From scattering towards multi-hadron weak decays

Felix Erben

Lattice 2024
Liverpool
30/7/2024

Resonances on the lattice

Hadron scattering: A few recent works

$K^*(892)$ & $\rho(770)$ at m_π^{phys}

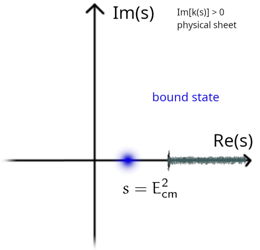
Electroweak matrix elements

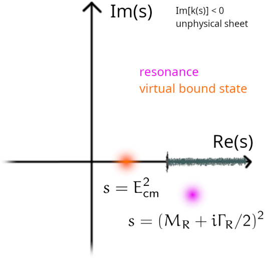
Outlook & Conclusions

- rich physics in QCD resonance states
- related to scattering phase-shift δ via

$$t(\sqrt{s}) = \frac{1}{\cot\delta(\sqrt{s}) - i}$$

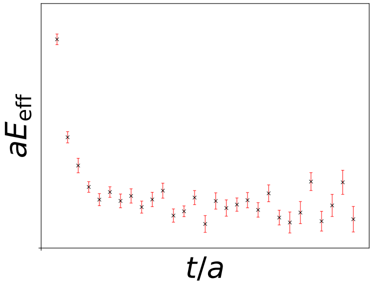
- Example $\pi\pi$ scattering: $k^2 = s/4 - M_\pi^2$
- physical sheet, $\mathrm{Im}[k(s)] > 0$
  - bound states on the real axis
- unphysical sheet, $\mathrm{Im}[k(s)] < 0$
  - virtual bound states on the real axis
  - resonances in the complex plane



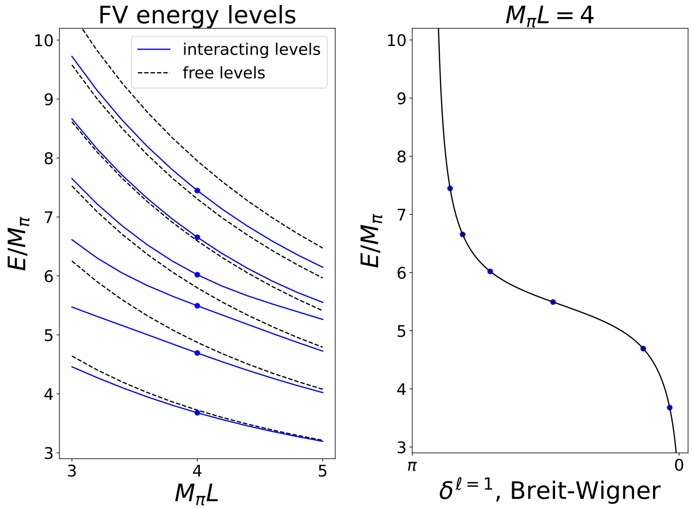


- for QCD-asymptotic states like $\pi, \mathrm{K}, \mathrm{D}, \mathrm{N}, \ldots$:
- define interpolator with correct quantum numbers
- compute 2-pt (n-pt) function
- extract asymptotic masses and matrix elements

Effective mass of a 2-pt function:



- for resonances (vast majority of QCD states):
- **no simple correspondence** between finite-volume energies and resonances
- resonances leave their marks as **imprints on the finite-volume energy spectrum**



Recipe for resonances in LQCD ($\rho \to \pi\pi$)

relevant interpolator basis

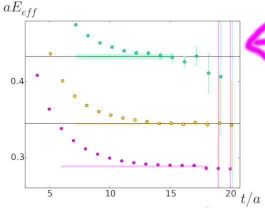
$$\rho(\vec{P}, t) = \sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}} (\bar{u}\gamma_i u - \bar{d}\gamma_i d)$$

$$(\pi\pi)(\vec{P}, t) = \pi^+(\vec{p}_1, t)\pi^-(\vec{p}_2, t) - \pi^-(\vec{p}_1, t)\pi^+(\vec{p}_2, t)$$

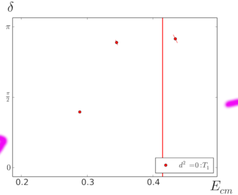
complete correlator matrix

$$\begin{pmatrix} \langle\rho(t)\rho^\dagger(t_0)\rangle & \langle\rho(t)(\pi\pi)_1^\dagger(t_0)\rangle & \langle\rho(t)(\pi\pi)_2^\dagger(t_0)\rangle & \cdots \\ \langle(\pi\pi)_1(t)\rho^\dagger(t_0)\rangle & \langle(\pi\pi)_1(t)(\pi\pi)_1^\dagger(t_0)\rangle & \langle(\pi\pi)_1(t)(\pi\pi)_2^\dagger(t_0)\rangle & \cdots \\ \langle(\pi\pi)_2(t)\rho^\dagger(t_0)\rangle & \langle(\pi\pi)_2(t)(\pi\pi)_1^\dagger(t_0)\rangle & \langle(\pi\pi)_2(t)(\pi\pi)_2^\dagger(t_0)\rangle & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

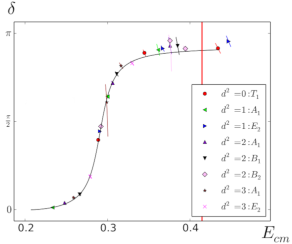
GEVP



finite-volume formalism



repeat for momenta, irreps



GEVP, FV formalism: complete (hopefully!) citations in backup slide. Plots from [FE et al.; PRD 20]

