A Confederacy of Anomalies

A personal recollection of early years in lattice gauge theory [1]

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[1] *not a balanced review*! free after
John Kennedy Toole, *A Confederacy of Dunces*, with one dunce

Prologue

1969: September, PhD student of S.A. Wouthuysen in Amsterdam

1970: started working on my own, e.g. on:

Glashow, Weinberg, Salam, Veltman, 'Higgs', Wilson, massive Yang-Mills, partons, renormalization group, chiral anomaly,. . .

1971:
my hero, Wilson, published article [2] which contained a non-perturbative approximation to a path integral (in the continuum)
- no longer a magic trick for deriving Feynman diagrams

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[2] K.G. Wilson, *Renormalization group and critical phenomena. 2. Phase space cell analysis of critical behavior, Phys.Rev.B4(1971)3184*

Prologue 1972

chiral anomalies should cancel [3] [4]

judged this artificial – wanted non-perturbative UV-regulator

Spring: scalar field on a lattice

August: moved to LA, student of Robert J. Finkelstein at UCLA

topic: massive YM in Schwinger's Source Theory

Christmas: non-Abelian gauge field on a lattice, 'Wilson action'

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[3] C. Bouchiat, J. Iliopoulos, Ph. Meyer, *An anomaly-free version of Weinberg's model, PLB38(1972)519*

[4] D.J. Gross, R. Jackiw, *Effect of Anomalies on Quasi-Renormalizable Theories,PRD6(1972)477*

1973

testing perturbative continuum limit at one loop on

$U(1)_V \times U(1)_A$ gauge-Higgs model coupled to one Dirac field:

$$\mathcal{L}_{\mathrm{F}} = \bar{\psi}\gamma^\mu \partial_\mu \psi + \bar{\psi}\,[g\,\gamma^\mu\,V_\mu + g_5\,i\gamma^\mu\gamma_5\,A_\mu + G(\,\sigma + i\gamma_5\,\pi)]\,\psi$$

'anomalous chiral gauge theory'

spatial cubic lattice, continuous time (just QM)

develop lattice methods

vector and axial-vector Ward-Takahashi identities exactly valid at finite lattice spacing

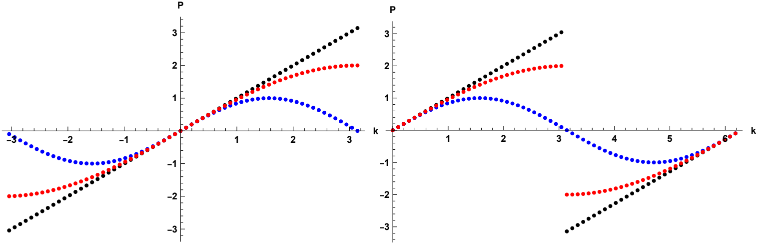
velocity-of-light counterterm etc.

vector selfenergy ('vacuum polarization'):
correct *Lorentz-invariant form* but factor 8 too large

VVA and $VV\pi$ triangle diagrams: *zero*

1973 (cont.)

propagators and vertices periodic in spatial momentum space



fermion propagator

$$\left[i\gamma^0 k_0 + i\gamma^j P_j(k) + m\right]^{-1}, \quad P_j(k) = \frac{1}{a}\sin(ak_j)$$

$8 = 2^3$ zeros of $P_j(k)$ at $ak_j = \{0, \pi\}$ contribute to continuum limit of vector selfenergy

Tried to cure by: $P_j(k) = \frac{2}{a}\sin\left(\frac{ak_j}{2}\right)$

discontinuous $\Rightarrow$ non-local

non-local, non-Lorentz covariant, UV divergent continuum limit

1973 (cont.)

