QCD Anderson transition

with overlap valence quarks on a twisted-mass sea

Robin Kehr

Institute for Theoretical Physics,
Justus Liebig University Giessen

Lattice Liverpool,
July 29, 2024

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

HFHF
Helmholtz Forschungsakademie Hessen für FAIR

[R. Kehr, D. Smith, L. von Smekal, PhysRevD.109.074512]

Motivation

Fundamental transitions in QCD

- Chiral restoration
- Deconfinement

Open question

Is there a relation between both transitions?

Motivation

Fundamental transitions in QCD

- Chiral restoration
- Deconfinement

Open question

Is there a relation between both transitions?

- QCD Anderson transition related to both phenomena
- In this work: Focus on relation to chiral restoration

Motivation

Fundamental transitions in QCD

- Chiral restoration
- Deconfinement

Open question

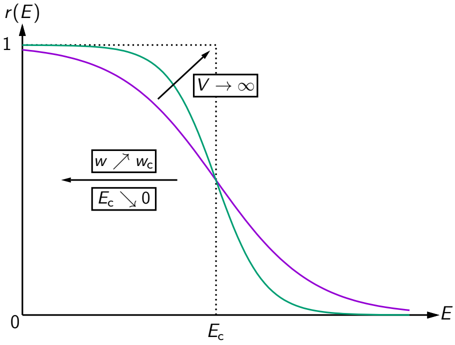
Is there a relation between both transitions?

- QCD Anderson transition related to both phenomena
- In this work: Focus on relation to chiral restoration

- Term *Anderson transition* originates from condensed matter physics
  [P. W. Anderson, 1958] [F. Evers, A. D. Mirlin, arXiv:0707.4378]
  - Describes metal-insulator transition in disordered solids
  - In metal phase low-lying eigenmodes of Hamiltonian delocalized
    $\Rightarrow$ Conductivity
  - Above critical disorder all eigenmodes localized
    $\Rightarrow$ No conductivity

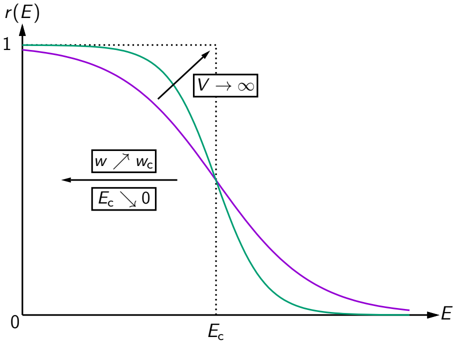
Anderson transition

- Delocalized modes separated from localized modes by energy threshold E_c (*mobility edge*)
- Above critical disorder strength w_c all modes localized



Anderson transition

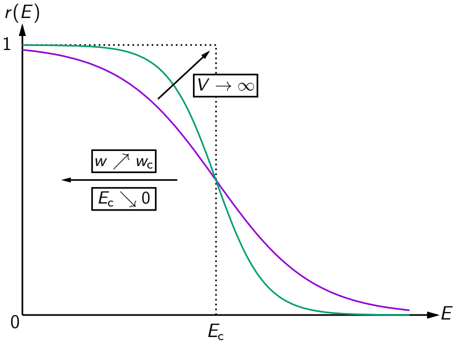
- Delocalized modes separated from localized modes by energy threshold E_c (*mobility edge*)
- Above critical disorder strength w_c all modes localized



- Analogous transition in QCD [M. Giordano, T. G. Kovács, arXiv:2104.14388]
  - Hamilton operator
    ↳ Dirac operator
  - Disorder strength
    ↳ Temperature

Anderson transition

- Delocalized modes separated from localized modes by energy threshold E_c (*mobility edge*)
- Above critical disorder strength w_c all modes localized



- Analogous transition in QCD [M. Giordano, T. G. Kovács, arXiv:2104.14388]
  - Hamilton operator
    ↳ Dirac operator
  - Disorder strength
    ↳ Temperature
  - Low-lying modes localized
  - Higher ones delocalized
  - Below T_0 all modes delocalized (no mobility edge)

Relation to . . .

. . . (de)confinement

- Eigenmodes tend to localize in sinks of Polyakov loop
  [L. Holicki, E.-M. Ilgenfritz, L. von Smekal, arXiv:1810.01130]
- Quenched QCD: T_0 coincides with deconfining phase transition
  [T. G. Kovács, R. Á. Vig, arXiv:1706.03562]

Relation to . . .