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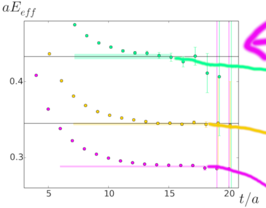
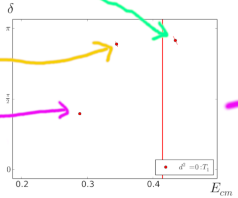
GEVP





finite-volume formalism

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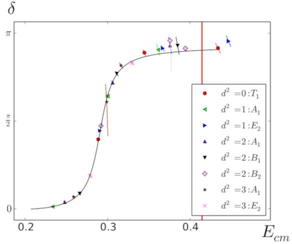
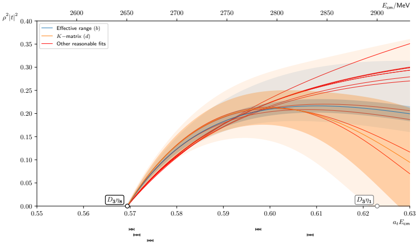


GEVP, FV formalism: complete (hopefully!) citations in backup slide. Plots from [FE et al.; PRD 20]

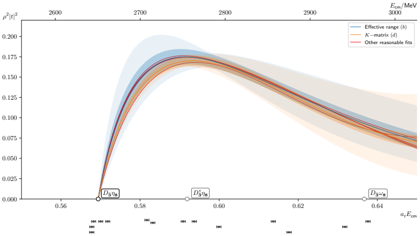
Hadron scattering: A few recent works

$SU(3)$ $D\pi/DK$ SCATTERING [YEO, THOMAS, WILSON; 2024]

- $D\pi/DK$ at the $SU(3)$ symmetric point
- Global flavour $SU(3)$ symmetry decomposes into $\overline{\mathbf{3}}, \mathbf{6}, \overline{\mathbf{15}}$ sectors
- three volumes, $M_\pi \approx 700$ MeV
- amplitudes presented as $\rho^2|t|^2$ as a function of E_{cm}
- more in Daniel's talk [Yeo; Thu 09:40]



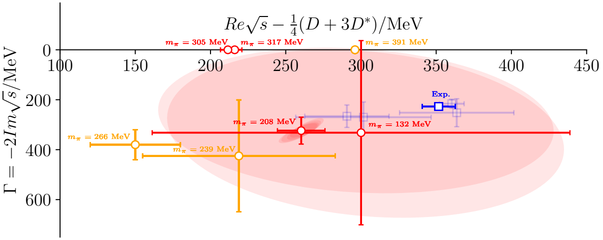
$\overline{\mathbf{3}}$: deeply bound state, $D^*_{s0}(2317)$



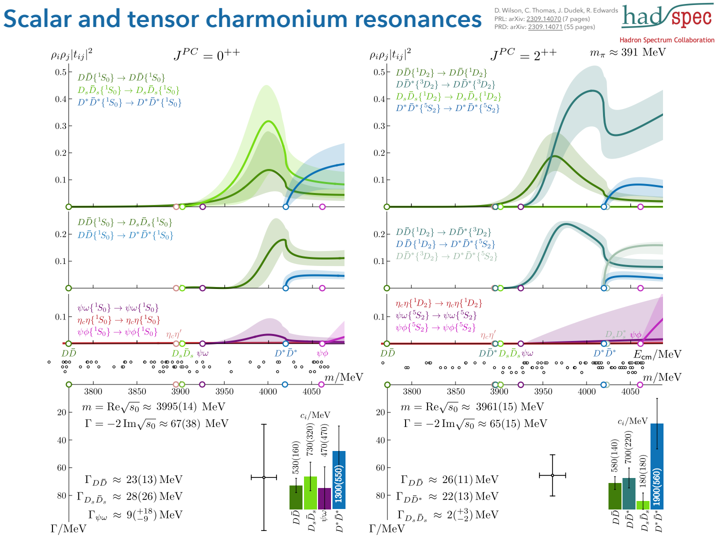
$\mathbf{6}$: virtual bound state at 2510 — 2610 MeV

Dπ SCATTERING, $D_0^*(2300)$ RESONANCE [YAN, LIU, LIU, MENG, XING; 2024]

- single-channel $\mathrm{I} = 1/2$ $\mathrm{D}\pi$ scattering
- 4 pion masses $m_\pi = 132 - 317$ MeV
- comparison to pole positions of earlier works [Mohler et al., PRD 13], [Moir et al., JHEP 16], [Gayer et al., JHEP 21]
- (virtual) bound states for $m_\pi \gtrsim 300$ MeV
- resonance for $m_\pi \lesssim 270$ MeV

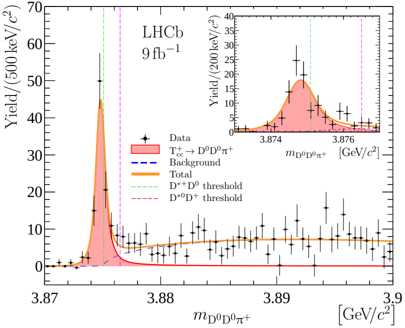


Charmonium χ_{c0}, χ_{c2} resonances [Wilson, Thomas, Dudek, Edwards; PRD 24], [PRL 24]



slide: David Wilson

- Experiment found new states, puzzles, possible exotics in charmonium energy region
- very challenging theoretically, high density of channels
- Lüscher method (and extensions) continue to work well
- Interactions in quark disconnected channels (e.g. $J/\Psi - \omega$) found to be small
- Interactions in quark-line connected channels (e.g. $D\bar{D}$, $D_s\bar{D}_s$) are larger
- A single resonance pole is found in 0^{++} and 2^{++} coupled to multiple channels
- So far unobserved (but not unexpected) 2^{-+} and 3^{++} states are also found
- more in David's talk [Wilson; Thu 09:00]



[LHCb, Nature Physics (2022)]

- Tetraquark state very close to D^*D threshold
- D^* is a narrow resonance
  - $M_{D^{*+}} \sim 2$ GeV
  - $\Gamma_{D^{*+}} \sim 80$ keV

$\Rightarrow$ for $M_\pi \gtrsim 150$ MeV, D^* will be stable

- $M_{T_{cc}^+} - (M_{D^{*+}} + M_{D^0}) = -0.27(6)$ MeV
- $D^0D^0\pi^+$ is the only decay mode ever seen
- $M_{T_{cc}^+} - (M_{D^{*0}} + M_{D^+}) = -1.68$ MeV

