# Low-Lying Spectrum of Two-Dimensional Adjoint QCD from the Lattice

William Jay, Manki Kim, Patrick Oare, Phiala Shanahan, Neill Warrington

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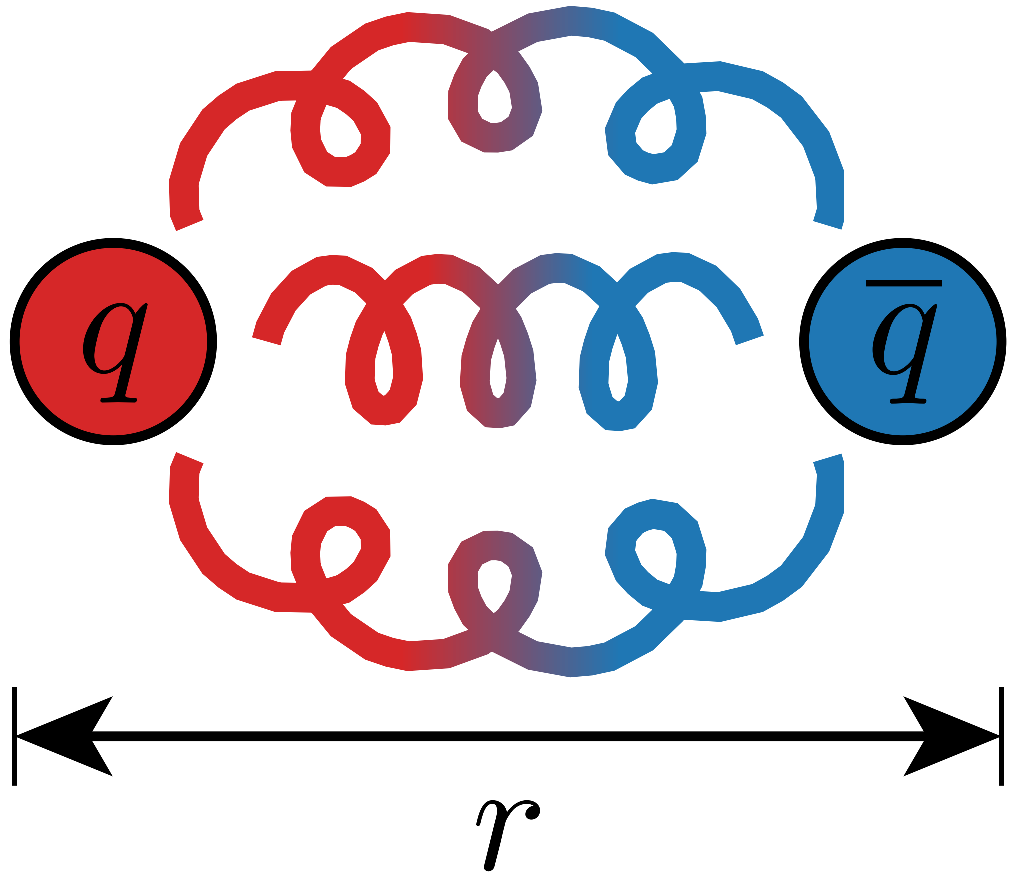
Lattice 2024, University of Liverpool

# Confinement in Gauge Theories

- A linear static quark potential $V(r)$ characterizes the confining phase of a gauge theory.
- With this definition, pure SU(N) gauge theory is confining.

$$V(r) =$$



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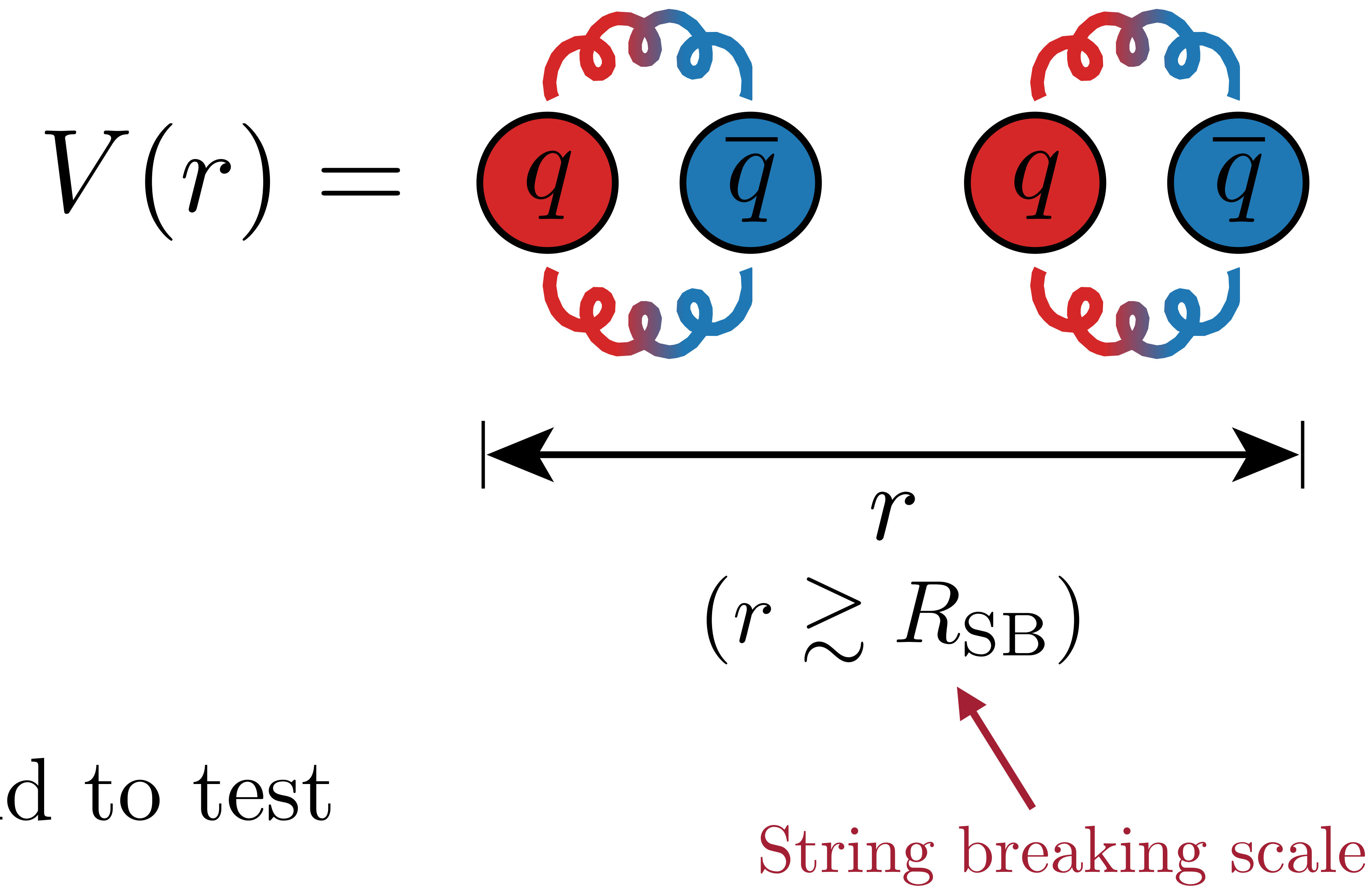

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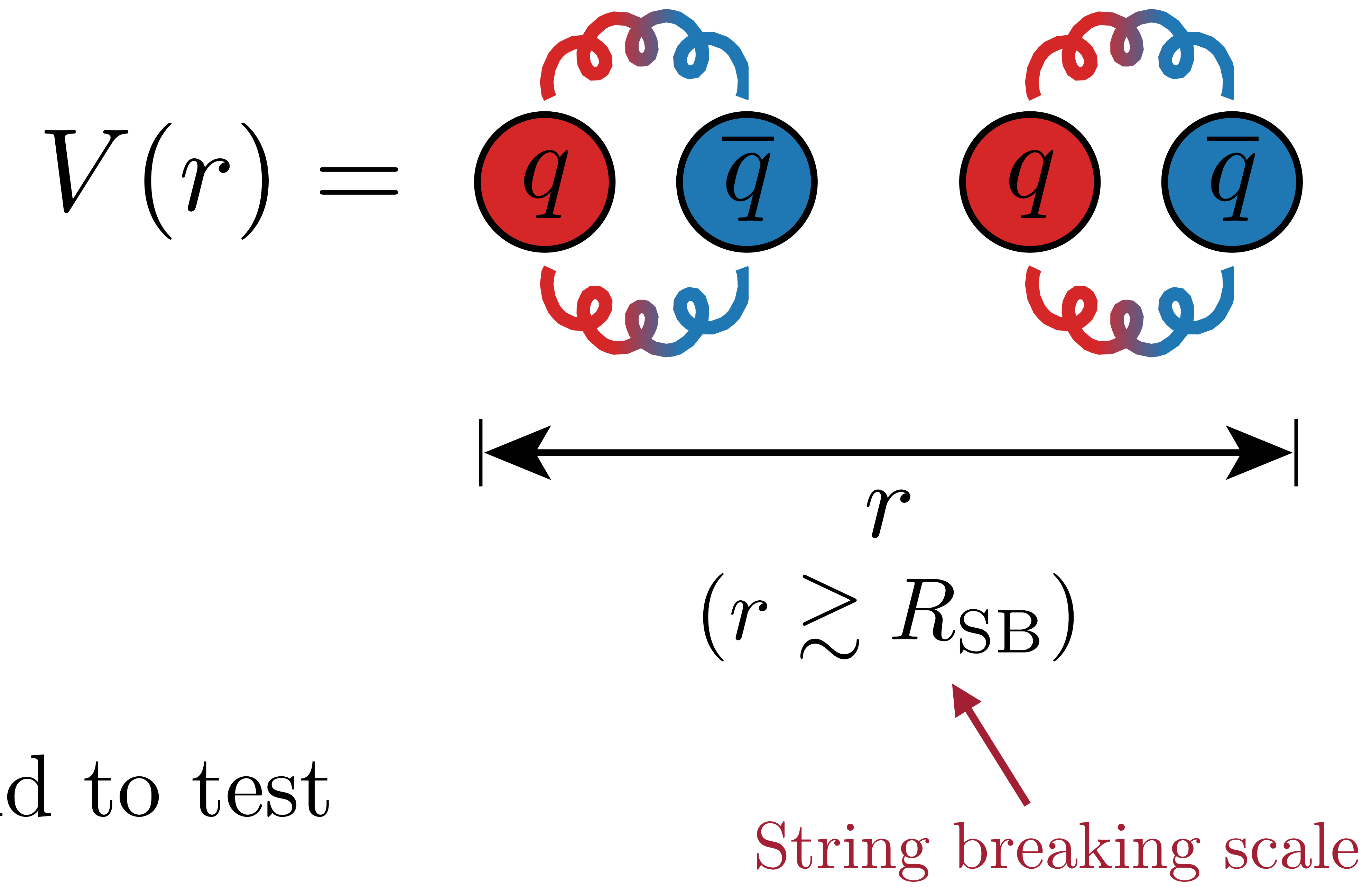
- QCD undergoes **string breaking** at large $r$.
  - Energetically favorable for sea quarks to bind to test charges and create a pair of mesons.
- How do adjoint fields influence the confining nature of a theory? Pure SU(N) gauge theory confines; what about theories with adjoint matter?



