Symmetric Mass Generation

David Tong
University of Cambridge

Lattice 2024

SIMONS FOUNDATION

Science & Technology Facilities Council

Things To Do With a Quantum Field Theory

Take a quantum field theory with a symmetry G. When can we do the following while preserving G:

- Give a mass to all fields?
- Put the theory on a space with boundary?
- Put the theory on a lattice (with on-site action of G)?

*omitting many caveats!

Things To Do With a Quantum Field Theory

Take a quantum field theory with a symmetry G. When can we do the following while preserving G:

- Give a mass to all fields?
- Put the theory on a space with boundary?
- Put the theory on a lattice (with on-site action of G)?

A naive answer: You can do this when the symmetry G is vector-like.

But it's harder when the symmetry is chiral.

Things To Do With a Quantum Field Theory

Take a quantum field theory with a symmetry G. When can we do the following while preserving G:

- Give a mass to all fields?
- Put the theory on a space with boundary?
- Put the theory on a lattice (with on-site action of G)?

A conjecture: we can do all of these when the ‘t Hooft anomaly for G vanishes.

(This is the same requirement that is needed if you want to gauge G.)

e.g. for G=U(1) the ‘t Hooft anomaly is $A = \sum_{\text{left}} q_i^3 - \sum_{\text{right}} \tilde{q}_i^3$

Symmetric Mass Generation

Idea: Give a mass to a bunch of fermions transforming under a *chiral* symmetry G *without breaking G.*

Note: $\bar{\psi}_R\psi_L$ does not work.

First Example: Chiral Fermion Parity

The first example was by Fidkowski and Kitaev ‘09, with an extension by Xiaoliang Qi.

Consider N massless Majorana fermions in d=1+1

$$S_{\rm Majorana} = \int d^2x \ \sum_{i=1}^{N} i\bar{\chi}_i \, \partial\!\!\!/ \chi_i$$

Can we gap the fermions preserving chiral fermion parity $\mathbf{Z}_2 = (-1)^{F_L}$?

Answer: Only if N is a multiple of 8.

For a review, see Tong and Turner ”*Notes on 8 Majorana Fermions*” ‘19

First Example: Chiral Fermion Parity

- Clearly a mass term $\mathcal{L}_{\text{mass}} = m \sum_i \bar{\chi}_i \chi_i$ breaks the $\boldsymbol{Z}_2$ symmetry.

- The Gross-Neveu term gives a mass but spontaneously breaks $\boldsymbol{Z}_2$

$$\mathcal{L}_{\text{GN}} = g \left(\sum_{i=1}^{8} \bar{\chi}_i \chi_i \right)^2$$

- However, there is a clever four-fermion term that does the job

$$\mathcal{L}_{\text{FK}} = g' \left(\sum_{i=1}^{8} \bar{\chi}_i \Gamma^A_{ij} \chi_j \right)^2$$

Another Example in d=1+1: The 3450 Model

Consider two left-moving Weyl fermions ψ and two right-moving Weyl fermions $\tilde{\psi}$

Take charges under a G = U(1) symmetry to be

	ψ_1	ψ_2	$\tilde{\psi}_1$	$\tilde{\psi}_2$
G	3	4	5	0

Can we gap these fermions preserving G?

Another Example in d=1+1: The 3450 Model

There are several ways to do this. One is to introduce a U(1) gauge field together with an additional scalar ϕ.

		ψ_1	ψ_2	$\tilde{\psi}_1$	$\tilde{\psi}_2$	ϕ
global →	G	3	4	5	0	0
gauged →	H	5	0	3	-4	1

- $m_\phi^2 \ll 0$ *H* is Higggsed. At low energies we have just the massless fermions.
- $m_\phi^2 \gg 0$ The scalar decouples and the *H* = U(1) confines, leaving behind two massless fermions

$$\Psi = \psi_2 \quad \text{and} \quad \tilde{\Psi} \sim \psi_1^2 (\tilde{\psi}^\dagger)^2 \tilde{\psi}_2$$