– multiplicity *not*-ignorable
– doublers real particles?

decided to leave it 'for a while'

November: article with Robert [5]

inspired by [6] investigated possibility of massive solution of massless YM, using Schwinger-Dyson equations [7]

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[5] Robert J. Finkelstein and JS, *Massive Gauge Field in Source Theory II, Ann.Phys.88(1974)157*

[6] J.M. Cornwall, R.E. Norton, *Spontaneous Symmetry Breaking without Scalar Mesons, Phys.Rev.D8(1973)3338*

[7] JS, *Possibility that massless Yang-Mills fields generate massive vector particles, Phys.Rev.D10(1974)2473*

1974

Spring/Summer: seminar at UCLA by Wilson [8] *bomb shell*

lattice, confinement, strong coupling!

after seminar I did *not* tel him about my own lattice work, but mentioned problem with fermions. Wilson's answer:

add term $\propto \partial_\mu \bar{\psi} \partial_\mu \psi$

chiral symmetry should come back in continuum limit

November: PhD, Thesis:

Massive Vector Particles with Yang-Mills Couplings

- Source Theory + the Schwinger-Dyson eqns. article

(lattice regularization only briefly mentioned)

[8] K.G. Wilson, Confinement of Quarks, Phys.Rev.D10(1974)2445

1975-1976

January: back to Amsterdam

- depressed – could not write article about my lattice results

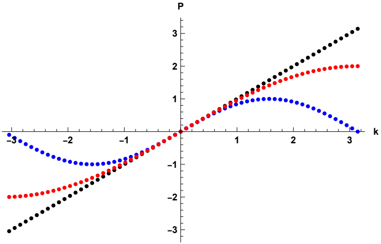
turned to Regge pole theory
description of experimental scattering results at CERN

together with new PhD student Luuk Karsten

1977

September: SLAC Summer Institute:
lecture by Sidney Drell on Lattice Field Theory

Avoid fermion doubling: [9] $P_j(k) = k_j\,, \quad ak_j \in \{-\pi, \pi\}$



I suggested problems (expected from $P_j(k) = \frac{2}{a}\sin\left(\frac{ak_j}{2}\right)$):

non-local divergences in continuum limit of gauge theory

Drell: *"You ought to write this up!"*

⇒ <u>returned to LGT, with Luuk</u> Karsten

[9] S.D. Drell, M. Weinstein, S. Yankielowicz, Strong Coupling Field Theories. 2. Fermions and Gauge Fields on a Lattice, Phys.Rev.D14(1976)1627

1978

catch up with exponentially rising number of publications
in particular [10] [11] [12] [13] [14] [15] (overlooked [16] [17])

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[10] K.G. Wilson, 13th International School of Subnuclear Physics: *New Phenomena in Subnuclear Physics*, CLNS-321 (1975)

[11] J.B. Kogut and L. Susskind, *Hamiltonian Formulation of Wilson's Lattice Gauge Theories*, Phys.Rev.D11(1975)395

[12] L. Susskind, *Lattice fermions*, Phys.Rev.D16(1977)3031

[13] B.E. Baaquie, *Gauge Fixing and Mass Renormalization in the Lattice Gauge Theory*, Phys.Rev.D16(1977)2612

[14] H.S. Sharatchandra, *The Continuum Limit of Lattice Gauge Theories in the Context of Renormalized Perturbation Theory*, Phys.Rev.D18(1978)2042

[15] J. Shigemitsu, *Spectrum calculations in lattice gauge theory using Wilson's fermion method*, Phys.Rev.D18(1978)1709

[16] M. Lüscher, Construction of a Selfadjoint, Strictly Positive Transfer Matrix for Euclidean Lattice Gauge Theories, Commun.Math.Phys.54(1977)283

[17] M. Creutz, *Gauge Fixing, the Transfer Matrix, and Confinement on a Lattice*, Phys.Rev.D15(1977)1128

1978 (cont.)

calculated continuum limit of ‘anomaly (triangle) diagrams’ with Drell-Weinstein-Yankielowicz (DWY) fermion method & find indeed non-local, non-covariant, divergent contributions [18]

... also in gauge-field selfenergy diagrams [19]