# Two-Dimensional Adjoint QCD ($QCD_2$)

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- The simplest two-dimensional theory with dynamical adjoint degrees of freedom is **two-dimensional adjoint QCD** ($QCD_2$).
  - Theory of a Majorana fermion coupled to an SU(N) gauge field in the adjoint representation with Minkowski action:

$$S_{\mathrm{QCD}_2}[\psi, G] = \int d^2x \, \mathrm{Tr}\left[\frac{1}{2g^2} G_{\mu\nu}(x)G^{\mu\nu}(x) + \overline{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x)\right]$$

Majorana fermion field
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- Toy model for confinement: confining for $m > 0$, deconfining for $m = 0$.

Z. Komargodski *et. al.* Symmetries and strings of adjoint QCD. *JHEP* 03 (2021) 103.

J. Donahue, *et. al.* Confining Strings, Infinite Statistics and Integrability. *Phys. Rev. D* **101** (2020) 8, 081901.

R. Dempsey *et. al.* Adjoint Majorana $QCD_2$ at Finite N. *JHEP* 04 (2023) 107.

(Computation of the spectrum with DLCQ at $N = 2,3,4$).

# $QCD_2$ on the lattice

G. Bergner, S. Piemonte, M. Ünsal
JHEP 07 (2024) 048.

- First study of **$QCD_2$** with LGT was published this year.
  - Computed Polyakov loop $\langle P \rangle$, static quark potential, chiral condensate, and string tension for a number of $(N, g, m)$.
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  - Used two discretizations: Wilson and reweighted overlap.
- Our calculation uses a Wilson discretization on a Euclidean $L \times T$ lattice, and the Wilson action for the gauge field $U_\mu(x)$.
  - Dirac operator:

adjoint links $V_\mu^{ab}(x) \equiv 2\,\mathrm{Tr}[U_\mu^\dagger(x) t^a U_\mu(x) t^b]$

$$D_W(x,y) = 1\,\delta_{x,y} - \kappa \sum_{\mu=1}^{2} \left[ V_\mu(x)(1-\gamma_\mu)\delta_{x+\hat{\mu},y} + V_\mu^T(y)(1+\gamma_\mu)\delta_{x-\hat{\mu},y} \right]$$

Hopping parameter $\kappa = \dfrac{1}{2m+4}$



I. Montvay. Majorana fermions on the lattice. hep-lat/0108011 (2001).

# LGT Setup ($N = 2$ colors)

| | $L$ | $T$ | $\beta$ | $\kappa$ | $N_{\text{cfgs}}$ |
|---|---|---|---|---|---|
| | 10 | 10 | 3.125 | 0.138889 | 90 |
| | 12 | 12 | 4.5 | 0.15 | 90 |
| | 14 | 14 | 6.125 | 0.159091 | 90 |
| | 16 | 16 | 8.0 | 0.166667 | 90 |
| | 18 | 18 | 10.125 | 0.173077 | 90 |
| ★ | 20 | 20 | 12.5 | 0.178571 | 90 |
| | 22 | 22 | 15.125 | 0.183333 | 90 |
| | 24 | 24 | 18.0 | 0.1875 | 76 |
| | 26 | 26 | 21.125 | 0.191176 | 58 |
| | 28 | 28 | 24.5 | 0.194444 | 45 |

# Majorana Fermions on the Lattice

$$\langle \mathcal{O}_1 \dots \mathcal{O}_k \rangle = \frac{1}{Z} \int DU\, D\psi\, D\overline{\psi}\, e^{-S_g[U] - \frac{1}{2}\int \overline{\psi} D_W \psi} \mathcal{O}_1 \dots \mathcal{O}_k$$

$$= \frac{1}{Z} \int DU\, e^{-S_g[U]}\, \mathrm{pf}\, D_W \langle \mathcal{O}_1 \dots \mathcal{O}_k \rangle_F$$

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- The Pfaffian $\mathrm{pf}\, D_W$ is *not necessarily positive*: how to interpret as a probability density for sampling?
  1. Check or prove that $\mathrm{pf}\, D_W > 0$.
  2. Reweight the probability measure.

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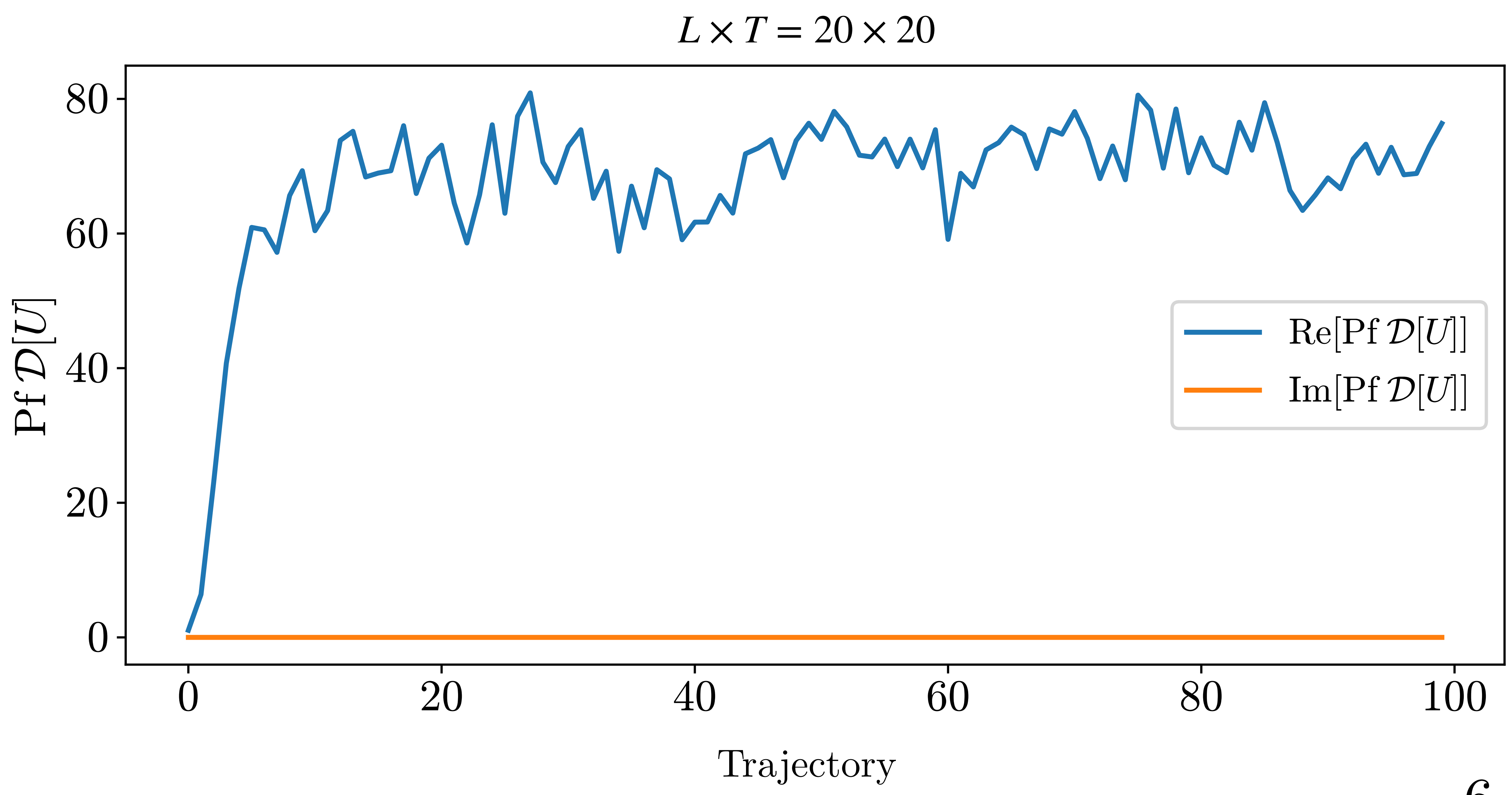
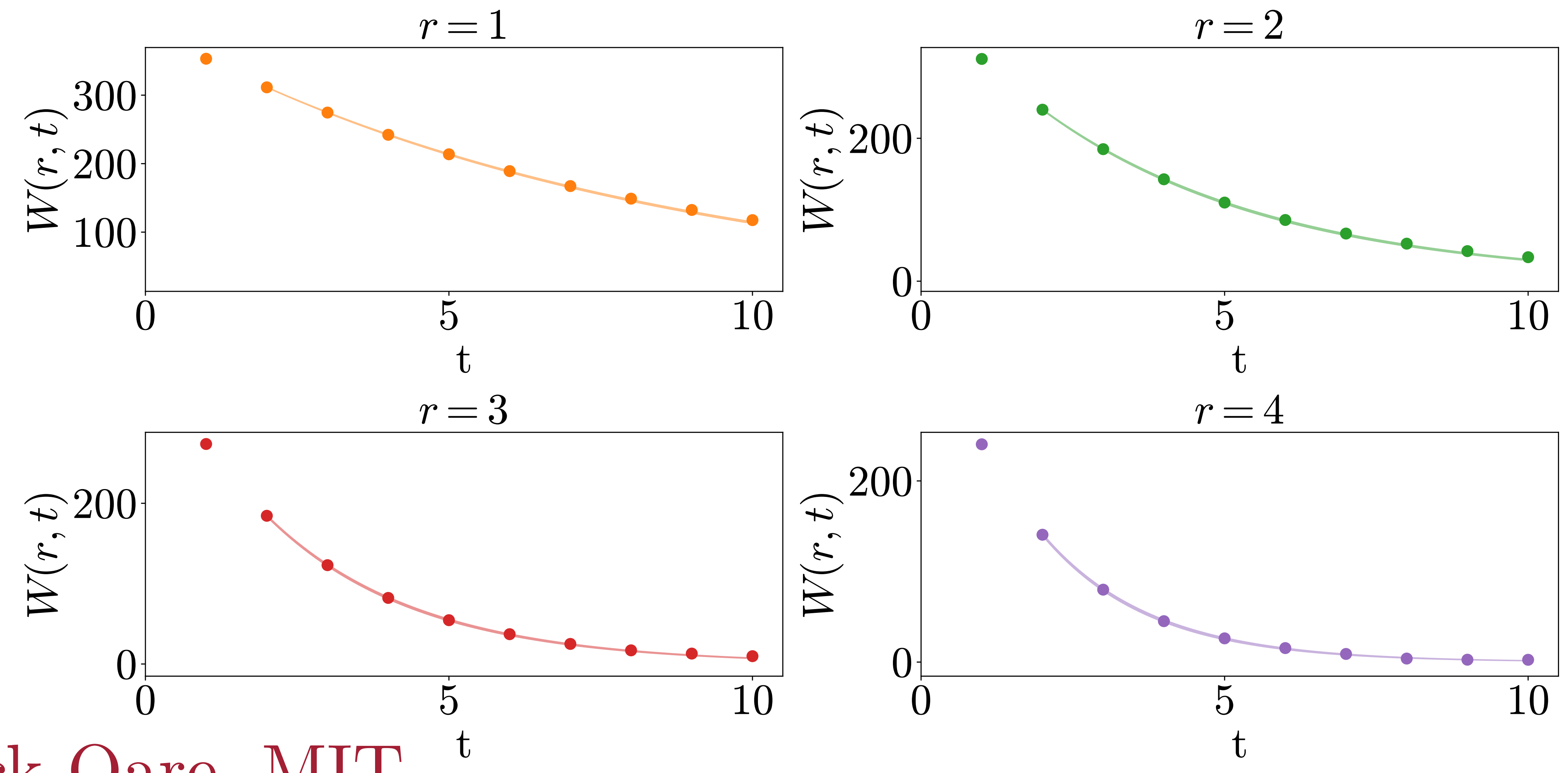