Haldane '95; Wang and Wen '18; Tong '21

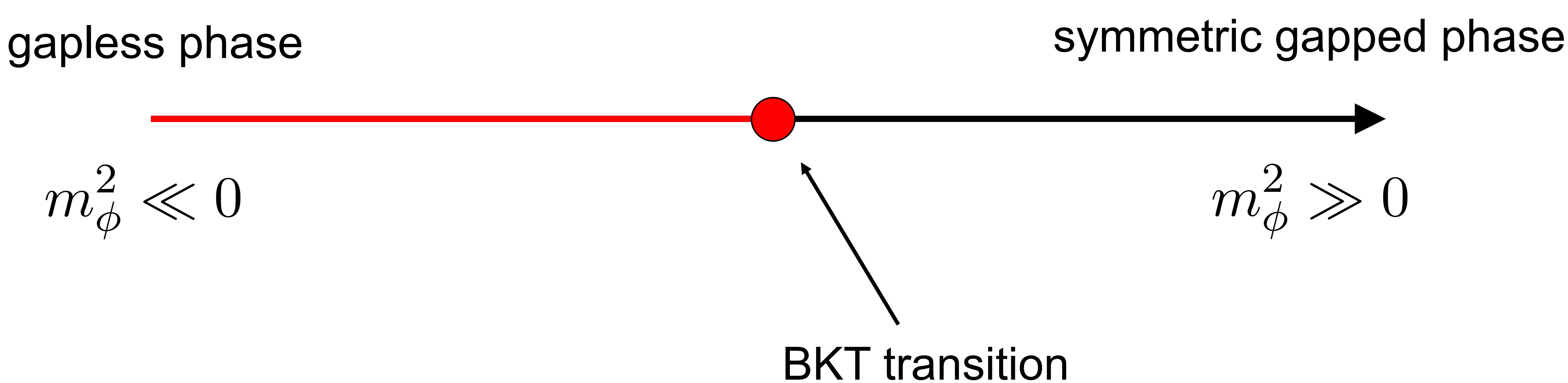
Another Example in d=1+1: The 3450 Model

Add the (dangerously) irrelevant operator

$$\mathcal{L}_{UV} \sim \psi_1^2 \psi_2 (\tilde{\psi}_1^\dagger)^2 \tilde{\psi}_2$$

In the confining phase, this flows to the symmetry-preserving mass term

$$\mathcal{L}_{IR} \sim \Psi \tilde{\Psi}$$



Symmetric Mass Generation in d=3+1

It is possible to gap one generation of the Standard Model
without breaking electroweak symmetry SU(2) x U(1)

Razamat and Tong ‘20

Symmetric Mass Generation on the Lattice

- Many examples of symmetric mass generation on the lattice

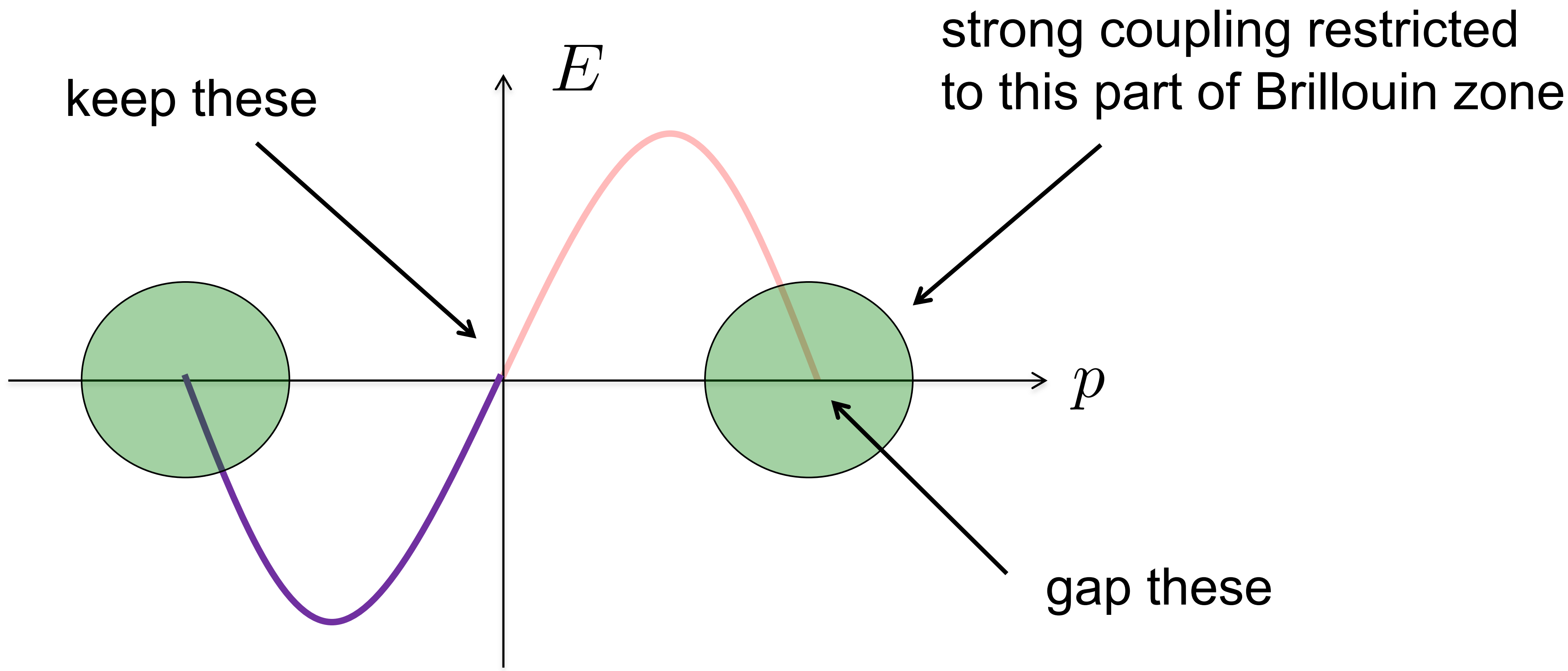
Ayyar and Chandrasekharan,
Catterall et al, Shaich, A. Hassenfratz