- ‘doubler fermions’ replaced by non-local & non-covariant contributions

[18] L.H. Karsten, JS, *Axial Symmetry in Lattice Theories*, Nucl.Phys.B144(1978)536

[19] L.H. Karsten, JS, *The Vacuum Polarization with SLAC Lattice Fermions*, Phys.Lett.B85(1979)100

1979

September: Cargèse Summer Institute

Wilson describes numerical RG computations [20]

& mentions MC computations by Creutz [21] (eye opener for me)

and strong coupling calculations, with Padé extrapolation to weak coupling, by Kogut, Pearson and Shigemitsu [22]

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[20] K.G. Wilson, *Monte Carlo Calculations for the Lattice Gauge Theory*, NATO Sci.Ser.B 59 (1980) 363-402

[21] M. Creutz, *Solving Quantized SU(2) Gauge Theory*, Brookhaven, September 1979; *Asymptotic Freedom Scales*, Phys.Rev.Lett.45(1980)313

[22] J.B. Kogut, R.B. Pearson, J. Shigemitsu, The QCD beta Function at Intermediate and Strong Coupling, Phys.Rev.Lett. 43(1979)484

1979 (cont.)

November: PhD Luuk Karsten. Thesis:

On Lattice Gauge Theories and On Backward Pion-Nucleon Scattering

- doubler fermions at $ak_\mu = \pi$ are genuine particles

- chiral charges: $g_5 \to g_5 Q_5$, $Q_5 = (-1)^n$,

$n =$ number of π s in four-vector ak giving $\sin(ak_\mu) = 0$

$$\sum Q_5 = 0$$

- anomalies cancel in triangle diagrams !

1979 (cont.)

in the $U(1)_V \times U(1)_A$ model a Wilson-type mass term made gauge invariant with the Higgs field $\sigma + i\pi$ led to the tree-graph level formula

$$m_{\mathrm{F}} = m + \frac{r}{a} \sum_{\mu} [1 - \cos(a k_\mu)]$$

with $m = G\langle\sigma\rangle$ and $r/a = G_{\mathrm{W}}\langle\sigma\rangle$ (Wilson's $r = 1$)

- in a continuum limit $a \to 0$ with $r = a\,\tilde{r}$ this would result in a mass spectrum:

$$m_{\mathrm{F}} = m + 2n\,\tilde{r}\,, \quad n = 1, 2, 3, 4$$

subsequently the doublers ($n \geq 1$) seem then be removable by sending $\tilde{r} \to \infty$, *except* for triangle diagrams where they become the chiral anomaly in the axial-vector WT identity (thesis and [23])

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[23] L.H. Karsten, *The Lattice Fermion Problem and Weak Coupling Perturbation Theory*, in Field Theoretical Methods in Elementary Physics, NATO Sci.Ser.B55(1980)235

1980

However, $\tilde{r} \to \infty$ would imply $G_{\mathrm{W}} \to \infty$ and one would be led to strong coupling dynamics . . .

Luuk moved to Stanford

we still had to write-up publish the results extend the calculations applying to QCD with Wilson fermions at fixed r

1980 (cont.)

February: Banks and Caashers stimulating paper on chiral symmetry breaking [24]

May: strong coupling paper [25] (≈ simultaneously with [26])
Hamiltonian formulation, N_f flavors, N colors, two orders in $1/N$

$$\begin{aligned} S_\mathrm{mass} &= \bar\psi M\psi - \bar\psi W\psi \\ \bar\psi W\psi &= \frac{ar}{2}\sum_{x,j}\left[\bar\psi(x)U_j(x)\psi(x+a_j) + \bar\psi(x+a_j)U_j^\dagger(x)\psi(x)\right] \end{aligned}$$

Greensite and me discovered that our write-ups nearly had same title)
- anti-ferromagnetic ground state
Nambu-Goldstone bosons (including vector mesons)

$$m^2 \propto M\langle\bar\psi\psi\rangle\,, \quad \langle\bar\psi\psi\rangle \propto N$$

