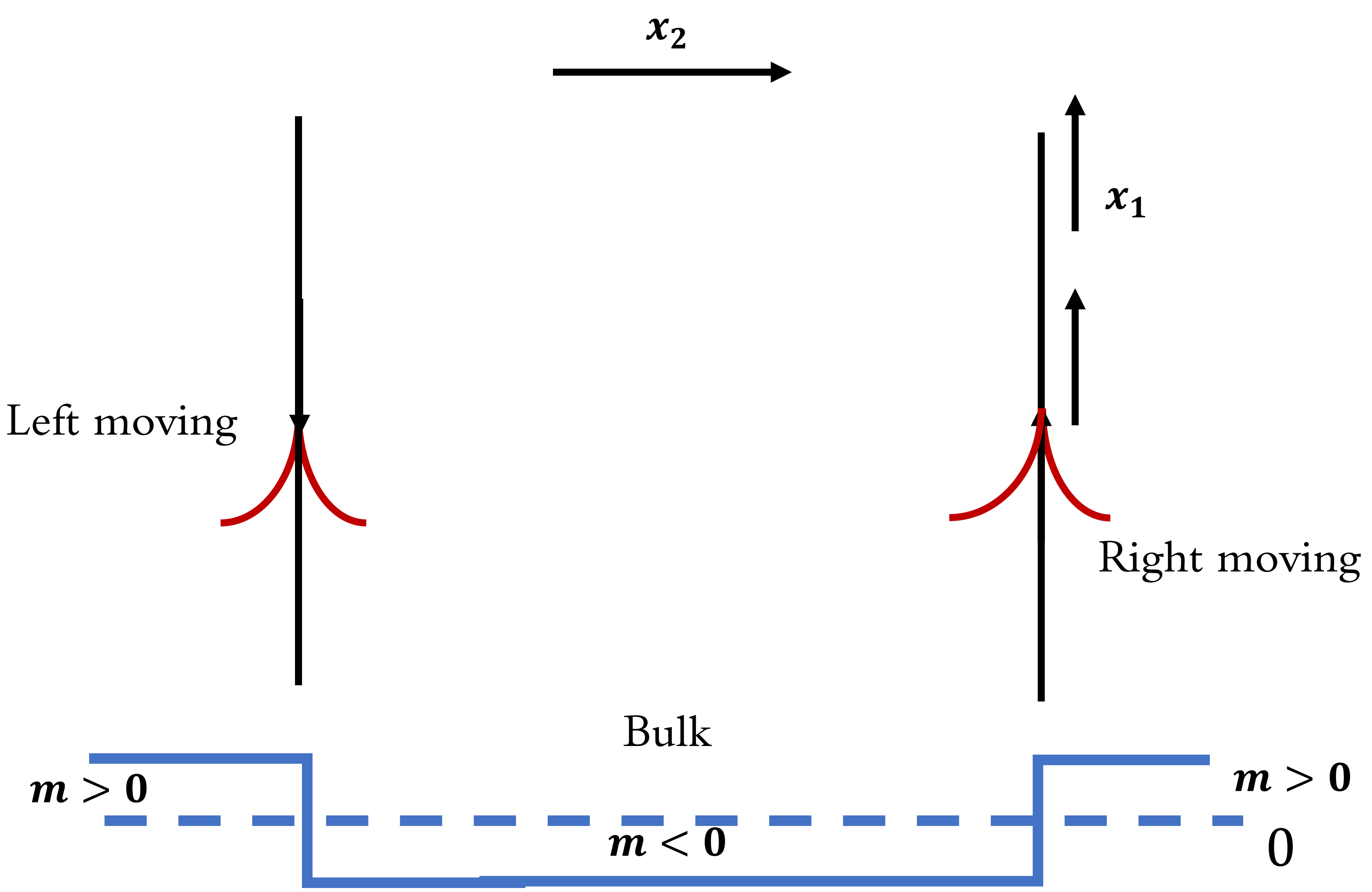


Figure credit: physicstoday



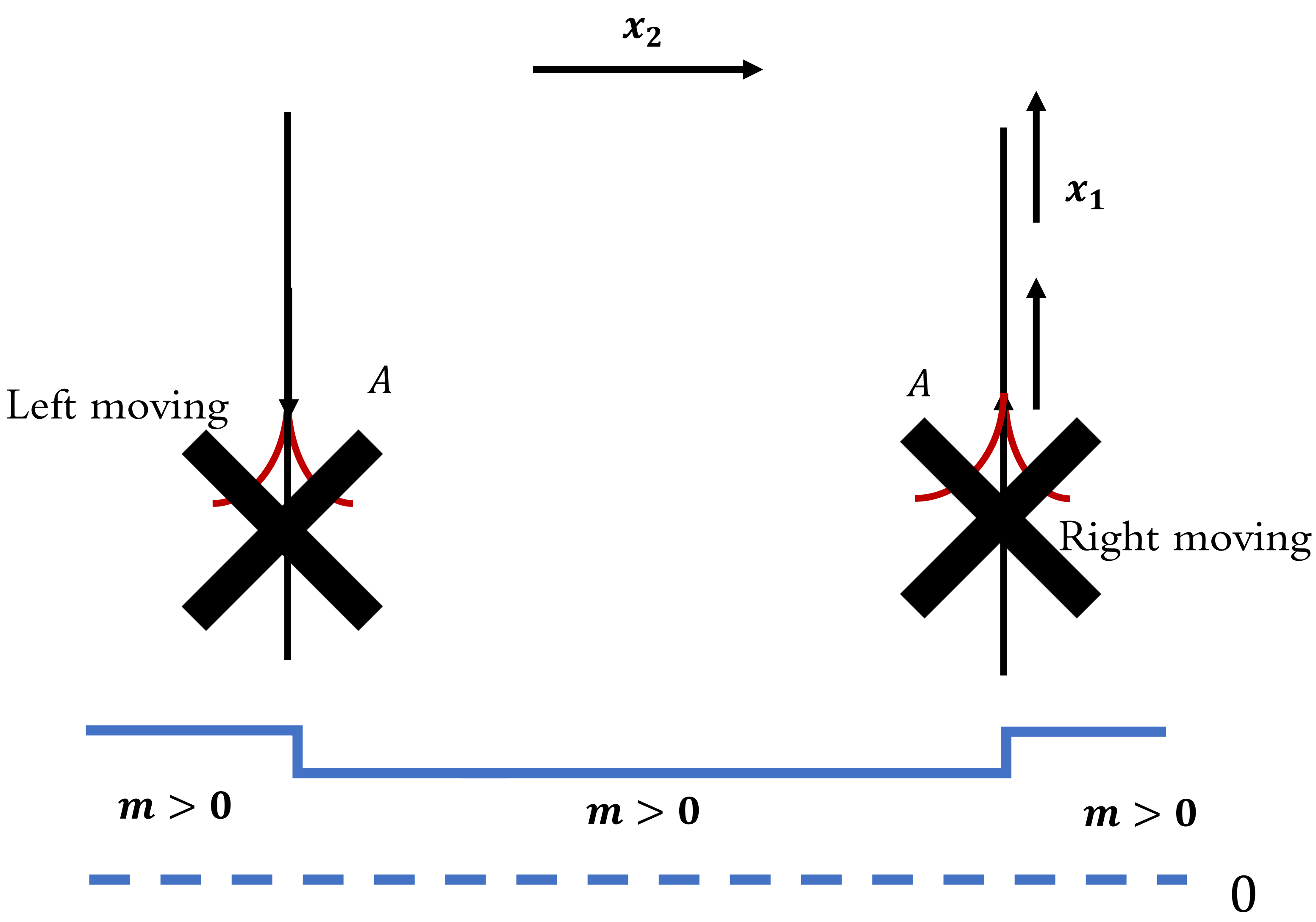
Relativistic fermion 2+1 D: Quantum Hall Effect(QHE)(Callan-Harvey 1984)



$$L_{2+1} = \bar{\psi}(i\gamma D - m)\psi$$

