Selected topics on the QCD phase diagram at finite temperature and density

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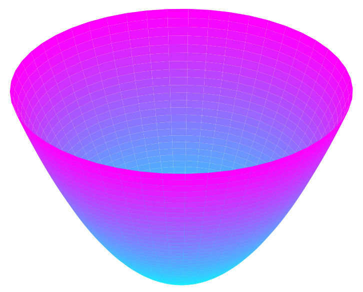
CRC-TR 211

PUNCH 4 NFDI

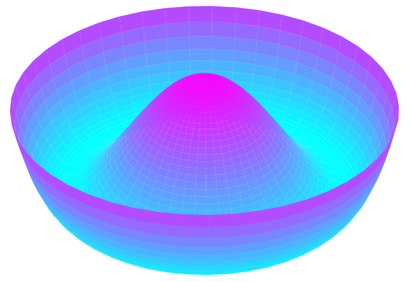
Chiral symmetry

$$\mathcal{L}_F = \bar{\psi}_L \not{D}(U)\psi_L + \bar{\psi}_R \not{D}(U)\psi_R$$

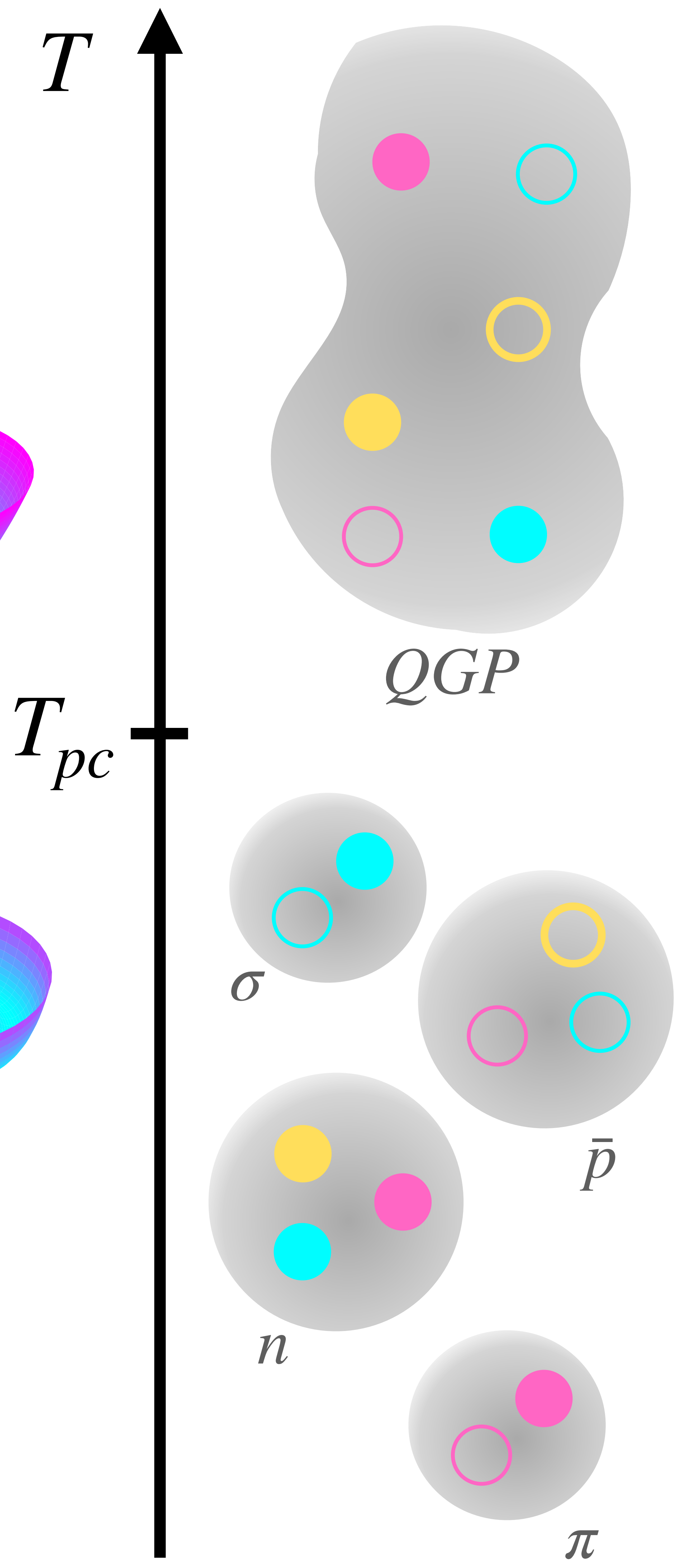
$$\mathrm{U}(1)_V \times \cancel{\mathrm{U}(1)}_A \times \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$$
$$\rightarrow \mathrm{U}(1)_V \times \cancel{\mathrm{U}(1)}_A \times \mathrm{SU}(N_f)_V \times \cancel{\mathrm{SU}(N_f)}_A$$



- Chiral symmetry is explicitly broken by the quark mass
- Chiral symmetry is spontaneously broken at low temperatures



- Assume O(4) symmetric effective theory [O(2) for staggered fermions]
  - ➡ **For $\mathrm{U}(1)_A$ restoration see plenary talk by T. Kovacs on Saturday and talk by M. Giordano on Monday**
- Make use of universal equation of state scaling functions to investigate the phase diagram



Confinement

- Only color-neutral states at low temperatures
- Mechanism of confinement still not well understood mathematically
- Ones hadrons percolate, quarks can move over macroscopic distances
- Free quarks only at asymptotic large temperatures or densities
- Typically investigated by studying the static quark potential
- Nontrivial observation: $T_{pc}^{\text{chiral}} = T_{pc}^{\text{deconfinemen}}$ (at $\mu = 0$, $m = m_{\text{phys}}$)
  - ➡ **Talk by R. Kehr: interplay between chiral and deconfinement transition based on Anderson transition and localisation of Dirac eigenmodes**

Chiral symmetry

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$$\to \mathrm{U}(1)_V \times \cancel{\mathrm{U}(1)}_A \times \mathrm{SU}(N_f)_V \times \cancel{\mathrm{SU}(N_f)}_A$$

Scaling fields

- Introducing the reduced temperature

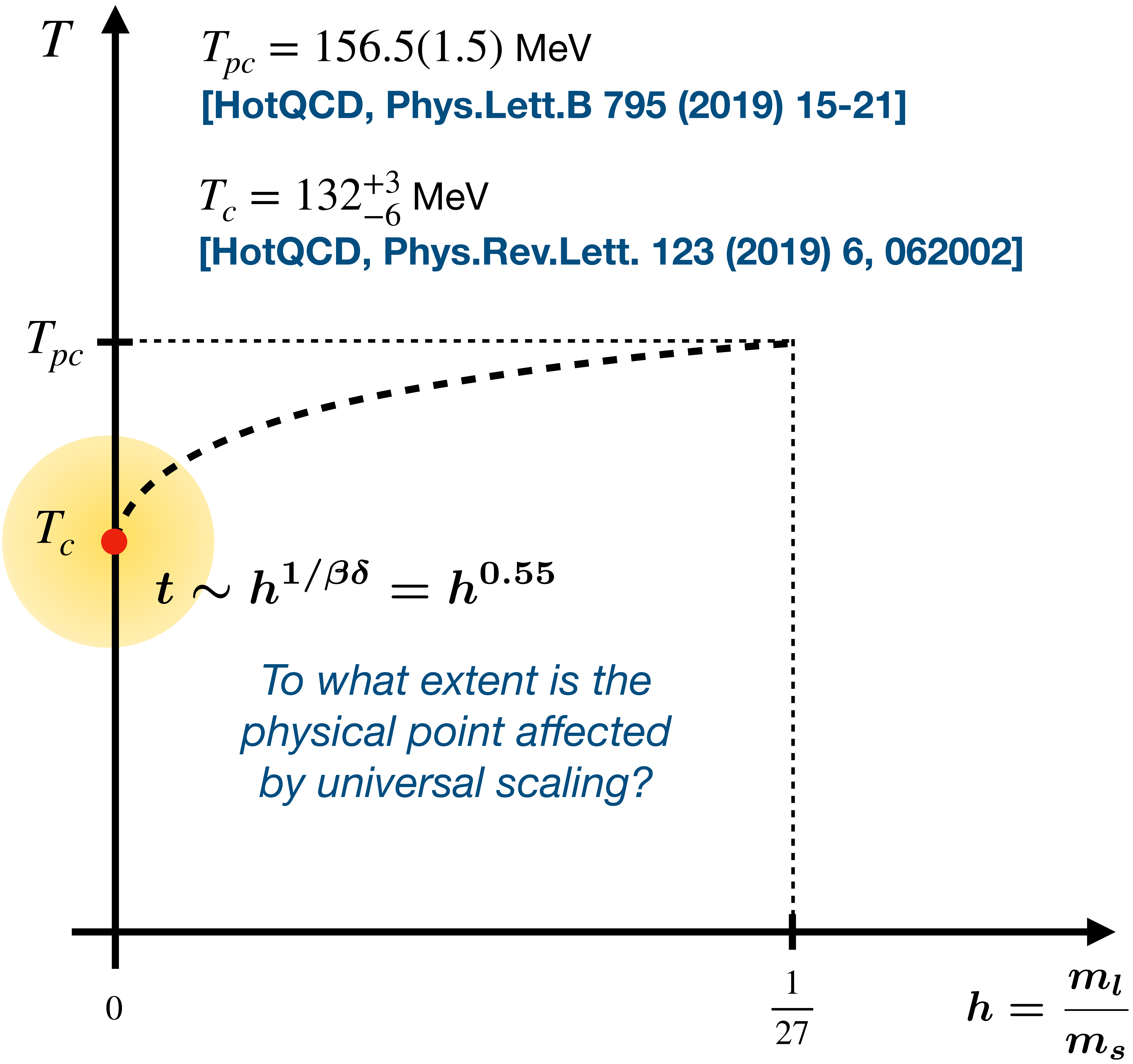
$$t = \frac{T - T_c}{T_c}$$

- Quark mass plays the role of the external symmetry-breaking field

$$h = \frac{m_l}{m_s}$$

$T_{pc} = 156.5(1.5)$ MeV
[HotQCD, Phys.Lett.B 795 (2019) 15-21]

$T_c = 132^{+3}_{-6}$ MeV
[HotQCD, Phys.Rev.Lett. 123 (2019) 6, 062002]



To what extent is the physical point affected by universal scaling?

Chiral symmetry

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Scaling fields

- Introducing the reduced temperature

$$t = \frac{T - T_c}{T_c} + \kappa_2^B \left(\frac{\mu_B}{T}\right)^2$$

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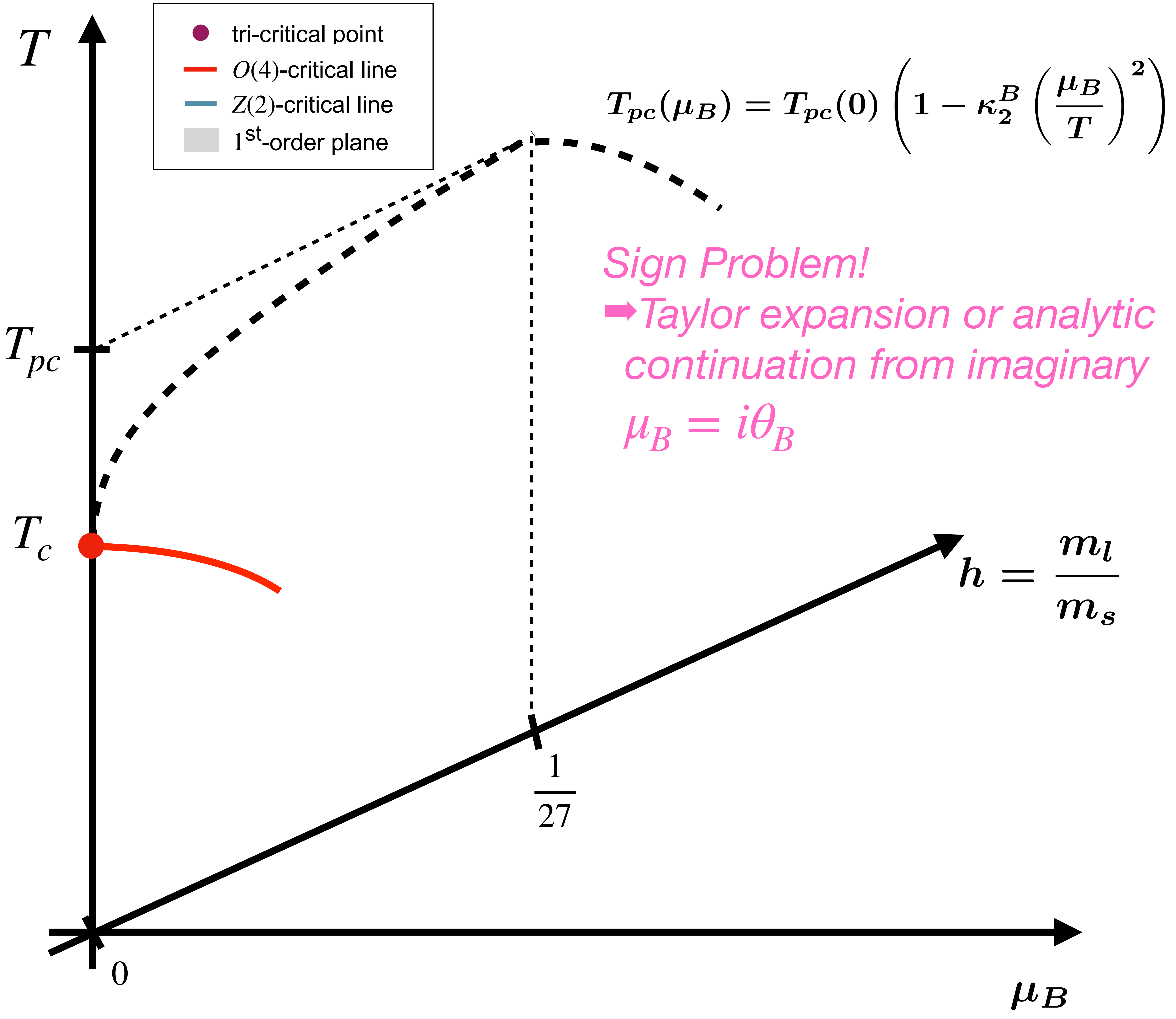
$$h = \frac{m_l}{m_s}$$



Sign Problem!
➡Taylor expansion or analytic continuation from imaginary $\mu_B = i\theta_B$

Chiral symmetry

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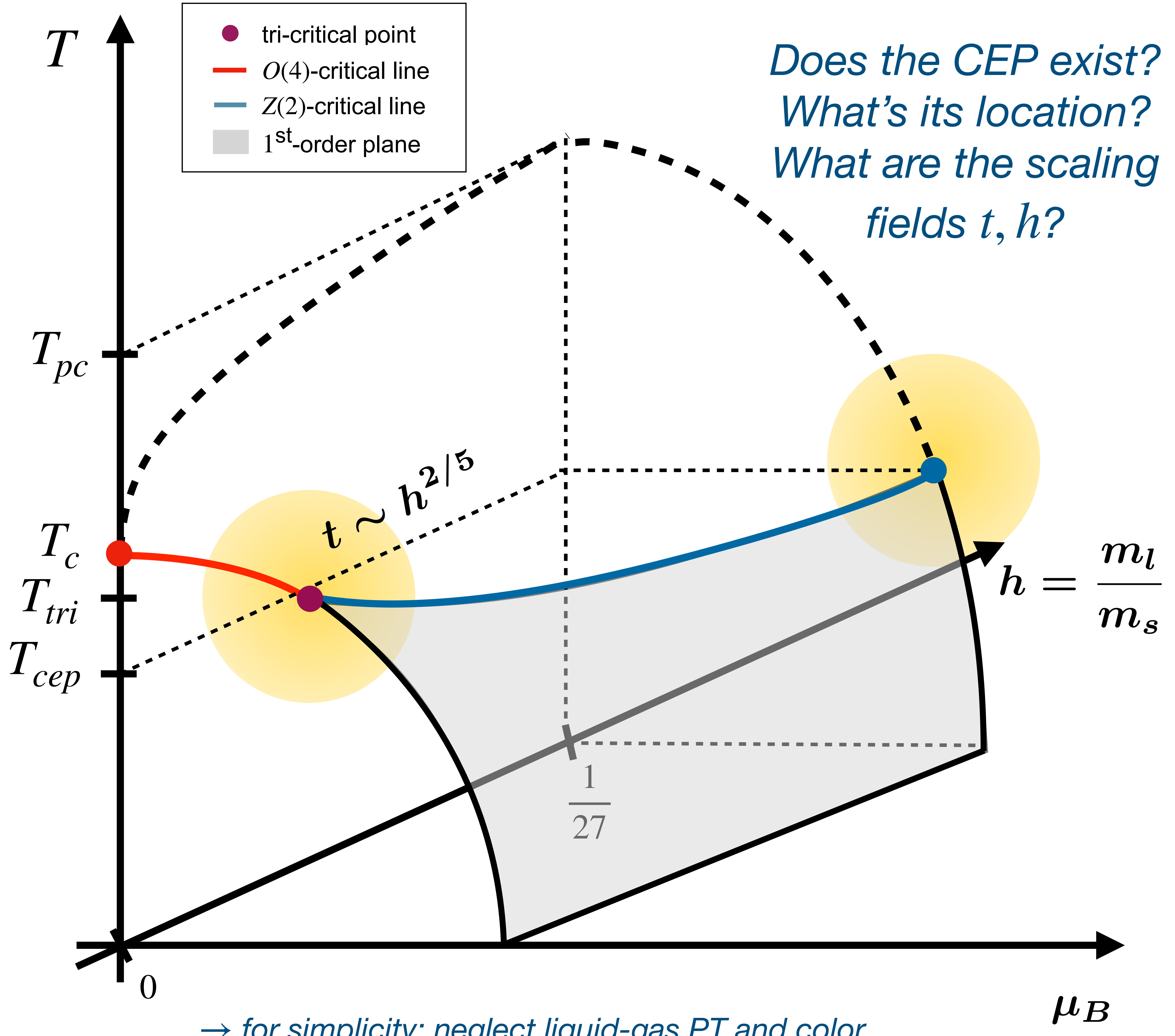
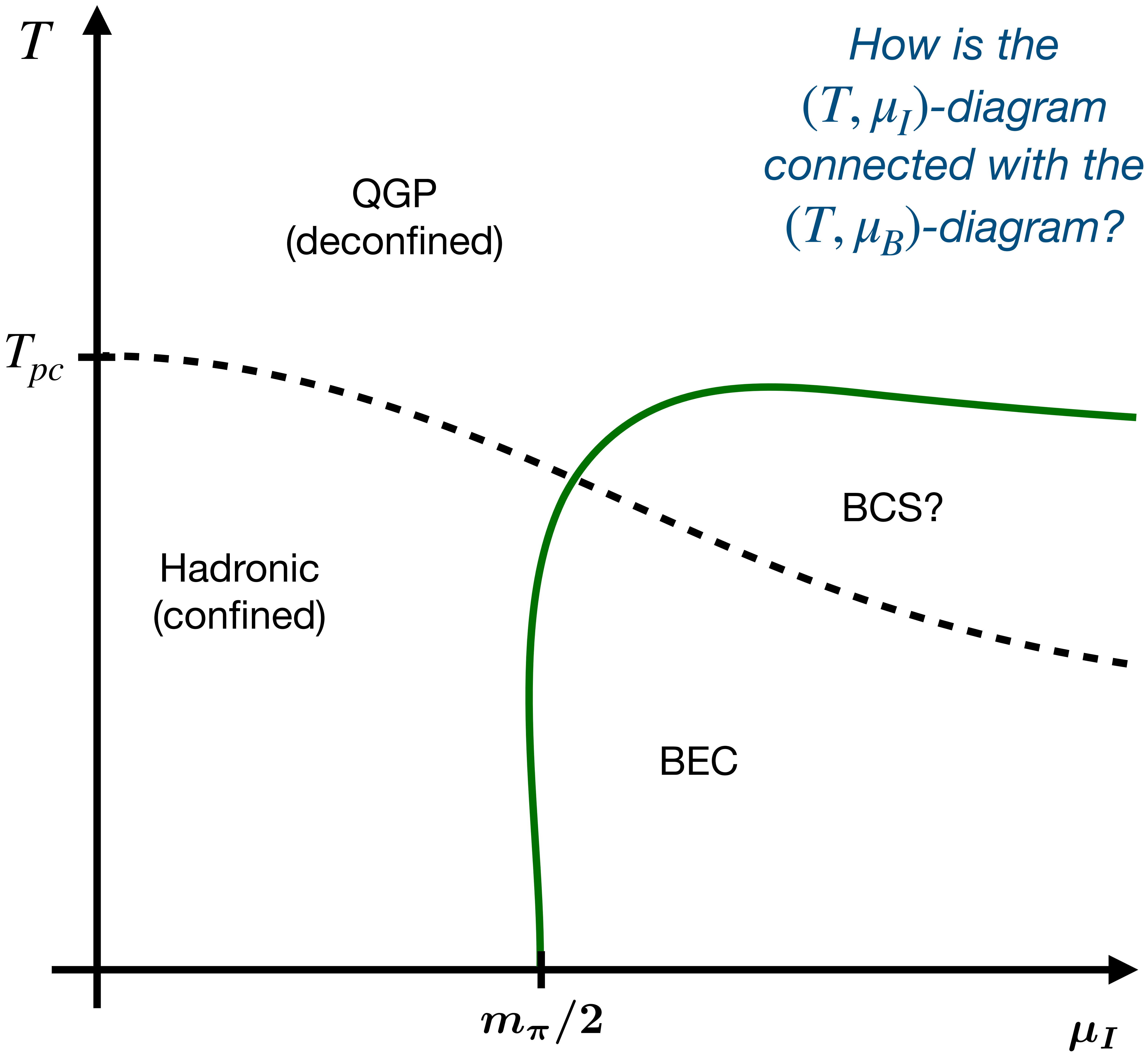


Does the CEP exist? What's its location? What are the scaling fields t, h?

→ for simplicity: neglect liquid-gas PT and color superconducting phases at large μ_B

Pion Condensation

- Pions are the Goldstone bosons of the chiral O(4) transition
- At sufficiently high density they condense into a Bose-Einstein-Condensate (BEC)
  - ➡ Pion condensate might have been triggered by large lepton asymmetry in the early universe **[Vovchenko et al. '21]**
  - ➡ Pion stars might define a class of new compact stars **[Brandt et al. '18]**
  - ➡ BEC - BCS crossover?



How is the (T, μ_I)-diagram connected with the (T, μ_B)-diagram?

Pion Condensation

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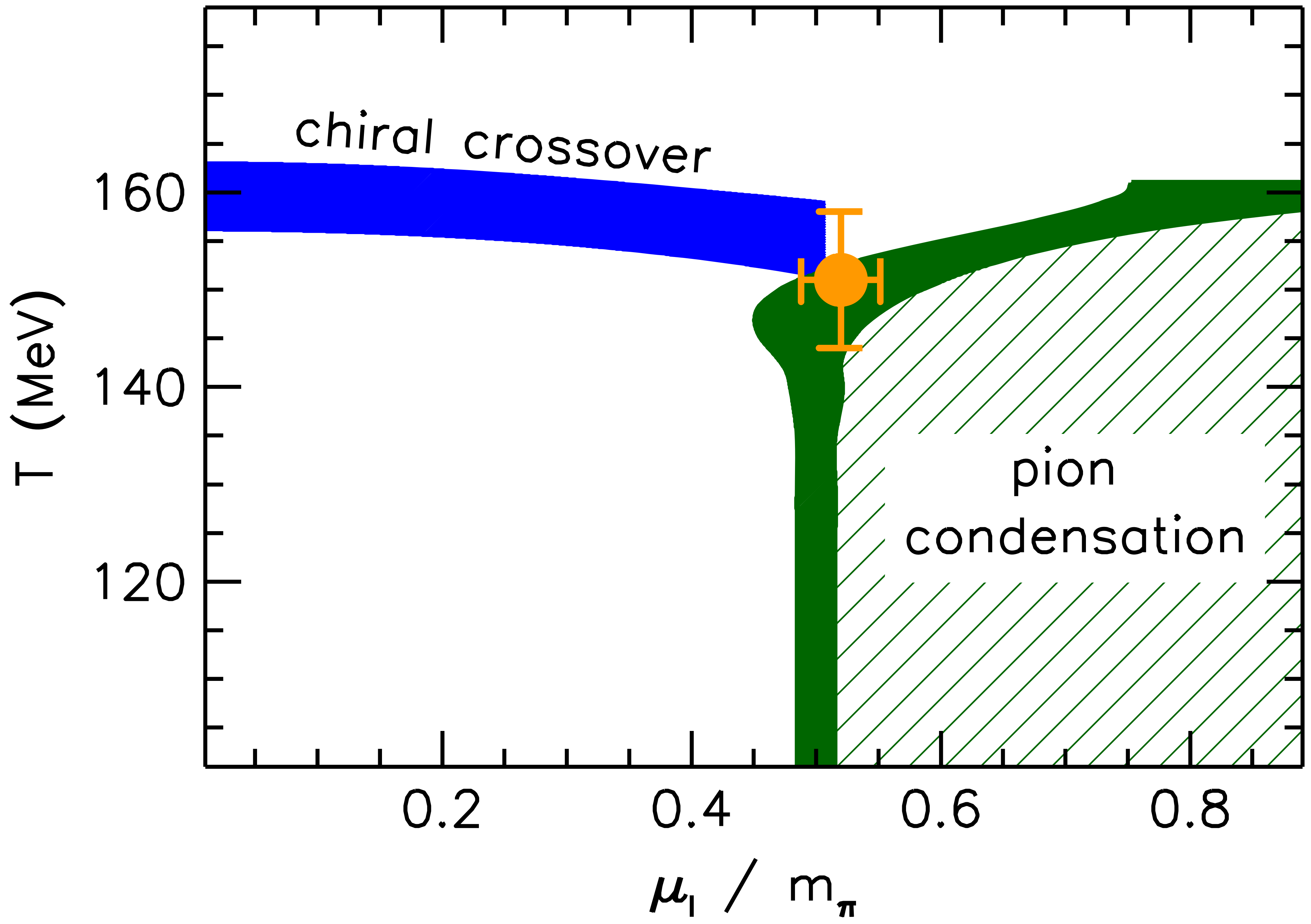


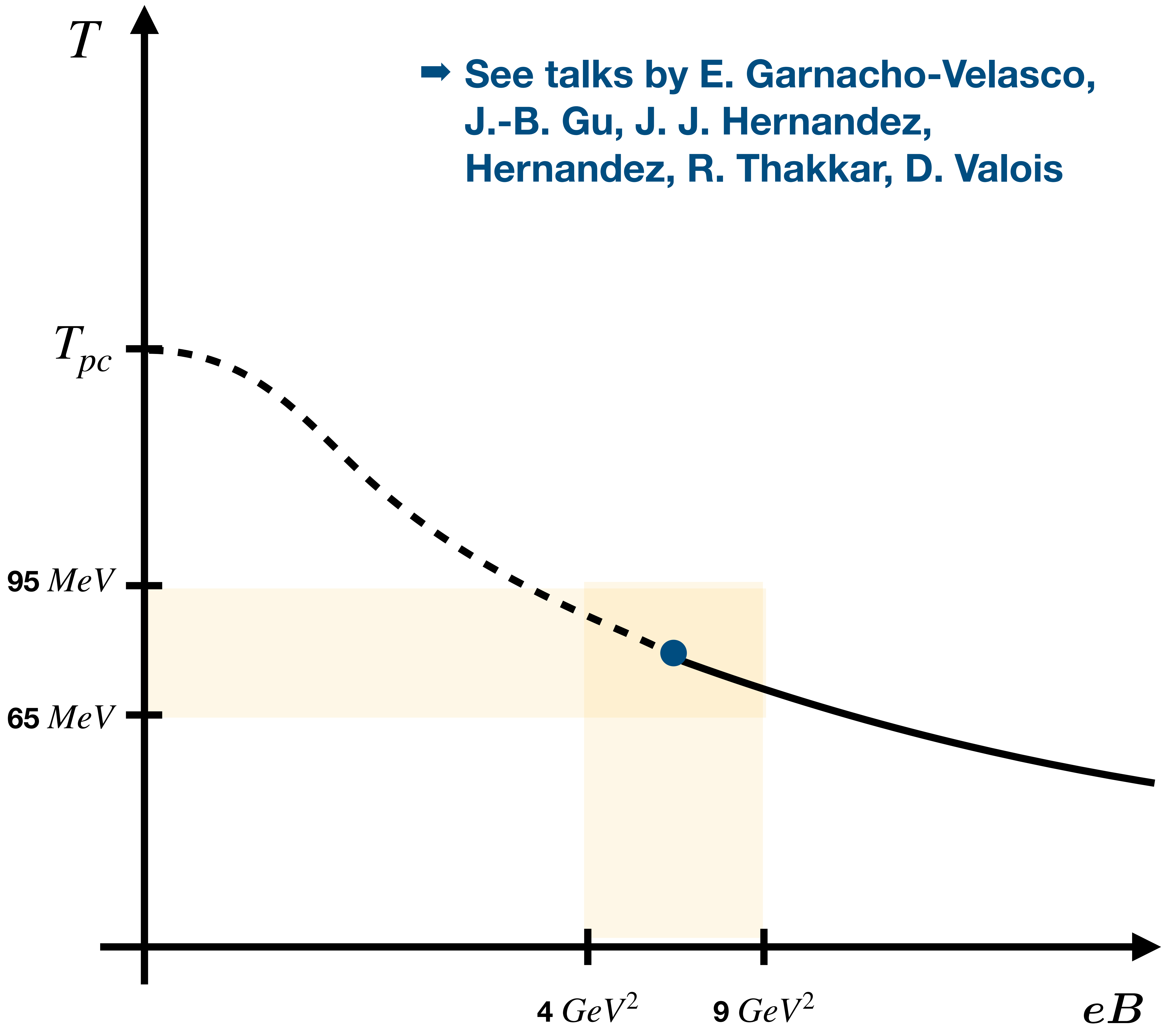
[Brandt, Endrodi, Schmalzbauer]

➡ **See talks by B. Brandt, W. Detmold, R. F. Basta., V. Chelnokov**

External magnetic field B

- Crossover temperature decreases with eB [Bali et al.'11, Bali et al.'12, Endrödi et al.'15]
  - ➡ This is in contrast to almost every effective theory [Andersen et al.'16]
- Critical endpoint recently observed [D'Elia et al.'22]
- At asymptotically large eB quarks might decouple from the dynamics [Miransky, Shovkovy'02]
  - ➡ First order deconfinement transition
  - ➡ $T_d \to 0$ with $B \to \infty$?

