BBGKY hierarchy for quantum error mitigation

Theo Saporiti (theo.saporiti@cea.fr), *Oleg Kaikov, Vasily Sazonov and Mohamed Tamaazousti*

Université Paris-Saclay, CEA, List, F-91120, Palaiseau, France

Motivation

Numerical simulation of **many-body** physical systems is classically difficult. Such systems can be studied for instance by

- **Monte Carlo** methods, but they suffer the **sign problem**
- **Hierarchies**, but their **truncation** introduces inaccuracies

Quantum computers to avoid both issues: they are **noisy**, but that can be **mitigated**. *How can the* ***BBGKY hierarchy*** *help to mitigate* ***quantum errors****?*

Method

BBGKY hierarchy [1]

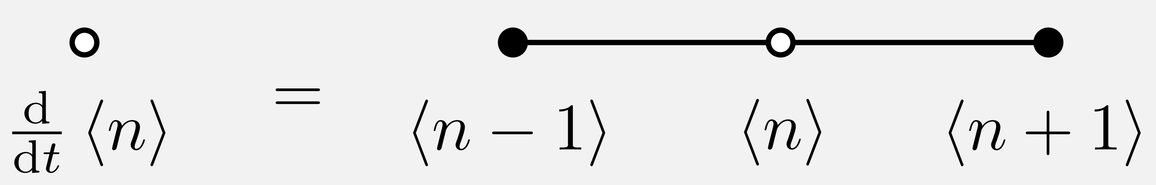
Consider a quantum spin model of sites $S = \{1, \dots, N\}$ with

$$H = \frac{1}{2}\sum_{i\in S} h_i^\mu \sigma_i^\mu + \frac{1}{4}\sum_{\substack{i,j\in S\\ i<j}} V_{ij}^{\mu\nu}\sigma_i^\mu\sigma_j^\nu$$

Let $A \subseteq S$ with $\{\mu_i\}_{i\in A}$ be a Pauli string. The **BBGKY equation** associated to A of $\{\mu_i\}_{i\in A}$ directions is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \prod_{i\in A}\sigma_i^{\mu_i}\right\rangle = \sum_{\substack{i,j\in A\\ i\neq j}} \frac{V_{ij}^{\mu_i\nu}}{2}\varepsilon_{\mu_j\nu\lambda}\left\langle \sigma_j^\lambda \prod_{k\in A\setminus\{i,j\}}\sigma_k^{\mu_k}\right\rangle + \sum_{i\in A} h_i^\lambda\, \varepsilon_{\mu_i\lambda\nu}\left\langle \sigma_i^\nu \prod_{j\in A\setminus\{i\}}\sigma_j^{\mu_j}\right\rangle + \sum_{\substack{i\in A\\ j\notin A}}\frac{V_{ij}^{\mu\nu}}{2}\varepsilon_{\mu_i\mu\lambda}\left\langle \sigma_i^\lambda\sigma_j^\nu \prod_{k\in A\setminus\{i\}}\sigma_k^{\mu_k}\right\rangle$$