Step function in Dirac mass m in x_2 (domain walls)

Transition to boring (non-topological)

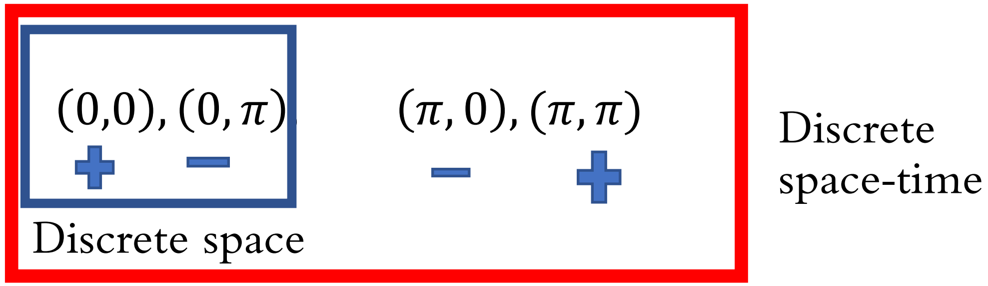


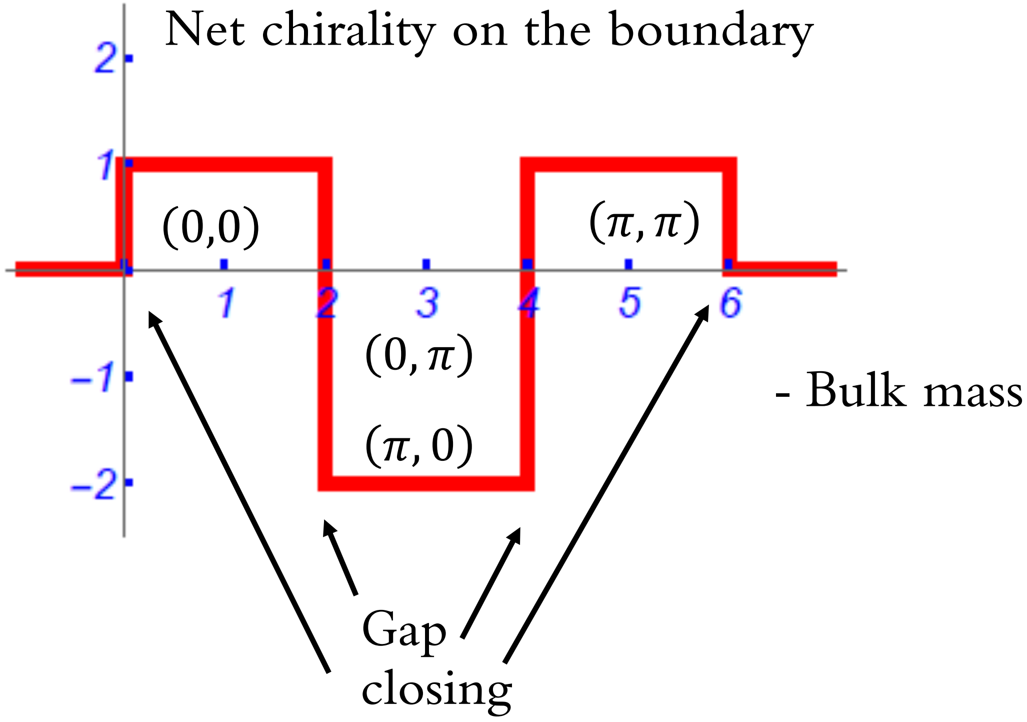
$$L_{2+1} = \bar{\psi}(i\gamma D - m)\psi$$

