QCD EoS in strong magnetic fields and non-zero baryon density

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Partially based on *Phys. Rev. Lett.* ***132****, 201903 (2024)* and ongoing work with Heng-Tong Ding, Jin-Biao Gu and Sheng-Tai Li

Nuclear Science Computing Center at CCNU

THE 41ST INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY

The University of Liverpool, UK
Jul 28 - Aug 03, 2024

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Equilibrium description of strong interacting matter

$$p, \epsilon, \sigma \equiv f(T, \mu, eB, \ldots)$$

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Early universe	Magnetars	Heavy ion-collision
Energy, evolution → Friedmann eq.	$m(r)$ of NS relations → TOV eq.	QGP → Hadronization → Freeze-out

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Equilibrium description of strong interacting matter
$p, \epsilon, \sigma \equiv f(T, \mu, eB, \dots)$

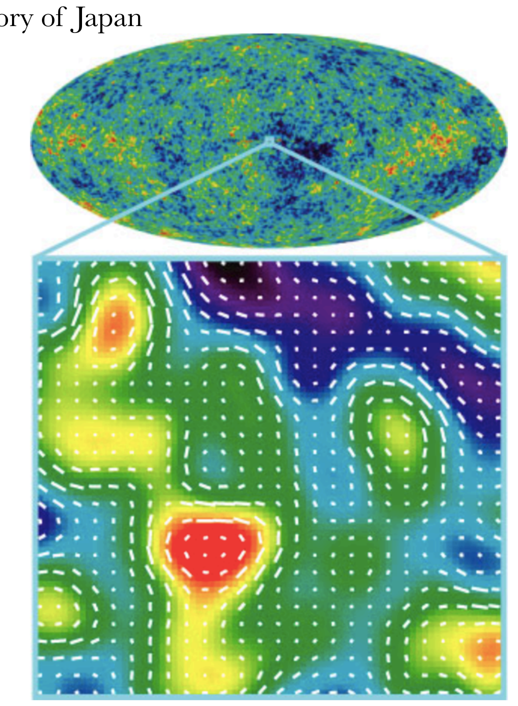
Early universe

Energy, evolution → Friedmann eq.

Cosmological Magnetic Field: a fossil of density perturbations in the early universe

January 6, 2006 | Science

National Astronomical Observatory of Japan



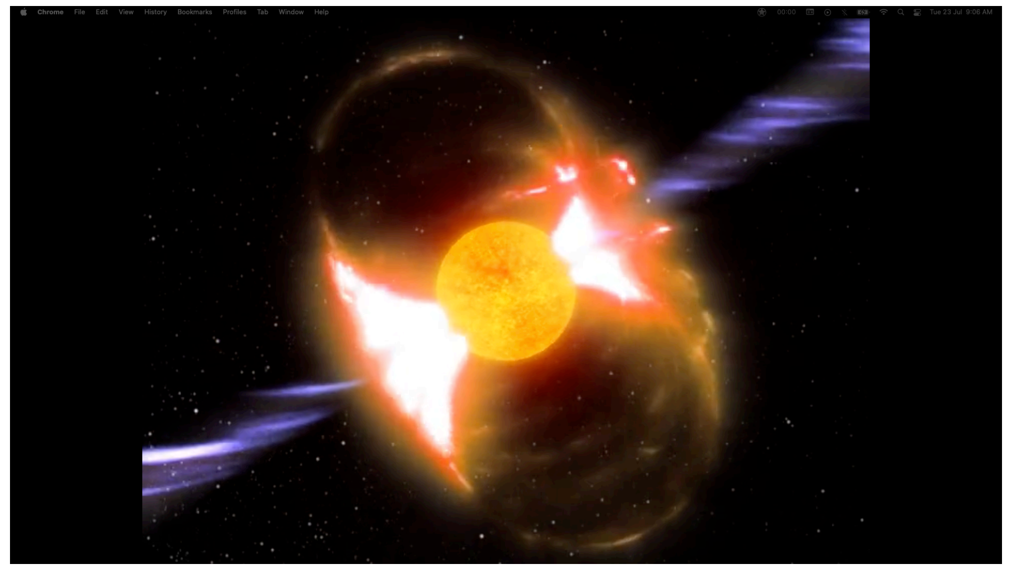
Ichiki *et al.*, *Science*, ***311***, *827-829, 2006*

Vachaspati, *Phys. Lett. B* ***265*** *(1991)*

Enqvist, *Phys. Lett. B* ***319*** *(1993)*

Magnetars

$m(r)$ of NS relations → TOV eq.