LEFT-HAND CUT, T_{cc}

- T_{cc} pole position lies close to the left-hand branch cut associated with one-pion exchange
- exponentially suppressed FV effects can be relatively large near the left-hand cut
- problem first encountered in H-dibaryon spectrum [Green et al.; PRL 21]
- modification of 2-particle formalism being more careful with on-shell projection of intermediate states

  [Raposo, Hansen; 2023]
- more details in André's talk

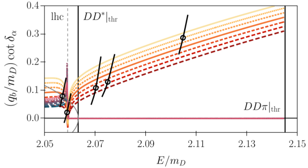
  [Raposo; Thu 11:50]

- 3-particle formalism automatically handles the problem T_{cc}

  [Hansen, Romero-López, Sharpe; JHEP 24]

- Three-body analysis of $T_{cc}^{+}(3875)$

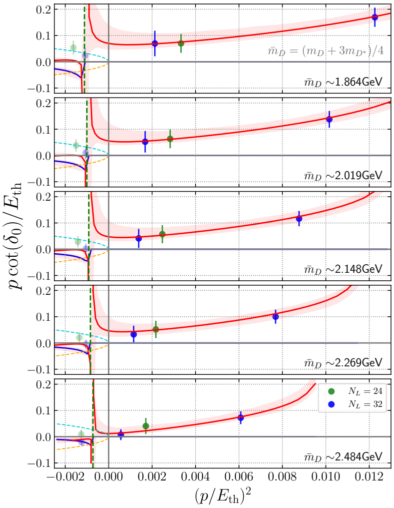
  [Dawid, Mon 15:35]

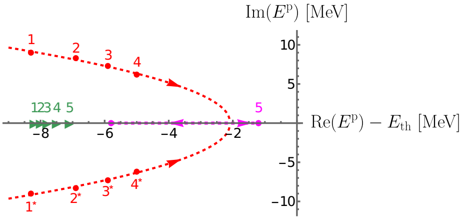


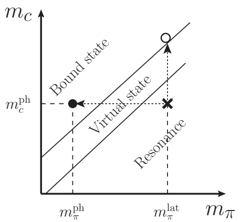
- more on the left-hand cut and the HALQCD method in talk by Sinya

  [Aoki; Thu 11:30]

QUARK MASS DEPENDENCE OF T_{cc} [Collins, Nefediev, Padmanath, Prelovsek; PRD 24]

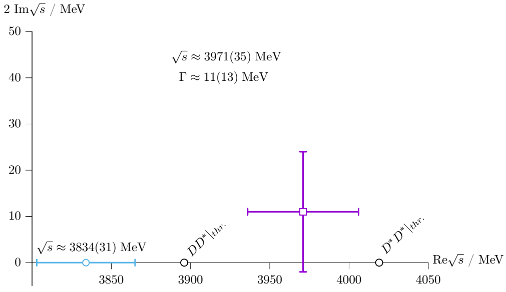


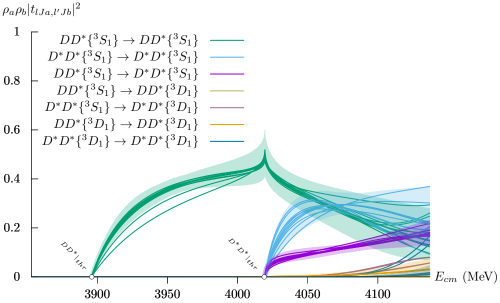




- DD^* scattering analysis at $m_\pi = 280$ MeV
- 5 values of m_Q, bracketing M_D
- S-wave fit to Lippman-Schwinger equation with EFT potential, taking left-hand cut into account
- resonance at m_c turning bound state for higher m_Q
- M_π dependence known from earlier T_{cc} lattice calculations [Padmanath, Prelovsek; PRL 22] [Chen et al.; PLB 22] [Lyu et al.; PRL 23]
- more in Sasa's talk [Prelovsek; Mon 14:55]

COUPLED-CHANNEL DD^*, D^*D^* SCATTERING, T_{cc} [WHYTE, WILSON, THOMAS; 2024]





- two poles observed in S-wave
  - a virtual bound state T_{cc}
  - a resonance T'_{cc} below D^*D^* threshold
- T'_{cc} predominatly couples to D^*D^* S-wave channel

- resonance pole enhances DD^* S-wave amplitude

$\Rightarrow$ cusp at D^*D^* threshold

- more in Travis' talk [Whyte; Thu 10:00]

T_{cc} RELATED TALKS [Mon 14:15 - 16:15] [Thu 09:00 - 11:00]

Quark mass dependence of doubly heavy tetraquark binding — *William Parrott* — 14:15 - 14:35

Strong decay of double charm tetra quark T_cc — *Subhasish Basak* — 14:35 - 14:55

Towards quark mass dependence of Tcc — *Sasa Prelovsek* — 14:55 - 15:15

T_{cc} via plane wave approach and including diquark-antidiquark operators — *Ivan Vujmilović* — 15:15 - 15:35

Three-body analysis of the tetraquark $T_{cc}^+(3875)$ — *Sebastian Dawid* — 15:35 - 15:55

Lattice QCD study of Ξ_{cc}-Ξ_{cc} interactions on the physical point — *Takumi Doi* — 15:55 - 16:15

$X(3872)$ relevant $D\bar{D}^*$ scattering in $N_f=2$ lattice QCD — *Chunjiang Shi* — 10:40 - 11:00

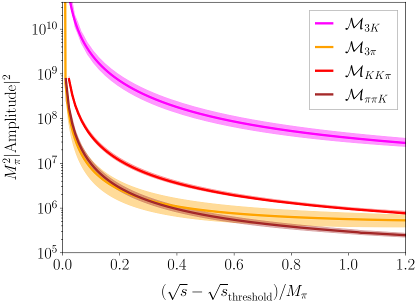
Near-threshold states in coupled $DD^{\ast}-D^{\ast}D^{\ast}$ scattering from lattice QCD — *Travis Whyte*