# Static Quark Potential

- The static quark potential $V(r)$ is extracted by fitting the $r \times t$ Wilson loop $W(r,t)$ to the functional form,

$$W(r,t) = Ce^{-V(r)t}$$



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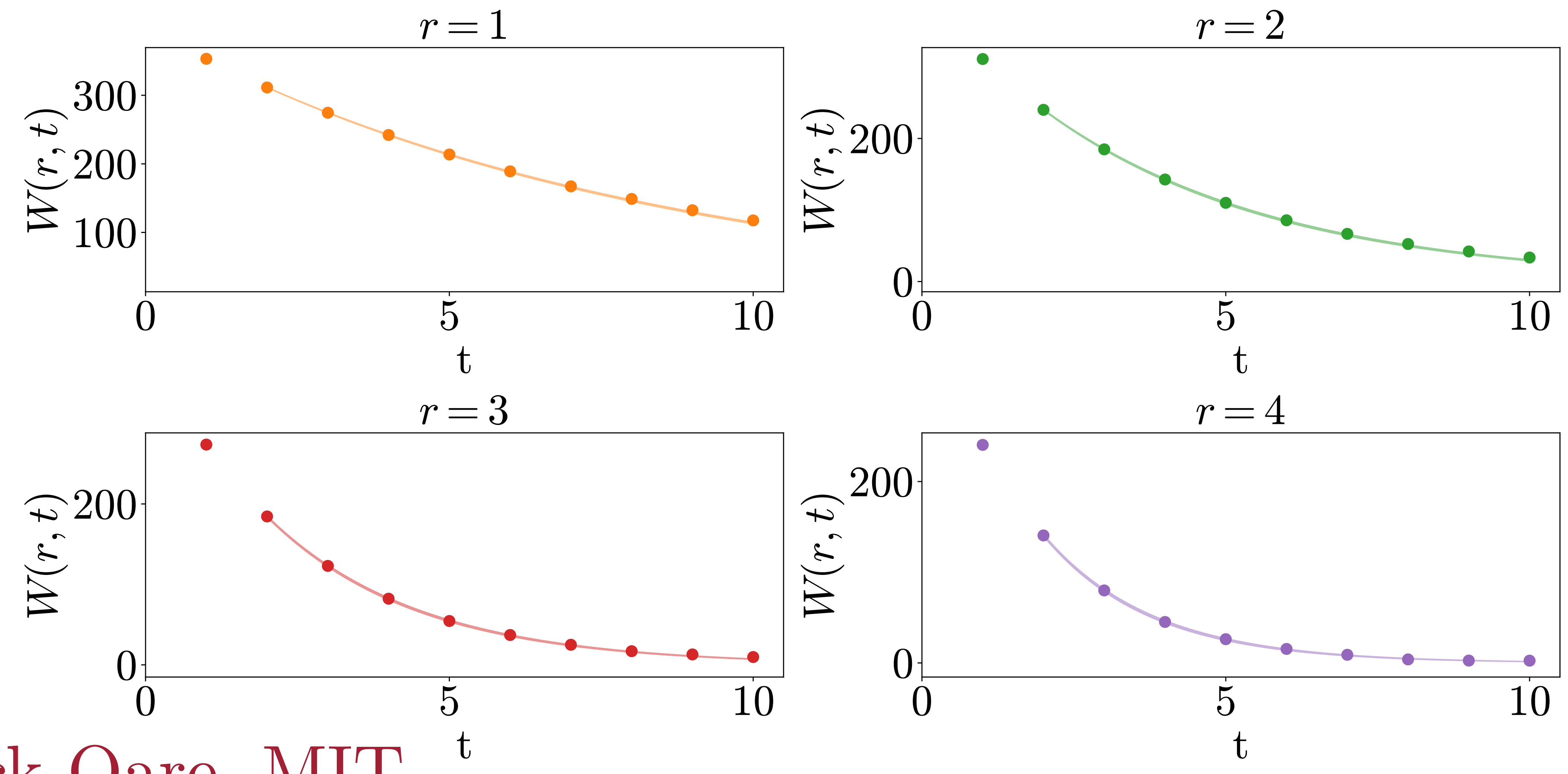
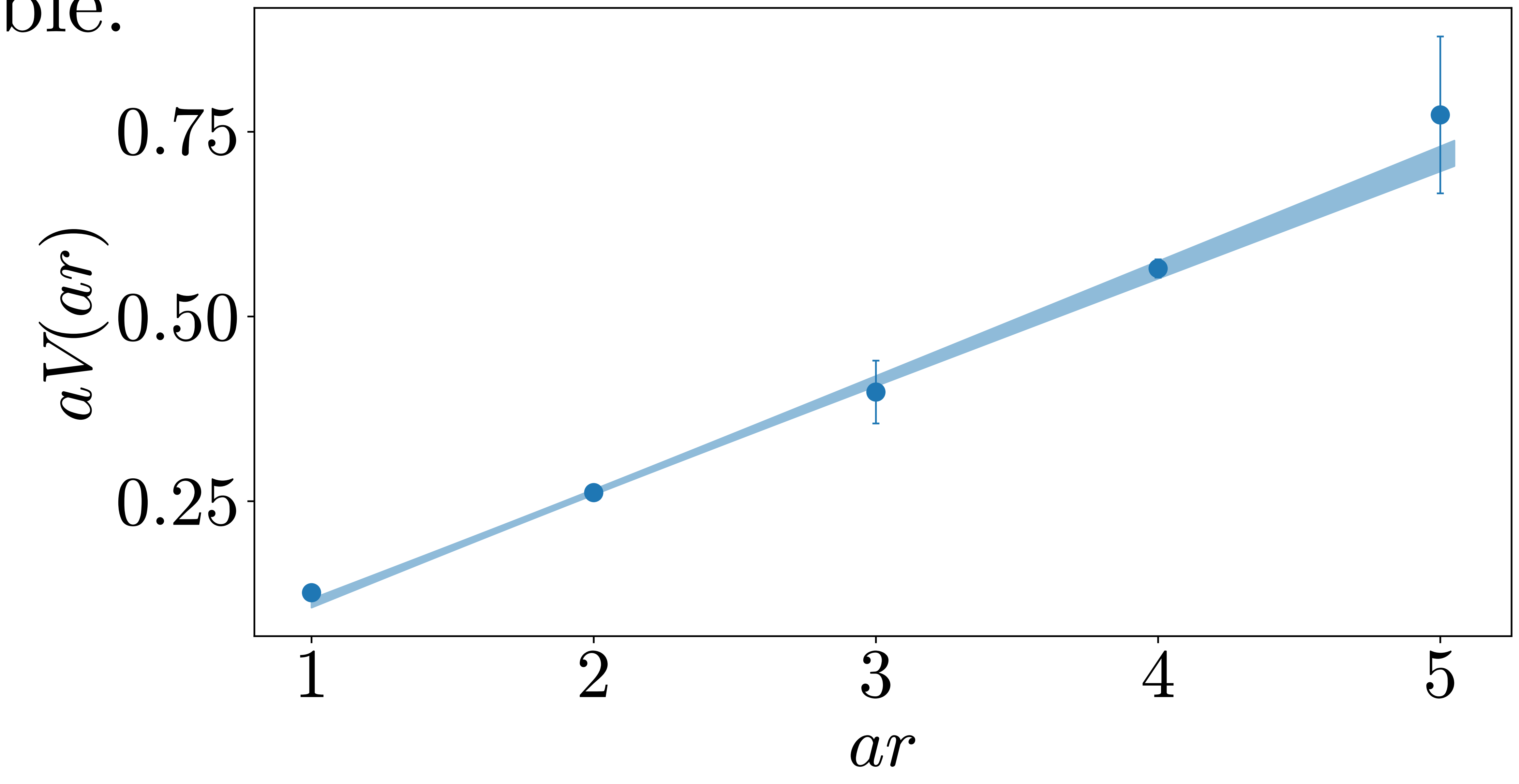
- $V(r)$ is fit to the linear ansatz $V(r) = A + \sigma r$ to extract the string tension $\sigma$.
  - $\sigma$ is used to set the scale of each ensemble.