- This is like the older PMS phase, but with a continuum limit.

Chiral Fermions on the Lattice

Old idea of Eichten and Preskill ’86:

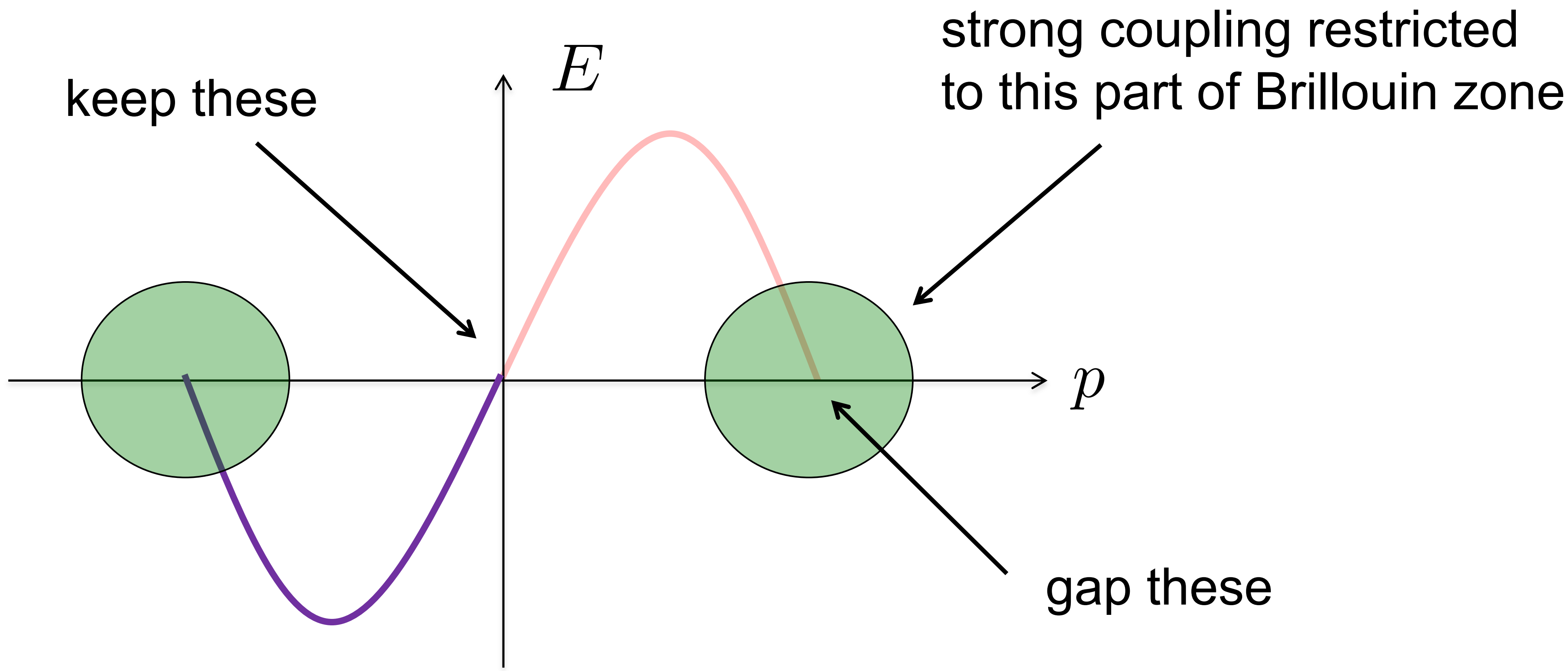
Turn on irrelevant 4-fermion coupling to gap the fermion doublers