Extraction of the S and P wave DD* scattering phase shifts using twisted boundary conditio... — *Masato Nagatsuka*

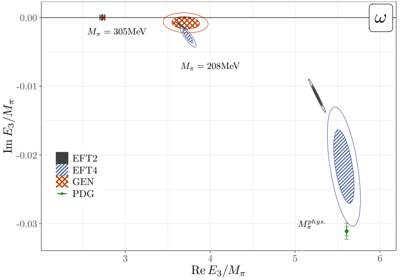
A lot more detail on T_{cc} and other exotic states in Nilmani's plenary [Mathur, Sat 09:00]

PROGRESS ON 3-PARTICLE SCATTERING

- 3-particle formalism has been extended to multiple channels of nondegenerate particles, $\eta\pi\pi + K\bar{K}\pi$ [Draper, Sharpe; JHEP 24] [Sharpe, Mon 12:15]
- Three neutrons in a finite volume [Schaaf, Tue 11:35]
- Relativistic-field-theory finite-volume formalism across all three-pion isospins [Alotaibi, Mon 11:55]



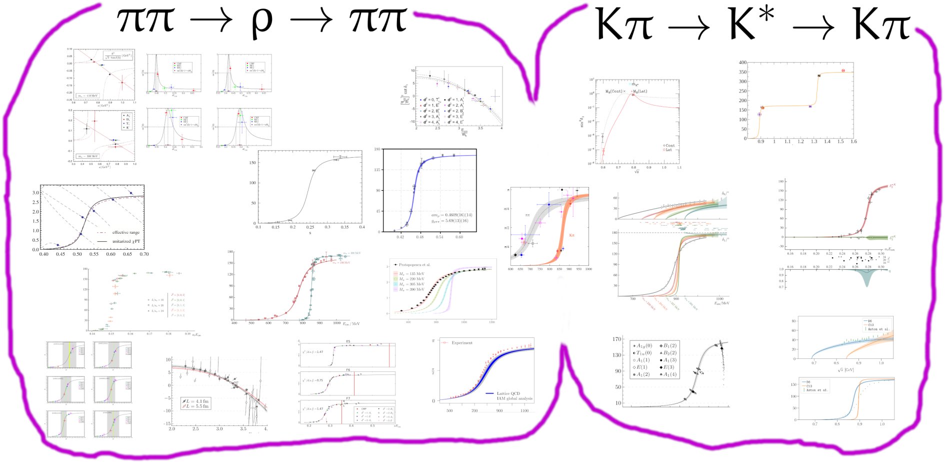
- Three-meson scattering amplitudes with physical quark masses [Romero-López, Mon 12:35]



- The ω meson from $\pi\pi\pi$ scattering [Yan et al.; arxiv:2407.16659] [Yan, Mon 12:55]

$K^*(892)$ & $\rho(770)$ at m_π^{phys}

HISTORY



[Aoki et al.; 07] [Aoki et al.; 11] [Feng et al.; 11] [Lang et al.; 11] [Fu, Fu; 12] [Pelissier, Alexandrou; 13] [Prelovsek et al.; 13] [Dudek et al.; 14]

[Wilson et al.; 15] [Wilson et al.; 15] [Bali et al.; 16] [Bulava et al.; 16] [Fu et al.; 16] [Alexandrou et al.; 17] [Andersen et al.; 18] [Brett et al.; 18]

[Wilson et al.; 19] [FE et al.; 20] [Werner et al.; 20] [Fischer et al.; 20] [Rendon et al.; 20]

$K^*(892)$ & $\rho(770)$ AT m_π^{phys} [BOYLE, FE, GÜLPERS, HANSEN, JOSWIG, LACHINI, MARSHALL, PORTELLI; 2024]

Physical-mass calculation of $\rho(770)$ and $K^*(892)$ resonance parameters via $\pi\pi$ and $K\pi$ scattering amplitudes from lattice QCD

Peter Boyle,[1,2] Felix Erben,[3,2] Vera Gülpers,[2] Maxwell T. Hansen,[2]
Fabian Joswig,[2] Nelson Pitanga Lachini,[4,2,*] Michael Marshall,[2] and Antonin Portelli[2,3,5]
[1]*Physics Department, Brookhaven National Laboratory, Upton NY 11973, USA*

[arXiv:2406.19193]

Light and strange vector resonances from lattice QCD at physical quark masses

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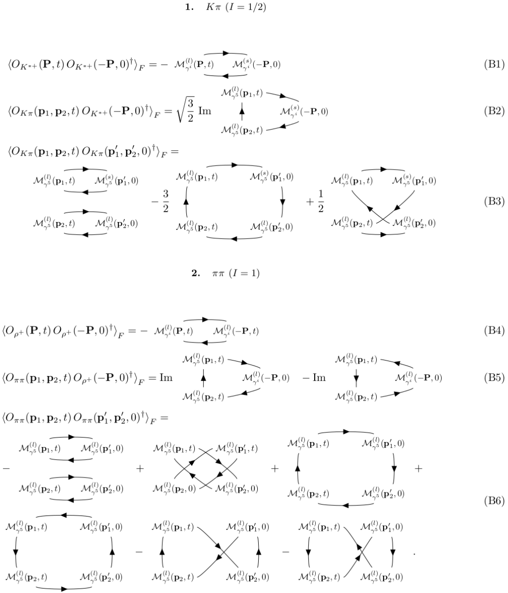
[1]*Physics Department, Brookhaven National Laboratory, Upton NY 11973, USA*
[2]*School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom*

[arXiv:2406.19194]

- computation of two of the most fundamanetal mesonic QCD resonances, $K^*(892)$ & $\rho(770)$
- at physical quark masses
- Domain-Wall fermions
- lattice spacing $a^{-1} \sim 1.7$ GeV
- full error budget for *statistical* and *systematic* errors
- all details in **Nelson's parallel talk** [Lachini, Mon 11:35]

