Hadron Spectroscopy from Lattice QCD:
Current Status and Future

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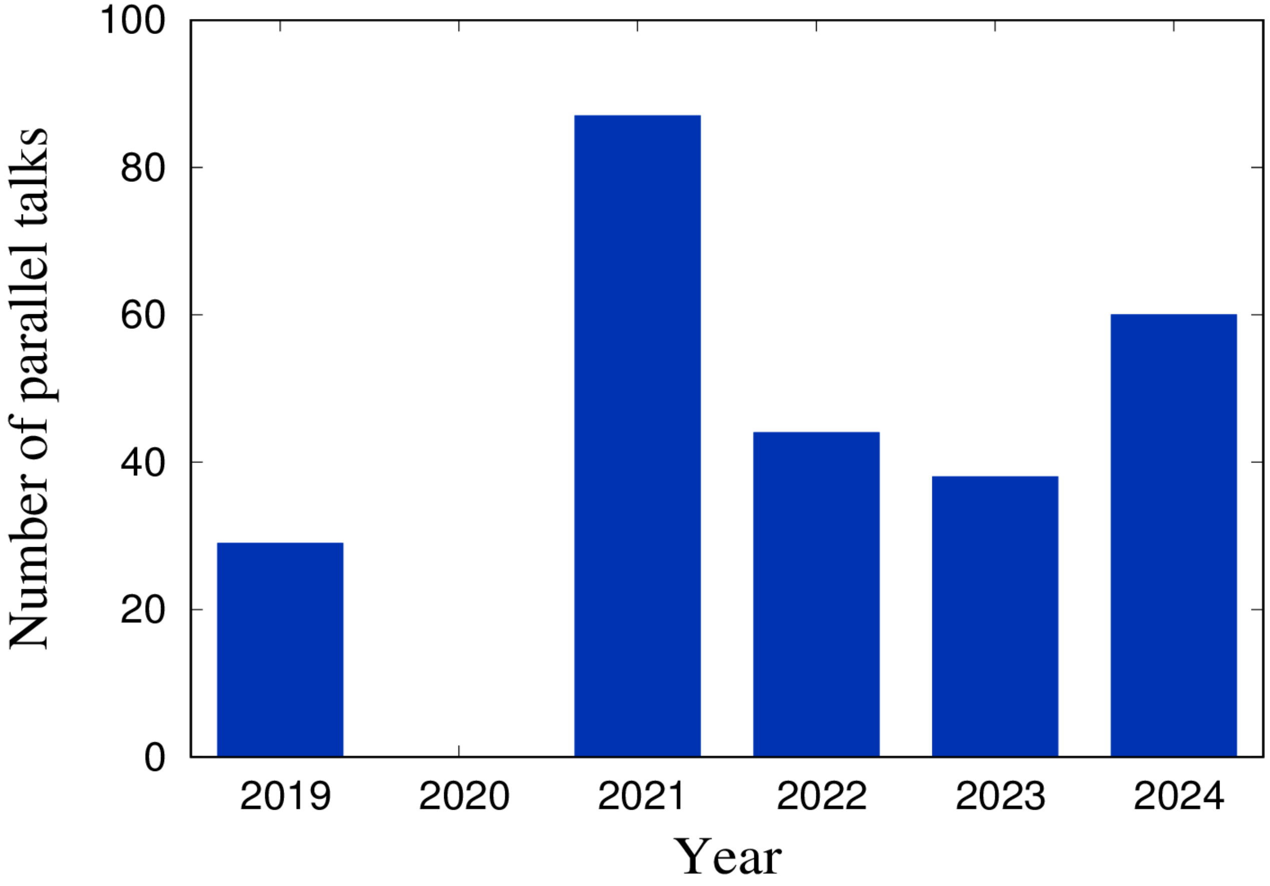
Disclaimer

Task: "contribute a 45-minute rapporteur-style talk on **Hadron Spectroscopy**"

In this conference 60 parallel talks + Talks on Hadron structures

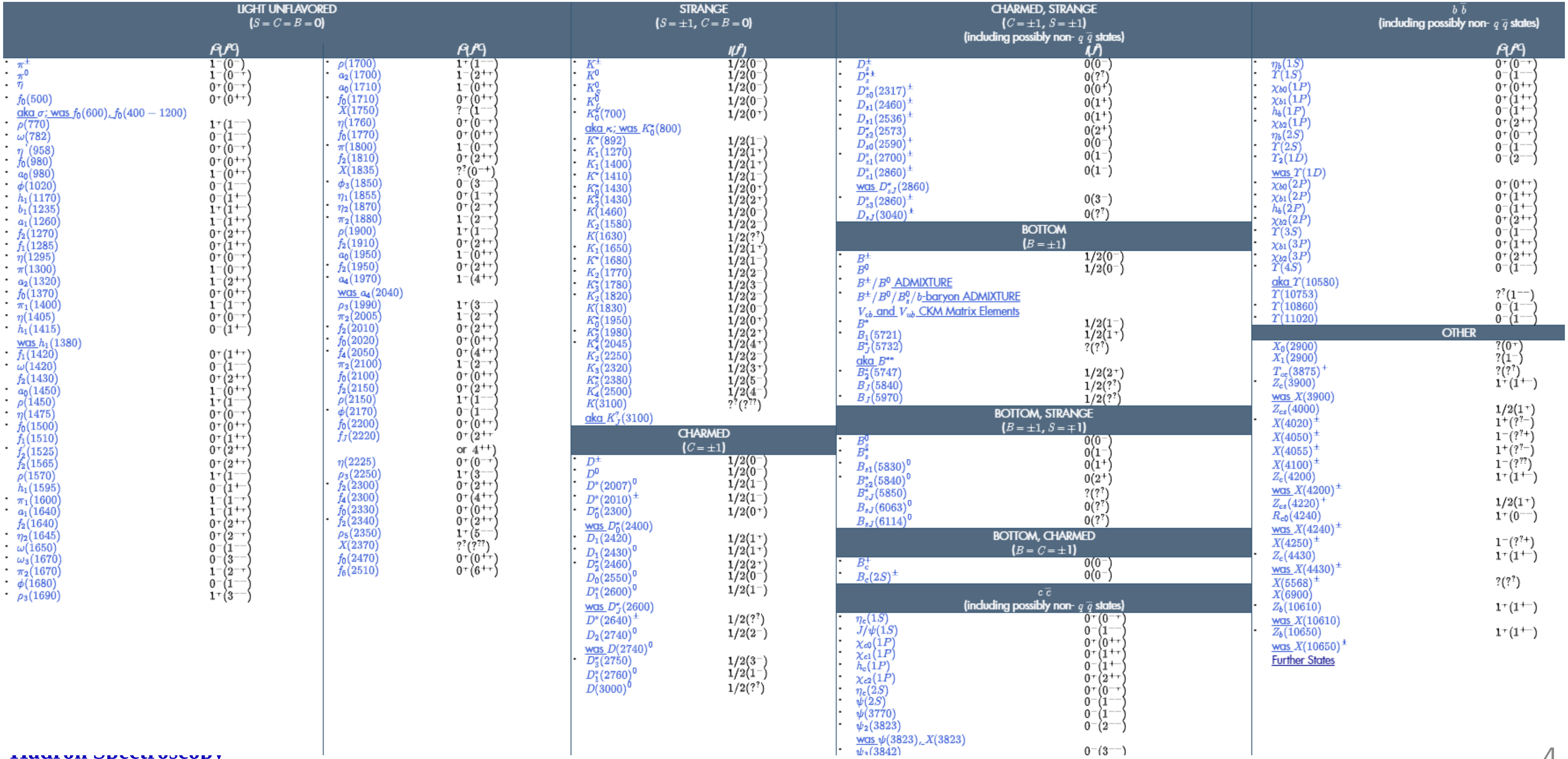
- Thanks to everyone who has sent their slides
- Apology for omitting some of those due to time constraint

Number of parallel talks on Hadron Spectroscopy in the last 5 lattice conferences



Vibrant area of research in lattice QCD

Mesons (quark-antiquark systems)



	LIGHT UNFLAVORED $(S = C = B = 0)$	$I^G(J^{PC})$
•	$\pi^\pm$	$1^-(0^-)$
•	π^0	$1^-(0^{-+})$
•	η	$0^+(0^{-+})$
•	$f_0(500)$	$0^+(0^{++})$
	aka σ; was $f_0(600)$, $f_0(400-1200)$	
•	$\rho(770)$	$1^+(1^{--})$
•	$\omega(782)$	$0^-(1^{--})$
•	$\eta'(958)$	$0^+(0^{-+})$
•	$f_0(980)$	$0^+(0^{++})$
•	$a_0(980)$	$1^-(0^{++})$
•	$\phi(1020)$	$0^-(1^{--})$
•	$h_1(1170)$	$0^-(1^{+-})$
•	$b_1(1235)$	$1^+(1^{+-})$
•	$a_1(1260)$	$1^-(1^{++})$
•	$f_2(1270)$	$0^+(2^{++})$
•	$f_1(1285)$	$0^+(1^{++})$
•	$\eta(1295)$	$0^+(0^{-+})$
•	$\pi(1300)$	$1^-(0^{-+})$
•	$a_2(1320)$	$1^-(2^{++})$
•	$f_0(1370)$	$0^+(0^{++})$
•	$\pi_1(1400)$	$1^-(1^{-+})$
•	$\eta(1405)$	$0^+(0^{-+})$
•	$h_1(1415)$	$0^-(1^{+-})$
	was $h_1(1380)$	
•	$f_1(1420)$	$0^+(1^{++})$
•	$\omega(1420)$	$0^-(1^{--})$
	$f_2(1430)$	$0^+(2^{++})$
•	$a_0(1450)$	$1^-(0^{++})$
•	$\rho(1450)$	$1^+(1^{--})$
•	$\eta(1475)$	$0^+(0^{-+})$
•	$f_0(1500)$	$0^+(0^{++})$
	$f_1(1510)$	$0^+(1^{++})$
•	$f_2'(1525)$	$0^+(2^{++})$
	$f_2(1565)$	$0^+(2^{++})$
	$\rho(1570)$	$1^+(1^{--})$
	$h_1(1595)$	$0^-(1^{+-})$
•	$\pi_1(1600)$	$1^-(1^{-+})$
•	$a_1(1640)$	$1^-(1^{++})$
	$f_2(1640)$	$0^+(2^{++})$
•	$\eta_2(1645)$	$0^+(2^{-+})$
•	$\omega(1650)$	$0^-(1^{--})$
•	$\omega_3(1670)$	$0^-(3^{--})$
•	$\pi_2(1670)$	$1^-(2^{-+})$
•	$\phi(1680)$	$0^-(1^{--})$
•	$\rho_3(1690)$	$1^+(3^{--})$
•	$\rho(1700)$	$1^+(1^{--})$
•	$a_2(1700)$	$1^-(2^{++})$
	$a_0(1710)$	$1^-(0^{++})$
•	$f_0(1710)$	$0^+(0^{++})$
	$X(1750)$	$?^-(1^{--})$
	$\eta(1760)$	$0^+(0^{-+})$
	$f_0(1770)$	$0^+(0^{++})$
•	$\pi(1800)$	$1^-(0^{-+})$
	$f_2(1810)$	$0^+(2^{++})$
	$X(1835)$	$?^?(0^{-+})$
•	$\phi_3(1850)$	$0^-(3^{--})$
	$\eta_1(1855)$	$0^+(1^{-+})$
•	$\eta_2(1870)$	$0^+(2^{-+})$
•	$\pi_2(1880)$	$1^-(2^{-+})$
	$\rho(1900)$	$1^+(1^{--})$
	$f_2(1910)$	$0^+(2^{++})$
	$a_0(1950)$	$1^-(0^{++})$
•	$f_2(1950)$	$0^+(2^{++})$
•	$a_4(1970)$	$1^-(4^{++})$
	was $a_4(2040)$	
	$\rho_3(1990)$	$1^+(3^{--})$
	$\pi_2(2005)$	$1^-(2^{-+})$
•	$f_2(2010)$	$0^+(2^{++})$
•	$f_0(2020)$	$0^+(0^{++})$
•	$f_4(2050)$	$0^+(4^{++})$
	$\pi_2(2100)$	$1^-(2^{-+})$
	$f_0(2100)$	$0^+(0^{++})$
	$f_2(2150)$	$0^+(2^{++})$
	$\rho(2150)$	$1^+(1^{--})$
•	$\phi(2170)$	$0^-(1^{--})$
	$f_0(2200)$	$0^+(0^{++})$
	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
	$\eta(2225)$	$0^+(0^{-+})$
	$\rho_3(2250)$	$1^+(3^{--})$
•	$f_2(2300)$	$0^+(2^{++})$
	$f_4(2300)$	$0^+(4^{++})$
	$f_0(2330)$	$0^+(0^{++})$
•	$f_2(2340)$	$0^+(2^{++})$
	$\rho_5(2350)$	$1^+(5^{--})$
	$X(2370)$	$?^?(?^{??})$
	$f_0(2470)$	$0^+(0^{++})$
	$f_6(2510)$	$0^+(6^{++})$

	STRANGE $(S = \pm1,\ C = B = 0)$	$I(J^P)$
•	$K^\pm$	$1/2(0^-)$
•	K^0	$1/2(0^-)$
•	K^0_S	$1/2(0^-)$
•	K^0_L	$1/2(0^-)$
•	$K_0^*(700)$	$1/2(0^+)$
	aka κ; was $K_0^*(800)$	
•	$K^*(892)$	$1/2(1^-)$
•	$K_1(1270)$	$1/2(1^+)$
•	$K_1(1400)$	$1/2(1^+)$
•	$K^*(1410)$	$1/2(1^-)$
•	$K_0^*(1430)$	$1/2(0^+)$
•	$K_2^*(1430)$	$1/2(2^+)$
•	$K(1460)$	$1/2(0^-)$
	$K_2(1580)$	$1/2(2^-)$
	$K(1630)$	$1/2(?^?)$
•	$K_1(1650)$	$1/2(1^+)$
•	$K^*(1680)$	$1/2(1^-)$
•	$K_2(1770)$	$1/2(2^-)$
•	$K_3^*(1780)$	$1/2(3^-)$
•	$K_2(1820)$	$1/2(2^-)$
	$K(1830)$	$1/2(0^-)$
	$K_0^*(1950)$	$1/2(0^+)$
•	$K_2^*(1980)$	$1/2(2^+)$
•	$K_4^*(2045)$	$1/2(4^+)$
	$K_2(2250)$	$1/2(2^-)$
	$K_3(2320)$	$1/2(3^+)$
	$K_5^*(2380)$	$1/2(5^-)$
	$K_4(2500)$	$1/2(4^-)$
	$K(3100)$	$?^?(?^{??})$
	aka $K_J^?(3100)$	

	CHARMED $(C = \pm1)$	$I(J^P)$
•	$D^\pm$	$1/2(0^-)$
•	D^0	$1/2(0^-)$
•	$D^*(2007)^0$	$1/2(1^-)$
•	$D^*(2010)^\pm$	$1/2(1^-)$
•	$D_0^*(2300)$	$1/2(0^+)$
	was $D_0^*(2400)$	
•	$D_1(2420)$	$1/2(1^+)$
•	$D_1(2430)^0$	$1/2(1^+)$
•	$D_2^*(2460)$	$1/2(2^+)$
	$D_0(2550)^0$	$1/2(0^-)$
	$D_1^*(2600)^0$	$1/2(1^-)$
	was $D_J^*(2600)$	
	$D^*(2640)^\pm$	$1/2(?^?)$
	$D_2(2740)^0$	$1/2(2^-)$
	was $D(2740)^0$	
•	$D_3^*(2750)$	$1/2(3^-)$
	$D_1^*(2760)^0$	$1/2(1^-)$
	$D(3000)^0$	$1/2(?^?)$

	CHARMED, STRANGE $(C = \pm1,\ S = \pm1)$ (including possibly non- $q\bar{q}$ states)	$I(J^P)$
•	$D_s^\pm$	$0(0^-)$
•	$D_s^{*\pm}$	$0(?^?)$
•	$D_{s0}^*(2317)^\pm$	$0(0^+)$
•	$D_{s1}(2460)^\pm$	$0(1^+)$
•	$D_{s1}(2536)^\pm$	$0(1^+)$
•	$D_{s2}^*(2573)$	$0(2^+)$
	$D_{s0}(2590)^+$	$0(0^-)$
•	$D_{s1}^*(2700)^\pm$	$0(1^-)$
	$D_{s1}^*(2860)^\pm$	$0(1^-)$
	was $D_{sJ}^*(2860)$	
•	$D_{s3}^*(2860)^\pm$	$0(3^-)$
	$D_{sJ}(3040)^\pm$	$0(?^?)$

	BOTTOM $(B = \pm1)$	$I(J^P)$
•	$B^\pm$	$1/2(0^-)$
•	B^0	$1/2(0^-)$
•	$B^\pm/B^0$ ADMIXTURE	
•	$B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
	V_{cb} and V_{ub} CKM Matrix Elements	
•	B^*	$1/2(1^-)$
•	$B_1(5721)$	$1/2(1^+)$
	$B_J^*(5732)$	$?(?^?)$
	aka B^{**}	
•	$B_2^*(5747)$	$1/2(2^+)$
	$B_J(5840)$	$1/2(?^?)$
•	$B_J(5970)$	$1/2(?^?)$

	BOTTOM, STRANGE $(B = \pm1,\ S = \mp1)$	$I(J^P)$
•	B^0_s	$0(0^-)$
•	B_s^*	$0(1^-)$
•	$B_{s1}(5830)^0$	$0(1^+)$
•	$B_{s2}^*(5840)^0$	$0(2^+)$
	$B_{sJ}^*(5850)$	$?(?^?)$
	$B_{sJ}(6063)^0$	$0(?^?)$
	$B_{sJ}(6114)^0$	$0(?^?)$

	BOTTOM, CHARMED $(B = C = \pm1)$	$I(J^P)$
•	B_c^+	$0(0^-)$
•	$B_c(2S)^\pm$	$0(0^-)$

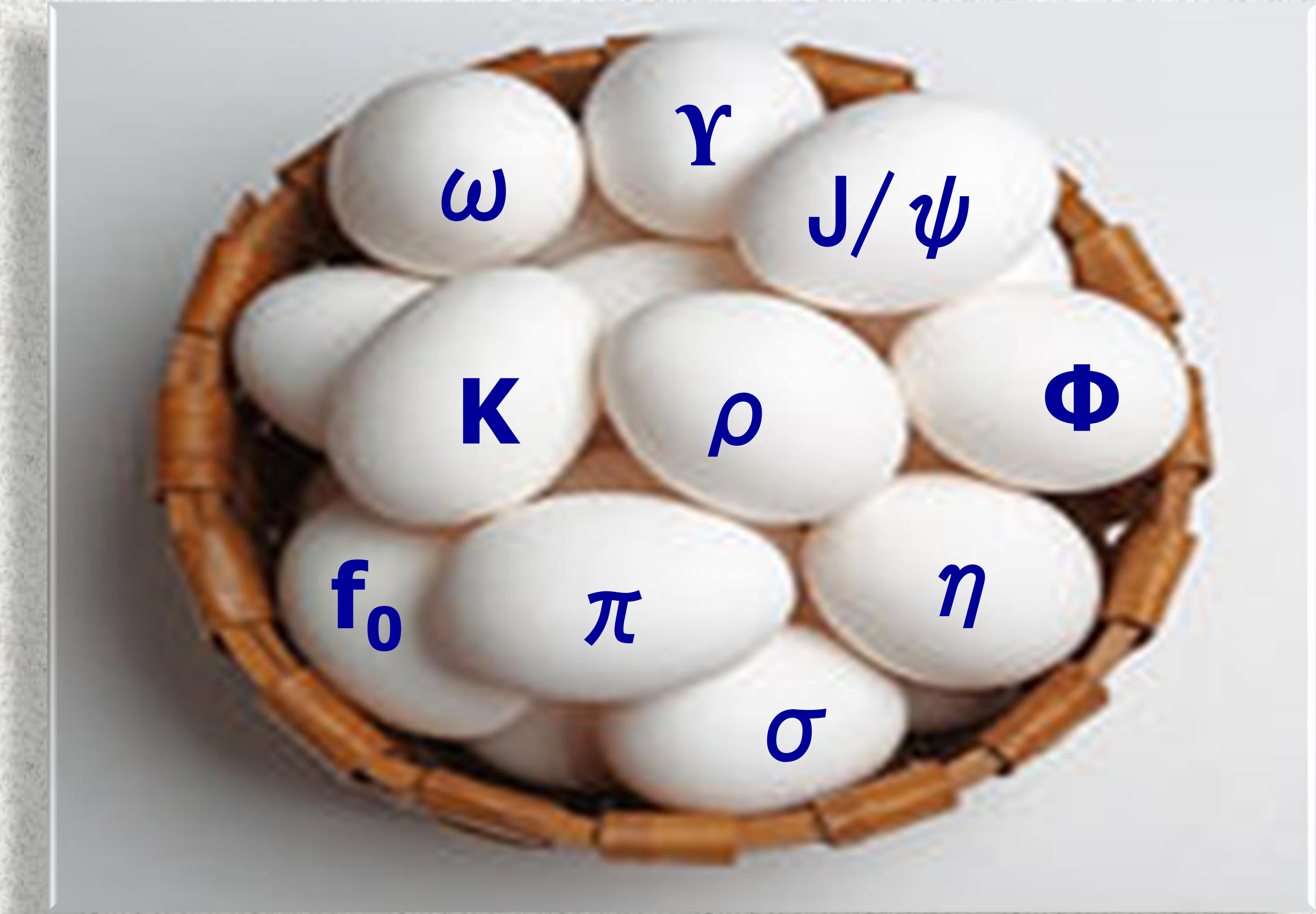
	$c\bar{c}$ (including possibly non- $q\bar{q}$ states)	$I^G(J^{PC})$
•	$\eta_c(1S)$	$0^+(0^{-+})$
•	$J/\psi(1S)$	$0^-(1^{--})$
•	$\chi_{c0}(1P)$	$0^+(0^{++})$
•	$\chi_{c1}(1P)$	$0^+(1^{++})$
•	$h_c(1P)$	$0^-(1^{+-})$
•	$\chi_{c2}(1P)$	$0^+(2^{++})$
•	$\eta_c(2S)$	$0^+(0^{-+})$
•	$\psi(2S)$	$0^-(1^{--})$
•	$\psi(3770)$	$0^-(1^{--})$
•	$\psi_2(3823)$	$0^-(2^{--})$
	was $\psi(3823)$, $X(3823)$	
•	$\psi_3(3842)$	$0^-(3^{--})$

	$b\bar{b}$ (including possibly non- $q\bar{q}$ states)	$I^G(J^{PC})$
•	$\eta_b(1S)$	$0^+(0^{-+})$
•	$\Upsilon(1S)$	$0^-(1^{--})$
•	$\chi_{b0}(1P)$	$0^+(0^{++})$
•	$\chi_{b1}(1P)$	$0^+(1^{++})$
•	$h_b(1P)$	$0^-(1^{+-})$
•	$\chi_{b2}(1P)$	$0^+(2^{++})$
	$\eta_b(2S)$	$0^+(0^{-+})$
•	$\Upsilon(2S)$	$0^-(1^{--})$
•	$\Upsilon_2(1D)$	$0^-(2^{--})$
	was $\Upsilon(1D)$	
•	$\chi_{b0}(2P)$	$0^+(0^{++})$
•	$\chi_{b1}(2P)$	$0^+(1^{++})$
•	$h_b(2P)$	$0^-(1^{+-})$
•	$\chi_{b2}(2P)$	$0^+(2^{++})$
•	$\Upsilon(3S)$	$0^-(1^{--})$
•	$\chi_{b1}(3P)$	$0^+(1^{++})$
•	$\chi_{b2}(3P)$	$0^+(2^{++})$
•	$\Upsilon(4S)$	$0^-(1^{--})$
	aka $\Upsilon(10580)$	
	$\Upsilon(10753)$	$?^?(1^{--})$
•	$\Upsilon(10860)$	$0^-(1^{--})$
•	$\Upsilon(11020)$	$0^-(1^{--})$

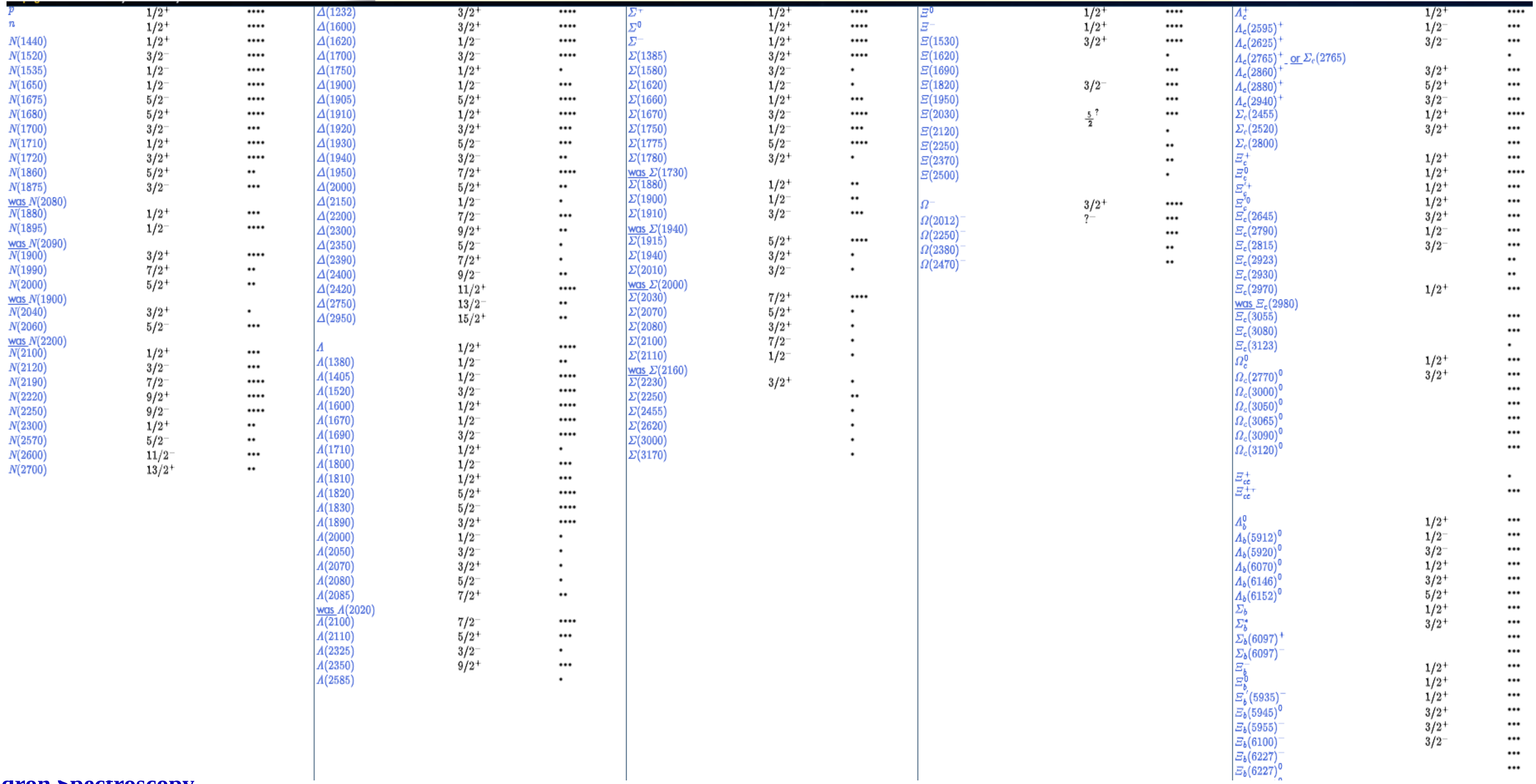
	OTHER	
	$X_0(2900)$	$?(0^+)$
	$X_1(2900)$	$?(1^-)$
	$T_{cc}(3875)^+$	$?(?^?)$
•	$Z_c(3900)$	$1^+(1^{+-})$
	was $X(3900)$	
	$Z_{cs}(4000)$	$1/2(1^+)$
•	$X(4020)^\pm$	$1^+(?^{?-})$
	$X(4050)^\pm$	$1^-(?^{?+})$
	$X(4055)^\pm$	$1^+(?^{?-})$
	$X(4100)^\pm$	$1^-(?^{??})$
	$Z_c(4200)$	$1^+(1^{+-})$
	was $X(4200)^\pm$	
	$Z_{cs}(4220)^+$	$1/2(1^+)$
	$R_{c0}(4240)$	$1^+(0^{--})$
	was $X(4240)^\pm$	
	$X(4250)^\pm$	$1^-(?^{?+})$
•	$Z_c(4430)$	$1^+(1^{+-})$
	was $X(4430)^\pm$	
	$X(5568)^\pm$	$?(?^?)$
	$X(6900)$	
•	$Z_b(10610)$	$1^+(1^{+-})$
	was $X(10610)$	
•	$Z_b(10650)$	$1^+(1^{+-})$
	was $X(10650)^\pm$	
	Further States	

Particle Data Group

Basket full of Mesons



Baryons (Three quark systems)



p	$1/2^+$	••••
n	$1/2^+$	••••
$N(1440)$	$1/2^+$	••••
$N(1520)$	$3/2^-$	••••
$N(1535)$	$1/2^-$	••••
$N(1650)$	$1/2^-$	••••
$N(1675)$	$5/2^-$	••••
$N(1680)$	$5/2^+$	••••
$N(1700)$	$3/2^-$	•••
$N(1710)$	$1/2^+$	••••
$N(1720)$	$3/2^+$	••••
$N(1860)$	$5/2^+$	••
$N(1875)$	$3/2^-$	•••
was $N(2080)$		
$N(1880)$	$1/2^+$	•••
$N(1895)$	$1/2^-$	••••
was $N(2090)$		
$N(1900)$	$3/2^+$	••••
$N(1990)$	$7/2^+$	••
$N(2000)$	$5/2^+$	••
was $N(1900)$		
$N(2040)$	$3/2^+$	•
$N(2060)$	$5/2^-$	•••
was $N(2200)$		
$N(2100)$	$1/2^+$	•••
$N(2120)$	$3/2^-$	•••
$N(2190)$	$7/2^-$	••••
$N(2220)$	$9/2^+$	••••
$N(2250)$	$9/2^-$	••••
$N(2300)$	$1/2^+$	••
$N(2570)$	$5/2^-$	••
$N(2600)$	$11/2^-$	•••
$N(2700)$	$13/2^+$	••

$\Delta(1232)$	$3/2^+$	••••
$\Delta(1600)$	$3/2^+$	••••
$\Delta(1620)$	$1/2^-$	••••
$\Delta(1700)$	$3/2^-$	••••
$\Delta(1750)$	$1/2^+$	•
$\Delta(1900)$	$1/2^-$	•••
$\Delta(1905)$	$5/2^+$	••••
$\Delta(1910)$	$1/2^+$	••••
$\Delta(1920)$	$3/2^+$	•••
$\Delta(1930)$	$5/2^-$	•••
$\Delta(1940)$	$3/2^-$	••
$\Delta(1950)$	$7/2^+$	••••
$\Delta(2000)$	$5/2^+$	••
$\Delta(2150)$	$1/2^-$	•
$\Delta(2200)$	$7/2^-$	•••
$\Delta(2300)$	$9/2^+$	••
$\Delta(2350)$	$5/2^-$	•
$\Delta(2390)$	$7/2^+$	•
$\Delta(2400)$	$9/2^-$	••
$\Delta(2420)$	$11/2^+$	••••
$\Delta(2750)$	$13/2^-$	••
$\Delta(2950)$	$15/2^+$	••
Λ	$1/2^+$	••••
$\Lambda(1380)$	$1/2^-$	••
$\Lambda(1405)$	$1/2^-$	••••
$\Lambda(1520)$	$3/2^-$	••••
$\Lambda(1600)$	$1/2^+$	••••
$\Lambda(1670)$	$1/2^-$	••••
$\Lambda(1690)$	$3/2^-$	••••
$\Lambda(1710)$	$1/2^+$	•
$\Lambda(1800)$	$1/2^-$	•••
$\Lambda(1810)$	$1/2^+$	•••
$\Lambda(1820)$	$5/2^+$	••••
$\Lambda(1830)$	$5/2^-$	••••
$\Lambda(1890)$	$3/2^+$	••••
$\Lambda(2000)$	$1/2^-$	•
$\Lambda(2050)$	$3/2^-$	•
$\Lambda(2070)$	$3/2^+$	•
$\Lambda(2080)$	$5/2^-$	•
$\Lambda(2085)$	$7/2^+$	••
was $\Lambda(2020)$		
$\Lambda(2100)$	$7/2^-$	••••
$\Lambda(2110)$	$5/2^+$	•••
$\Lambda(2325)$	$3/2^-$	•
$\Lambda(2350)$	$9/2^+$	•••
$\Lambda(2585)$		•

Σ^+	$1/2^+$	••••
Σ^0	$1/2^+$	••••
Σ^-	$1/2^+$	••••
$\Sigma(1385)$	$3/2^+$	••••
$\Sigma(1580)$	$3/2^-$	•
$\Sigma(1620)$	$1/2^-$	•
$\Sigma(1660)$	$1/2^+$	•••
$\Sigma(1670)$	$3/2^-$	••••
$\Sigma(1750)$	$1/2^-$	•••
$\Sigma(1775)$	$5/2^-$	••••
$\Sigma(1780)$	$3/2^+$	•
was $\Sigma(1730)$		
$\Sigma(1880)$	$1/2^+$	••
$\Sigma(1900)$	$1/2^-$	••
$\Sigma(1910)$	$3/2^-$	•••
was $\Sigma(1940)$		
$\Sigma(1915)$	$5/2^+$	••••
$\Sigma(1940)$	$3/2^+$	•
$\Sigma(2010)$	$3/2^-$	•
was $\Sigma(2000)$		
$\Sigma(2030)$	$7/2^+$	••••
$\Sigma(2070)$	$5/2^+$	•
$\Sigma(2080)$	$3/2^+$	•
$\Sigma(2100)$	$7/2^-$	•
$\Sigma(2110)$	$1/2^-$	•
was $\Sigma(2160)$		
$\Sigma(2230)$	$3/2^+$	•
$\Sigma(2250)$		••
$\Sigma(2455)$		•
$\Sigma(2620)$		•
$\Sigma(3000)$		•
$\Sigma(3170)$		•

Ξ^0	$1/2^+$	••••
Ξ^-	$1/2^+$	••••
$\Xi(1530)$	$3/2^+$	••••
$\Xi(1620)$		•
$\Xi(1690)$		•••
$\Xi(1820)$	$3/2^-$	•••
$\Xi(1950)$		•••
$\Xi(2030)$	$\frac{5}{2}^?$	•••
$\Xi(2120)$		•
$\Xi(2250)$		••
$\Xi(2370)$		••
$\Xi(2500)$		•
Ω^-	$3/2^+$	••••
$\Omega(2012)^-$	$?^-$	•••
$\Omega(2250)^-$		•••
$\Omega(2380)^-$		••
$\Omega(2470)^-$		••

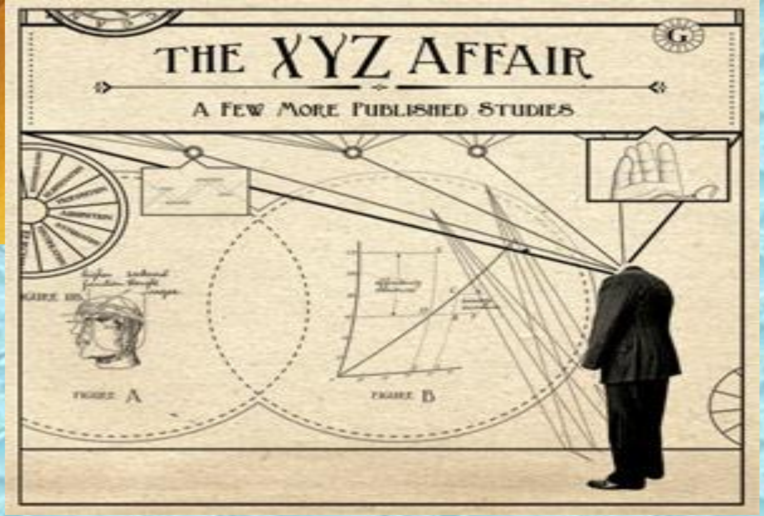
Λ_c^+	$1/2^+$	••••
$\Lambda_c(2595)^+$	$1/2^-$	•••
$\Lambda_c(2625)^+$	$3/2^-$	•••
$\Lambda_c(2765)^+$ or $\Sigma_c(2765)$		•
$\Lambda_c(2860)^+$	$3/2^+$	•••
$\Lambda_c(2880)^+$	$5/2^+$	•••
$\Lambda_c(2940)^+$	$3/2^-$	•••
$\Sigma_c(2455)$	$1/2^+$	••••
$\Sigma_c(2520)$	$3/2^+$	•••
$\Sigma_c(2800)$		•••
Ξ_c^+	$1/2^+$	•••
Ξ_c^0	$1/2^+$	••••
$\Xi_c^{\prime +}$	$1/2^+$	•••
$\Xi_c^{\prime 0}$	$1/2^+$	•••
$\Xi_c(2645)$	$3/2^+$	•••
$\Xi_c(2790)$	$1/2^-$	•••
$\Xi_c(2815)$	$3/2^-$	•••
$\Xi_c(2923)$		••
$\Xi_c(2930)$		••
$\Xi_c(2970)$	$1/2^+$	•••
was $\Xi_c(2980)$		
$\Xi_c(3055)$		•••
$\Xi_c(3080)$		•••
$\Xi_c(3123)$		•
Ω_c^0	$1/2^+$	•••
$\Omega_c(2770)^0$	$3/2^+$	•••
$\Omega_c(3000)^0$		•••
$\Omega_c(3050)^0$		•••
$\Omega_c(3065)^0$		•••
$\Omega_c(3090)^0$		•••
$\Omega_c(3120)^0$		•••
Ξ_{cc}^+		•
Ξ_{cc}^{++}		•••
Λ_b^0	$1/2^+$	•••
$\Lambda_b(5912)^0$	$1/2^-$	•••
$\Lambda_b(5920)^0$	$3/2^-$	•••
$\Lambda_b(6070)^0$	$1/2^+$	•••
$\Lambda_b(6146)^0$	$3/2^+$	•••
$\Lambda_b(6152)^0$	$5/2^+$	•••
Σ_b	$1/2^+$	•••
Σ_b^*	$3/2^+$	•••
$\Sigma_b(6097)^+$		•••
$\Sigma_b(6097)^-$		•••
Ξ_b^-	$1/2^+$	•••
Ξ_b^0	$1/2^+$	•••
$\Xi_b'(5935)^-$	$1/2^+$	•••
$\Xi_b(5945)^0$	$3/2^+$	•••
$\Xi_b(5955)^-$	$3/2^+$	•••
$\Xi_b(6100)^-$	$3/2^-$	•••
$\Xi_b(6227)^-$		•••
$\Xi_b(6227)^0$		•••