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  - In $d = 2$, $\{1, \gamma^1\}$ are parity even, and $\{\gamma_5, \gamma^0\}$ are parity odd.

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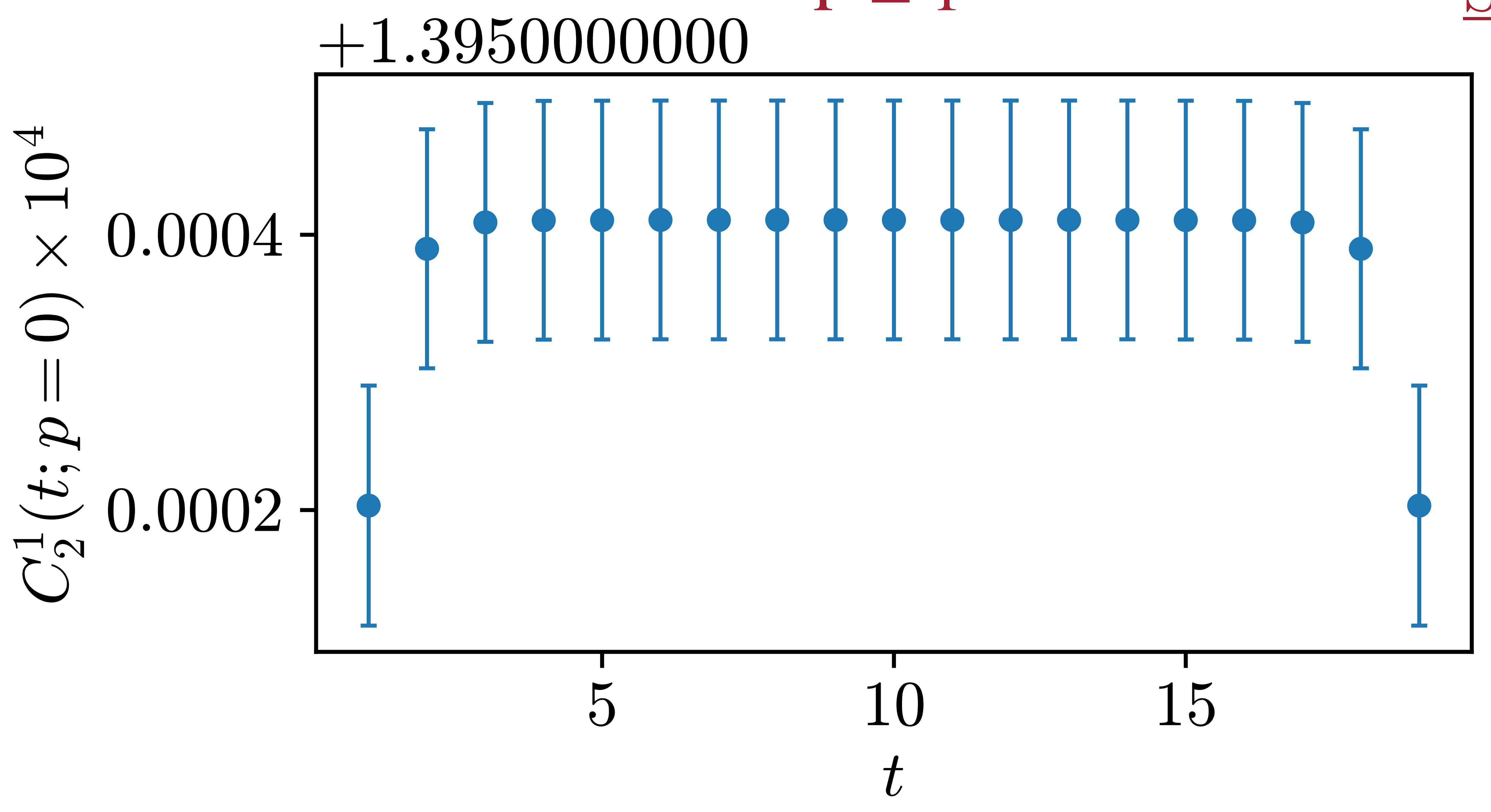
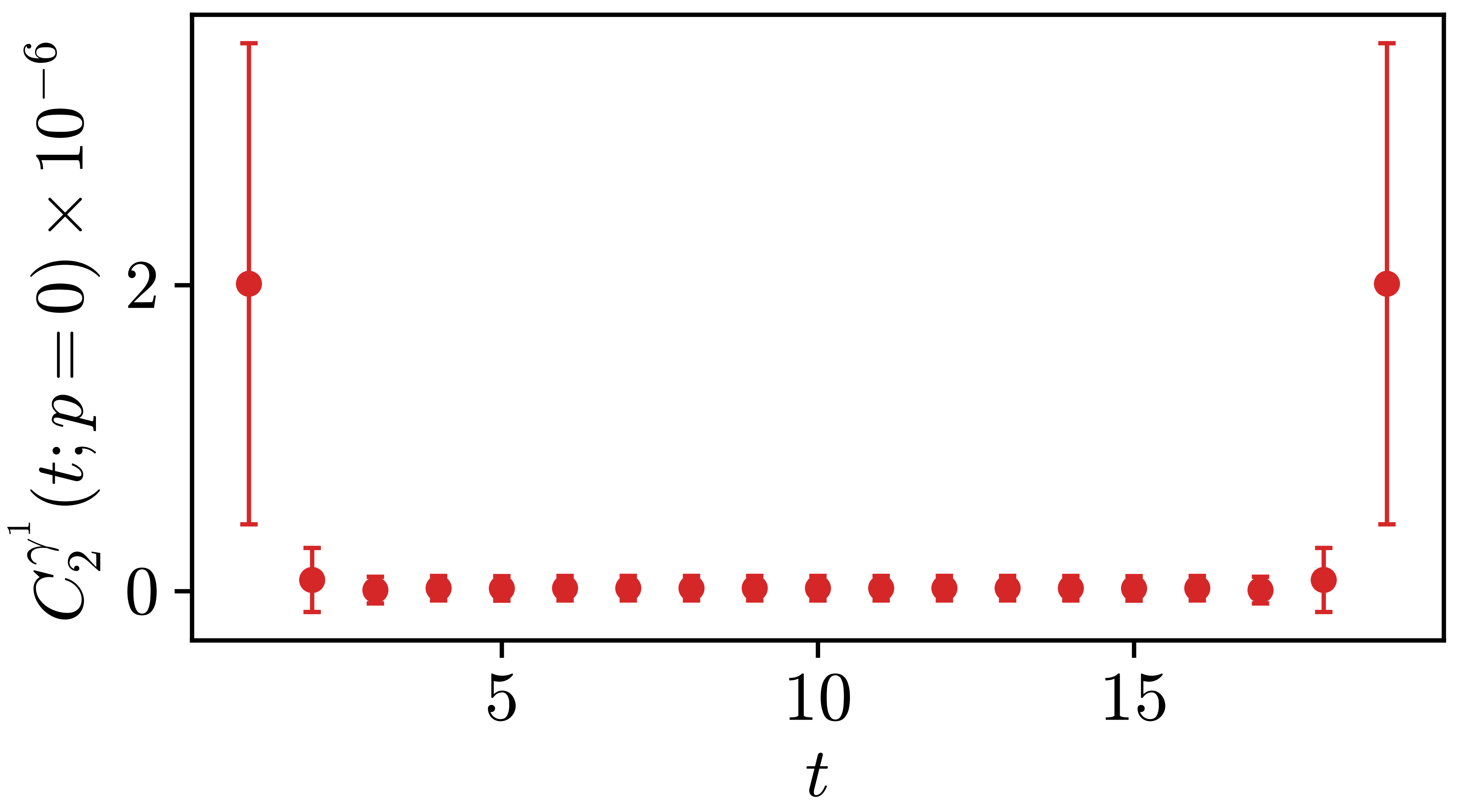
- Compute two-point correlators at fixed momentum p to extract the ground state in each sector.

$$C_2^\Gamma(t;p) = \frac{1}{T}\sum_s \langle \chi_\Gamma(t+s;p)\overline{\chi}_\Gamma(s;p)\rangle.$$

