Progress on the QCD chiral phase transition for various numbers of flavors and imaginary chemical potential

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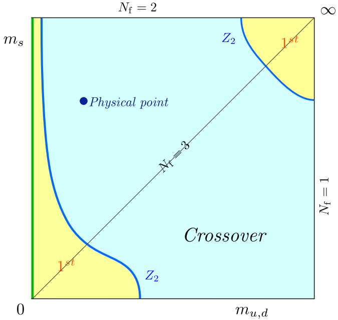
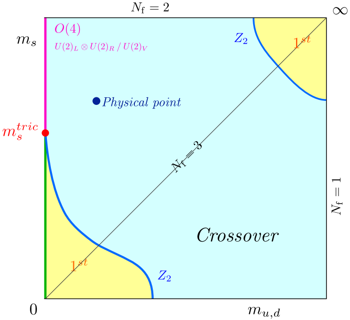
30/07/2024 - 14:05

QCD at non-zero density

Lattice 2024 - Liverpool

The (old) QCD Columbia plot



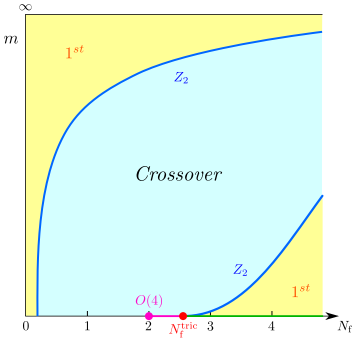


1. order scenario, Fig. from (Cuteri, Philipsen, and Sciarra 2021)

2. order scenario, Fig. from (Cuteri, Philipsen, and Sciarra 2021)

- at physical point: analytic, smooth crossover (Aoki et al. 2006)
- two possible scenarios predicted from linear sigma models (Pisarski and Wilczek 1984)
- results from coarse lattices seemed to support scenarios (Brown et al. 1990) (Iwasaki et al. 1996)
- change of transition from first-order to second-order implies a tricritical point

Columbia plot for N_f mass-degenerate quarks



Columbia plot for mass-degenerate quarks, from (Cuteri, Philipsen, and Sciarra 2021)

Strategy from our group: (Cuteri, Philipsen, and Sciarra 2021)

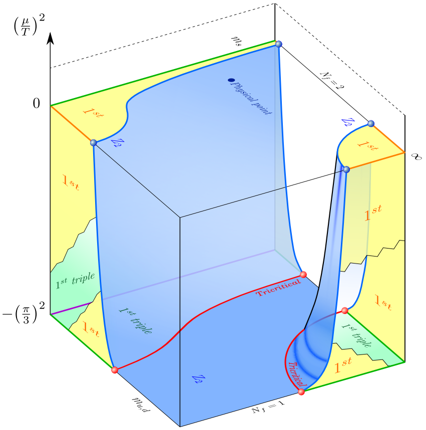
- analytic continuation to continuous, non-integer values of N_f with degenerate mass m
- tricritical point is guaranteed to exist
- second order phase boundary enters tricritical point, exhibiting tricritical scaling

Goal

Study the chiral critical surface as a function of N_f and the lattice spacing a with staggered LQCD.

The 3D-Columbia plot

- investigate phases of QCD for negative μ^2
- purely imaginary chemical potential $\mu = i\mu_i$ keeps action real
- Roberge-Weiss-plane at $\mu_i = \frac{\pi T}{3}$ with mass-dependent Roberge-Weiss transition
- coarse lattices: chiral first order region grows towards Roberge-Weiss plane (e.g. (Forcrand and Philipsen 2007) (Bonati, Cossu, et al. 2011))



From (Philipsen and Sciarra 2020)

Extended goal

Study the chiral critical surface at fixed $\mu_i = 0.81\frac{\pi T}{3}$ applying the same strategy.