Chiral Fermions on the Lattice

Old idea of Eichten and Preskill '86:

Turn on irrelevant 4-fermion coupling to gap the fermion doublers

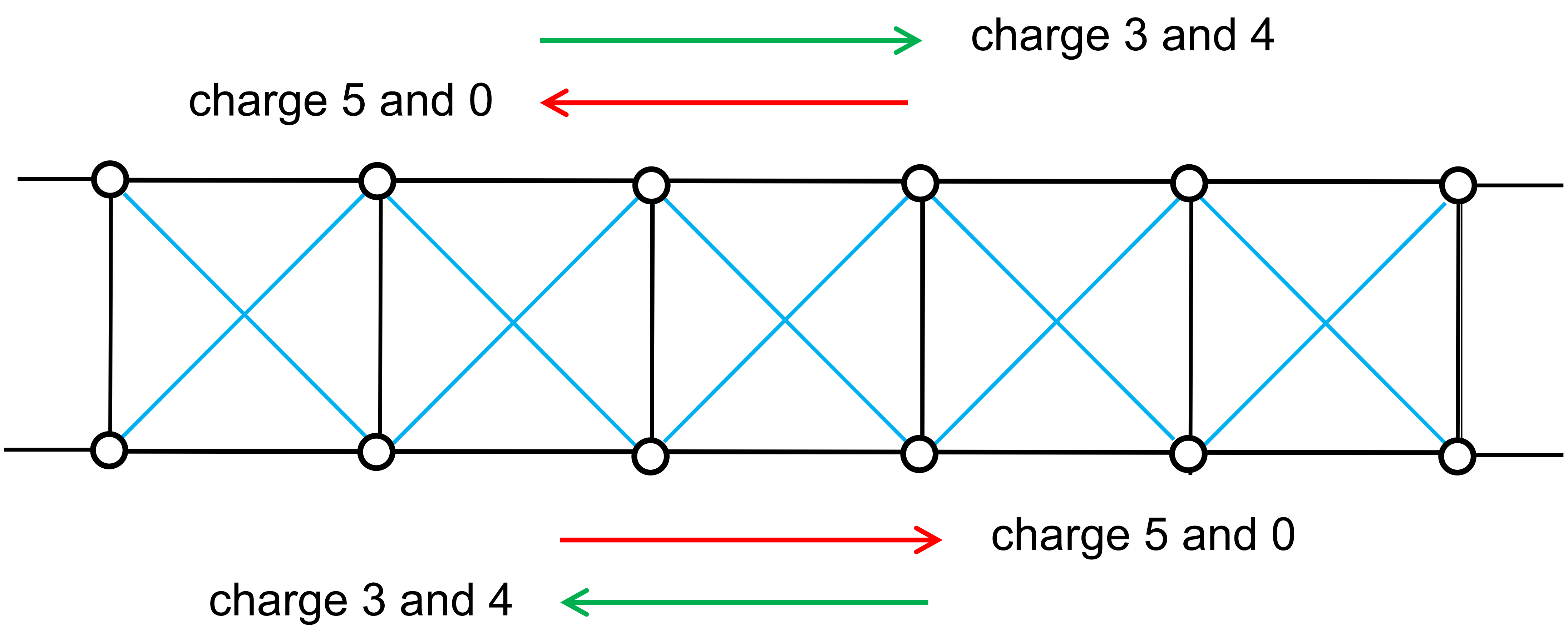


Problem: it doesn't work!

Golterman, Pechter and Rivas '93, Shamir '93, Chen, Giedt, Poppitz '07

0345 Model on the Lattice

This was recently revisited for the 3450 model.