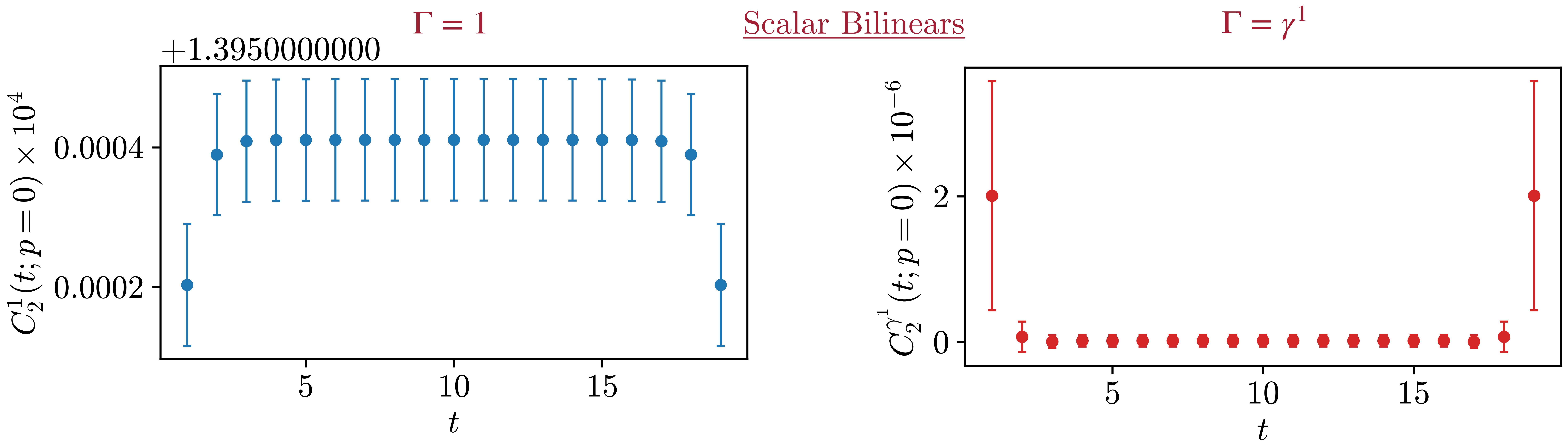
$\Gamma = 1$



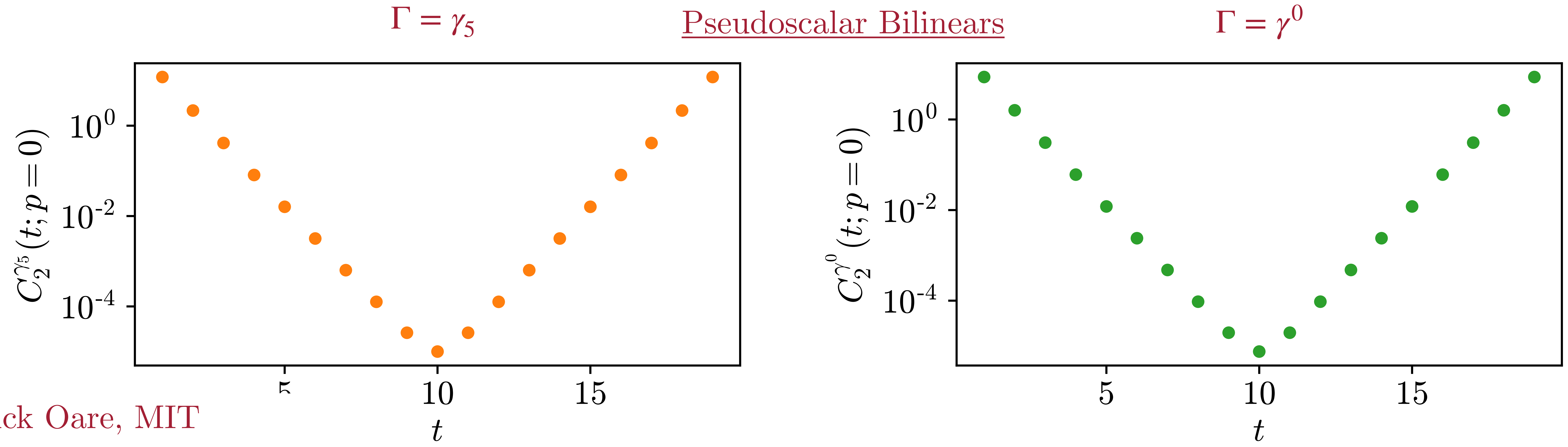

$\Gamma = \gamma^1$



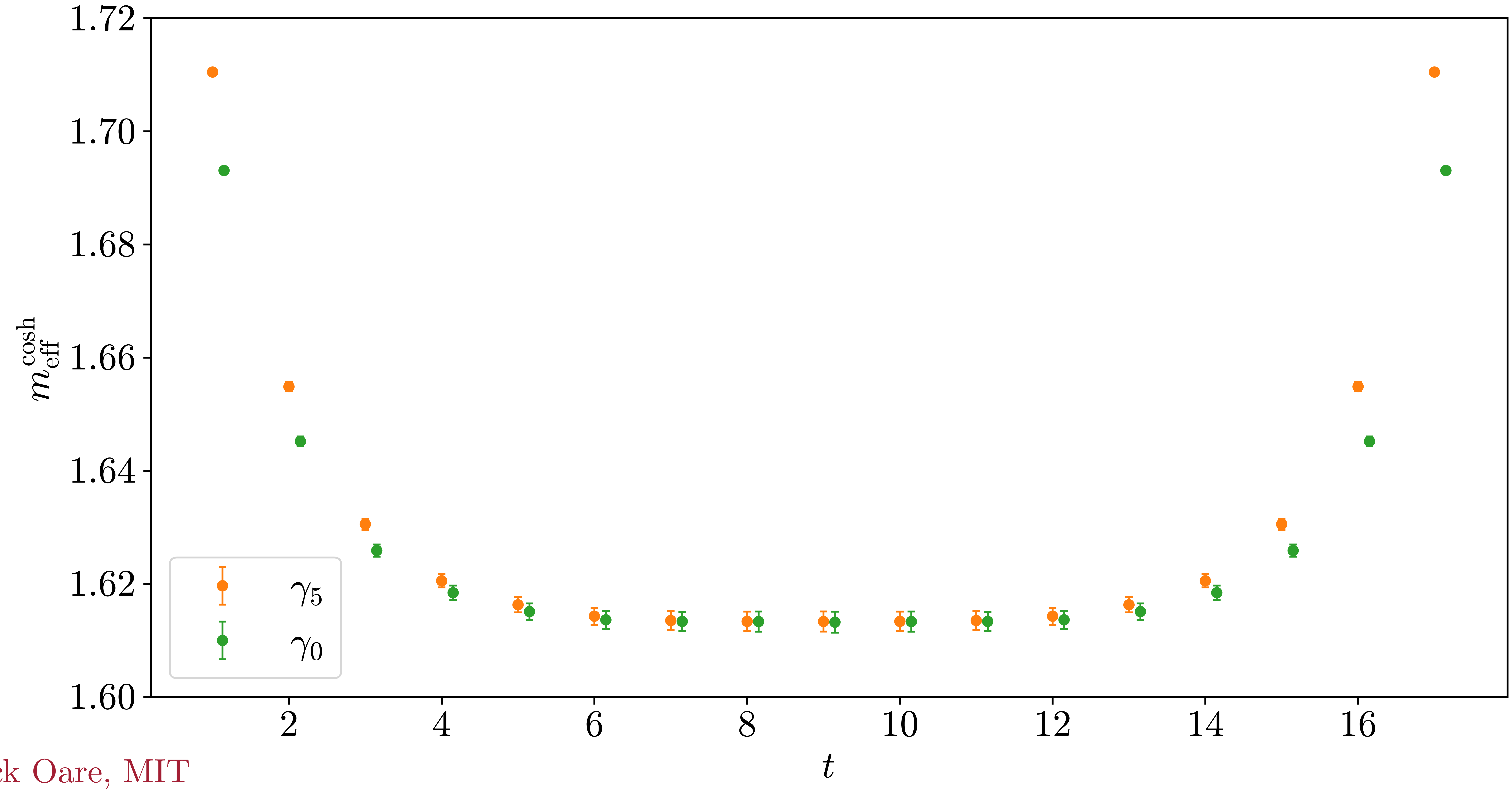
## Scalar Bilinears




## Pseudoscalar Bilinears



# Effective Mass, Pseudoscalar Sector



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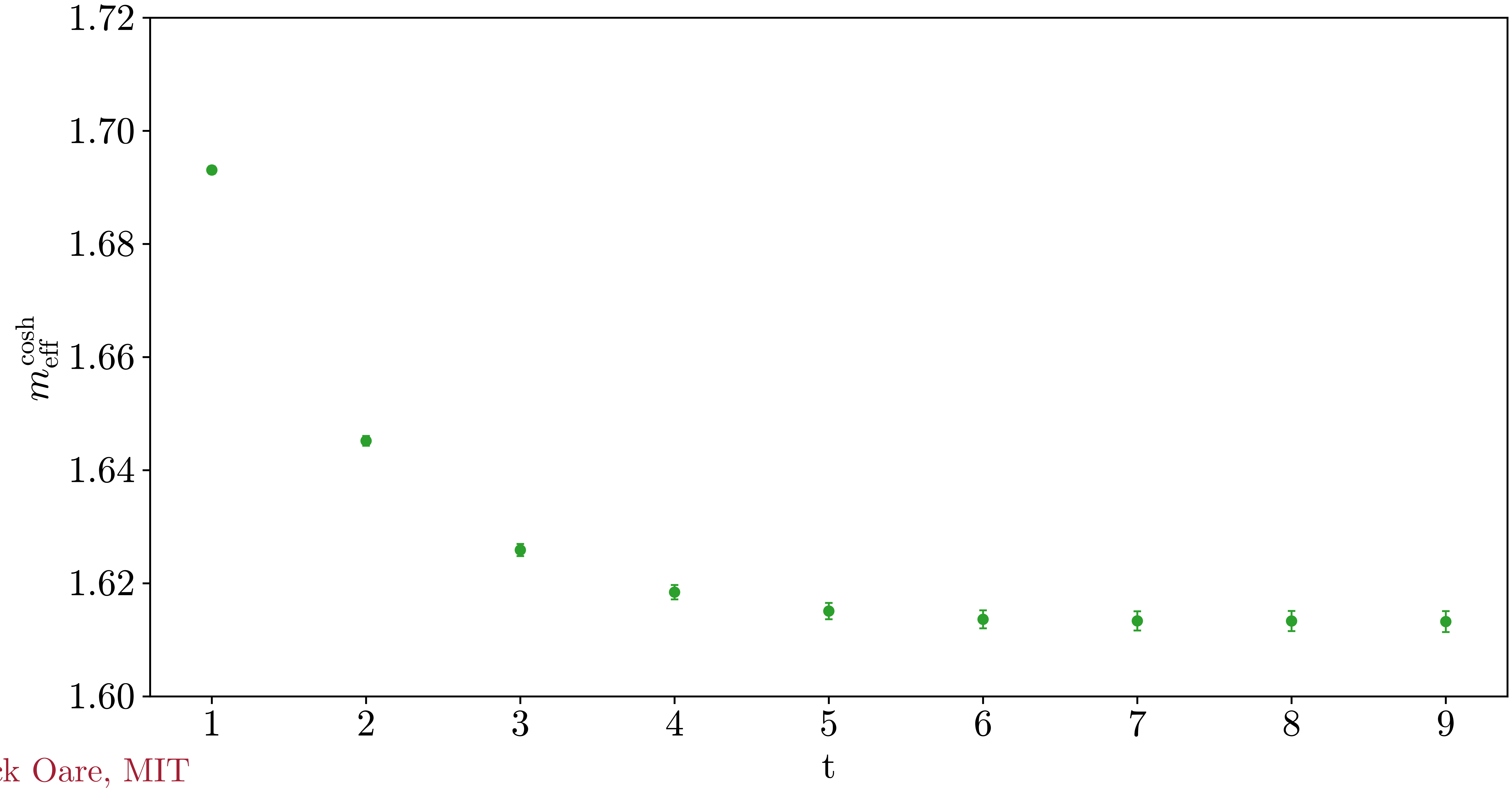