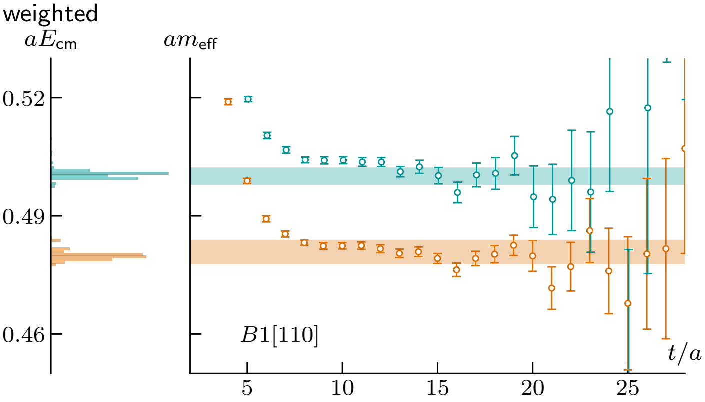
DISTILLATION [PEARDON ET AL.; PRD 09] [MORNINGSTAR ET AL.; PRD 11] CODE



GRID Hadrons

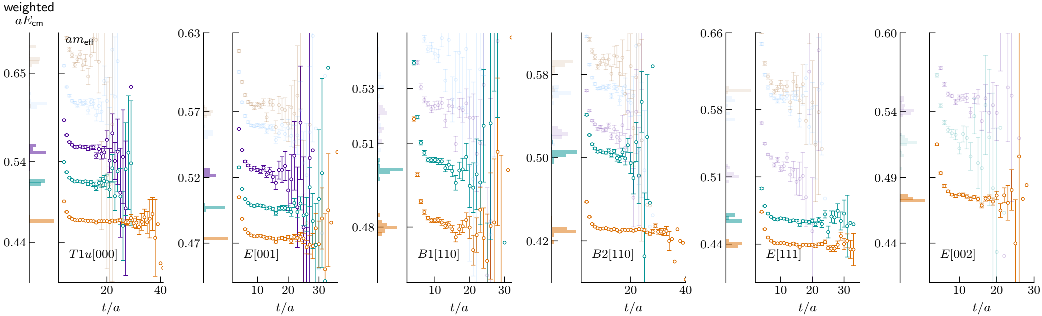
[github:paboyle/grid] [github:aportelli/hadrons]

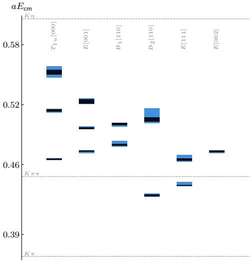
- Distillation code written for Grid and Hadrons
  - free & open source
  - code documentation: [Hadrons/MDistil]
  - exact distillation, $N_\nu = 64$, $N_c = 90$, all $N_t = 96$ source times
- Correlator data published on CERN Document Server [CDS, 2024]
  - easy mapping to paper notation



- data-driven error estimate
- histogram weighted by AIC

FV spectrum $K\pi$





- full FV spectrum determined, including *statistical* and *systematic* error

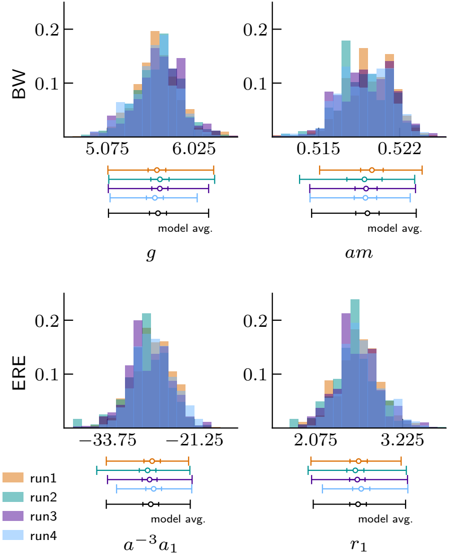
DATA-DRIVEN SYSTEMATIC ERROR $K\pi$

- next step: solve Lüscher condition $\cot[\delta(E)] = -\cot[\phi(E)]$, fit to phase-shift model
- 13 individual energy levels, hundreds of fit variations each
  - naive propagation of systematic error unfeasible

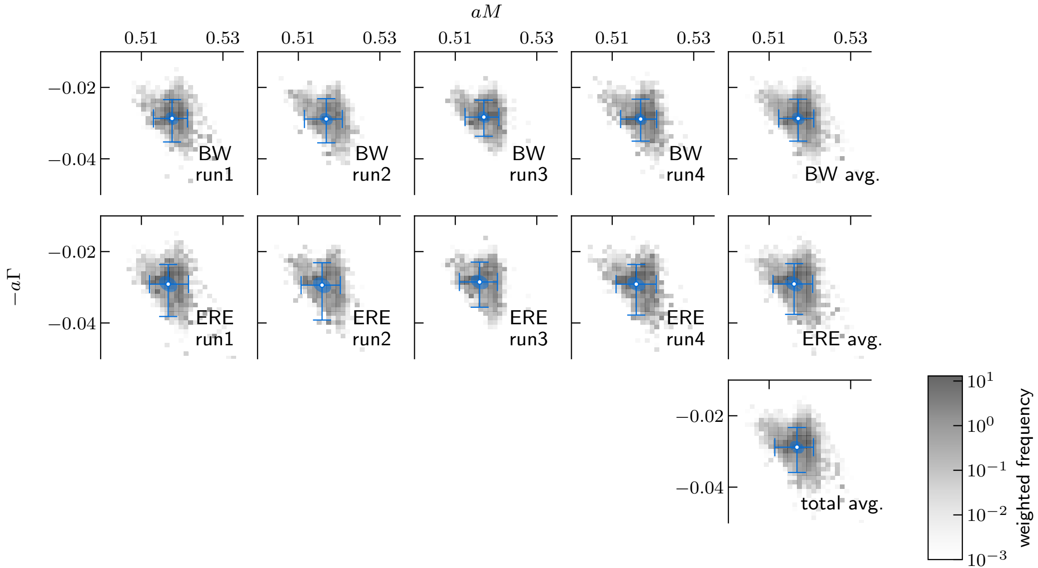
$\Rightarrow$ *sample* one level each, AIC determines likeliness to be drawn, fit δ