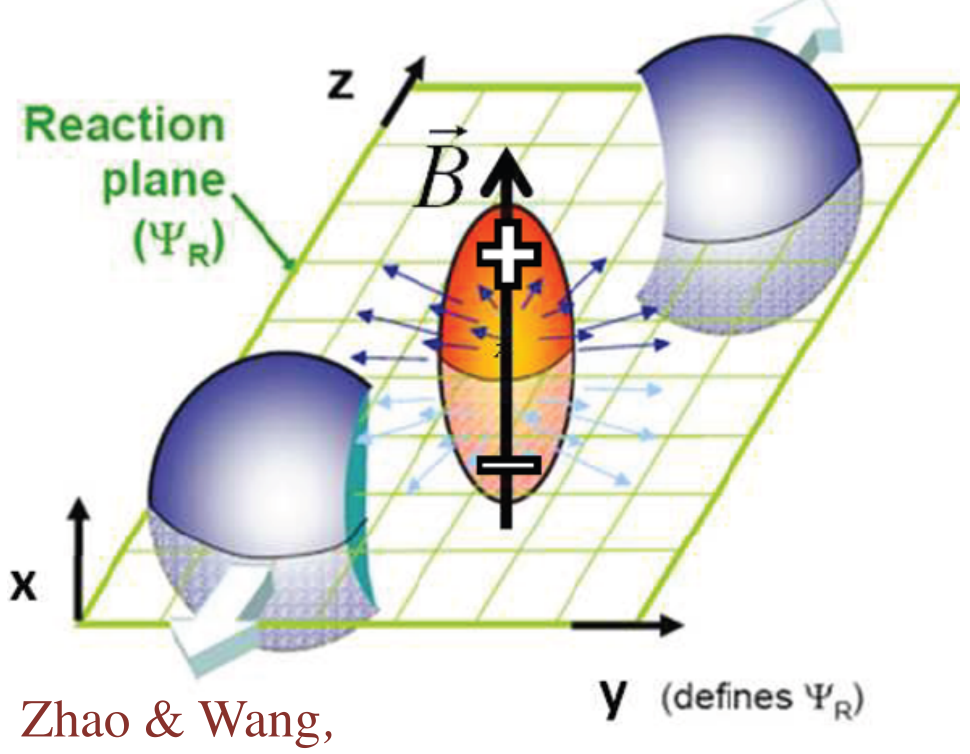
Schematic of XTE J1810-197

Duncan & Thompson, *Astrophys. J. Lett.* ***392*** *(1992) L9*

Anderson et al., *Phys. Rev. Lett.* ***100****, 191101*

Heavy ion-collision

QGP → Hadronization → Freeze-out



Zhao & Wang, *Prog. Part. Nucl. Phys.* ***107*** *(2019)*

Kharzeev et al., *Nucl.Phys.A* ***803*** *(2008)*

Bali et al., *JHEP* ***07*** *(2020) 183*

Astrakhantsev et al., *PRD* ***102*** *(2020) 054516*

EoS and interplay with magnetic fields is ubiquitous!

QCD EoS in strong magnetic fields and non-zero baryon density

★ Interest in rich QCD phase structure at finite T and non-zero μ!