Particle Data Group

Basket full of Baryons

Newly Found Hadrons

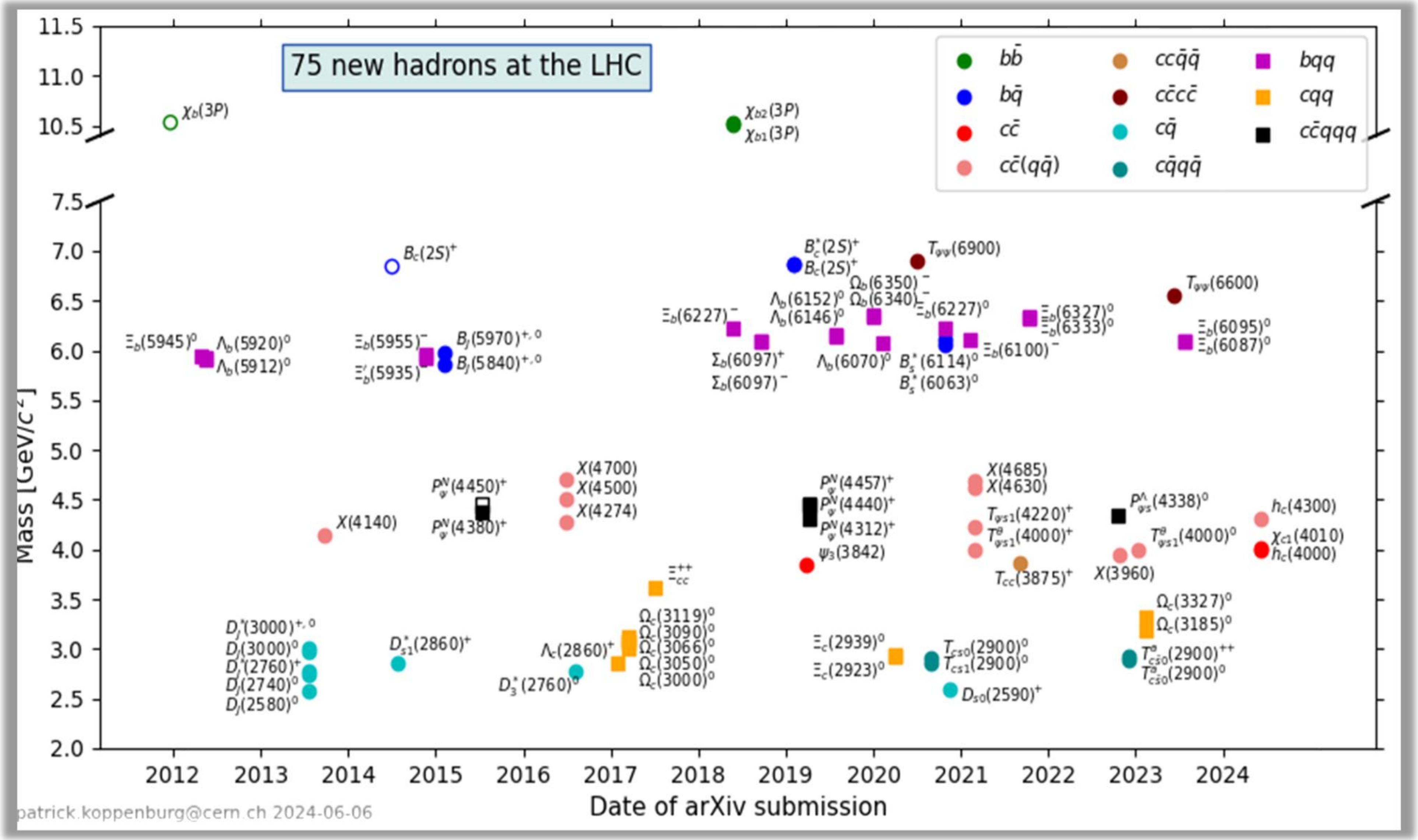
Exciting Discovery



THE XYZ AFFAIR

A FEW MORE PUBLISHED STUDIES

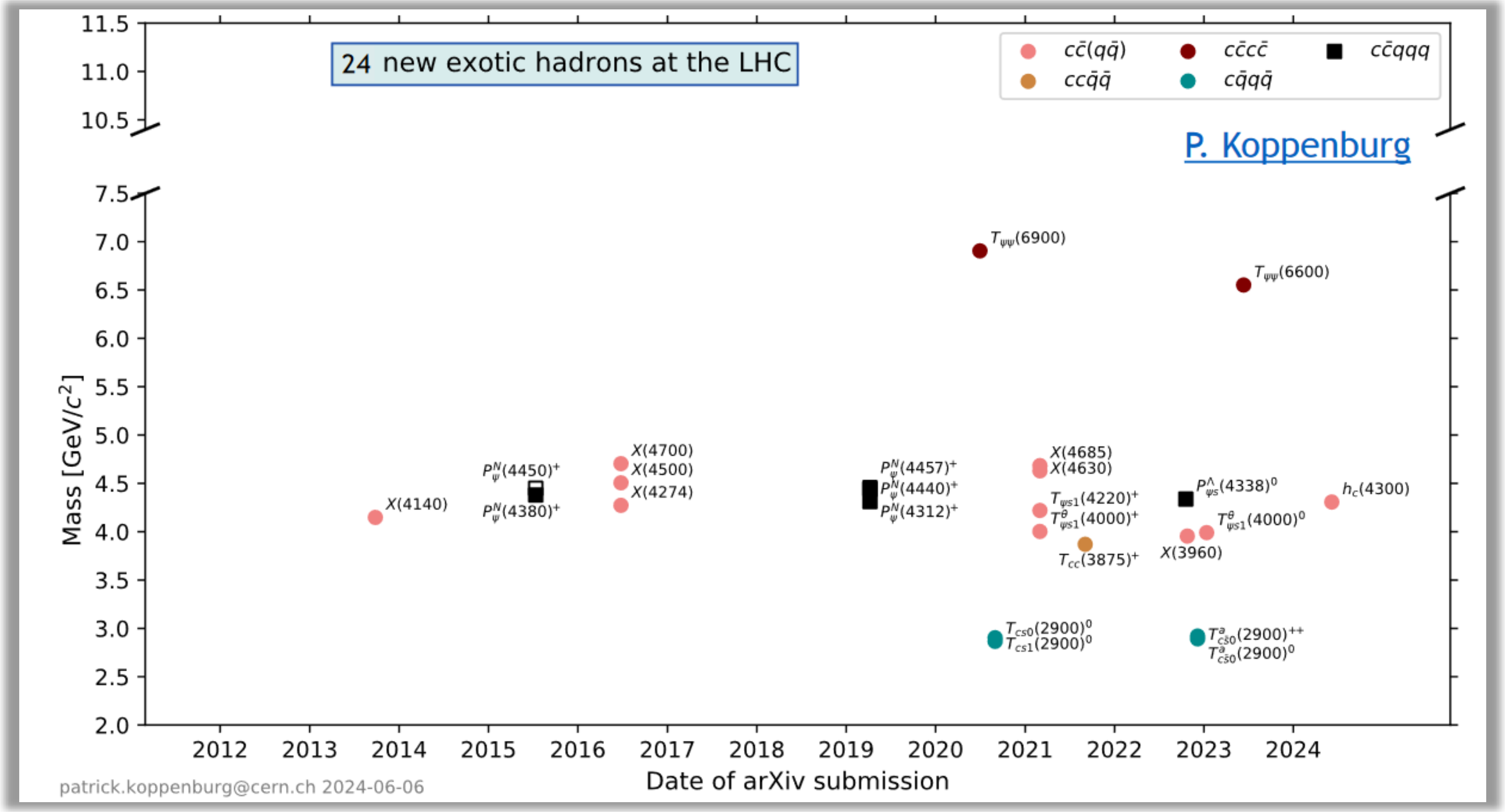
New Hadrons



From: Patrick Koppenburg@NIKHEF

….And so at Belle, BES III, COMPASS, JLAB

Exotic hadrons at LHC



@LHCb

….And so at Belle, BES III, COMPASS, CMS, ALICE, ATLAS, JLAB

VOLUME 47, NUMBER 25 PHYSICAL REVIEW LETTERS 21 DECEMBER 1981

Numerical Estimates of Hadronic Masses in a Pure SU(3) Gauge Theory

H. Hamber
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

and

G. Parisi
Istituto Nazionale di Fisica Nucleare, Frascati, Italy, and Istituto di Fisica della Facoltà di Ingegneria, Rome, Italy

(Received 2 October 1981)

$6^3 \times 10$

$a^{-1} = 1120$ MeV

$$m_\rho = 800 \pm 100, \quad m_p = 950 \pm 100,$$

$$m_\delta = 1000 \pm 100, \quad m_\Delta = 1300 \pm 100,$$

$$m_{A_1} = 1200 \pm 100, \quad f_\pi = 95 \pm 10.$$

$$m_G = (3.6 \pm 0.5)\sqrt{k} = 1500 \pm 200 \text{ MeV}$$

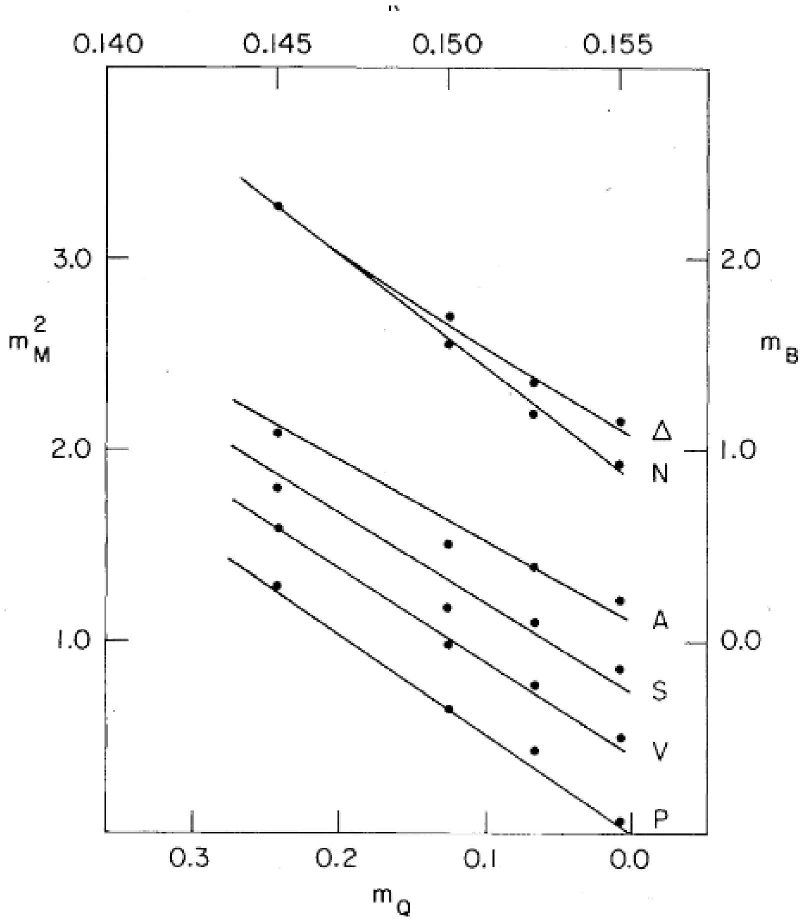
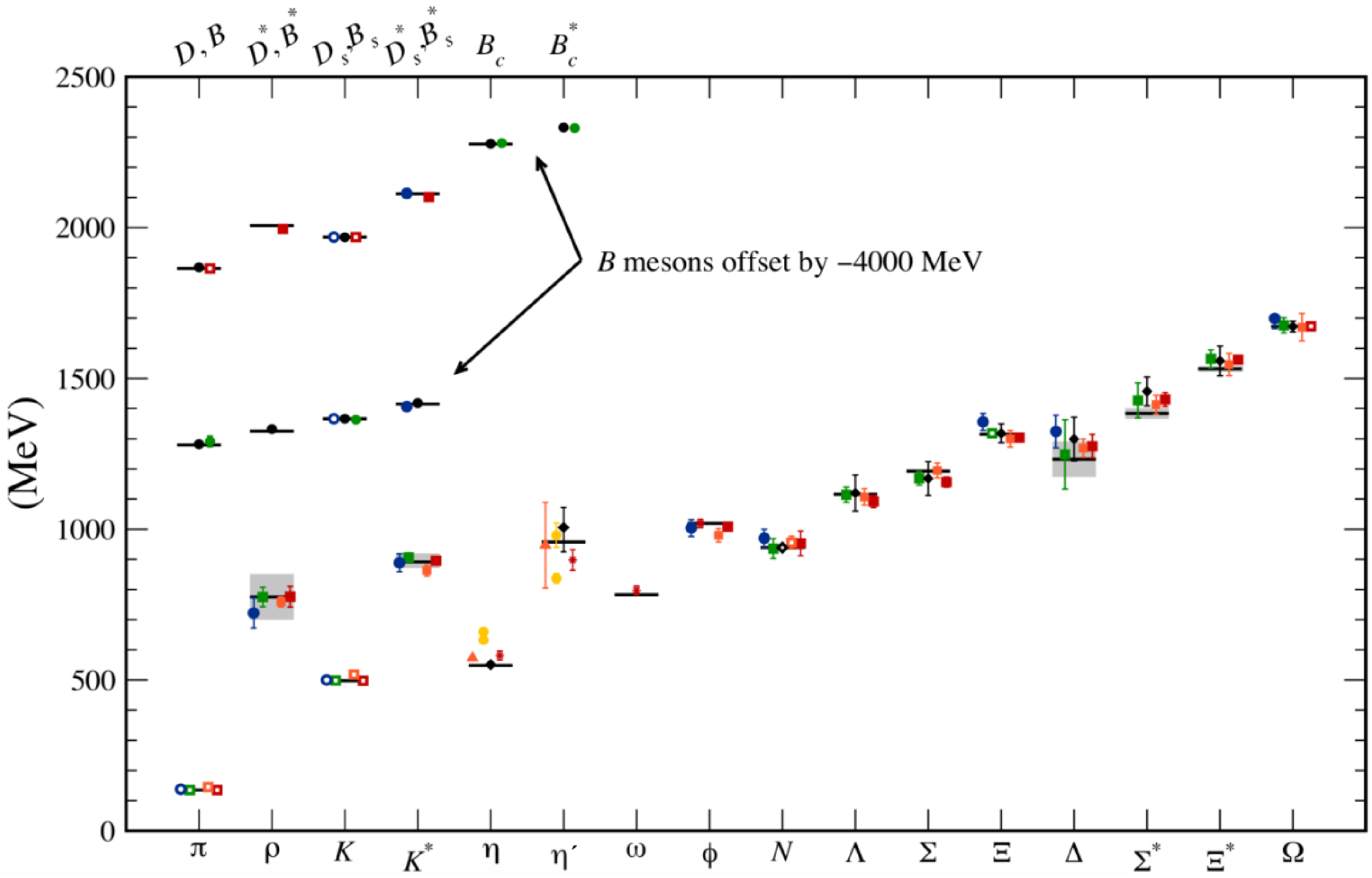


FIG. 2. Meson masses squared and baryon masses as a function of k and the bare quark mass $m_Q = (k_c - k)/2k_c^{\,2}$ obtained with use of the Wilson fermion action ($r = 1$) at $\beta = 6$. P, V, S, and A stand for pseudoscalar ($J^{PC} = 0^{-+}$), vector (1^{--}), scalar (0^{++}), and axial vector (1^{++}) masses. N and Δ stand for nucleon ($\frac{1}{2}^+$) and delta ($\frac{3}{2}^+$) masses.

About a decade ago



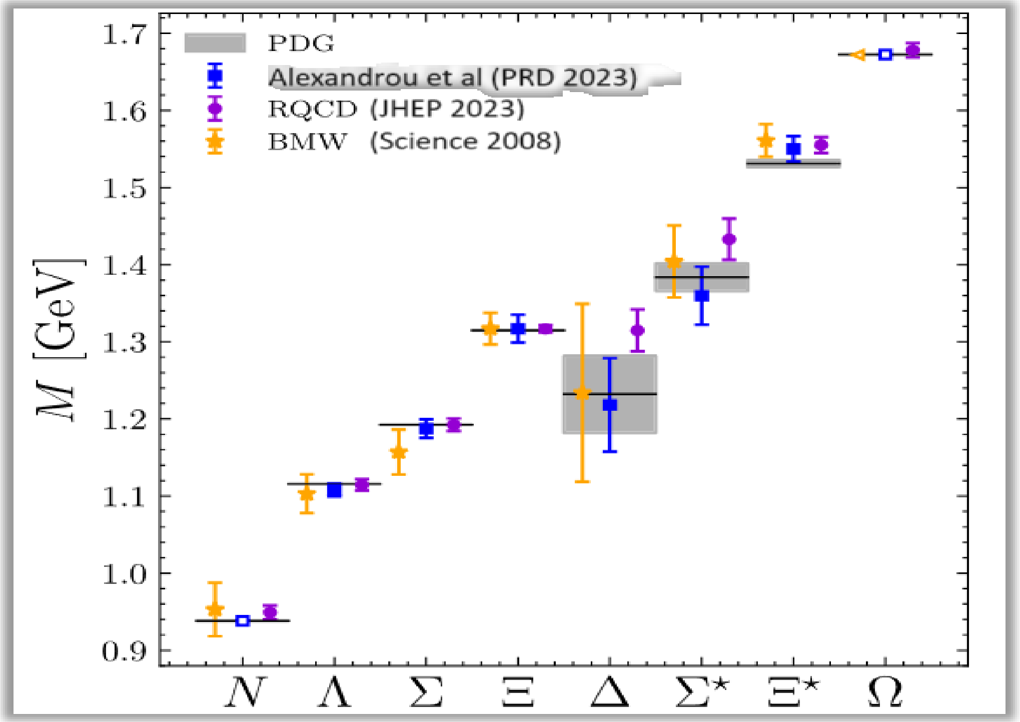
MILC (RMP 2010)
PAC-CS (PRD2019)
BMW (Science 2008)
QCDSF (PRD 2011)
ETM (PRD2014)
UKQCD (PRD2012)

η, η': RBC & UKQCD (PRL 2010)
HADSPEC (PRD 2011)
Heavy-light:
FERMILAB-MILC (PRD 2011)
HPQCD (PRD 2011, 12)
Mohler et al (PRD 2011)
Open symbols: fixing parameters

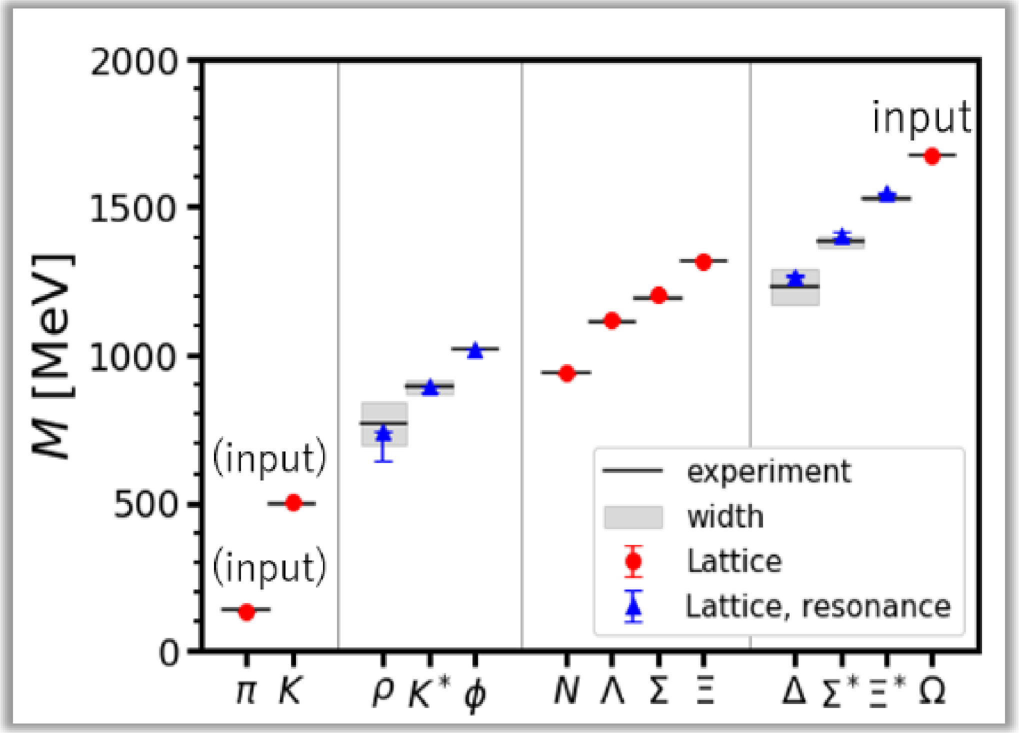
Circles: staggered
Squares: Wilson
Diamonds: Chiral

A. Kronfeld: Ann.Rev.Nucl.Part.Sci. 62 (2012) 265-284
also PDG

Some up-to-date results (incomplete list)



Alexandrou et al: PRD 108, 094510 (2023)



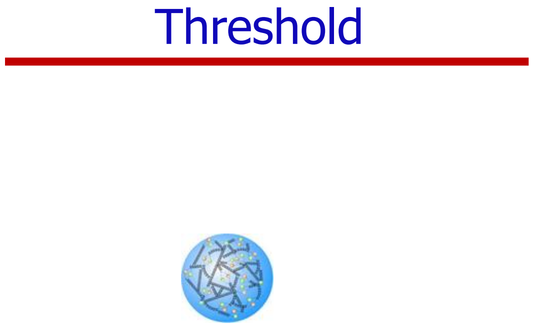
HAL QCD Collaboration: arXiv: 2406.16665

Hadron Spectra: What can Lattice QCD do?

Relatively Easy	Moderately difficult	Difficult
Threshold  Stable under strong interactions, Far away below threshold $\pi, K, D, p, n, \Lambda\ etc.$ ground states		

Hadron Spectra: What can Lattice QCD do?

Relatively Easy

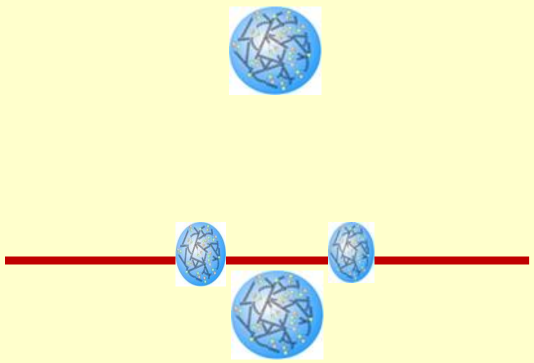


Threshold

Stable under strong interactions,
Far away below threshold

π, K, D, p, n, Λ *etc.*
ground states

Moderately difficult



- Close to threshold loosely bound,
- Elastic resonances with two-body decays

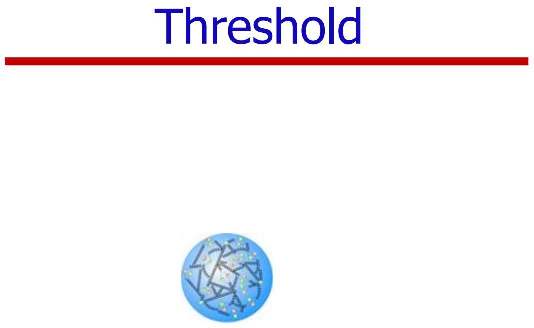
$\rho, \Delta, D^*, D_{s0}^*$, deuteron,
H-dibaryon, and other hadronic
resonances with only 2-body decays

Difficult

Power law volume corrections
Need rigorous finite volume amplitude analysis with a large basis of operators

Hadron Spectra: What can Lattice QCD do?

Relatively Easy



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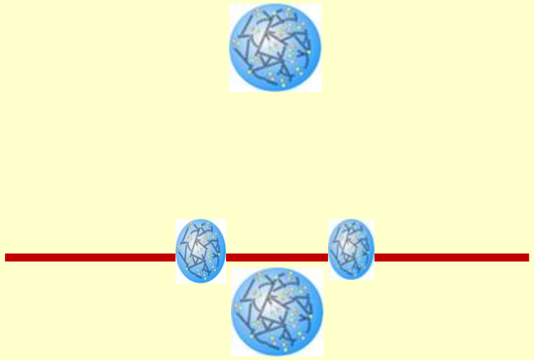

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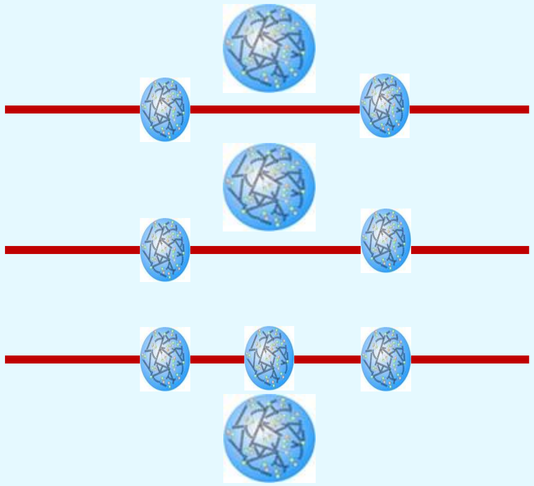

- Close to multiple thresholds, loosely bound
- Inelastic resonances with multiple two-body decays, three or more body

X, Y, Z, P_c Pentaquarks, $1^{-+}, \sigma, glueballs$,
higher nuclei

Power law volume corrections

Need rigorous finite volume amplitude analysis with a large basis of operators

Scattering -- continuum

Pole Singularities

Existence of a composite particle with angular momentum l ➡ Pole singularities in scattering amplitudes

$$\sigma(E) \propto |\mathcal{M}(E)|^2 \qquad \mathcal{M}_l = \frac{g^2}{\mathcal{S}_0 - \mathcal{S}}$$

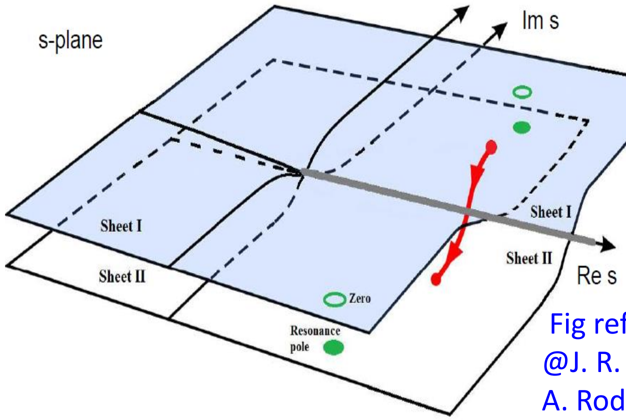

Fig ref.
@J. R. Peláez,
A. Rodas

Scattering -- continuum

Pole Singularities

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$$\sigma(E) \propto |\mathcal{M}(E)|^2 \qquad \mathcal{M}_l = \frac{g^2}{\mathcal{S}_0 - \mathcal{S}}$$

Allowed Poles:

1. **Poles on the real axis below threshold on the physical sheet**
   These are bound state poles

Bound states ➡

$$\mathcal{M}_l \propto \frac{1}{E_{cm}^2 - m_B^2} \qquad \mathcal{M}_l(E_{cm} = m_B) \to \infty$$

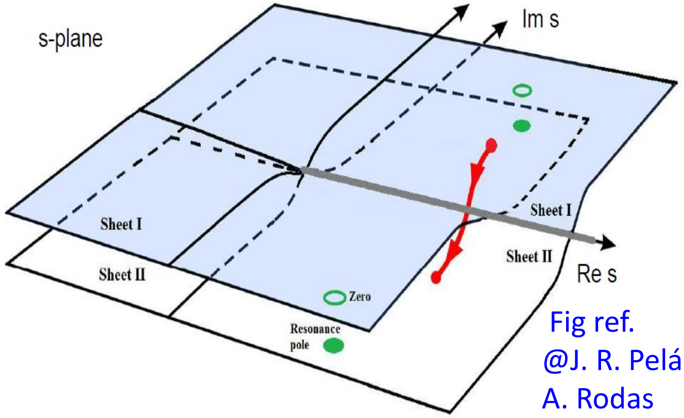

Fig ref.
@J. R. Peláez,
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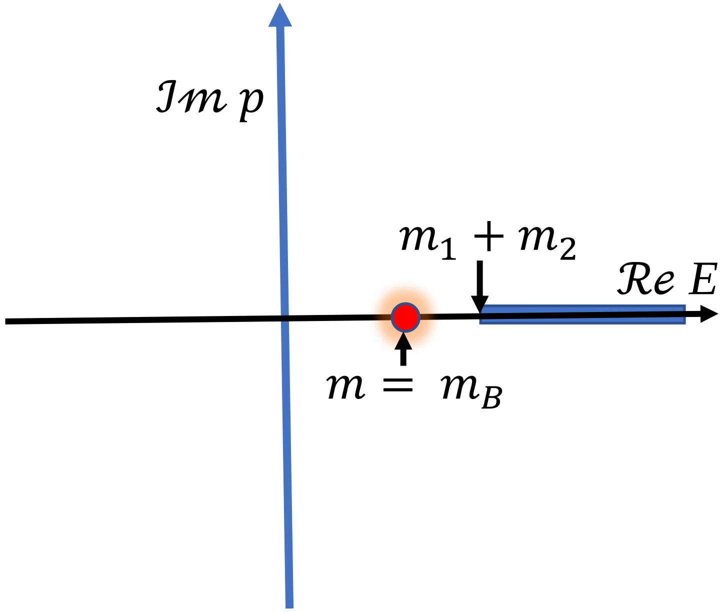

Scattering -- continuum

Pole Singularities

Existence of a composite particle with angular momentum l ⟶ Pole singularities in scattering amplitudes

$$\sigma(E) \propto |\mathcal{M}(E)|^2 \qquad \mathcal{M}_l = \frac{g^2}{\mathcal{S}_0 - \mathcal{S}}$$

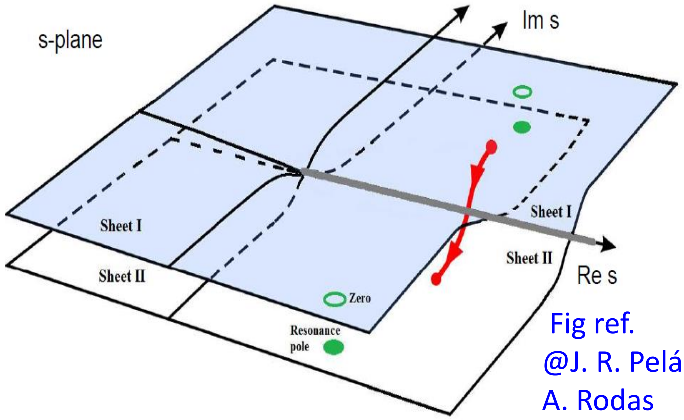

Fig ref. @J. R. Peláez, A. Rodas

Allowed Poles:

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Bound states ⟶

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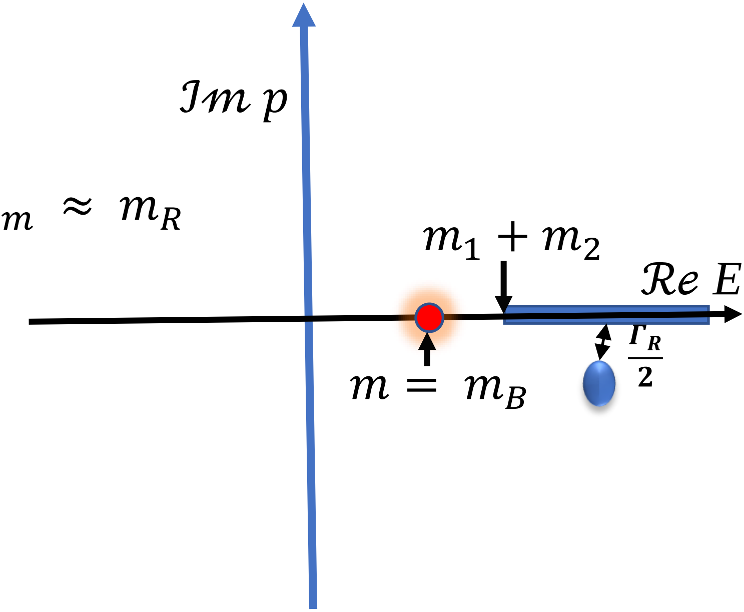
2. **Poles *off* the real axis but on the unphysical sheet**
   They appear as complex conjugate pair. $\sqrt{\mathcal{S}_0} = m_R \pm i\Gamma_R$
   Proximity of unphysical sheet to the real energy axis $m_R - i\Gamma_R$
   For not so big Γ_R, cross-section can have a bump at the energy around $E_{cm} \approx m_R$
   These poles correspond to Resonances.

Resonances ⟶



Scattering -- continuum

Pole Singularities

Existence of a composite particle with angular momentum l → Pole singularities in scattering amplitudes

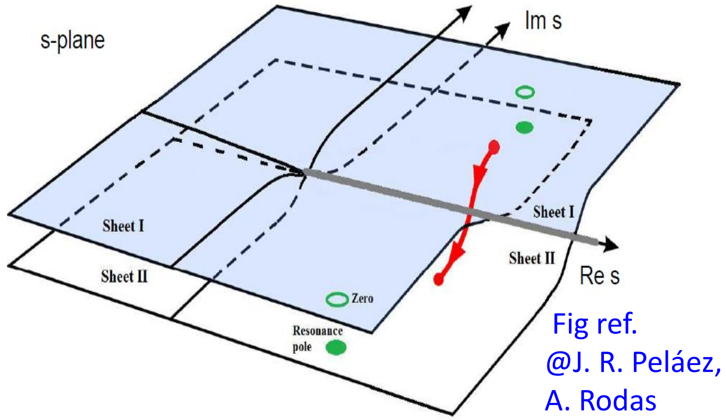


Fig ref. @J. R. Peláez, A. Rodas

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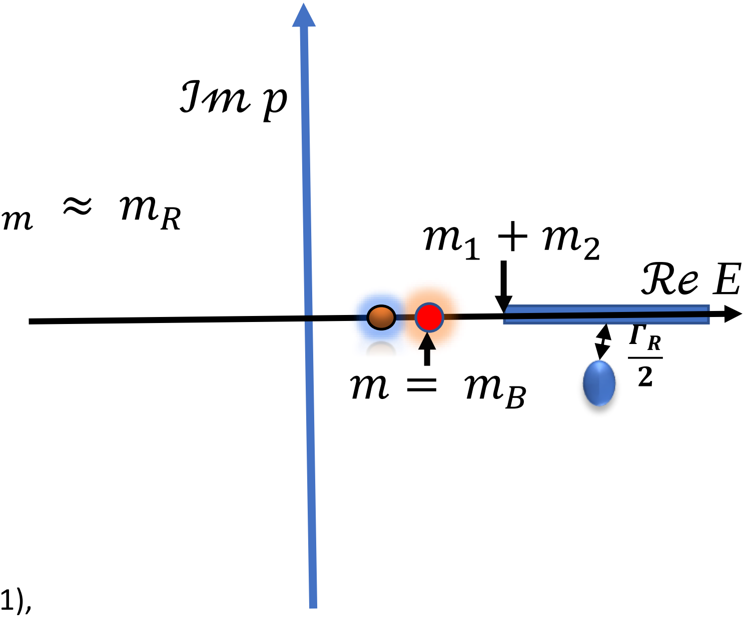
Resonances →



3. **Pole on the real energy axis below threshold but on the unphysical sheet**

   For such pole $p = -i|p|$

   Such poles correspond to virtual bound states (no asymptotic state)

   Attraction is too weak to form a bound state

   Ex: Spin-singlet dineutron in NN-scattering I. Matuschek et al, Eur. Phys. J. A 57, 101 (2021), P. Reinert, Eur. Phys. J. A 54, 86 (2018)

Virtual bound states →

Scattering -- continuum

Pole Singularities

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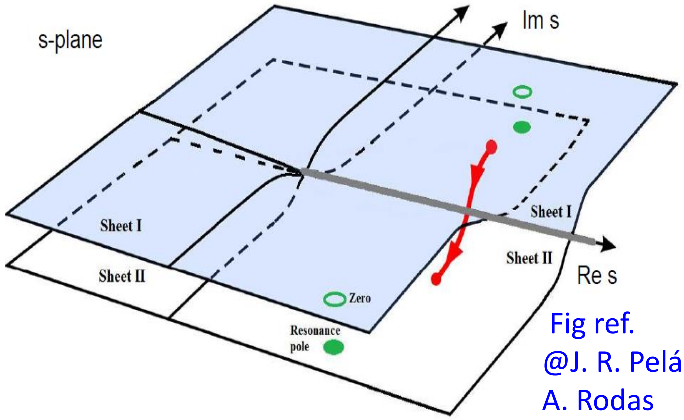

Fig ref.
@J. R. Peláez,
A. Rodas

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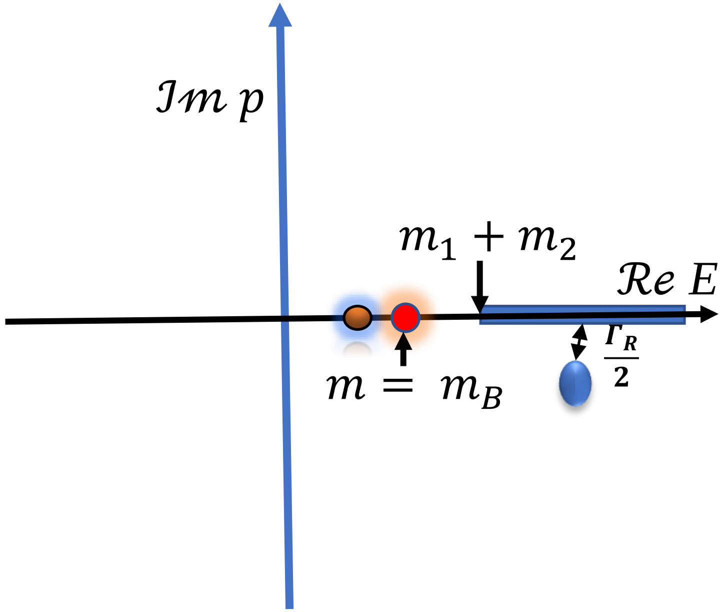
   Bound states