. . . (de)confinement

- Eigenmodes tend to localize in sinks of Polyakov loop
  [L. Holicki, E.-M. Ilgenfritz, L. von Smekal, arXiv:1810.01130]
- Quenched QCD: T_0 coincides with deconfining phase transition
  [T. G. Kovács, R. Á. Vig, arXiv:1706.03562]

. . . chiral symmetry restoration/breaking

- Previous work suggests $T_0 = T_{\text{pc}}$ (pseudocritical temperature of chiral crossover, pion mass $m_\pi \neq 0$)
- No Goldstone bosons in chiral limit, if near-zero modes localized
  [M. Giordano, arXiv:2206.11109]
  $\Rightarrow T_0 \geq T_{\text{c}}$ (temperature of chiral phase transition, $m_\pi \to 0$)

Relation to ...

...(de)confinement

- Eigenmodes tend to localize in sinks of Polyakov loop
  [L. Holicki, E.-M. Ilgenfritz, L. von Smekal, arXiv:1810.01130]
- Quenched QCD: T_0 coincides with deconfining phase transition
  [T. G. Kovács, R. Á. Vig, arXiv:1706.03562]

...chiral symmetry restoration/breaking

- Previous work suggests $T_0 = T_{\mathrm{pc}}$ (pseudocritical temperature of chiral crossover, pion mass $m_\pi \neq 0$)
- No Goldstone bosons in chiral limit, if near-zero modes localized
  [M. Giordano, arXiv:2206.11109]
  $\Rightarrow T_0 \geq T_{\mathrm{c}}$ (temperature of chiral phase transition, $m_\pi \to 0$)
- Near-zero modes produce chiral condensate (Banks-Casher relation)
  [T. Banks, A. Casher, 1980]

Mixed action setup

Chiral lattice fermions

- Compute low-lying eigenmodes of overlap operator:

$$D_{\text{ov}} = \frac{1+s}{a}\left(1 + \operatorname{sgn} K\right)$$

- Wilson kernel: $K = aD_{\text{W}} - (1+s)$
- Optimize locality with parameter s

Mixed action setup

Chiral lattice fermions

- Compute low-lying eigenmodes of overlap operator:

$$D_{\rm ov} = \frac{1+s}{a}\,(1 + \operatorname{sgn} K)$$

- Wilson kernel: $K = aD_{\rm W} - (1+s)$
- Optimize locality with parameter s

Gauge configurations

- From *twisted mass at finite temperature* collaboration
  [F. Burger, E.-M. Ilgenfritz, M. P. Lombardo, A. Trunin, arXiv:1805.06001]
  - Twisted mass Wilson fermions at maximal twist, Iwasaki gauge action
  - $N_{\rm f} = 2+1+1$: two degenerate light, physical strange & charm quarks
  - $T_{\rm pc}$ from disconnected chiral susceptibility
  - Lattice spacing a from nucleon mass [C. Alexandrou et al., arXiv:1406.4310]

Overview of configurations

Set of ensembles	N_s	N_t	$T\,/\,T_{pc}$
A370 $a = 0.0936(13)$ fm $m_\pi = 364(15)$ MeV $T_{pc} = 185(8)$ MeV	24	4	2.85(13)
		5	2.28(10)
		6	1.90(9)
		7	1.63(7)
		8	1.42(6)
		9	1.27(6)
		10	1.14(5)
		11	1.04(5)
		12	0.95(4)
	32	13	0.88(4)
		14	0.81(4)
D370 $a = 0.0646(7)$ fm $m_\pi = 369(15)$ MeV $T_{pc} = 185(4)$ MeV	32	3	5.50(13)
		6	2.75(7)
		14	1.18(3)
		16	1.03(2)
	40	18	0.92(2)
	48	20	0.83(2)
D210 $a = 0.0646(7)$ fm $m_\pi = 213(9)$ MeV $T_{pc} = 158(5)$ MeV	48	4	4.83(16)
		6	3.22(11)
		8	2.42(8)
		10	1.93(6)
		12	1.61(5)
		14	1.38(5)
		16	1.21(4)
		18	1.07(4)

- N_s: Number of lattice sites in each space direction
- Volume $V = L^3$:

$$L = aN_s$$

- N_t: Number of lattice sites in temporal direction
- Temperature:

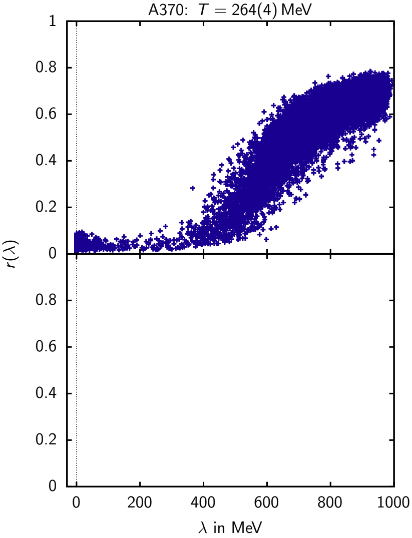
$$T = \frac{1}{aN_t}$$

Localization measure

- Relative volume of eigenmode to eigenvalue λ:

$$r(\lambda) = \frac{P^{-1}(\lambda)}{N_s^3 N_t}$$

- $P(\lambda)$: *inverse participation ratio*



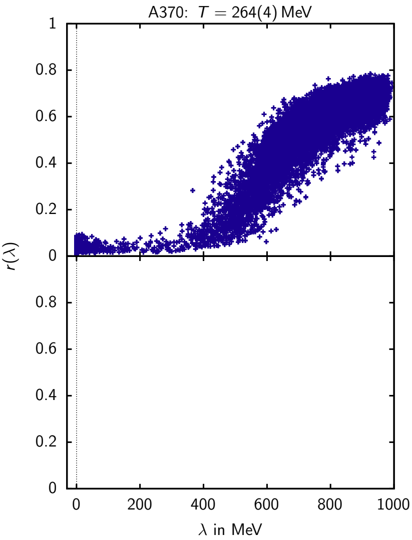
Localization measure

- Relative volume of eigenmode to eigenvalue λ:

$$r(\lambda) = \frac{P^{-1}(\lambda)}{N_s^3 N_t}$$

- $P(\lambda)$: *inverse participation ratio*

- Inflection point λ_c as estimate for mobility edge



Localization measure

- Relative volume of eigenmode to eigenvalue λ:

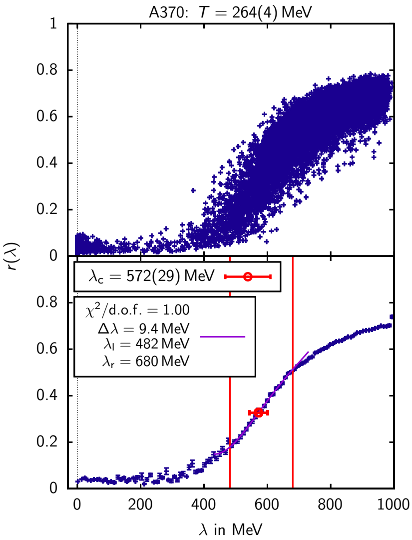
$$r(\lambda) = \frac{P^{-1}(\lambda)}{N_s^3 N_t}$$

- $P(\lambda)$: *inverse participation ratio*

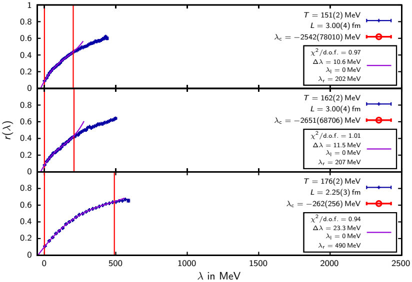
- Inflection point λ_c as estimate for mobility edge

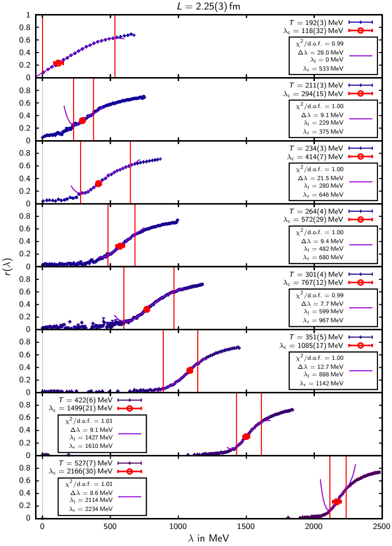
Fit Taylor polynomial

$$r(\lambda) = r_c + b\,(\lambda - \lambda_c) + 0\,(\lambda - \lambda_c)^2 + c\,(\lambda - \lambda_c)^3 + d\,(\lambda - \lambda_c)^4$$