$\Rightarrow$ repeat this process 50,000 times

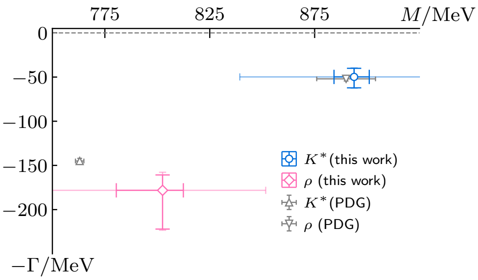
$\Rightarrow$ histogram for phase-shift parameters



Pole position Kπ



ρ, K^* AT M_π^{phys}: SUMMARY



- pole positions of ρ, K^*, including statstical and systematic error
- M_π^{phys}, single lattice spacing

$\Rightarrow$ faint error: naive esimation accounting for use of single lattice spacing

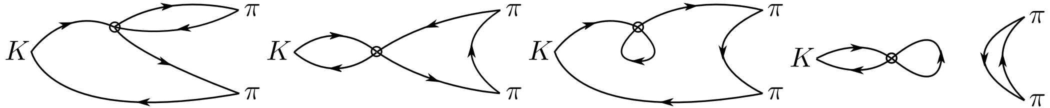
- Agreement with PDG at the level of $< 1\sigma$ for all but Γ_ρ, where agreement is at the level of 1.6σ

Why going through all this effort of using Domain-Wall Fermions at M_π^{phys}?

$\Rightarrow$ Answers in the remainder of the talk!

Electroweak matrix elements

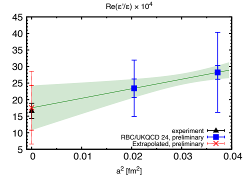
RBC/UKQCD K $\to \pi\pi$ [Abbott et al.; PRD 20]

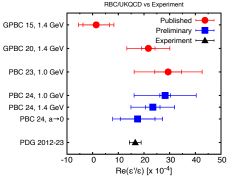


- One project where DWF at M_π^{phys} led to success: K $\to \pi\pi$
- Domain-Wall Fermions lead to **clean** 4**-quark-operator renormalization**
- long-standing puzzle $\text{Re}(A_0)/\text{Re}(A_2) = 22.45$ - factor 10 larger than perturbatively
- cancellation at M_π^{phys} between Wick contractions of $\text{Re}(A_2)$, **very sensitive to** m_l

$\Rightarrow$ $\text{Re}(A_0)/\text{Re}(A_2) = 19.9(2.3)(4.4)$

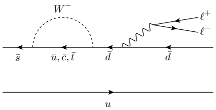
- Progress on direct Kaon CPV parameter ϵ'
  - second lattice spacing added to existing approach (G-parity boundary), presented by Chris [Kelly; Thu 09:00]
  - Peridoic boundary conditions, excited state from variational techniques, see talk by Masaaki [Tomii; Fri 12:35]

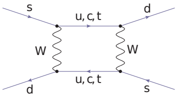


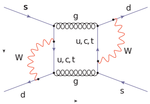


Electroweak matrix elements

- To extract long-range electroweak matrix elements, strongly-coupled intermediate multi-hadron states need to be understood
- Formalisms first developed for rare decays $K \to \pi \ell^+ \ell^-$ [Christ et al.; PRD 15] and Kaon mixing [Christ et al.; PRD 15]
- general formalism [Briceño et al.; PRD 20]







- applied in practice for $K \to \pi \ell^+ \ell^-$ [Christ et al.; PRD 16]

  [FE et al.; PRD 23]

- CP-violating parameter ϵ_K from kaon mixing

  [Bai et al.; PRD 24]

New developments at this conference:

- Split-even approach to $K \to \pi \ell^+ \ell^-$ [Hodgson; Fri 11:35]
- progress on $K_L \to \mu^+ \mu^-$ [Chao; Fri 11:55]
- $B \to \mu^+ \mu^- \gamma, B_s \to \phi \gamma$

  [Sanfilippo; Tue 16:15]

$\Sigma^+ \to p\ell^+\ell^-$ amplitude

$$\mathcal{A}_\mu^{rs} = \int d^4x \, \langle p, r | T[\mathcal{H}_W(x) J_\mu(0)] | \Sigma^+, s \rangle$$

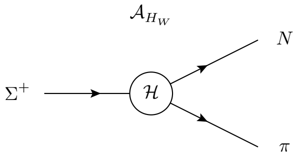
its FV estimator contains poles in volume L, when $E_n(L) \to E_\Sigma$

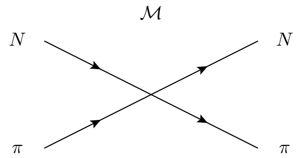
$$F_\mu(\mathbf{k}, \mathbf{p})_L = \sum_n \frac{C_{n,\mu}}{2E_n(L)\big(E_\Sigma - E_n(L)\big)} + \dots$$

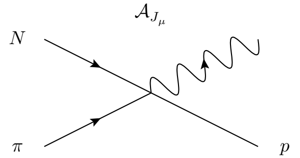
which must be cancelled *exactly* by FV correction

$$\tilde{\mathcal{A}}_\mu(k, p) = F_\mu(\mathbf{k}, \mathbf{p})_L + \Delta F_\mu(\mathbf{k}, \mathbf{p})_L$$

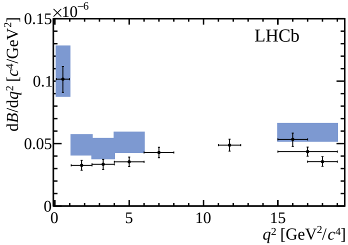
$$\Delta F_\mu(\mathbf{k}, \mathbf{p})_L = i\mathcal{A}_{J_\mu}(E_\Sigma, \mathbf{k}, \mathbf{p}) \mathcal{F}(E_\Sigma, \mathbf{k}, L) \mathcal{A}_{H_W}(E_\Sigma, \mathbf{k})$$