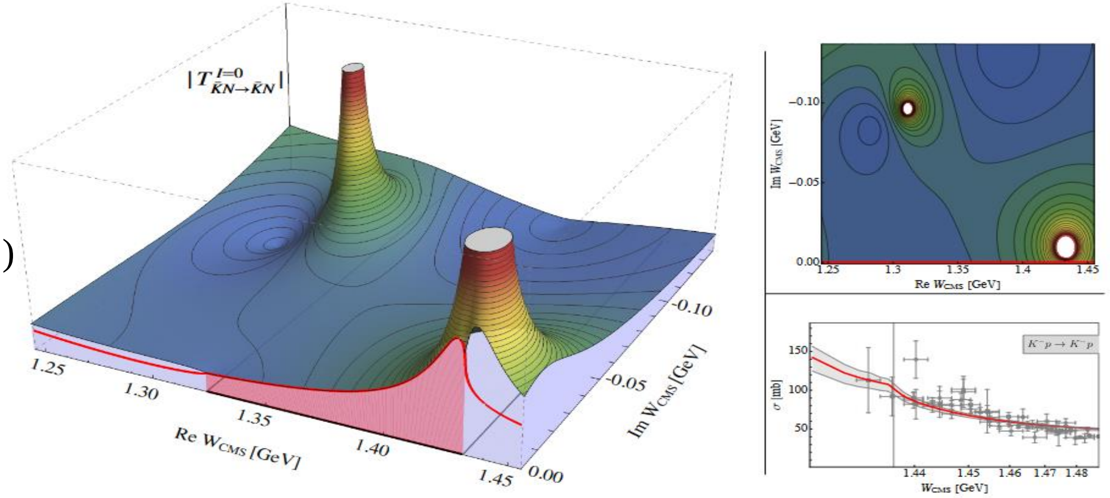
2. **Poles *off* the real axis but on the unphysical sheet**

   Resonances

3. **Pole on the real energy axis below threshold but on the unphysical sheet**

   Virtual bound states





M. Mai: Eur.Phys.J.ST 230 (2021) 6, 1593-1607

Luscher’s Method

Obtain infinite volume scattering information from finite volume energy spectra

Luscher’s quantization condition for 2 → 2 scattering processes:

$$\boxed{\mathbf{det}\left[\boldsymbol{M}(\boldsymbol{E}) + \boldsymbol{F}^{-1}(\boldsymbol{E},\boldsymbol{p};\boldsymbol{L})\right] = \mathbf{0}}$$

M. Luscher, Commun. Math. Phys. 104, 177 (1986),
Commun. Math. Phys. 105, 153 (1986),
Nucl. Phys. B 354, 531 (1991)

Rev.Mod.Phys. 90 (2018) 2, 025001
Brecino, Dudek, Young
(see other references related to Luscher’s method in this reference)

Caveats:

- Assumes continuum energies and ignore exponentially smaller contributions
- Truncated at some max ℓ
- Only valid above left-hand cut and below 3 (or 4) particle threshold

Alternatives:

- HAL QCD potential method [S. Aoki & T. Doi 2003.10730]
- Spectral functions from Euclidean correlators

[J. Bulava & M. Hansen 1903.11735]

…Discussed Lat 2023 (Hanlon)

Today's common practice

Construct the correlator matrix with your favorite interpolators

$$\begin{bmatrix} C(t)_{00} & C(t)_{01} & \cdots \\ \vdots & \ddots & \\ C(t)_{N0} & & C(t)_{NN} \end{bmatrix} v_a = \lambda_a(t,t_0) \begin{bmatrix} C(t_0)_{00} & C(t_0)_{01} & \cdots \\ \vdots & \ddots & \\ C(t_0)_{N0} & & C(t_0)_{NN} \end{bmatrix} v_a$$

Distillation:
[Peardon etal. PRD 09]
[Morningstar et al
PRD 11]
CODE: Chroma,
[github:paboyle/grid]
[github:aportelli/hadrons]

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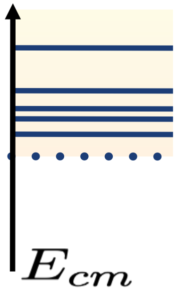
PRD 11]
CODE: Chroma,
[github:paboyle/grid]
[github:aportelli/hadrons]

Fit the principal correlators and extract the finite volume spectra

$$\lambda_\alpha(t,t_0) \sim e^{-E_\alpha(t-t_0)}$$



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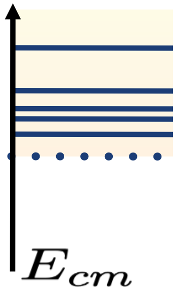
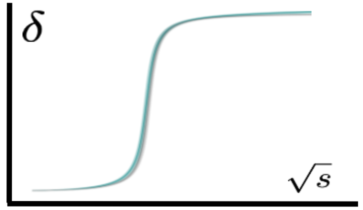


Solve for the quantization condition,
Analytically continue the amplitudes,
look for poles in the complex energy plane

$$\det\left[F^{-1}+\mathcal{M}\right]=0 \longrightarrow$$

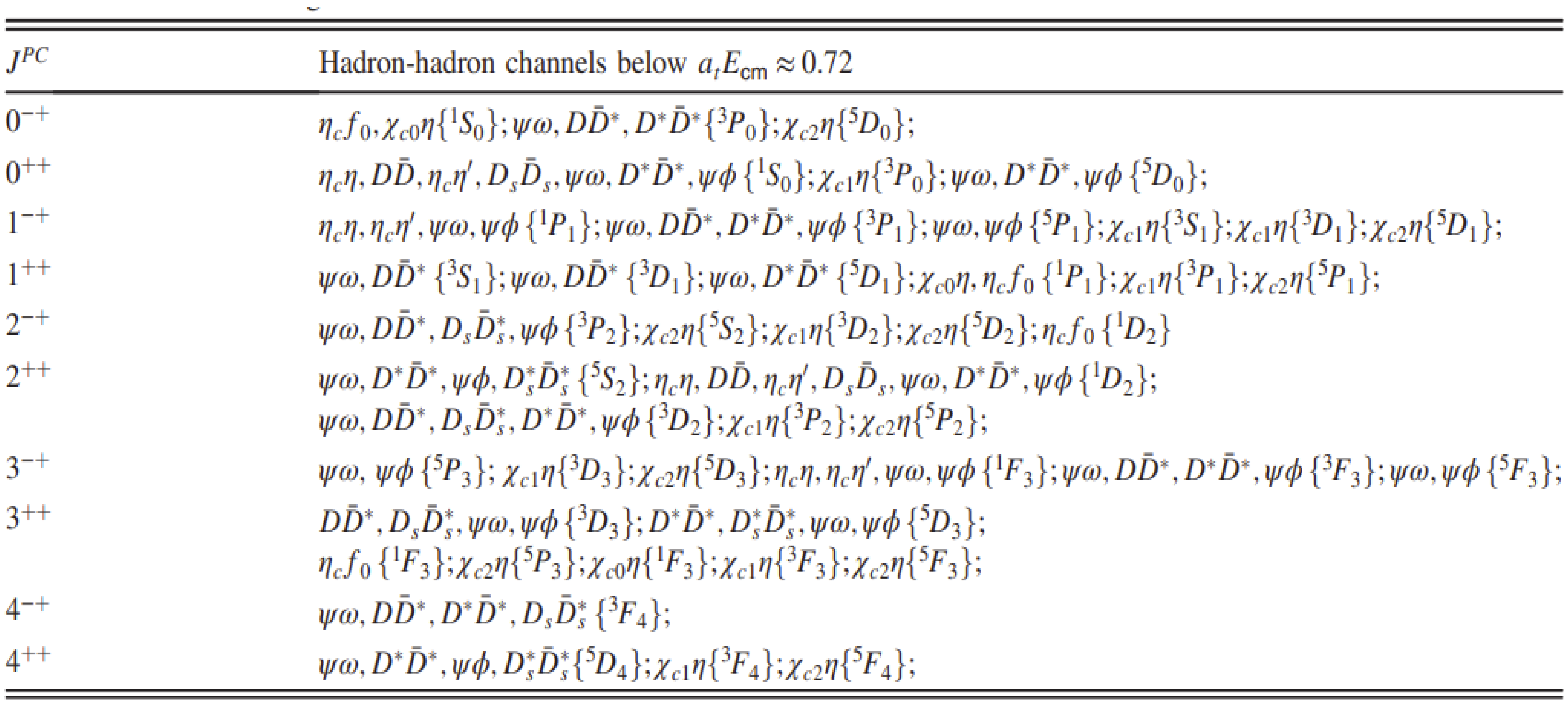


Phase sifts, pole positions

Example of codes: Morningstar et al, ``TwoHadronsInBox", NPB924, 477 (2017); Chroma etc

Example of a state-of-the-art calculation

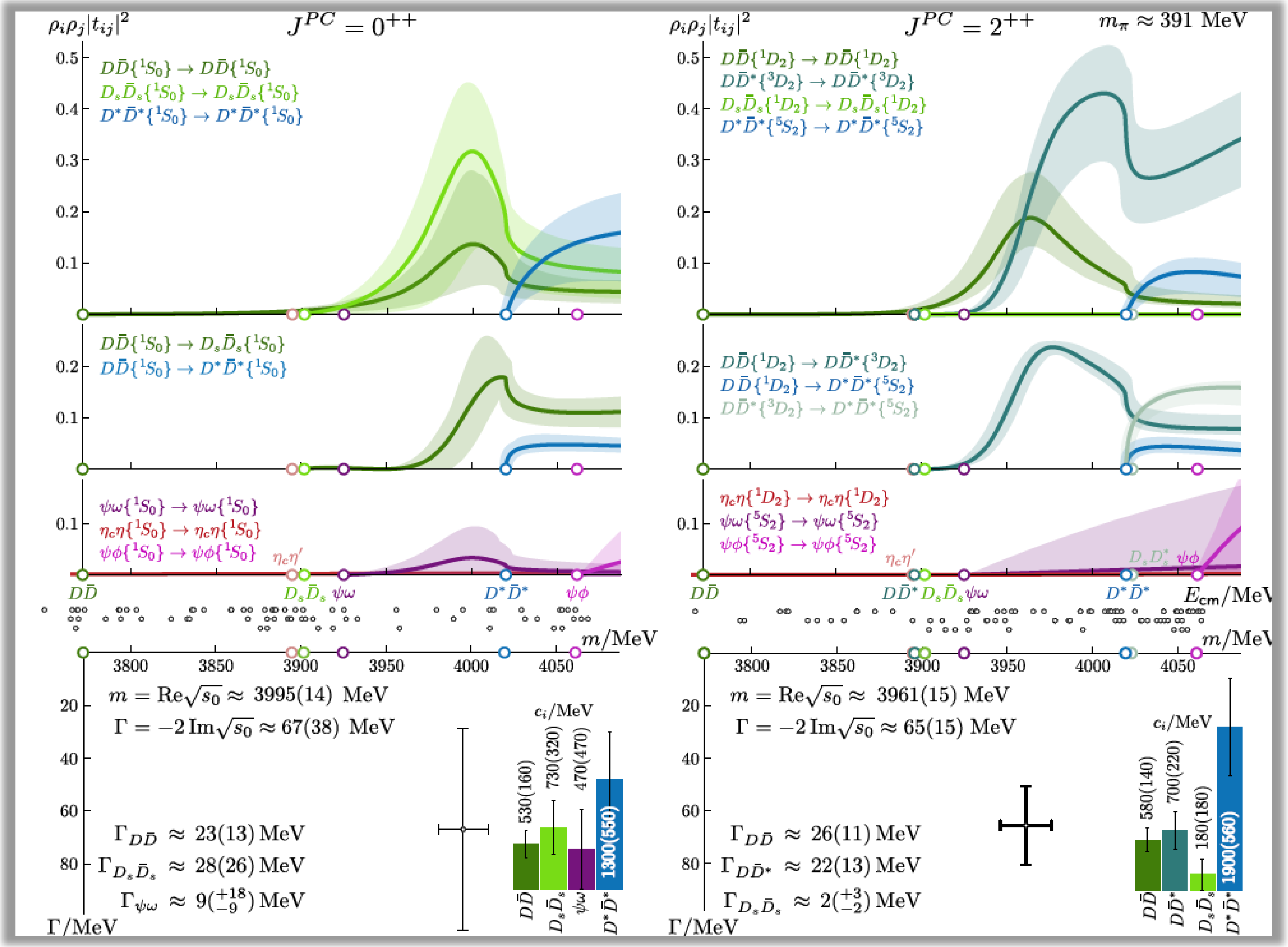
Hidden-charm scalar and tensor resonances: J^{PC}: $0^{++}, 2^{++}$ ($m_\pi \approx 391$ MeV)



J^{PC}	Hadron-hadron channels below $a_t E_{\mathrm{cm}} \approx 0.72$
0^{-+}	$\eta_c f_0, \chi_{c0}\eta\{{}^1S_0\}; \psi\omega, D\bar{D}^*, D^*\bar{D}^*\{{}^3P_0\}; \chi_{c2}\eta\{{}^5D_0\};$
0^{++}	$\eta_c\eta, D\bar{D}, \eta_c\eta', D_s\bar{D}_s, \psi\omega, D^*\bar{D}^*, \psi\phi\,\{{}^1S_0\}; \chi_{c1}\eta\{{}^3P_0\}; \psi\omega, D^*\bar{D}^*, \psi\phi\,\{{}^5D_0\};$
1^{-+}	$\eta_c\eta, \eta_c\eta', \psi\omega, \psi\phi\,\{{}^1P_1\}; \psi\omega, D\bar{D}^*, D^*\bar{D}^*, \psi\phi\,\{{}^3P_1\}; \psi\omega, \psi\phi\,\{{}^5P_1\}; \chi_{c1}\eta\{{}^3S_1\}; \chi_{c1}\eta\{{}^3D_1\}; \chi_{c2}\eta\{{}^5D_1\};$
1^{++}	$\psi\omega, D\bar{D}^*\,\{{}^3S_1\}; \psi\omega, D\bar{D}^*\,\{{}^3D_1\}; \psi\omega, D^*\bar{D}^*\,\{{}^5D_1\}; \chi_{c0}\eta, \eta_c f_0\,\{{}^1P_1\}; \chi_{c1}\eta\{{}^3P_1\}; \chi_{c2}\eta\{{}^5P_1\};$
2^{-+}	$\psi\omega, D\bar{D}^*, D_s\bar{D}_s^*, \psi\phi\,\{{}^3P_2\}; \chi_{c2}\eta\{{}^5S_2\}; \chi_{c1}\eta\{{}^3D_2\}; \chi_{c2}\eta\{{}^5D_2\}; \eta_c f_0\,\{{}^1D_2\}$
2^{++}	$\psi\omega, D^*\bar{D}^*, \psi\phi, D_s^*\bar{D}_s^*\,\{{}^5S_2\}; \eta_c\eta, D\bar{D}, \eta_c\eta', D_s\bar{D}_s, \psi\omega, D^*\bar{D}^*, \psi\phi\,\{{}^1D_2\};$ $\psi\omega, D\bar{D}^*, D_s\bar{D}_s^*, D^*\bar{D}^*, \psi\phi\,\{{}^3D_2\}; \chi_{c1}\eta\{{}^3P_2\}; \chi_{c2}\eta\{{}^5P_2\};$
3^{-+}	$\psi\omega, \psi\phi\,\{{}^5P_3\}; \chi_{c1}\eta\{{}^3D_3\}; \chi_{c2}\eta\{{}^5D_3\}; \eta_c\eta, \eta_c\eta', \psi\omega, \psi\phi\,\{{}^1F_3\}; \psi\omega, D\bar{D}^*, D^*\bar{D}^*, \psi\phi\,\{{}^3F_3\}; \psi\omega, \psi\phi\,\{{}^5F_3\};$
3^{++}	$D\bar{D}^*, D_s\bar{D}_s^*, \psi\omega, \psi\phi\,\{{}^3D_3\}; D^*\bar{D}^*, D_s^*\bar{D}_s^*, \psi\omega, \psi\phi\,\{{}^5D_3\};$ $\eta_c f_0\,\{{}^1F_3\}; \chi_{c2}\eta\{{}^5P_3\}; \chi_{c0}\eta\{{}^1F_3\}; \chi_{c1}\eta\{{}^3F_3\}; \chi_{c2}\eta\{{}^5F_3\};$
4^{-+}	$\psi\omega, D\bar{D}^*, D^*\bar{D}^*, D_s\bar{D}_s^*\,\{{}^3F_4\};$
4^{++}	$\psi\omega, D^*\bar{D}^*, \psi\phi, D_s^*\bar{D}_s^*\{{}^5D_4\}; \chi_{c1}\eta\{{}^3F_4\}; \chi_{c2}\eta\{{}^5F_4\};$

Hadspec: Wilson, Thomas, Dudek, and Edwards

PRL 132, 241901 (2024)
PRD 109, 114503 (2024)



$$a_t^{-1} = 5667 \text{ MeV}$$

$$\xi = a_s/a_t \approx 3.5$$

$$N_f = 2+1$$

- Above the ground state χ_{c0}, no other scalar bound states or near-$D\bar{D}$ threshold resonances are found
- No χ_{c0} state between 3700 and 3860 MeV
- The single 2^{++} resonance found in this calculation decays to $D\bar{D}$ and $D\bar{D}^*$, but has at most weak coupling to $D_s\bar{D}_s$ and closed-charm final states

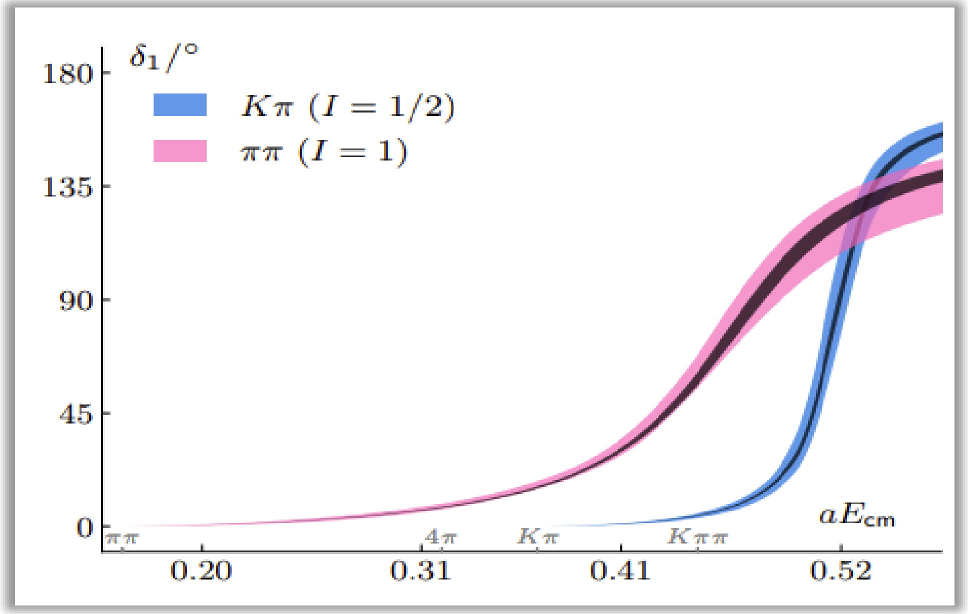
Need to repeat this calculation at the physical pion mass

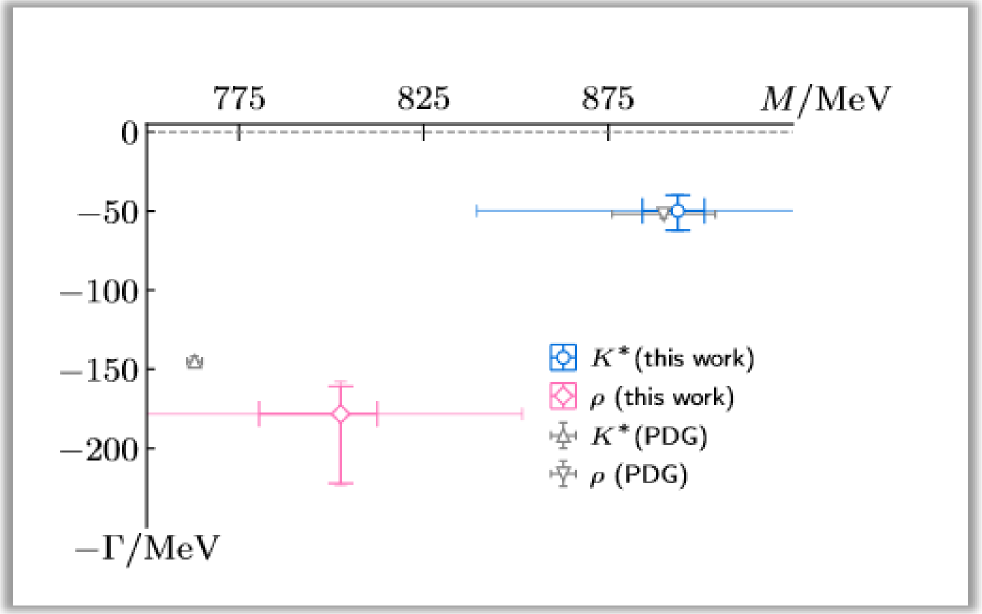
Hadspec: Wilson, Thomas, Dudek, and Edwards

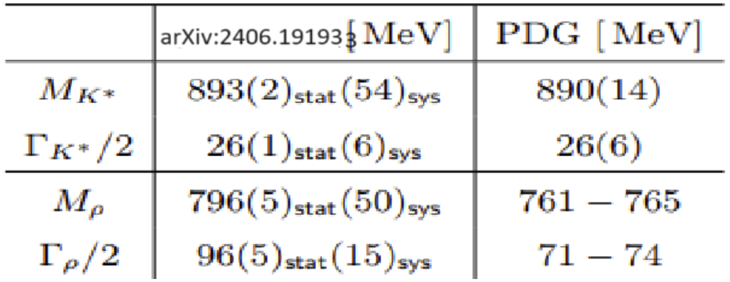
PRL 132, 241901 (2024)
PRD 109, 114503 (2024)

Example of a state-of-the-art calculation

$\rho(770)$ and $K^*(892)$ resonance parameters







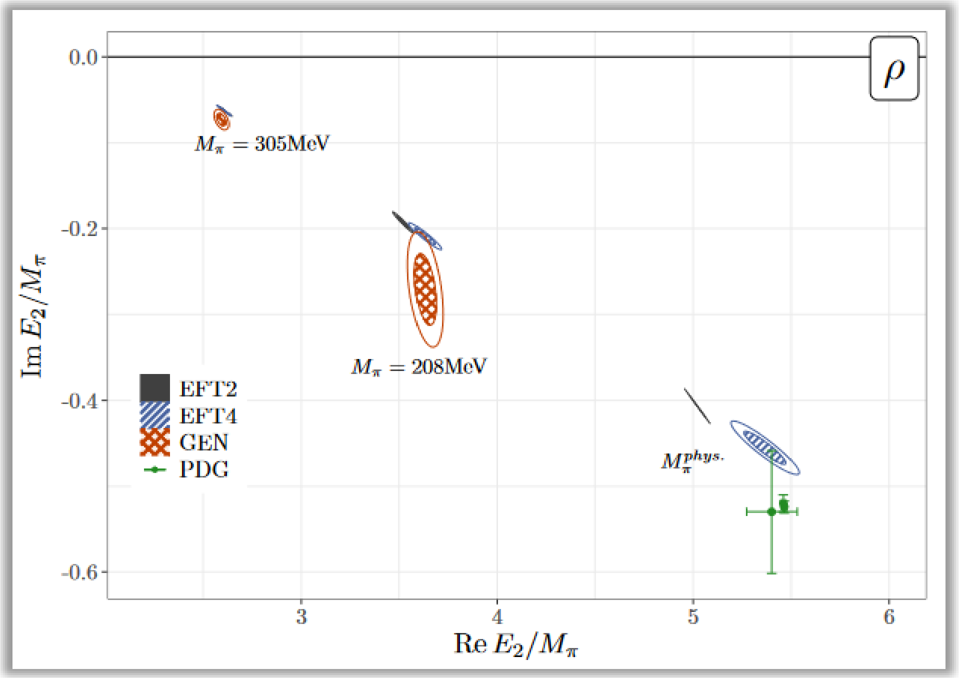
	arXiv:2406.19193 [MeV]	PDG [MeV]
M_{K^*}	$893(2)_{\mathrm{stat}}(54)_{\mathrm{sys}}$	890(14)
$\Gamma_{K^*}/2$	$26(1)_{\mathrm{stat}}(6)_{\mathrm{sys}}$	26(6)
M_ρ	$796(5)_{\mathrm{stat}}(50)_{\mathrm{sys}}$	761 − 765
$\Gamma_\rho/2$	$96(5)_{\mathrm{stat}}(15)_{\mathrm{sys}}$	71 − 74

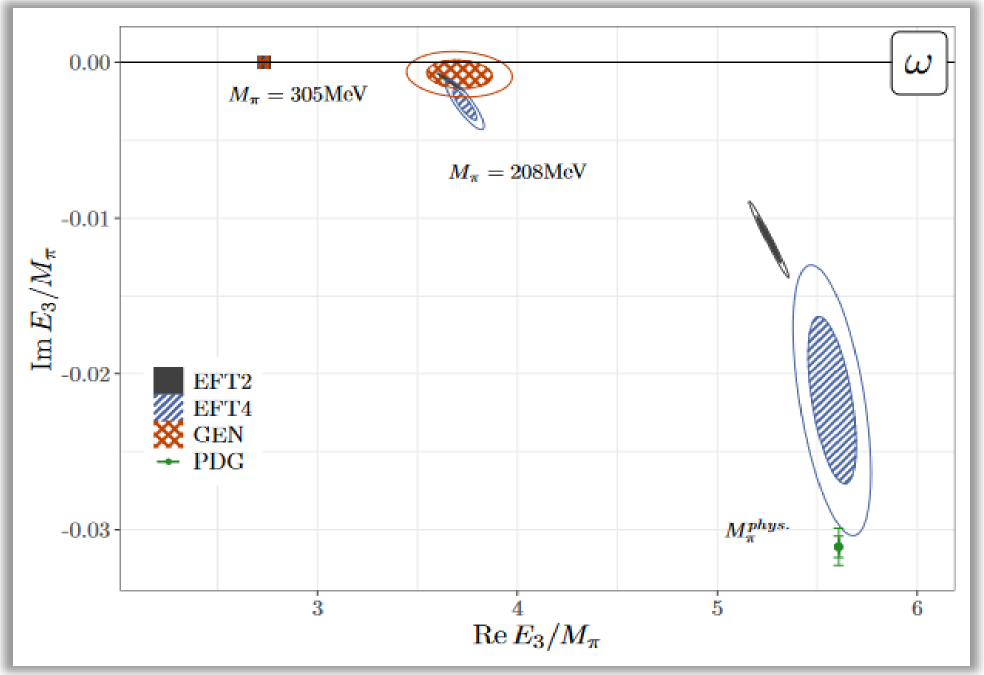
2 + 1 flavor RBC/UKQCD Mobius domain-wall
a^{-1}= 1. 7295(38) GeV, V = $48^3 \times 96$
Physical pion mass

Boyle et al, arXiv: 2406.19193v1
Plenary talk (Tuesday): Felix Erben

Example of a state-of-the-art calculation

The *ω*-meson





$$a = 0.07746(18)\ \mathrm{fm}$$

$$\sqrt{s_\rho} = (748.9(10.0) - i63.5(1.8))\ \mathrm{MeV}$$

$$\sqrt{s_\omega} = (778.0(11.2) - i3.0(5))\ \mathrm{MeV}$$

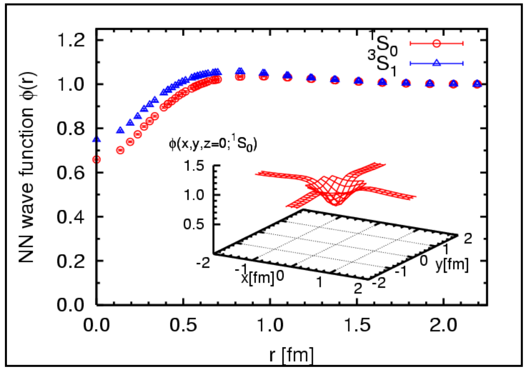
$$m_\omega - m_\rho = 29(15)\ \mathrm{MeV}$$

H. Yan et al. Monday [arXiv:2407.16659v1]

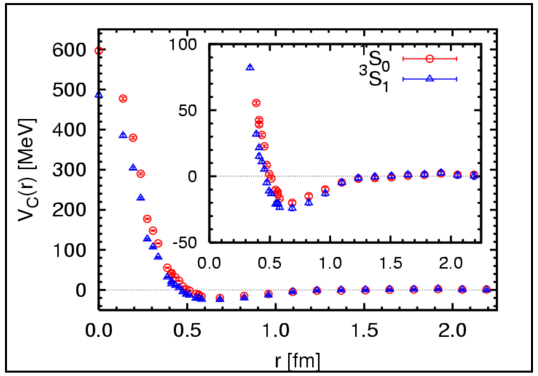
HAL QCD Method

Lattice QCD

NBS wave func.



Lattice Hadron Force



$$\psi_{\rm NBS}(\vec{r}) = \langle 0|H_1(\vec{r})H_2(\vec{0})|H_1(\vec{k})H_2(-\vec{k}), in\rangle \simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

$$(k^2/m_N - H_0)\,\psi_{\rm NBS}(\vec{r}) = \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi_{\rm NBS}(\vec{r}')$$

- **Interacting kernel (“potential”) from spatial & temporal correlation**
  - Potential → Phase Shifts

Aoki-Hatsuda-Ishii PTP123(2010)89

- **Energy-independent potential**

N.Ishii et al. (HAL Coll.) PLB712(2012)437

  - “Signal” from (elastic) excited states → Ground state saturation NOT required

- **Coupled Channel formalism**

S. Aoki et al. (HAL Coll.) Proc.Jpn.Acad.B87(2011)509

  - Physics above inelastic threshold → Essential for Hyperon-forces, Exotics

- **Extension to N-body systems (N>=3)**

S. Aoki et al. (HAL Coll.) PRD88(2013)014036
(non-rela approx.)

Sample of channels studied by various lattice groups (an incomplete list …apology)

$\boldsymbol{\pi\pi}(\mathbf{1},\mathbf{2})$ many many groups

$\boldsymbol{\pi\pi}\ (0,1,2), \boldsymbol{K\bar{K}}\ (0,1), \boldsymbol{K\pi}(1/2,\ 3/2), \boldsymbol{K\eta}(1/2), \boldsymbol{\eta\pi}(1), \boldsymbol{\eta'\pi}, \boldsymbol{\eta\eta}(0), \boldsymbol{\omega\pi}(1), \boldsymbol{\phi\pi}(1), \boldsymbol{\pi\pi\pi}(3)$

$\boldsymbol{D\pi}(1/2\ , 3/2), \boldsymbol{D\eta}(1/2), \boldsymbol{D_s\bar{K}}(1/2), \boldsymbol{DK}(0), \boldsymbol{D\bar{K}}(0,1), \boldsymbol{D^*\pi}(1/2), \boldsymbol{\eta_c\eta}(0), \boldsymbol{\eta_c\eta'}(0),$
$\boldsymbol{D\bar{D}}(0), \boldsymbol{D_S\bar{D}_S}(0), \boldsymbol{J/\psi\omega}(0), \boldsymbol{D\bar{D}^*}(0), \boldsymbol{D^*\bar{D}^*}(0), \boldsymbol{J/\psi\phi}(0), \boldsymbol{D^*D}(0), \boldsymbol{D^*D^*}\ (0)$ Hadspec

KN, BB, NN, NNN, ππ, DD*s, **Ω Ω** , πJ/ψ, $\boldsymbol{\eta_c}$ρ, D$\underline{D}$*, ΞΞ,ΞΣ,ΞΛ,
ΞΣ, $\boldsymbol{\Omega_{ccc}\Omega_{ccc}}$, $N\,J/\psi$, NΛ_c, N **Ω**, ΛΛ,NΞ,**ΣΣ**, ΔΔ HALQCD

NN, NNN, *ΛΛ* …NPLQCD

$\boldsymbol{D\bar{D}^*, D_sD_s^*, DD, D_sD_s, D^*D,}$
$\boldsymbol{D^*D^*, J/\psi\phi, J/\psi\rho, J/\psi\pi, \eta_c\pi, \eta_c\rho, BD, B^*D}$
$\boldsymbol{N\pi, N\sigma, \Lambda\Lambda, N\Xi, \Sigma\Sigma, \Omega_{bbb}\Omega_{bbb}}$

Prelovsek, Padmanath, Christian, Mohler, Colins, Stefano, Morningstar, Green, Mathur and others

ππ, Kπ, Dπ, DK, D*K, πρ, πω, Bsπ, J/ψπ, ψ2Sπ, ψ1Dπ,
DD¯*, D*D¯*, ηcρ, BK, B*K, Nηc, N-Jpsi

Prelovsek, Lang, Daniel, Luka, Woloshyn

Σπ,$\underline{K}$N, Kπ, Kππ, KKπ, Dπ, Nπ, ππ, NN, ***ΛΛ*, *NΞ***
ΣΣ, πππ, KKK

Morningstar, Bulava

ππ, πππ, KN, … Maxim/Michael

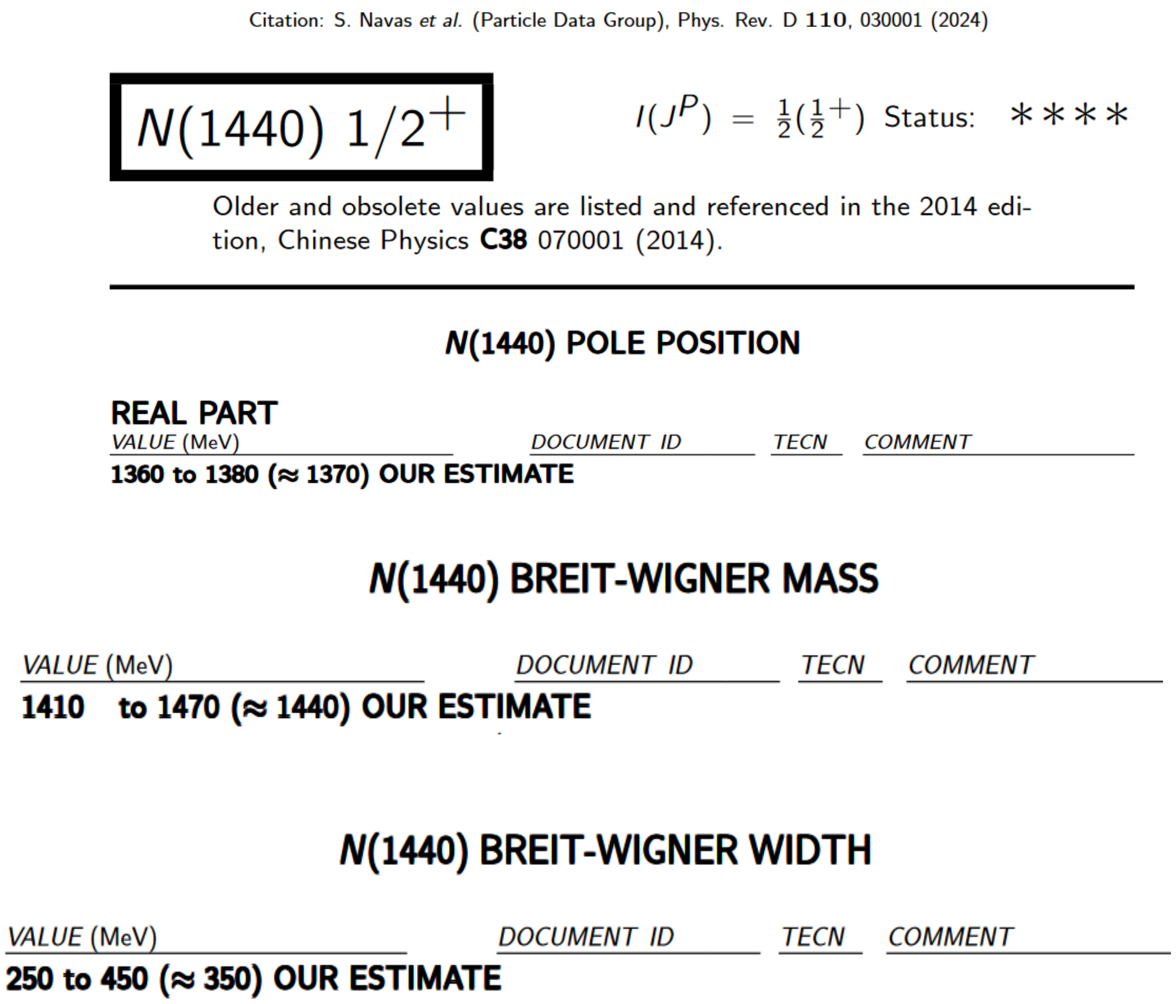
BB*, BBs*, $\underline{B}$*D, $\underline{B}$D, ππ, Kπ, Nπ, Marc/Stefan

Roper Resonance [N(1440) $J^P: \frac{1}{2}^+$]

- ❑ The Roper is the lightest excitation of the proton
- ❑ Discovery in 1963 (Adelman, 1964; Auvil et al., 1964; Bareyre et al., 1964; Roper, 1964; Roper, Wright, and Feld, 1965)

Motivation:

- Understanding the first excited state of proton
- Assessing the excited state contamination to control the systematic errors of the nucleon matrix elements and the πN contribution in calculations such as neutrino-nucleon scattering.



Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)

$N(1440)\ 1/2^+$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ∗∗∗∗

Older and obsolete values are listed and referenced in the 2014 edition, Chinese Physics **C38** 070001 (2014).