Computational Strategy

Lattice setup

- unimproved Wilson gauge action
- unimproved staggered fermion action with N_f degenerate flavors
- bare parameters: lattice gauge coupling β, quark mass am
- lattice of size $N_\tau \times N_\sigma^3$ with lattice spacing $a(\beta)$
- finer lattices: $N_\tau = \frac{1}{aT} \to \infty$

Numerical tools

- LQCD code: $\mathtt{CL^2QCD}$ (Sciarra et al. 2021) based on OpenCL
- run on Virgo cluster at GSI, Darmstadt (AMD GPUs of type MI100)
- handle thousands of simulations: `BaHaMAS` (Sciarra 2021)
- analysis: python scripts bundled in PLASMA

Analysis of the Chiral Transition

- order parameter $\mathcal{O}$: chiral condensate $\langle\bar{\psi}\psi\rangle$
- standardized moments: $B_n = \frac{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^n\rangle}{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^2\rangle^{n/2}}$
- phase boundary β_{pc}: $B_3(\beta_{\text{pc}}; am, N_\sigma) = 0$
- information on the order of the transition: $B_4(\beta_{\text{pc}}; am, N_\sigma)$
- $B_4(N_\sigma \to \infty)$ values:

1. order	$Z(2)$ 2. order	crossover
1	1.604	3

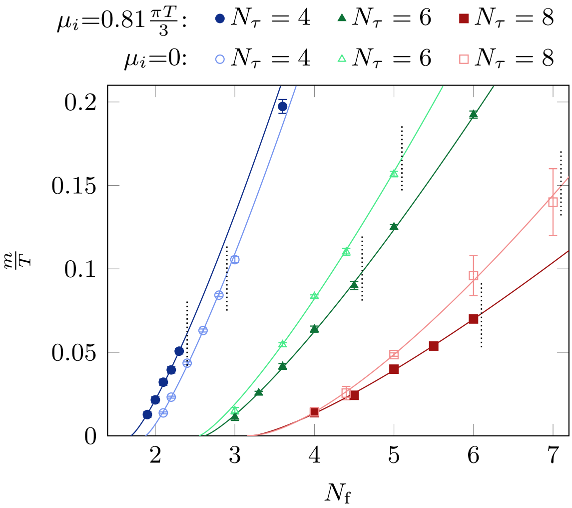
Finite size scaling formula of B_4 (Jin et al. 2017)

$$B_4(\beta_{\mathsf{pc}}; am, N_\sigma) = (1.604 + Bx + \dots)\left(1 + CN_\sigma^{y_t - y_h} + \dots\right)$$

$y_t = 1/\nu$, y_h: Ising 3D critical exponents,
$x = (am - am_c)N_\sigma^{1/\nu}$: scaling variable

- fit finite size scaling formula to $B_4(\beta_{\text{pc}}; am, N_\sigma)$ values
- determine critical mass am_c as fit parameter

Results - am-N_f-plane

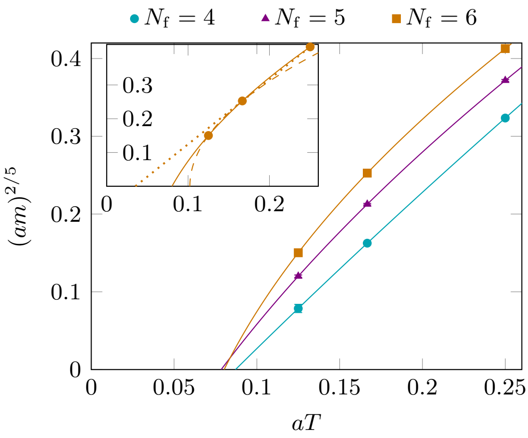


Tricritical scaling formula for N_f^c

$$N_\mathrm{f}^\mathrm{c}(am, N_\tau) = N_\mathrm{f}^\mathrm{tric}(N_\tau) + \mathcal{D}_1(N_\tau)(am)^{2/5} + \mathcal{D}_2(N_\tau)(am)^{4/5}$$

- critical lines separate crossover from first-order regions
- tricritical scaling for both μ_i values for small am
- 1. order region **grows** with increasing N_f
- 1. order region **shrinks** with decreasing a

Results - am-aT-plane

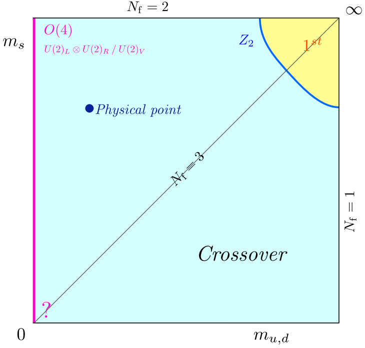


Tricritical scaling formula for $(aT)_\mathrm{c}$

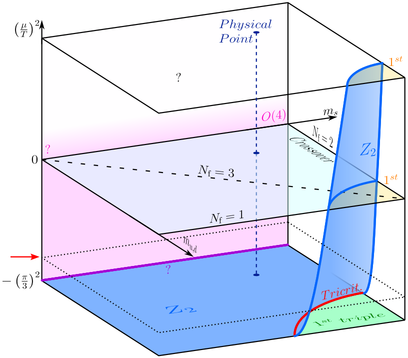
$$(aT)_\mathrm{c}(am, N_\mathrm{f}) =$$
$$(aT)_\mathrm{tric}(N_\mathrm{f}) + \mathcal{E}_1(N_\mathrm{f})(am)^{2/5} + \mathcal{E}_2(N_\mathrm{f})(am)^{4/5}$$