Step function in Dirac mass m in x_2 (domain walls)

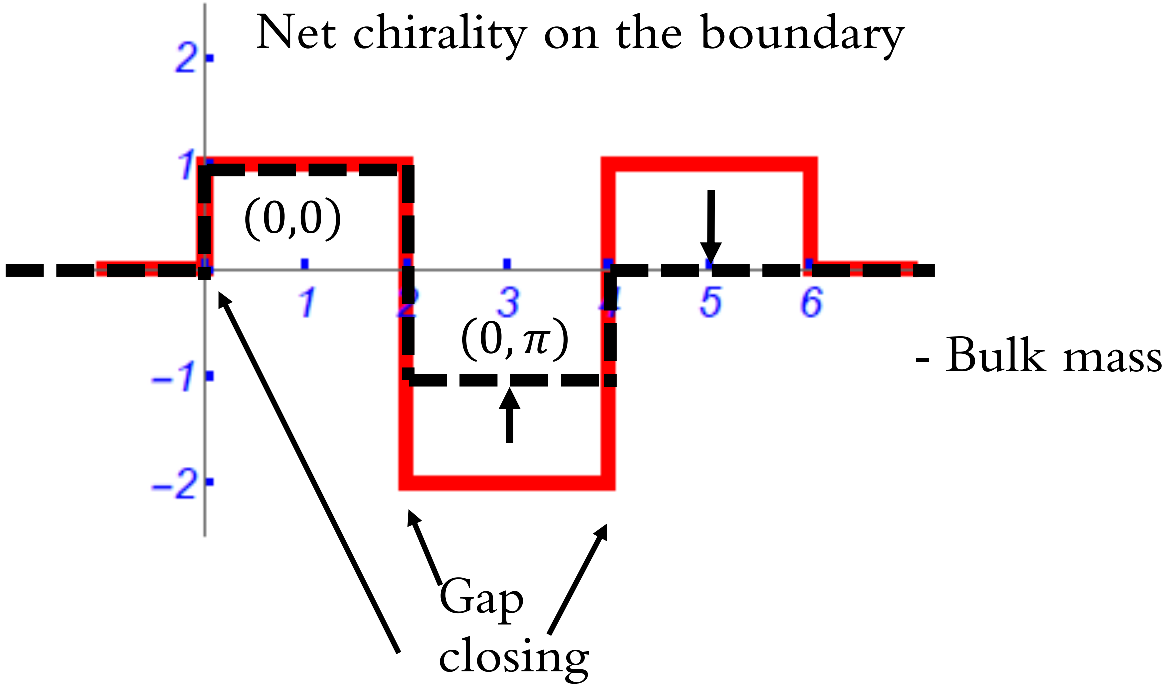
Discrete space-time Euclidean, discrete space continuous time

Weyl Fermion modes discrete space-time:





Wilson-Dirac model with a domain wall
(Phys.Lett. B301 (1993) 219-223)



BHZ model (Wilson-Dirac with continuous time) with a domain wall (Similar to TKNN, PhysRevLett.49.405)

1+1 D, discrete real time

Minkowski space discrete time solutions to Dirac equation: $\sin p_0 - \epsilon = 0$

where, e.g. $\epsilon = \pm\sqrt{p_1^2 + m^2}$

Point to note: For every zero at $\sin^{-1}\epsilon$, there is another at $\pi - \sin^{-1}\epsilon$.

(time lattice spacing set to 1)

Takeaway: there is no time doubling for continuous time systems. Present only for discrete time.

But, something curious happens for periodically driven systems.

Curious case of Floquet insulators (free fermion)

What's needed for this talk :

Continuous time but periodically driven.

Can exhibit novel phases: similar to undriven case.

Topological transition associated with gap closing.

Rudner, Levin, Lindner, Sondhi, Else, Monroe, Nayak, Sondhi, Vishwanath, Yao, Berg, Zhang, Keyserlingk

Curious case of Floquet insulators

What's needed for this talk :

Continuous time but periodically driven.

Can exhibit novel phases: similar to undriven case.

Topological transition associated with gap closing.

What does this mean? Energy is not conserved.

Rudner, Levin, Lindner, Sondhi, Else, Monroe, Nayak, Sondhi, Vishwanath, Yao, Berg, Zhang

Curious case of Floquet insulators

Driving a Hamiltonian over period T.

Observe the system at integer multiples of T.

Define quasi energy:

Time evolution operator $U_F(T)$. Quasi energy is the $\frac{i}{T}\log U_F(T)$.

Conserved.

Curious case of Floquet insulators

Identify phase boundaries by considering gap closing in quasi energy.

Interestingly, we observe boundary modes of quasi energy: $\frac{\pi}{T}$.

Reminiscent of time doublers in lattice field theory.