★ QCD pressure Taylor expanded as fluctuations of conserved charges $\mathscr{C} \in \{\mathrm{B}, \mathrm{Q}, \mathrm{S}\}$,

$$\hat{p}(T, eB, \hat{\mu}) \equiv \frac{p}{T^4} = \frac{1}{VT^3} \ln \mathscr{Z}_{\mathrm{GC}}(T, eB, V, \hat{\mu}_{\mathscr{C}})$$
$$= \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \boxed{\chi_{ijk}^{\mathrm{BQS}}} \hat{\mu}_{\mathrm{B}}^{\,i}\, \hat{\mu}_{\mathrm{Q}}^{\,j}\, \hat{\mu}_{\mathrm{S}}^{\,k}$$

(Lattice computable)

$$\chi_{ijk}^{\mathrm{BQS}} \equiv \chi_{ijk}^{\mathrm{BQS}}(T, eB) = \frac{\partial^{i+j+k}}{\partial \hat{\mu}_{\mathrm{B}}^{i} \partial \hat{\mu}_{\mathrm{Q}}^{j} \partial \hat{\mu}_{\mathrm{S}}^{k}} \hat{p}(T, eB, \hat{\mu}) \Big|_{\hat{\mu}=0}$$

Sign-Problem

↓

Taylor Expand

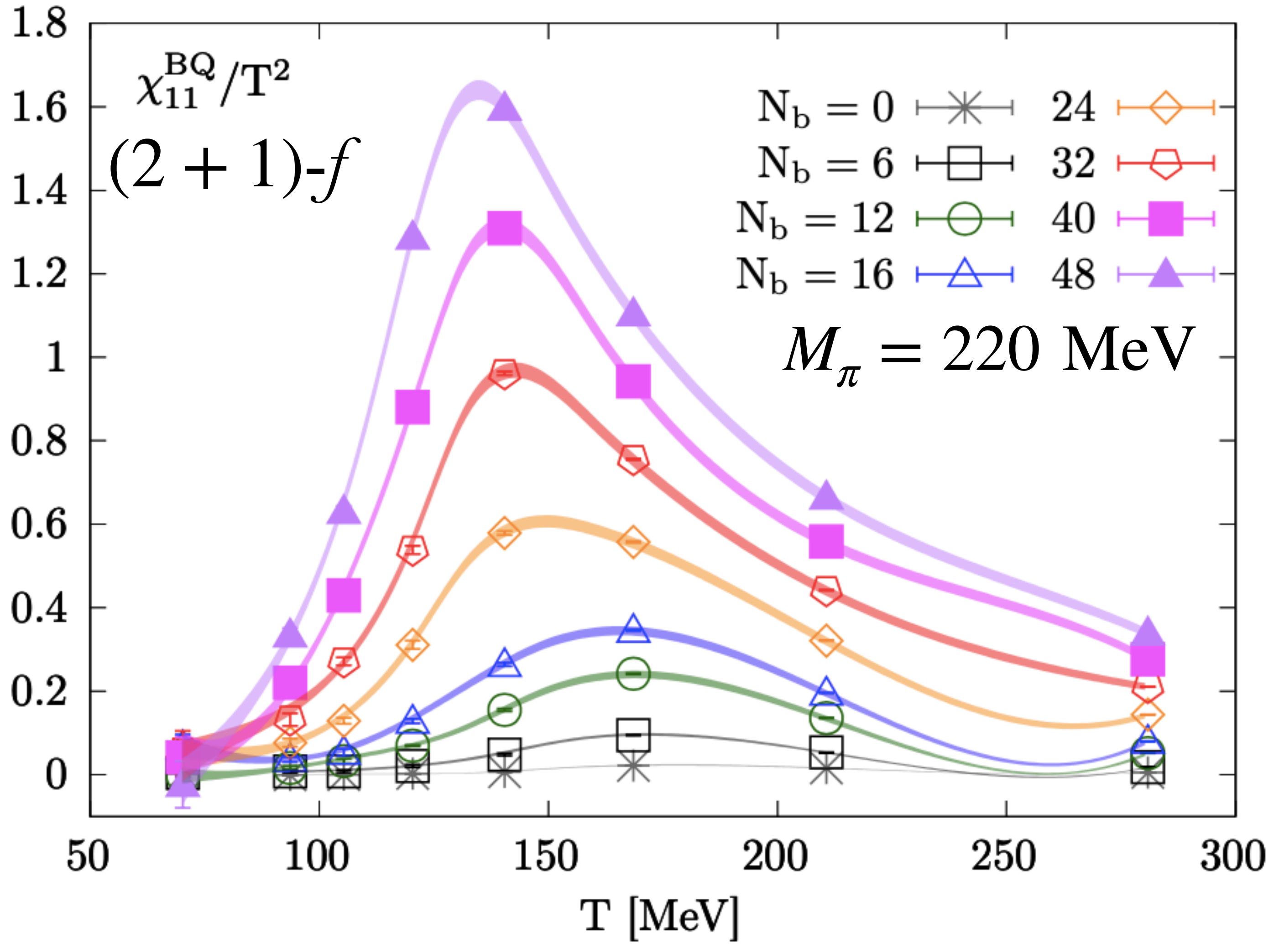
$\mu_f \longleftrightarrow \mu_{\mathscr{C}}$ $\qquad$ $\chi_{ijk}^{uds} \longleftrightarrow \chi_{ijk}^{\mathrm{BQS}}$