➡ See talks by E. Garnacho-Velasco, J.-B. Gu, J. J. Hernandez, Hernandez, R. Thakkar, D. Valois

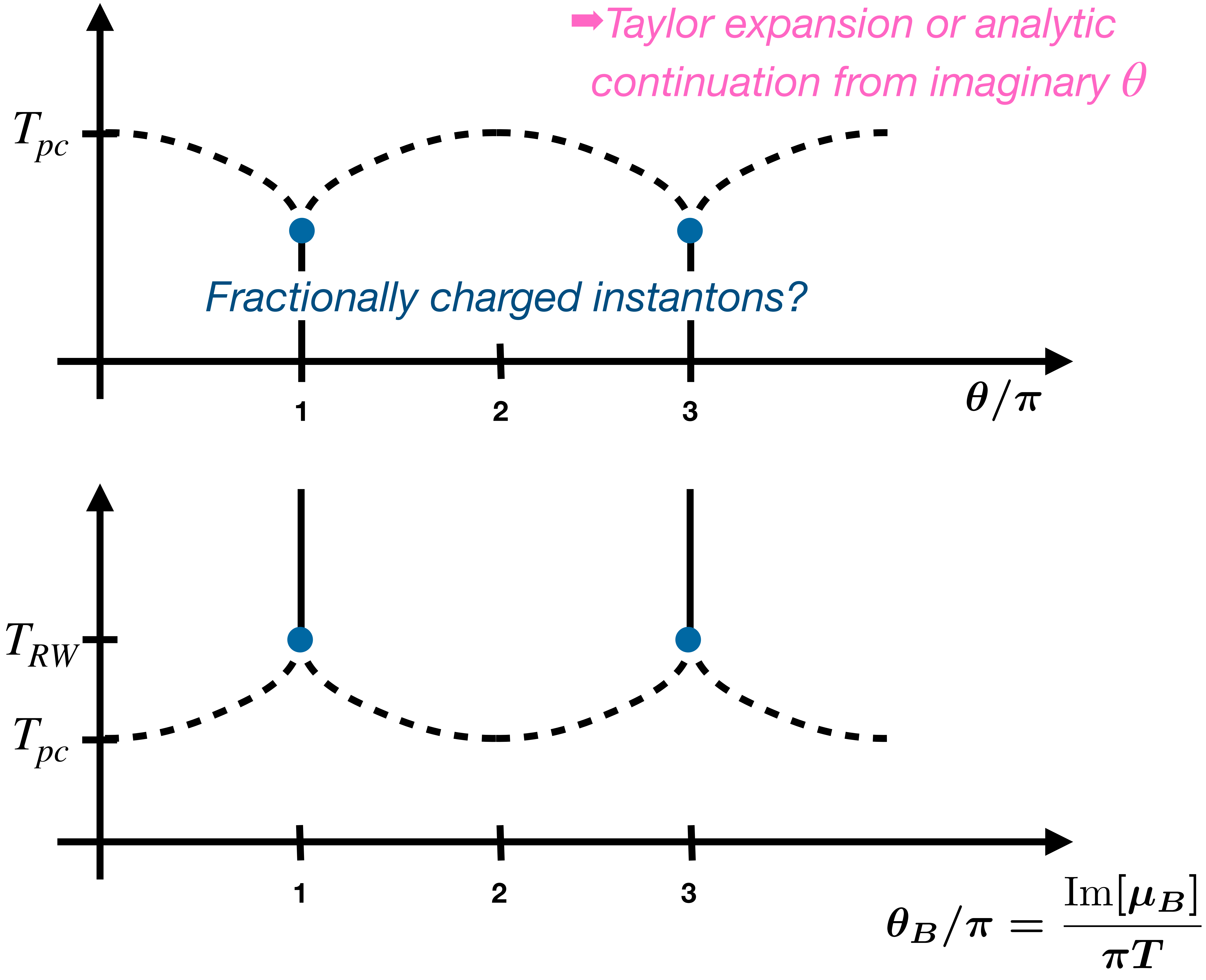


Vacuum alinement angle θ

$$\mathcal{L}_\theta = -i\theta q(x) = -i\theta \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

- θ couples to the topological charge density $q(x)$
  - ➡ Breaks C and CP symmetries
  - ➡ Experimental constraints $\theta < 10^{-10}$ (strong CP problem)
- Phenomenological very interesting: axion physics, η' mass, $U_A(1)$ problem, … [Grilli di Cortona et al. '16]
- $T-\theta$ phase diagram analogous to the $T-\theta_B$ phase diagram? [D'Elia, Negro '13]

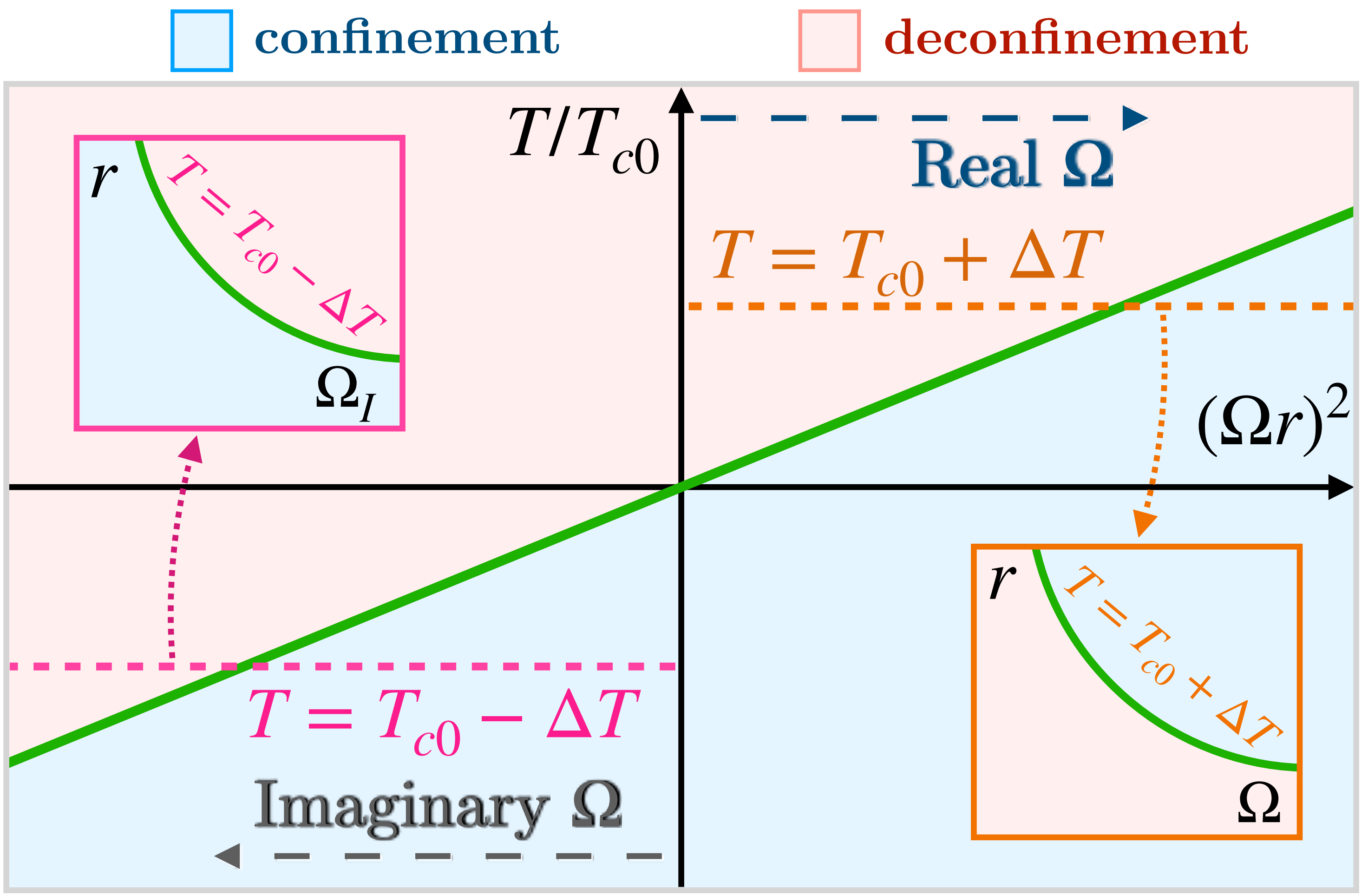
Sign Problem!
➡Taylor expansion or analytic continuation from imaginary θ



Angular velocity Ω

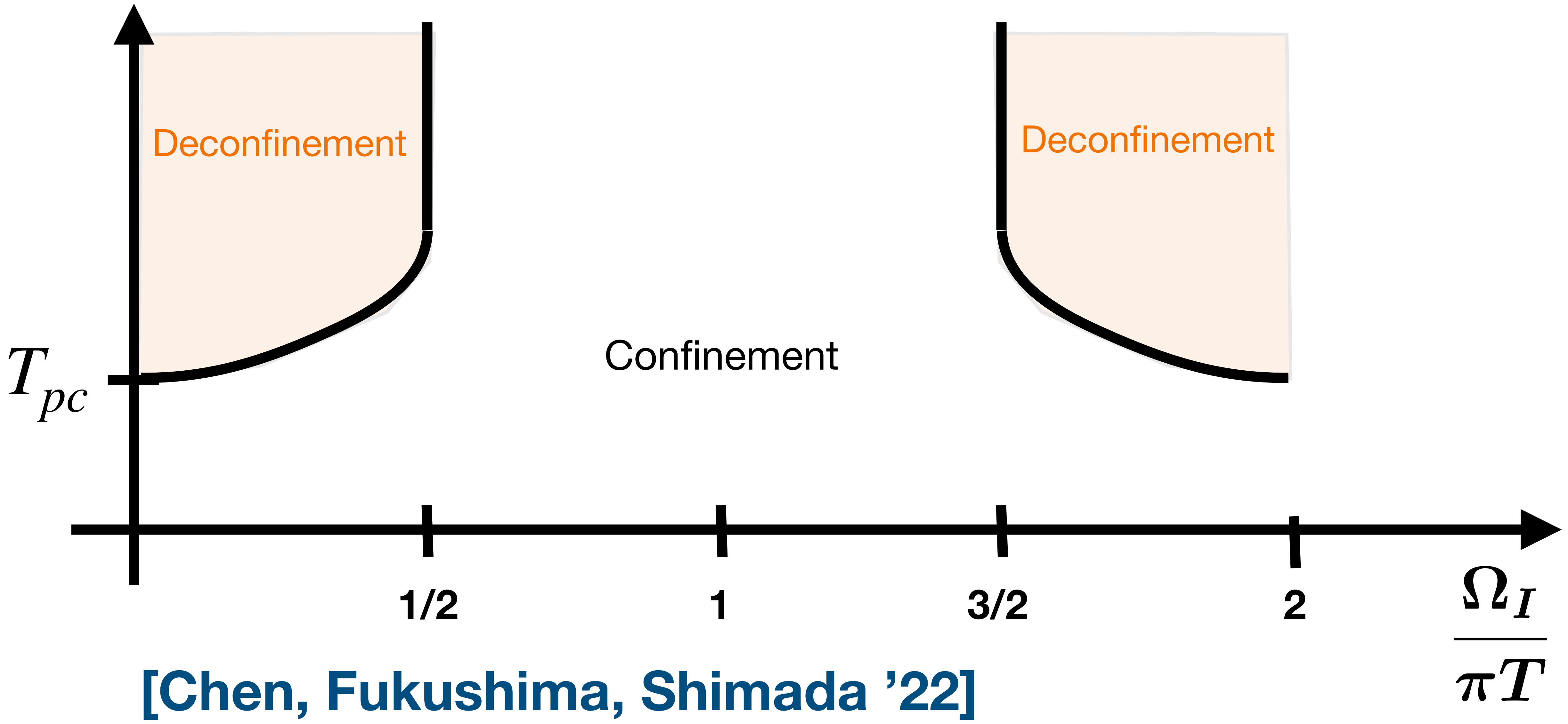
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega \\ 0 & 1 & 0 & -x\Omega \\ 0 & 0 & 1 & 0 \\ y\Omega & -x\Omega & 0 & 1 + r^2\Omega^2 \end{pmatrix}$$

- Calculate in co-rotating frame [Yamamoto, Hirono '13]
- Impose boundary conditions to avoid causality violations.
- Deconfinement temperature increases with rotation velocity?
  ➡ Partial inconsistency with perturbative calculations
- Rich inhomogeneous phase structure



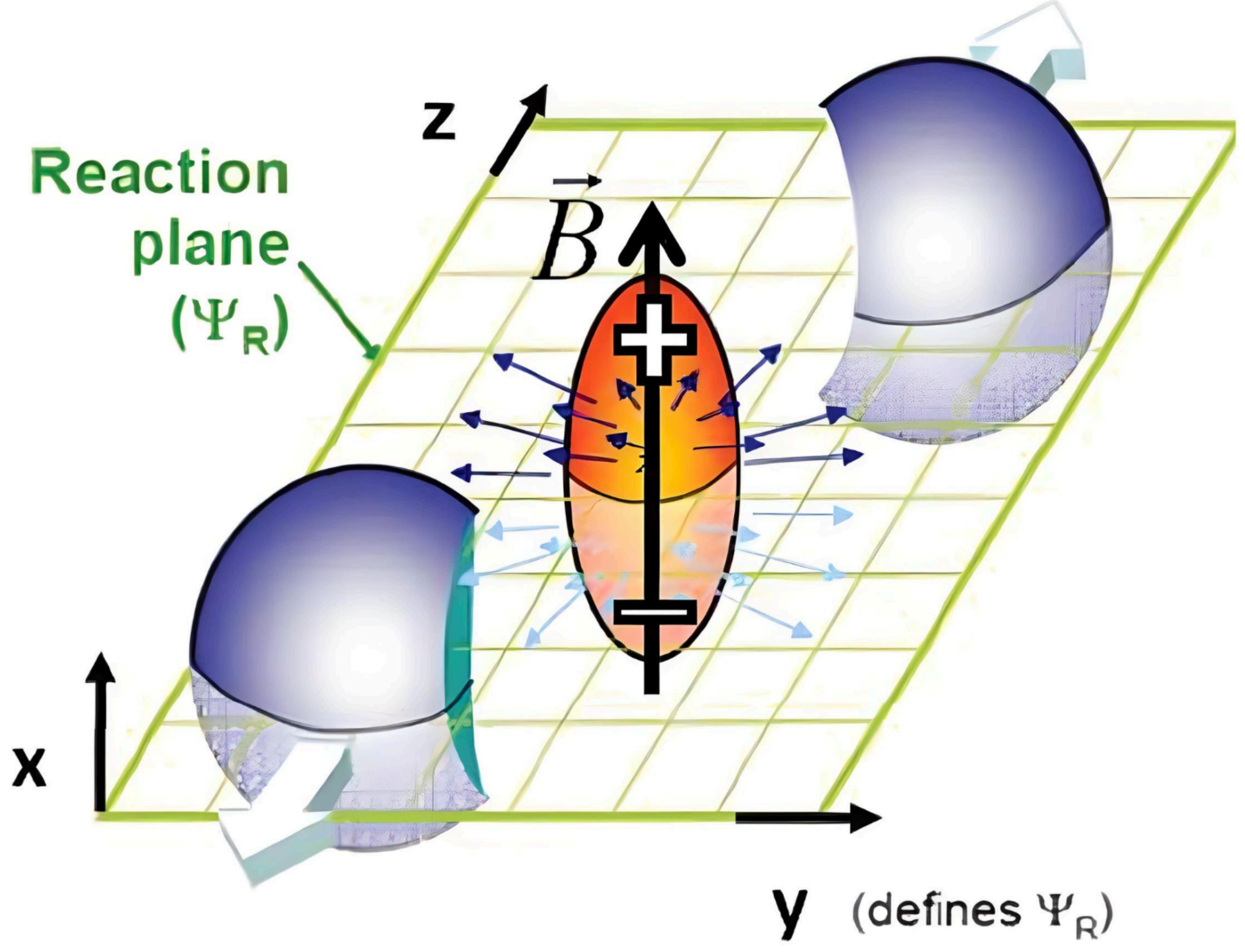
[Braguta, Chernodub, Roenko '24]

Sign Problem!
➡Taylor expansion or analytic continuation from imaginary Ω



[Chen, Fukushima, Shimada '22]

Heavy Ion collisions



$T \lesssim 200$ MeV

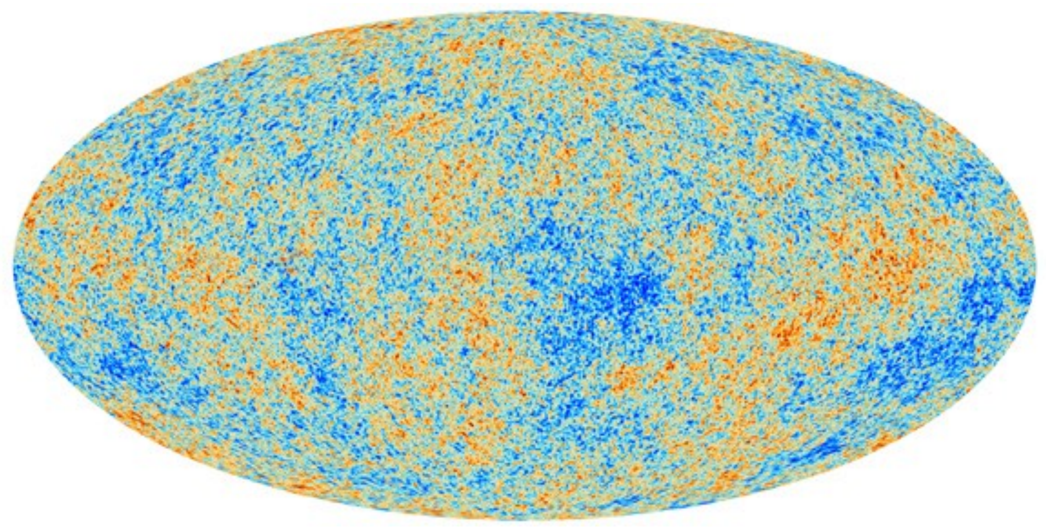
$\mu_B/T \lesssim 3$ (RHIC Beam Energy Scan, collider mode)

$eB \lesssim 5\, m_\pi^2 \approx 0.1\, GeV^2$ (RHIC, fast decaying)

$|J_0| \sim 10^6$ (initial angular momentum, impact parameter dependent)

Can we explain the BES data for cumulants of conserved charges?

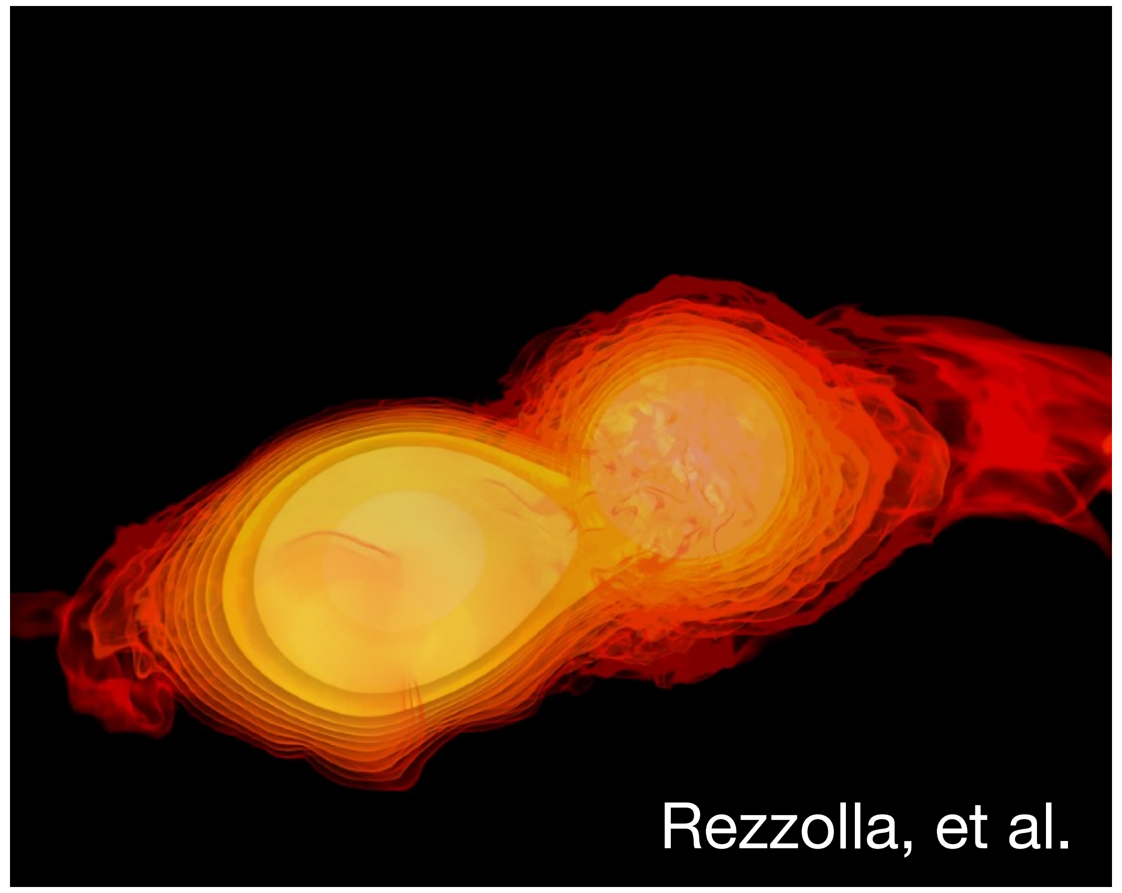
Cosmic trajectory



- μ_B, μ_Q range depends on the lepton flavour asymmetry, pion condensation is possible **[Middeldorf-Wygas et al. '22]**

Can we explain the observed baryonic anisotropy?

Neutron star mergers

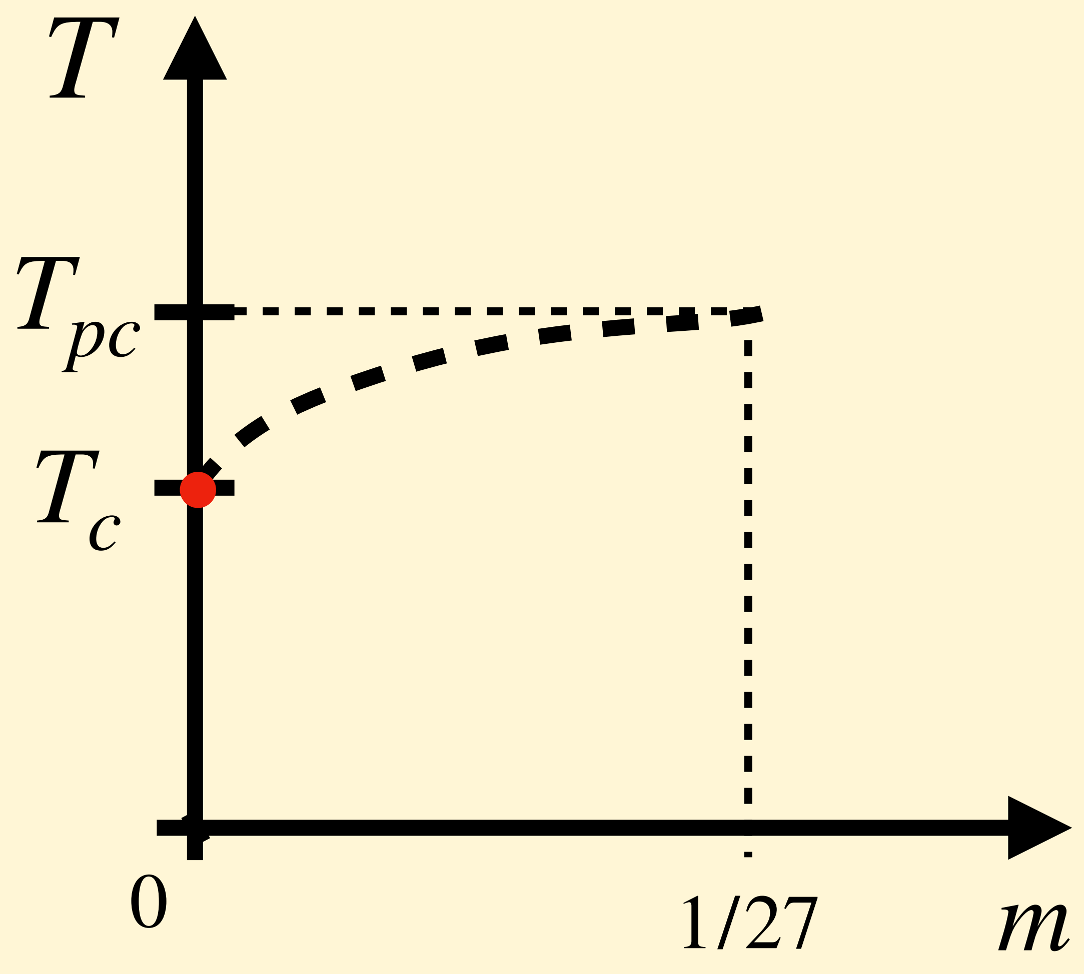


- Neutron Stars: $T \lesssim 1$ keV, Magnetars: $B \sim 10^{12} T$
- Mergers: $T \leq 50$ MeV
- Details of the GW-spectrum depend on the EoS

The QCD phase diagram is very high dimensional, observable systems take very different trajectories!