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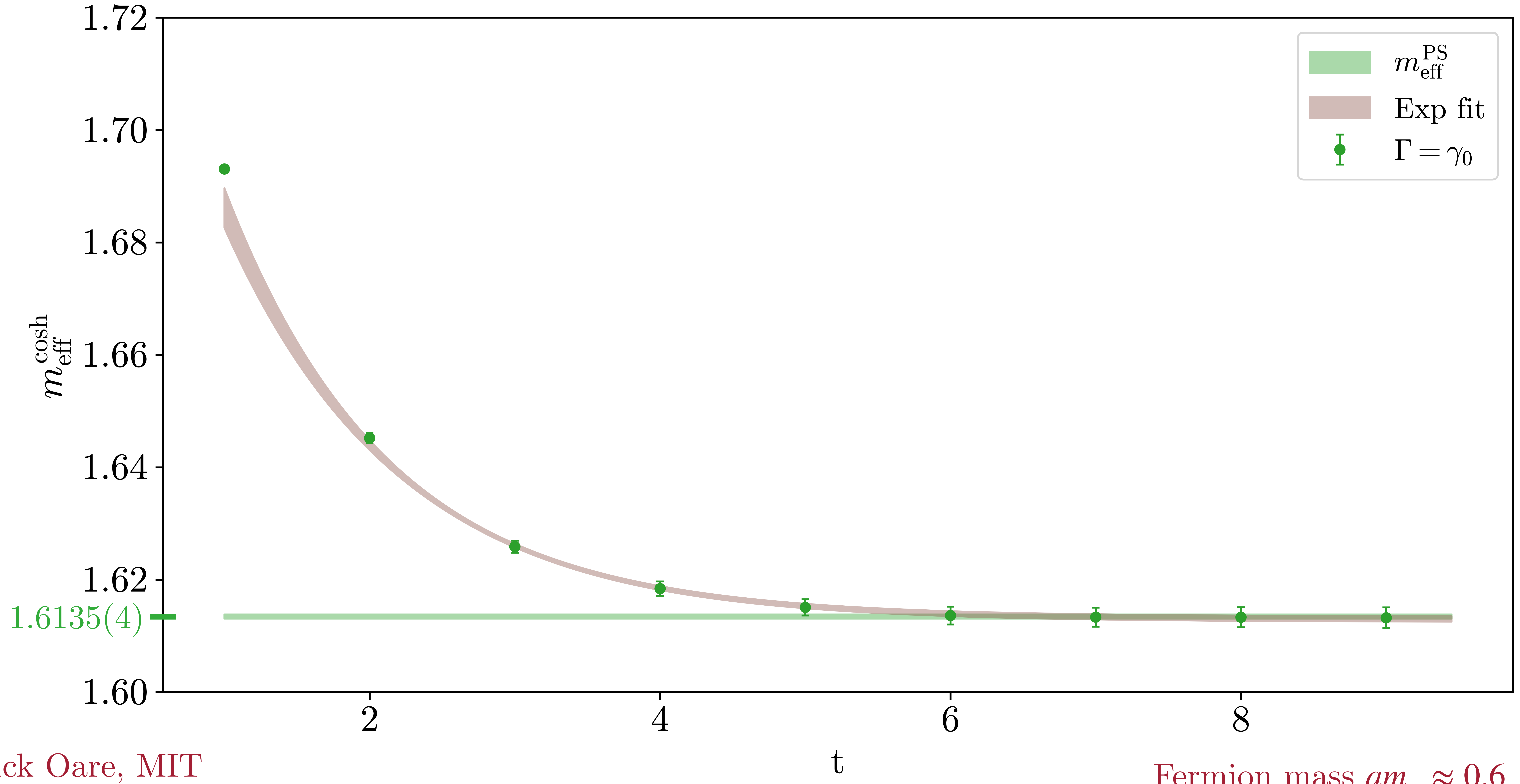




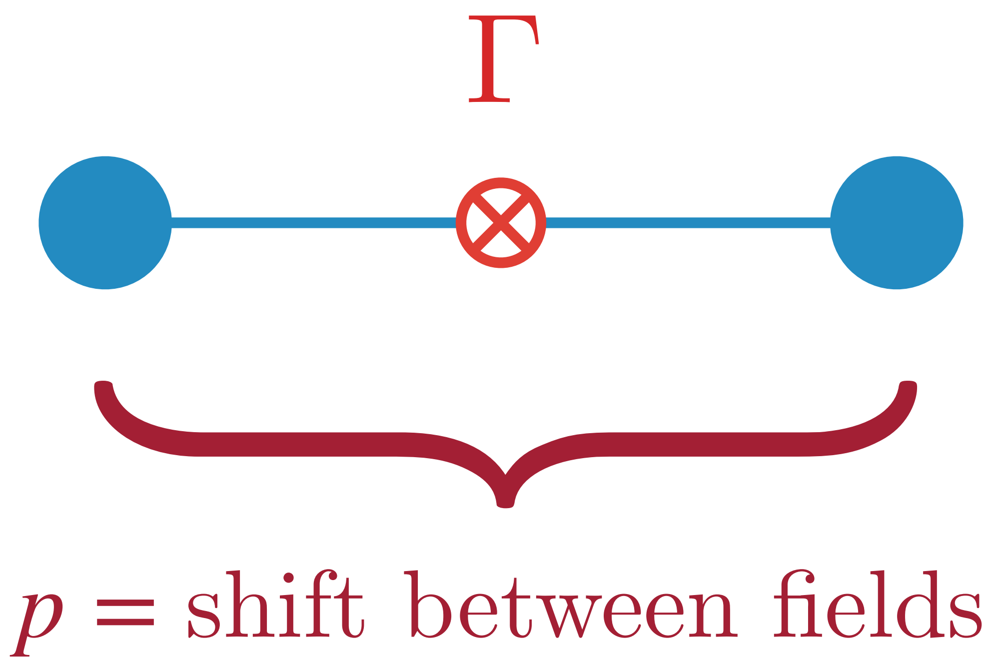

Fermion mass $am_\psi \approx 0.6$

# Low-Lying Spectrum and the GEVP

- Compute the low-lying spectrum with the operator basis,

$$B_{\Gamma}^{p}(t) = \sum_{x} \overline{\psi}(x) W_{\text{Adj}}(x, x + p\hat{0}) \Gamma \psi(x + p\hat{0})$$

  projected onto lattice irreps to yield operator basis $\mathcal{B}_{p}^{\Gamma}(t)$.



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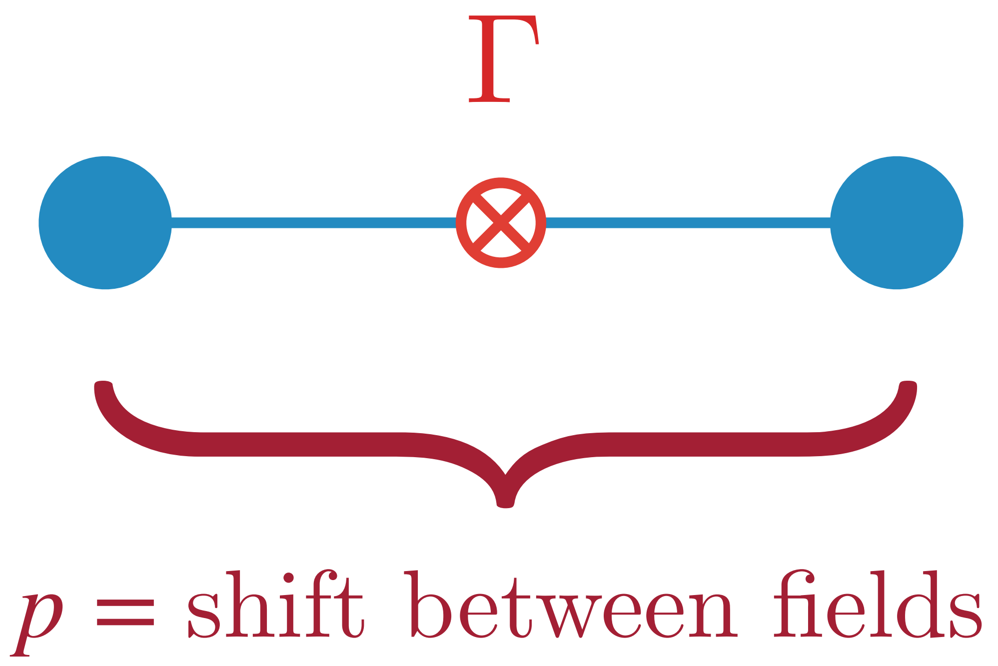


- Construct the correlator matrix,

$$C^{\Gamma}_{pq}(t) = \frac{1}{T} \sum_{s} \langle \mathcal{B}^{\Gamma}_{p}(t+s) (\mathcal{B}^{\Lambda,\Gamma}_{q})^{\dagger}(s) \rangle$$

and solve the Generalized Eigenvalue Problem (GEVP) to evaluate variational bounds $E^{(\mathrm{eff})}_{n}$ on the low-lying spectrum, $\lambda^{(k)}(t, t_0) \sim e^{-E_k(t-t_0)}$

$$C(t)\vec{v}^{(k)}(t, t_0) = \lambda^{(k)}(t, t_0) C(t_0) \vec{v}^{(k)}(t, t_0)$$