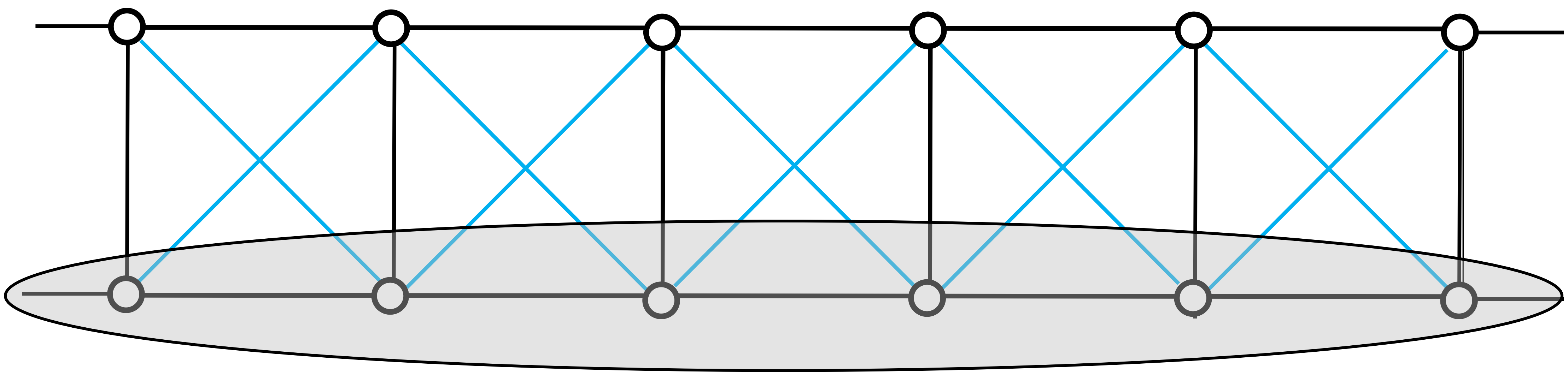
$$H_{\text{free}} = \sum_{I=1}^{4} \sum_{i,j} (t_{I,ij} \psi_{I,i}^{\dagger} \psi_{I,j} + \text{h.c.})$$

Zheng, Zou, Wang, You '22

0345 Model on the Lattice

Turn on interactions only on one edge

$$H_{\text{int}} = g \sum_{\text{one edge}} \psi_{1,i}\psi^{\dagger}_{2,i}\psi^{\dagger}_{2,i+1}\tilde{\psi}_{1.i}\tilde{\psi}_{2,i}\tilde{\psi}_{2,i+1}$$