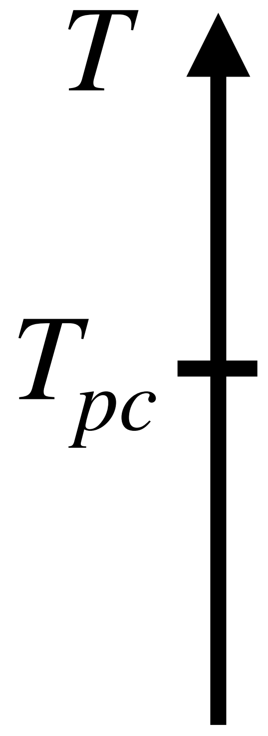
1.

Universal scaling and chiral transition at $\mu = 0$



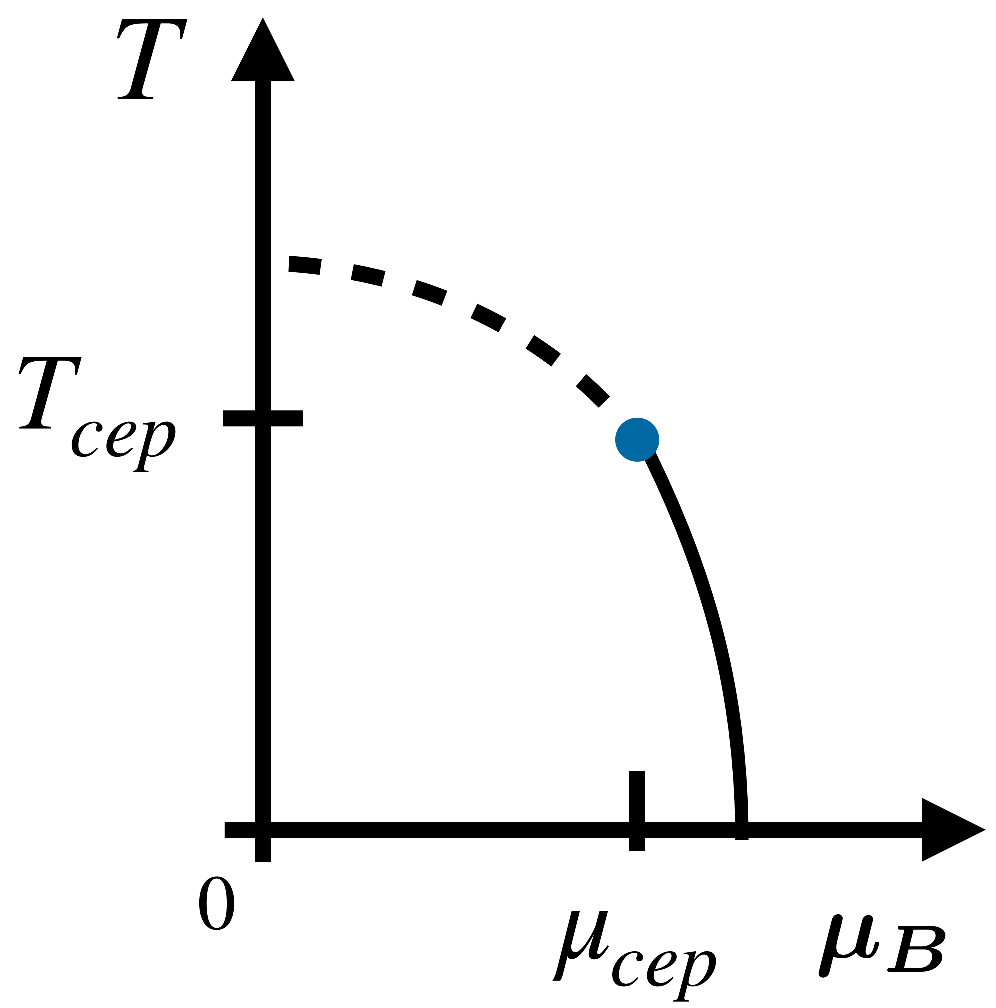
2.

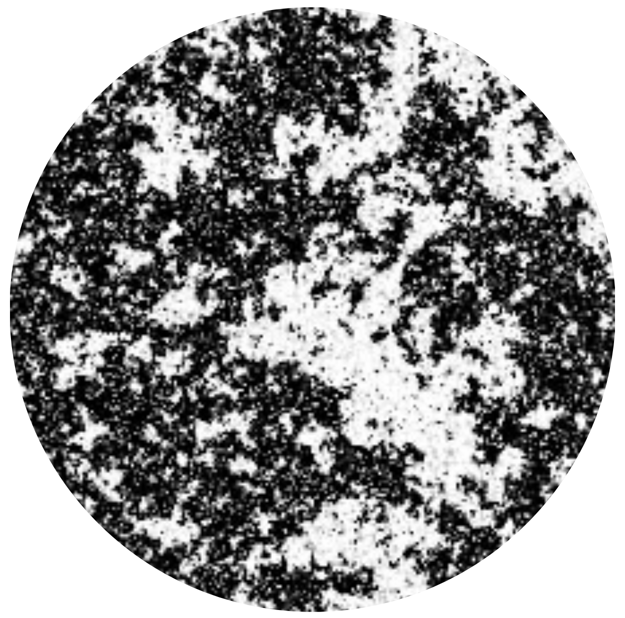
Aspects of deconfinement and melting of hadrons



3.

Beam energy scan results and the QCD critical point





Scaling hypotheses:

Free energy: $f_s(t,h,L) = b^{-d} f_s(b^{y_t} t, b^{y_h} h, L^{-1} b)$

Effective model O(4)/O(2)/Z(2):

Scaling fields

t reduced temperature

h reduced symmetry breaking field

L^{-1} inverse system size

map QCD to the effective model ↔ controlled by non-universal parameters:

t_0, h_0, l_0
T_c, H_c

(2+1)-flavor QCD:

Scaling fields

$$t = \frac{1}{t_0}\left(\Delta T + \kappa_2^l \hat{\mu}_l^2 + \kappa_2^s \hat{\mu}_s^2 + 2\kappa_{11}^{ls} \hat{\mu}_l \hat{\mu}_s\right)$$

$$\Delta T = \frac{T - T_c}{T_c}$$

$$h = \frac{1}{h_0}(H - H_c), \quad H = \frac{m_l}{m_s}$$

$$l = l_0 L^{-1}$$

- We aim on the determination of the parameters, including $\kappa_2^l, \kappa_2^s, \kappa_{11}^{ls}$
- So far no evidence for $H_c > 0$ in (2+1)-flavor QCD

➡ We assume $H_c = 0$ and determine $T_c \equiv T_c^0$

Order parameter

Remove multiplicative UV divergences

$$M_l = \frac{m_s}{f_K^4} \frac{T}{V} \frac{\partial \ln Z}{\partial m_l}$$

$$\frac{\partial}{\partial m_l} = \frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}$$

Magnetic susceptibility

$$\chi_l = m_s \frac{\partial}{\partial m_l} M_l$$

Renormalized order parameter

$$M = M_l - H\chi_l$$

Equation of state

$$M_l = h^{1/\delta} f_G(z)$$

+ sub-leading

With scaling variable

$$z = t/h^{1/\beta\delta}$$

Equation of state

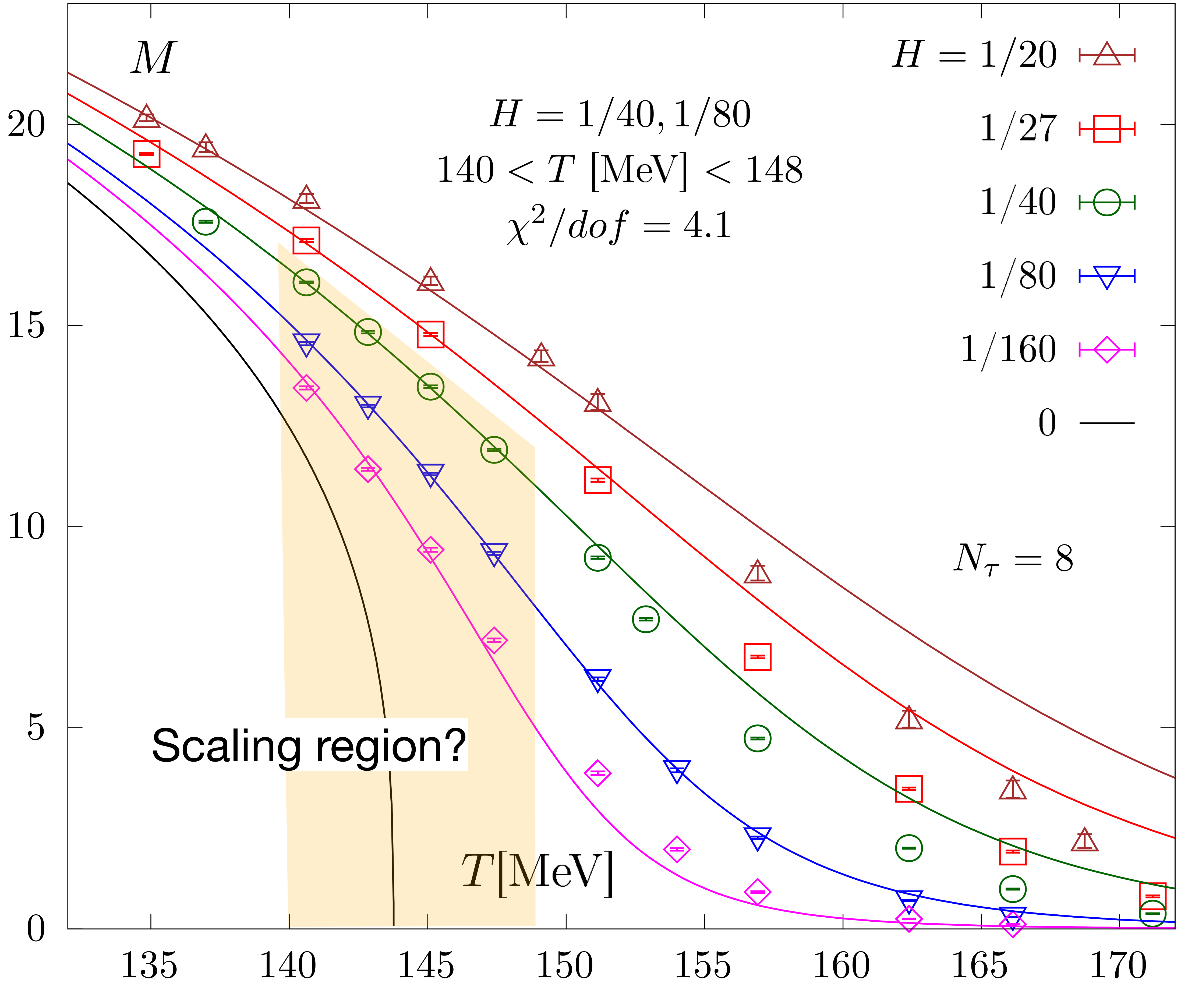
$$M = h^{1/\delta} \left(f_G(z) - f_\chi(z)\right)$$

+ sub-leading

- $f_G(z)$ and $f_\chi(z)$ are the well known universal scaling functions of a single scaling variable z [Karsch, Neumann, Sarkar ’23]
- This version of the renormalised order parameter has advantages:
  - ➡ No explicit contribution from the strange condensate
  - ➡ Direct relation to the scaling function of the free energy

  [Kotov et. al, '21]

$$M = h_0^{-1/\delta} H^{1/\delta} \left(f_G(z) - f_\chi(z) \right)$$



[Ding et al. (HotQCD) '24]

- $N_\tau = 8$ (with updated statistics)
- Corresponding pion masses: $m_\pi \simeq$ 180 MeV, 140 MeV, 110 MeV, 80 MeV, 55 MeV.
- Use O(2) scaling functions and exponents due to staggered fermions
- Fit results for $N_\tau = 8$

  $T_c^0 = 143.7(2)$ MeV

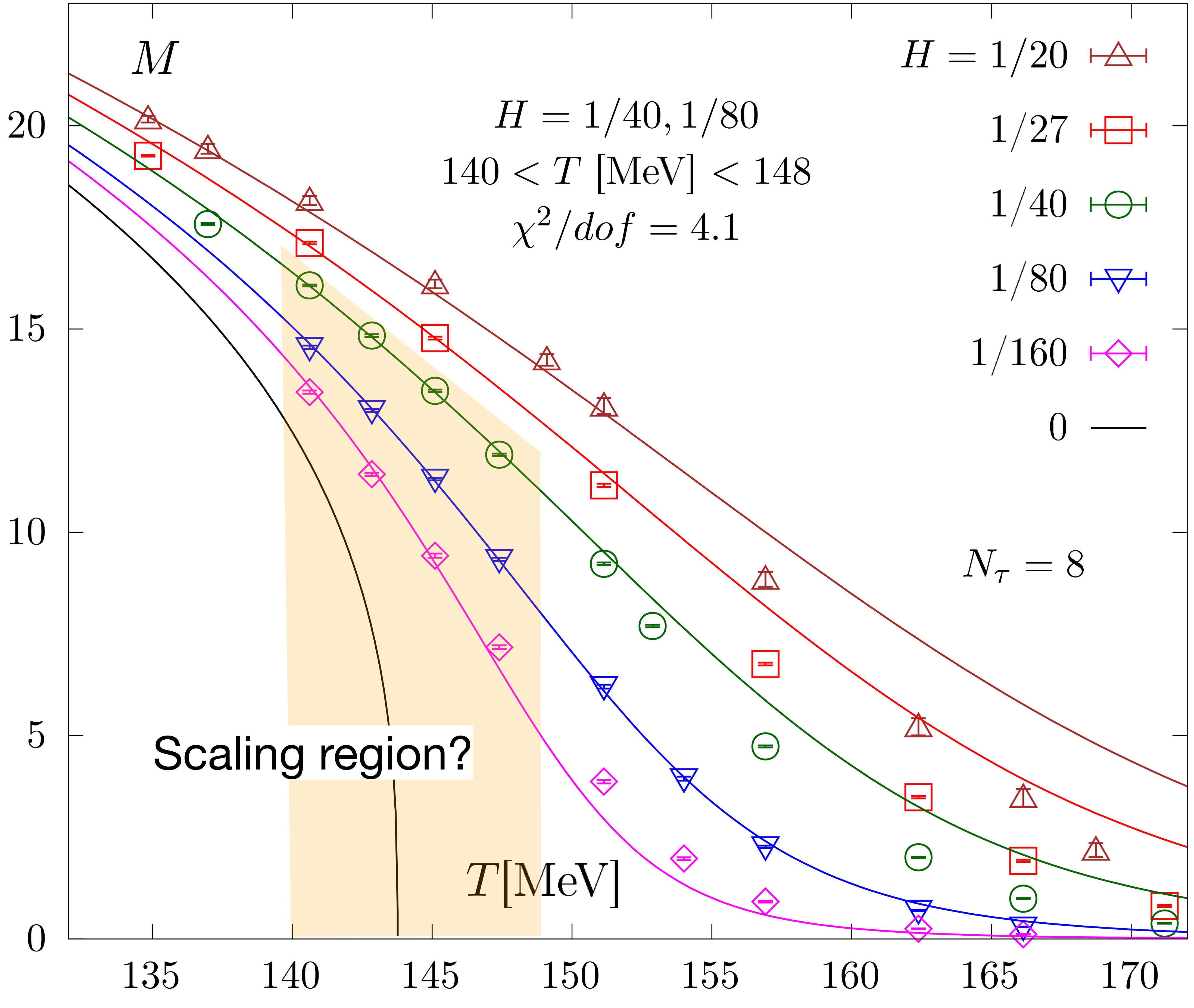
  $z_0 = 1.42(6)$

  $h_0^{-1/\delta} = 39.2(4)$

  [Ding et al. (HotQCD) '24]
- Continuum estimate: $T_c^0 = 132^{+2}_{-6}$ MeV

  [Ding et al. (HotQCD) '19]

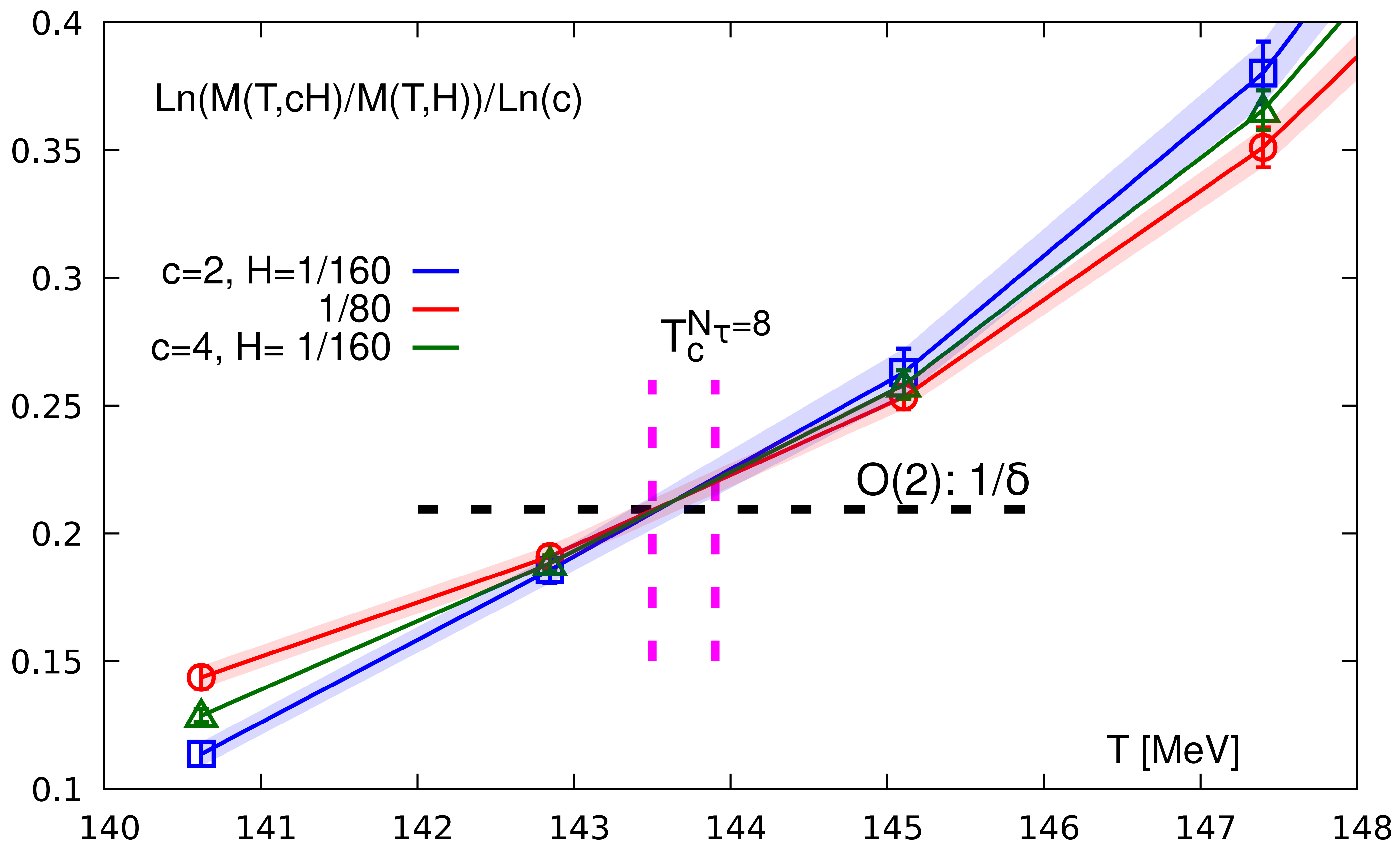
EoS fit: universality class is assumed



[Ding et al. (HotQCD) '24]

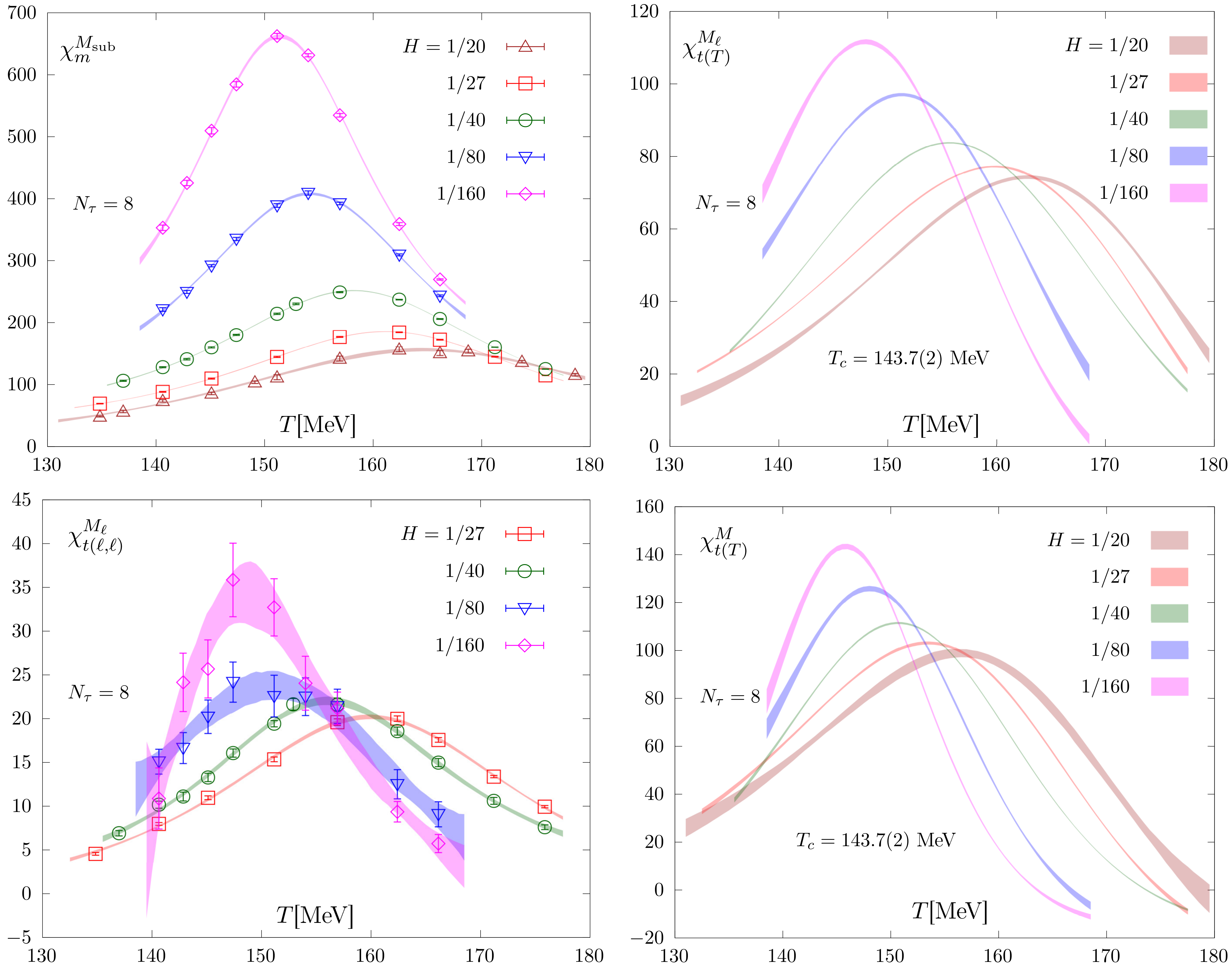
New parameter free method

➡ **[Talk by S. Mitra:] specific ratios of the renormalized order parameter can be used to define define T_c and δ**



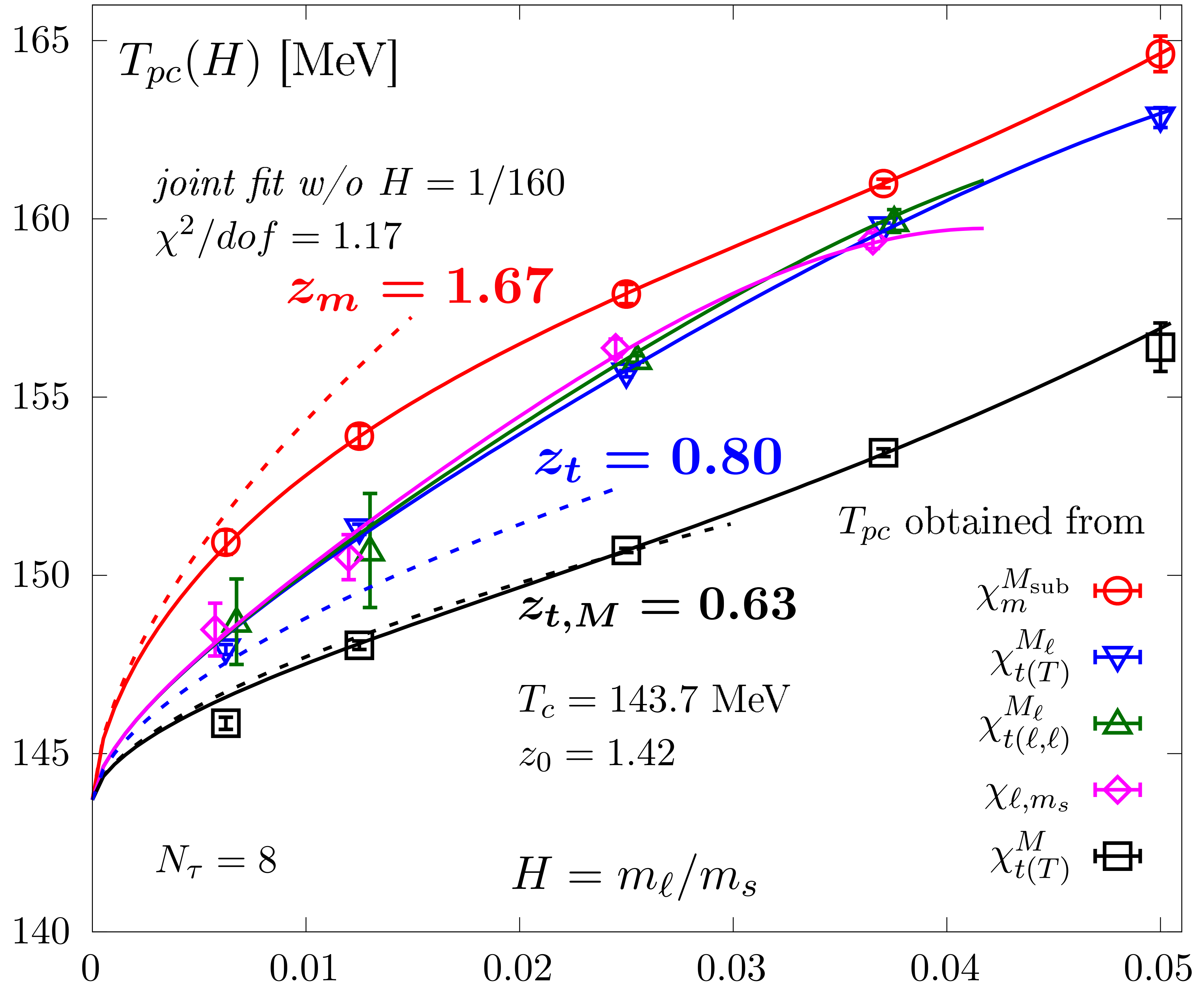
- Intersection at $(T_c, 1/\delta)$
- Obtained T_C and δ consistent with EoS fit and O(2) universality class

- Determine peak positions of various susceptibilities
  - ➡ Definition of pseudo critical line (constant z value)