N(1440) POLE POSITION

REAL PART

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
1360 to 1380 (≈ 1370) OUR ESTIMATE			

N(1440) BREIT-WIGNER MASS

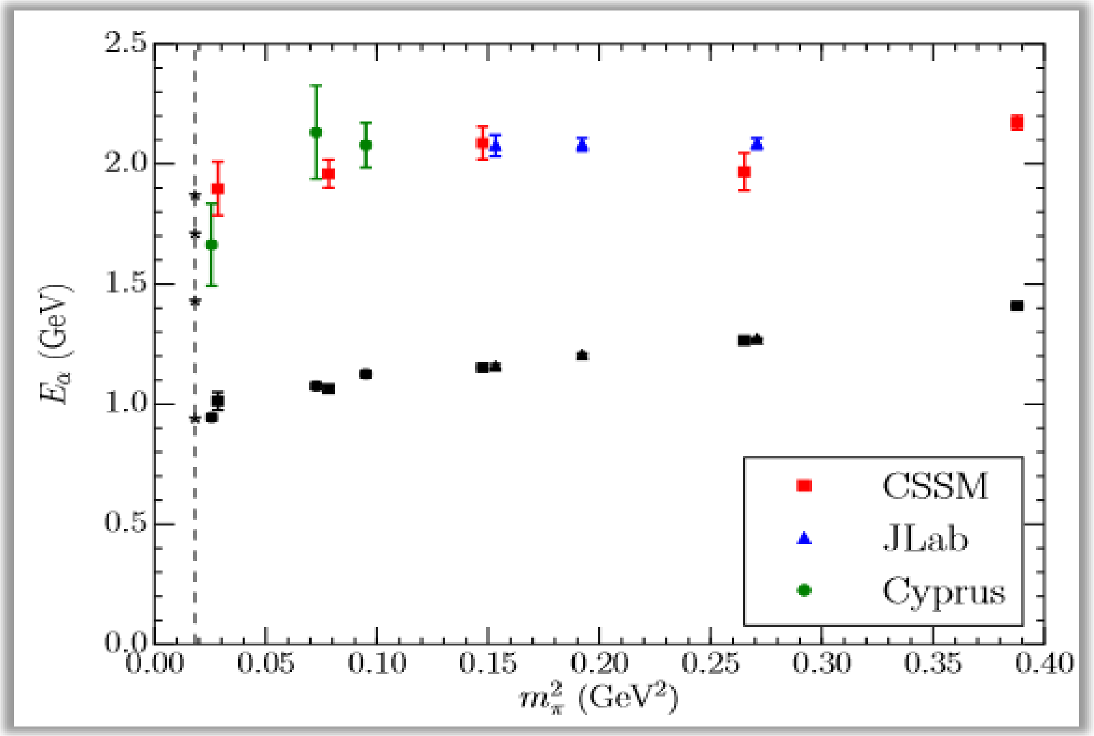
VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
1410 to 1470 (≈ 1440) OUR ESTIMATE			

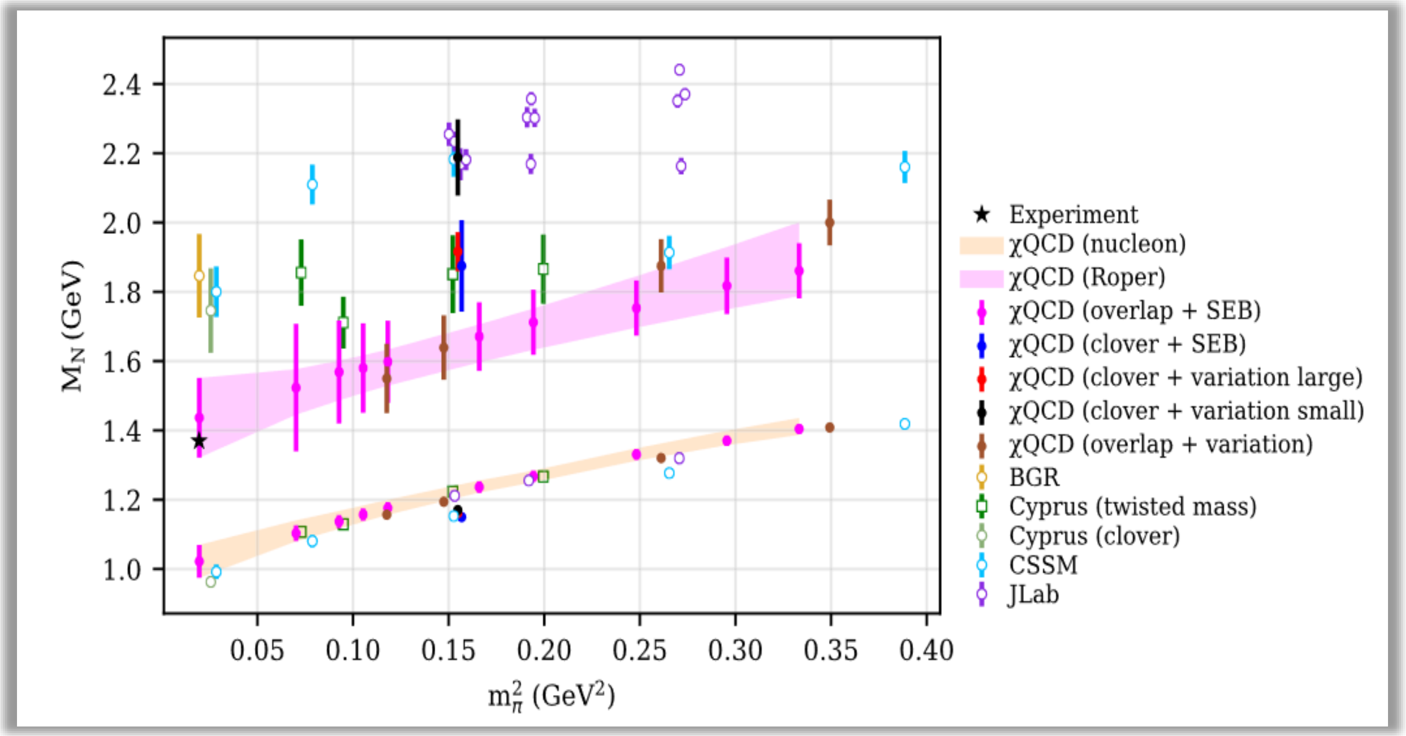
N(1440) BREIT-WIGNER WIDTH

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
250 to 450 (≈ 350) OUR ESTIMATE			

Recent review: Burkert and Roberts: RMP, Volume 91 (2019)

Roper: an enigma?

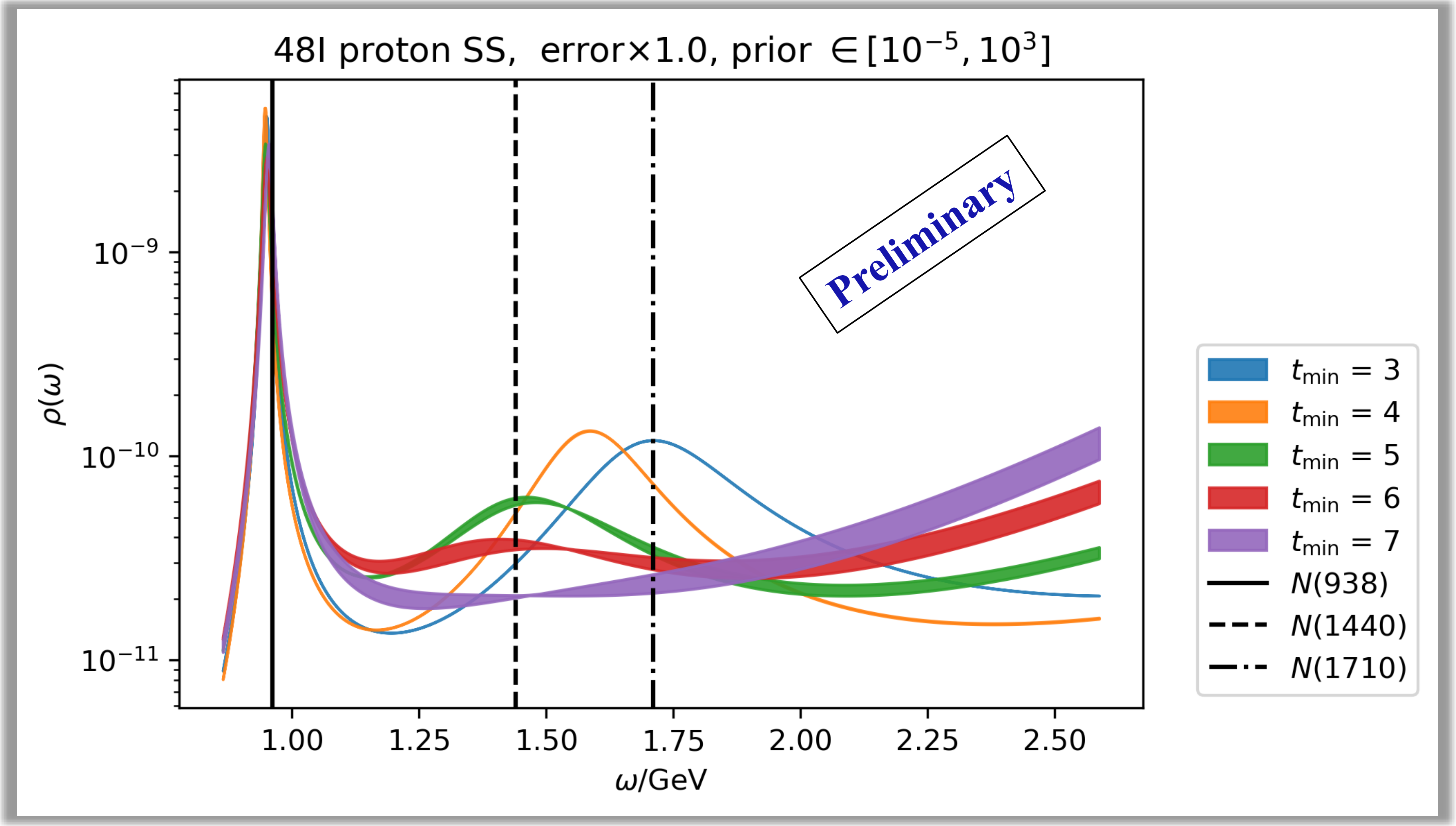




D. Leinweber, NSTAR 2024
No presence of Roper state

Importance of Chiral fermions:
χQCD (Sun et al). PRD 101, 054511 (2020)

Evidence of the Roper state? χQCD Collaboration



Barkus-Gilbert (BG),

Maximum Entropy,

Bayesian Reconstruction (BR)

BG good for broad distributions

BR good for discrete states

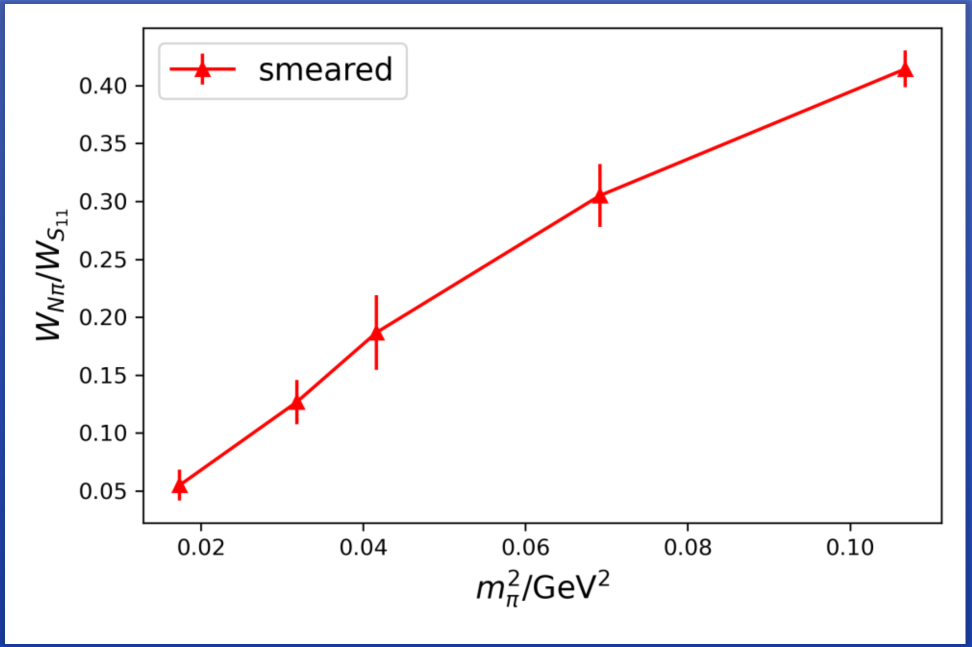
BR -- Y. Burnier and A. Rothkopf,

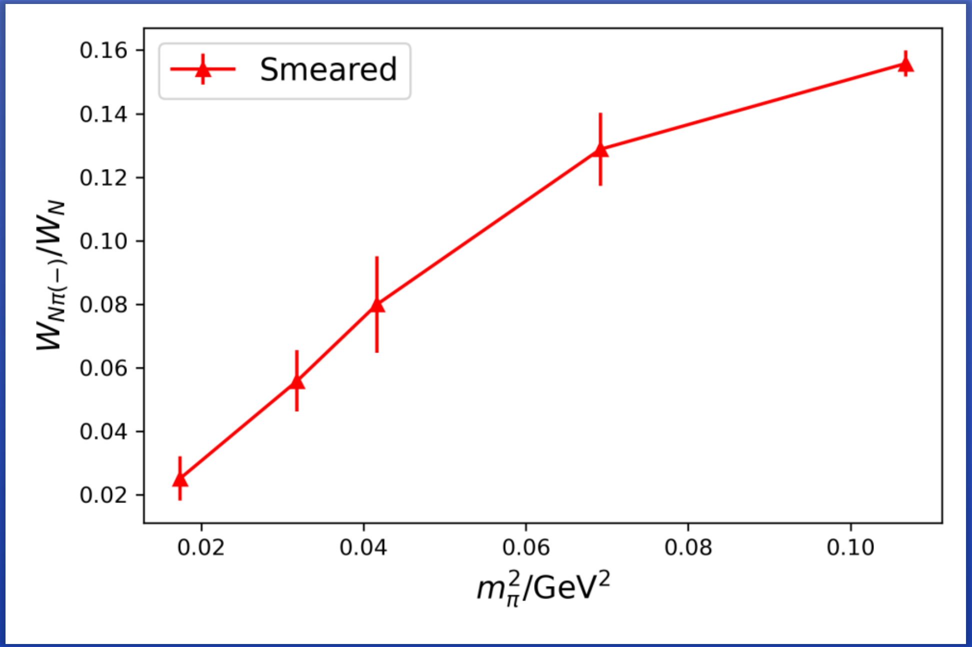
PRL 111, 182003 (2013)

- Prior for asymptotic distribution
- No priors for the excited states positions and spectral weight

K.-F. Liu: INT Workshop, July 2024

Ratio of Spectral Weights





$$\frac{C_{\pi N}(t)(1/2^-)}{C_N(t)(1/2^+)} \approx \frac{3}{8 f_\pi^2 m_\pi L^3} \sum_{\vec{p}} \frac{m_\pi}{E_\pi} \frac{E_N + M_N}{2E_N} \left[1 - g_A \frac{E_{\text{tot}} - M_N}{E_{\text{tot}} + M_N}\right]^2 e^{-(E_{\text{tot}}-M_N)t} = \mathrm{R}\, e^{-(E_{\text{tot}}-M_N)t}$$

O. Bär – 1503.03649

When $m_\pi = 139$ MeV, L = 5.5 fm ($m_\pi L \sim 4$), R = 1.6%

K.-F. Liu: INT Workshop, July 2024

A detail calculation on Roper needs:

- $\pi N, \sigma N, \pi\Delta$ channels
- $\sigma N, \pi\Delta$: resonant contribution from the three body $\pi\pi N$ channel

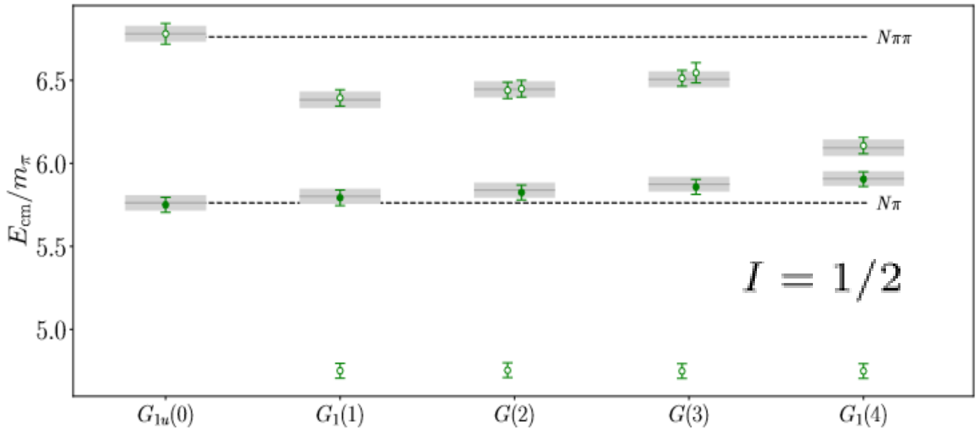
- Reasonably large volume > 5 fm

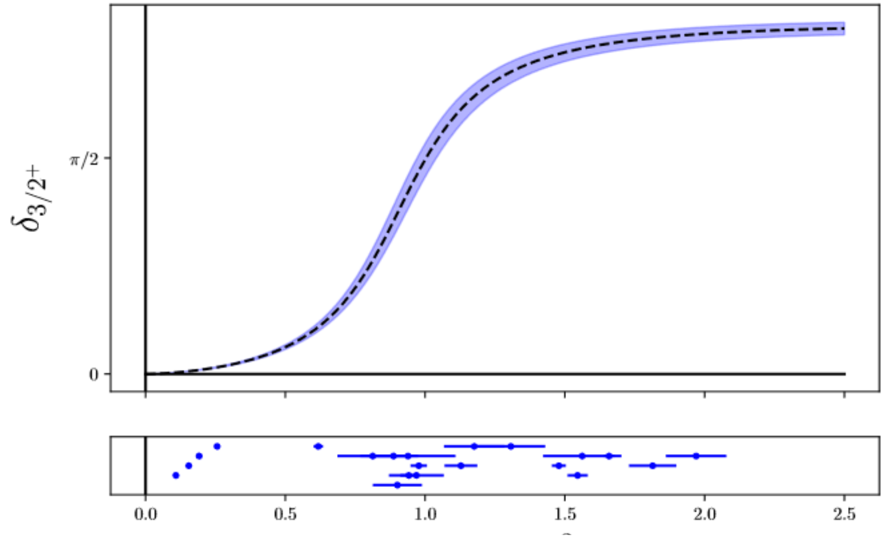
- Proper amplitude analysis

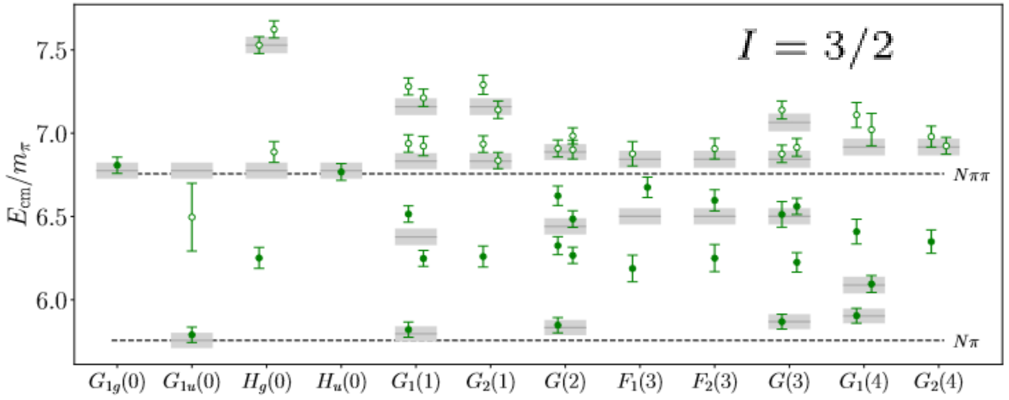
Such calculations can be performed in the next 5 years

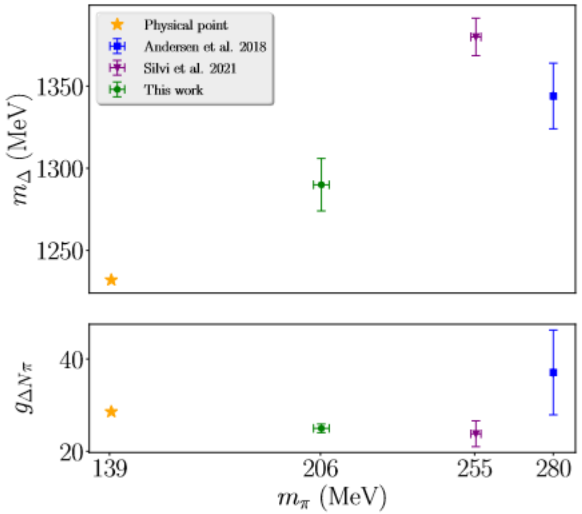
$N\pi$ (S-wave)

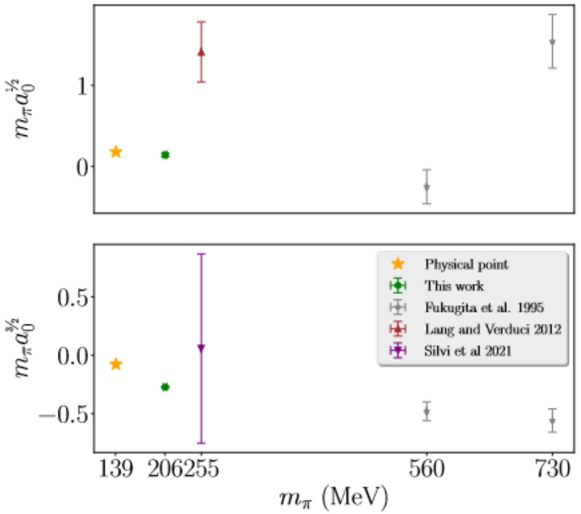
$N_f = 2 + 1$ dynamical quark flavors and m_π = 200 MeV, single ensemble



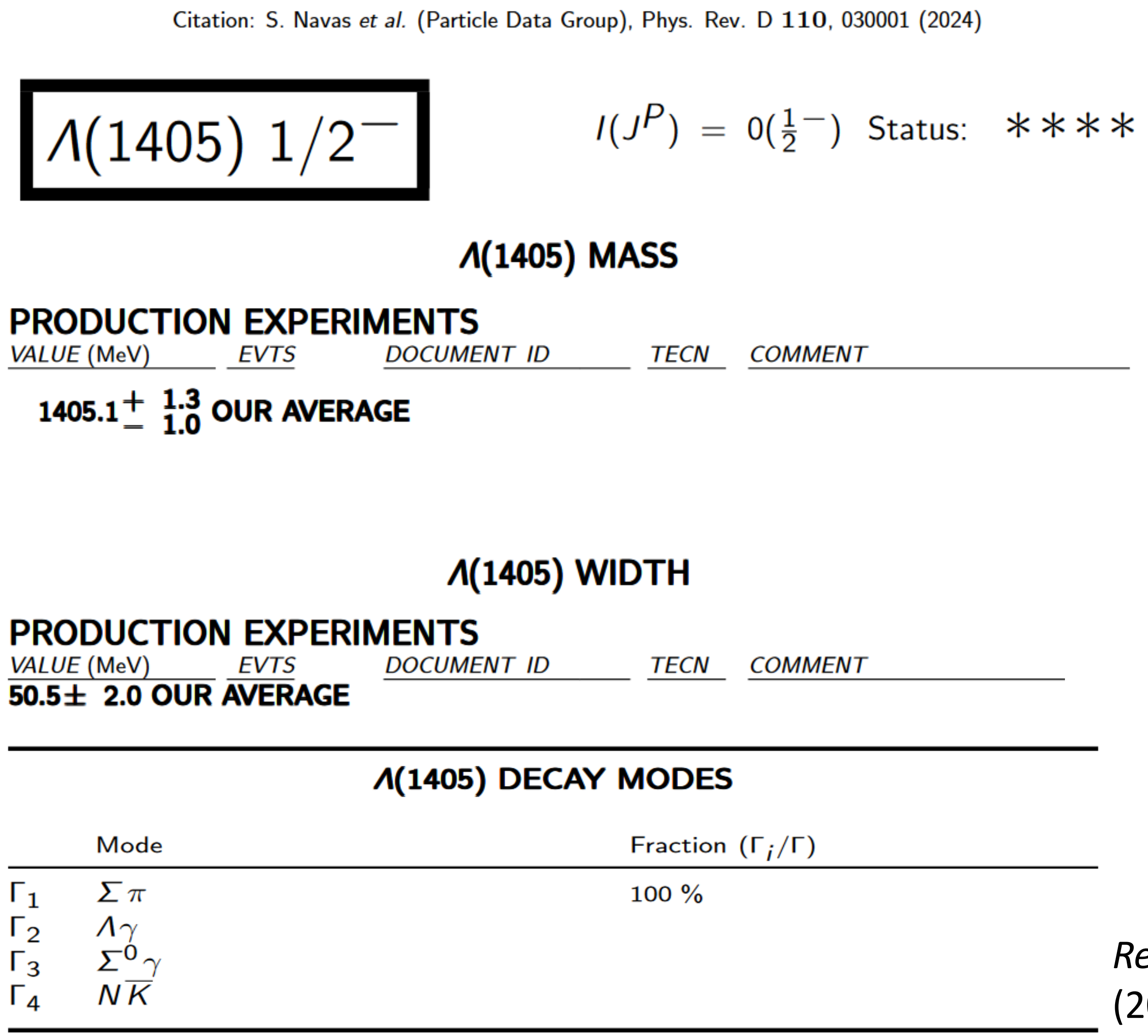








Bulava et al, NPB 987 (2023) 116105



Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)

$\Lambda(1405)\ 1/2^-$

$I(J^P) = 0(\frac{1}{2}^-)$ Status: $\ast\ast\ast\ast$

Λ(1405) MASS

PRODUCTION EXPERIMENTS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
$1405.1^{+1.3}_{-1.0}$ OUR AVERAGE				

Λ(1405) WIDTH

PRODUCTION EXPERIMENTS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
50.5± 2.0 OUR AVERAGE				

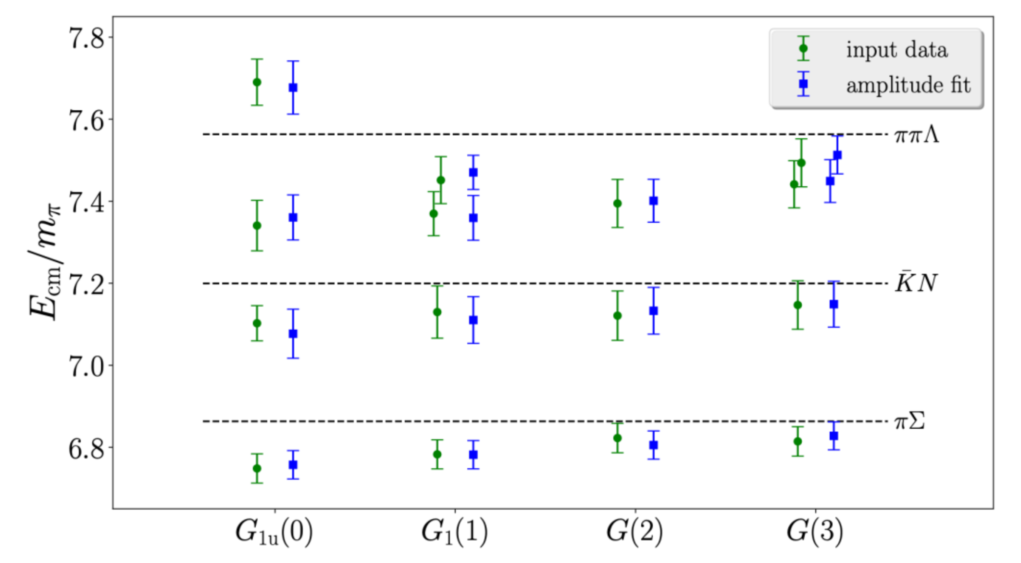
Λ(1405) DECAY MODES

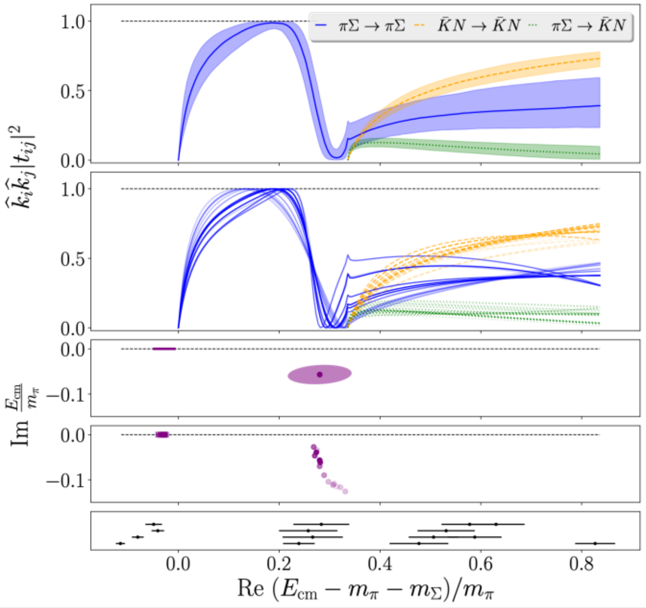
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\Sigma\pi$	100 %
Γ_2	$\Lambda\gamma$	
Γ_3	$\Sigma^0\gamma$	
Γ_4	$N\overline{K}$	

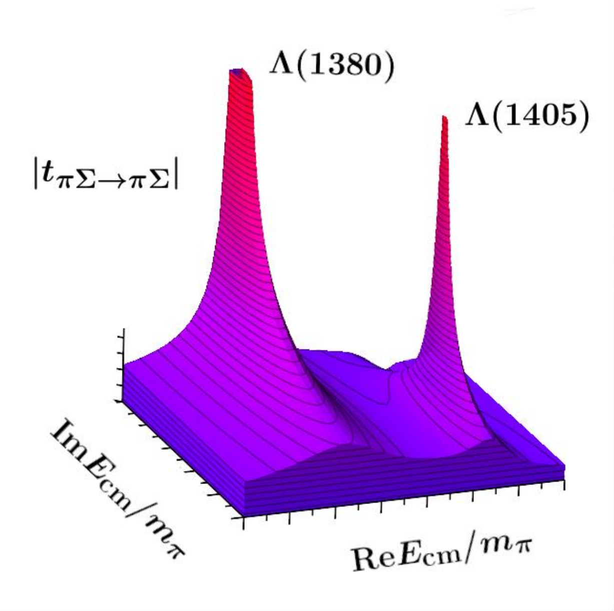
Review: M. Mai: Eur.Phys.J.ST 230 (2021) 6, 1593-1607

On the two-pole nature of the $\Lambda(1405)$ from $\pi\Sigma - \overline{K}N$ scattering

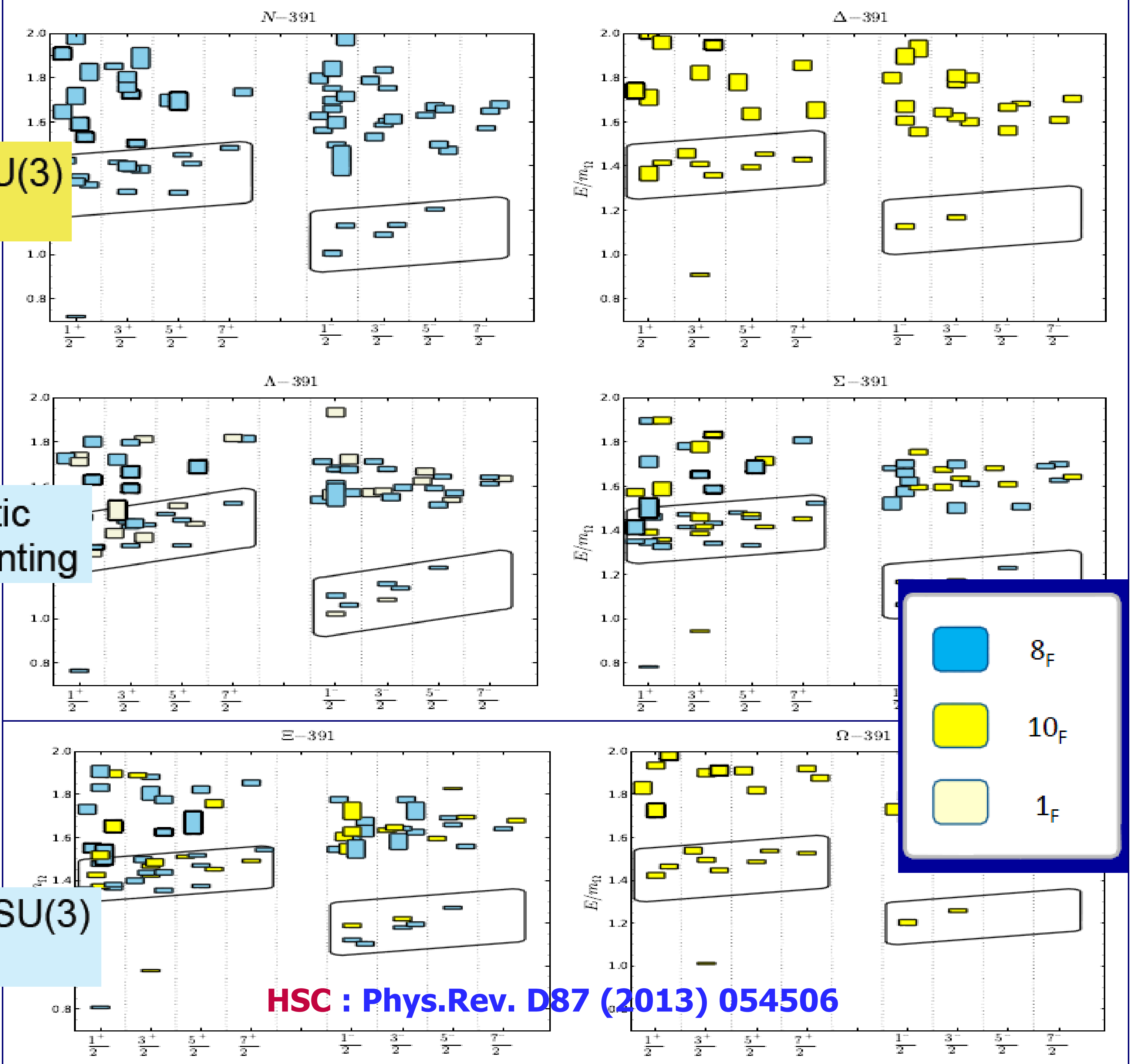
Bulava, Cid-Mora, Hanlon, Hörz, Mohler, Morningstar, Moscoso, Nicholson, Romero-López, Skinner, Walker-Loud
PRL 132 (2024) [2307.10413] & PRD 109 (2024) [2307.13471], *Editor's Suggestion*







- D200 CLS ensemble; $m_\pi \approx 200$ MeV, $m_K \approx 487$ MeV, $a \approx 0.064$ fm, $L/a = 64$
- stochastic LapH method [1104.3870] used to construct basis of operators, which are rotated with GEVP
- M,B & MB spectrum analyzed (LEFT) and LQC used to determine coupled channel scattering amplitudes (MIDDLE)
- Several parameterizations of the amplitude were studied (MIDDLE: 2nd pannel), all of which identified two poles
    - A complex pole below the $\overline{K}N$ thrshold
    - A virtual bound state (real valued pole) below the $\pi\Sigma$ threshold
- Calculations at $m_\pi^{\rm phys}$ underway



Light quarks – SU(3) flavor broken

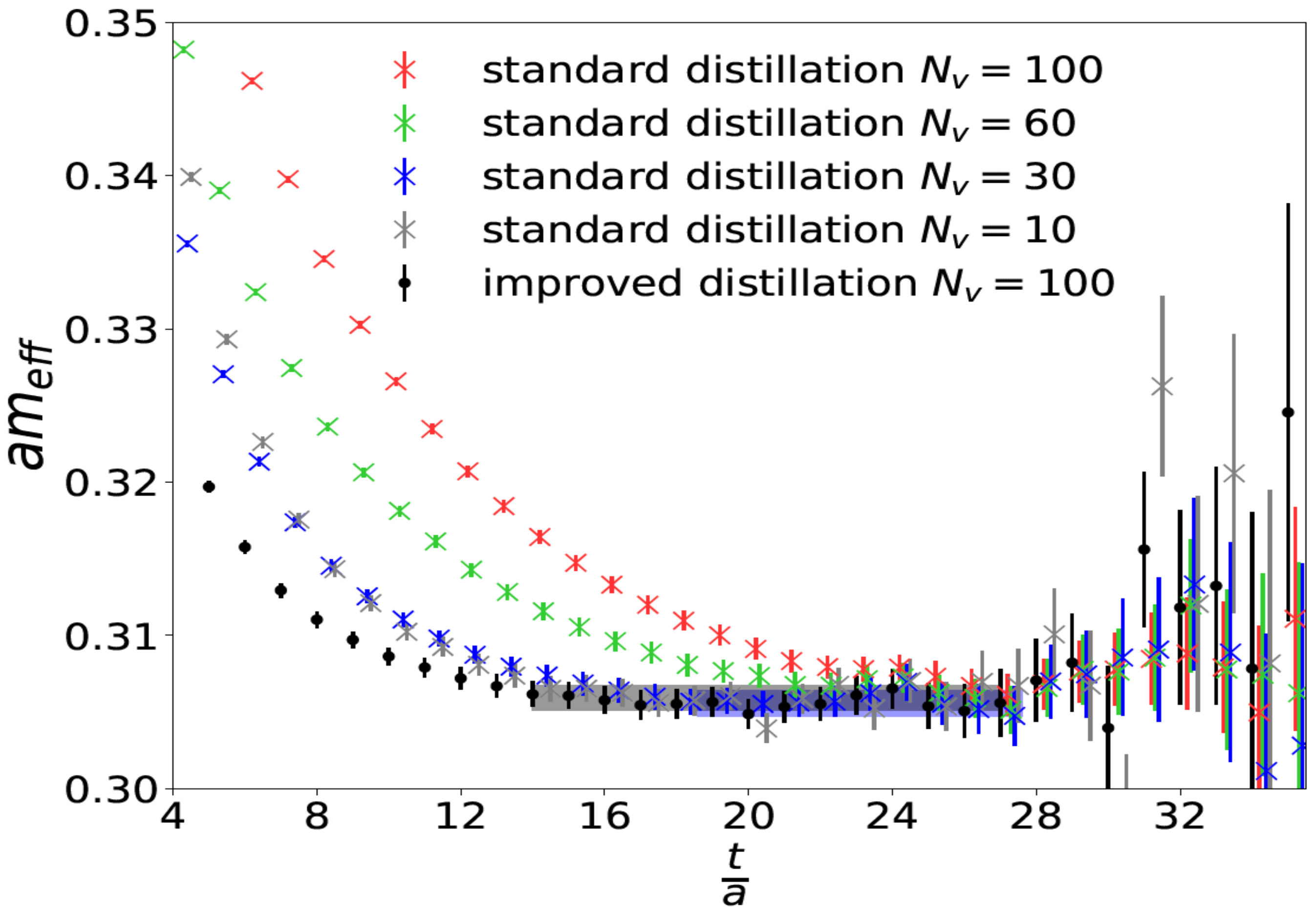
Full non-relativistic quark model counting

Some mixing of SU(3) flavor irreps

HSC : Phys.Rev. D87 (2013) 054506

Light and strange baryons: Excited states

- For baryons, current distillation procedures are computationally quite expensive



Improved distillation:
F. Knechtli et al.
Phys. Rev. D 106, 034501 (2022)

Comparison of static-light ground state S-wave effective mass using improved distillation

Talk by Laura Struckmeier on Tuesday, 11:35 in session “Hadronic and nuclear spectrum and interactions”.