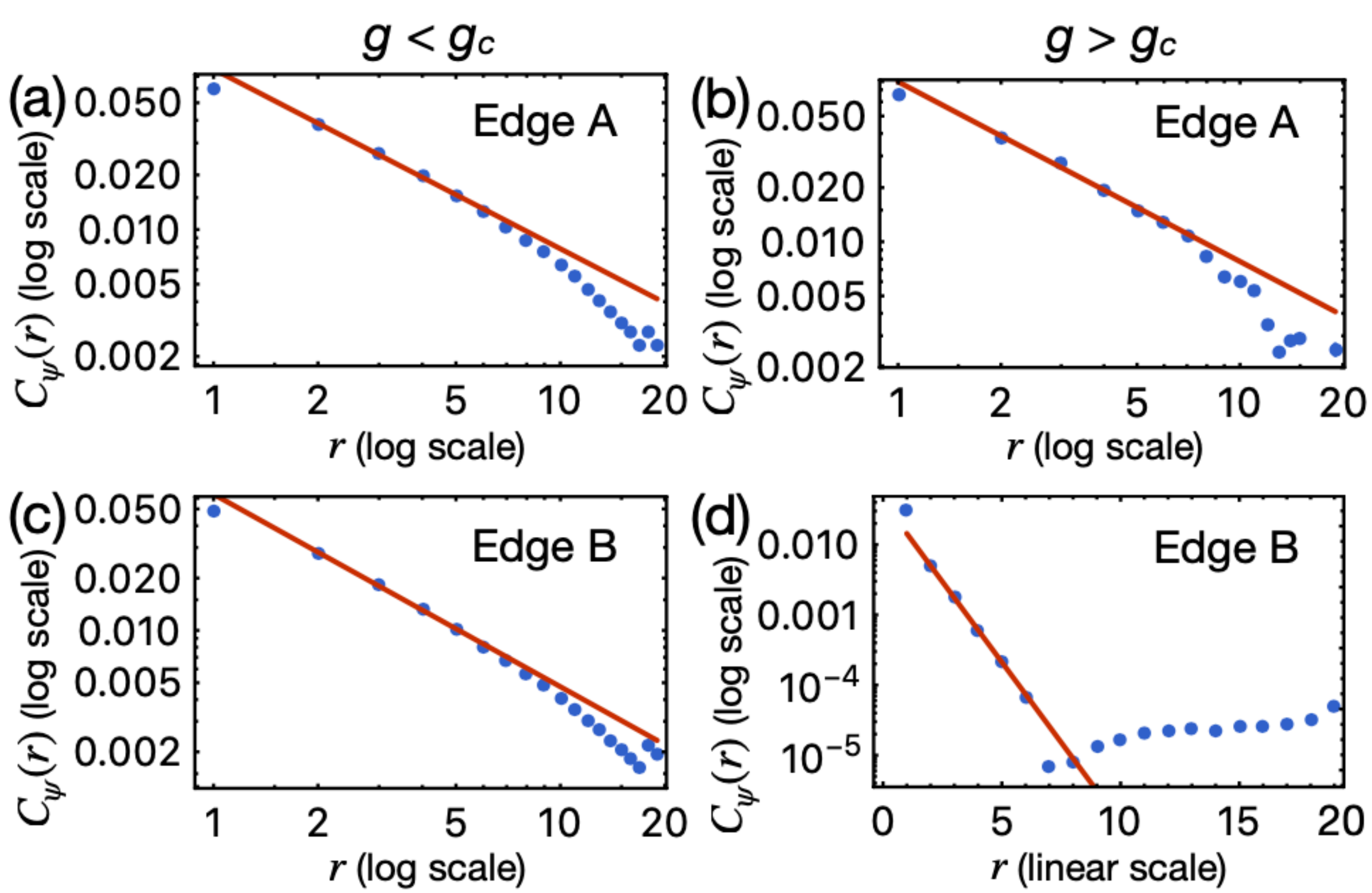


Simulate using tensor networks (DMRG) to avoid sign problem

Zheng, Zou, Wang, You ‘22

0345 Model on the Lattice: Results

Correlation function $C(r) = \langle \psi^\dagger_{I,i+r} \psi_{I,i} \rangle$

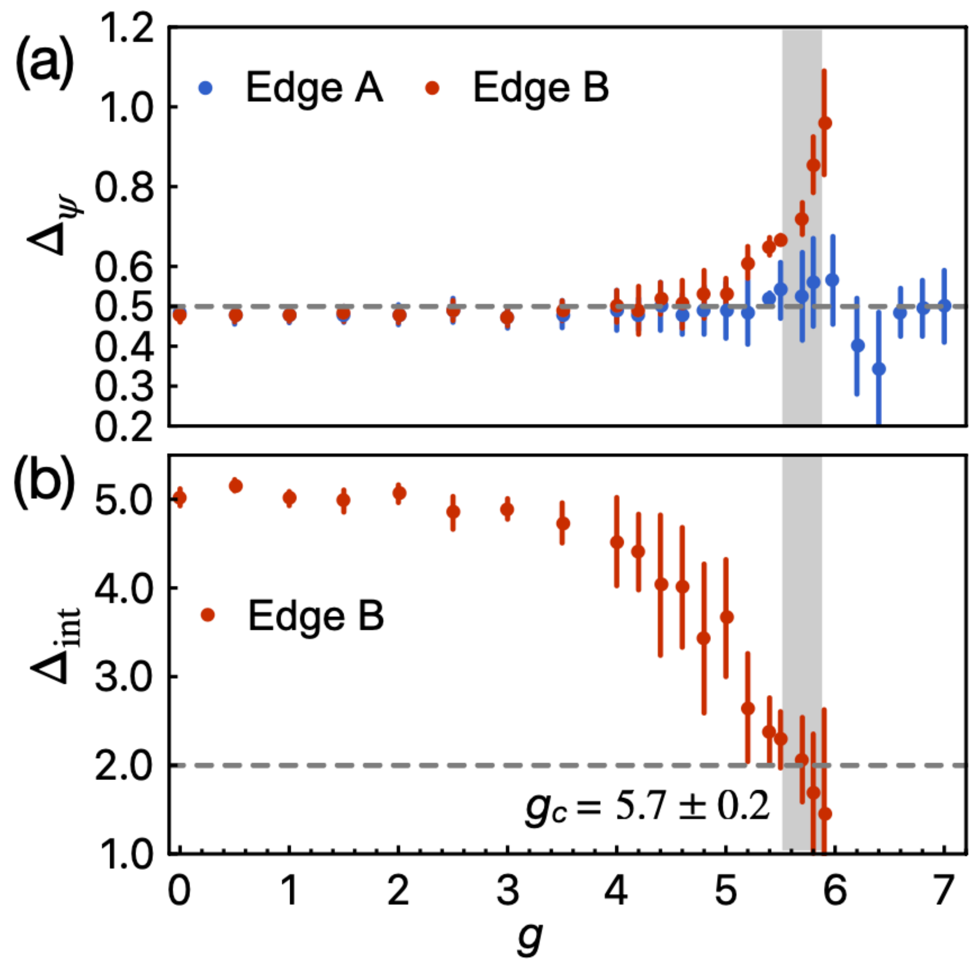


Zheng, Zou, Wang, You ‘22

Thank you for your attention

0345 Model on the Lattice: Results

Dimensions of operators



Zheng, Zou, Wang, You ‘22

An Example: The Standard Model

$$G = SU(3)\times SU(2)\times U(1)$$

Razamat and Tong '20

(left-handed)^c		right-handed		
leptons	quarks	electron	up quark	down quark
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

(left-handed)^c		right-handed			
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{+3}$				$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- Add three further pairs of fermions

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

(left-handed)^c		right-handed			
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{+3}$				$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- Add three further pairs of fermions
- Gauge the *H* = SU(2) symmetry

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

(left-handed)^c		right-handed			
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{+3}$				$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- Add three further pairs of fermions
- Gauge the *H* = SU(2) symmetry
- Supersymmetrize.
  - Add scalar superpartners for all fermions, and a *H* = SU(2) gaugino

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

	L	Q	E	U	D	N
	leptons	quarks	electron	up quark	down quark	neutrino
	$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_0$
	$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_0$