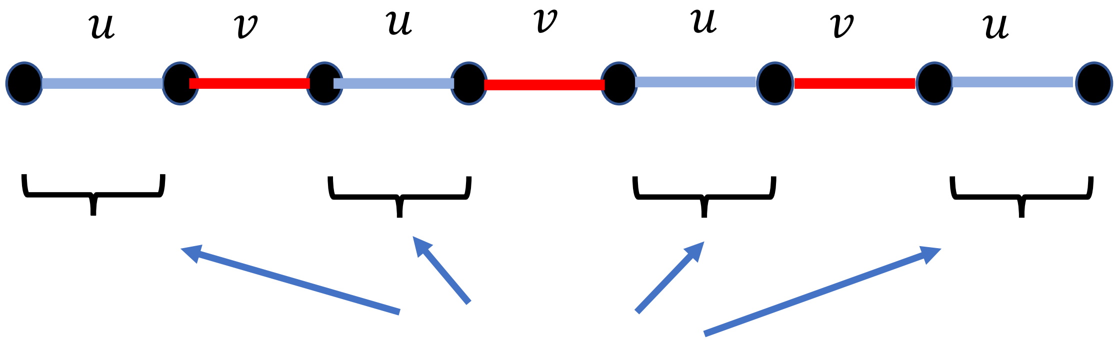
Is there an explicit way to connect Floquet insulators to discrete time systems?

Can the Floquet spectrum be reinterpreted as a time lattice theory of some undriven Hamiltonian (with time lattice spacing T)?

But boundary mode of π/T alone doesn't imply it can be thought of as a discrete time theory.

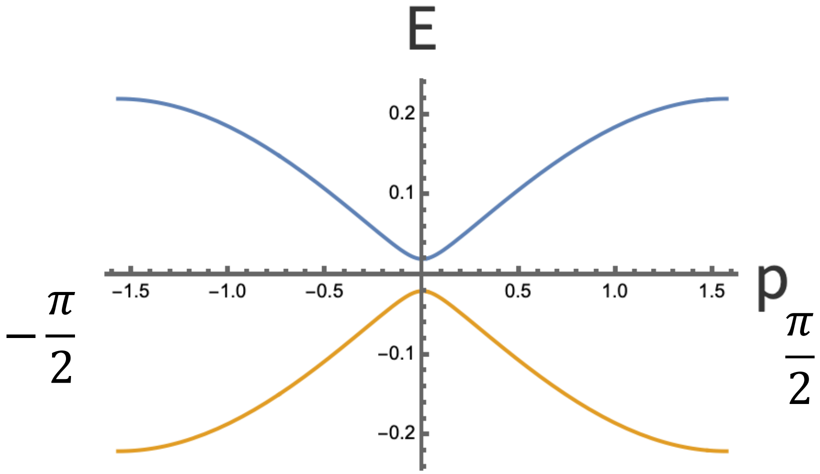
Even if this was the case, what kind of undriven Hamiltonian would those be?

SSH model (very similar to lattice staggered fermion: Dirac)



spatial lattice

sublattice,
two sites –> two components
of Dirac fermion

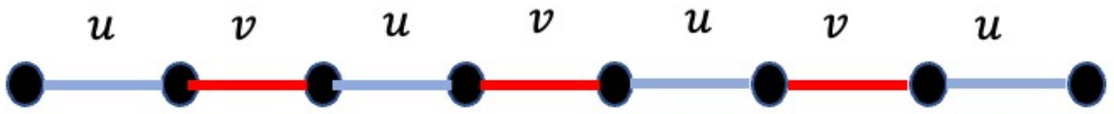


With PBC the spectrum is: $E(p) = \pm\sqrt{u^2 + v^2 - 2uv\cos 2p}\,,\quad \frac{\pi}{2} > p > -\frac{\pi}{2}$

(periodic boundary) $= \pm\sqrt{(u-v)^2 + 4uv\sin^2 p}$

Su–Schrieffer–Heeger (SSH) model spectrum

SSH model: Static topological Hamiltonian



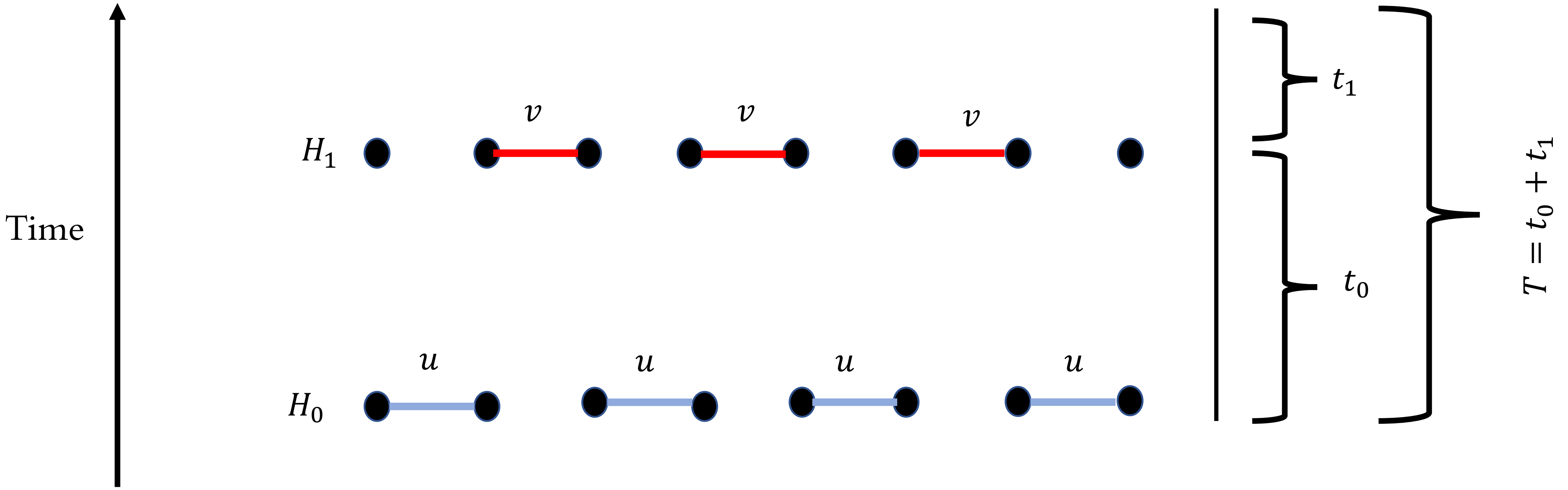
PBC: $E(p) = \pm\sqrt{(u-v)^2 + 4uv\sin^2 p}$

$u - v$ is Dirac mass.