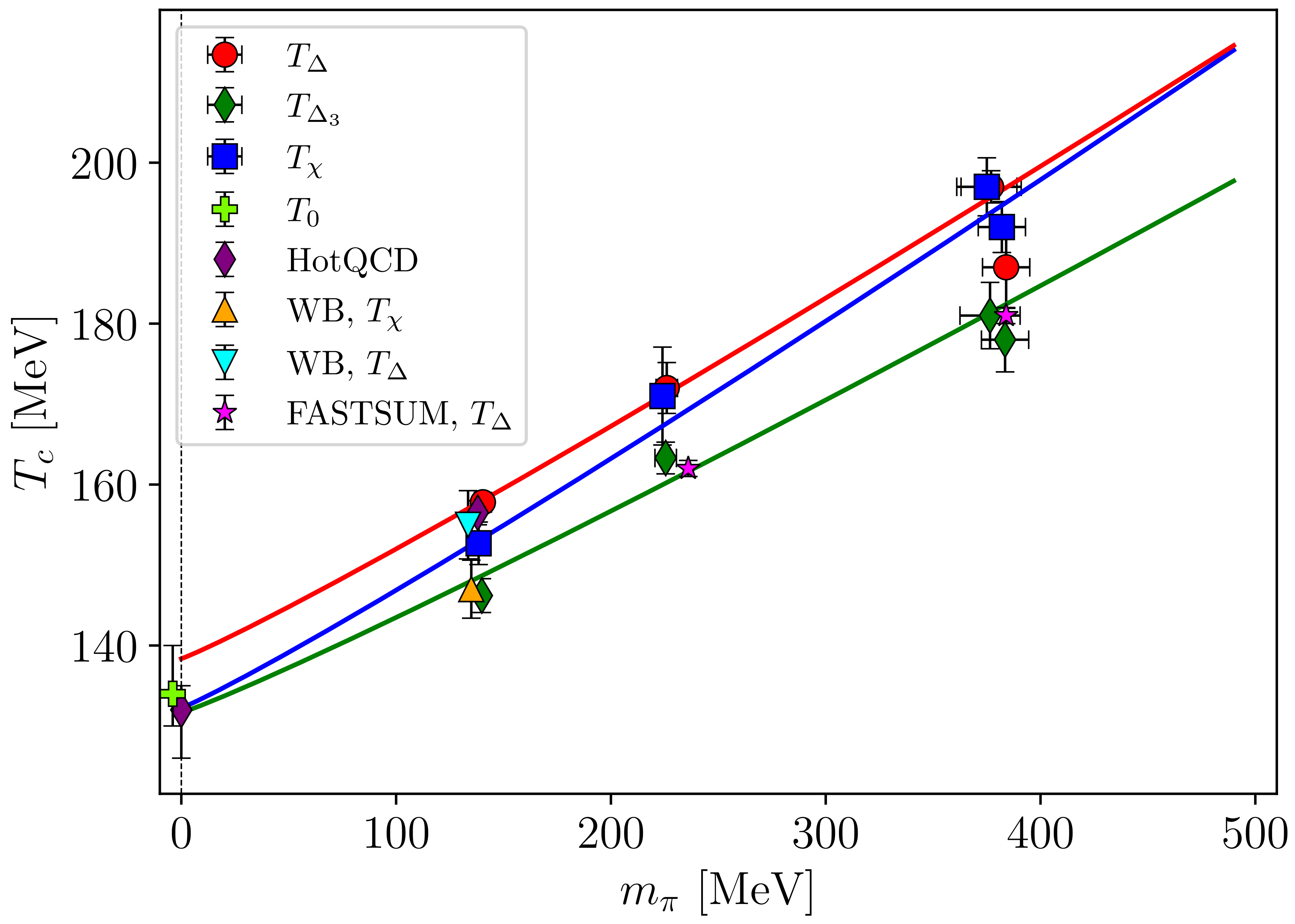
[Ding et al. (HotQCD) '24]

$$T_{pc,x} = T_c \left(1 + \frac{z_x}{z_0} H^{1/\beta\delta} + \text{corrections to scaling}\right)$$



- Fit to peak positions give T_c in good agreement with EoS fits

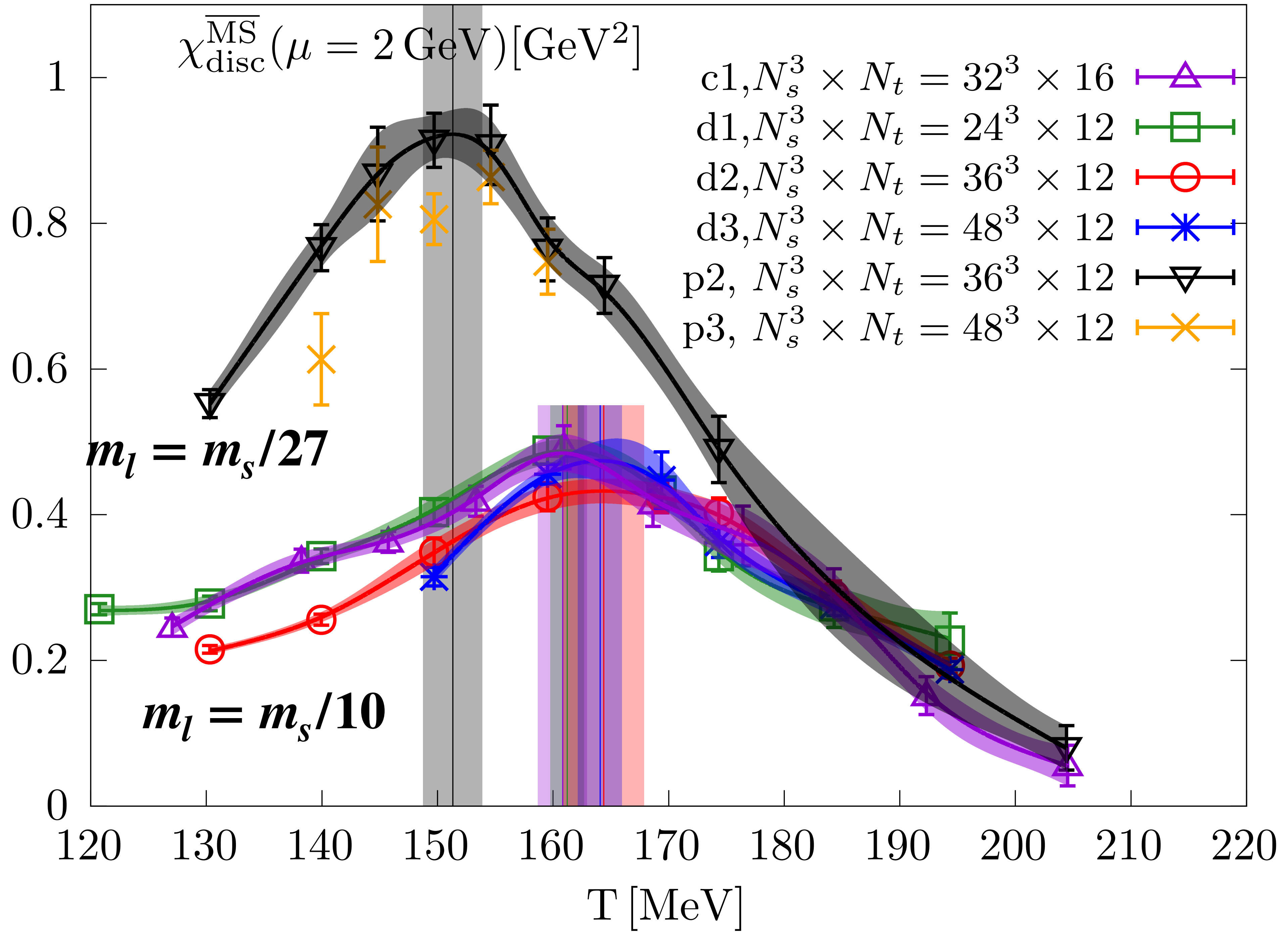
[Kotov et. al, '21]



❖ Continuum extrapolations with HISQ and tmW in agreement:

$$T_c^0 = \begin{cases} 132^{+2}_{-6} \text{ MeV, HISQ [HotQCD '19]} \\ 134^{+6}_{-4} \text{ MeV, tmW [Kotov et. al, '21]} \end{cases}$$

➡ Talk by Y. Aoki: preliminary results on (2+1)-flavour MDWF



❖ Crossover at physical point likely

❖ $T_{pc} = 151(3)$ MeV (preliminary) on $36^3 \times 12$, to be compared with

$$T_{pc} = \begin{cases} 156.5(1.5) \text{ MeV} \quad \text{[HotQCD '19]} \\ 158.0(0.6) \text{ MeV} \quad \text{[BW '20]} \end{cases}$$

- Temperature like scaling field

$$t = \frac{1}{t_0}\left(\Delta T + \kappa_2^l \hat{\mu}_l^2 + \kappa_2^s \hat{\mu}_s^2 + 2\kappa_{11}^{ls}\hat{\mu}_l\hat{\mu}_s\right)$$

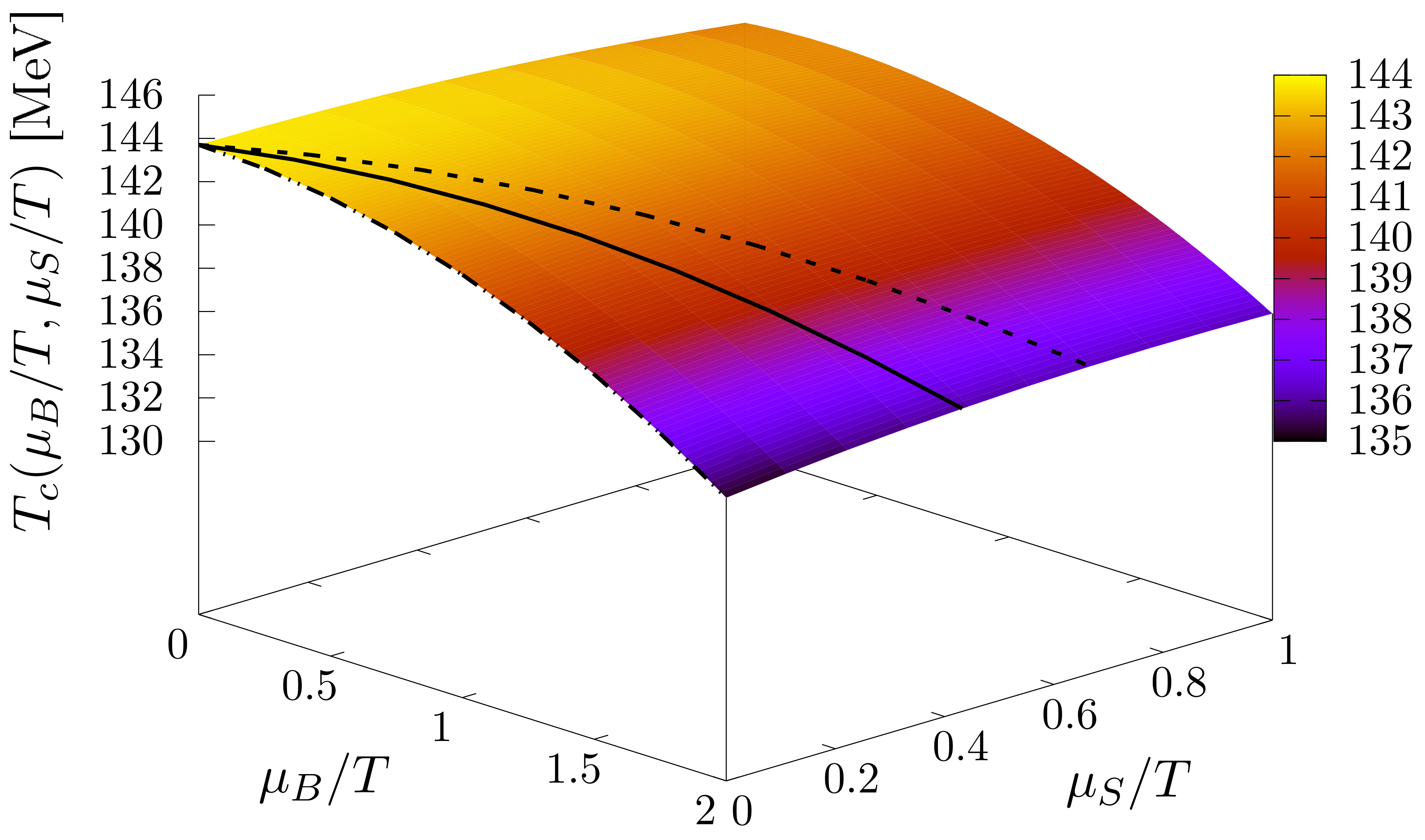
- Ratio of mixed susceptibilities are related to the curvature coefficients

$$\kappa_2^l = \frac{1}{2T_c}\left(\frac{\partial^2 M_l/\partial\hat{\mu}_l^2}{\partial M_l/\partial T}\right)\Bigg|_{(T=T_c,\vec{\mu}=0)}$$

$$\kappa_{11}^{ls} = \frac{1}{2T_c}\left(\frac{\partial^2 M_l/\partial\hat{\mu}_l\hat{\mu}_s}{\partial M_l/\partial T}\right)\Bigg|_{(T=T_c,\vec{\mu}=0)}$$

$$\kappa_2^s = \frac{1}{2T_c}\left(\frac{\partial^2 M_l/\partial\hat{\mu}_s^2}{\partial M_l/\partial T}\right)\Bigg|_{(T=T_c,\vec{\mu}=0)}$$

- results may be transformed to the hadronic basis



[Ding et al. (HotQCD) '24]

$m_l = 0$ ($N_\tau = 8$)

$$\kappa_2^{B,\hat{\mu}_S=0} \equiv \kappa_2^B = 0.015(1)$$

$$\kappa_2^{B,n_S=0} = 0.893(35)\kappa_2^B$$

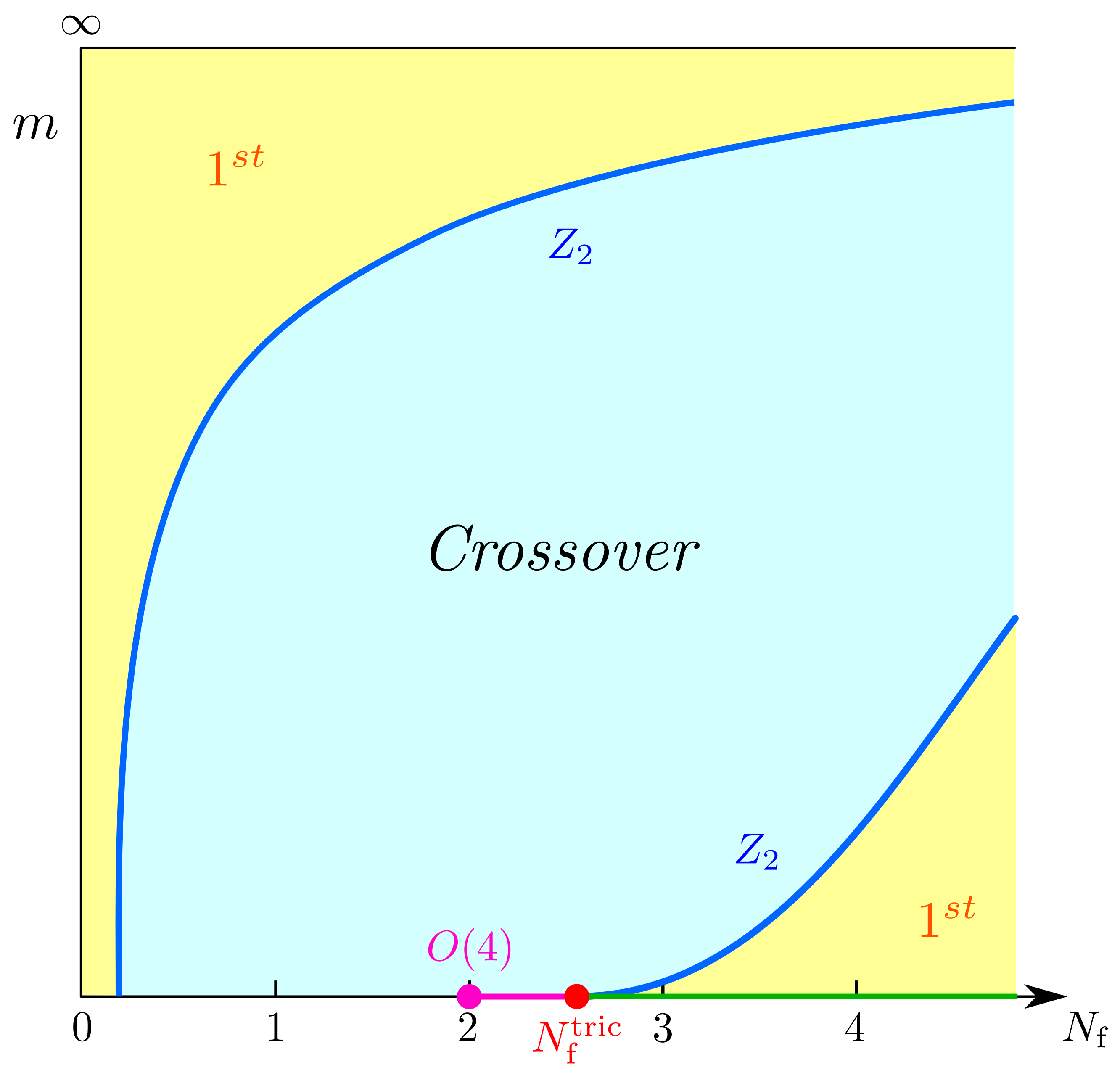
$$\kappa_2^{B,\hat{\mu}_s=0} = 0.968(23)\kappa_2^{n_S=0}$$

[Ding et al. (HotQCD) '24]

$m_l = m_s/27$ (cont.)

$$\kappa_2^{B,n_S=0} = \begin{cases} 0.012(4) & \text{[HotQCD '19]} \\ 0.0153(8) & \text{[BW '20]} \end{cases}$$

- First-order transition in 3-flavor QCD was predicted [Pisarski, Wilczek, '83]

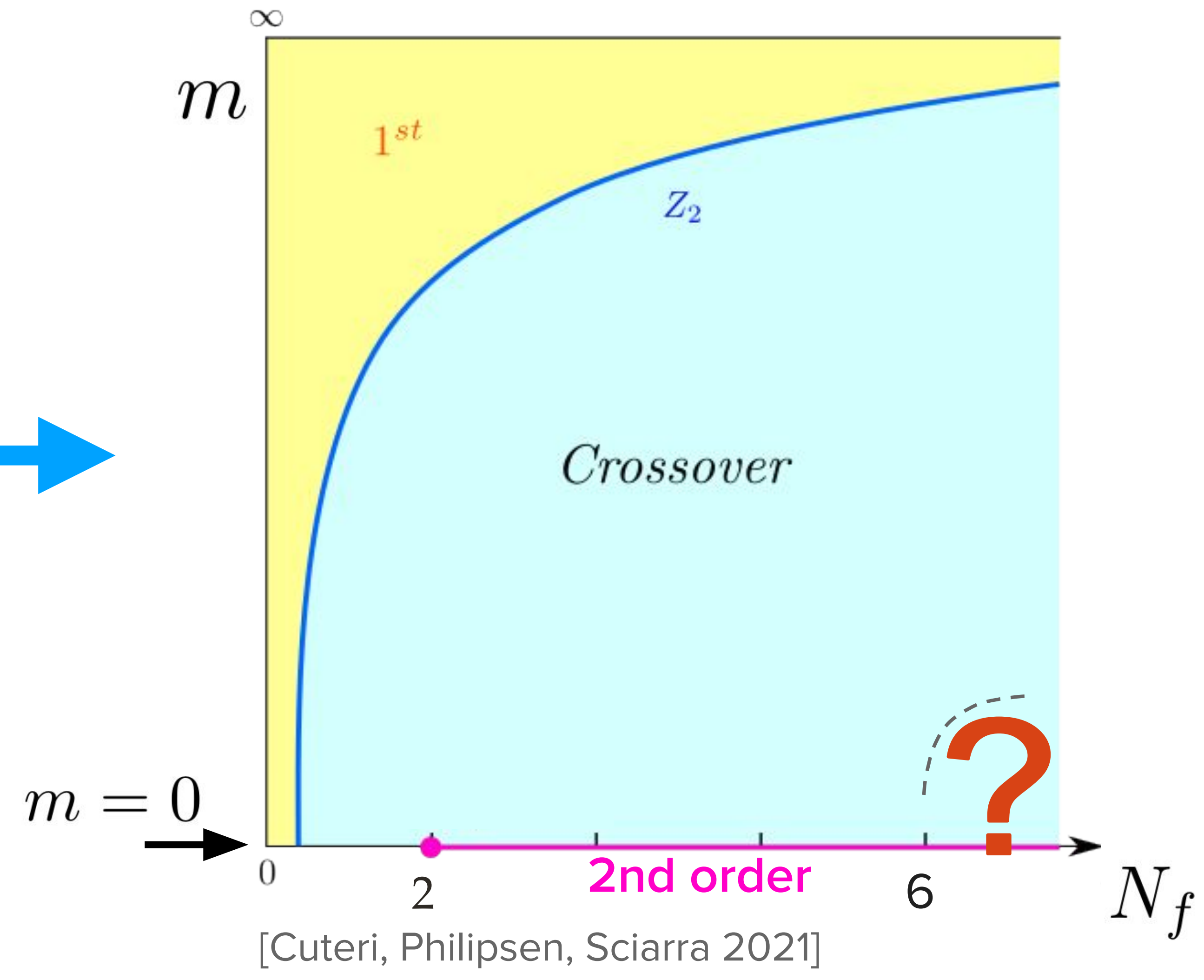


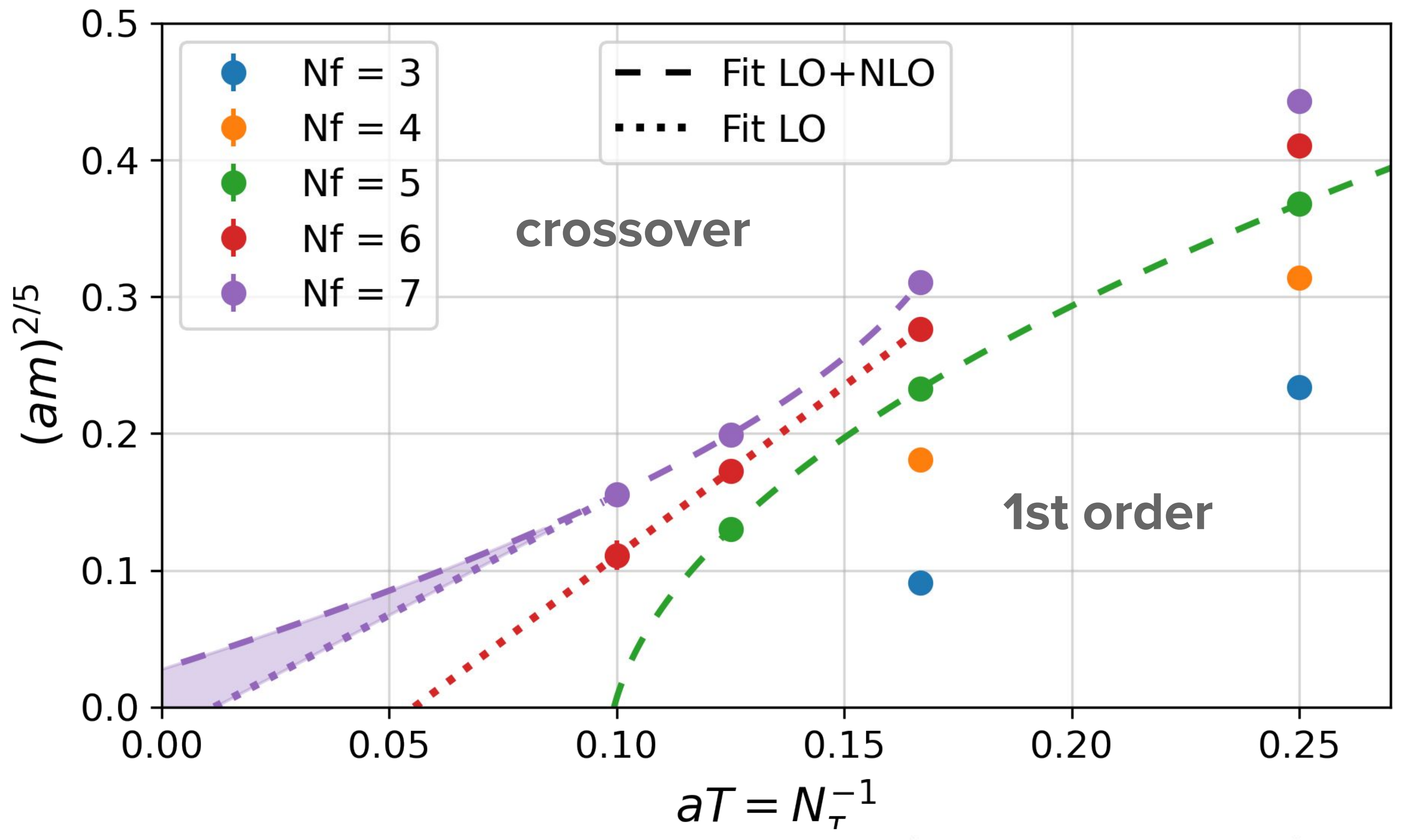
- First order region found on course unimproved lattices but is shrinking towards the continuum [Cuteri et al. '21]
- Evidence of a second order transition in the chiral limit of (2+1)-flavor and 3-flavour with HISQ [HotQCD '19][Dini et al. '22]

➡ **Talk by Y. Zhang: no evidence in 3-flavours of MDWF**

➡ **Talk by J.P. Klinger: chiral transition at the tri-critical point**

➡ **Talk by R. Kaiser: same analysis at imaginary μ**





$$T_c(m) = T_{tric} + A \cdot m^{2/5} + B \cdot m^{4/5}$$

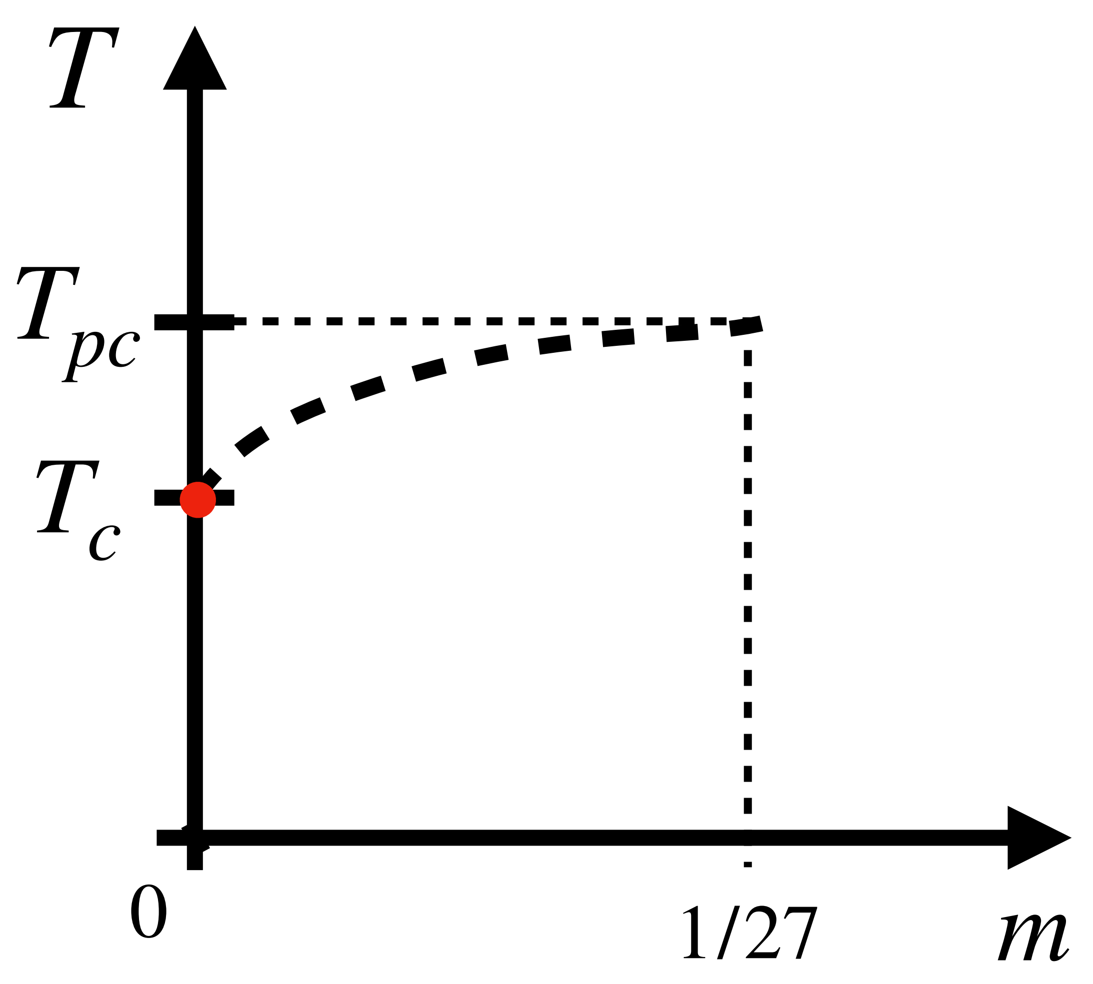
- The chiral limit is of second order for $N_f \leq 7$
- Transition temperature decreases with N_f
- Same conclusion for finite imaginary μ
- Chiral transition might be second order for all N_f all the way to the conformal window ($T_c \approx 0$ for $N_f = 8$)

1.