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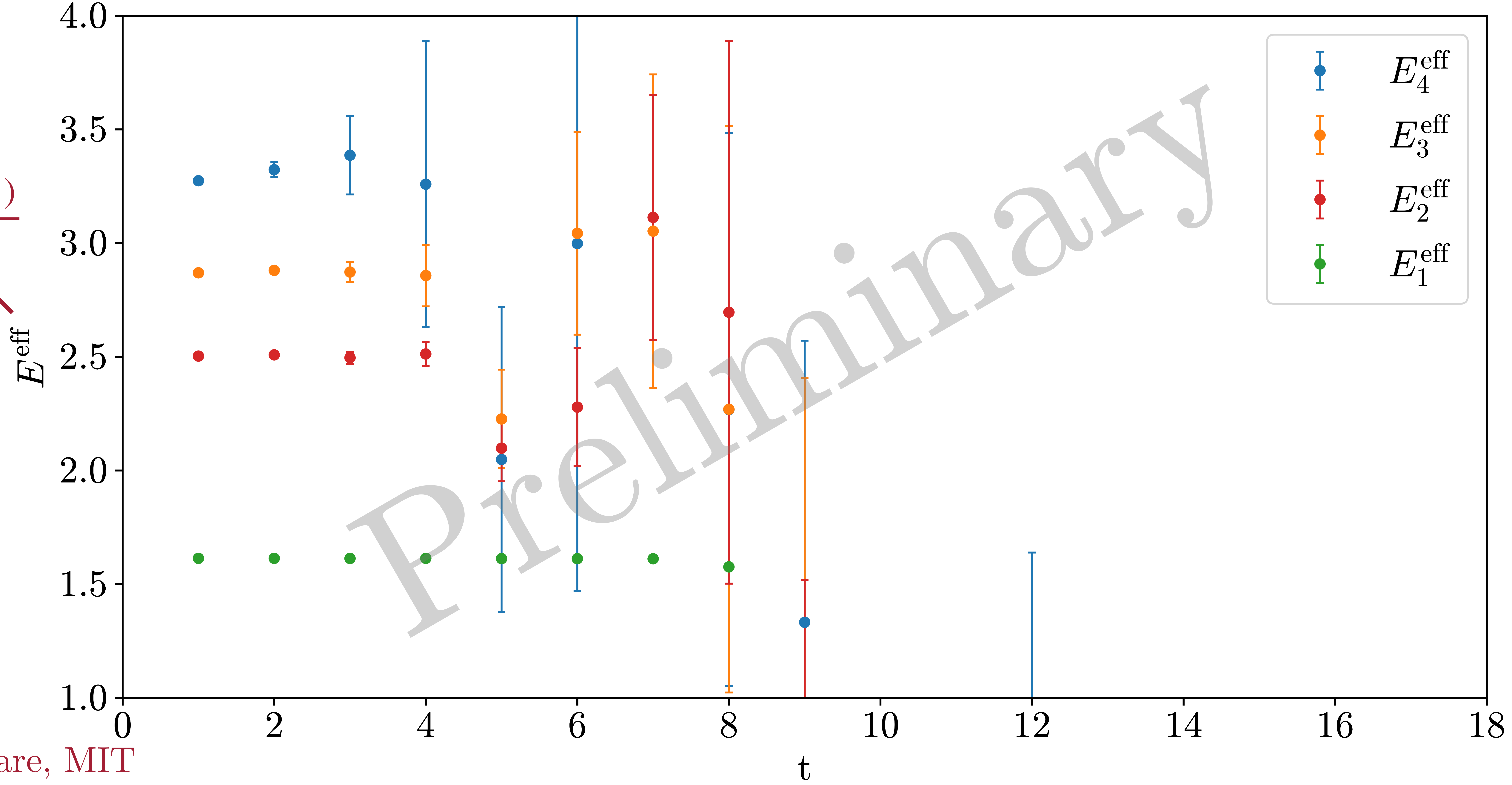
$\Gamma = \gamma_5$, $t_0 = 0$, maximum shift 3.

$$-\log \frac{\lambda(t+1)}{\lambda(t)}$$



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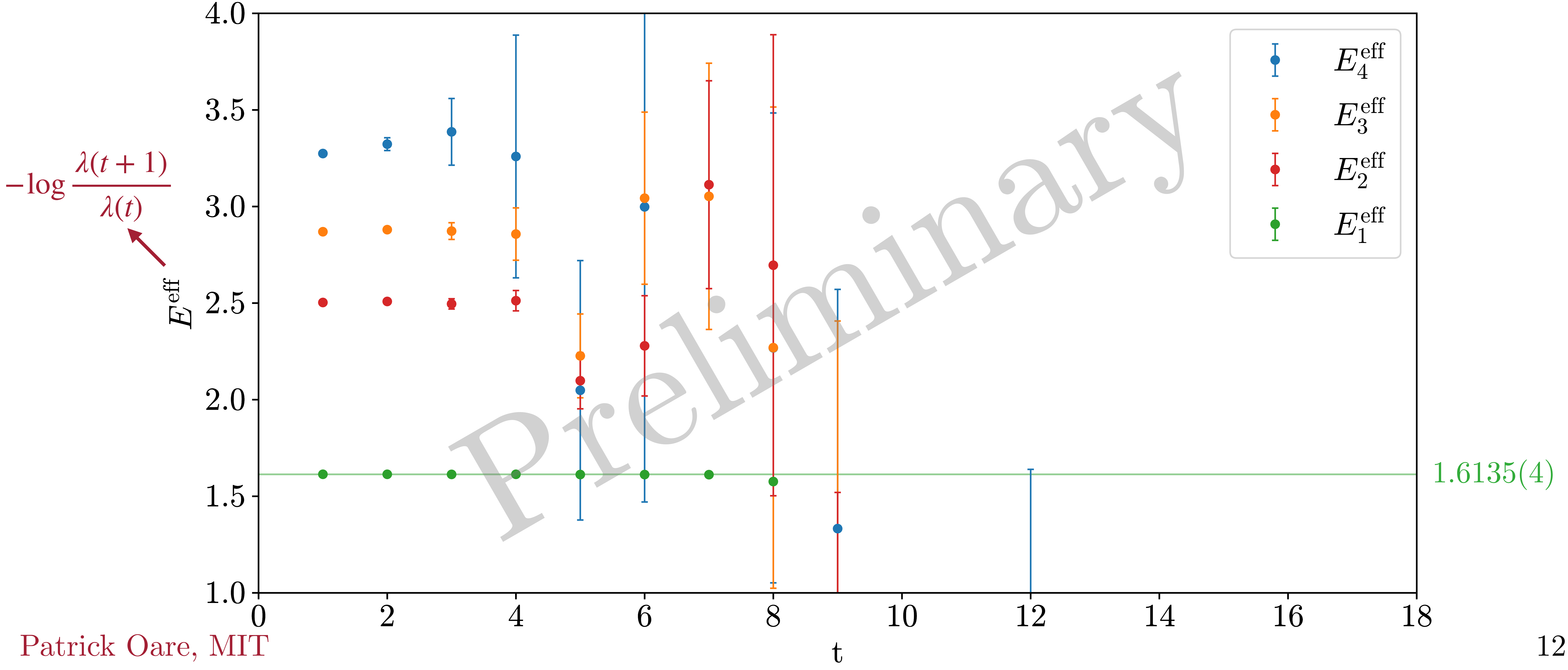

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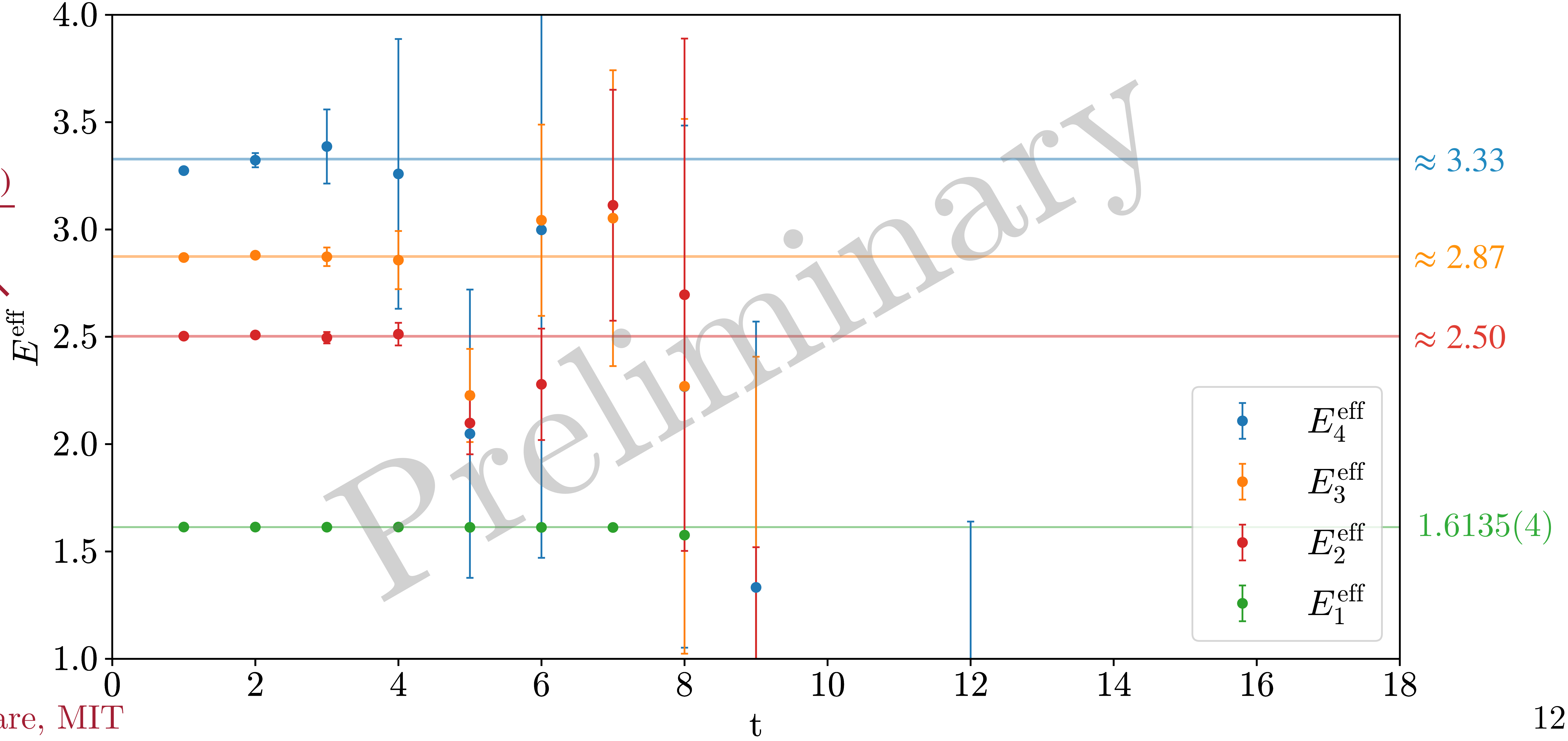