$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}$$
$$\mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}$$
$$\mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}$$

Allton et al., *Phys. Rev. D* ***66*** *(2002) 074507*
HotQCD, *Phys. Rev. D* ***95*** *(2017) 054504*

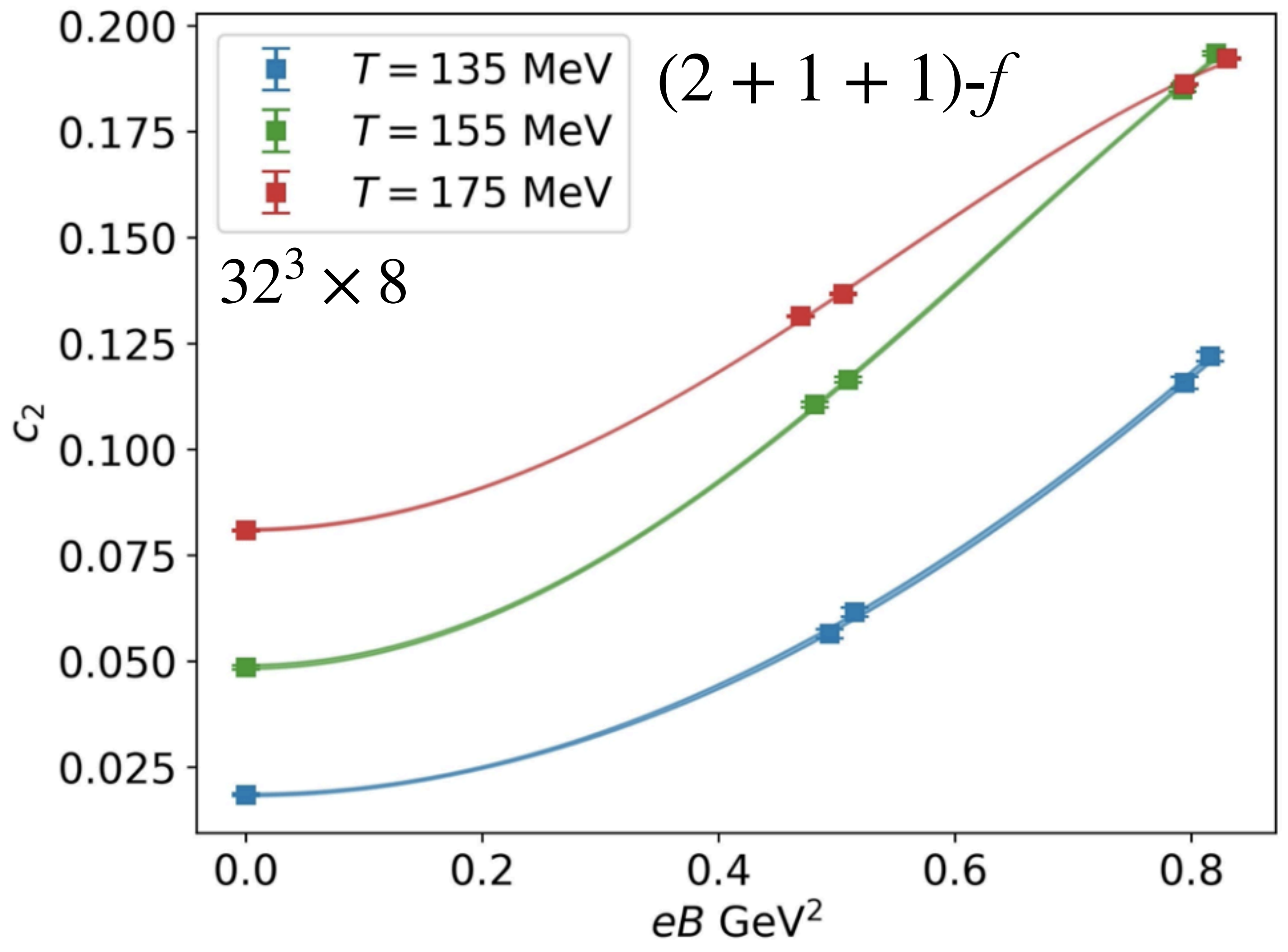
Recent Works:

Conserved Charges in Magnetic fields



Ding, Li, Shi & Wang, *Eur. Phys. J.A* ***57*** *(2021) 6, 202*

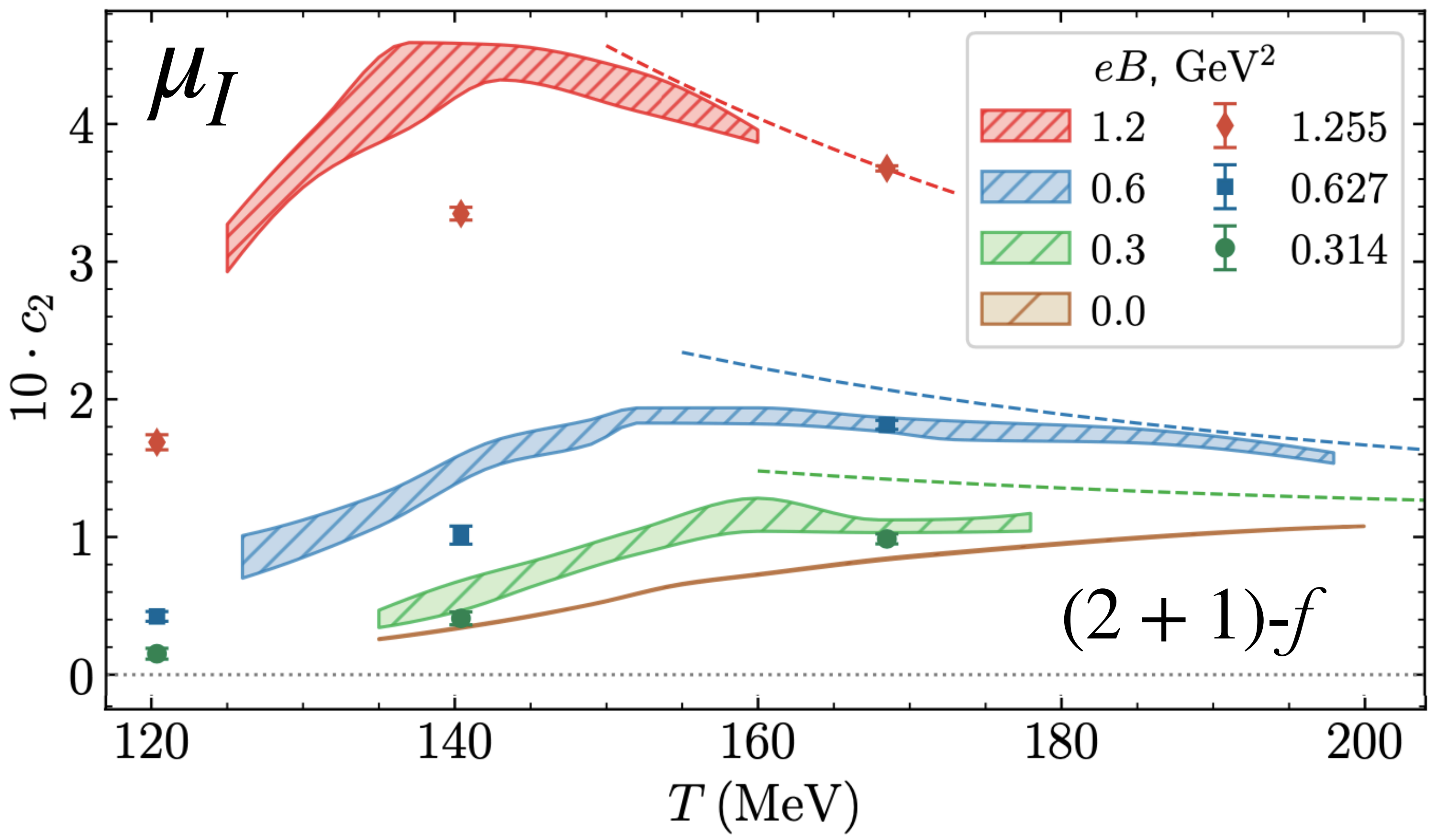
Borsanyi et al., *PoS* ***LATTICE2023*** *(2024) 164*