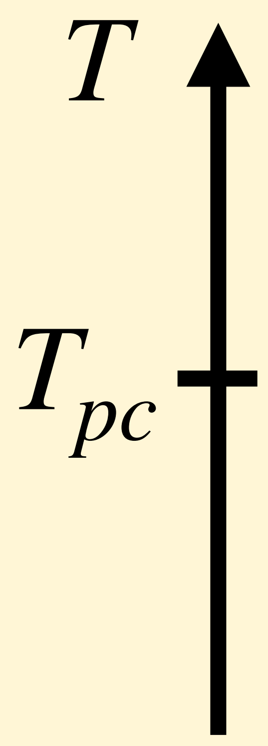
2.

3.

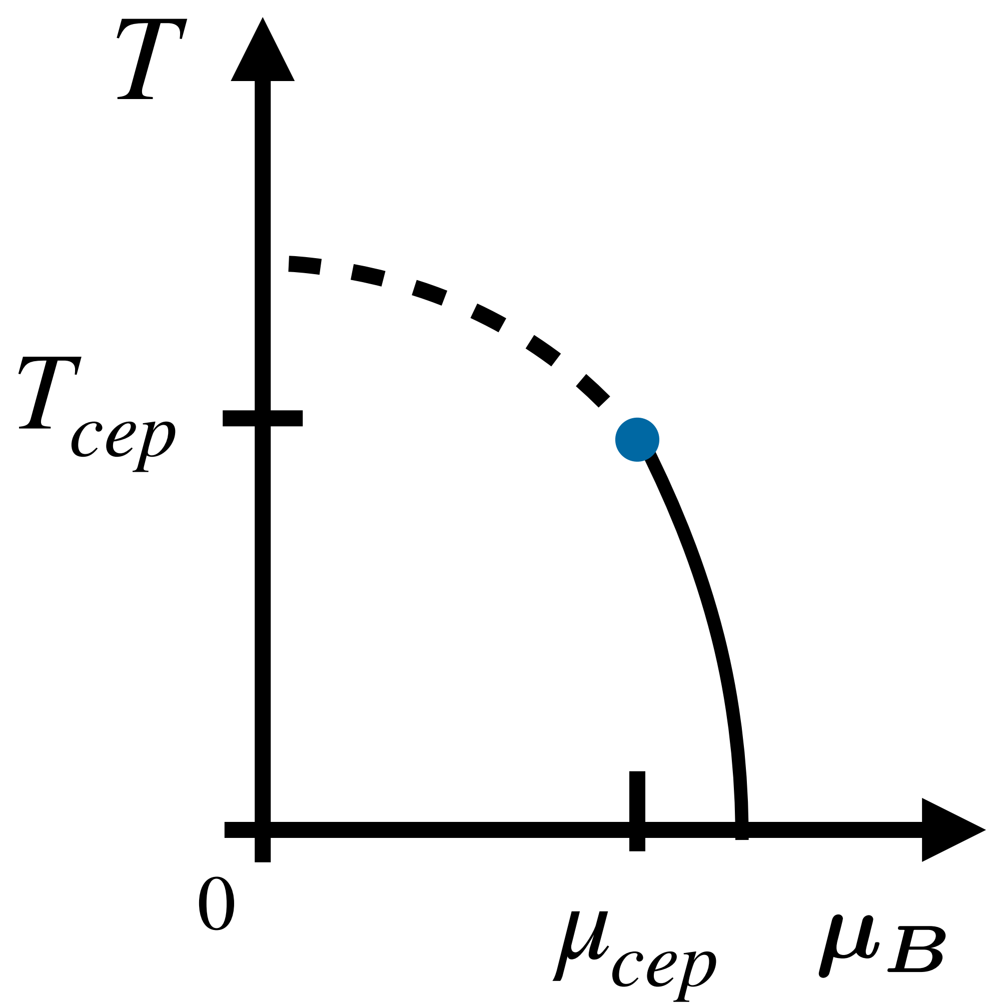
Universal scaling and chiral transition at $\mu = 0$



Aspects of deconfinement and melting of hadrons



Beam energy scan results and the QCD critical point



Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k,l=0}^{\infty} \frac{1}{i!j!k!l!} \chi_{i,j,k,l}^{BQSC} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_C}{T}\right)^l$$

$X = B, Q, S, C$: net-conserved charges

Lattice

$$\chi_{i,j,k,l}^{BQSC} = \left.\frac{\partial^{(i+j+k+l)}(p/T^4)}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k \partial\hat{\mu}_C^l}\right|_{\vec{\mu}=0}$$

generalized susceptibilities

$\Rightarrow$ **To be calculated at $\mu_X = 0$**

Experiment

$$VT^3 \chi_2^X = \langle(\delta N_X)^2\rangle$$
$$VT^3 \chi_4^X = \langle(\delta N_X)^4\rangle - 3\langle(\delta N_X)^2\rangle^2$$
$$VT^3 \chi_6^X = \langle(\delta N_X)^6\rangle - 15\langle(\delta N_X)^4\rangle\langle(\delta N_X)^2\rangle + 30\langle(\delta N_X)^2\rangle^3$$

cumulants of net-charge fluctuations

$$\delta N_X \equiv N_X - \langle N_X\rangle$$

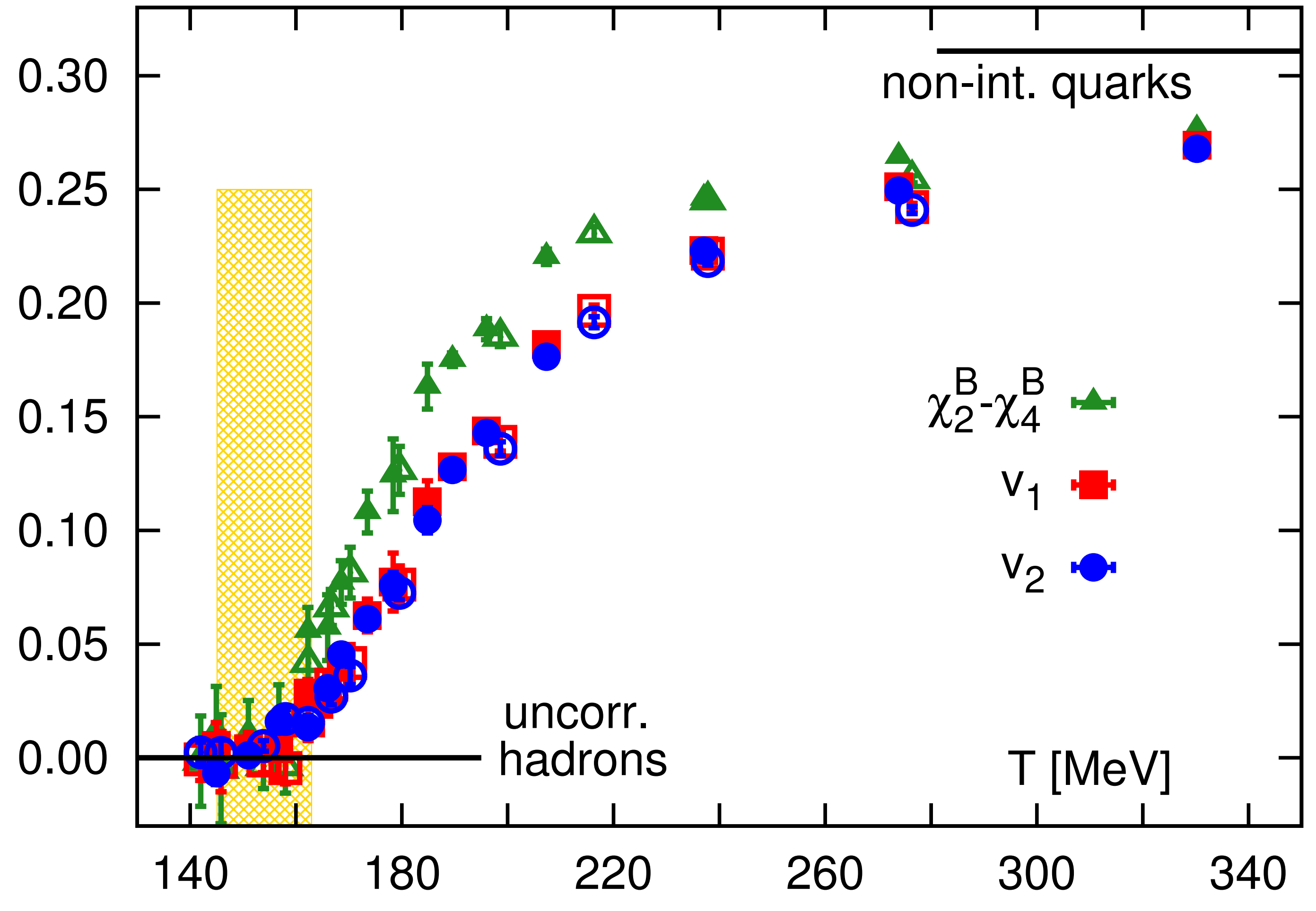
$\Rightarrow$ Measured at freeze-out (T^f, μ_B^f)

Strangeness carriers

The pressure obtains contributions from **4 different (B,S)-sectors:**

$$\frac{p}{T^4} = f_{(0,-1)}(T)\cosh(-\hat{\mu}_S) + f_{(1,-1)}(T)\cosh(\hat{\mu}_B - \hat{\mu}_S) + f_{(1,-2)}(T)\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + f_{(1,-3)}(T)\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

- Use $\chi_2^S, \chi_4^S, \chi_{11}^{BS}, \chi_{13}^{BS}, \chi_{31}^{BS}, \chi_{22}^{BS}$ to project onto the sectors
- Find 3 linear independent contributions that have to vanish as long as these changes are sufficient (constraints)



[Bazavov et al. '13]

- Melting of strange hadrons starts right after T_{pc}
- BS correlations are perturbative at $T \lesssim 200$ MeV

Charm carriers

The pressure obtains contributions from **3 different (B,C)-sectors:**

$$\frac{p}{T^4} = f_{(0,1)}(T)\cosh(\hat{\mu}_C) + f_{(1,1)}(T)\cosh(\hat{\mu}_B + \hat{\mu}_C) + f_{(\frac{1}{3},1)}(T)\cosh\left(\frac{1}{3}\hat{\mu}_B + \hat{\mu}_C\right)$$

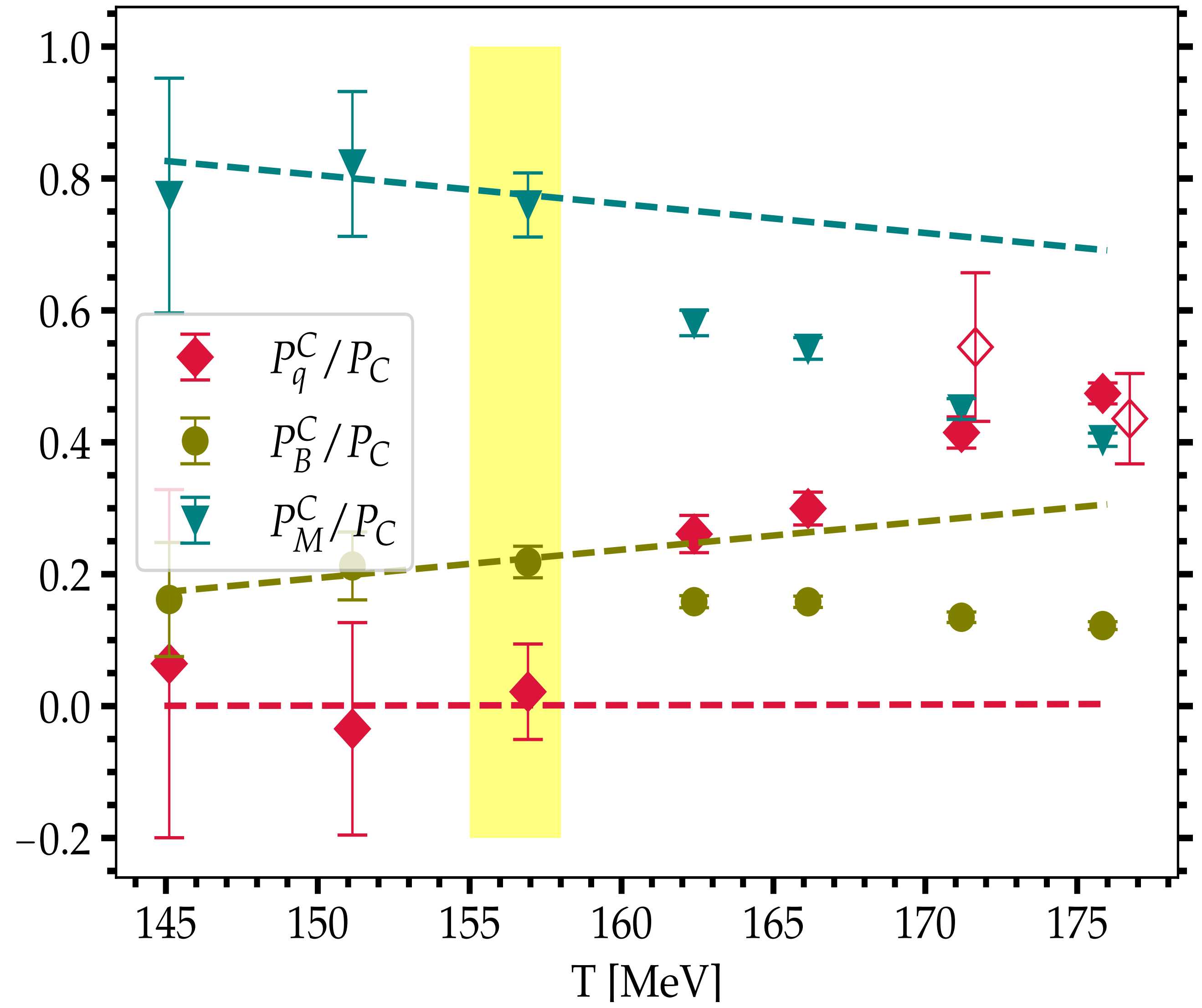
Mesons

Baryons

Quarks

- Use $\chi_2^C, \chi_4^C, \chi_{11}^{BC}, \chi_{13}^{BC}, \chi_{31}^{BC}, \chi_{22}^{BC}$ to project onto the sectors
- Find 4 constraints

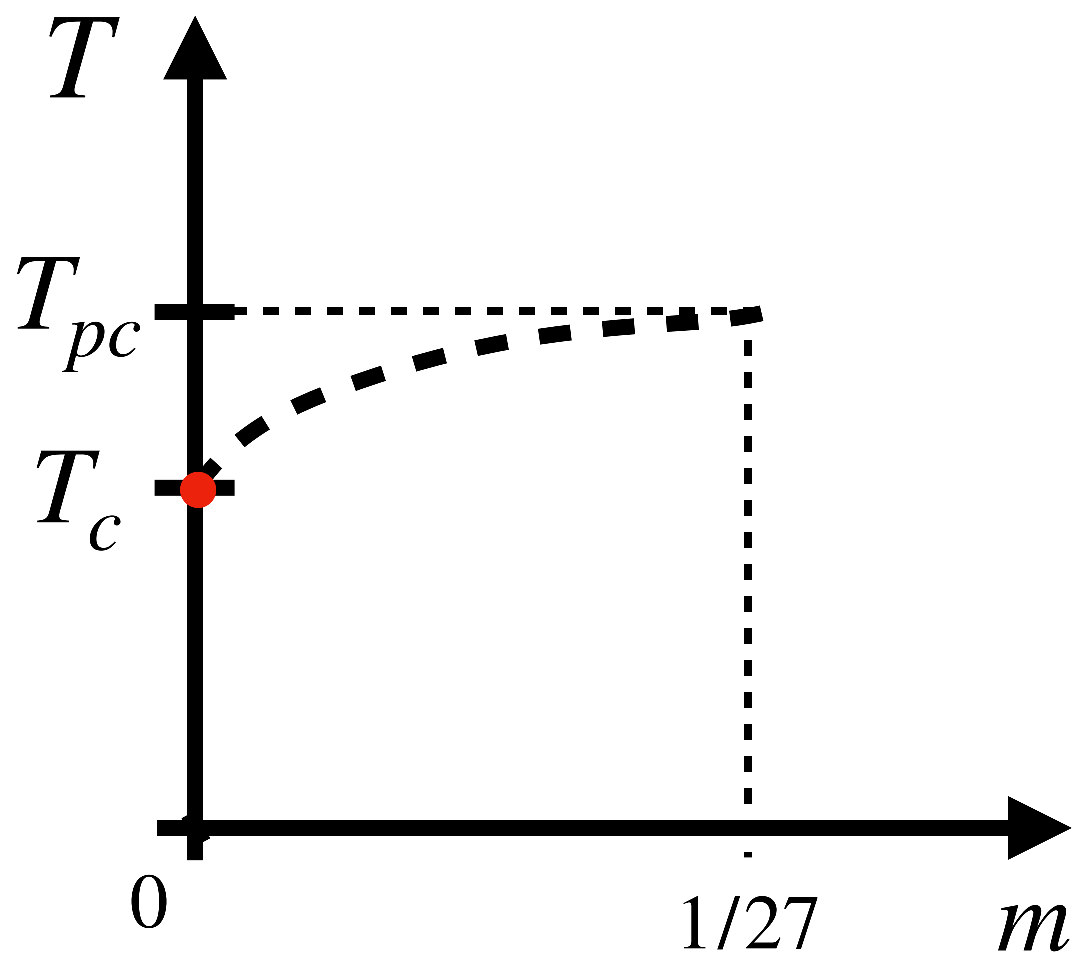
➡ Talk by S. Sharma [Bazavov et al. '24]



- Evidence of deconfinement in terms of presence of charm quark-like excitations
- At 1.1 T_{pc} quarks contribute already 50% to the total charm pressure

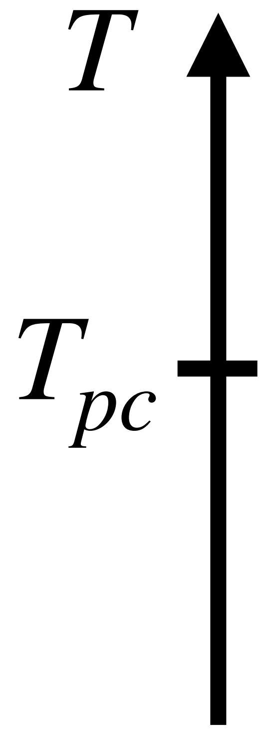
1.

Universal scaling and chiral transition at $\mu = 0$



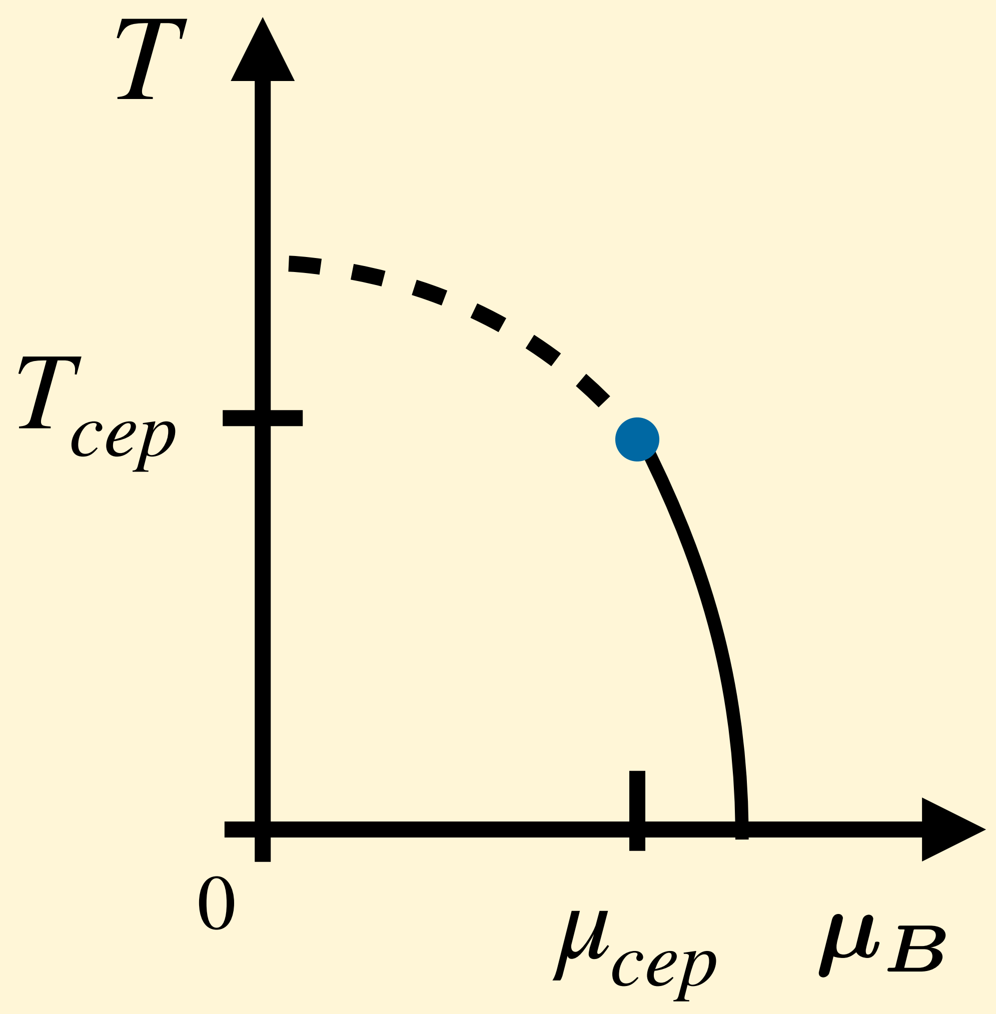
2.

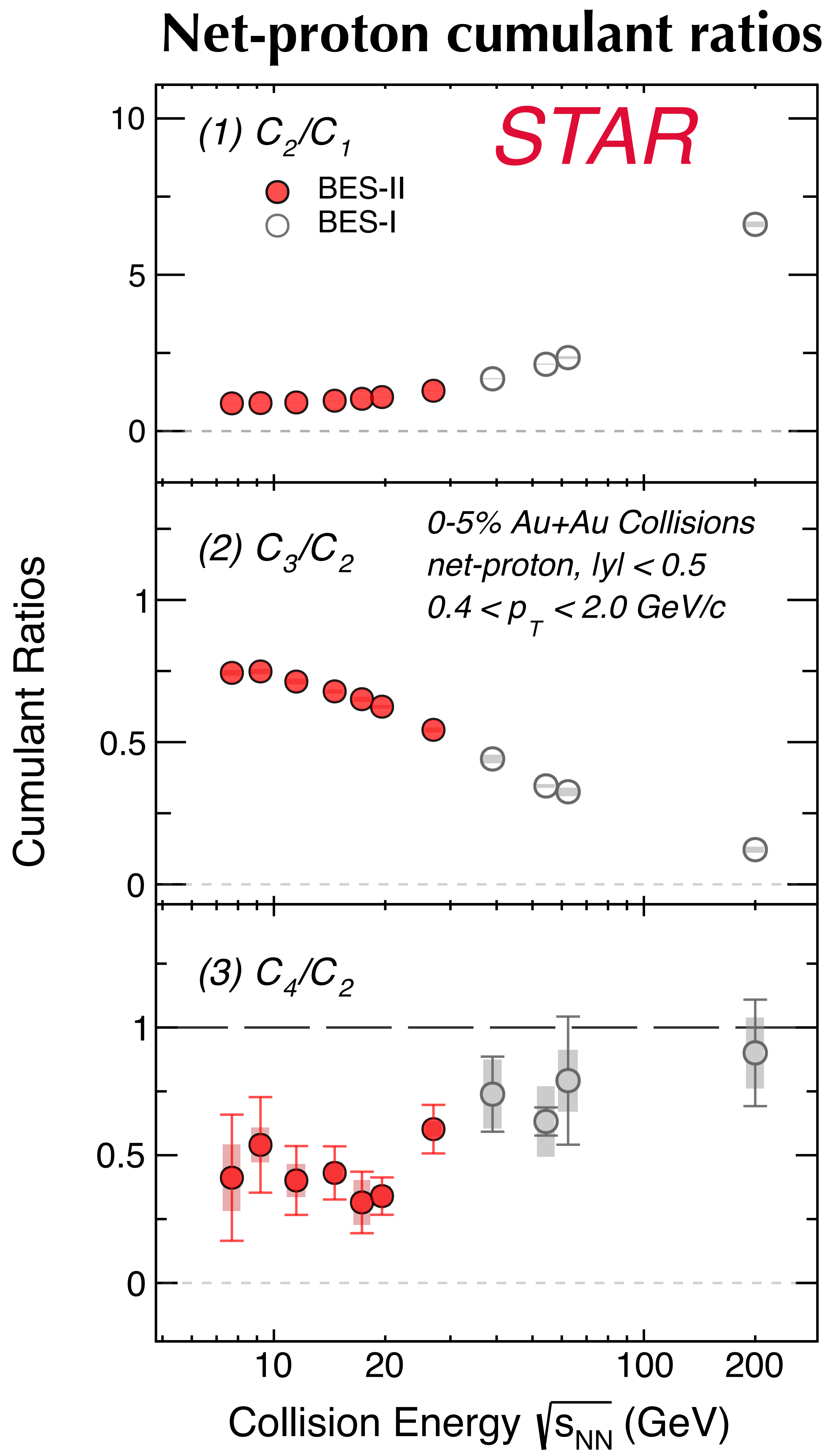
Aspects of deconfinement and melting of hadrons

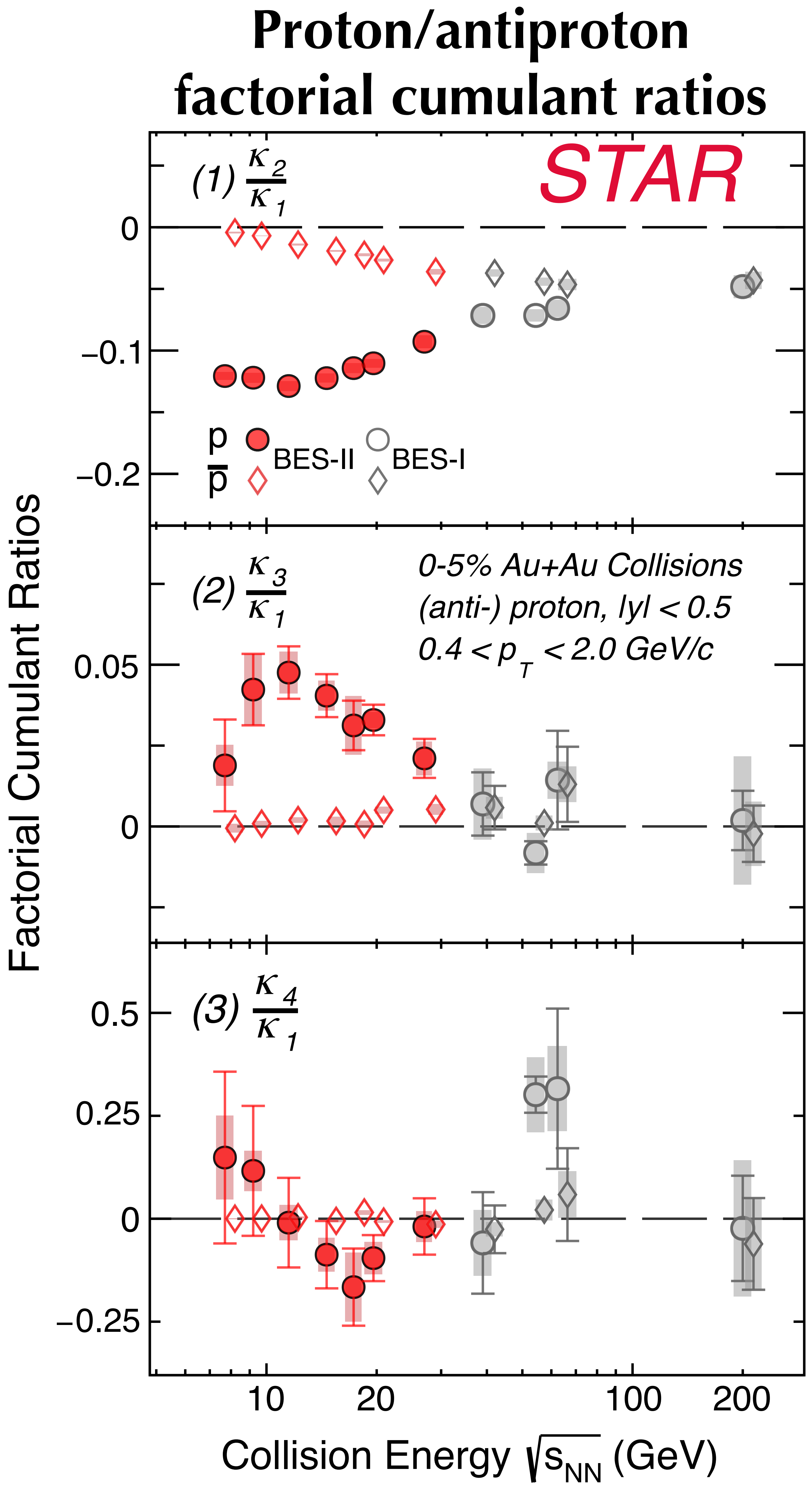


3.