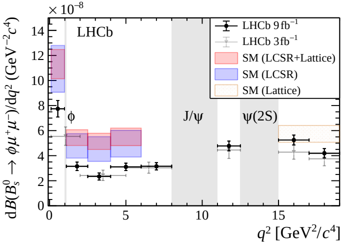




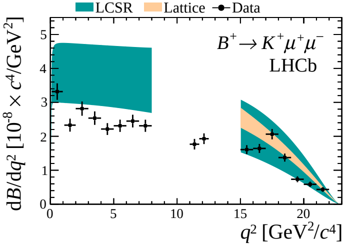
B-anomalies



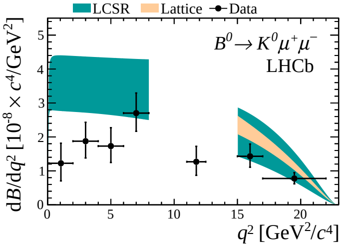
$B^0 \to K^{*0}$ [LHCb, JHEP 16]



$B_s \to \phi$ [LHCb, PRL 21]



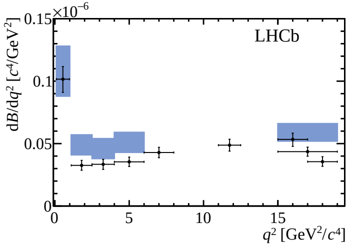
$B^+ \to K^+$ [LHCb, JHEP 14]



$B^0 \to K^0$ [LHCb, JHEP 14]

- most recent LHCb measurements of $b \to s\ell^+\ell^-$ branching fractions
- low-q^2: LCSR
- high-q^2: lattice
- experiment consitently lower than theory, particularly at low q^2
- tension reinforced by HPQCD $B \to K\ell^+\ell^-$
- can lattice contribute to $B \to K^*\ell^+\ell^-$?

B-ANOMALIES



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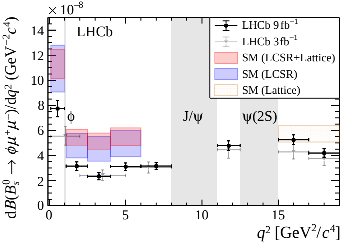

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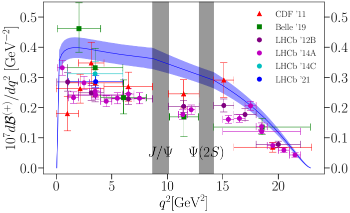
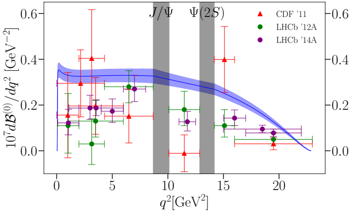


$B^+ \to K^+$ [Parrot et al.; PRD 23]



$B^0 \to K^0$ [Parrot et al.; PRD 23]

- most recent LHCb measurements of $b \to s\ell^+\ell^-$ branching fractions
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$1 + \mathcal{J} \to 2$ TRANSITIONS

- recipe for $1 + \mathcal{J} \to 2$ transitions:

  [Briceño, Dudek, Leskovec; PRD 21]
- "FV form factor" decomposition

$$\langle n, \mathbf{P}_f | J^\mu | \mathbf{P}_i \rangle_L = \frac{1}{L^3} \frac{1}{\sqrt{2E_i}} \frac{1}{\sqrt{2E_n}} \mathcal{K}^\mu \mathcal{F}_L$$

- optimized resonance state $\langle n, \mathbf{P}_f|$ from GEVP
- related to infinite-volume form factor via Lellouch-Lüscher factor

$$\mathcal{F}_L = \sqrt{-2E_n^*/\mu_0^{*\prime}}\, \mathbf{w}_0^T \mathcal{F}$$

- applied already to $\pi\gamma \to \pi\pi$ and $K\gamma \to K\pi$ transition amplitudes

  [Briceño et al.; PRD 16] [Alexandrou et al.; PRD 18]

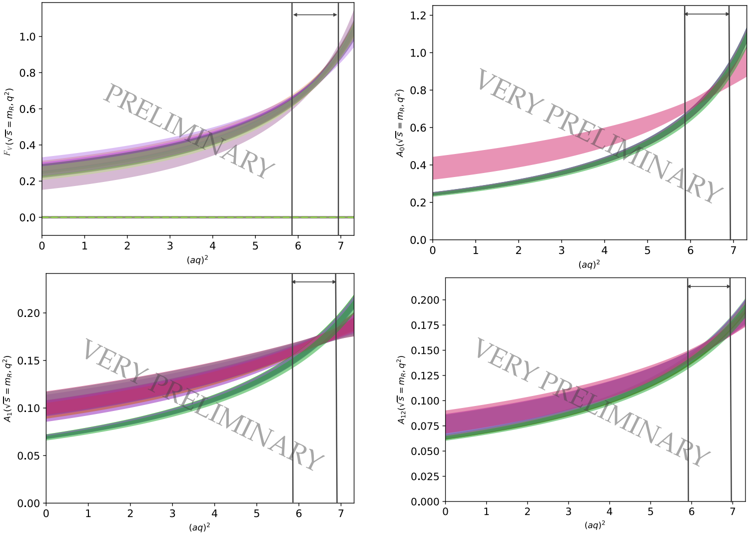
  [Radhakrishnan et al.; PRD 22]
- progress on 4 form factors of $B \to \rho\ell\nu$

$$\langle n; \mathbf{p}_{\pi\pi} | J_V | B, \mathbf{p}_B \rangle_L = \mathcal{C}_V^\mu F_V(L)$$

$$\langle n; \mathbf{p}_{\pi\pi} | J_A | B, \mathbf{p}_B \rangle_L = \sum_{i=0}^{2} \mathcal{C}_{A_i}^\mu F_{A_i}(L)$$