★ Recent review article:

"QCD with background electromagnetic fields on the lattice: a review"

Endrodi, arXiv:2406.19780



Astrakhantsev et al., *Phys.Rev.D* ***109*** *(2024) 9, 094511*

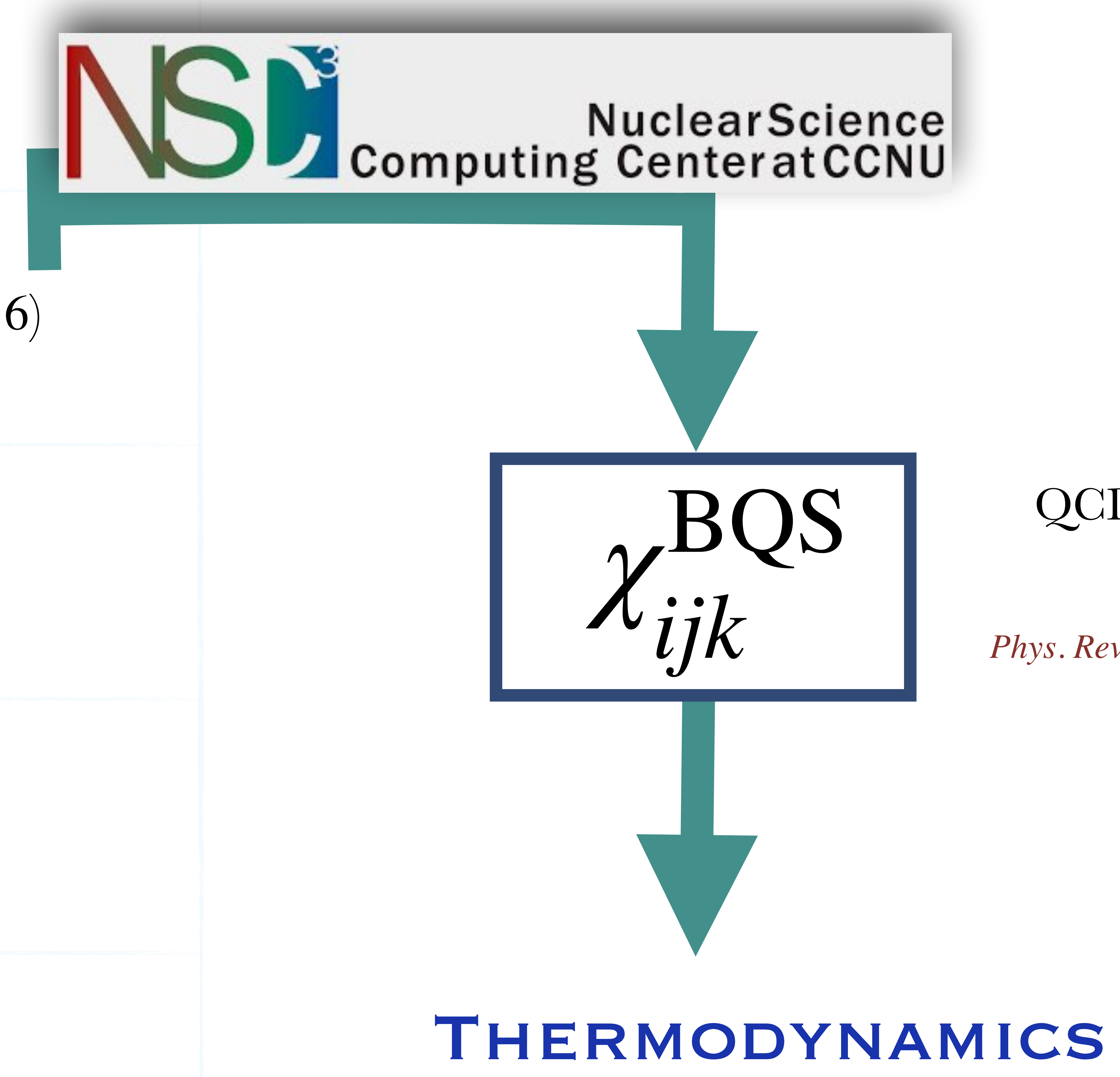
(2+1)-flavor QCD Lattice Ingredients

- HISQ & tree-level improved Symanzik gauge action
- Lattice: $N_\sigma / N_\tau = 4$ and $N_\tau = 8,\ 12 \rightarrow$ cont. est.
  (one additional $N_\tau = 16$)
- Non-zero μ and T: Taylor expansion, around T_{pc}
  $T \equiv [145 - 166)$ MeV
- Physical pion mass: $m_s^{\text{phy}} / m_{u/d} = 27$,
  $M_\pi \approx 135$ MeV
- Magnetic field: No sign-problem! Fixed U(1) phase factor with PBC, Bali et al., *JHEP 02 (2012) 044*

$$eB = 6\pi N_b\, a^{-2} N_\sigma^{-2}$$

ranging: $N_b = [1 - 32]$

$$[M_\pi^2 - \ 45 M_\pi^2) \sim [0.02 - 0.8)\ \text{GeV}^2$$



QCD magnetometer χ_{11}^{BQ}
Phys. Rev. Lett. **132**, 201903 (2024)

Initial Nuclei Conditions

$$\hat{p}(T, eB, \hat{\mu}) = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \boxed{\chi_{ijk}^{BQS}} \ \hat{\mu}_B{}^i \ \hat{\mu}_Q{}^j \ \hat{\mu}_S{}^k \rightarrow \hat{\mu}_B{}^{i+j+k}$$

✓

Strangeness neutrality : $n^S = 0$ + Isospin symmetry : $n^Q / n^B = r$

$$\hat{\mu}_{Q/S} \equiv \hat{\mu}_{Q/S}(T, eB, \hat{\mu}_B)$$

q_{2k-1}, s_{2k-1} →

$$\mu_Q/\mu_B = q_1 + q_3\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4) + \dots$$

$$\mu_S/\mu_B = s_1 + s_3\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4) + \dots$$