- critical lines are LO+NLO interpolations
- results are non-zero $(aT)_\mathrm{tric}(N_\mathrm{f})$
- first-order region not continuously connected to continuum limit up to $N_\mathrm{f} = 6$
- implies second-order chiral phase transition in the continuum
- identical qualitative behavior for $\mu_i = 0.81\pi T/3$ and $\mu_i = 0$

Results - The resulting Columbia plot



From (Cuteri, Philipsen, and Sciarra 2021)



- no chiral first-order region in the continuum limit
- continuum chiral phase transition is of second order
- chiral critical surface moves to zero mass plane towards the continuum limit
- consistent with results from simulations in the Roberge-Weiss-plane (Bonati, Calore, et al. 2019), (Cuteri, Goswami, et al. 2022)

A Ginzburg-Landau approach

Idea

Find the Ginzburg-Landau functional that describes the lattice QCD data in the tricritical region.

Tricritical functional:

$$\Omega(\eta) = -m\eta + \frac{1}{2}a\eta^2 - \frac{1}{4}b\eta^4 + \frac{1}{6}C\eta^6.$$

- order parameter $\eta = \langle\bar{\psi}\psi\rangle$
- symmetry breaking field m is bare quark mass
- a, b depend on non-ordering fields (N_{f}, μ, aT, β)

- conditions for second-order wing line (Hatta and Ikeda 2003):

$$\Omega'(\eta_c) = a_c\eta_c - b_c\eta_c^3 + c\eta_c^5 - m = 0$$
$$\Omega''(\eta_c) = a_c - 3b_c\eta_c^2 + 5c\eta_c^4 = 0$$
$$\Omega'''(\eta_c) = 2\eta_c\left(-3b_c + 10c\eta_c^2\right) = 0$$

Goal

Determine the Landau coefficients from second-order conditions and the tricritical scaling fit coefficients.

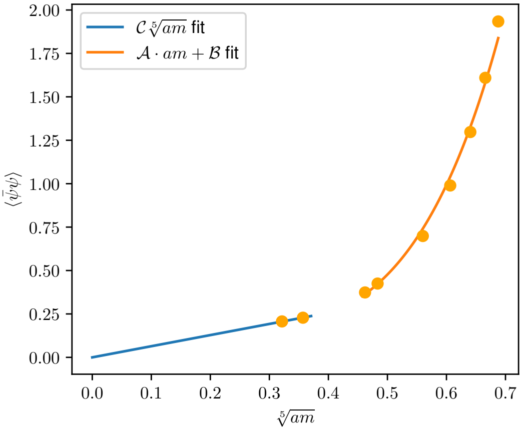
First numerical results for the Ginzburg-Landau approach

- second-order conditions yield

$$\eta_c = \frac{\sqrt[5]{12}}{2\sqrt[5]{C}}\sqrt[5]{m}$$

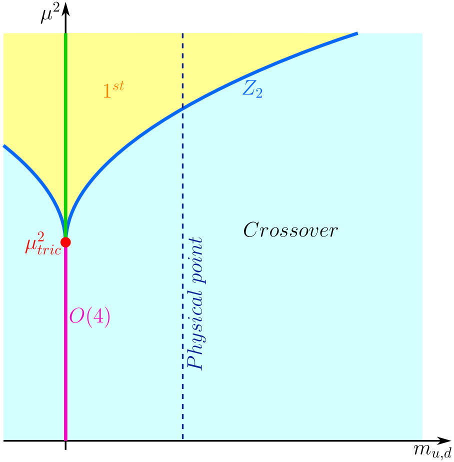
- fit η_c to lattice data of $\langle\bar{\psi}\psi\rangle$ at the critical point
- $\sqrt[5]{am}$-region is small, only two masses could be simulated

Fit for $\mu = 0$, $N_\tau = 4$:



Conclusions

- Columbia plot scenarios with a chiral first-order region have been ruled out
- qualitatively the same Columbia plot for $\mu_i = 0.81\pi T/3$
- order of the chiral phase transition does not change with imaginary μ
- Ginzburg-Landau possibly requires smaller masses
- open question: behavior of chiral critical surface for real μ
- results from DSE: 2. order chiral PT persists for small values of real μ (Bernhardt and Fischer 2023)



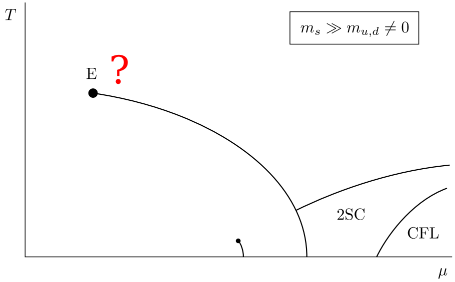
Possible scenario for the existence of a critical endpoint

Thank you for your attention!

Backup

Connection between QCD phase diagram and Columbia plot

- QCD phase diagram is mostly conjectured
- large coupling prohibits perturbative methods
- sign problem restricts lattice QCD to real $\mu = 0$
- location/existence of critical endpoint is of particular interest



Conjectured QCD phase diagram, from (Rajagopal and Wilczek 2000)

Columbia plot

Represents the order of thermal transition at $\mu = 0$ as a function of the 3 lightest quark masses $m_{u,d}$, m_s.

Analysis of the chiral transition in finite volumes

- order parameter $\mathcal{O}$: chiral condensate $\langle\bar{\psi}\psi\rangle$
- standardized moments: $B_n = \frac{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^n\rangle}{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^2\rangle^{n/2}}$
- phase boundary $\beta_{\rm pc}$: $B_3(\beta_{\rm pc}; am, N_\sigma) = 0$
- order of the transition: $B_4(\beta_{\rm pc}; am, N_\sigma)$
- $B_4(N_\sigma \to \infty)$ values:

1. order	$Z(2)$ 2. order	crossover
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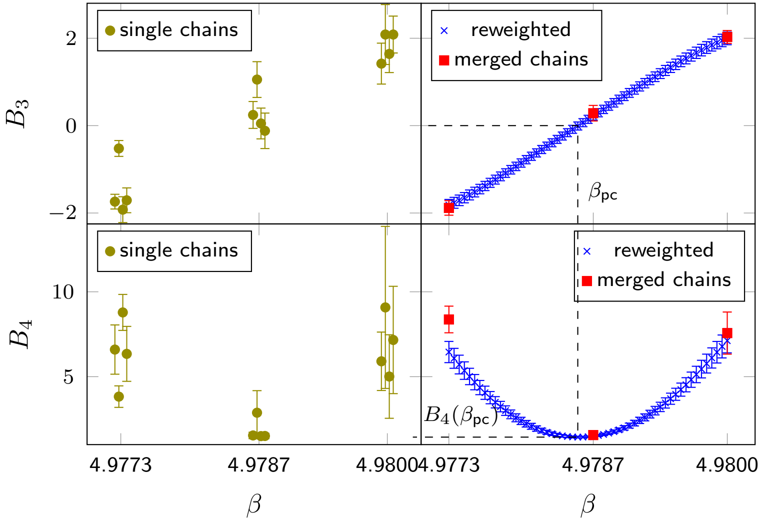


Analysis for fixed μ_i, $N_{\rm f}$, N_τ, am and N_σ.

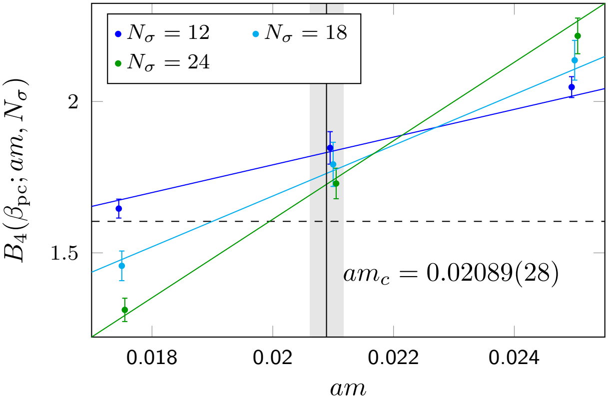
Kurtosis finite size scaling

Finite size scaling formula of B_4

$$B_4(\beta_{\text{pc}}; am, N_\sigma) =$$
$$(1.604 + Bx + \dots)\left(1 + C N_\sigma^{y_t - y_h} + \dots\right)$$