$v - u > 0$: topological phase with zero energy edge mode for OBC (open boundary)

$u - v > 0$: non-topological phase with zero energy edge mode for OBC (open boundary)

Driven SSH model

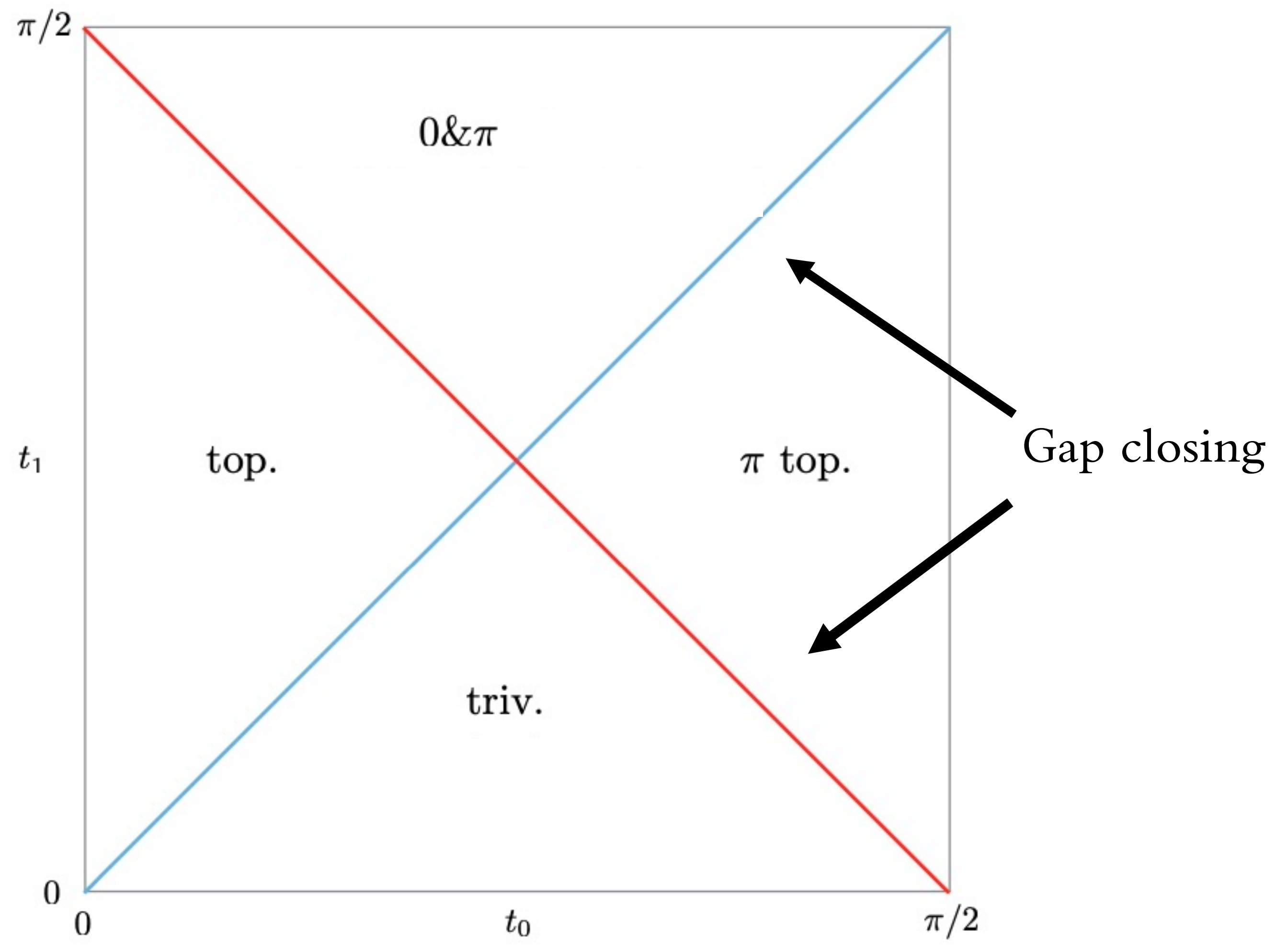


$$U_F(T) = e_0^{-iH_1 t_1} e^{-i\,H_0 t_0} \equiv e^{-i\,H_F T}$$

Get the quasi-energy ϵ by taking a log

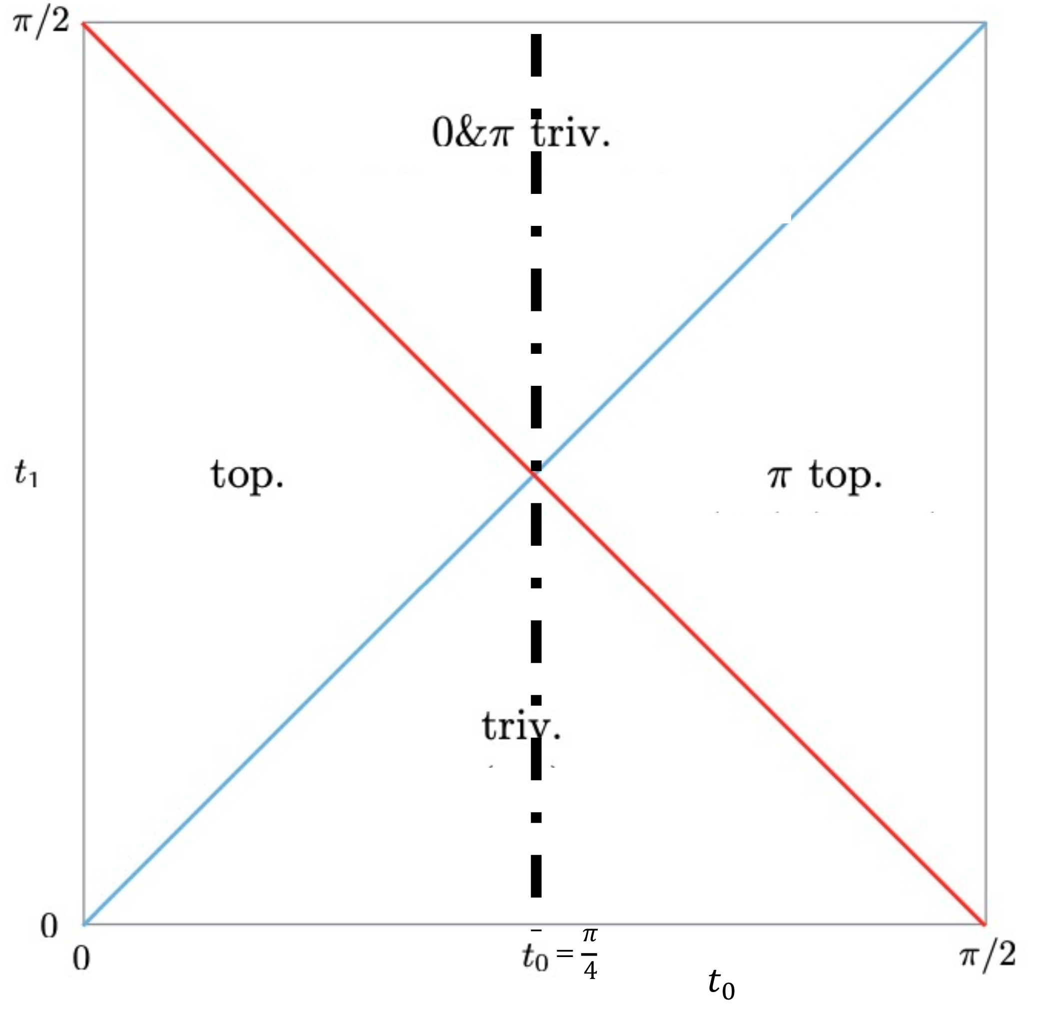
Inspired by Keyserlingk and Sondhi, Majorana model

Quasi-energy Phase diagram periodic and open boundary



Axes in lattice units

Energy eigenvalues with PBC



Along that line energy eigenvalues come in pairs.

i.e. ϵ and $\frac{\pi}{T} - \epsilon$ appear together.

π pairing or fermion doubling.

Appears mappable to a discrete time lattice Hamiltonian.

What is this Hamiltonian like?