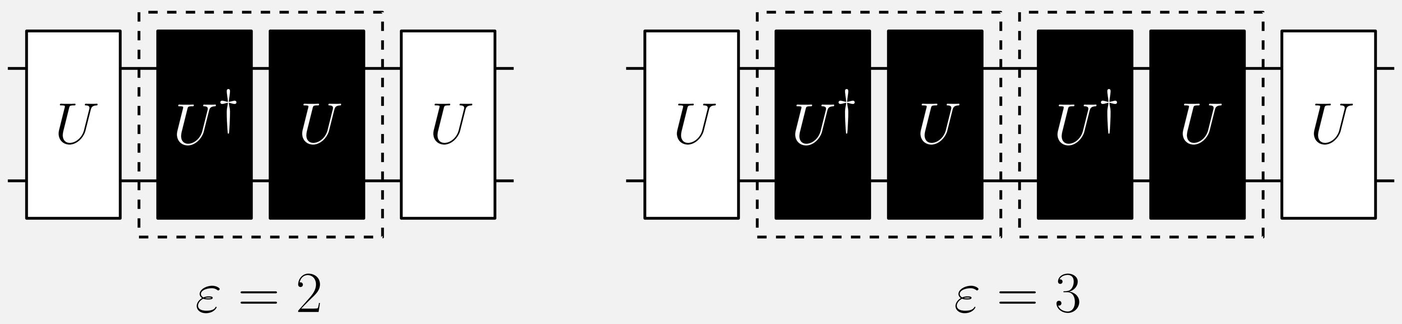
Schematically, for $n = |A|$,



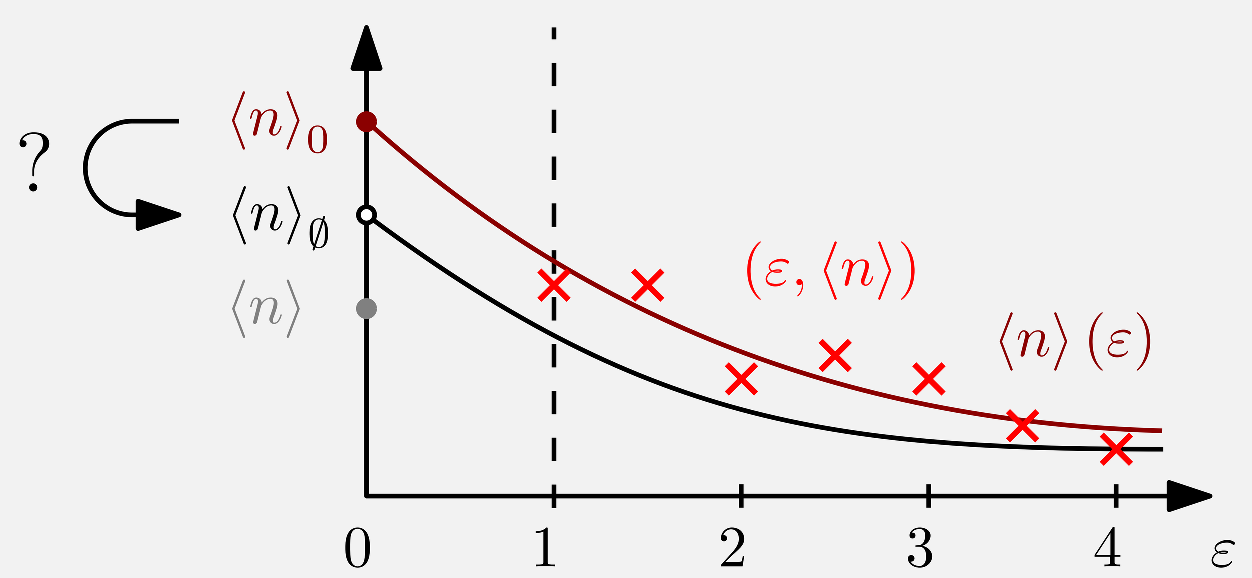
Different choices of A generate a **hierarchy** of equations, each of which composed of a **polynomial** in n amount of terms, which in principle can be used to mitigate the effects of quantum errors.

Zero noise extrapolation (ZNE) [2]

The time evolution of the system is obtained by **Trotterization**, with time steps U of duration Δt.



Different **unitary foldings** give different realizations of the same quantum circuit. Each realization produces a noisy observable measurement $\langle n\rangle$ at **noise level** ε.