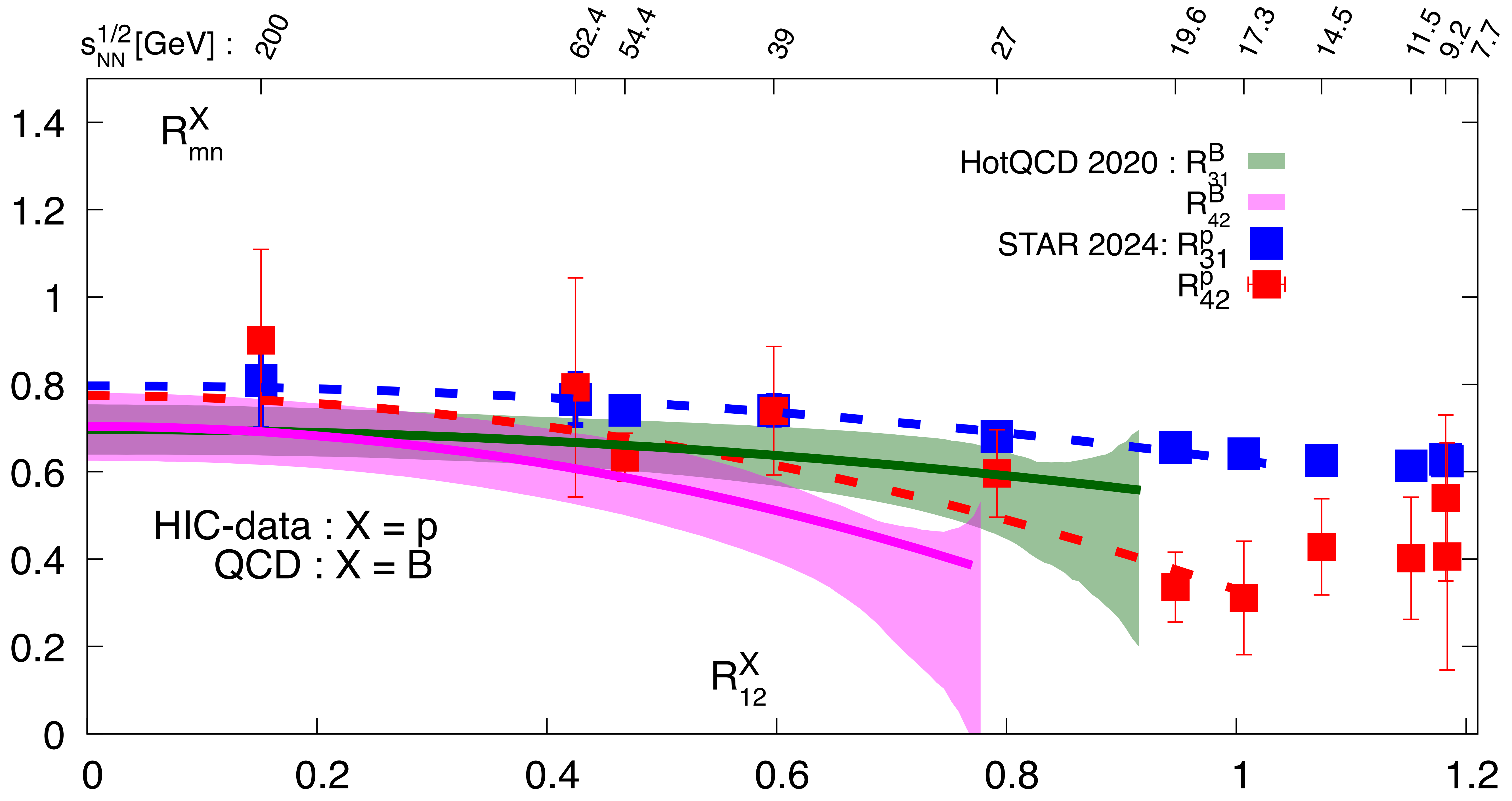
Beam energy scan results and the QCD critical point



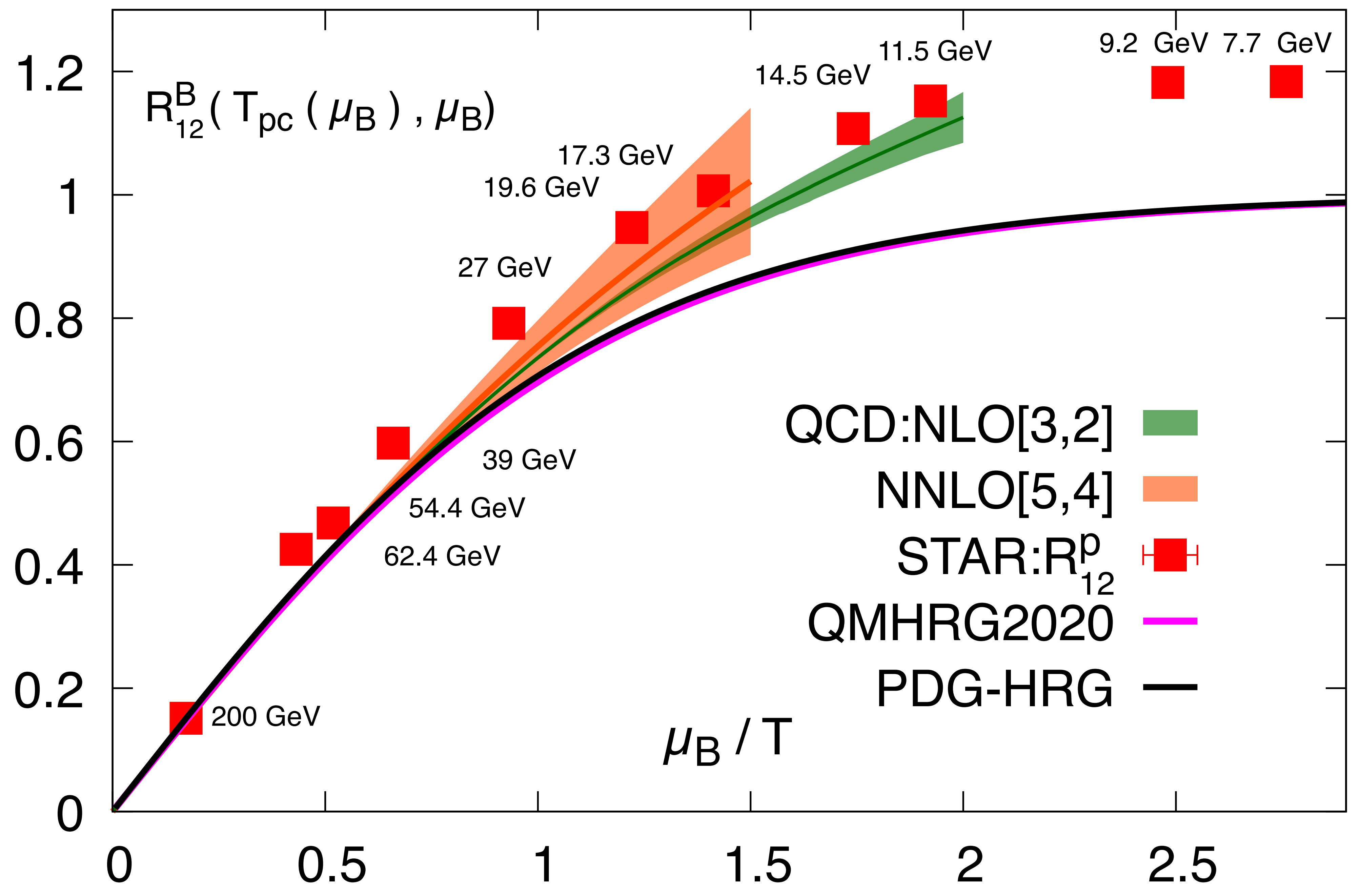




- Final BES-II data on proton number cumulants (in collider mode) was presented in CPOD '24 **[A. Pandav, CPDOD '24]**
- Error is significant reduced compared to the BES-I results
- Non-monotonic behaviour in C_4/C_2 is considered a signal for the QCD critical point **[Stephanov et al. '99]**
- Significance of non-monotonicity still needs to be estimated
- General structure is in agreement with the expectation: first a dip then a bump **[Stephanov '11]**



[Karsch, Goswami, XQCD '24]



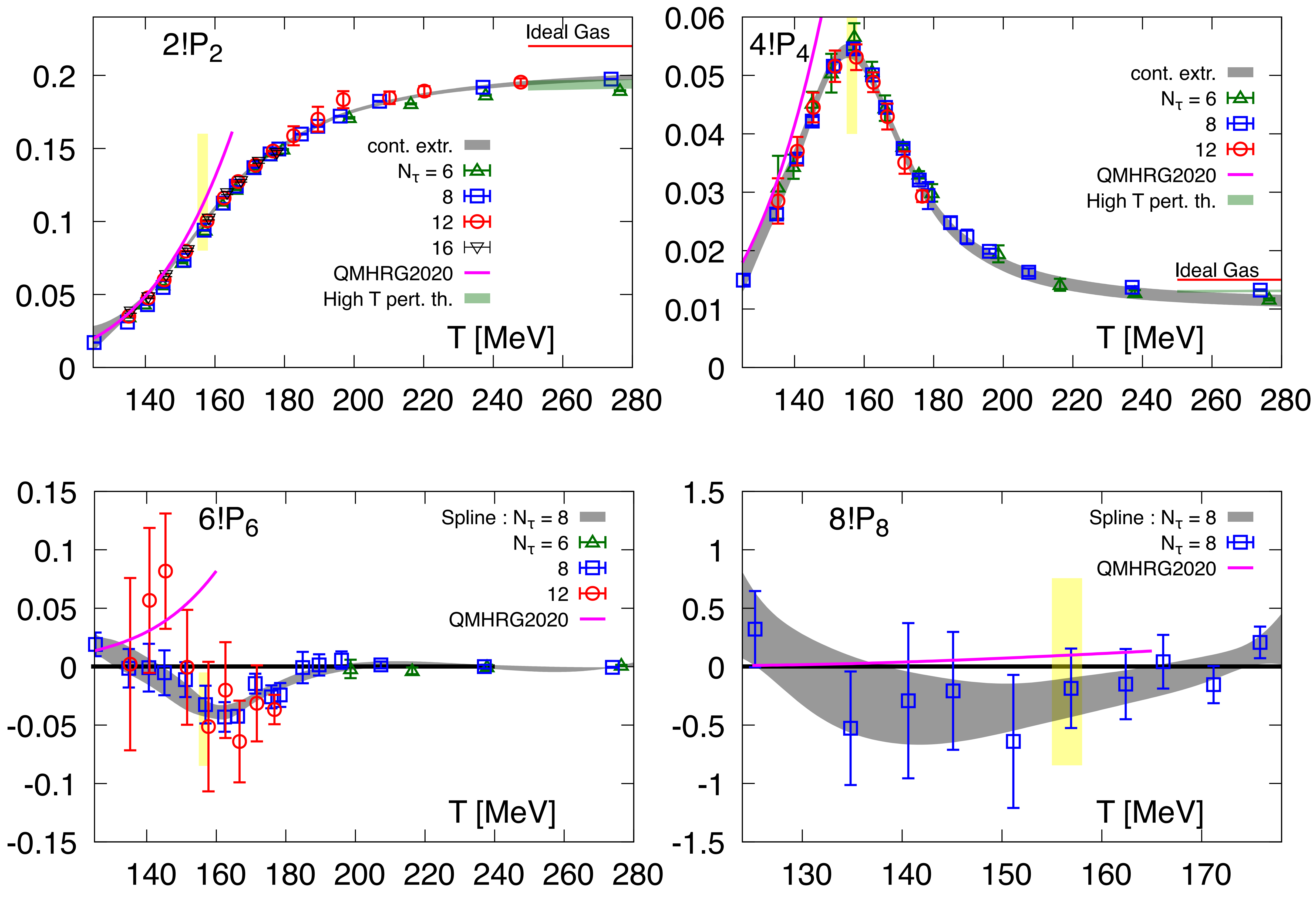
[Bollweg et al. '24]

- QCD and STAR results are in good agreement for $\sqrt{s_{NN}} < 19.6\ GeV$
- Slight vertical shift might suggest that freeze-out temperature is slightly below T_{pc}

- HRG model calculations based on noninteracting, point-like hadrons will always lead to $R^B_{12} < 1$
➡ HRG can not reproduce STAR data

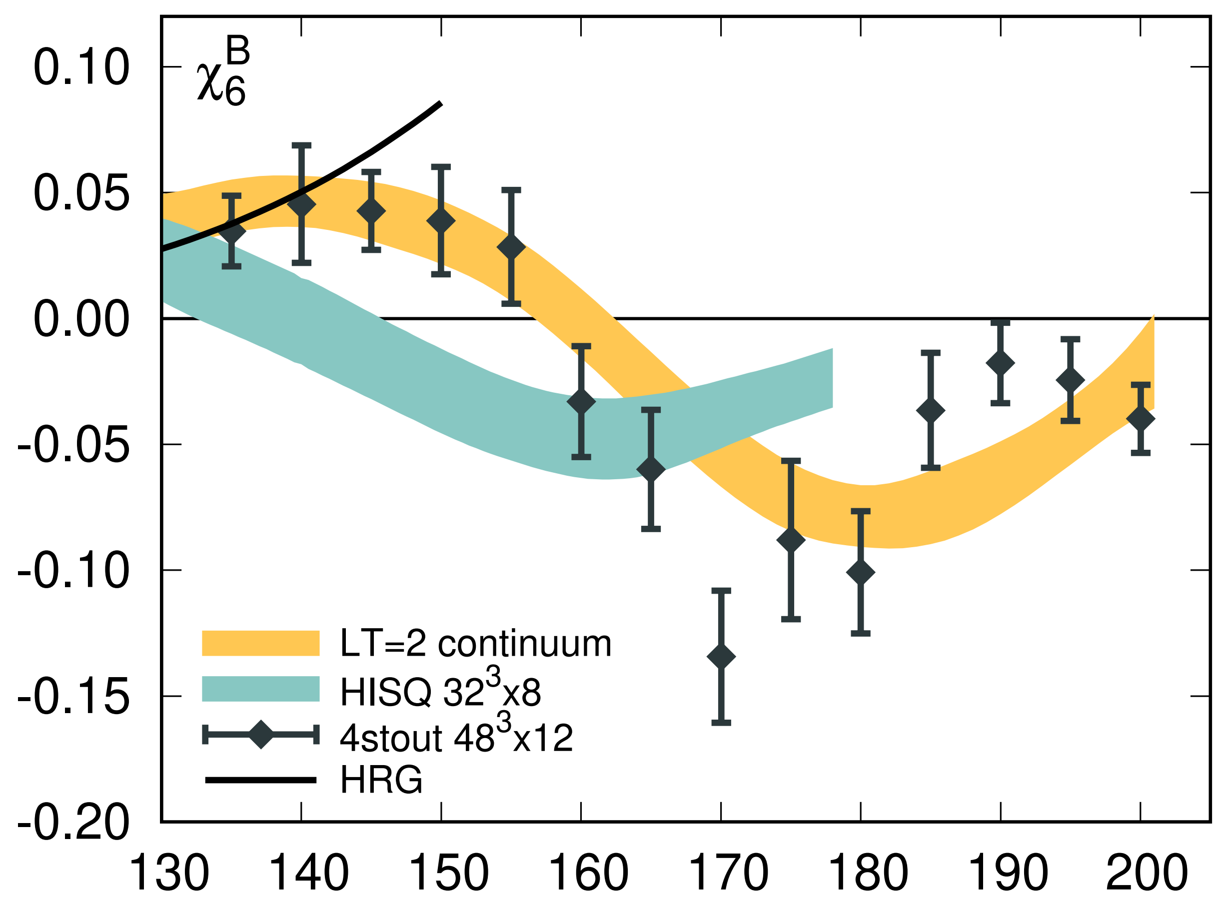
- Detecting phase transitions via Padé and post-Padé approximants has a long history in statistical and high energy physics
- They are often used in combination with perturbation theory
- QCD is non-perturbative in the vicinity of the phase
- The numerical calculation of the pressure series in μ_B is difficult

$$\Delta\hat{p} \equiv \frac{p(T,\mu_B)}{T^4} - \frac{p(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\hat{\mu}_B^{2k}$$



[HotQCD, PRD 108 (2023) 1, 014510, arXiv: 2212.09043]

➡ Talk by J. Guenther on continuum extrapolated χ_6^B and χ_8^B at LT=2



[Borsanyi et al. '24]

- Discrepancy between HotQCD and BW of unclear origin

➡ Talk by L. Pirelli on finite volume effects of chiral and deconfinement observables

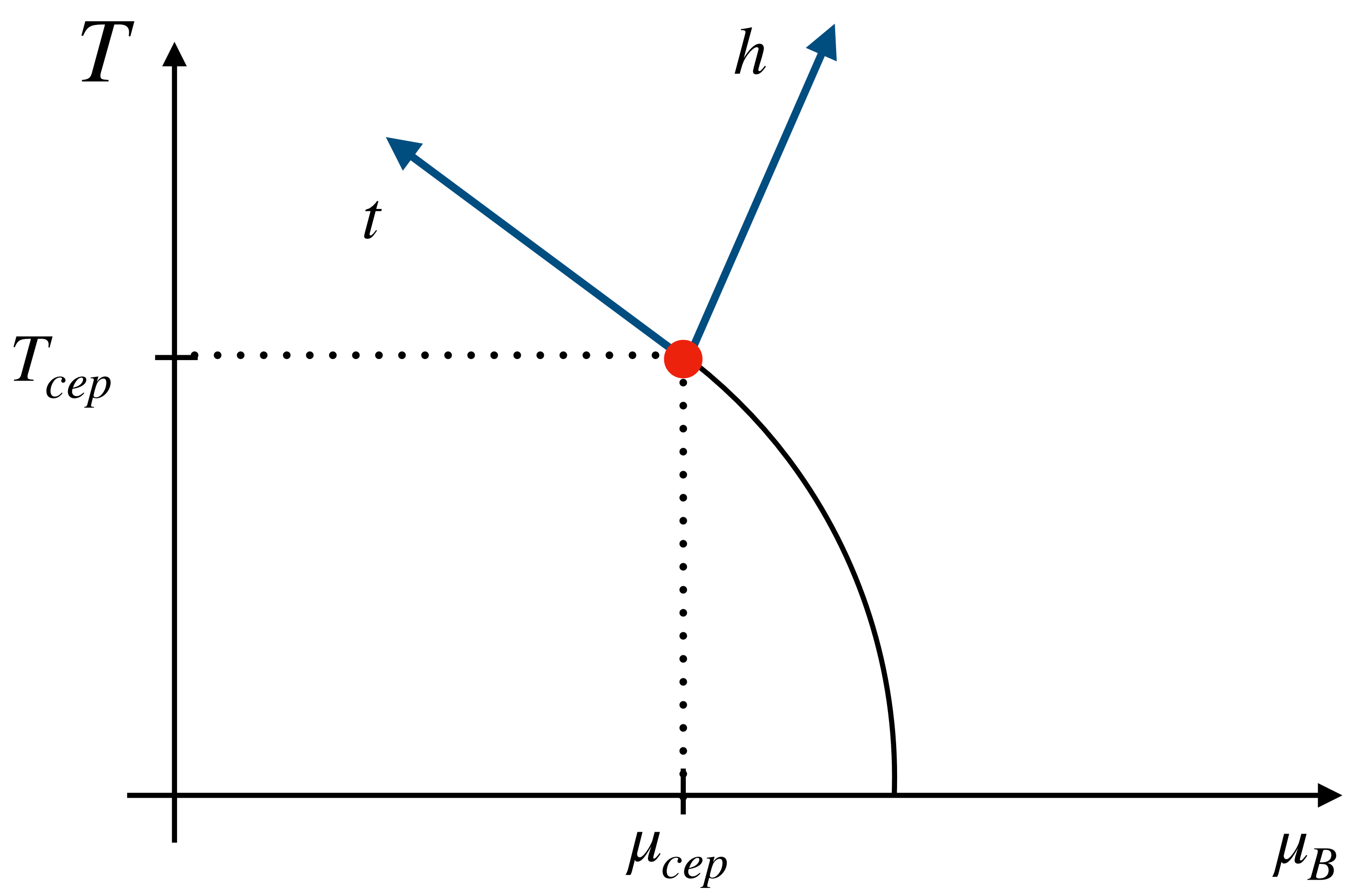
➡ Talk by J. Goswami on electric charge fluctuations with MDWF

Mixing of scaling fields:

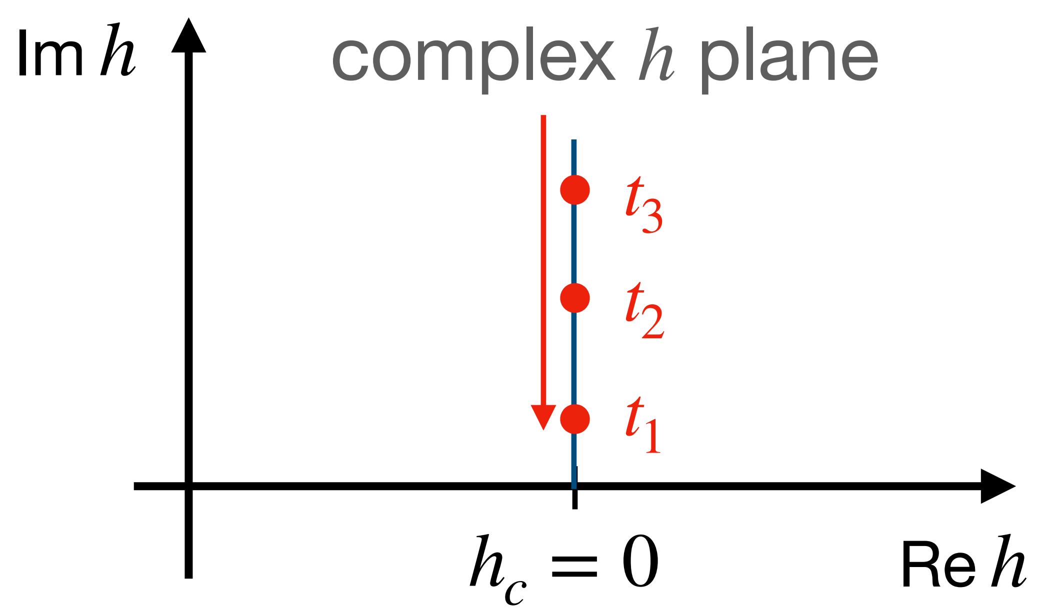
- Scaling fields are unknown, a frequently used ansatz is given by a linear mixing of T, μ_B

$$t = A_t \Delta T + B_t \Delta \mu_B,$$
$$h = A_h \Delta T + B_h \Delta \mu_B,$$

with $\Delta T = T - T^{\mathrm{CEP}}$ and $\Delta \mu_B = \mu_B - \mu_B^{\mathrm{CEP}}$



Lee-Yang edge:



- Poles approach critical point along imaginary h-axis **[Yang,Lee'59]**
- $t/h^{1/\beta\delta} = z_c$ is const. and universal

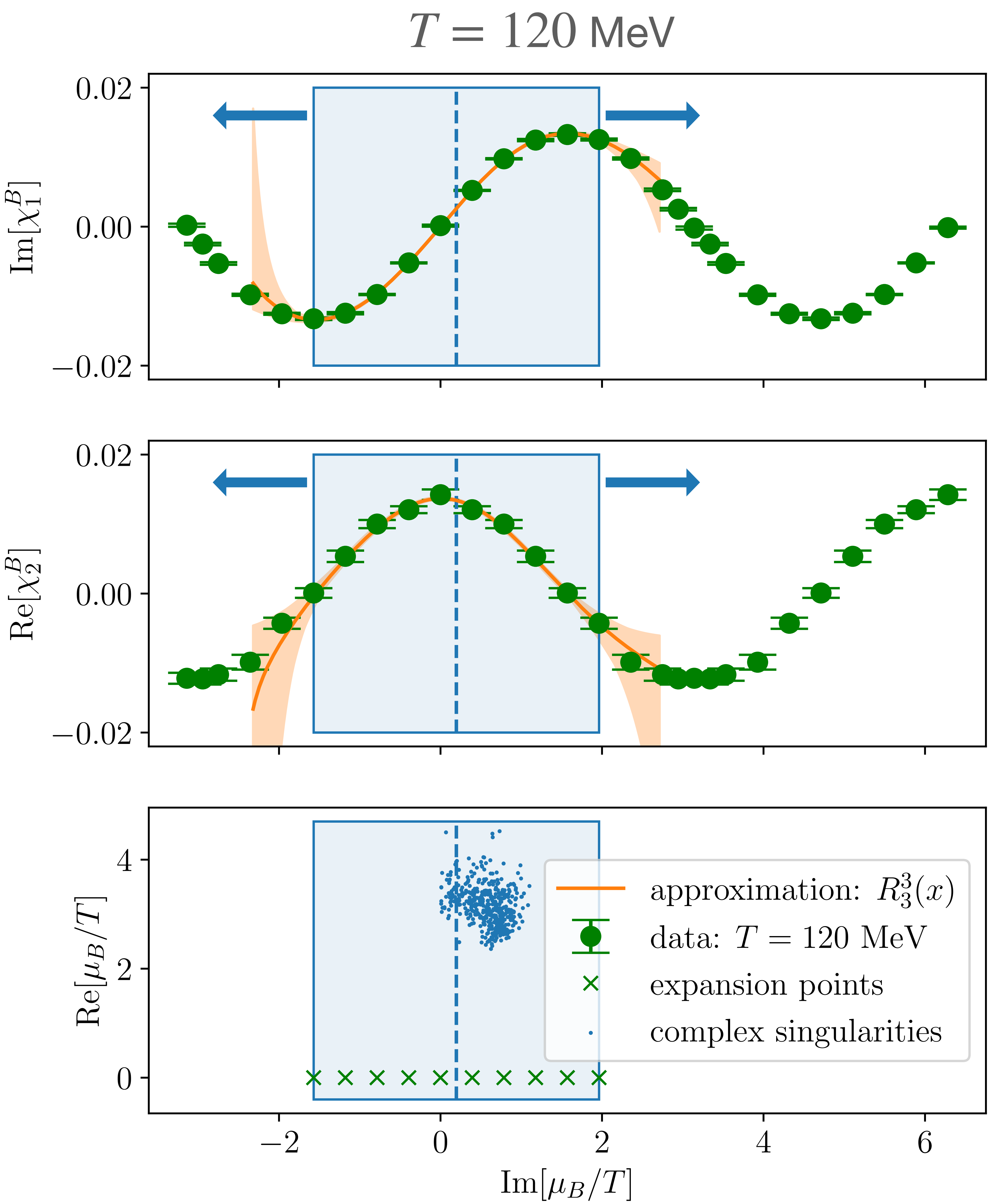
Fit Ansatz:

- For a constant $z = z_c$ we obtain

$$\mathrm{Re}[\mu_{\mathrm{LYE}}] = \mu_B^{\mathrm{CEP}} + c_1 \Delta T + c_2 \Delta T^2 + O(\Delta T^3)$$
$$\mathrm{Im}[\mu_{\mathrm{LYE}}] = c_3 \Delta T^{\beta\delta},$$

[Stephanov, Phys. Rev. D, 73.9, 094508 (2006)]

- The fit parameter c_1 gives the (inverse) slope of the 1st order line at the critical point: $c_1 = -A_h/B_h$

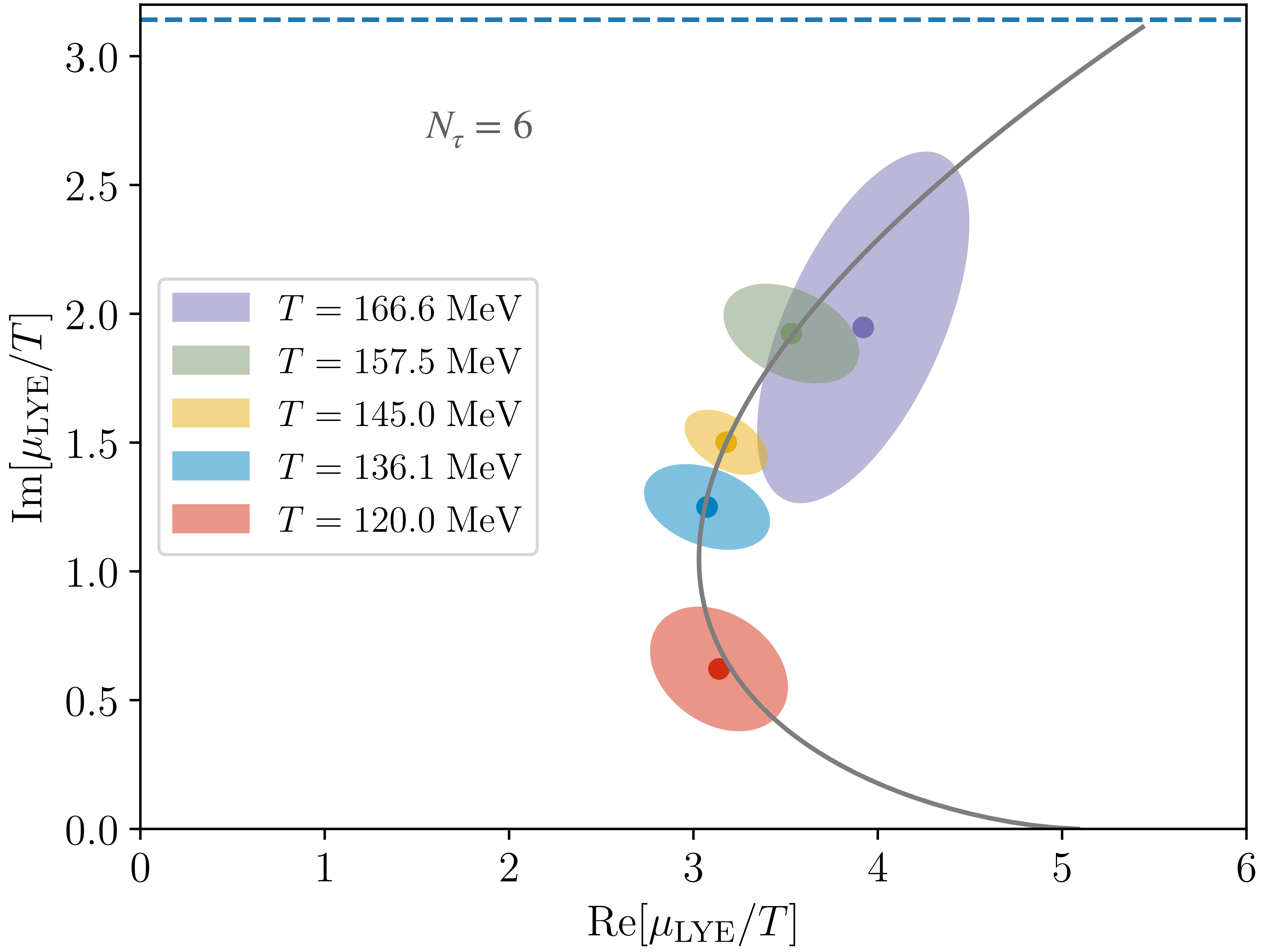


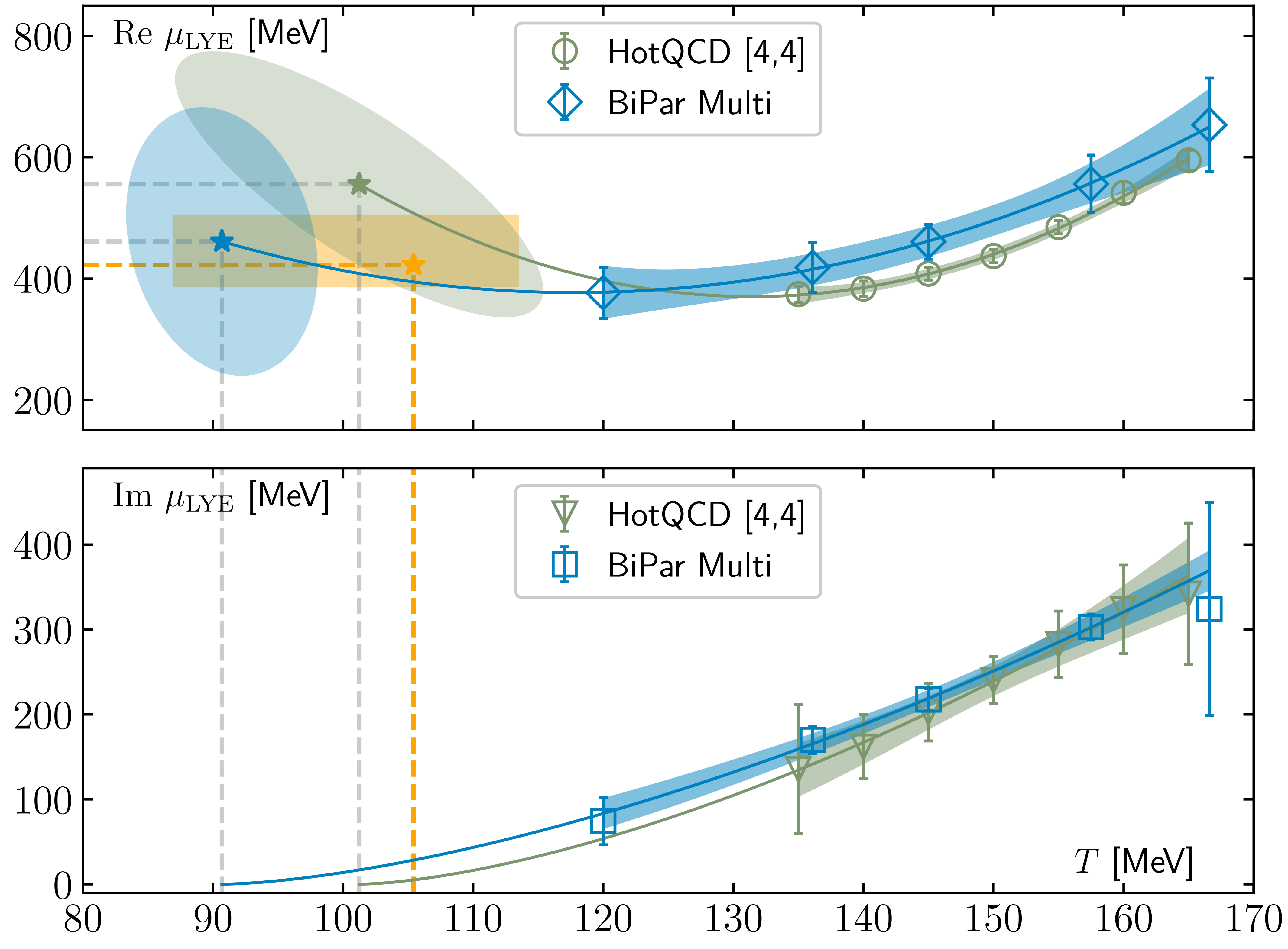
[Clark et al. (BiePar), arXiv: 2405.10196]

Procedure:

- Perform simultaneous fits to χ_1^B and χ_2^B for each temperature
- Use [3,3]-Padé
- Varry length of the fit interval in $[\pi, 2\pi]$ and the center of the interval in $[-\pi/2, +\pi/2]$
- bootstrap over the data by assuming independent and normal distributed errors
- Calculate roots of the denominator and keep only roots in the first quadrant
- Collect all the results for Lee-Yang scaling fits. We have 55 different intervals per temperature.

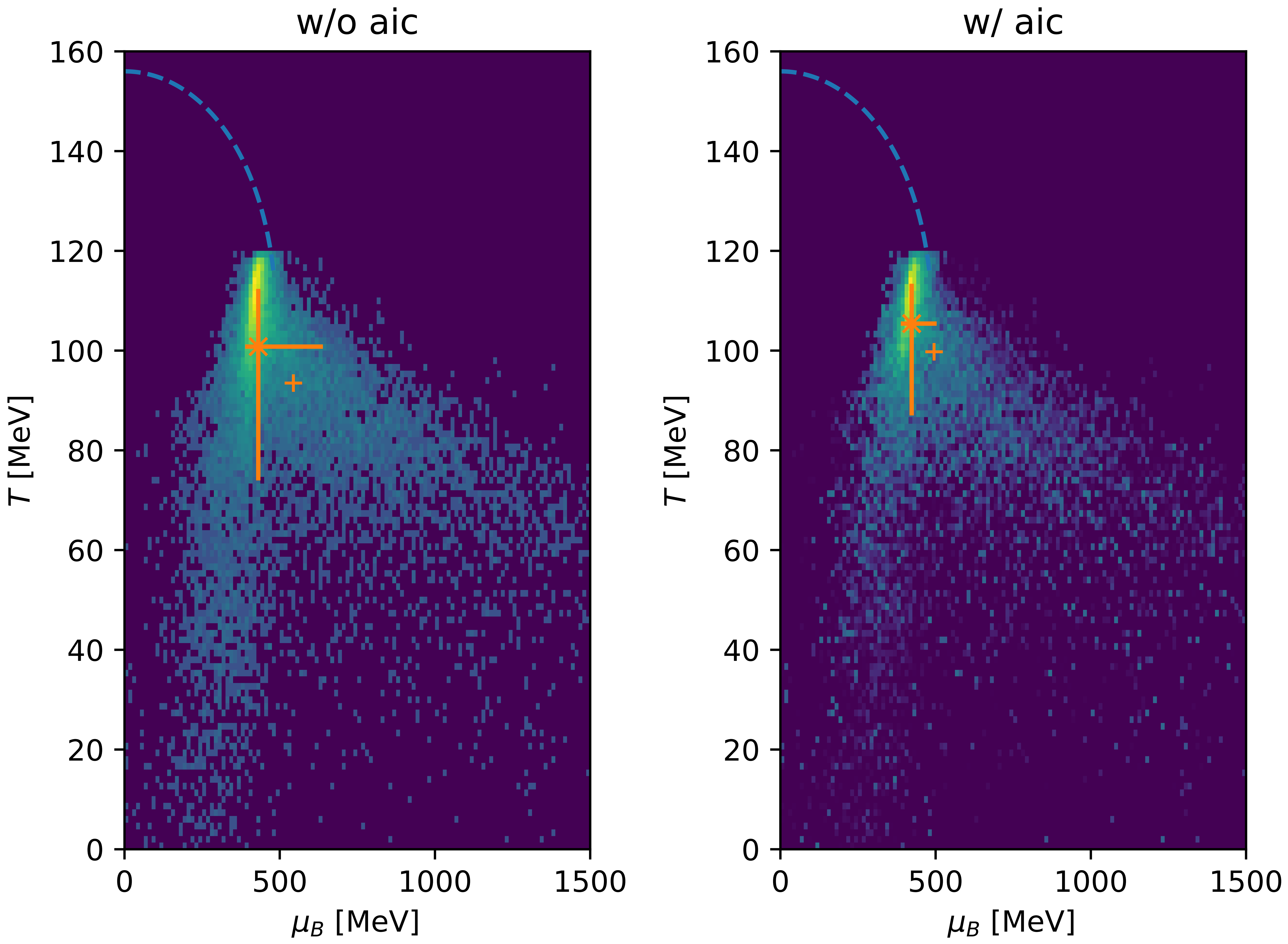
- Perform one fit for $N_\tau = 8$ (LT=4) and $\mathcal{O}(10^5)$ fits for $N_\tau = 6$ (LT=6)





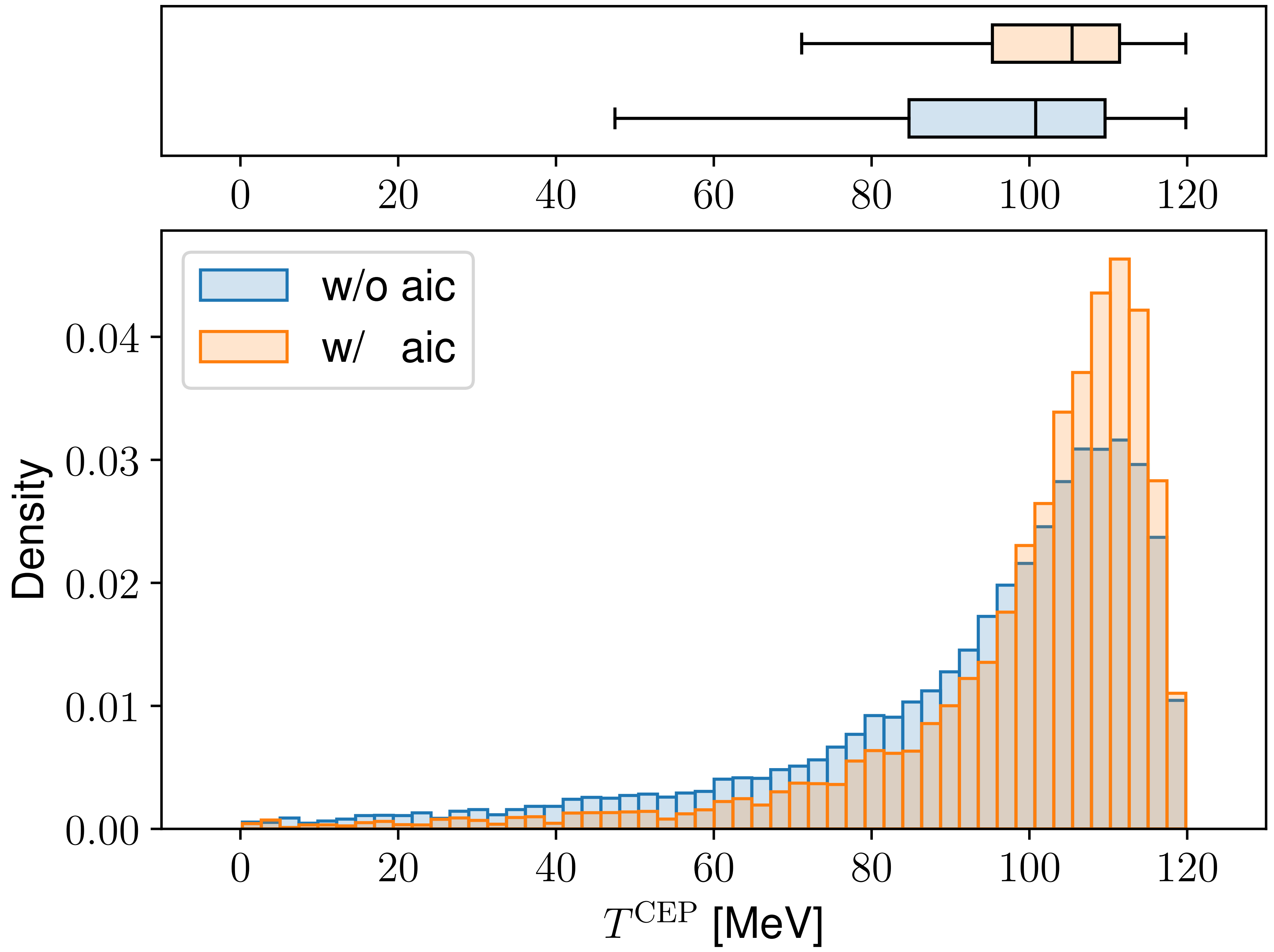
$$\mathrm{Re}[\mu_{\rm LYE}] = \mu_B^{\rm CEP} + c_1 \Delta T + c_2 \Delta T^2$$
$$\mathrm{Im}[\mu_{\rm LYE}] = c_3 \Delta T^{\beta\delta},$$

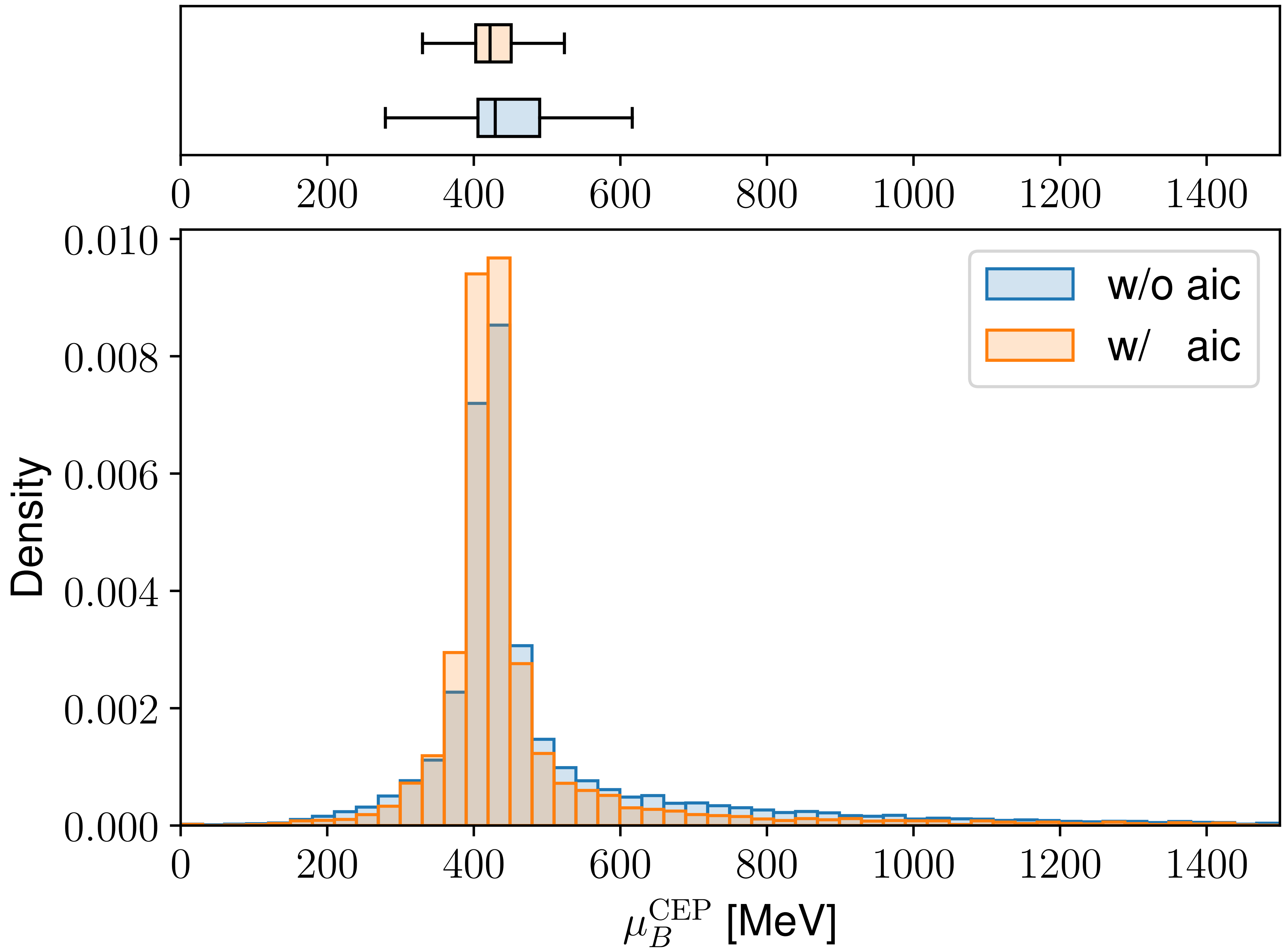
- Ellipses show 1σ confidence region, using the Pearson correlation coefficient
- $N_\tau = 6$ singularities show here are chosen on the basis of the χ^2 of the scaling fit ("best fit")
- Orange box shows the AIC weighted result for $N_\tau = 6$, based on $\mathcal{O}(10^5)$ scaling fits



- Histogram over the T^{CEP} and μ_B^{CEP} from the $\mathcal{O}(10^5)$ fits
- Error bars are based on the inner 68-percentile
- Observe interesting structure
- Dashed line indicates the continuum extrapolated crossover line

[Clark et al. (BiePar), arXiv: 2405.10196]





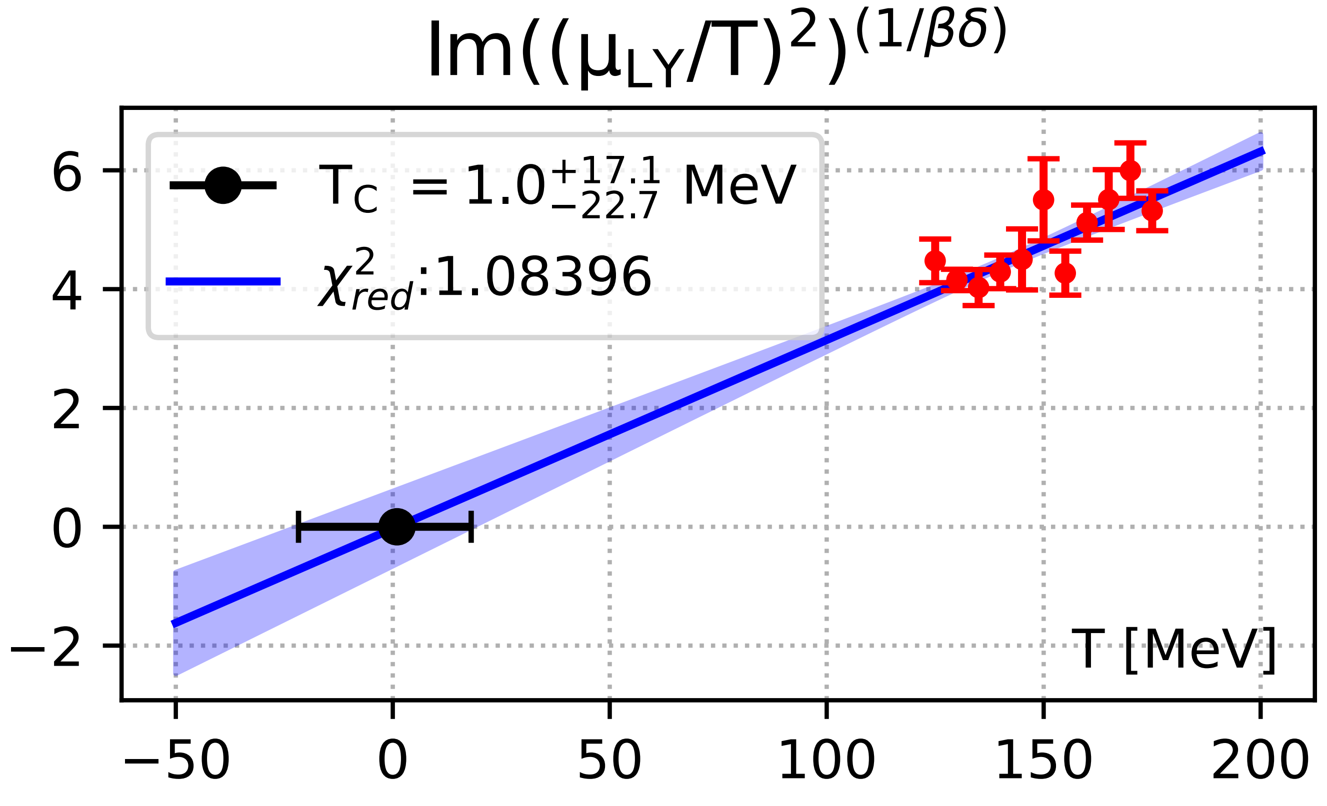
	$\mathbf{N_\tau = 6}$ **multi-point Padé**			$\mathbf{N_\tau = 8}$ **[4,4]-Padé**		
	T^{CEP} [MeV]	μ_B^{CEP} [MeV]	μ_B/T	T^{CEP} [MeV]	μ_B^{CEP} [MeV]	μ_B/T
best fit	90.7 ± 7.7	461.2 ± 220	5.09 ± 0.68	101 ± 15	560 ± 140	5.5 ± 1.7
weight-1	$105.4 + 8.0 - 18.4$	$422.9 + 80.5 - 34.9$	$3.92 + 1.52 - 0.24$			
weight-2	$100.8 + 11.6 - 26.8$	$430.9 + 208.2 - 42.2$	$4.20 + 4.13 - 0.47$			
	c_1	c_2	c_3	c_1	c_2	c_3
best fit	-6.2 ± 9.2	0.115 ± 0.090	0.424 ± 0.086	-12.3 ± 8.1	0.203 ± 0.059	0.55 ± 0.25

- For $N_\tau = 8$: similar results by Basar, based on the same HotQCD data **[Basar, arXiv: 2312.06952]**