- similar approach with 3 extra tensor form factors for $B \to K^* \ell^+ \ell^-$

$B \to (\rho \to \pi\pi)\ell\nu$

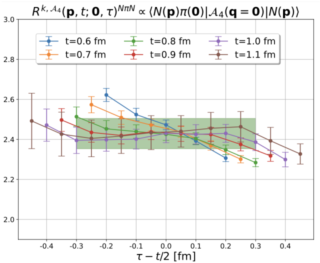


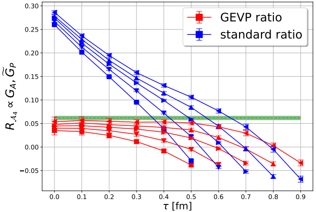
- plots by slide by Luka Leskovec, presented earlier this month

  [Leskovec, Lattice@CERN 24]

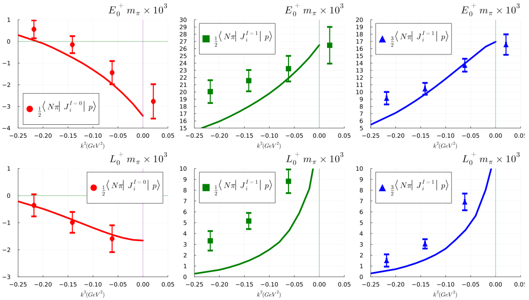
- vector form factor well described by many parametrizations
- axial form factors more dependent on parametrizations
- lattice data available between black lines

$N\gamma \to N\pi$





- Progress on computation of $\langle N\pi|\mathcal{J}|N\rangle$ matrix elements
  [Barca, Bali, Collins; 2405.20875]
- $a = 0.1$ fm, $M_\pi = 420$ MeV
- GEVP dramatically improves extraction of matrix elements
- Relevant for DUNE, Hyper-Kamiokande and other experiments

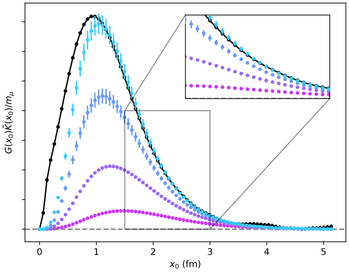


- independent effort by Xu Feng, Yu-Sheng Gao [Gao, Mon 14:35]
- two ensembles at M_π^{phys}, $a = 0.14$ fm
- comparison of lattice data to ANL-Osaka experimental data

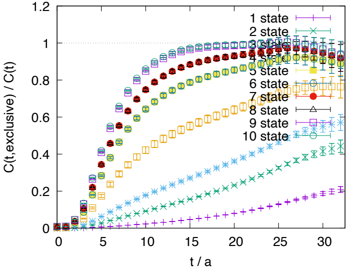
- $\pi\pi$ scattering states can reconstruct vector-vector correlator [Della Morte et al. 17] [Bruno et al. 19]

$$G(t)=\sum_{n=0}^{N}\langle E_n,L|J(t)|0\rangle e^{-E_n t}$$

- FV energy states $\langle E_n, L|$ constructed with GEVP
- very precise way to estimate long-distance contribution from the lattice
- addresses exponential signal-to-noise problem and useful to study volume dependence



Mainz: blinded a_μ^{HVP} integrand, $M_\pi = 132$ MeV, $a = 0.0635$ fm presented in [Miller, Tue 16:15] [Kuberski, Mon 12:35]



RBC/UKQCD: $G(t)$ saturation, $M_\pi = 131$ MeV, $a^{-1} = 2.69$ GeV presented in [Lehner, Mon 11:55] [McKeon, Mon 12:15]

More details on $g-2$ in Christine's plenary talk right after this one [Davies, Tue 09:45]

Outlook & Conclusions

Outlook

- Hadron scattering can play a crucial role in computations
- $K \to \pi\pi$ decays and $g-2$ are two examples of complete calculations controlling
  - continuum extrapolation
  - chiral extrapolation
  - full systematic error budget
- We have a powerful and extensive toolbox at our hand to study multi-hadron states
  - $2 \to 2$ scattering
  - $1 + \mathcal{J} \to 2$
  - $2 + \mathcal{J} \to 2$ [Briceño, Hansen; PRD 16] [Baroni et al.; PRD 19] [Briceño, Hansen, Jackura; PRD 20]
  - processes involving 3 particles
- As soon as electroweak matrix elements are involved, FLAG criteria become important

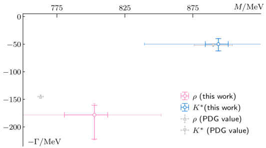
$\Rightarrow$ Ambitious, but not unreasonably difficult calculations like $B \to K^*\ell^+\ell^-$ form factors could be achieved soon

CONCLUSIONS FOR FV WORKFLOW

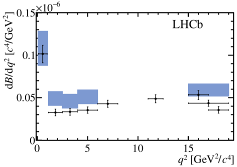
- many new and challenging hadron scattering calculations
- many new calculations and exploratory works involving electroweak matrix elements
- new calculation of K^* and ρ resonances at physical quark masses using Domain-Wall fermions

$\Rightarrow$ crucial milestone towards semileptonic $\mathrm{B} \to \mathrm{K}^* \ell^+ \ell^-$

$\Rightarrow$ potential avenue towards finding new physics in B anomalies



a long way



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Backup

Lüscher formalism and its extensions:

[Lüscher; 1986]

[Lüscher; 1991]

[Rummukainen, Gottlieb; 1995]

[Kim, Sachrajda, Sharpe; 2005]

[Christ, Kim, Yamazaki; 2005]

[He, Feng, Liu; 2005]

[Beane, Detmold, Savage; 2007]

[Tan; 2008]

[Leskovec, Prelovsek; 2012]

[Hansen, Sharpe; 2012]

[Briceño, Davoudi; 2012]

[Li, Liu; 2013]

[Briceño; 2014]

Generalized Eigenvalue Problem:

[Lüscher, Wolff; 1990]

[Blossier et al.; 2009]

[Fischer et al.; 2020]