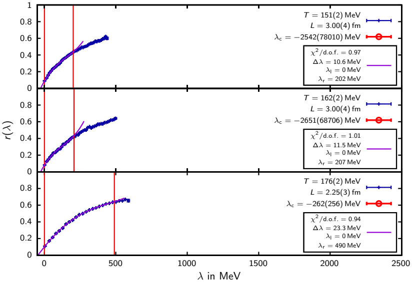
A370: $a = 0.0936(13)$ fm, $m_\pi = 364(15)$ MeV

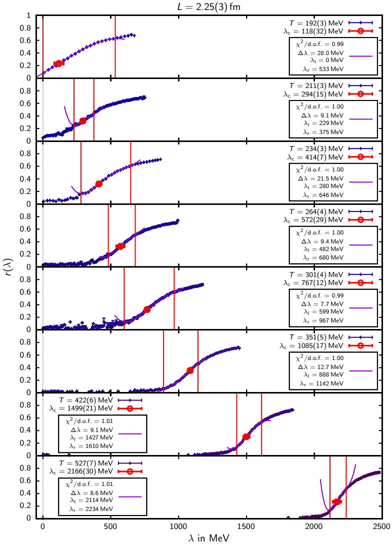




- Mobility edge vanishes around $T_{\text{pc}} = 185(8)$ MeV

A370: $a = 0.0936(13)\,\text{fm}$, $m_\pi = 364(15)\,\text{MeV}$





- Mobility edge vanishes around
  $$T_{\text{pc}} = 185(8)\,\text{MeV}$$
- Consistent with earlier work
  [M. Giordano et al., arXiv:1410.8392]
  [L. Holicki, E.-M. Ilgenfritz, L. von Smekal, arXiv:1810.01130]

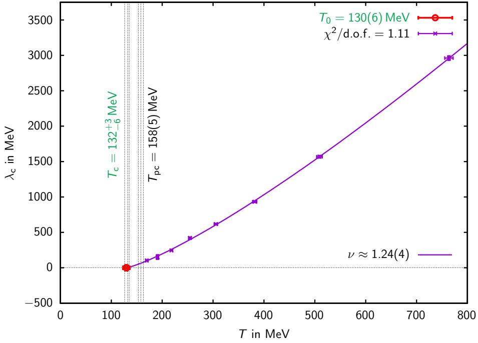
D210: $a = 0.0646(7)\,\text{fm}$, $m_\pi = 213(9)\,\text{MeV}$

- Reduce a, m_π, T_{pc}
- Increase volume: $L = 3.10(3)\,\text{fm}$

- Zero coincides with T_c [H.-T. Ding et al., arXiv:1903.04801]

Scaling fit

$$\lambda_c(T) = b\,(T - T_0)^\nu$$



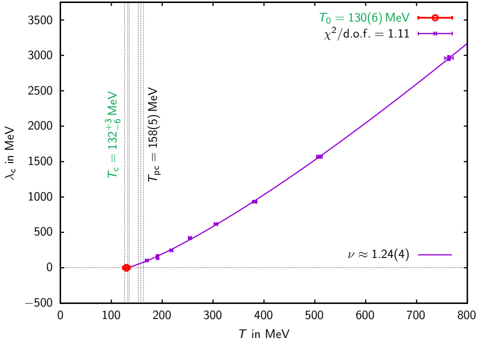
D210: $a = 0.0646(7)\,\mathrm{fm}$, $m_\pi = 213(9)\,\mathrm{MeV}$

- Reduce a, m_π, T_{pc}
- Increase volume: $L = 3.10(3)\,\mathrm{fm}$

- Zero coincides with T_{c} [H.-T. Ding et al., arXiv:1903.04801]
- $\nu \approx 1.44$ for unitary Anderson model [L. Ujfalusi, I. Varga, arXiv:1501.02147]

Scaling fit

$$\lambda_{\mathrm{c}}(T) = b\,(T - T_0)^{\nu}$$

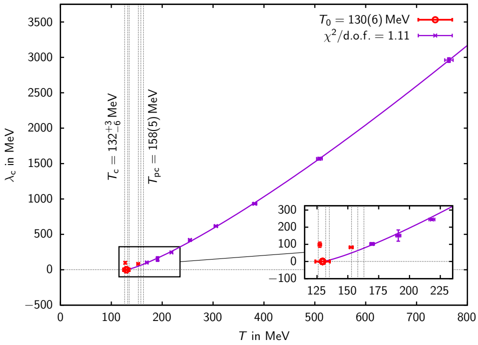


No vanishing at/above T_c?