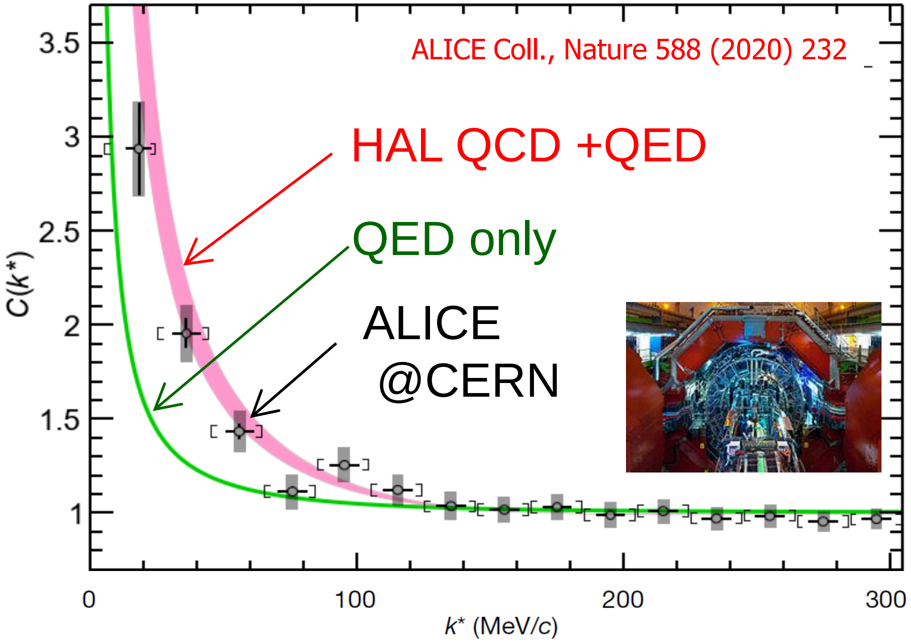
Light and strange baryons: Excited states

- For baryons, current distillation procedures are computationally quite expensive
- Need dedicated efforts
- Strange baryon excited states are not known that much → lattice can provide input
- Problem of missing resonances, parity doublets

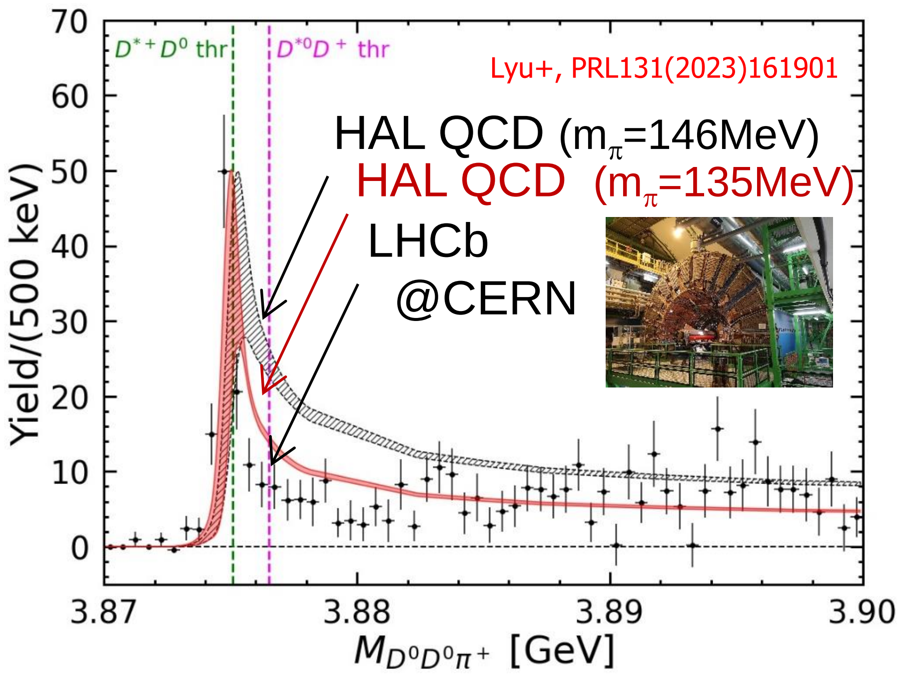
Some of the Results from HAL QCD

NΞ interactions predicted by HAL

← confirmed by LHC ALICE

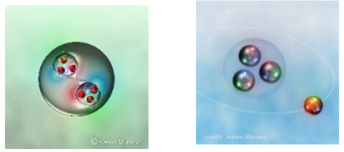


Nature of T_{cc} revealed by HAL



Various Exotic hadrons / nuclei predicted

- Dibaryons: NΩ, ΩΩ, $\Omega_{ccc}\Omega_{ccc}$
- Ξ hyper-nuclei: (ΞNNN), (ΞNαα)
- Nϕ state ← combined analysis of HAL & LHC ALICE



HAL QCD (2018-)
Hiyama+(2020-)
ALICE-HAL (2024-)

HAL @ mπ=146MeV (+ small extrapolation) so far → HAL @ mπ=137MeV on-going!

LQCD for heavy quark physics

Requirement: lattice quark mass $ma << 1$

discretization error

$$\frac{1}{L} \ll m_\pi \ll m_H \ll \frac{1}{a}$$

Λ_{IR}

Λ_{UV}

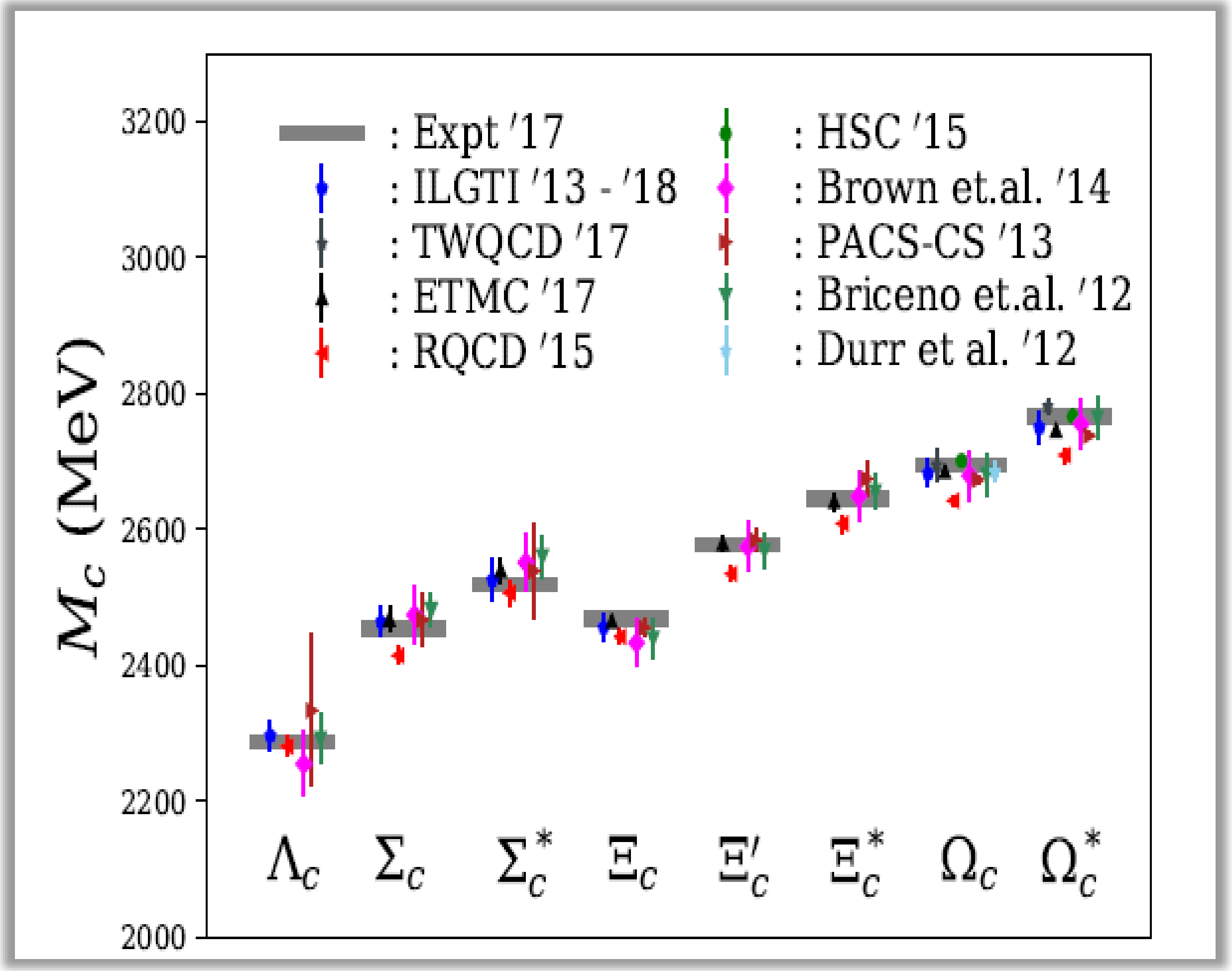
$$\boxed{L \geq \frac{4}{m_\pi} \sim 6\ \mathrm{fm}}$$

$$m_c a(0.3) \sim 0.045\ \mathrm{fm}$$

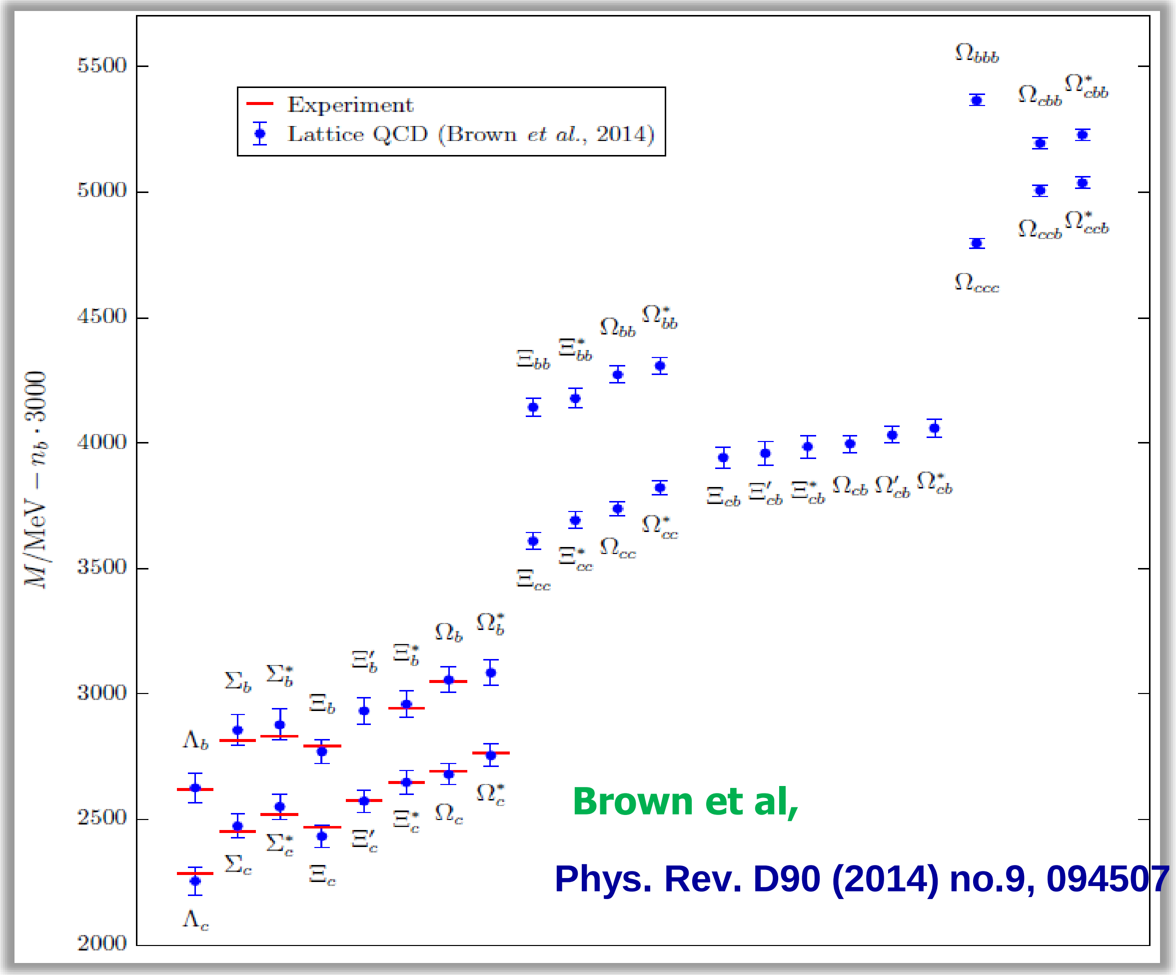
$$m_b a(0.5) \sim 0.0235\ \mathrm{fm}$$

Effective Action	Relativistic Action
NRQCD, FermiLab, HQET, RHQ tune $m_b \rightarrow m_b^{phy}$, but could be difficult to estimate systematic for precise results	HISQ, Clover, TM, DWF Charm physics can be done. Bottom: Currently unavailable, expensive but should be the way forward in the exascale era

Charmed Baryons

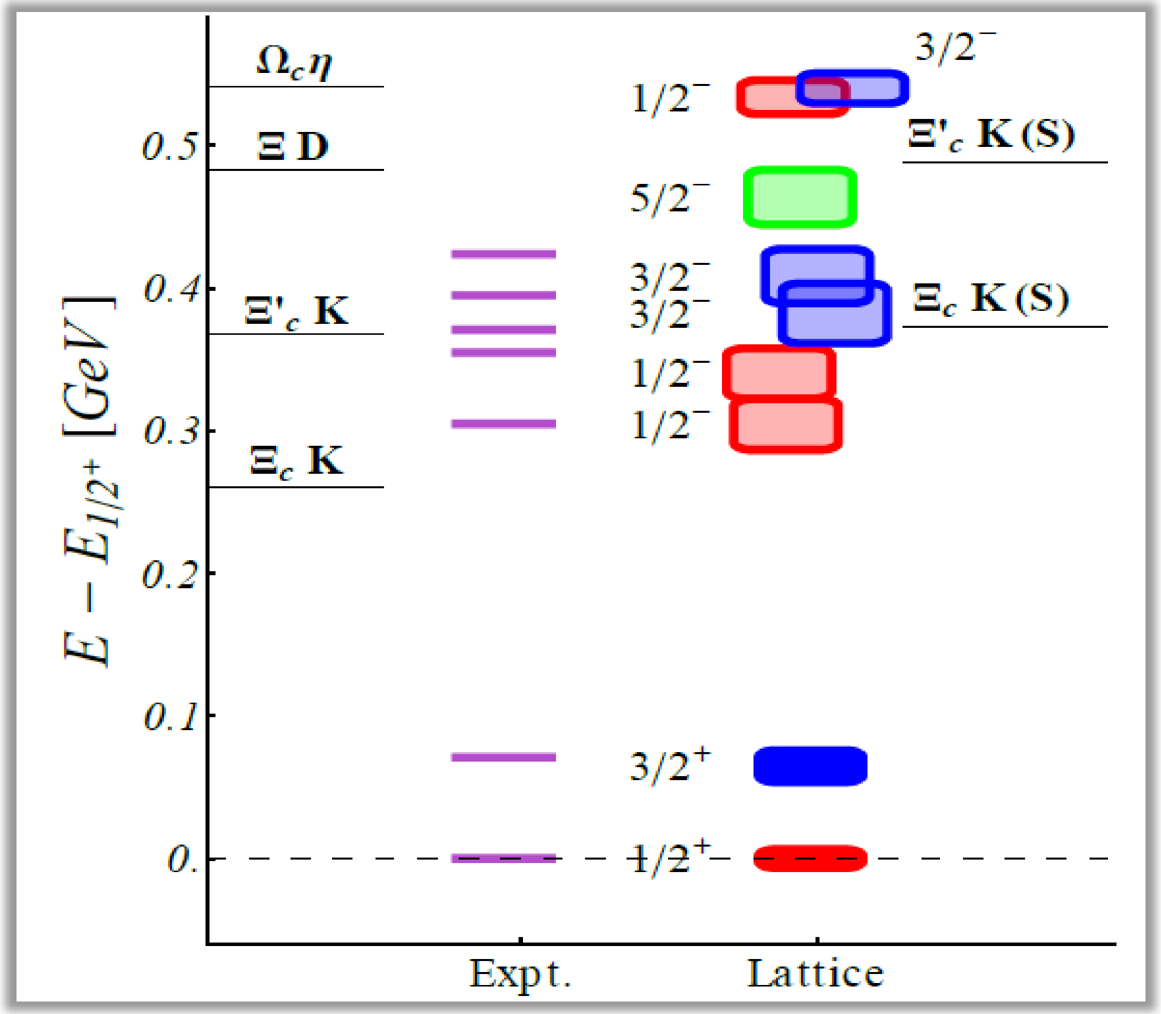


Padmanath NM, Lat'17, Charm 21



Brown et al,

Phys. Rev. D90 (2014) no.9, 094507



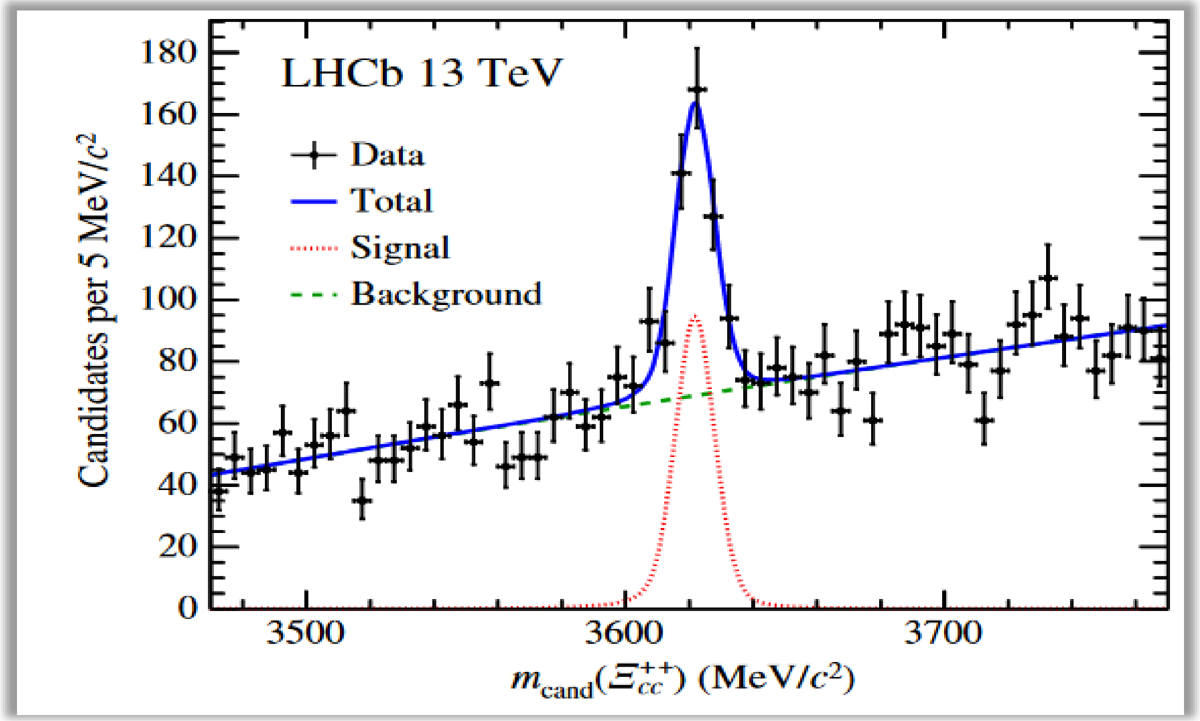
Energy Splittings (ΔE)	Experiment ΔE (MeV)	Experiment J^P (PDG)	Lattice ΔE (MeV)	Lattice J^P
$E_{\Omega_c^0} - \frac{1}{2}E_{\eta_c}$	1203(2)	$1/2^+$	1209(7)	$1/2^+$
$\Delta E_{\Omega_c^0(2770)}$	70.7(1)	$3/2^+$	65(11)	$3/2^+$
$\Delta E_{\Omega_c^0(3000)}$	305(1)	?	304(17)	$1/2^-$
$\Delta E_{\Omega_c^0(3050)}$	355(1)	?	341(18)	$1/2^-$
$\Delta E_{\Omega_c^0(3066)}$	371(1)	?	383(21)	$3/2^-$
$\Delta E_{\Omega_c^0(3090)}$	395(1)	?	409(19)	$3/2^-$
$\Delta E_{\Omega_c^0(3119)}$	422(1)	?	464(20)	$5/2^-$

Here $\Delta E^n = E^n - E^0$.
The new states correspond to the excited p-wave states.

Padmanath and NM: Phys. Rev. Lett. 119 (2017) no.4, 042001

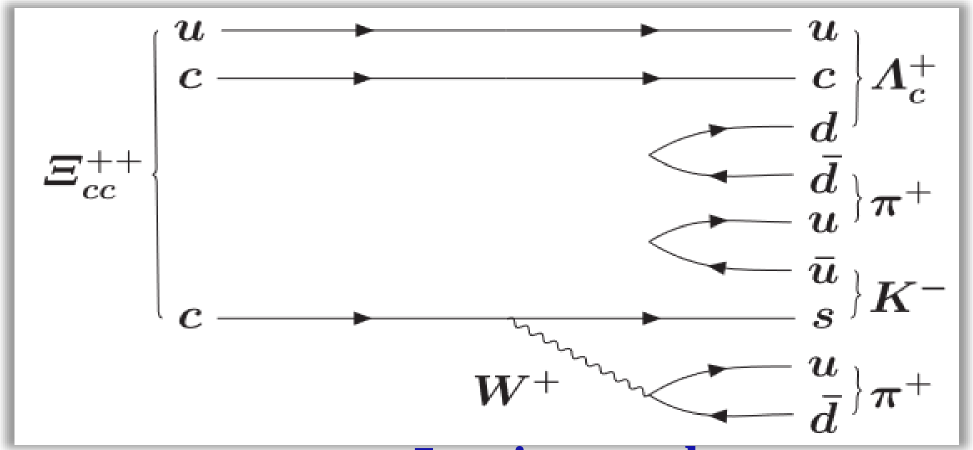
$\Xi_{cc}^{++}(ccu)$

$3621.55 \pm 0.23 \pm 0.30$ MeV

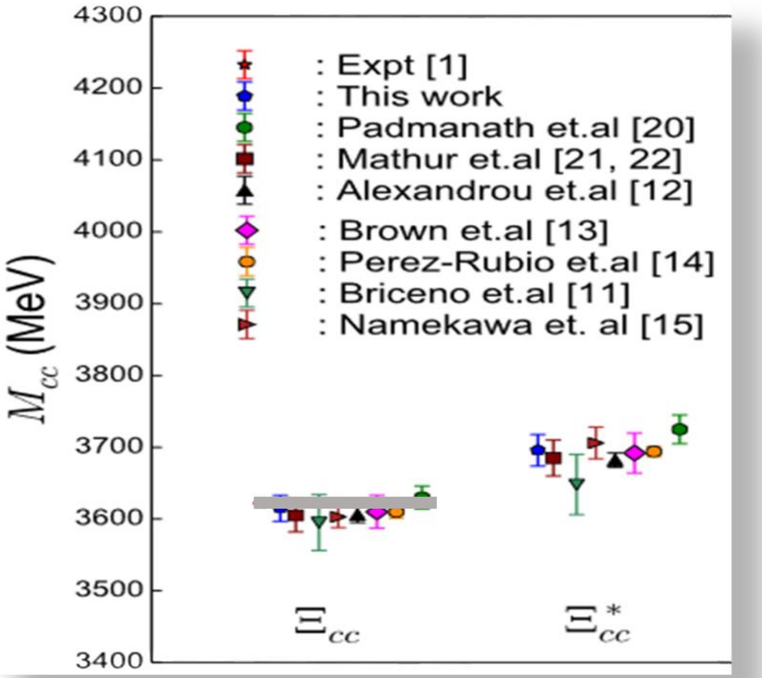


$$3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)\ \text{MeV}/c^2$$

LHCb: PRL 119, 112001 (2017)

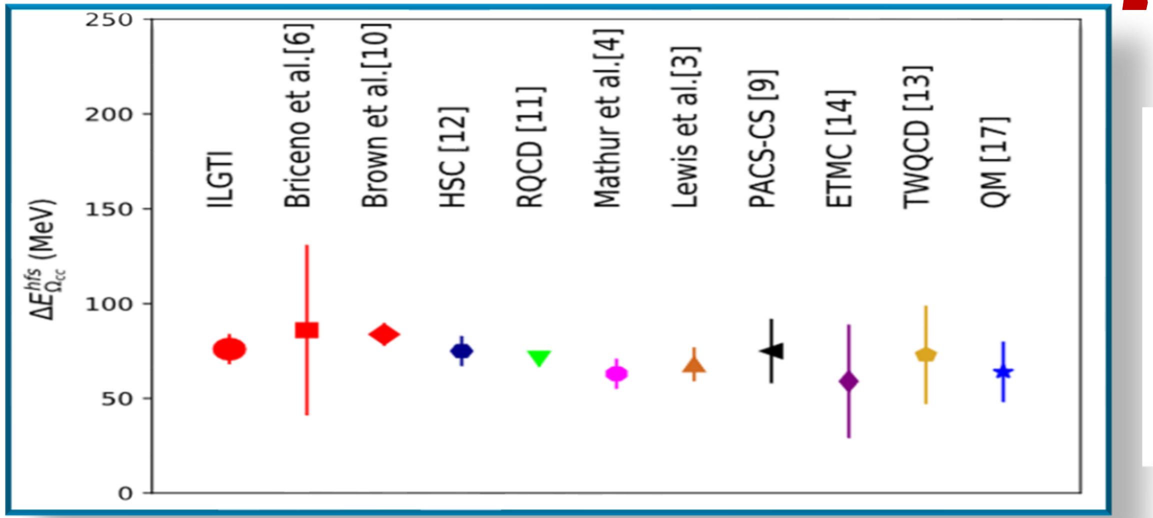


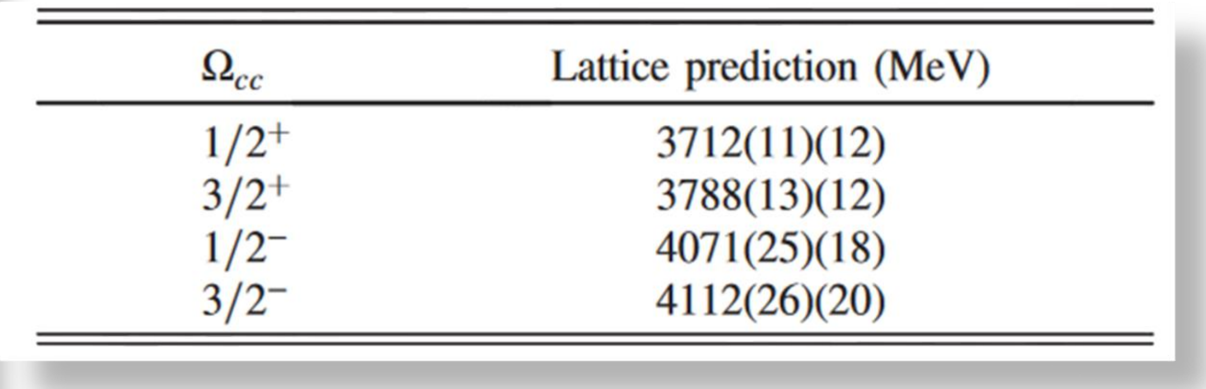
Lattice results



These lattice results were prediction!

$\Omega_{cc}(ccs)$



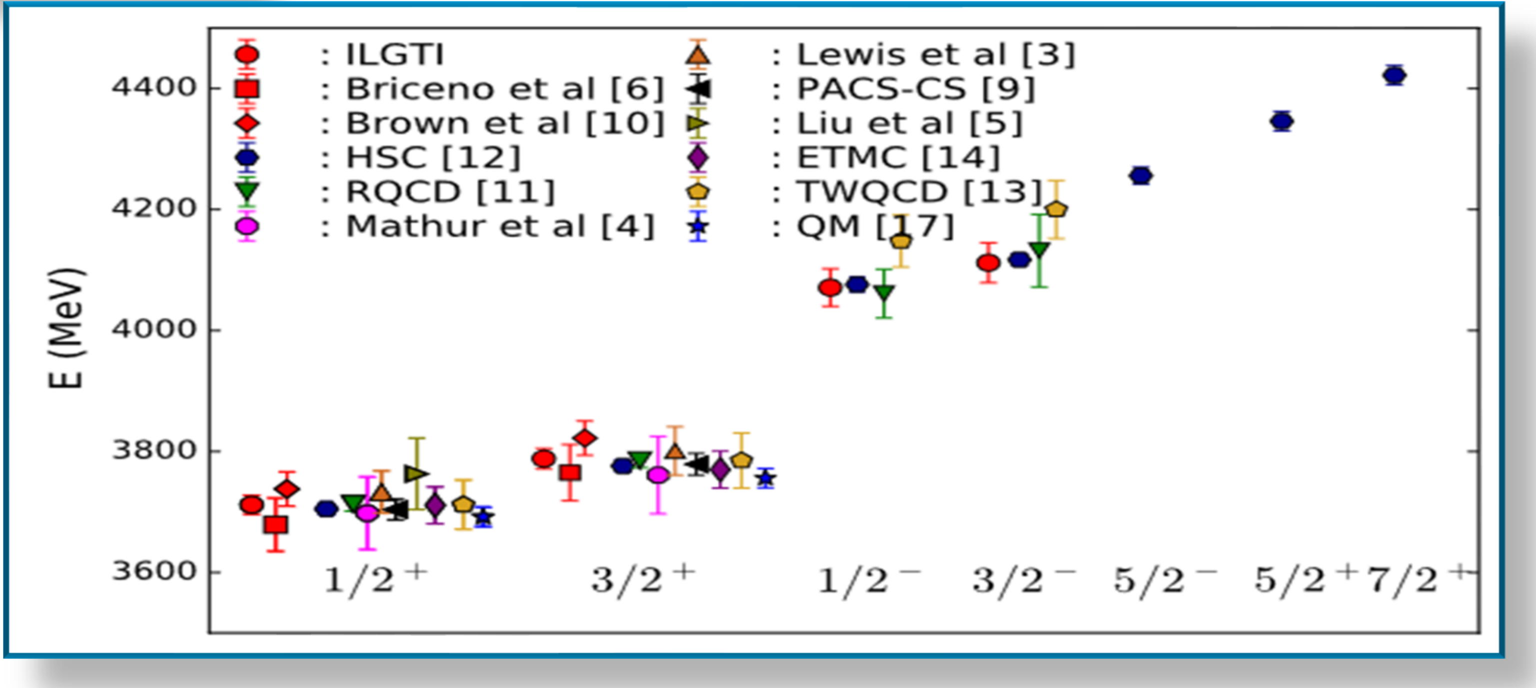


Ω_{cc}	Lattice prediction (MeV)
$1/2^+$	3712(11)(12)
$3/2^+$	3788(13)(12)
$1/2^-$	4071(25)(18)
$3/2^-$	4112(26)(20)

Is it the next doubly charmed baryon to be discovered?

Decay to : $\Xi^0 K^+ \pi^+ \pi^+$

and $\Omega_c \pi^+$



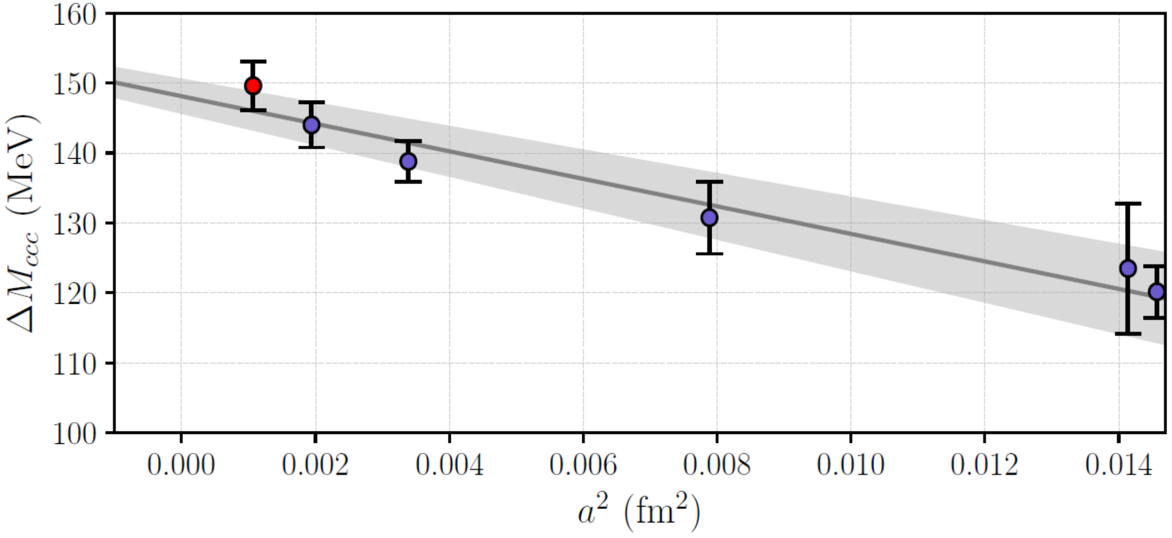
NM and Padmanath:
PHYSICAL REVIEW D 99, 031501(R) (2019)

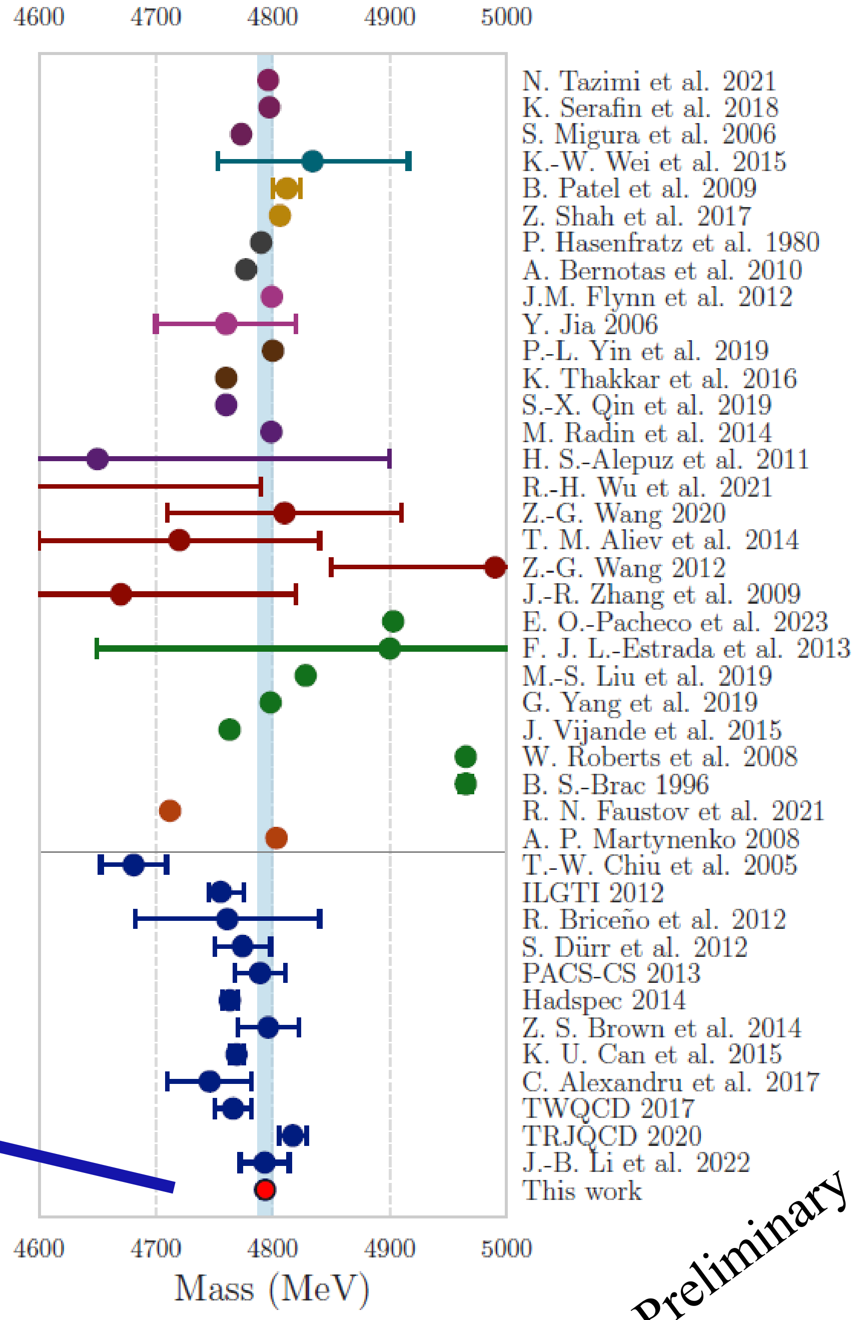
Hadron Spectroscopy
Lattice 2024, Liverpool

Ω_{ccc}

A new window for understanding the structure of baryons

…Bjorken (FERMILAB-CONF-85/69)

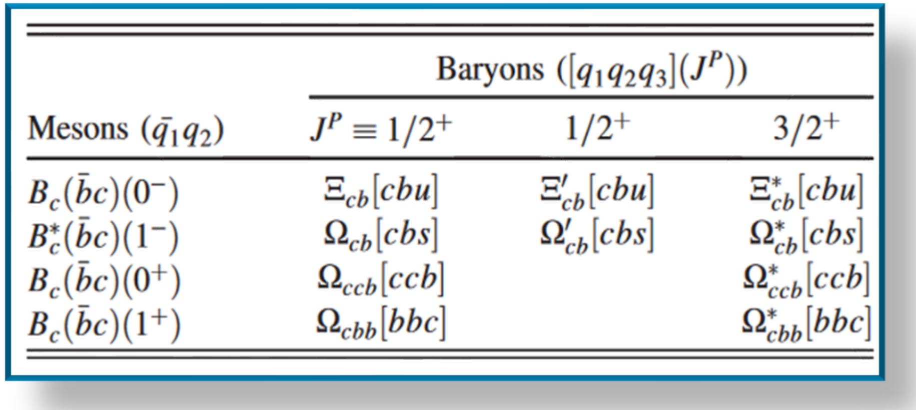




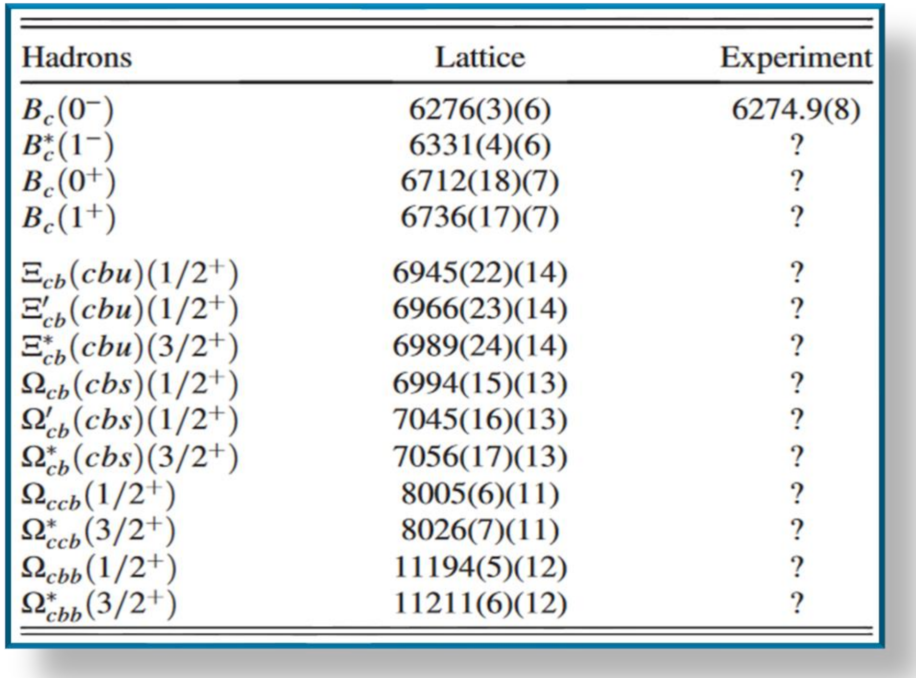
Preliminary

Radhakrishnan, Chakraborty, Dhindsa, Padmanath and NM

Charmed-bottom hadrons

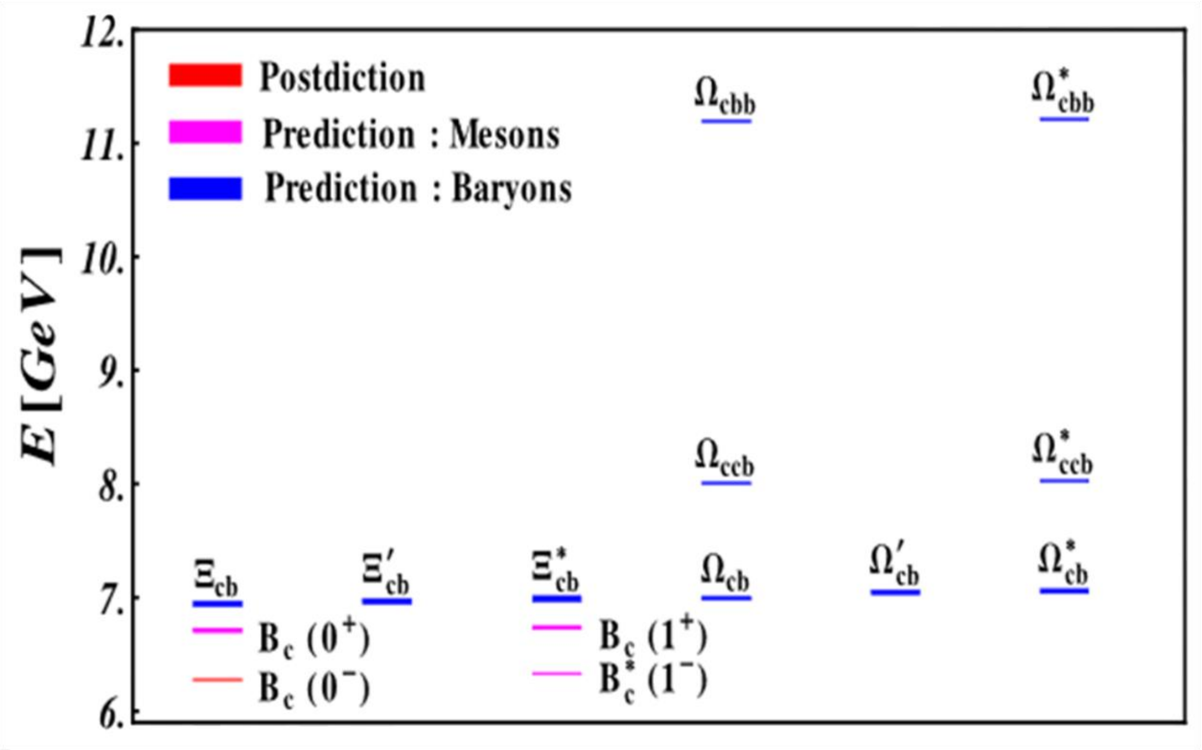


Mesons ($\bar{q}_1 q_2$)	Baryons ($[q_1q_2q_3](J^P)$)		
	$J^P \equiv 1/2^+$	$1/2^+$	$3/2^+$
$B_c(\bar{b}c)(0^-)$	$\Xi_{cb}[cbu]$	$\Xi'_{cb}[cbu]$	$\Xi^*_{cb}[cbu]$
$B^*_c(\bar{b}c)(1^-)$	$\Omega_{cb}[cbs]$	$\Omega'_{cb}[cbs]$	$\Omega^*_{cb}[cbs]$
$B_c(\bar{b}c)(0^+)$	$\Omega_{ccb}[ccb]$		$\Omega^*_{ccb}[ccb]$
$B_c(\bar{b}c)(1^+)$	$\Omega_{cbb}[bbc]$		$\Omega^*_{cbb}[bbc]$



Hadrons	Lattice	Experiment
$B_c(0^-)$	6276(3)(6)	6274.9(8)
$B^*_c(1^-)$	6331(4)(6)	?
$B_c(0^+)$	6712(18)(7)	?
$B_c(1^+)$	6736(17)(7)	?
$\Xi_{cb}(cbu)(1/2^+)$	6945(22)(14)	?
$\Xi'_{cb}(cbu)(1/2^+)$	6966(23)(14)	?
$\Xi^*_{cb}(cbu)(3/2^+)$	6989(24)(14)	?
$\Omega_{cb}(cbs)(1/2^+)$	6994(15)(13)	?
$\Omega'_{cb}(cbs)(1/2^+)$	7045(16)(13)	?
$\Omega^*_{cb}(cbs)(3/2^+)$	7056(17)(13)	?
$\Omega_{ccb}(1/2^+)$	8005(6)(11)	?
$\Omega^*_{ccb}(3/2^+)$	8026(7)(11)	?
$\Omega_{cbb}(1/2^+)$	11194(5)(12)	?
$\Omega^*_{cbb}(3/2^+)$	11211(6)(12)	?