[24] T. Banks, A. Casher, *Chiral Symmetry Breaking in Confining Theories*, NPB169(1980)103

[25] JS, Chiral Symmetry Breaking in QCD: Mesons as Spin Waves, NPB175(1980)307

[26] Jeff Greensite and Joel Primack, Pions as Spin Waves, NPB180(1981)170

1980 (cont.)

increasing r: increase $M > M_{\rm cr}$, critical mass $M_{\rm cr}(r,g) \propto r^2$,

(avoid broken parity phase when $M < M_{\rm cr}$)

for $0.7 < r \leq 1$ vector mesons (ρ, K^*) have lost their NGB character, but *not* the charged pseudo scalars ($\pi^\pm$, K)

'current quark mass' of flavor a: $m_a = M_a - M_{\rm cr}$

$$m_{ab}^2 \propto (m_a + m_b)\,, \quad a \neq b$$

neutral: $m_{aa}^2 > m_{ab}^2$ in next order $1/N$
quantitatively: effective strength of anomaly ('$U(1)$ problem')

'dynamical quark mass' of oldfashioned quark model $m_{\rm dyn}$

$$m_V \simeq 2\, m_{\rm dyn} + m_a + m_b$$

1980 cont.

for $M = M_{\rm cr}$, chiral *non-singlet* charge Q^5_{ab} commutes with effective Hamiltonian and creates zero energy GB state $Q^5_{ab}|0\rangle$ out of vacuum

chiral *singlet* charge Q^5_{aa} is anomalous for $r > 0$

currents were derived from a lattice version of the Standard Model that generated the r-parameter in terms of a 'Wilson-Yukawa coupling' $G_{\rm W}$ and the vev of the Higgs field Φ: $\tilde{r} = G_{\rm W}\,\langle\Phi\rangle$

decoupling doublers would require $G_{\rm w} \to \infty$??

matrix elements of currents (meson decay constants f_π, γ_ρ, etc) did not come out very well (neither very badly)

1980 (cont.)

anomaly paper with Luuk [27] (errata in [28])

- $\bar{\psi}(M_{\rm cr} - W)\psi \to$ flavor-$U(1)$ anomaly

first of No-Go proofs by Nielsen and Ninomiya [29] [30] [31]

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[27] Luuk H. Karsten, JS, *Lattice fermions: species doubling, chiral invariance and the triangle anomaly*, NPB183(1981)103

[28] R. Groot, J. Hoek, JS, *Normalization of Currents in Lattice QCD*, NPB237(1984)111

[29] H.B. Nielsen, M. Ninomiya, *Absence of Neutrinos on a Lattice. 1. Proof by Homotopy Theory*, NPB185(1981)20; NPB195(1982)541 (erratum)

[30] *Absence of Neutrinos on a Lattice. 2. Intuitive Topological Proof*, NPB193(1981)173

[31] *A No-Go Theorem for regularizing chiral fermions*, PLB105(1981)219

1981

No-Go proof by Karsten [32] using the Poincaré-Hopf theorem which relates the index of a vector field on a manifold (in casu $P_\mu(k)$) to the Euler number of the manifold (momentum space T^4)

- weak coupling calculations with *staggered fermions* by Sharatchandra, Thün and Weisz, including chiral anomaly and ratio of Λ-scales [33]

- papers by Lüscher, Münster and Weisz concerning the roughening transition (causing a $-\pi/(12r)$ term in the potential), e.g. [34] [35] [36]

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[32] L.H. Karsten, *Lattice fermions in Euclidean space-time*, PL104B(1981)315

[33] H.S. Sharatchandra, H.J. Thun, P. Weisz, *Susskind Fermions on a Euclidean Lattice*. NPB192(1981)205

[34] G. Münster, P. Weisz, *On the Roughening Transition in Nonabelian Lattice Gauge Theories*, NPB180(1981)330

[35] M. Lüscher, G. Münster, P. Weisz, *How Thick Are Chromoelectric Flux Tubes ?*, NPB180(1981)1