- Include new data to $T \approx T_{pc}$ & $T \approx T_c$
- Inflection points at 83(3) & 98(15) MeV

Scaling fit

$$\lambda_c(T) = b\,(T - T_0)^{\nu}$$



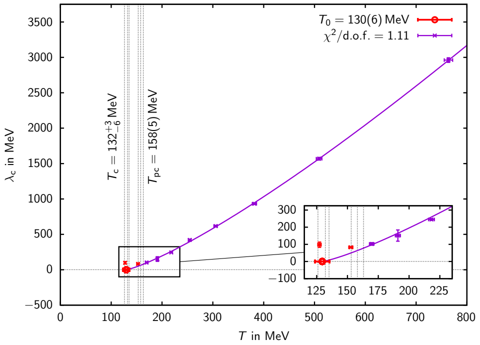
No vanishing at/above T_c?

- Include new data to $T \approx T_{pc}$ & $T \approx T_c$
- Inflection points at 83(3) & 98(15) MeV

- Tendency of IP being higher for T_c
- Even with gradient flow & better statistics
- Already seen for D370

Scaling fit

$$\lambda_c(T) = b\,(T - T_0)^\nu$$



Why?

- Systematic error of estimate via inflection point
- Lattice artifacts: finite volume, spacing, unphysical pion mass
- Probably does not explain qualitative behavior of mobility edge

Why?

- Systematic error of estimate via inflection point
- Lattice artifacts: finite volume, spacing, unphysical pion mass
- Probably does not explain qualitative behavior of mobility edge

Localization measure

- Modes with low relative volume might scale with L^d, where $d \in (0, 3]$
  $\Rightarrow$ Not localized
- Determine d [A. Alexandru, I. Horváth, arXiv:2103.05607]
  $\Rightarrow$ Second mobility edge $\lambda_{\mathrm{IR}} = 0$ above $T_{\mathrm{IR}} \in (200, 250)\,\mathrm{MeV}$

Why?

- Systematic error of estimate via inflection point
- Lattice artifacts: finite volume, spacing, unphysical pion mass
- Probably does not explain qualitative behavior of mobility edge

Localization measure

- Modes with low relative volume might scale with L^d, where $d \in (0, 3]$
  $\Rightarrow$ Not localized
- Determine d [A. Alexandru, I. Horváth, arXiv:2103.05607]
  $\Rightarrow$ Second mobility edge $\lambda_{\mathrm{IR}} = 0$ above $T_{\mathrm{IR}} \in (200, 250)\,\mathrm{MeV}$
- Modes below λ_{IR} delocalized, higher ones localized
- For lower temperatures λ_{IR} might rise and annihilate λ_c

Conclusion and outlook

- Inflection point of $r(\lambda)$ does not vanish at T_c
- Annihilation of both mobility edges possible scenario
  $\Rightarrow$ Without near-zero modes T_0 could still be at T_c
  $\Rightarrow$ More interesting quantity would be intersection point

Conclusion and outlook

- Inflection point of $r(\lambda)$ does not vanish at T_c
- Annihilation of both mobility edges possible scenario
  $\Rightarrow$ Without near-zero modes T_0 could still be at T_c
  $\Rightarrow$ More interesting quantity would be intersection point

- Estimate quality of inflection point via finite-size analysis for low N_t
- Configurations with physical pion masses available
  $\Rightarrow$ Reduce computational costs: UV-smoothing of configurations
- Determination of λ_{IR} computationally still very expensive
  $\Rightarrow$ Estimate intersection point by other means, e.g. via ULSD

Conclusion and outlook

- Inflection point of $r(\lambda)$ does not vanish at T_c
- Annihilation of both mobility edges possible scenario
  $\Rightarrow$ Without near-zero modes T_0 could still be at T_c
  $\Rightarrow$ More interesting quantity would be intersection point

- Estimate quality of inflection point via finite-size analysis for low N_t
- Configurations with physical pion masses available
  $\Rightarrow$ Reduce computational costs: UV-smoothing of configurations
- Determination of λ_{IR} computationally still very expensive
  $\Rightarrow$ Estimate intersection point by other means, e.g. via ULSD

Thank you!