NM, Padmanath, Mandal:
Physical Review Letters 121, 202002 (2018)



Exotic Hadrons

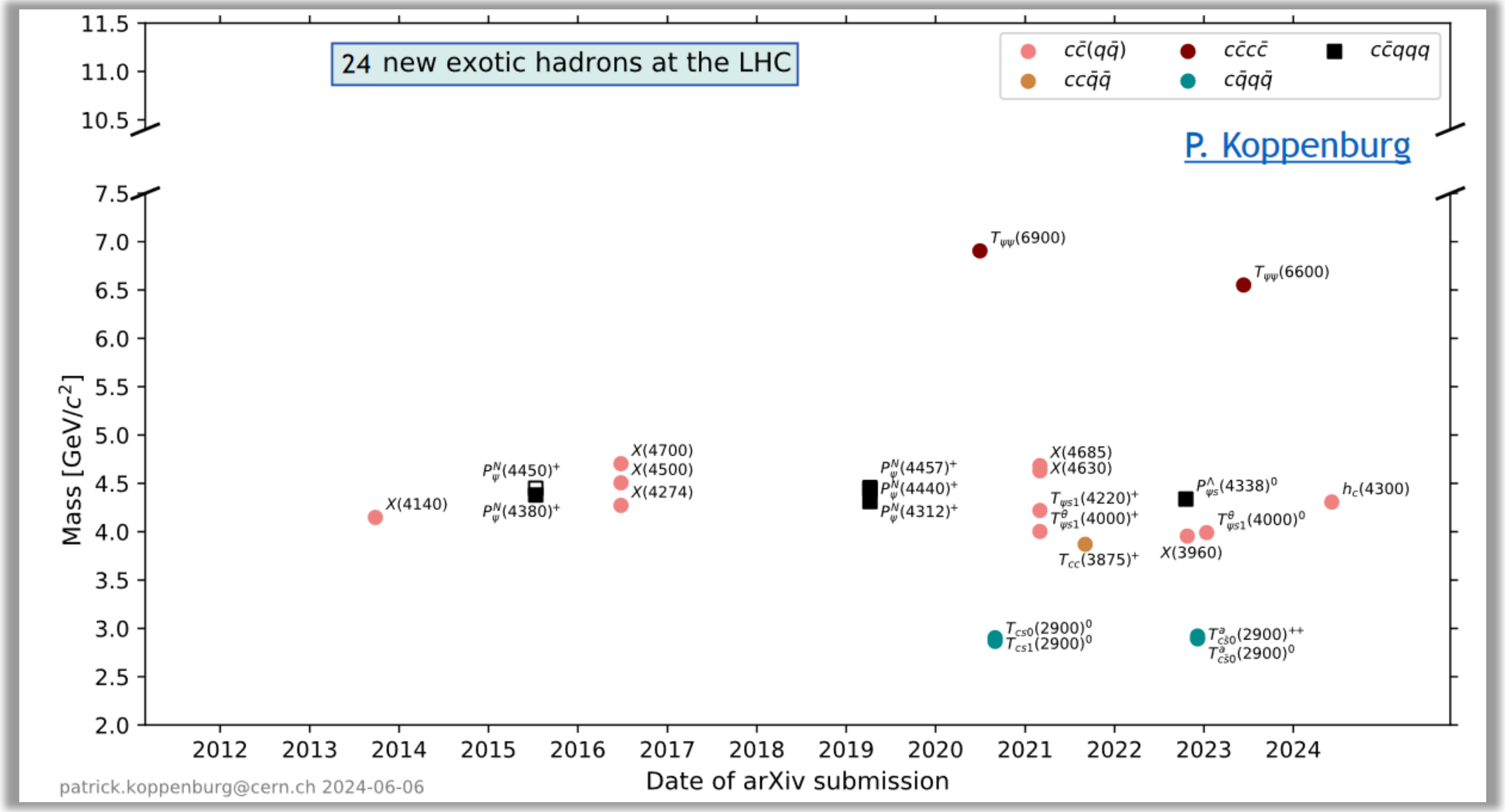
- Hadrons whose quantum numbers require a valence quark content beyond **qqq** or $q\bar{q}$ are called as **"exotics"**, e.g. $cc\bar{u}\bar{d}$, **glueball**

(glueball will be discussed by Morningstar)

- Hadrons whose spin, parity and charge conjugation are forbidden in the non-relativistic quark model are also often termed "exotics" (spin exotics)

- Cryptoexotics :
  - mass/width does not fit with meson or baryon spectra
  - overpopulation of the spectra
  - production or decay properties incompatible with standard mesons/baryons

Exotic hadrons at LHC



@LHCb

….And so at Belle, BES III, COMPASS, CMS, ALICE, ATLAS, JLAB

A constituent picture of Hadrons

- QCD : Fundamental degrees of freedoms are quarks (6 flavours) and gluons (8 degrees of freedom)
- Confinement conjecture: quarks and gluons must be combined into colour-neutral combinations of hadrons

Constituents	Combinations	Naming convention (quark model)
$\mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{8}$	Meson
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$	Baryon
$\mathbf{8} \otimes \mathbf{8}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Glueball
$\bar{\mathbf{3}} \otimes \mathbf{8} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Hybrid
$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Tetraquark/molecule
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27} \oplus \mathbf{35} + \cdots$	Pentaquark
………..	………..	?

A constituent model of hadrons

- However, there can be strong mixings between different hadrons with the same quantum numbers

Exotic hadrons and lattice QCD

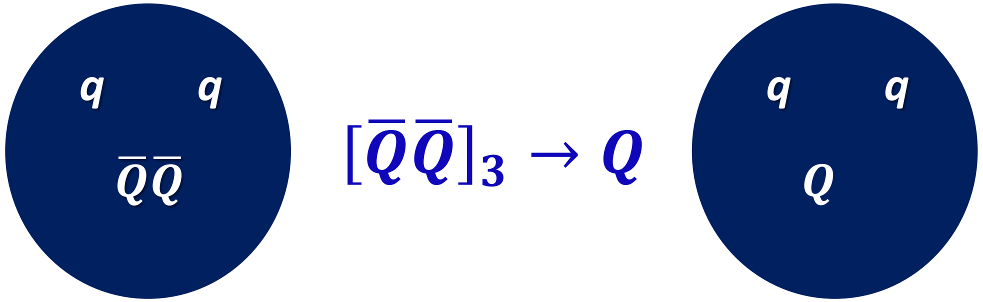
- Tetraquark and pentaquark hadrons have been observed experimentally with heavy quark contents. ….LHC, Belle, BES

- Are their possibilities to discover more of those? And other multiquark states?

- What are the structures and properties of these exotic hadrons?

- What can lattice studies do?
  - **Can predict more exotic states with their possible energy ranges and possible valence structures**
  - **Can decipher structures and properties of exotic hadrons**

Heavy four-quark states



$[\bar{Q}\bar{Q}]_3 \rightarrow Q$

A possible structure:

How about ? $(qC\gamma_5 q')(\bar{Q}C\gamma_i\bar{Q}')$

$\downarrow \{qq'\}$ $\downarrow \{\bar{Q}\bar{Q}'\}$

Possible states? : $\bar{b}\bar{b}ud, \bar{b}\bar{b}us, \bar{b}\bar{b}uc, \bar{b}\bar{b}sc,$
$\bar{b}\bar{c}ud, \bar{b}\bar{c}us$ $etc.$

$J = 1, l_1 l_2\ \bar{Q}\bar{Q}$

$\swarrow \{\bar{3}_c, J = 0, F_A\}$ $\searrow \{3_c, J = 1, F_S\}$

$J = 0, ll\ \bar{Q}\bar{Q}$

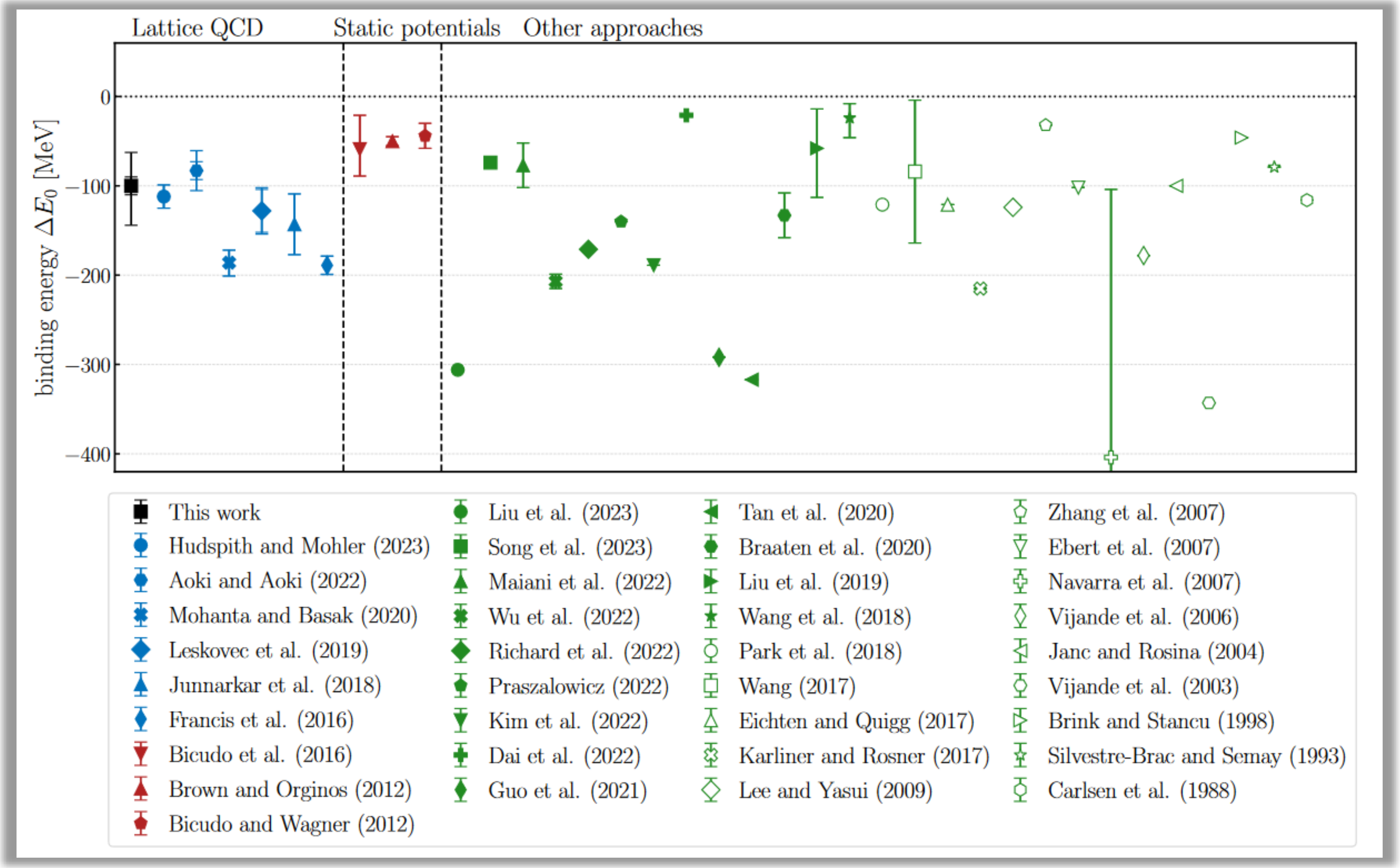
$\swarrow \{6_c, J = 0, F_A\}$ $\searrow \{\bar{6}_c, J = 0, F_S\}$

LQCD: bound states of

$T_{bb}(bb\bar{u}\bar{d}), T_{bbs}(bb\bar{u}\bar{s}), T_{bc}(bc\bar{u}\bar{d}), T_{cc}(cc\bar{u}\bar{d})$

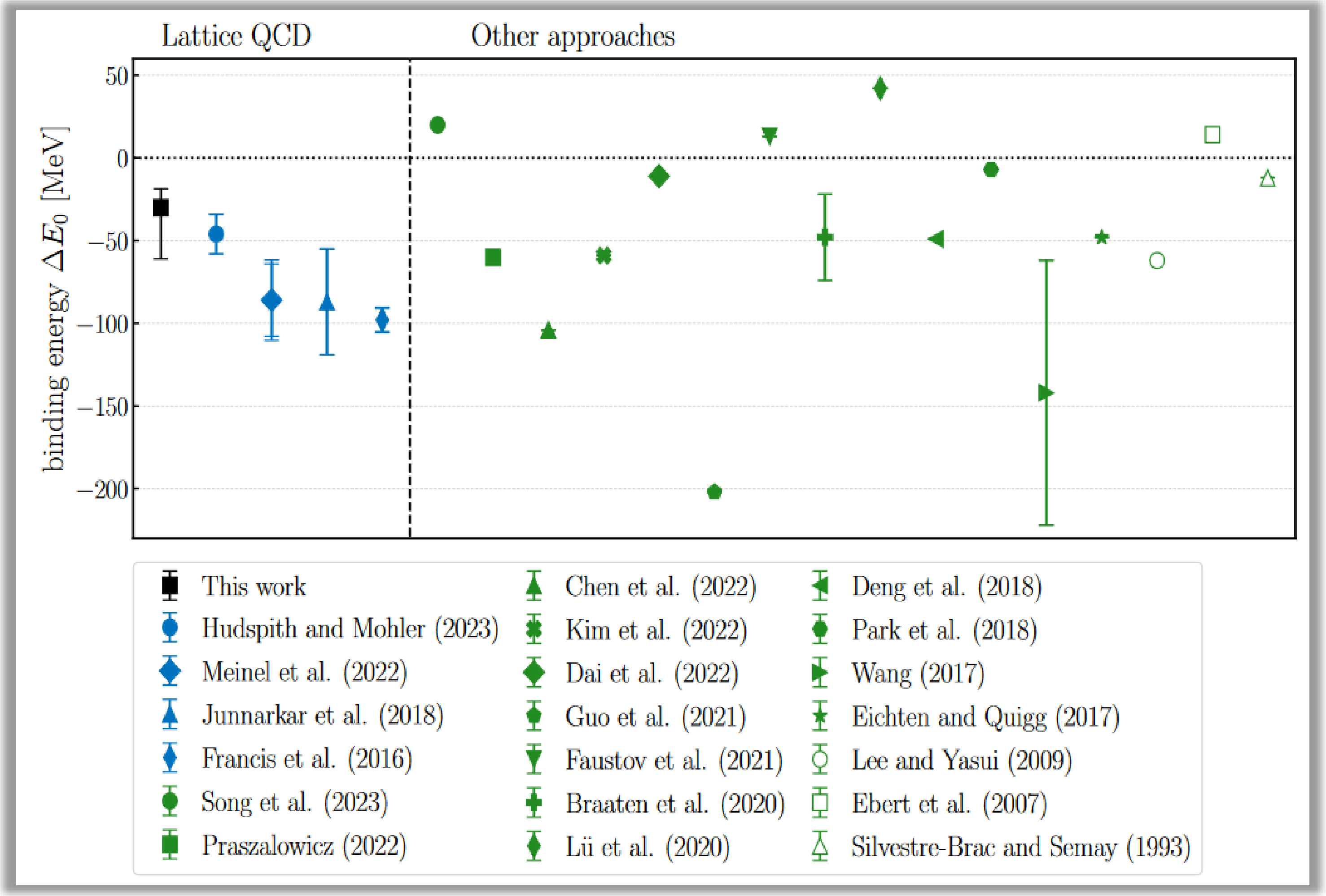
Expt: $T_{cc}(cc\bar{u}\bar{d})$

$T_{bb}(bb\bar{u}\bar{d})$



Summary by Alexandrou et al: arXiv:2404.03588

$T_{bbs}(bb\bar{u}\bar{s})$

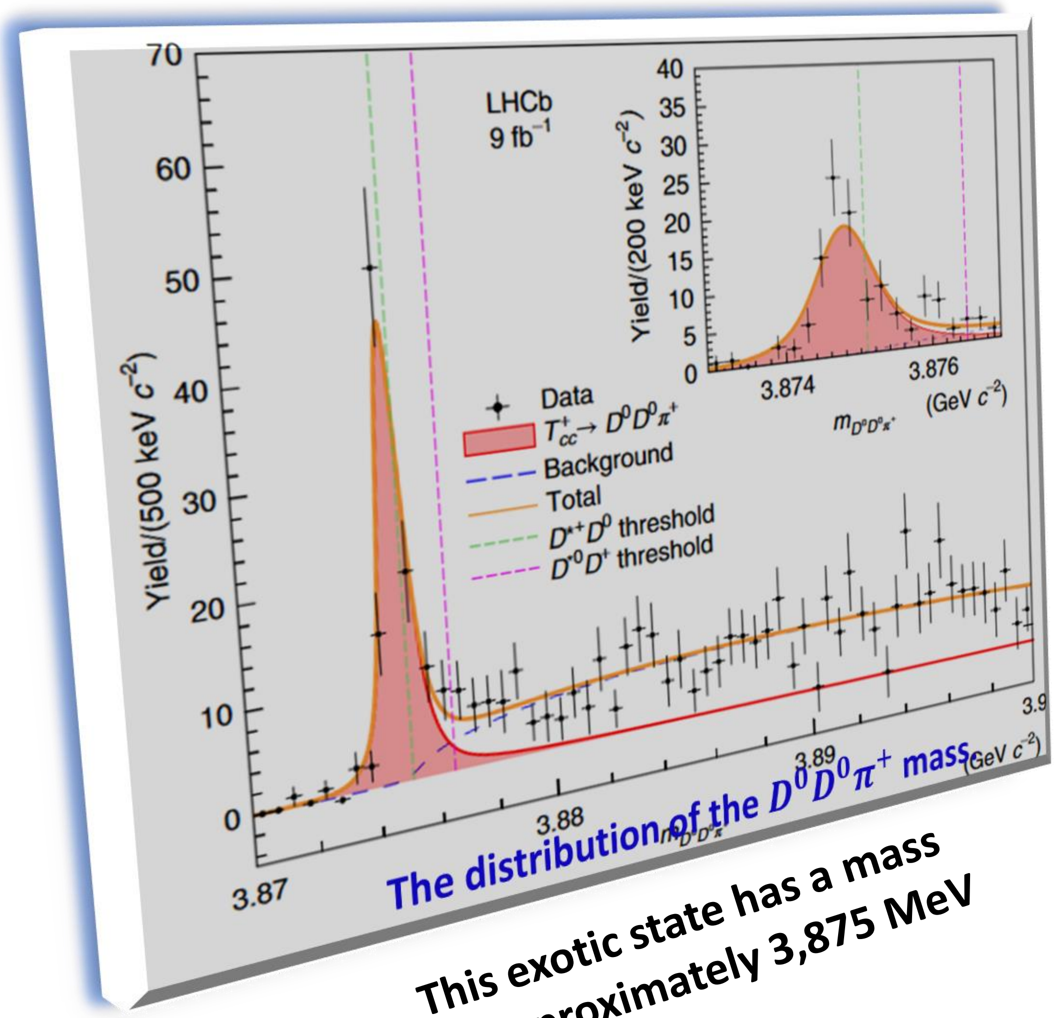


Summary by Alexandrou et al: arXiv:2404.03588

$D^*(c\bar{d})D^0(c\bar{u})$

$T_{cc}^+(cc\bar{u}\bar{d})$

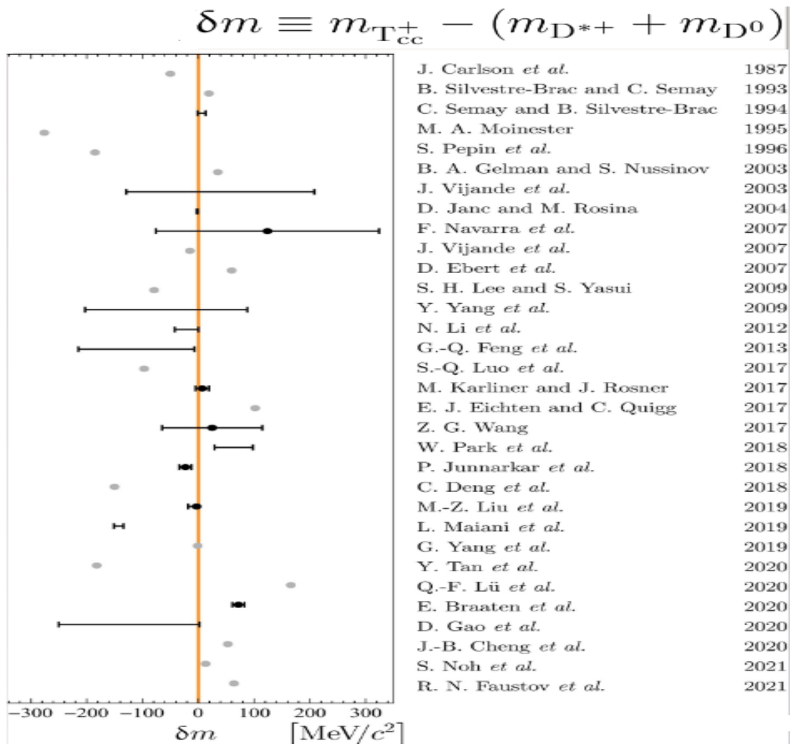
Nature Physics,18, 751(2022) @ LHCb



The distribution of the $D^0D^0\pi^+$ mass

This exotic state has a mass of approximately 3,875 MeV

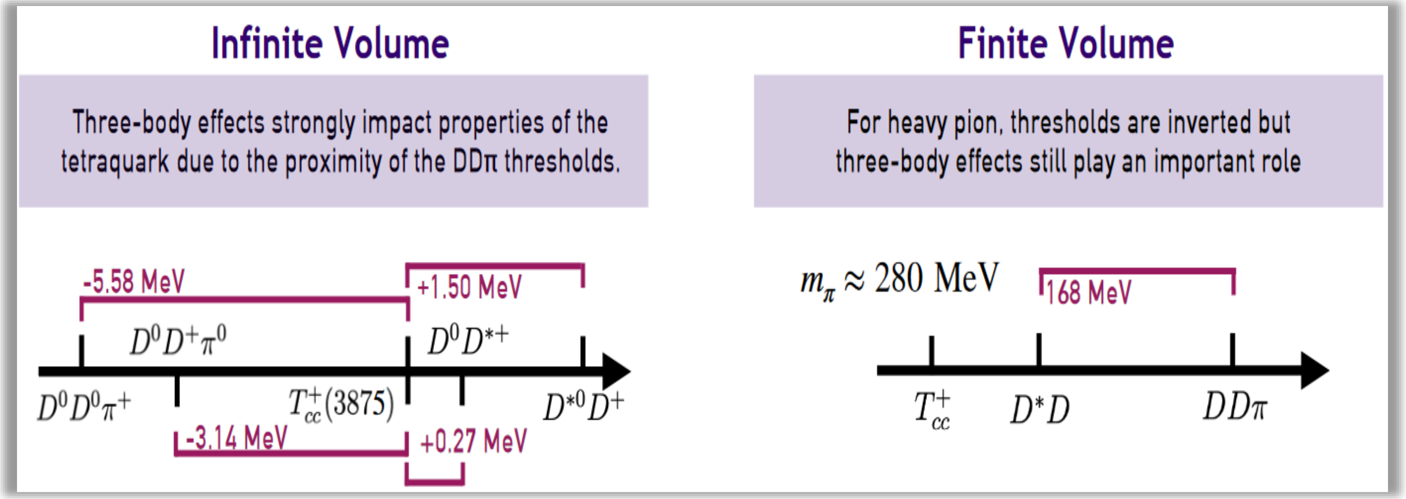
Parameter	Value
N	117 ± 16
δm_{BW}	-273 ± 61 keV c^{-2}
Γ_{BW}	410 ± 165 keV



T_{bb}, T_{cc} and T_{bc}

- *Quark mass dependence of doubly heavy tetraquark binding:* William Parrott (Monday)
- *Strong decay of double charm tetraquark* T_{cc}: Subhasish Basak (Monday)
- *Towards quark mass dependence of* T_{cc} : Sasa Prelovsek (Monday)
- T_{cc} *via plane wave approach and including diquark-antidiquark operators*: Ivan Vjmilovic (Monday)
- *Three body analysis of the tetraquark* T_{cc} : Sebatian Dawid (Monday)
- *Beautiful exotics in a non-perturbatively tuned Lattice NRQCD setup*: Daniel Mohler (Tuesday)
- *Antistatic-antistatic-light-light tetraquark potentials with u, d and s quarks from lattice QCD*: Pedro Bicudo (Tuesday)
- *Tetraquarks* $\bar{b}\bar{b}ud, I(J^P) = 0\ (1^-)$ *and* $\bar{b}\bar{b}ud, I(J^P) = 0\ (0^+), 0\ (1^+)$ *from Lattice QCD Static Potentials*: Jakob Hoffmann (Tuesday)
- *Exotic* T_{bc} *tetraquarks from Lattice QCD*: Archana Radhakrishnan (Tuesday)
- *Left-hand cut and the HAL QCD method: S. Aoki (Thrusday)*
- *Distillation and position-space sampling for local multiquark interpolators*: Andres Stump (Friday)

T_{cc} $cc\bar{u}\bar{d}$: Analysis is complicated energy levels below the left-hand branch point cannot be used



Infinite Volume

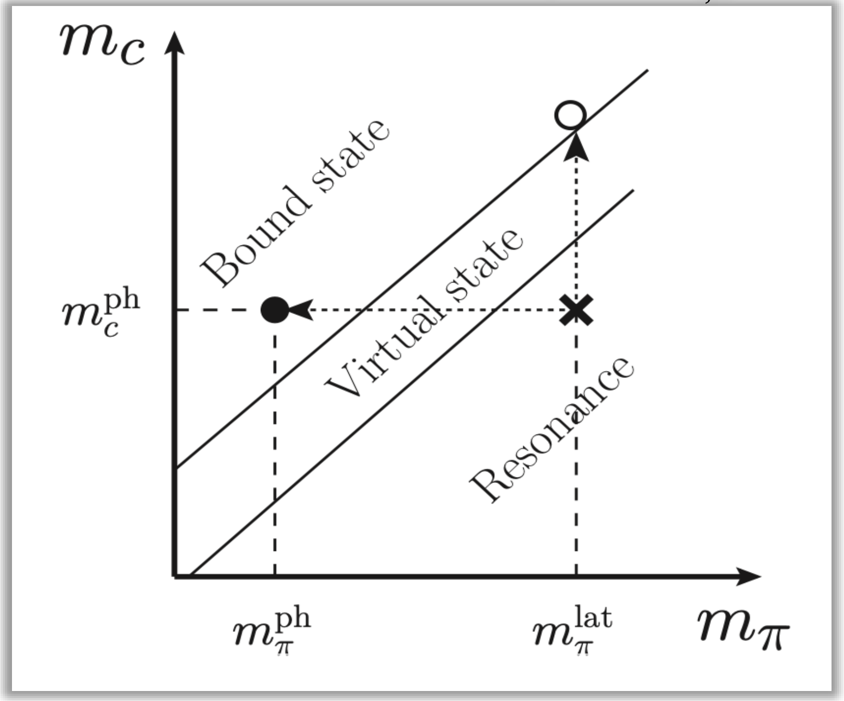
Three-body effects strongly impact properties of the tetraquark due to the proximity of the DDπ thresholds.

Finite Volume

For heavy pion, thresholds are inverted but three-body effects still play an important role

S. Dawid (Monday)

Dependence on m_c and $m_{u,d}$



Sasa Prelovsek @ Spectrum (Mon)

Also,

S. Sharpe (Monday)

S. Aoki (Thursday)

A. Raposo (Thursday)

A. Rusetsky (Thursday)

$T_{bc} \equiv \bar{b}\bar{c}ud$

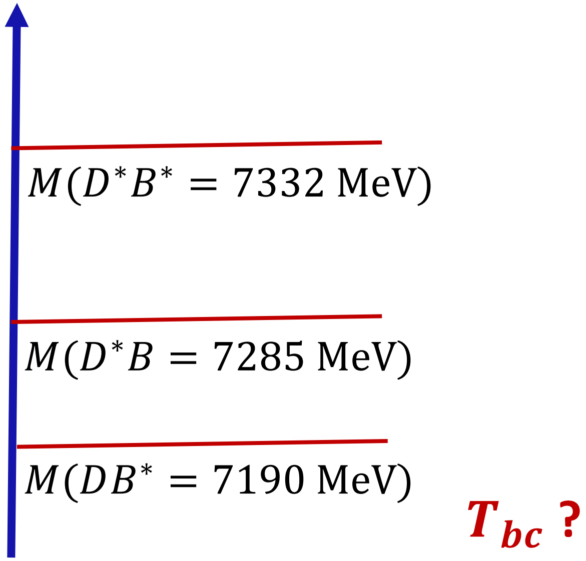
$I(J^P): 0(1^+)$

$(\bar{c}u)\ (\bar{b}d)$

$D \quad B^* \quad 0 \oplus 1 \rightarrow 1$

$D^* \quad B \quad 1 \oplus 0 \rightarrow 1$

$D^* \quad B^* \quad 1 \oplus 1 \rightarrow 1$



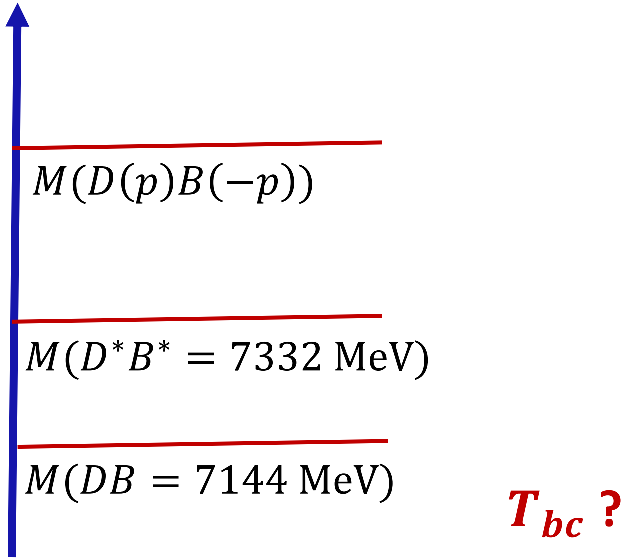
$I(J^P): 0(0^+)$

$(\bar{c}u)\ (\bar{b}d)$

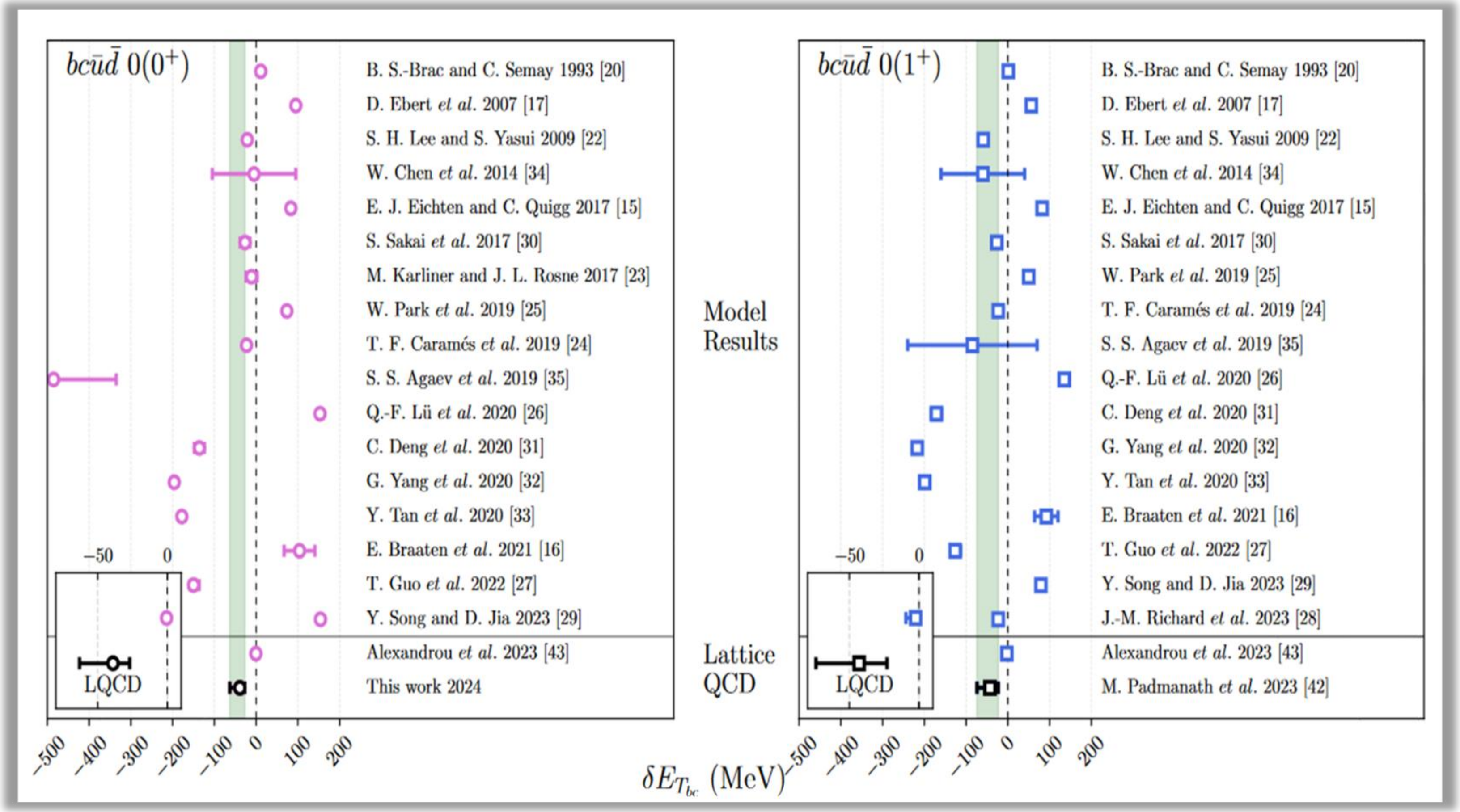
$D \quad B \quad 0 \oplus 0 \rightarrow 0$