[36] M. Lüscher, *Symmetry Breaking Aspects of the Roughening Transition in Gauge Theories*, NPB180(1981)317

1981 (cont.)

with Kawamoto, Euclidean effective action at strong coupling (staggered- and Wilson-fermions): [37] [38]

meson and baryon fields, large N approximation

- for $r = 0$ transformation to staggered-fermion form exhibits

$U(4N_\mathrm{f}) \times U(4N_\mathrm{f})$ symmetry, broken spontaneously to $U(4N_\mathrm{f})$

with NG bosons and massive baryons

- for $r = 1$ effective action has chiral symmetry at $M = M_\mathrm{cr}$ up to 2nd order in the NG fields, but at 4th order the symmetry is broken, as shown in the π-π scattering amplitude

similar results (naive fermions): [39]

[37] N. Kawamoto, JS, *Effective Lagrangian and Dynamical Symmetry Breaking in Strongly Coupled Lattice QCD*, NPB192(1981)100

[38] J. Hoek, N. Kawamoto, JS, *Baryons in the Effective Lagrangian of Strongly Coupled Lattice QCD*, NPB199(1982)495

[39] H. Kluberg-Stern, A. Morel, O. Napoly, B. Petersson, *Spontaneous Chiral Symmetry Breaking for a U(N) Gauge Theory on a Lattice*, NPB190(1981)504

1983

spin, flavor and symmetry group of ('reduced') staggered fermions with method for loop calculation [40]

related work [41] [42] [43]

PhD Jaap Hoek [44] [45] [46]

---

[40] Cees van den Doel, JS, *Dynamical Symmetry Breaking in Two Flavor $SU(N)$ and $SO(N)$ Lattice Gauge Theories*, NPB228(1983) 122

[41] F. Gliozzi, *Spinor Algebra of the One Component Lattice Fermions*, NPB204(1982)419

[42] H. Kluberg-Stern, A. Morel, O. Napoly and B. Petersson, *Flavors of Lagrangian Susskind Fermions*, NPB220 (1983)447

[43] P. Becher & H. Joos, *Geometric fermions*

[44] Thesis: *Effective Action Calculation in Lattice QCD*, Amsterdam, 1983

[45] *Strong Coupling Expansion of the Generating Functional for Gauge Systems on a Lattice with Arbitrary Sources*, J.Comp.Phys. 49(1983)265

[46] *Strong Coupling Expansion of the SU(3) and $U(3)$ Effective Actions*, J.Comp.Phys. 54(1984)245

1983 (cont.)

September: Cargèse Summer Institute

Wilson lectured [47]

– I mentioned to him my work on LGT at UCLA in 1972-1973

– Wilson's generous answer: ask your promotor to send me a description of what you did and I shall mention it in future reviews

and so it was done

[47] K.G. Wilson, *Monte Carlo Renormalization Group and the Three-Dimensional Ising Model*, Progress in Gauge Field Theory, NATO Sci.Ser.B 115 (1984) 589-604

1984

Swift in [48] used essentially the same method as I did (cf. [49]) for deriving currents for the 'spin wave paper'

a $U(1)$ version was analyzed a few years later by Hands and Carpenter [50]

[48] P.V.D. Swift, *The Electroweak Theory on the Lattice*, PLB145(1984)256
[49] JS, *Fermions on a lattice*, Act.Phys.Pol.B17(1986)531
[50] S.J. Hands, D.B. Carpenter, *Lattice Sigma Model and Fermion Doubling*, NPB266(1986)285

$\gtrsim$ 1984

lots of inspiring papers were being published, on Monte-Carlo-RG scaling, string tension and potential, hadron spectrum, critical temperature

- desire for similar computational work in Amsterdam and the arrival of the Cyber 205 supercomputer led to taking part in larger collaborations [51] [52]

Peter Hasenfratz sent me gauge-field configurations:

"they are beautiful !"