# Four-Fermion Operators

- Two additional operators may be added to the $\mathbf{QCD_2}$ action consistent with its symmetries ($\mathcal{O}_1 = \mathcal{O}_2$ for $N = 2$ colors):

A. Cherman *et al.*, SciPost Phys. 8 (2020) 5, 072.

$$\mathcal{O}_1 = (\mathrm{Tr}\,\overline{\psi}\psi)^2 \qquad\qquad \mathcal{O}_2 = \mathrm{Tr}\,(\overline{\psi}\gamma^\mu\psi\overline{\psi}\gamma_\mu\psi)$$

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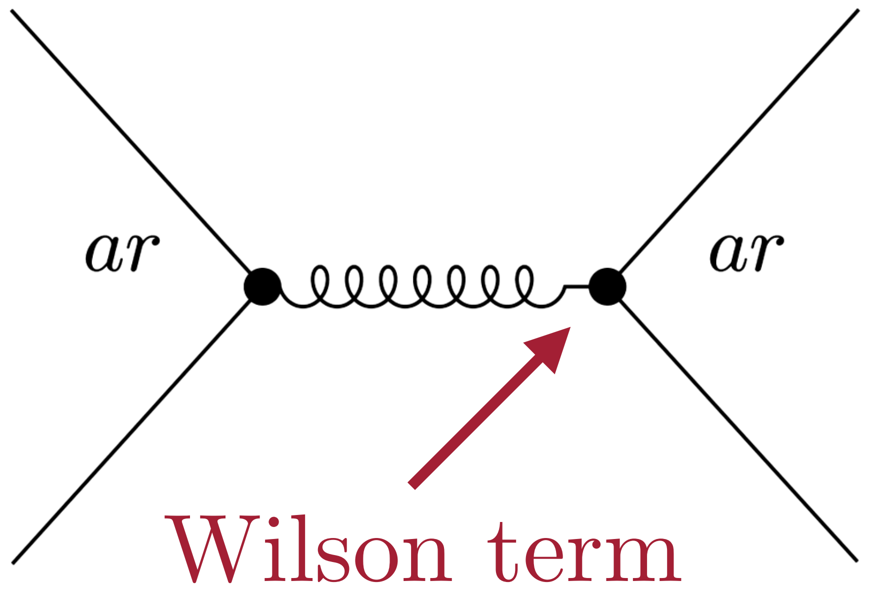
  - Non-zero $\langle \mathcal{O}_1 \rangle$ indicates that a coupling for $\mathcal{O}_1$ is generated at finite $a$.

A. Cherman, M. Neuzil. Phys. Rev. D 109 (2024) 10, 105014.



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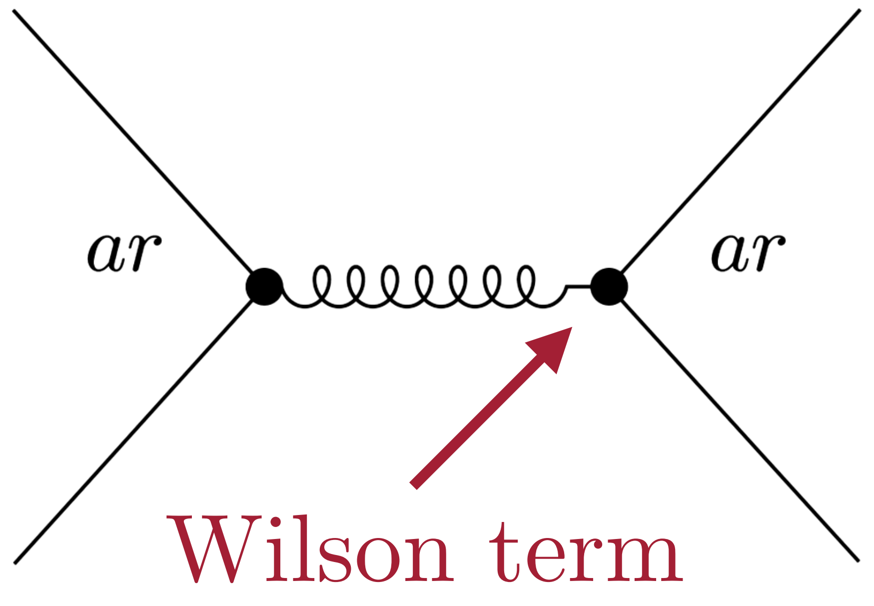


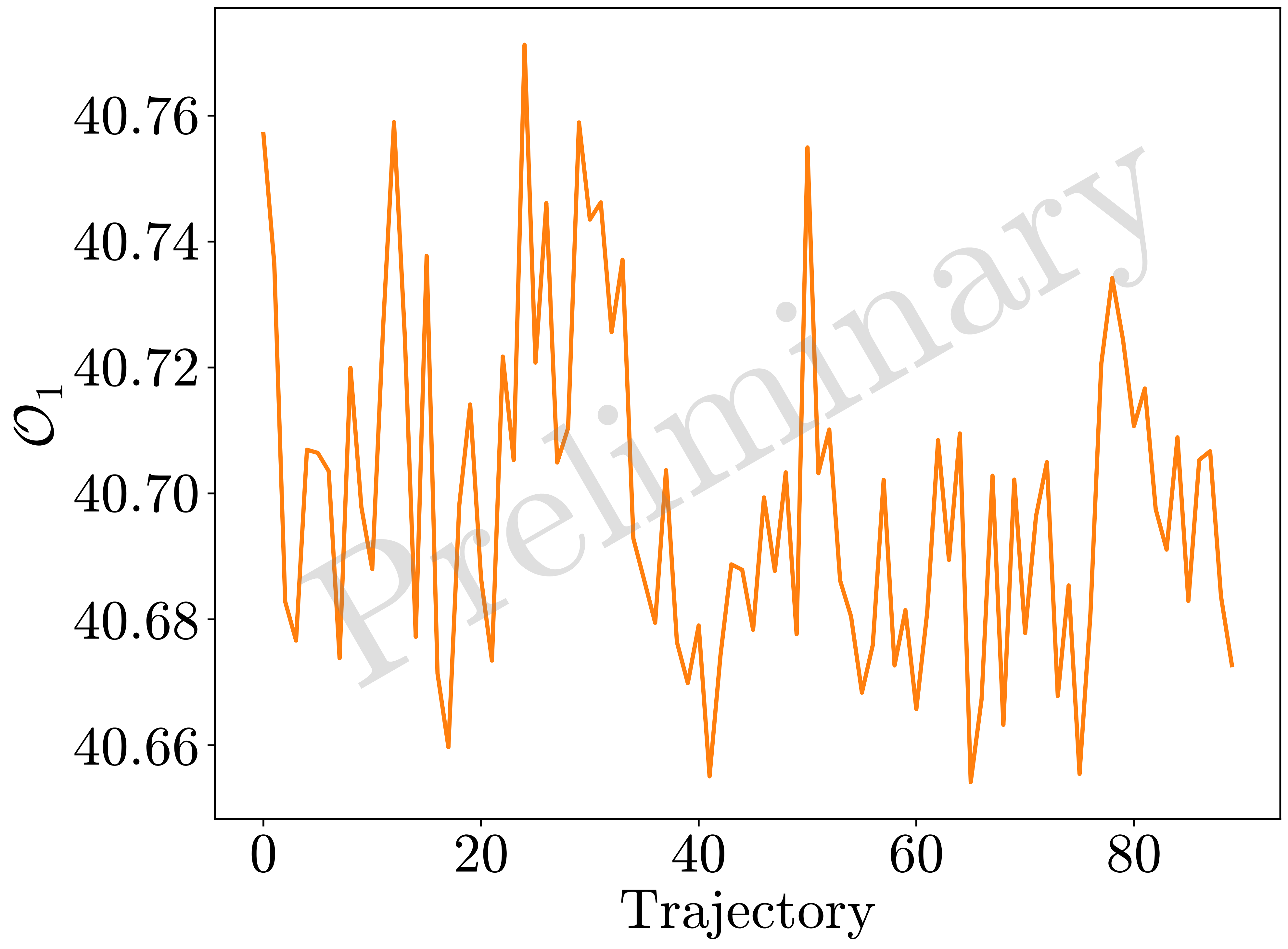

# Conclusion

- We have presented an ongoing computation of the low-lying spectrum of $QCD_2$ using Lattice Gauge Theory.
  - Extrapolation to the continuum limit is still required.
- We are currently in the process of scaling up the calculation.
  - Generating additional ensembles with larger numbers of lattice sites and numbers of colors.
  - Generating additional configurations on each ensemble used in this work.
- Further investigation of the four-fermion operators $\mathcal{O}_1$ and $\mathcal{O}_2$ are required: how does $\langle \mathcal{O}_i \rangle$ scale as $a \to 0$?

# Backup Slides

# Finite-Volume Dispersion