$L' \longrightarrow$	$(\mathbf{1},\mathbf{2})_{+3}$			$D' \longrightarrow$	$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- The H = SU(2) gauge theory is coupled to six doublets.
- This confines *without* breaking the global symmetry. Seiberg '94
- The low-energy physics consists of 15 free mesons:

$$\epsilon_{ab}L^aL^b \qquad \epsilon_{ijk}D^iD^j \qquad L^aD^i \qquad L^aN \qquad D^iN$$

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

L	Q	E	U	D	N
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$L' \longrightarrow (\mathbf{1},\mathbf{2})_{+3}$				$D' \longrightarrow (\bar{\mathbf{3}},\mathbf{1})_{-2}$	

If we add the superpotential

$$\mathcal{W}_{UV} = \epsilon_{ab} L^a L^b E + \epsilon_{ijk} D^i D^j U^k + \epsilon_{ab} L^a D^i Q^b_i + \epsilon_{ab} L^a N L'^b + D^i N D'_i$$

But, in the infra-red, this becomes

$$\mathcal{W}_{IR} = \widetilde{E} E + \widetilde{U}_k U^k + \widetilde{Q}^i_b Q^b_i + \widetilde{L}^b L'^b + \widetilde{D}_i D'_i$$

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

L	Q	E	U	D	N
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1}$					$\mathbf{1})_0$
$(\mathbf{1}$					$\mathbf{1})_0$

The theory is gapped while preserving G = SU(3) x SU(2) x U(1)!

$L' \longrightarrow (\mathbf{1}, \mathbf{2})_{+3}$ $D' \longrightarrow (\bar{\mathbf{3}}, \mathbf{1})_{-2}$

If we add the superpotential

$$\mathcal{W}_{UV} = \epsilon_{ab} L^a L^b E + \epsilon_{ijk} D^i D^j U^k + \epsilon_{ab} L^a D^i Q_i^b + \epsilon_{ab} L^a N L'^b + D^i N D'_i$$

But, in the infra-red, this becomes

$$\mathcal{W}_{IR} = \widetilde{E} E + \widetilde{U}_k U^k + \widetilde{Q}_b^i Q_i^b + \widetilde{L}^b L'^b + \widetilde{D}_i D'_i$$