The zero-eigenvalue map

$(i\partial_t - H_F) = 0$ to the discrete time operator $(i\nabla_t - H)$

$p_0 - H_F = 0$

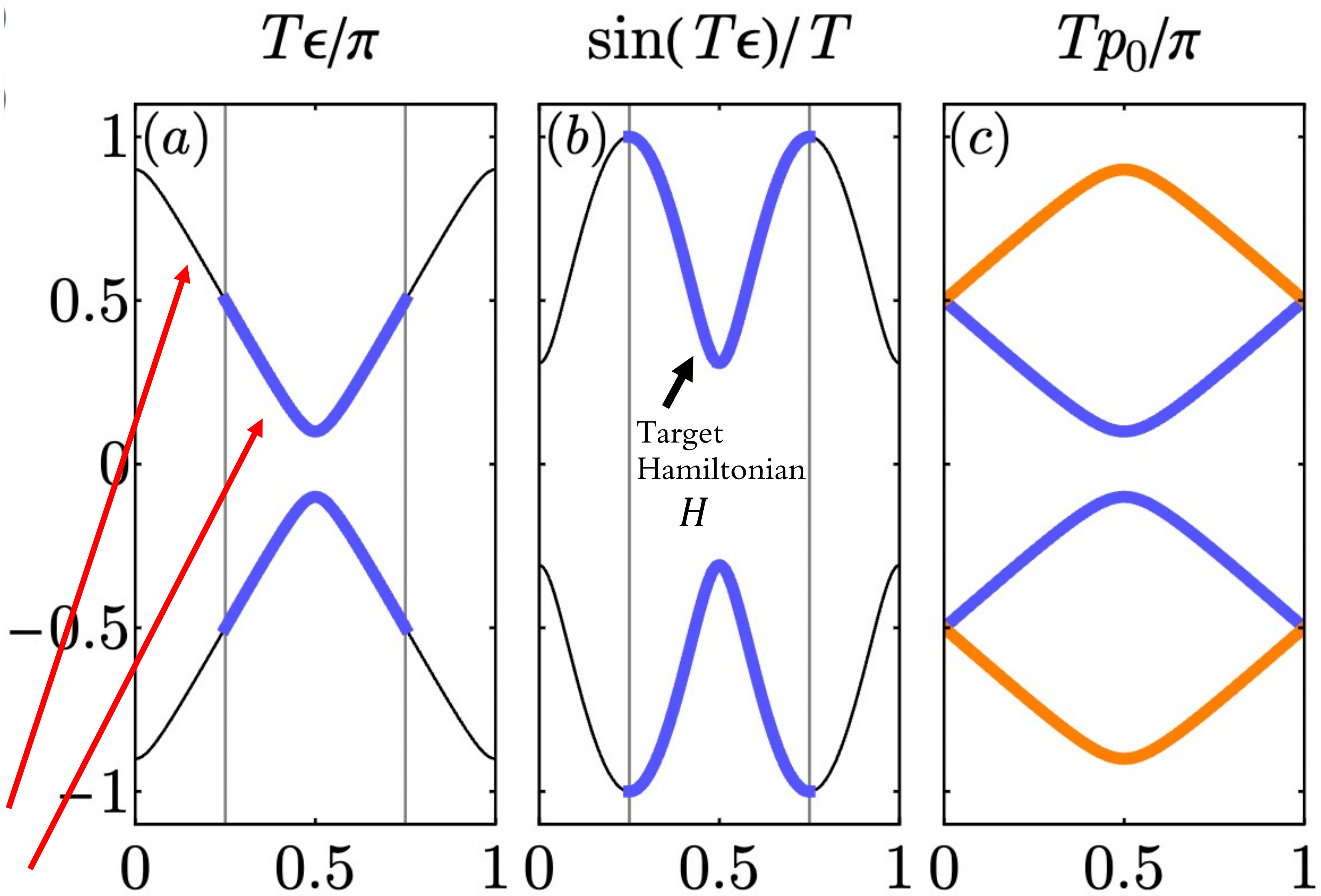
$\sin\, p_0 - H = 0$

Solutions for p_0 match.

Clearly, H has half the dimensions as that of H_F.

What is H ?

$$H \to \sin H_F \; ?$$

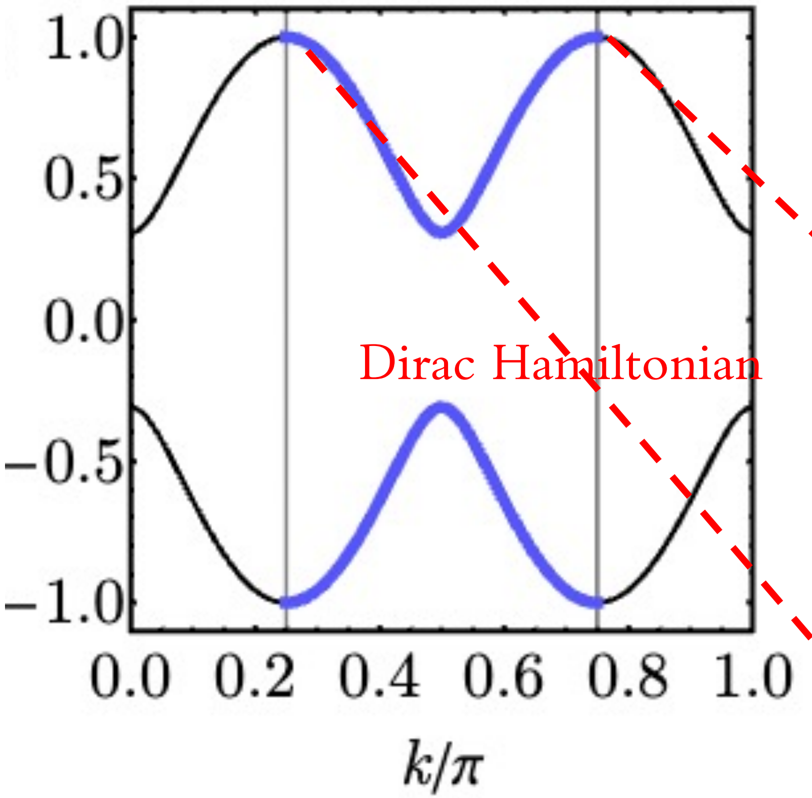


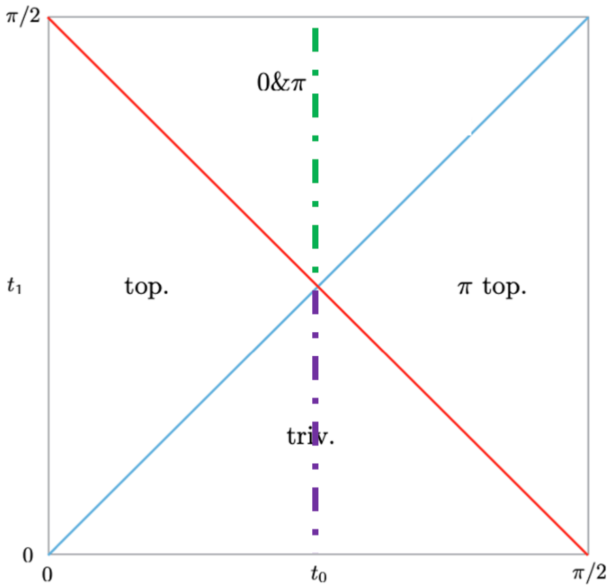
The blue and black regions are π paired with each other.