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[51] A. König, K.H. Mütter, K. Schilling, JS, *Large distance propagators for hadrons on a* 56×16^3 *lattice, PLB157(1985)421*

[52] K.C. Bowler, F. Gutbrod, P. Hasenfratz, U. Heller, F. Karsch, R.D. Kenway, I. Montvay, G.S. Pawley, JS, D.J. Wallace, *The* β*-function and potential at* $\beta = 6.0$ *and* 6.3

1984 - 1986

more on staggered fermions: [53] [54] [55] [56] [57]

related work: [58] [59]

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[53] Maarten F.L. Golterman, JS, *Relation Between QCD Parameters on the Lattice and in the Continuum*, PLB140(1984)392

[54] M.F.L. Golterman, JS, *Self-Energy and Flavor Interpretation of Staggered Fermions*, NPB245(1984)61

[55] M.F.L. Golterman, JS, *Lattice Baryons With Staggered Fermions*, NPB255(1985)328

[56] M.F.L. Golterman, *Staggered Mesons*, NPB273(1986)663

[57] M.F.L. Golterman, *Irreducible Representations of the Staggered Fermion Symmetry Group*, NPB278(1986)417

[58] G.W. Kilcup, S.R. Sharpe, *A Tool Kit for Staggered Fermions*, NPB283(1987)493

[59] A. Coste, C. Korthals Altes, O. Napoly, *Calculation of the Nonabelian Chiral Anomaly on the Lattice*, NPB289(1987)645

$\gtrsim$ 1986 $U(1)$ problem and topological charge

can instantons explain the large mass of the η' meson?

anomalous divergence of the axial flavor-singlet current

$$\partial_\mu \left(\bar\psi i\gamma_\mu\gamma_5\psi\right) = 2m\,\bar\psi i\gamma_5\psi + 2iN_\mathrm{f}\,q \qquad U(1)\text{ current}$$

$$q = \frac{1}{32\pi^2}\,\epsilon_{\kappa\lambda\mu\nu}\,\mathrm{Tr}\,G_{\kappa\lambda}G_{\mu\nu} \qquad \text{topological charge density}$$

derived lattice version [60] of the Witten-Veneziano relation

$$m^2_{\eta'} - \frac{1}{2}\,m^2_\eta - \frac{1}{2}\,m^2_{\pi^0} = \frac{6}{f^2_\pi}\,\chi \qquad N_\mathrm{f} = 3$$

$$\chi = \int d^4x\,\langle q(x)\,q(0)\rangle|_\mathrm{quenched}$$

$$\simeq (180\,\mathrm{MeV})^4 \qquad \text{topological susceptibility}$$

[60] J.C. Vink, JS, *Neutral Pseudoscalar masses in Lattice QCD*, NPB284(1987)234; *Remnants of the Index Theorem on the Lattice*, NPB286(1987)485

$U(1)$ problem and topological charge (cont.)

which led to a 'fermionic determination' of topological charge [61] [62]

$$\overline{Q} = \mathrm{Tr}\, \kappa_P\, m\, \gamma_5\, (\not{D} + m + M_{\rm cr} - W)^{-1} \qquad \text{Wilson fermions}$$

$$= \frac{1}{4}\, \mathrm{Tr}\, \kappa_P\, m\, \Gamma_5\, (\not{D} + m)^{-1} \qquad \text{staggered fermions}$$

[61] J.C. Vink, *Staggered Fermions, Topological Charge and Topological Susceptibility in Lattice QCD*, PLB212(1988)483; and references there-in

[62] M.L. Laursen, JS, J.C. Vink, *Small-Scale Instantons, Staggered Fermions and the Topological Susceptibility*, NPB343(1990)522; and references there-in

$U(1)$ problem and topological charge (cont.)

other definitions: geometrical [63] [64] 'cooling' e.g. [65] [66] [67]

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[63] M. Lüscher, *Topology of Lattice Gauge Fields*, CMP85(1982)39

[64] A. Philips, D. Stone, *Lattice Gauge Fields, Principal Bundles and the Calculation of Topological Charge*, CMP103(1986)399