$D(p)B(-p) \quad 0 \oplus 0 \rightarrow 0 \qquad \vec{p} = \dfrac{2\pi\vec{n}}{La}$

$D^* \quad B^* \quad 1 \oplus 1 \rightarrow 0$

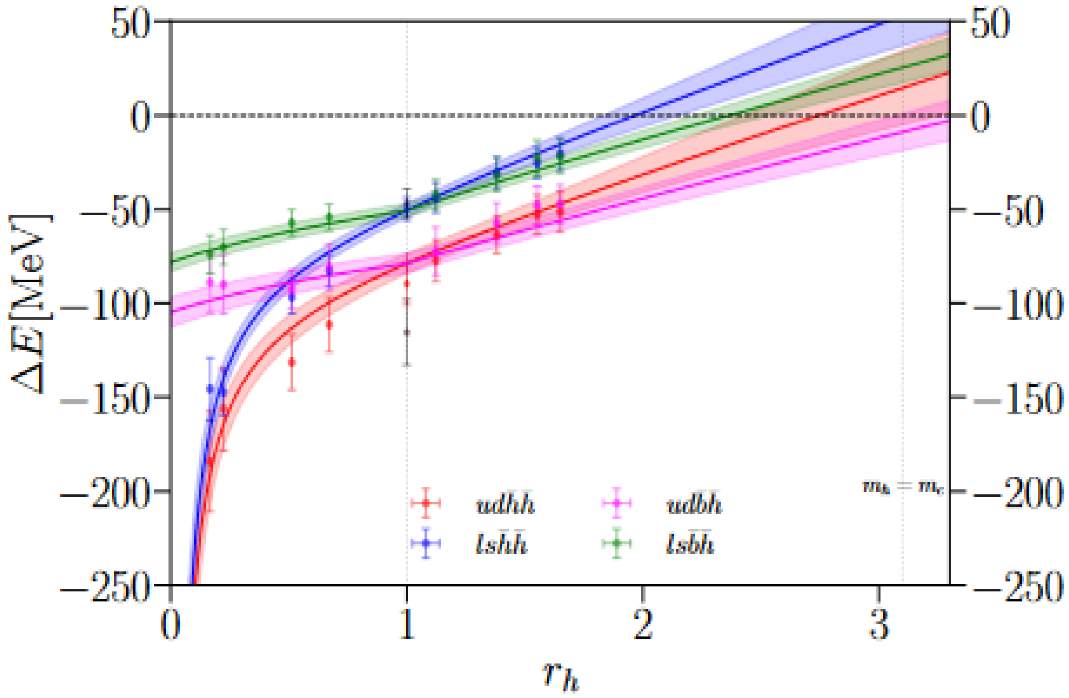


$T_{bc}(bc\bar{u}\bar{d})$



Radhakrishnan, Padmanath and NM: PRL 132 (20), 201902 (2024), and PRD (2024) arXiv:2404.08109

Alexandrou et al: PRL 132 (15), 151902 (2024), arXiv:2404.03588



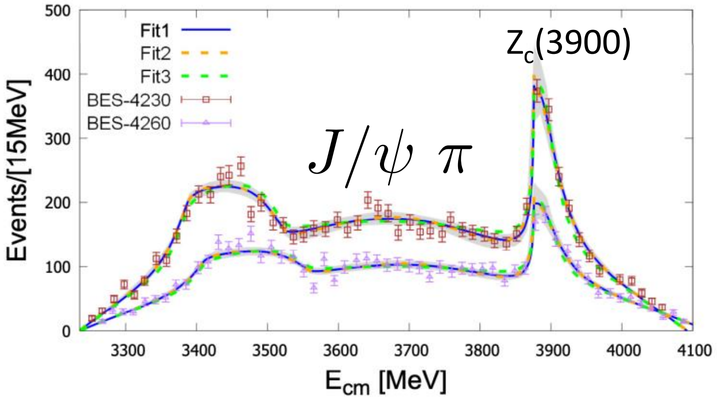
William Parrott (Monday)

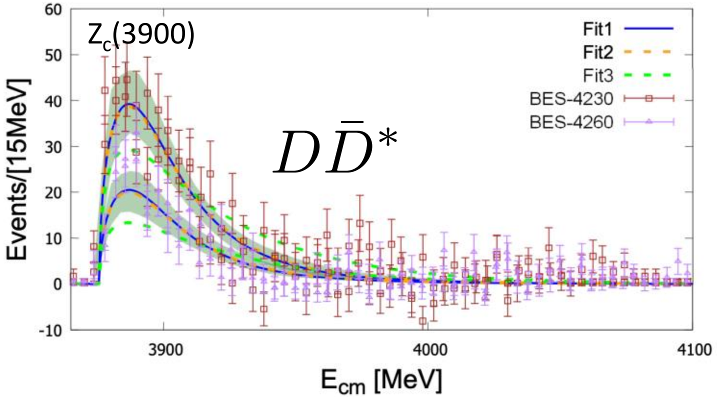
- ➢ **Heavier the heavy quark masses, deeper the binding**
- ➢ **Lighter the light quark masses, deeper the binding**

Z$_c$(3900) $\bar{c}c\bar{d}u$

Sadl, Collins, Z.H. Guo, Padmanath, Prelovsek, L.W. Yan, 2406.09842

L.W. Yan, Z.H. Guo, F.K. Guo, D.L. Yao, Z.Y. Zhou, 2307.12283





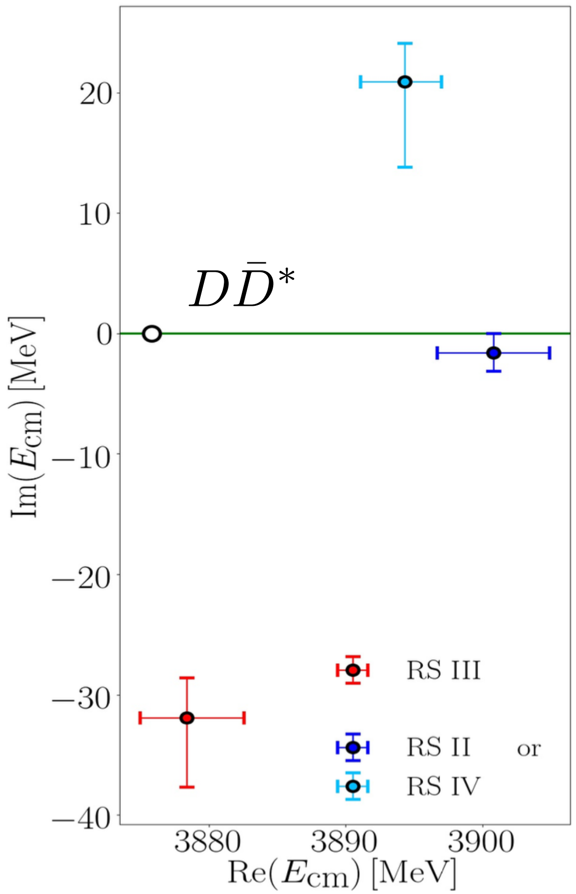
coupled-channels

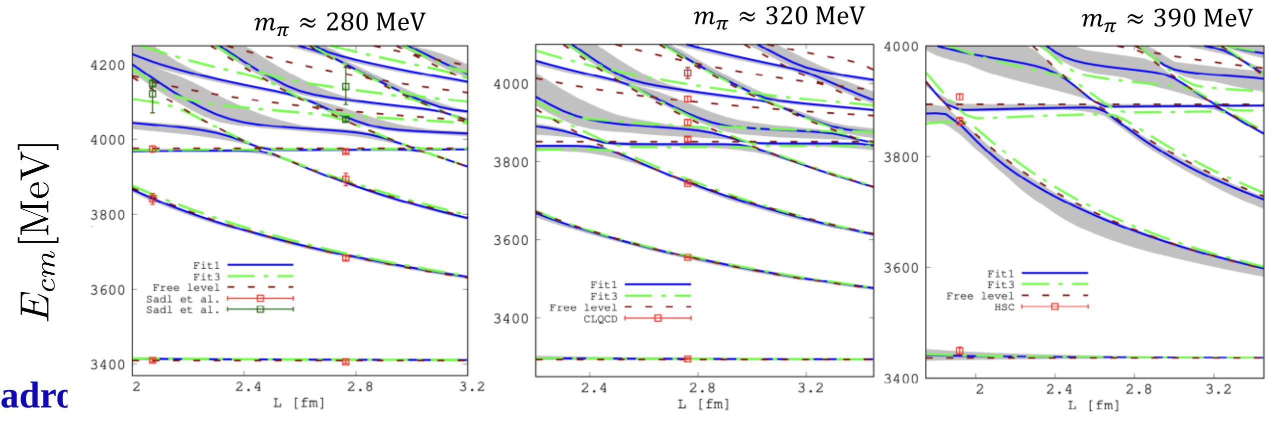
$$D\bar{D}^* \ - \ J/\psi\pi$$

EFT

Lipmann Schwinger

Luscher's formalism





Sadl et al. [2406.09842] CLQCD [1907.03371] HadSpec [1709.01417]

- two poles near DD* threshold
- significant off-diagonal int. $D\bar{D}^*–J/\psi\pi$ in agreement with HALQCD, PRL 2016

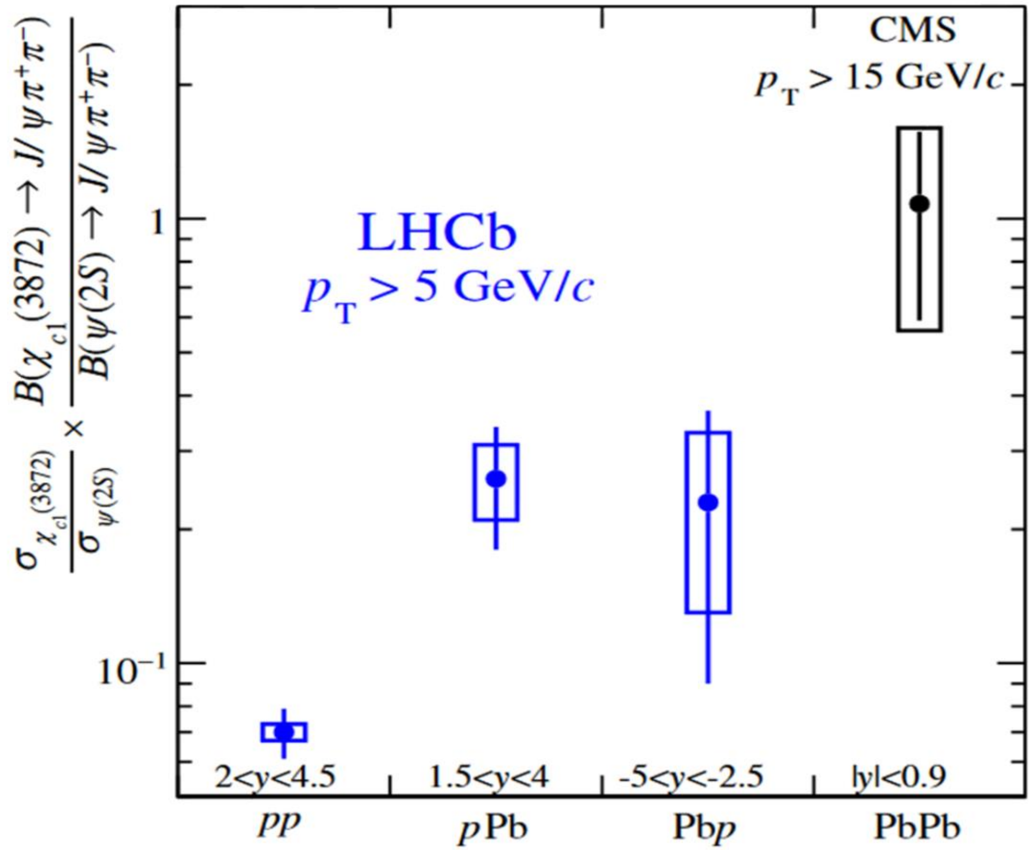


FIG. 2. Ratio of $\chi_{c1}(3872)$ to $\psi(2S)$ cross sections in the $J/\psi\pi^+\pi^-$ decay channel, measured in pp [13], pPb, Pbp, and PbPb [16] data. The error bars (boxes) represent the statistical (systematic) uncertainties on the ratio.

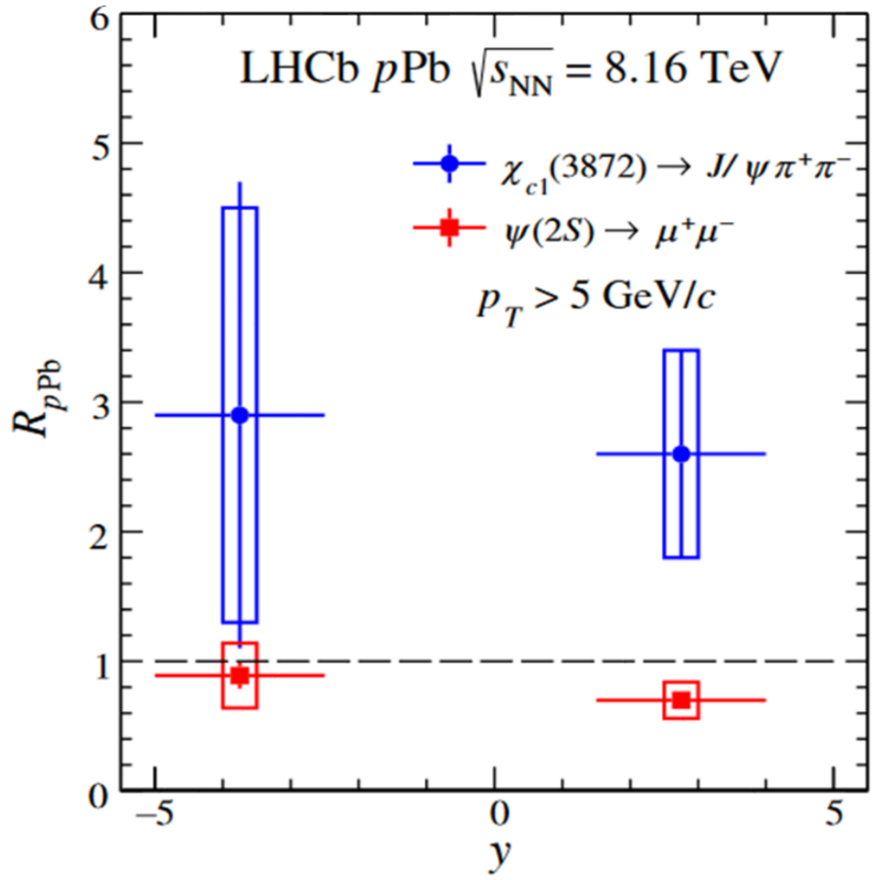


FIG. 3. Nuclear modification factor $R_{p\text{Pb}}$ for $\chi_{c1}(3872)$ and $\psi(2S)$ hadrons [56]. The error bars (boxes) represent the statistical (systematic) uncertainties.

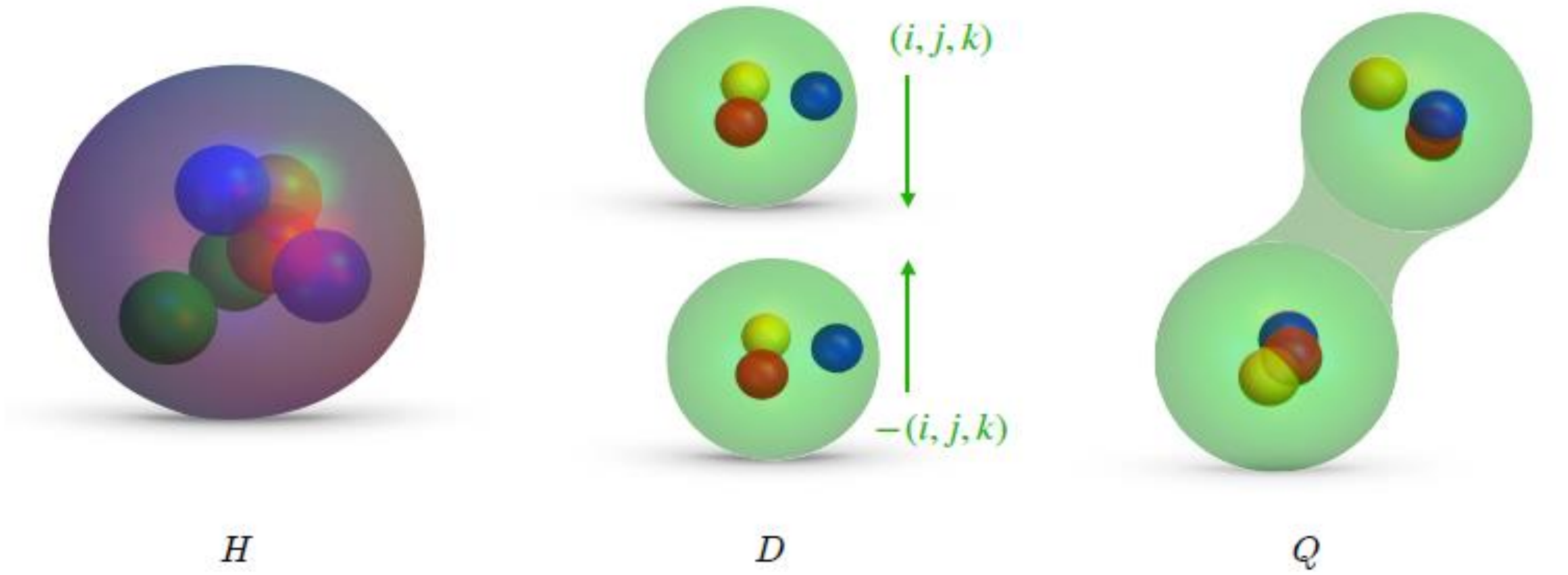
PRL 132, 242301 (2024) @LHCb

Nuclear Frontier

Challenges in Nuclear LQCD calculations

Constructing nuclear correlation functions	Signal-to-noise problem	Analysis
▪ Naïve Wick contraction explodes ▪ GEVP will be very expensive in naïve methods ▪ Large possibilities of interpolating fields. Which one to choose as the Wick contractions are expensive. These problems may possibly be overcome till C-12 with clusters of nuclei interpolating fields (and with symmetry and help of AI tools)	▪ Stochastic noise grows with the number of nucleons $SNR(C_i) \sim \frac{\langle C_i \rangle}{\sqrt{\langle \lvert C_i \rvert^2 \rangle}} \sim e^{-A\left(M_N - \frac{3}{2} m_\pi\right)t}$ M. Wagman arXiv:2406.20009 and his talk on Thursday P. Bedaque, H. Oh Phys. Rev. D 109, 094519	▪ Large number of closely spaced states (become worse with increasing volume which is necessary for nuclei) ▪ Presence of multiple thresholds and many possible multibody decays ▪ Quantization conditions more complicated (left hand cut) ▪ Infinite volume amplitudes have a larger class of singularities.

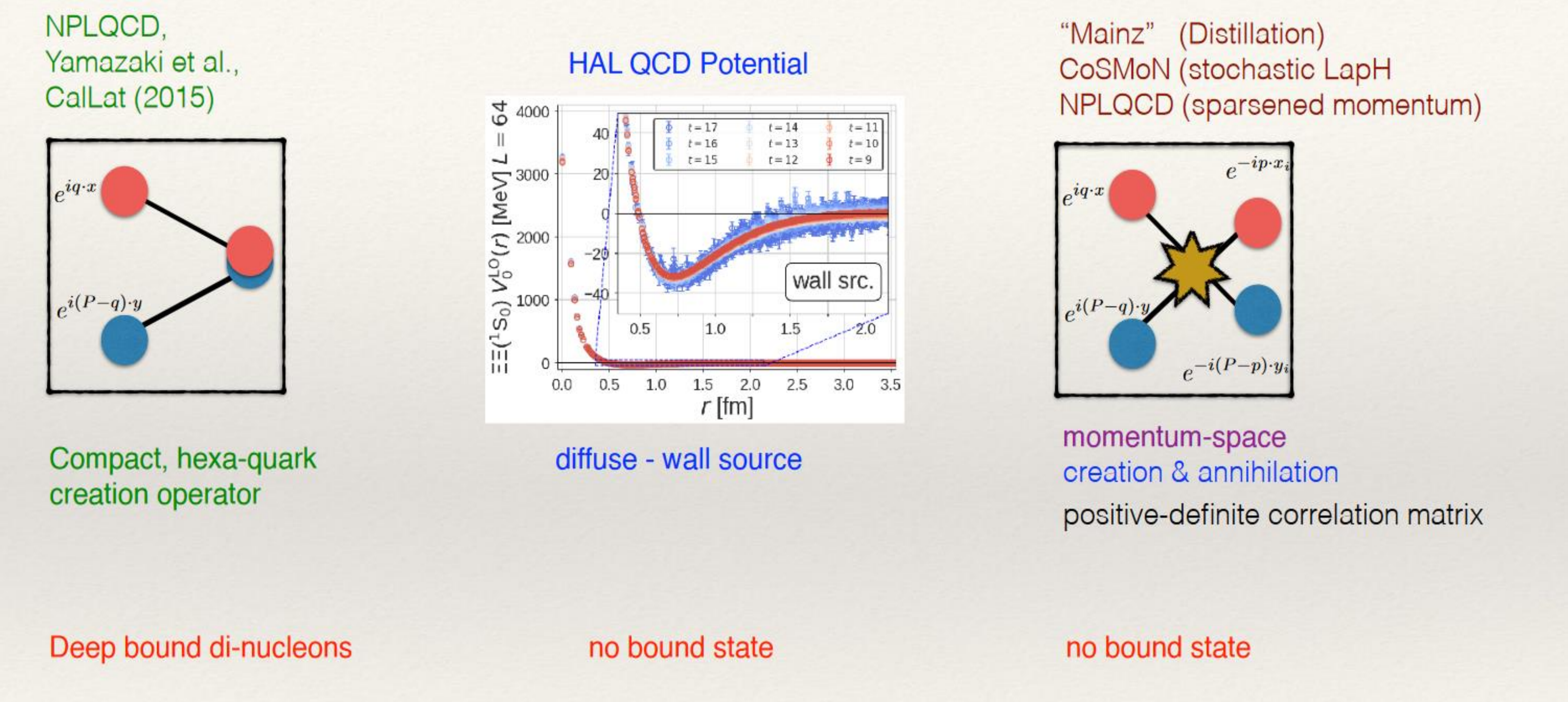
Briceno et al,
Few Body Syst. 63 (2022) 4, 67



and more

Fig from: Detmold et al (NPLQCD): arXiv:2404.12039

Binding of two-nucleon systems (SU(3)$_F$ point)



NPLQCD,
Yamazaki et al.,
CalLat (2015)

Compact, hexa-quark
creation operator

Deep bound di-nucleons

HAL QCD Potential

diffuse - wall source

no bound state

“Mainz” (Distillation)
CoSMoN (stochastic LapH
NPLQCD (sparsened momentum)

momentum-space
creation & annihilation
positive-definite correlation matrix

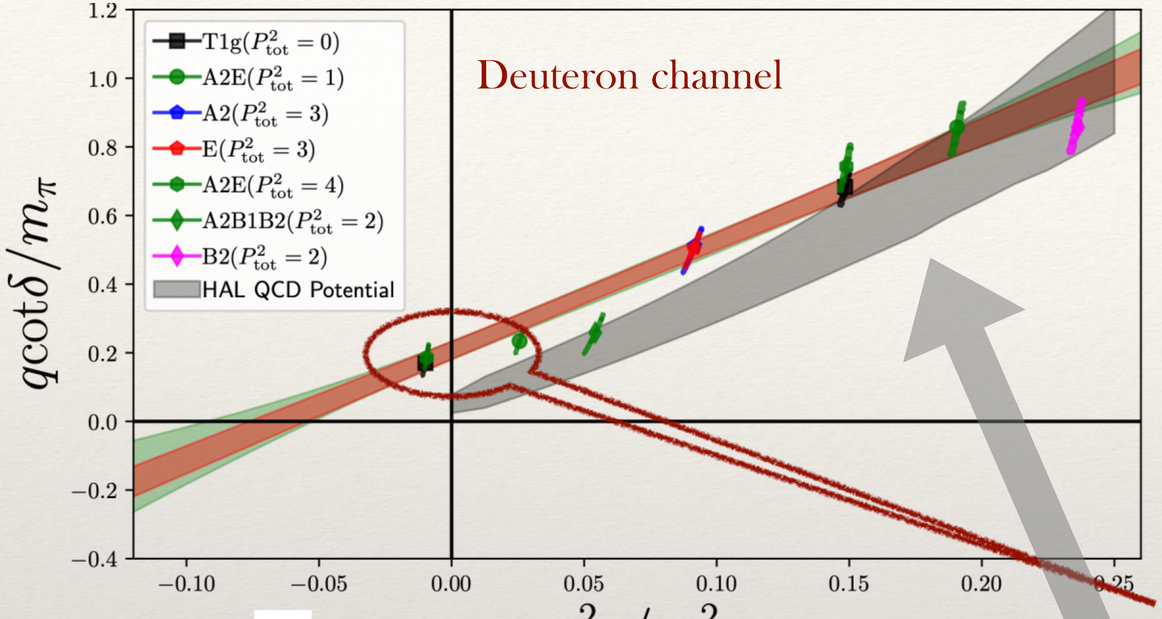
no bound state

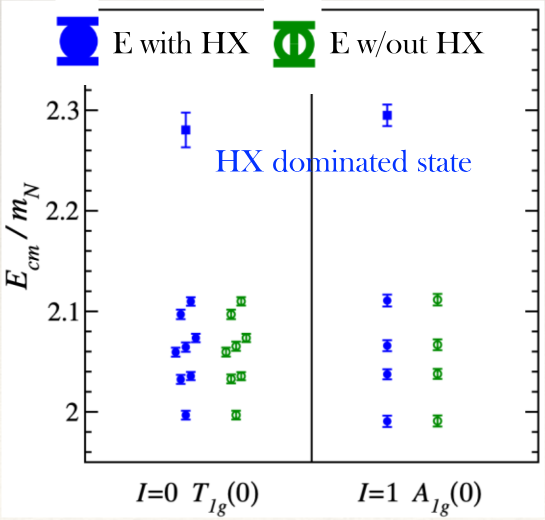
A. Walker-Loud, Lattice 2023

NN Scattering @ heavy SU(3) symmetric point

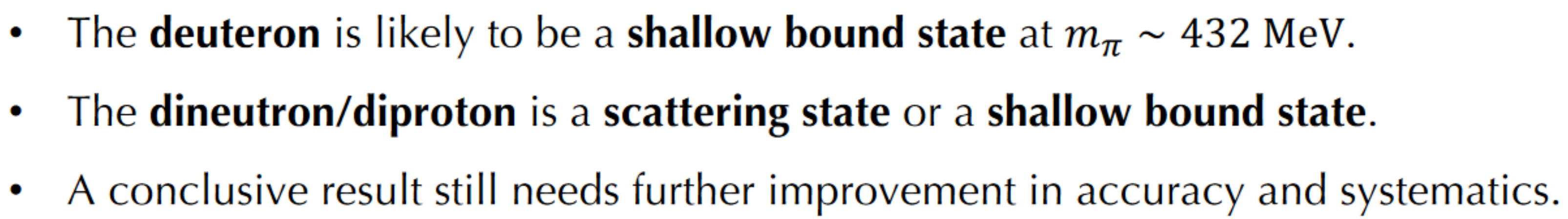
Bulava, Clark, Gambhir, Hanlon, Hörz, Joó, Körber, Meyer, Monge-Camacho, Morningstar, Moscoso, Nicholson, Romero-López, Rrapaj, Shindler, Skinner, Vranas, Walker-Loud

in preparation, update of Hörz et al., PRC 103 (2021) [2009.11825]





- CLS action; $m_\pi = m_K \approx 714$ MeV, $a \approx 0.086$ fm, $L/a = 48$
- Designed to address decade-long controversy on whether or not NN form bound states at heavy pion mass
- stochastic LapH method [1104.3870] used to construct basis of operators, which are rotated with GEVP
- N and NN spectrum analyzed and Lüscher Quantization Condition used to determine scattering phase shifts
- Our amplitude analysis rules out a bound state $\gtrsim 5\sigma$ on this ensemble [$q\cot\delta|_{q=0} = 0.207(26)(\text{sys})$]
- The HAL QCD potential was computed on the same ensemble
  - phase shift from large-t extrapolation of potential
  - resulting phase shift is qualitatively consistent
  - do not expect discretization errors to be the same in both methods
- Found that hexa-quark (HX) operators do not influence the spectrum



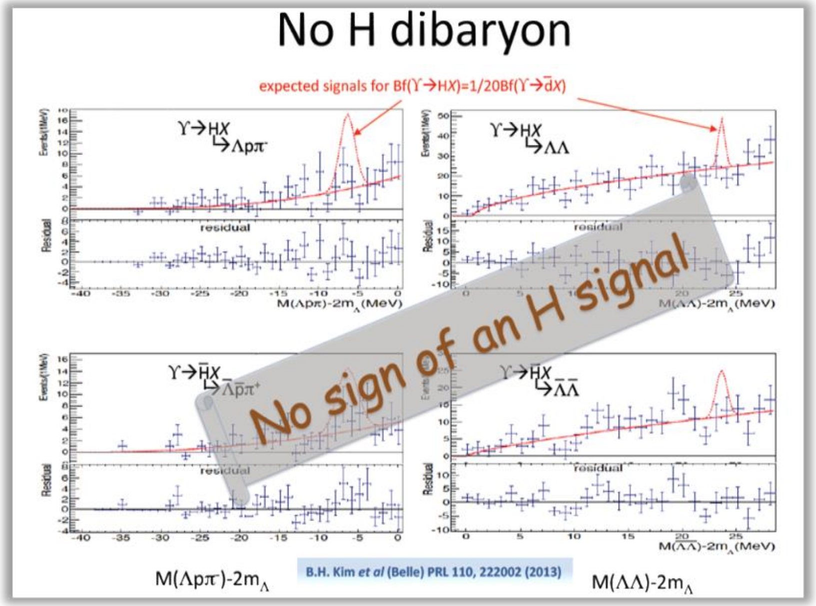
- The **deuteron** is likely to be a **shallow bound state** at $m_\pi \sim 432$ MeV.
- The **dineutron/diproton** is a **scattering state** or a **shallow bound state**.
- A conclusive result still needs further improvement in accuracy and systematics.

…Zi-Yu Wang et al: , Wednesday parallel talk

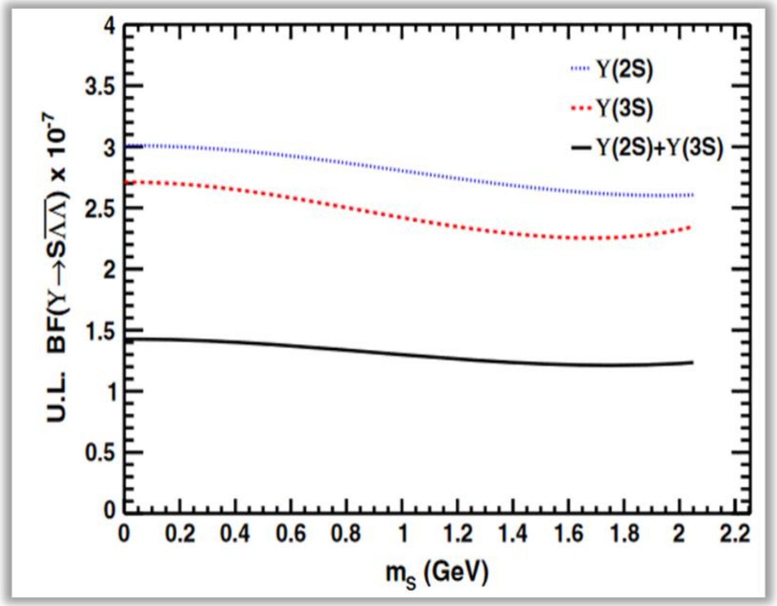
H Dibaryon

Bound state of two Λ $\Lambda\Lambda\ (udssud)$ Proposed by Jaffe (1976)

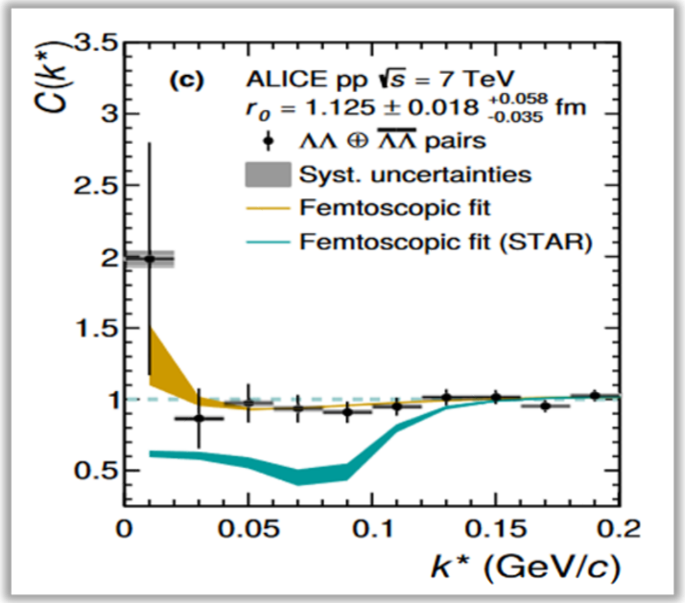
- If the ground state is below the two-Lambda threshold, then it will be bound
- If it exists (below 2P) it will be stable and could be a candidate for SM dark matter?
  (May not be as oxygen will not get produced with that! @S. Reddy)



Belle: PRL 110,222002(2013)



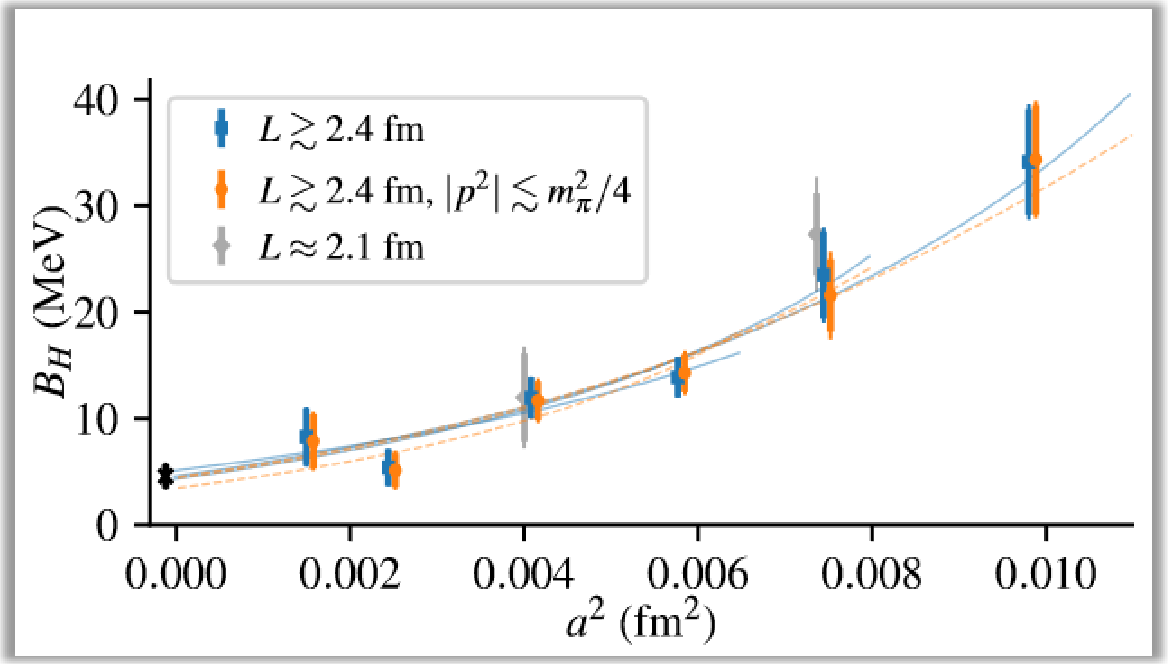
BABAR Collaboration:
Phys. Rev. Lett. 122, 072002 (2019)
No signal is observed
in Υ **decays** (90% confidence limit)



Alice: Phys. Rev. C 99, 024001 (2019)

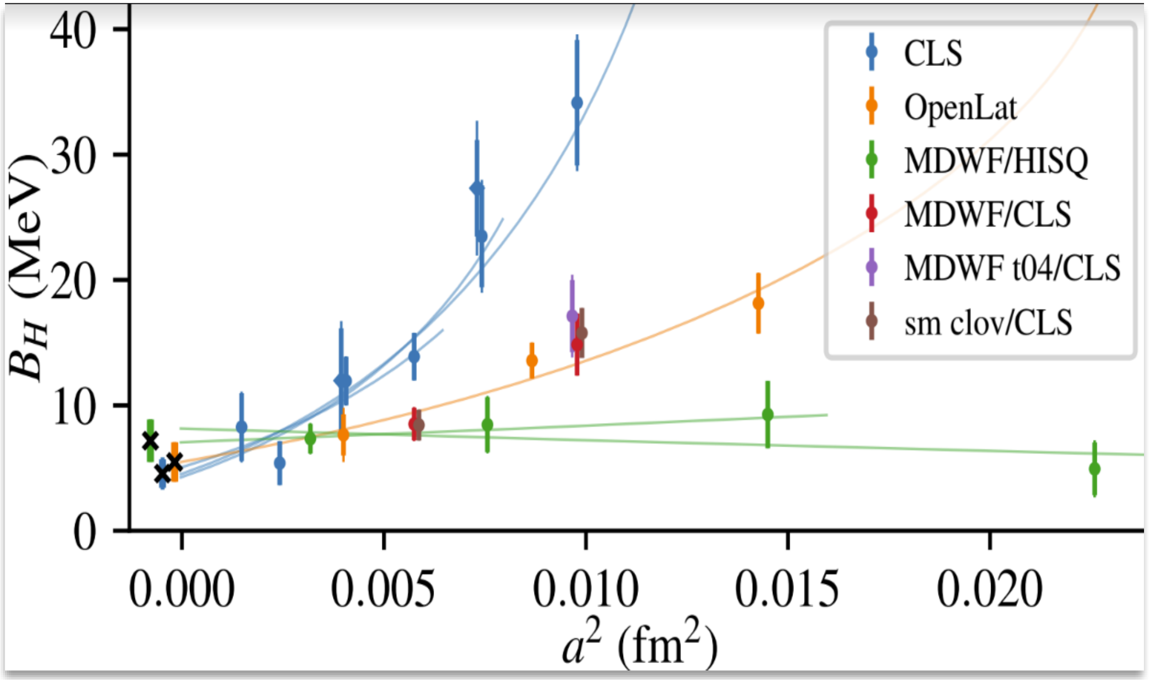
New results from JPARC soon?
Fumiya Oura@NSTAR'24
for the J-PARC E42 collaboration