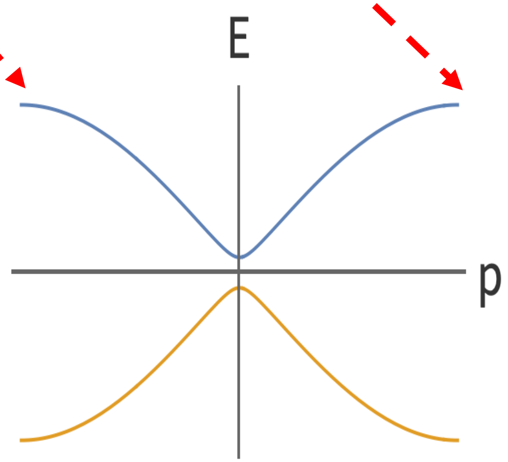
Keep the blue line, discard the black one.

This works. But more is true..

Surprise

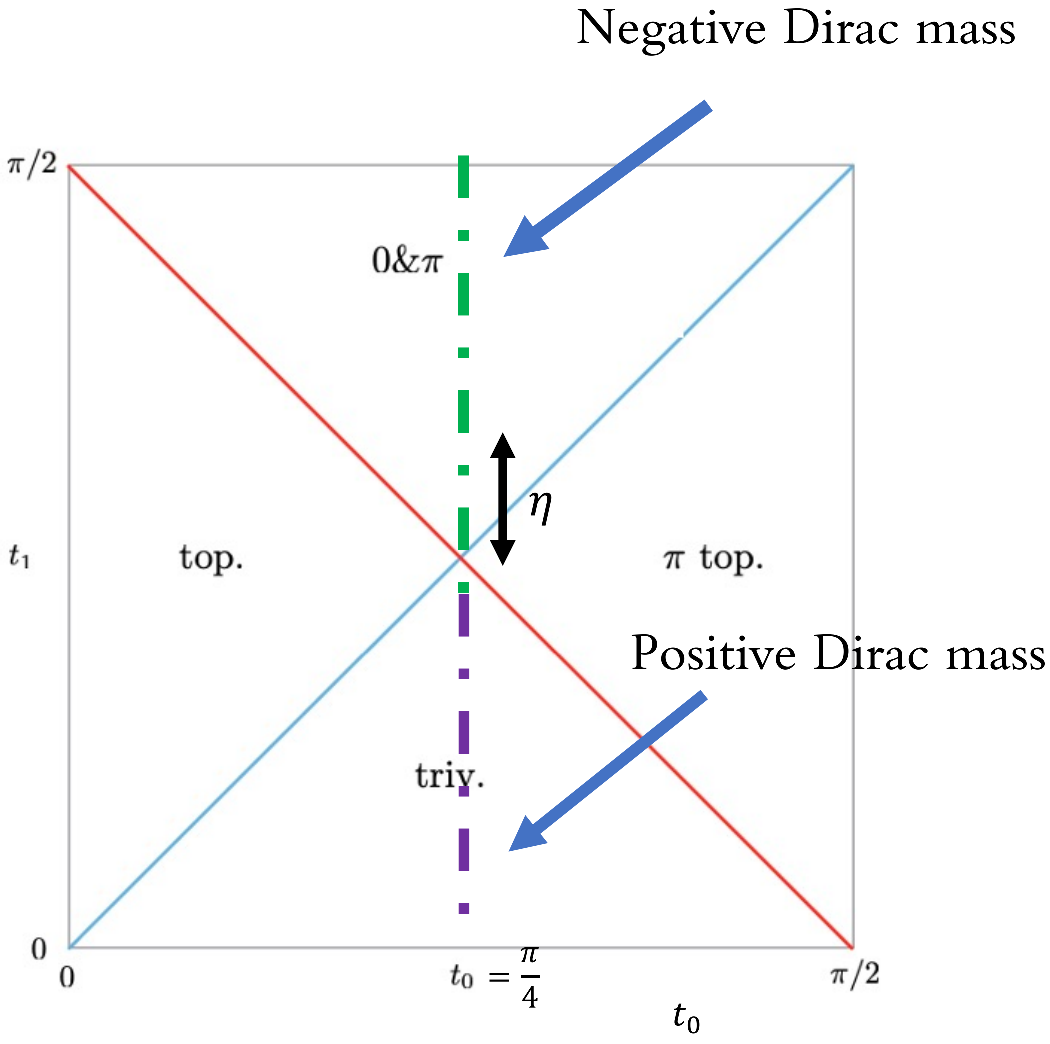






SSH/staggered

The map



SSH:

$$u = \frac{1 + \sin 2\eta}{2T} \qquad v = \frac{1 - \sin 2\eta}{2T}$$

As η goes from +ve to –ve, u and v switch.

With OBC, one gives you a zero mode, the other doesn't. Discretizing time gives you a pi mode for the former and none for the other.

Topological to non-topological transition

Iadecola, Sen, Sivertsen *Phys.Rev.Lett.* 132 (2024) 13, 136601

Summary

Periodically driven systems are sometimes termed as discrete time systems in a loose way.

This comparison can be been made concrete. Thus far in $1+1$ D.

The target Hamiltonian H ends up being a topological Hamiltonian itself !!

And there is an exact analytical correspondence between the two.

The Floquet transition maps to a topological transition of a static Hamiltonian with discrete time.

Open/recently answered questions

- Is there a way to make this comparison off the $t_0 = \frac{\pi}{4}$ line? Yes, see *Phys.Rev.Res.* 6 (2024) 1, 013098
- Does the correspondence hold for interacting theories?
- Why is there an exact analytic match? (Who ordered that?)
- What happens in higher dimensional examples?
- Ties Bulk boundary correspondence of the two cases?

Big picture

Many common threads between different areas of physics. Sometimes recognized in hindsight.

The ties of topological phases and fermion field theories go deep.

We understand these ties for equilibrium, free and some interacting theories.

Exploring such ties between periodically driven systems and lattice field theory may provide new perspective in both.