[65] Y. Iwasaki, T. Yosie,*Instantons and Topological Charge in Lattice Gauge Theory*, PLB131(1983)159

[66] M. Teper, *Instantons in the Quantized SU(2) Vacuum: A Lattice Monte Carlo Investigation*, PLB162(1985)357; *The Topological Susceptibility in SU(2) Lattice Gauge Theory: An Exploratory Study*, PLB171(1986)81

[67] E-M. Ilgenfritz, M.L. Laursen, G. Schierholz, M. Muller-Preussker, H. Schiller, *First Evidence for the Existence of Instantons in the Quantized SU(2) Lattice Vacuum*, NPB268(1986)693

$\gtrsim$ 1986 Standard Model: Wilson-Yukawa

in a model with only quarks or leptons with (possibly!) decoupled doublers, anomalies may be carried by the Higgs field [68]

decoupling of doublers was studied by focusing on Higgs-fermion interaction without gauge fields

[68] E. D'Hoker and E. Fahri, *Decoupling a fermion in the standard electro-weak theory*, NPB248(1984)77

$\gtrsim$ 1986 Standard Model: Wilson-Yukawa (cont.)

$SU(2)$ invariant combinations $\Phi^\dagger P_{\rm L}\Psi$ and $\bar\Psi\Phi P_{\rm R}$ put in the Wilson-Yukawa 'mass term'

$$
\begin{aligned}
S_{\rm Fmass} &= (y+4w)\bar\Psi_x(\Phi_x P_{\rm R} + \Phi_x^\dagger P_{\rm L})\Psi_x \\
&\quad -\frac{w}{2}\sum_\mu \left[\bar\Psi_x\Phi_x P_{\rm R}\Psi_{x+\hat\mu} + \bar\Psi_x\Phi^\dagger_{x+\hat\mu}P_{\rm L}\Psi_{x+\hat\mu} + \text{h.c.}\right]
\end{aligned}
$$

- continuum limit at the weak coupling FM(W)-PMW phase boundary: as expected, doublers are heavier than target fermions but not really removable because of triviality [69]

[69] W. Bock, A.K. De, C. Frick, K. Jansen, T. Trappenburg, *Search for an upperbound of the renormalized Yukawa coupling in a lattice fermion-Higgs model*, NPB371(1992)683

$\gtrsim$ 1986 Standard Model: Wilson-Yukawa (cont.)

- at the disjoint strong coupling FM(S)-PMS phase boundary: lots of papers by groups of aficionados (& a nice Amsterdam-Jülich collaboration) with the conclusion:

- removing doublers possible *but on the way out they become fermion-Higgs bound-state* $SU(2)$ *singlets* ! [70] [71]

- only at the FM(W)-PMW phase boundary it is possible to *approximate* the physics of the Standard Model (doublers still present)

mirror fermion models appear to fare better in this respect [72]

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[70] M.F.L. Golterman, D. Petcher, JS, *Fermion-interactions in models with Strong Wilson-Yukawa Couplings*, NPB370(1992)51 and references therein

[71] W. Bock, A.K. De, JS, *Fermion masses at strong Wilson-Yukawa Couplings in the Symmetric Phase*, NPB388(1992)243 and references therein

[72] C. Frick, L. Lin, I. Montvay, G. Münster, M. Plagge, T. Trappenberg, H. Wittig, *Numerical simulation of heavy fermions in an* $SU(2)_{\mathrm{L}} \otimes SU(2)_{\mathrm{R}}$ *symmetric Yukawa model*, NPB397(1993)431

$\gtrsim$ 1992 Standard Model: staggered fermion

. . . it did not work out well

$\gtrsim$ 1992 changing the game

D.B. Kaplan: domain walls (Y. Shamir use for QCD)

R. Narayanan & H. Neuberger: overlap

Ginsparg-Wilson: Lüscher

meanwhile

we enjoyed work, e.g. by

... too many names, gave up