H-dibaryon at $\mathrm{SU}(3)_{\mathrm{F}}$ symmetric point



$$\check{B}_H^{\mathrm{SU}(3)_{\mathrm{f}}} = 4.56 \pm 1.13_{\mathrm{stat}} \pm 0.63_{\mathrm{syst}}\ \mathrm{MeV}.$$

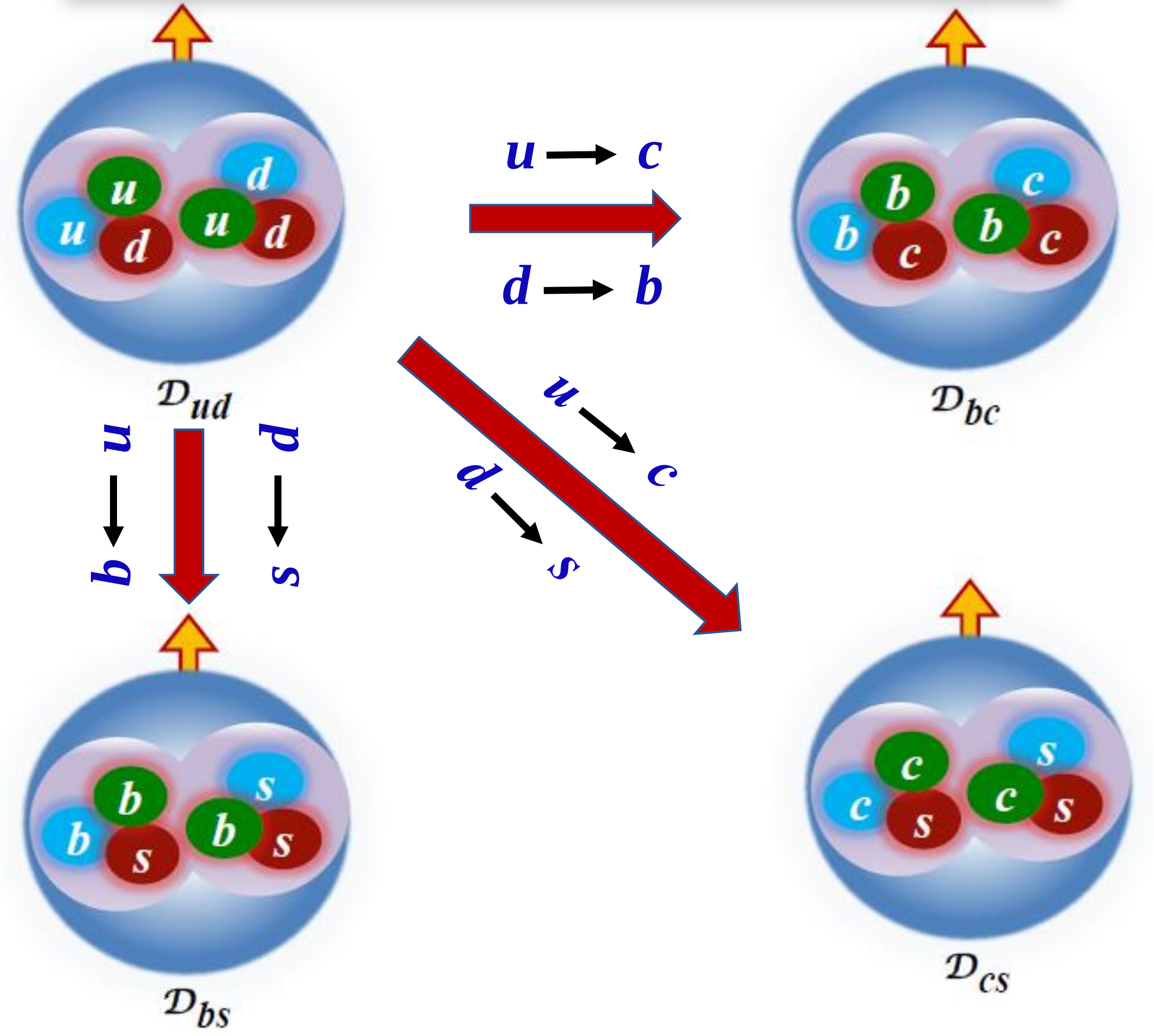
Green et al : *Phys. Rev. Lett.* 127 (2021) 24, 242003



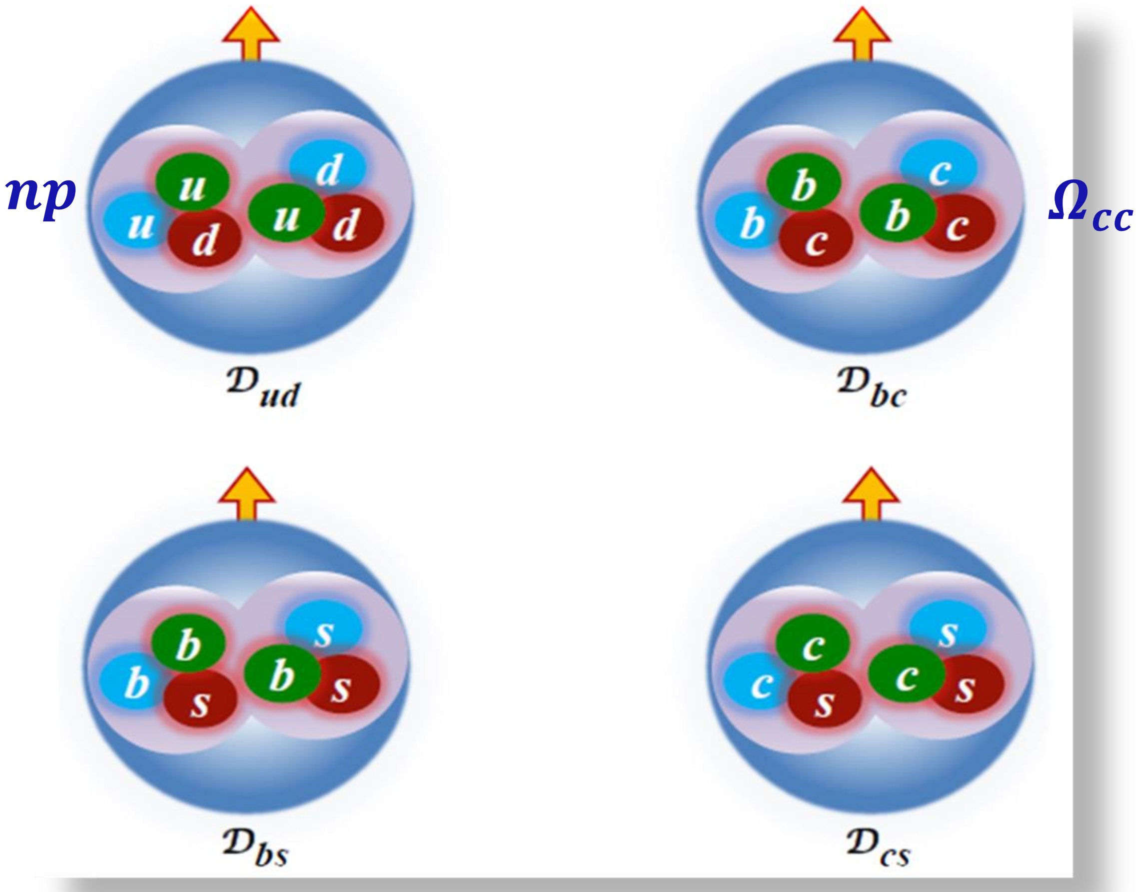
Green (parallel: Friday)

Are there heavy Dibaryons?

Deuteron-like heavy dibaryons



Deuteron-like heavy dibaryons



np

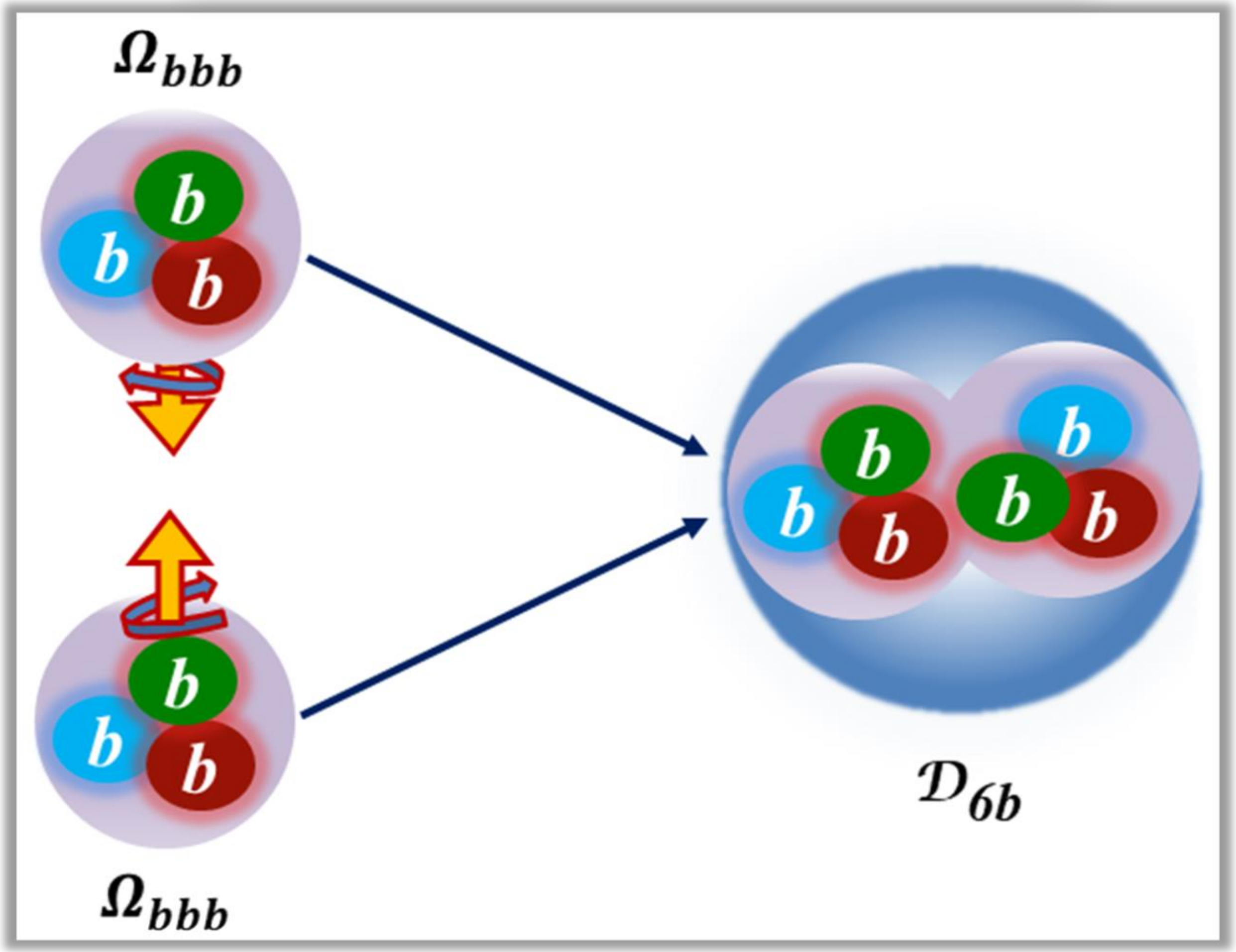
$\Omega_{ccb}\Omega_{cbb}$

$\Omega_b\Omega_{bb}$

$\Omega_c\Omega_{cc}$

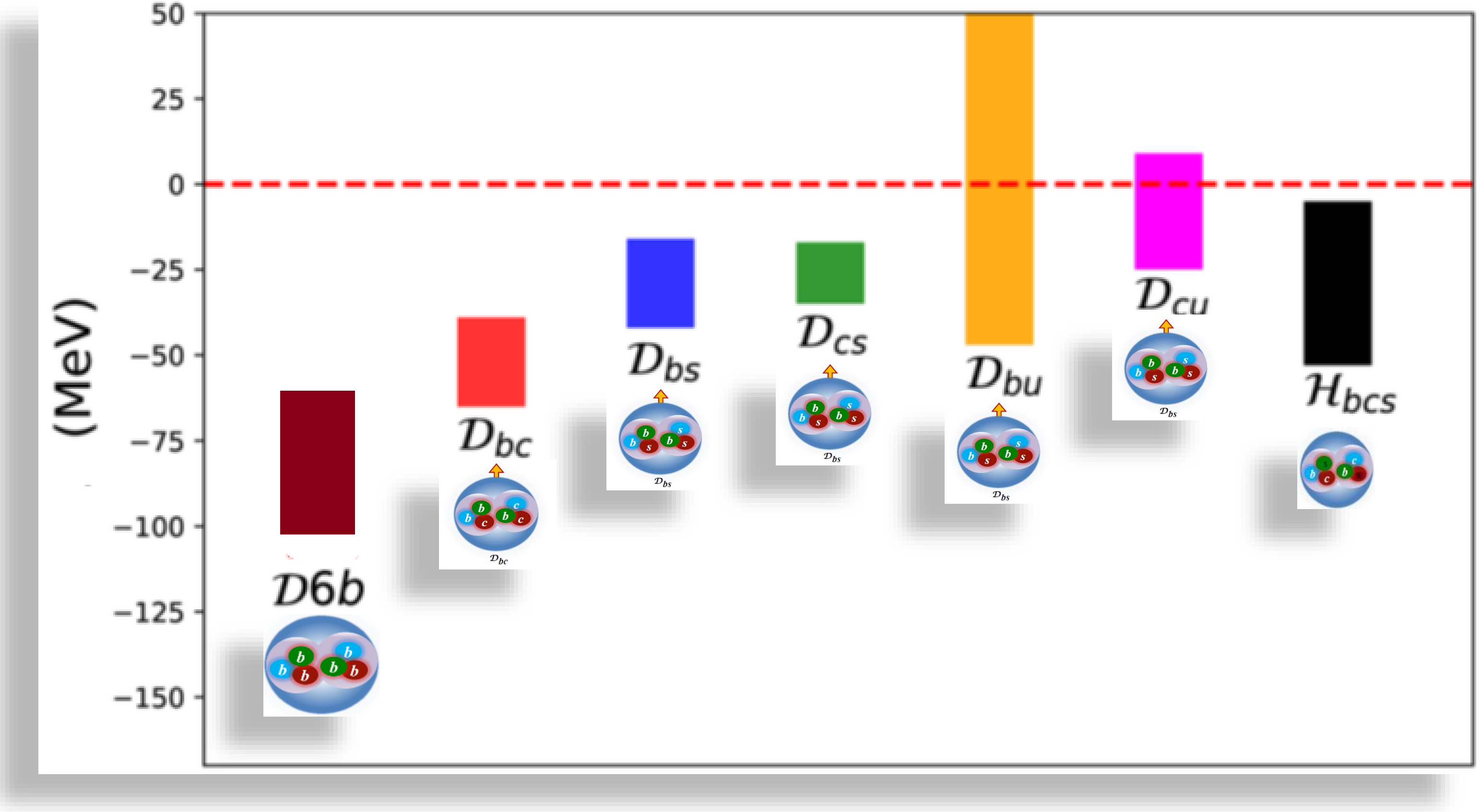
Junnarkar and NM : Phys. Rev. Lett. 123, 162003(2019)

Most beautiful dibaryons!



NM, Padmanath and Chakraborty: PRL 130, 111901 (2023)

Heavy Dibaryon Candidates?



PRL 123,162003 (2019): Junnarkar, NM

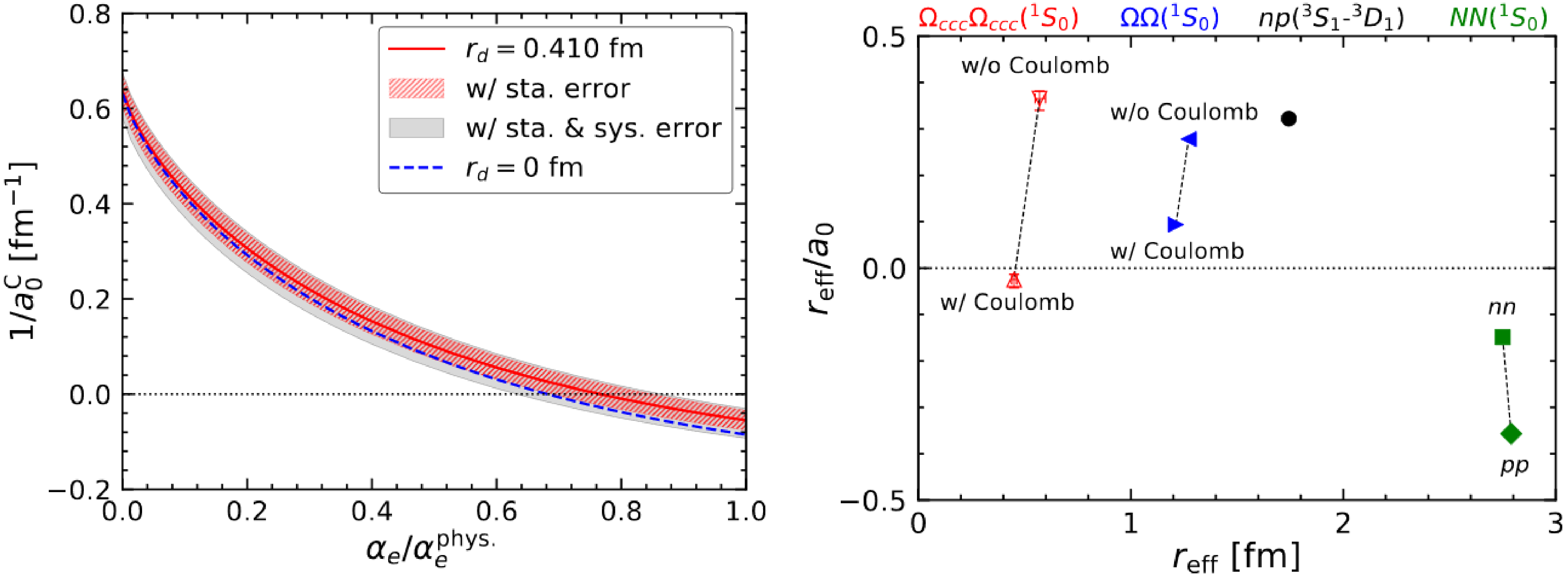
PRL 130, 111901 (2023): NM, Padmanath and Chakraborty

PRD 106, 054511 (2022): Junnarkar, NM

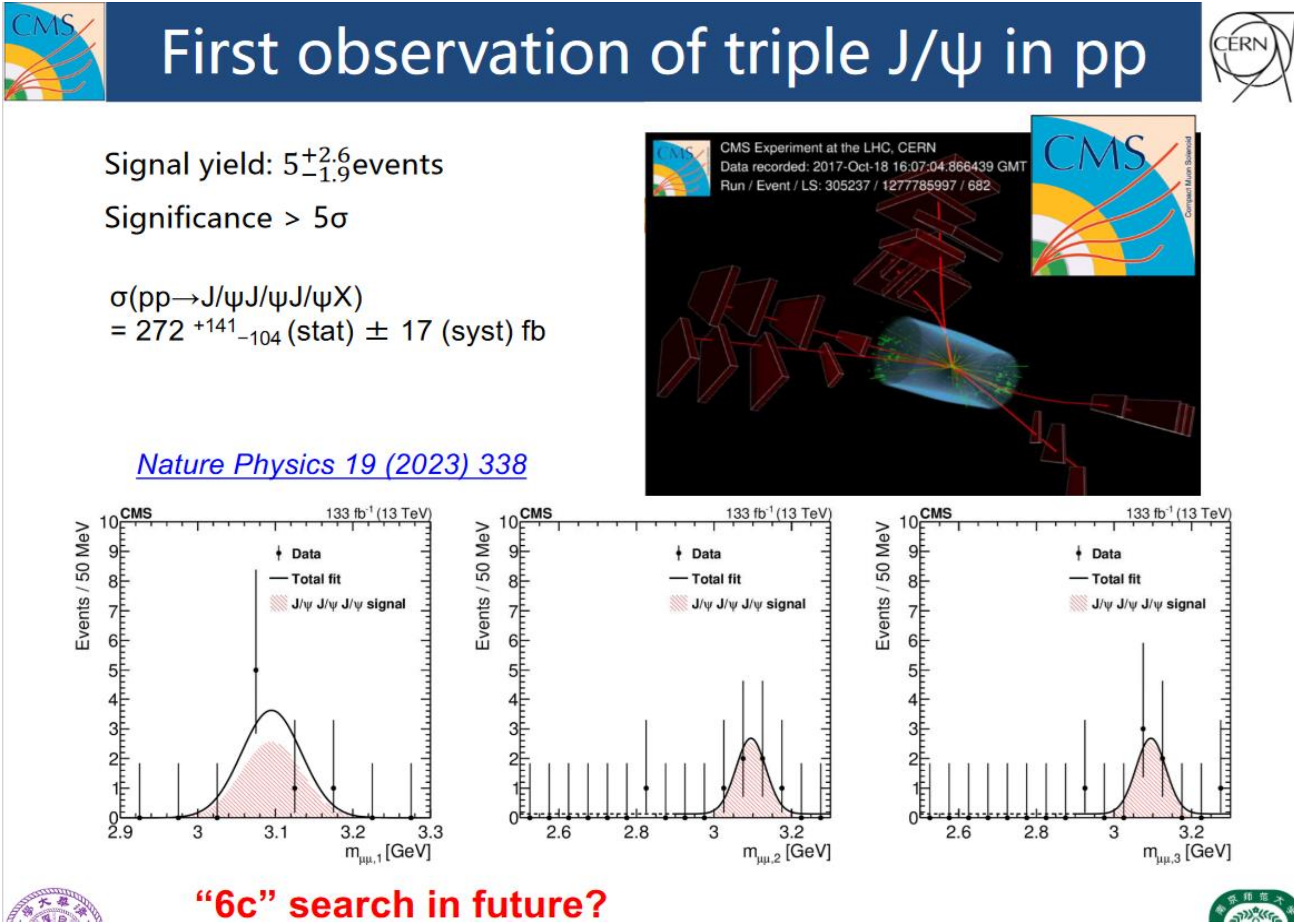
Also, Dhindsa (Friday, parallel talk)

Need to study these with rigorous amplitude analysis

$\Omega_{ccc}\,\Omega_{ccc}(^1S_0)$



HALQCD:PRL,127, 072003 (2021)



First observation of triple J/ψ in pp

Signal yield: $5^{+2.6}_{-1.9}$events

Significance > 5σ

$\sigma(pp \to J/\psi J/\psi J/\psi X)$
$= 272\ ^{+141}_{-104}$ (stat) ± 17 (syst) fb

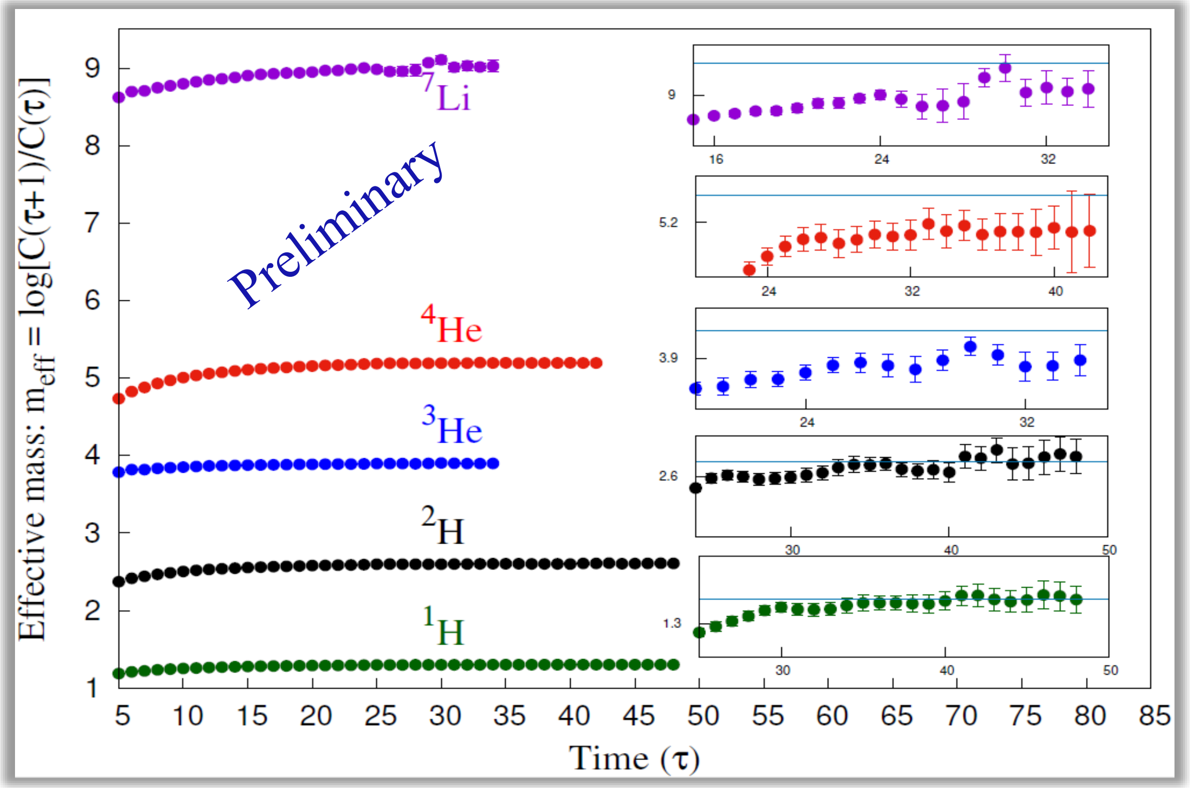
Nature Physics 19 (2023) 338

"6c" search in future?

Zhen Hu @ NSTAR2024

Heavy Nuclei (unphysical)

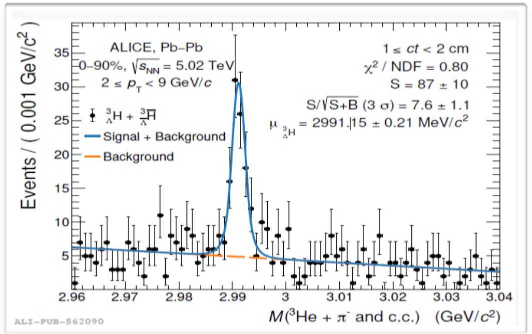
Nuclei with $m_u = m_d = m_c$



- The strong binding mechanism for such fictitious states could be very different.
- They could be very small in spatial sizes
- They could be bound by gluon exchanges and such models can be developed to check if that is indeed true.

Exotic Nuclei !

Nuclei with one or more hyperon, charmed or bottom baryons



ALICE: PRL 131, 102302 (2023)

$$npn \rightarrow np\Lambda, np\Xi\ , np\Lambda_c, np\Xi_c, , np\Lambda_b, np\Xi_b\ \ etc.$$

- Similar to hypernuclei, such as $^{3}_{\Lambda}H$

  charmed-nuclei, bottom-nuclei
  (one or more quarks in a nucleon is (are) replaced by charm or bottom quarks)

$$^{3}_{\Lambda_c}H, \quad ^{3}_{\Xi_{cc}}H \qquad etc.$$

- And in general, $^{A}_{X_Q}N,\ ^{A}_{X_{QQ}}N,\quad etc.$

Outlook

- Repeat the previously performed calculations with realistic lattices (spacings and volume)
    - To match/confront experiments (Proof of principle to gold plated results)
    - Learn the movement of pole positions *w.r.t.* dialing the quark masses → binding mechanisms

- Exotics:
    - Predicting more exotics with gold standard (aiding discovery)
    - Understanding the pole structures of exotics
    - Understanding the structure of exotics: radiative transitions, form factors?

- Study of strongly-coupled multihadron states to extract long-range electroweak matrix elements.

Plenary: Felix Erben

- Mapping the baryon resonances: baryon-mesons interactions

- Studies on multi-baryons:
    - Baryon-baryon interactions, scatterings → nuclear frontier (Are we ready?)
    - Towards calculating matrix elements of nuclei

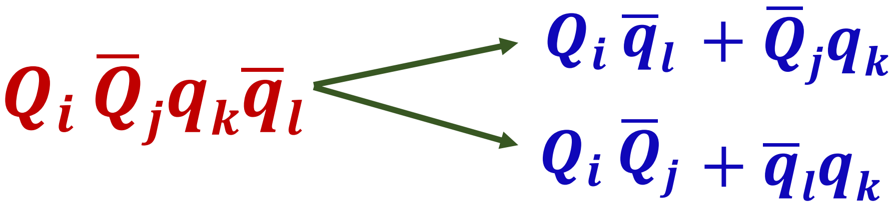
Acknowledgements

Padmanath, Junnarkar, Radhakrishnan, Chakraborty, Bali, Liu, Prelovsek, and all others for sending their results

X, Y, Z-type four quarks (LHCb, Belle, BES)

$$Q_i\,\bar{Q}_j q_l \bar{q}_k$$

Ex: $Z_b = b\bar{b}u\bar{d}$, $Z_c = c\bar{c}u\bar{d}$ etc.



$Q_i\,\bar{Q}_j q_k \bar{q}_l$

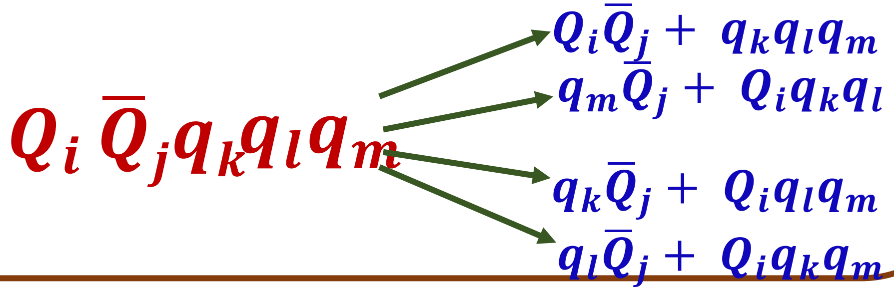
$Q_i\,\bar{q}_l + \bar{Q}_j q_k$

$Q_i\,\bar{Q}_j + \bar{q}_l q_k$

Pentaquarks: LHCb (2015,2019,2022)

$$Q_i\,\bar{Q}_j q_k q_l q_m$$

Ex: $P_c = c\bar{c}uud$



$Q_i\,\bar{Q}_j q_k q_l q_m$

$Q_i\bar{Q}_j + q_k q_l q_m$

$q_m\bar{Q}_j + Q_i q_k q_l$

$q_k\bar{Q}_j + Q_i q_l q_m$

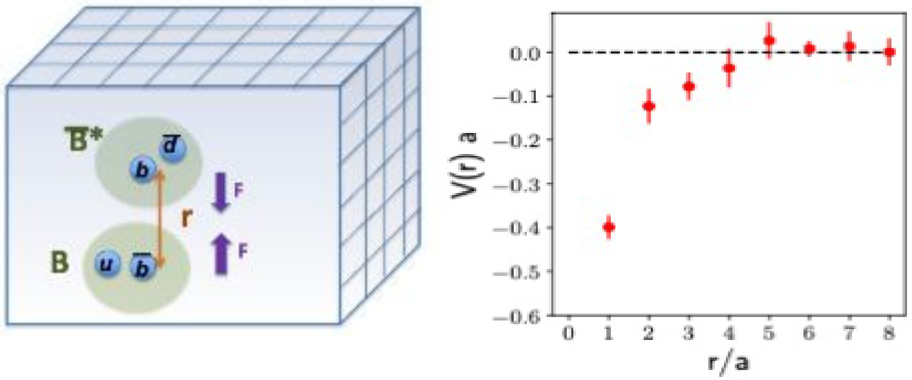
$q_l\bar{Q}_j + Q_i q_k q_m$

- Multiple decay modes
- Close to thresholds
- Presence of disconnected diagrams
- Both heavy and light quark together (multiple lattices with bigger volumes and smaller lattice spacings)

Difficult for lattice QCD precision study

$\boldsymbol{Z_b = b\bar{b}u\bar{d}}$ Belle(2011)

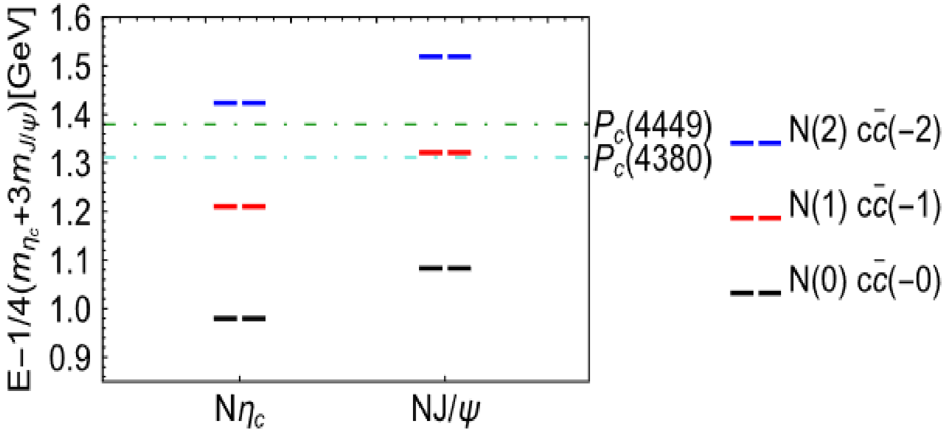
$$b\bar{b}u\bar{d} \rightarrow BB^*, \Upsilon\pi$$



- Born-Openheimer approach with static b-quarks
- Attraction between B and B^* was found

Prelovsek et al: Phys. Lett. B 805 (2020) 135467
Bicudo et al: Phys. Rev. D 101, 034503 (2020)

$\boldsymbol{P_c = c\bar{c}uud}$ LHCb (2015, 2019)



No interaction was found in NJ/ψ channel

Skerbis et al: Phys. Rev. D 99, 094505 (2019)

Strong-decay thresholds for dibaryons

Charm

$$M_{\Sigma_c}^{\frac{1}{2}} + M_{\Xi_{cc}}^{\frac{1}{2}} < M_{\Delta_u}^{\frac{3}{2}} + M_{\Omega_{ccc}}^{\frac{3}{2}}$$

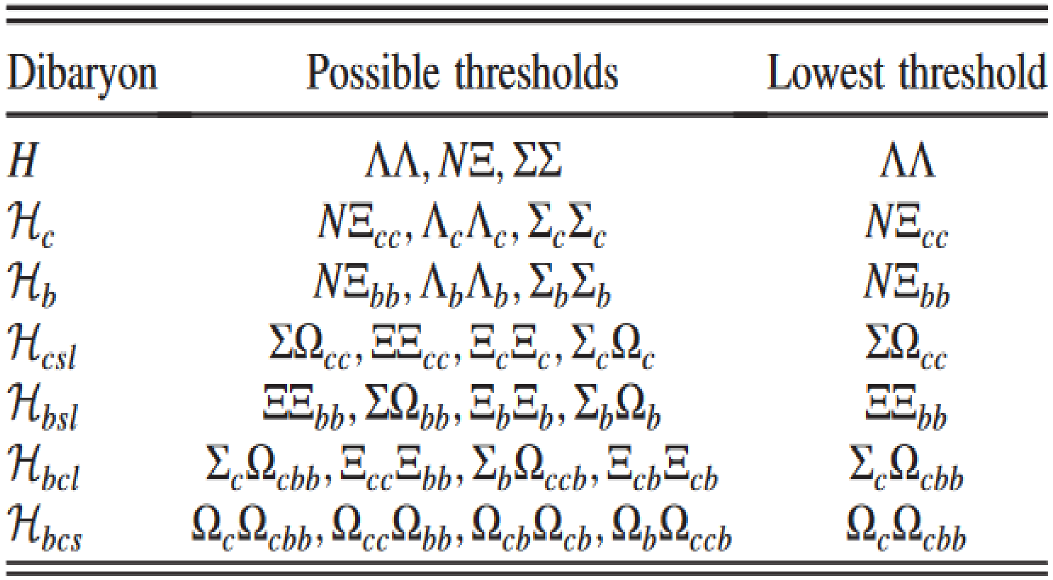
$$M_{\Omega_c}^{\frac{1}{2}} + M_{\Omega_{cc}}^{\frac{1}{2}} < M_{\Omega_s}^{\frac{3}{2}} + M_{\Omega_{ccc}}^{\frac{3}{2}}$$

Bottom

$$M_{\Sigma_b}^{\frac{1}{2}} + M_{\Xi_{bb}}^{\frac{1}{2}} > M_{\Delta_u}^{\frac{3}{2}} + M_{\Omega_{bbb}}^{\frac{3}{2}}$$

$$M_{\Omega_b}^{\frac{1}{2}} + M_{\Omega_{bb}}^{\frac{1}{2}} > M_{\Omega_s}^{\frac{3}{2}} + M_{\Omega_{bbb}}^{\frac{3}{2}}$$

$$M_{\Omega_{ccb}}^{\frac{1}{2}} + M_{\Omega_{cbb}}^{\frac{1}{2}} > M_{\Omega_{ccc}}^{\frac{3}{2}} + M_{\Omega_{bbb}}^{\frac{3}{2}}$$



Dibaryon	Possible thresholds	Lowest threshold
H	$\Lambda\Lambda, N\Xi, \Sigma\Sigma$	$\Lambda\Lambda$
$\mathcal{H}_c$	$N\Xi_{cc}, \Lambda_c\Lambda_c, \Sigma_c\Sigma_c$	$N\Xi_{cc}$
$\mathcal{H}_b$	$N\Xi_{bb}, \Lambda_b\Lambda_b, \Sigma_b\Sigma_b$	$N\Xi_{bb}$
$\mathcal{H}_{csl}$	$\Sigma\Omega_{cc}, \Xi\Xi_{cc}, \Xi_c\Xi_c, \Sigma_c\Omega_c$	$\Sigma\Omega_{cc}$
$\mathcal{H}_{bsl}$	$\Xi\Xi_{bb}, \Sigma\Omega_{bb}, \Xi_b\Xi_b, \Sigma_b\Omega_b$	$\Xi\Xi_{bb}$
$\mathcal{H}_{bcl}$	$\Sigma_c\Omega_{cbb}, \Xi_{cc}\Xi_{bb}, \Sigma_b\Omega_{ccb}, \Xi_{cb}\Xi_{cb}$	$\Sigma_c\Omega_{cbb}$
$\mathcal{H}_{bcs}$	$\Omega_c\Omega_{cbb}, \Omega_{cc}\Omega_{bb}, \Omega_{cb}\Omega_{cb}, \Omega_b\Omega_{ccb}$	$\Omega_c\Omega_{cbb}$