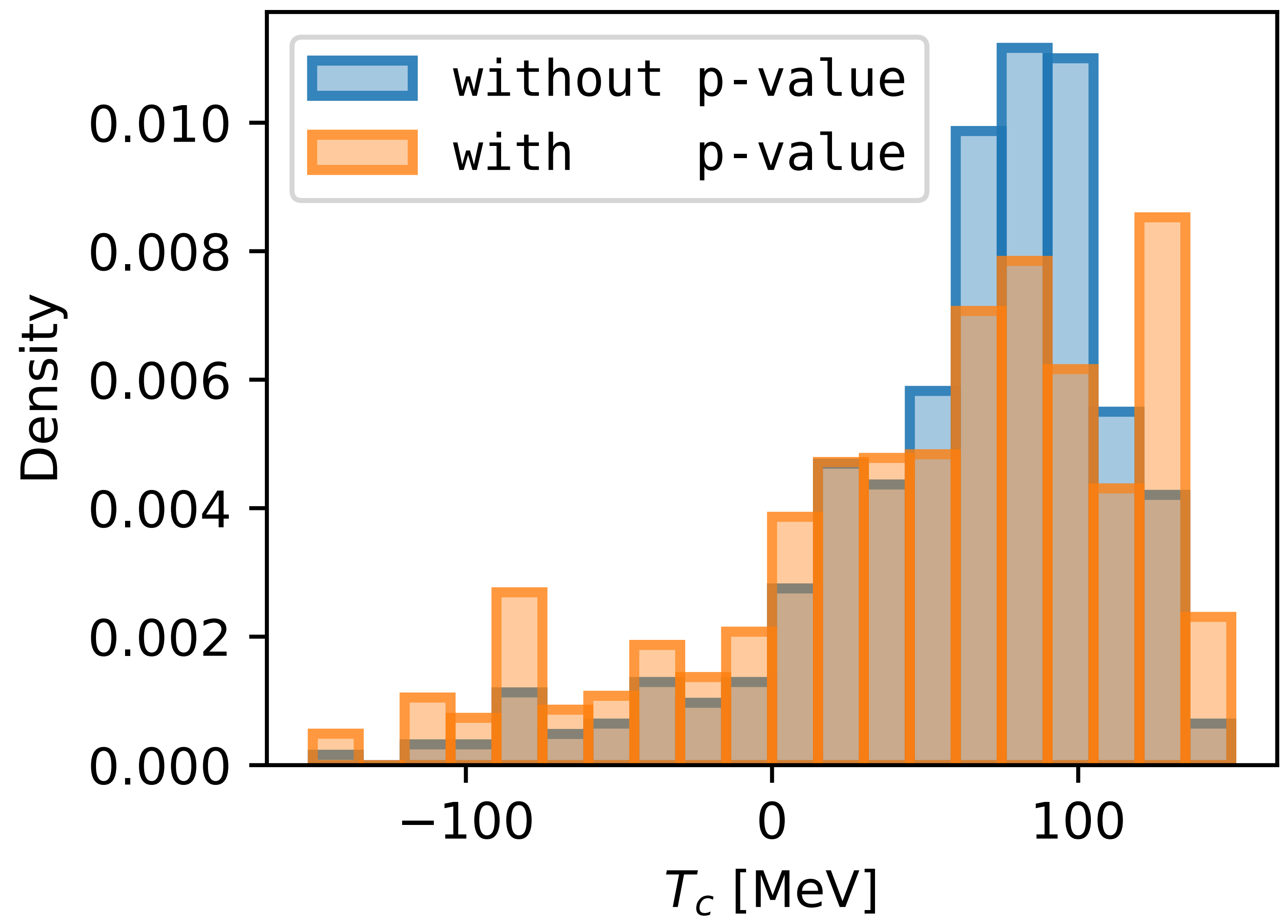
➡ **Talk by A. Adam: single point Padé approximation of the pressure based on $\chi_2^B, \chi_4^B, \chi_6^B, \chi_8^B$ (LT=2)**

- Interesting to check results with the Budapest-Wuppertal data
- Preliminary results for single Point Padé analysis on $16^3 \times 8$ lattices, multi-Point is work in progress
- Preliminary result on the transition temperature based on extrapolations 432 on different approximations and fit ranges
- T^{CEP} around 90 MeV, in agreement with [BiePar, 2405.10196]
- Results are very sensitiv to noise

One example of the extrapolations



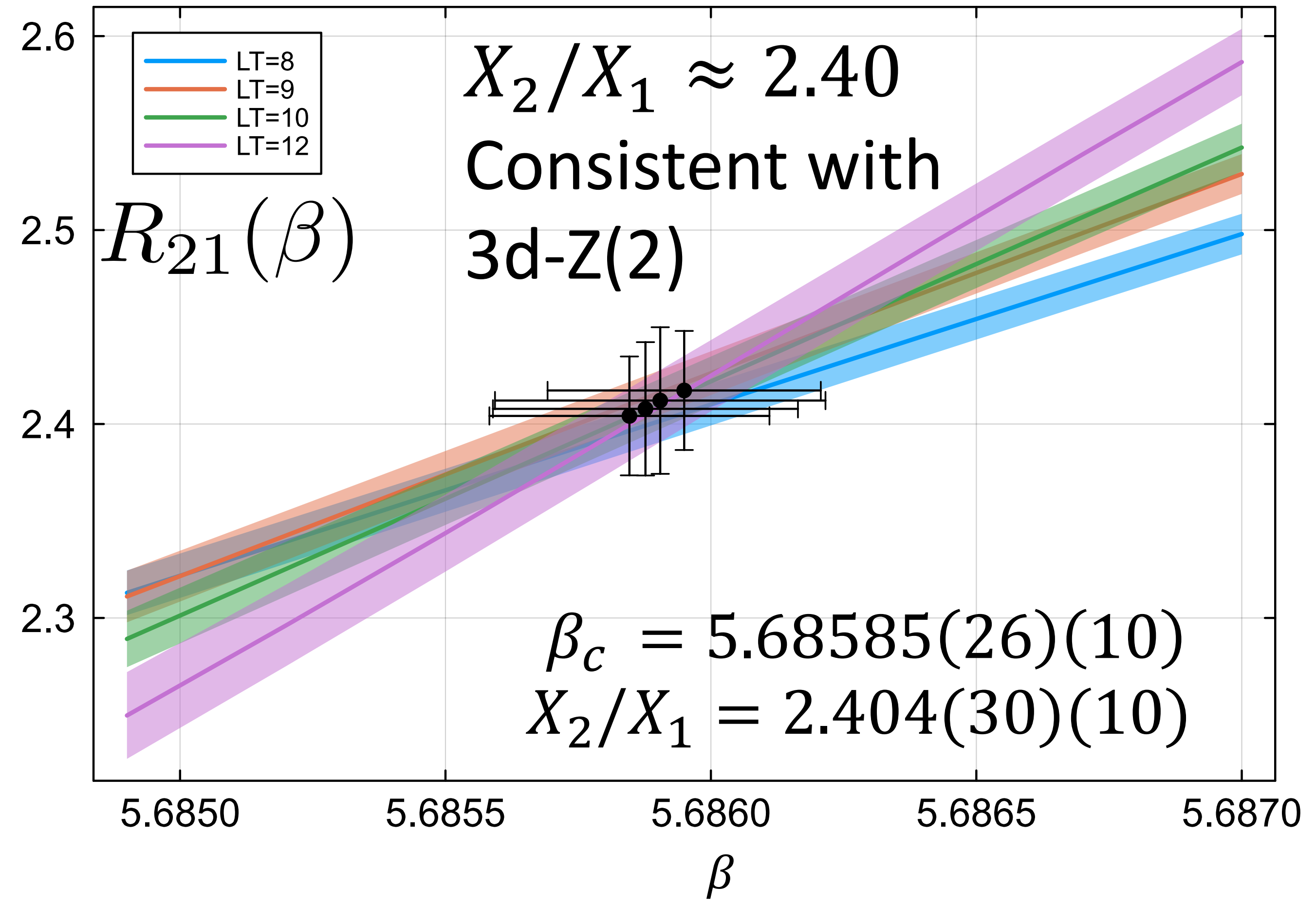
Histogram of the T^{CEP} results



➡ **Talk by T. Wada: Finite size scaling of Lee-Yang zeros in 3d Pots model and heavy-quark QCD**

- Construct ratios of the LYZ locations
- Scaling is in accordance with a ratio of scaling function: non-universal pre-factors cancel, intersection point of different volumes is universal.
- Ratios show reduced corrections to scaling and regular parts
- T^{CEP} is shifted to higher values, results from extrapolation of first LYZ can serve as a lower bound

2nd/1st LYZ Ratio

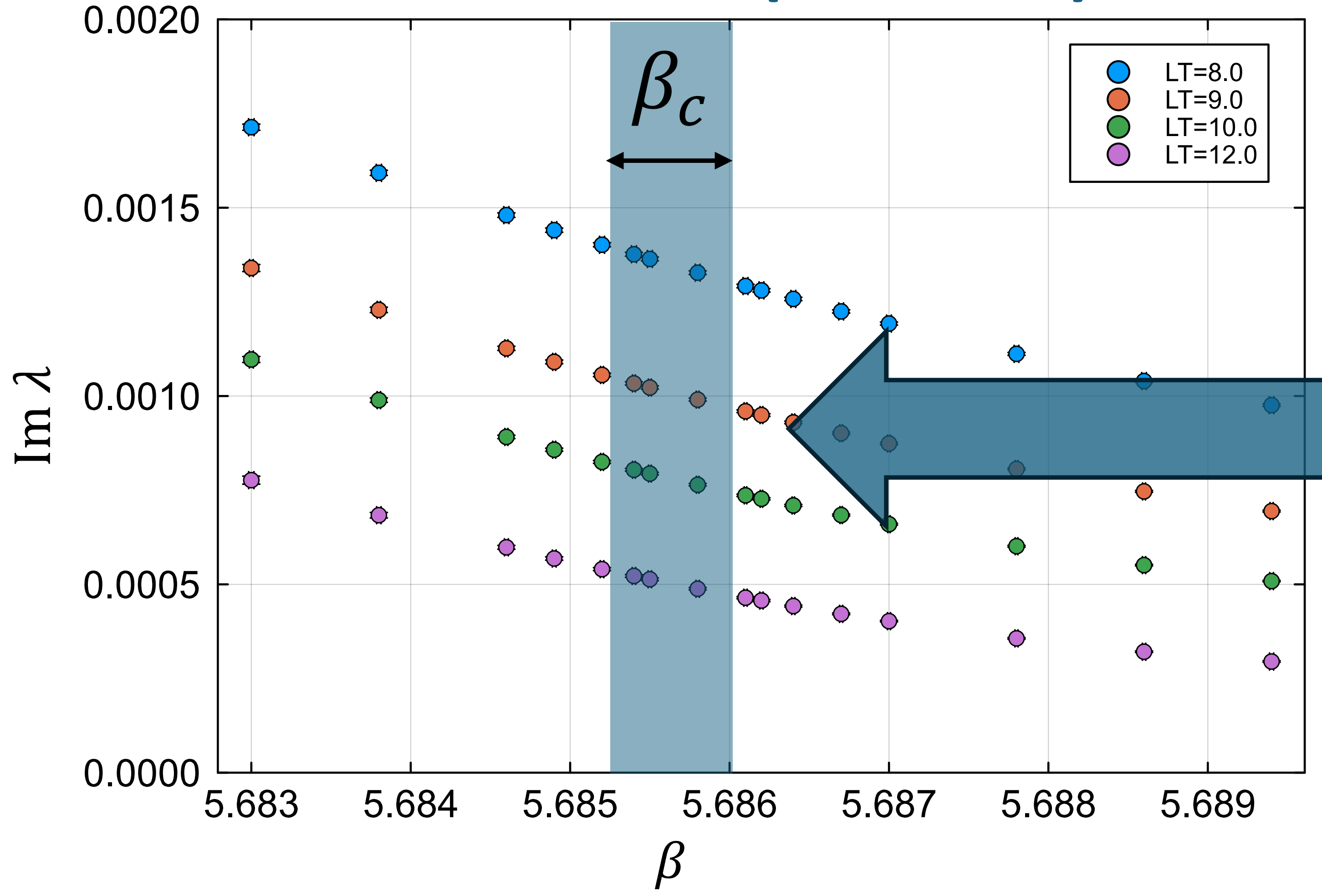


$$R_{12}(t) = \frac{h_{LY}^{(2)}(t)}{h_{LY}^{(1)}(t)}$$

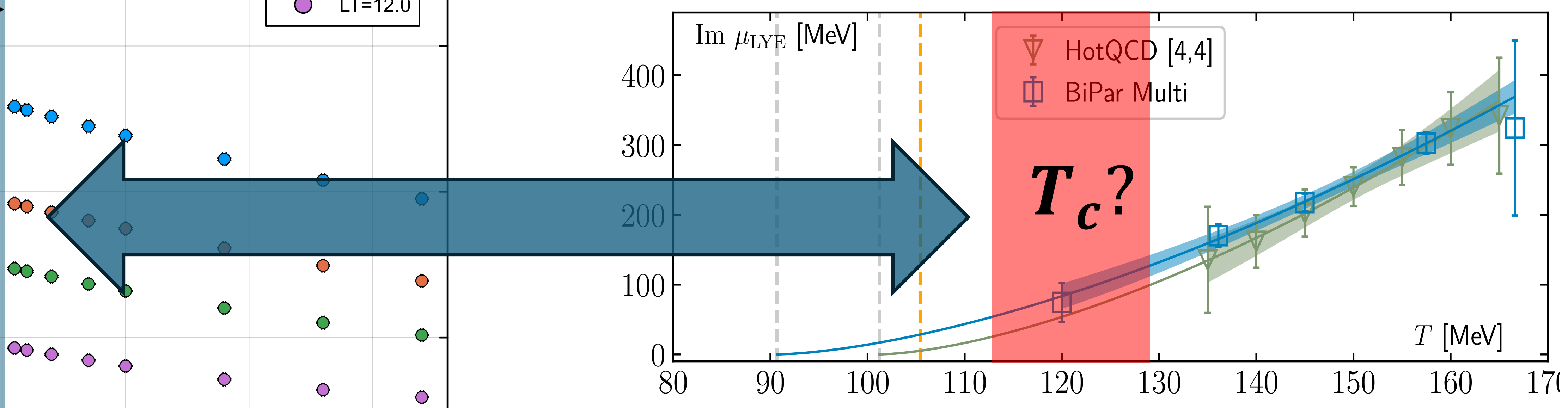
$$= \frac{X_1}{X_2}(1 + C(tL^{y_t}))\underbrace{(1 + D(tL^{2(y_t - y_h)}))}_{\text{mixing with temperature like}}$$

+ higher orders

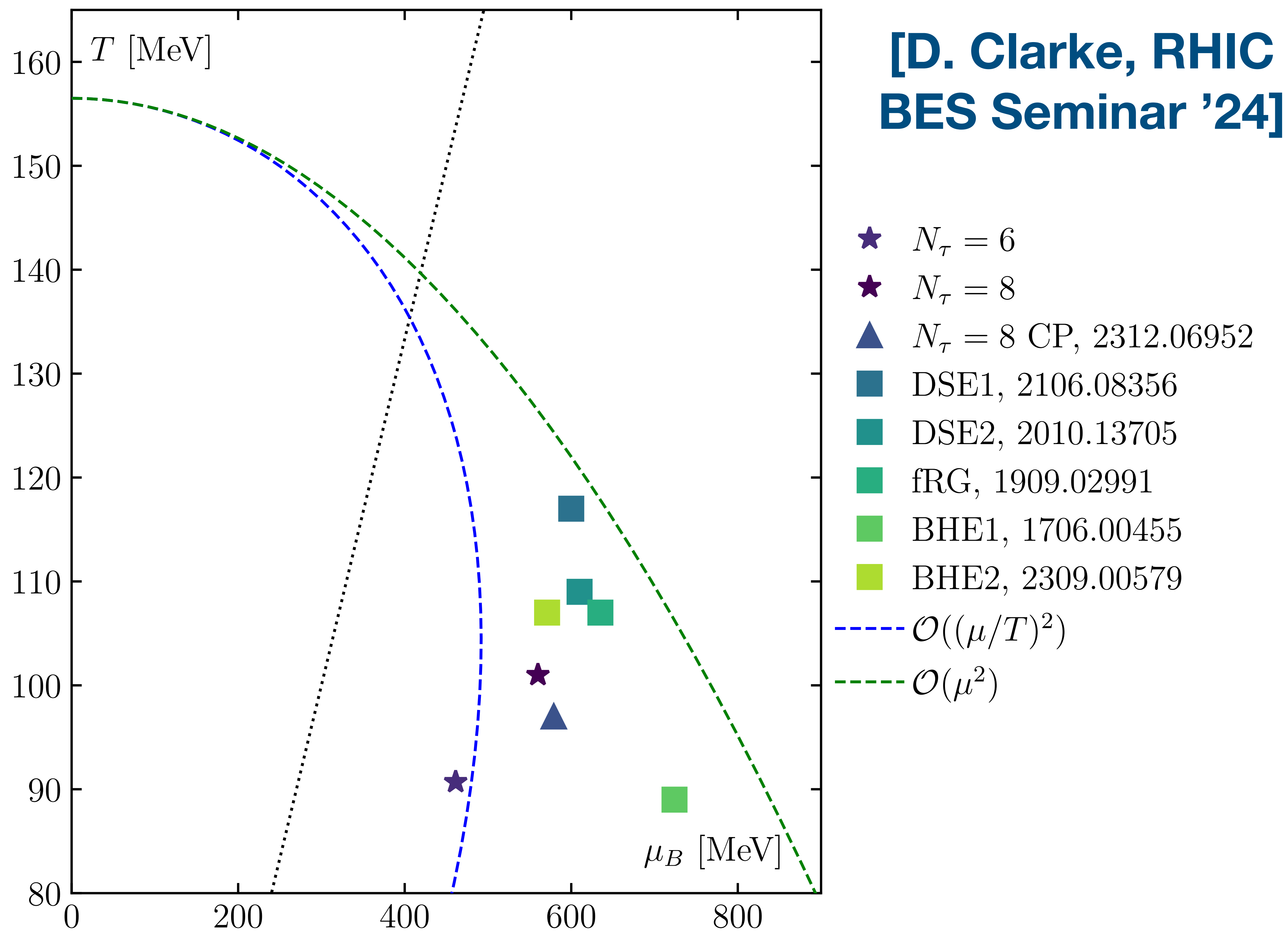
1st LYZ data (HQ-QCD)



$N_f = 2 + 1$ (Clarke, *et al.*)



- No continuum estimate of the Lee-Yang zero extrapolation yet (possible large systematic errors)
  ➡ Finite volume and finite cut-off effects will increase both T^{CEP} and μ_B^{CEP}
- Results from other approaches seam to cluster in a narrow region (DSE, fRG, BHE)



[D. Clarke, RHIC BES Seminar '24]

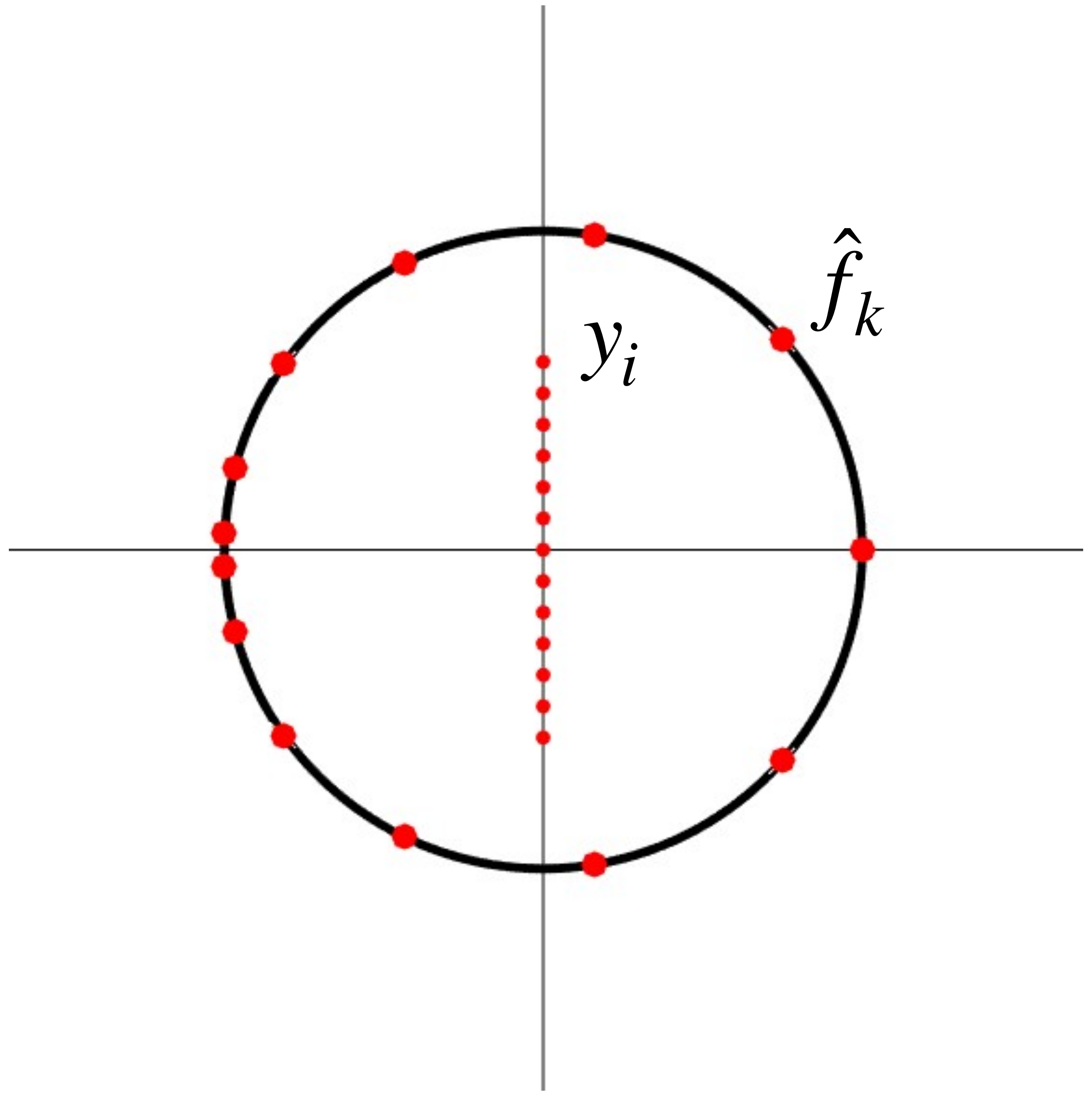
Parametrizations of the crossover line:

1.) $$T_{\rm pc}(\mu_B) = T_{\rm pc}(0)\left[1 + \kappa_2^B\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^B\left(\frac{\mu_B}{T}\right)^4\right]$$

2.) $$T_{\rm pc}(\mu_B) = T_{\rm pc}(0)\left[1 + \bar{\kappa}_2^B\left(\frac{\mu_B}{T_{\rm pc}(0)}\right)^2 + \bar{\kappa}_4^B\left(\frac{\mu_B}{T_{\rm pc}(0)}\right)^4\right]$$

- $\kappa_2 = \bar{\kappa}_2 = -0.015(1)$ **[HotQCD, 2403.09390]**
- Many results seem to favour a small $\bar{\kappa}_4 \approx -0.0002(1)$

- ➡ Talk by M. Aliberti on Taylor coefficients from imaginary μ data
- ➡ Talk by F. Di Renzo on analytic continuations from imaginary μ data



- New method to calculate Taylor coefficients and analytic continuations from discreet data at imaginary μ, based on Cauchy's formula

$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0} \, dz$$

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} \frac{f(R\, e^{i\theta})\, R\, e^{i\theta}}{R\, e^{i\theta} - z_0} \, d\theta \simeq \frac{1}{2\pi} \sum_{k=1}^{n} w_k \frac{f(R\, e^{i\theta_k})\, R\, e^{i\theta_k}}{R\, e^{i\theta_k} - z_0}$$

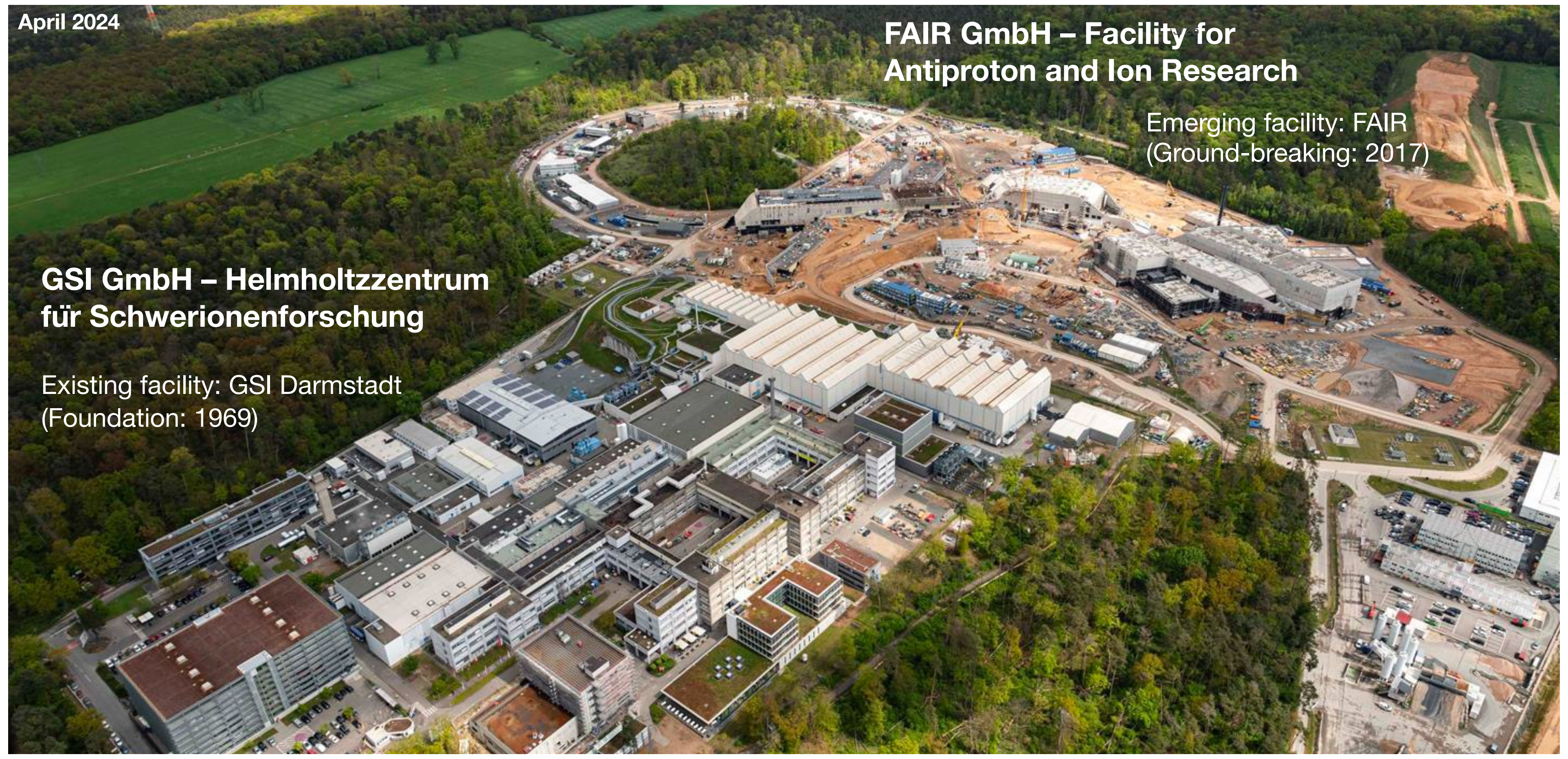
- Once the function on the contour is known, every point in the interior is known
  - ➡ Replace the integral by a Quadrature rule
  - ➡ Use Legendre weights
  - ➡ Set up a linear system and solve for the function values on the contour

$$y_i = \frac{1}{2\pi} \sum_{k=1}^{n} w_k \frac{R\, e^{i\theta_k}}{R\, e^{i\theta_k} - z_i} \hat{f}_k \, , \quad i = 1, 2, \ldots, n$$

- New experimental results are expected to arrive BES-II (fixed target), CMB@FAIR, …,
- We start to calculate multi-dimensional phase diagrams
- Universal scaling and Lee-Yang zeros are powerful tools to explore the QCD phase diagram
  - ➡ New techniques are still being designed
- However, for finite μ_B we still rely on extrapolatory methods
  - ➡ we will need very precise data

installation of SIS100 dipoles Apr'24

CBM cave, Jun'24



[T. Galatyu, Criticality II '24]

Apologies to all speakers of the finite temperature and finite density sessions, how's talks I could not highlight

Thank you for your attention!