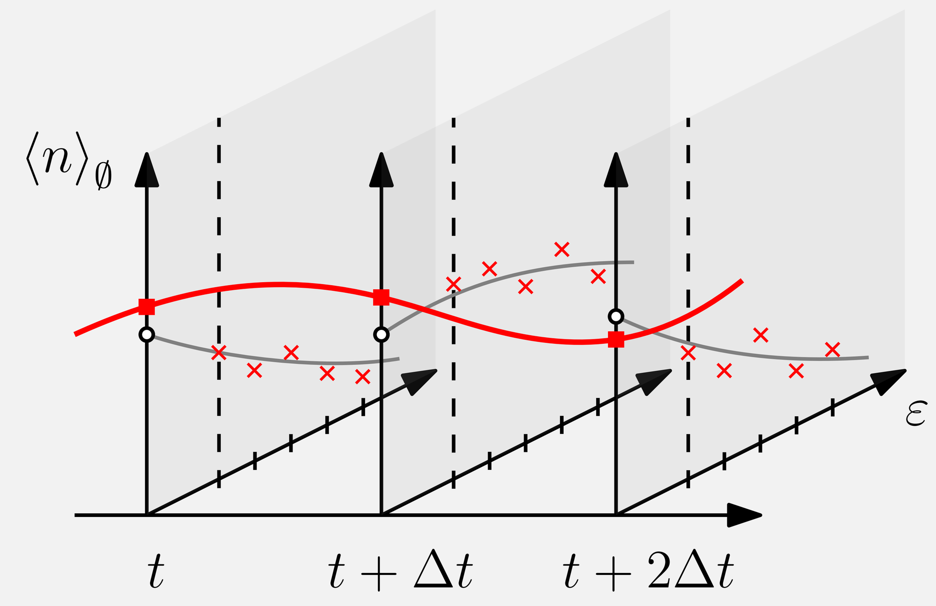


The zero noise **extrapolation** $\langle n\rangle_0$ is obtained as the **noiseless limit** $\varepsilon \to 0$ of the least squares polynomial of degree d

$$\langle n\rangle(\varepsilon) = \langle n\rangle_0 + a_1\varepsilon + \dots + a_d\varepsilon^d \xrightarrow{\varepsilon\to 0} \langle n\rangle_0 \approx \langle n\rangle$$

Improving ZNE with BBGKY

The proposed $\langle n\rangle_0$ are interpolated in time with a Bernstein polynomial, giving access to time derivatives. The former are then required to satisfy the additional BBGKY **constraints**, producing an **improved** $\langle n\rangle_\emptyset$ extrapolation.