$y_t = 1/\nu$, y_h: Ising 3D critical exponents,
$x = (am - am_c) N_\sigma^{1/\nu}$: scaling variable

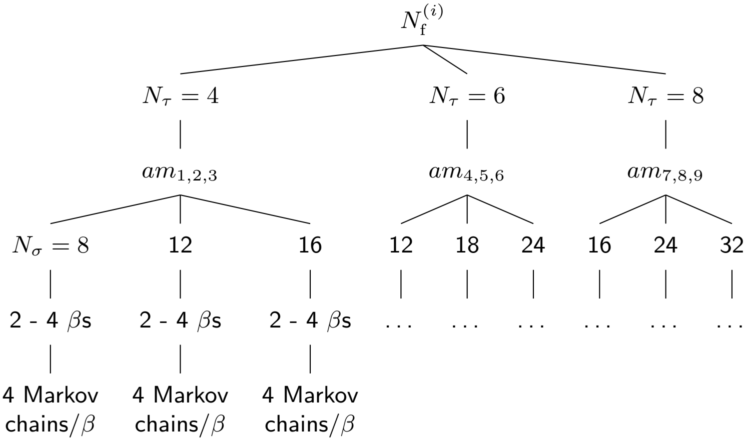
- fit finite size scaling formula to $B_4(\beta_{\text{pc}}; am, N_\sigma)$ values
- determine critical mass am_c as fit parameter



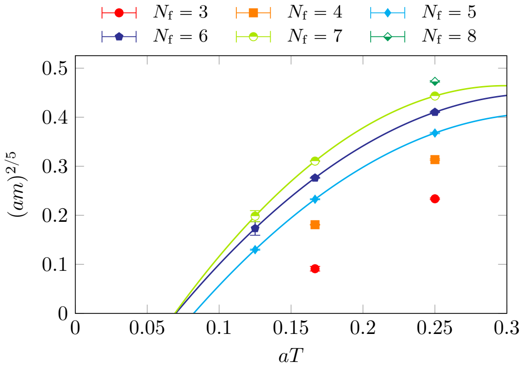
Linear fit with correction
for $\mu_i = 0.81\frac{\pi T}{3}$, $N_{\text{f}} = 5.0$ and $N_\tau = 6$

Procedure to collect data

- data collection for one value of μ_i
- thousands of separate Monte Carlo simulations
- more than 100 million trajectories generated
- few simulations are still running



Results - am-aT-plane: $\mu_i = 0$ (Cuteri, Philipsen, and Sciarra 2021)



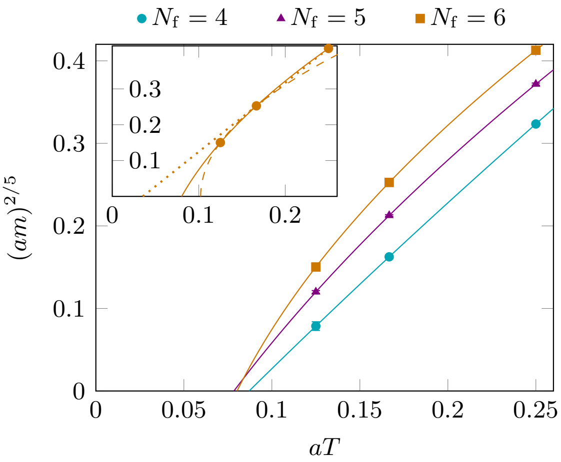
From (Cuteri, Philipsen, and Sciarra 2021)

Tricritical scaling formula for $(aT)_c$

$$(aT)_c(am, N_f) =$$
$$(aT)_{\text{tric}}(N_f) + \mathcal{E}_1(N_f)(am)^{2/5} + \mathcal{E}_2(N_f)(am)^{4/5}$$

- critical lines are LO+NLO interpolations
- results are non-zero $(aT)_{\text{tric}}(N_f)$
- first-order region not continuously connected to continuum limit up to $N_f = 7$
- implies second-order chiral phase transition in the continuum

Results - am-aT-plane: $\mu_i = 0.81\pi T/3$



- same qualitative behavior as for $\mu_i = 0$
- conservative error estimation of $(aT)_{\mathrm{tric}}(N_\mathrm{f})$:
  - upper bound: NLO interpolation
  - lower bound: LO interpolation
- non-scaling polynomial fits rule out first-order transition in continuum

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