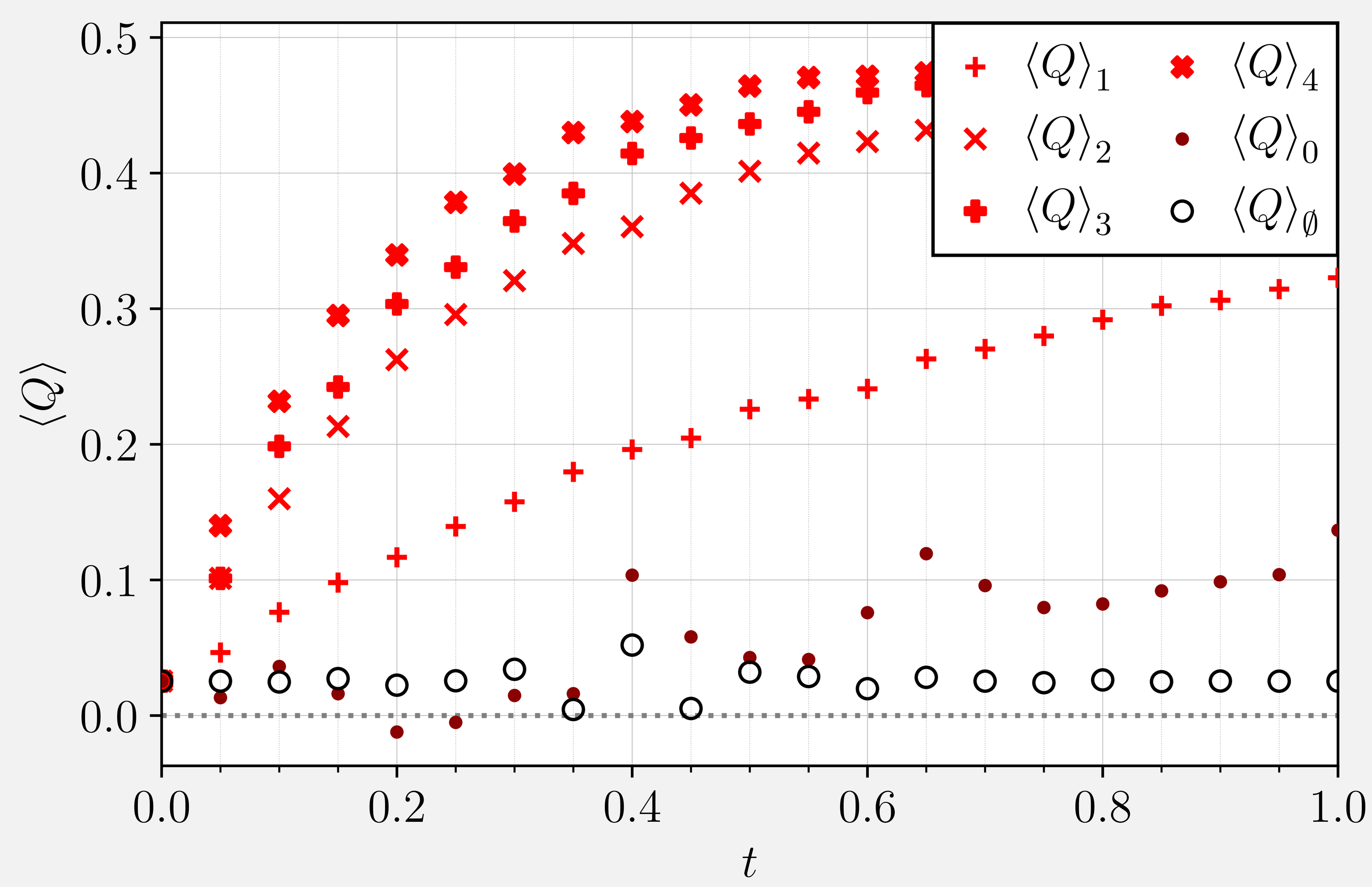
Preliminary results

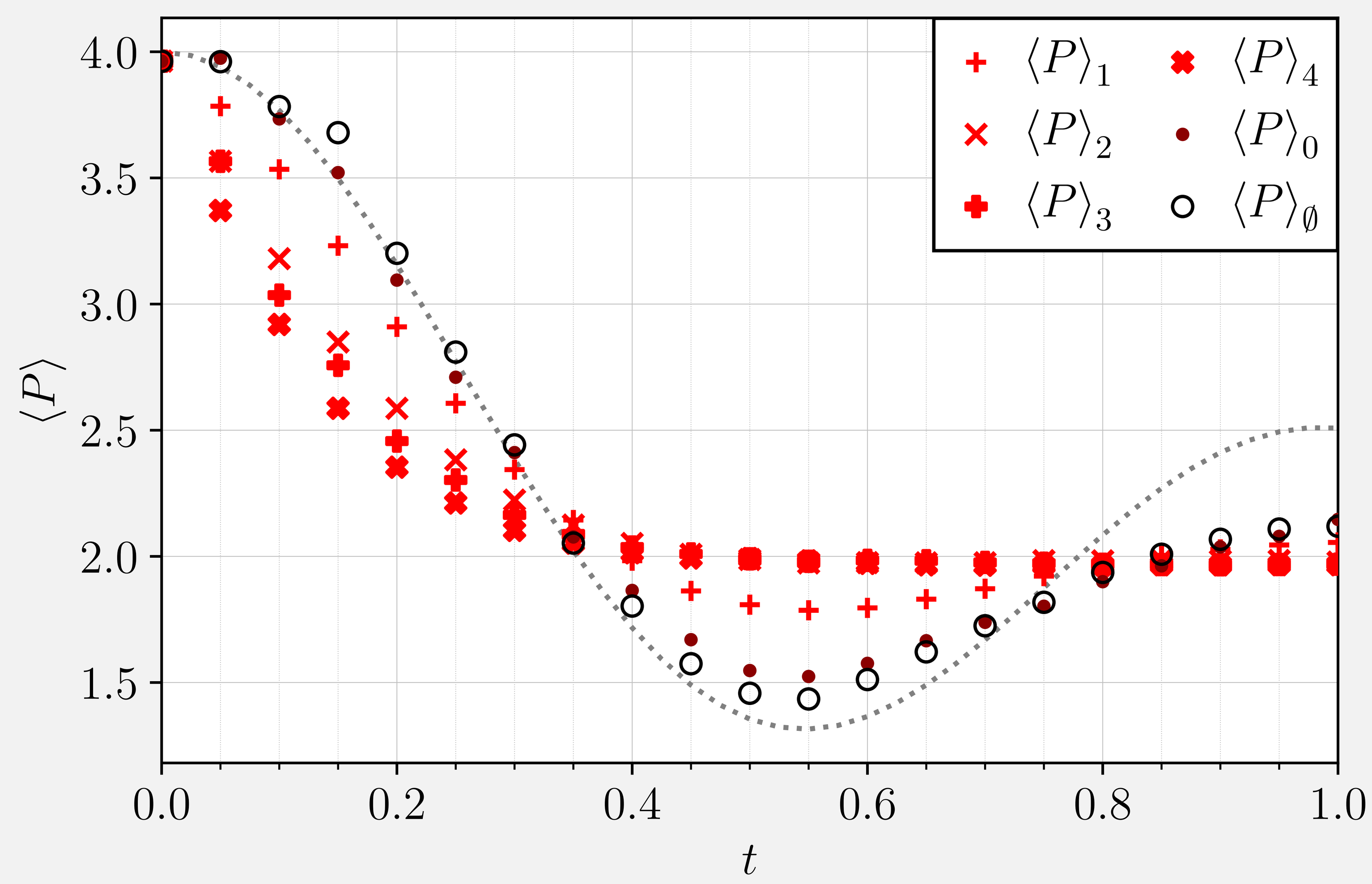
Consider the lattice Schwinger model [3] of $N = 4$ sites at initial state $|0101\rangle$

$$H = \frac{x}{2}\sum_{i=1}^{N-1}\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y\right) + \frac{1}{2}\sum_{\substack{i,j\in S\\ i<j}}(N-j+\lambda)\sigma_i^z\sigma_j^z + \sum_{i=1}^{N-1}\left(\frac{N}{4} - \frac{1}{2}\left\lceil\frac{i-1}{2}\right\rceil + l_0(N-i)\right)\sigma_i^z - \frac{m}{g}\sqrt{x}\sum_{i\in S}(-1)^i\sigma_i^z$$

with parameters $(x, \lambda, l_0, m/g) = (2, 1, 0, 10^{-3})$. Measure the **total charge** and **particle number** observables

$$Q = \frac{1}{2}\sum_{i\in S}\sigma_i^z \qquad P = \frac{N}{2} - \frac{1}{2}\sum_{i\in S}(-1)^i\sigma_i^z$$





References

[1] T. Cox. *Aspects of Decoherence in Qubit Systems.* PhD thesis, The University of British Columbia, 2019.
[2] K. Temme S. Bravyi J.M. Gambetta. Error Mitigation for Short-Depth Quantum Circuits. *Phys. Rev. Lett.*, 119(18):180509, 2017.
[3] T. Angelides P. Naredi A. Crippa K. Jansen S. Kühn I. Tavernelli D.S. Wang. First-Order Phase Transition of the Schwinger Model with a Quantum Computer. 12 2023.