$r = 0.5$
Isospin symmetric

$r = 0.4$ ✓
HIC ~ Isospin asymmetry

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

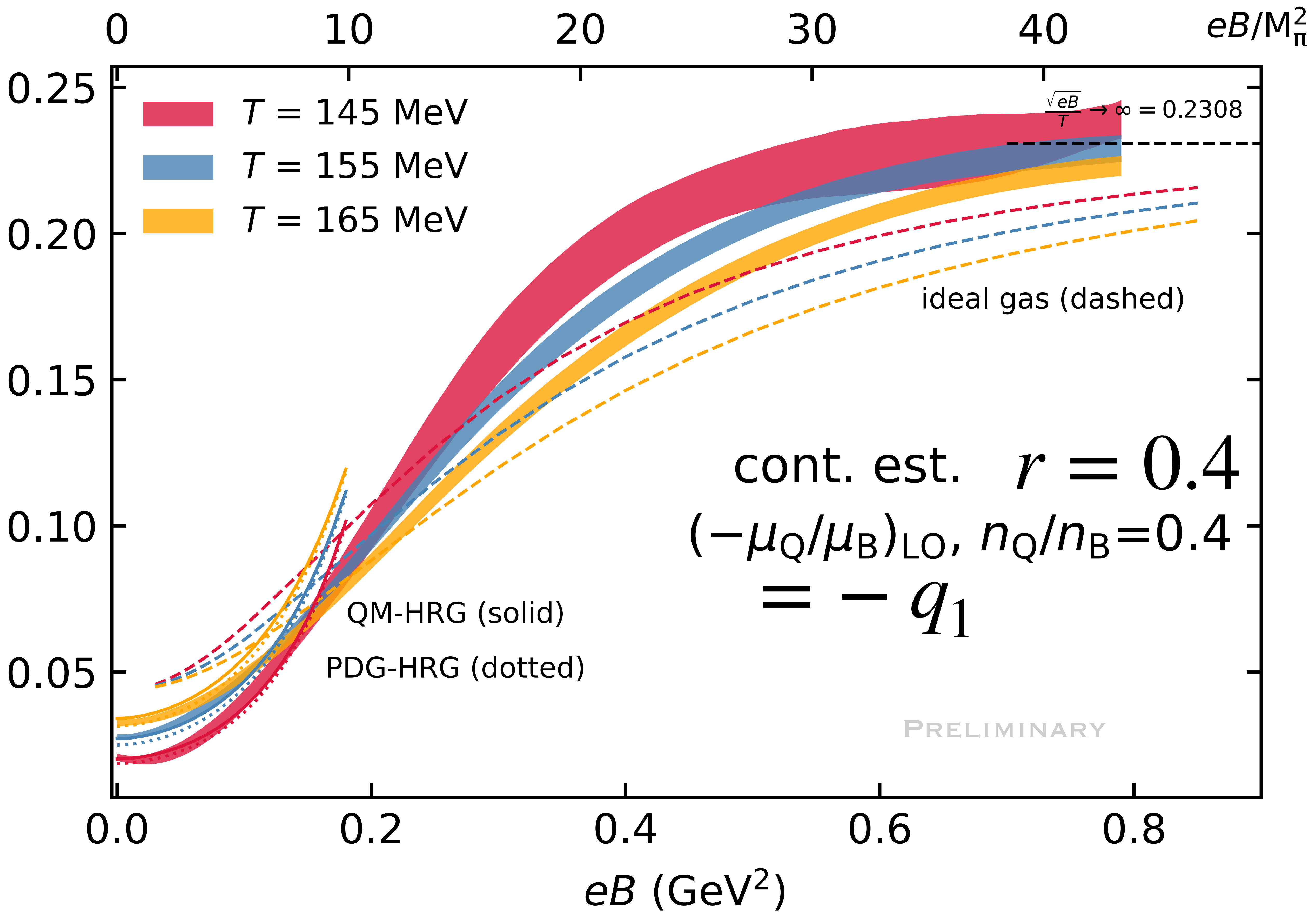
$$s_1 = -\frac{\left(\chi_{11}^{BS} + q_1\chi_{11}^{QS}\right)}{\chi_2^S}$$

HotQCD, *Phys. Rev. Lett.* ***109*** *(2012) 192302*

★ $P_2 \equiv f(\chi_{ijk}^{BQS}, q_1, s_1)$

μ_Q/μ_B IN THE PRESENCE OF eB

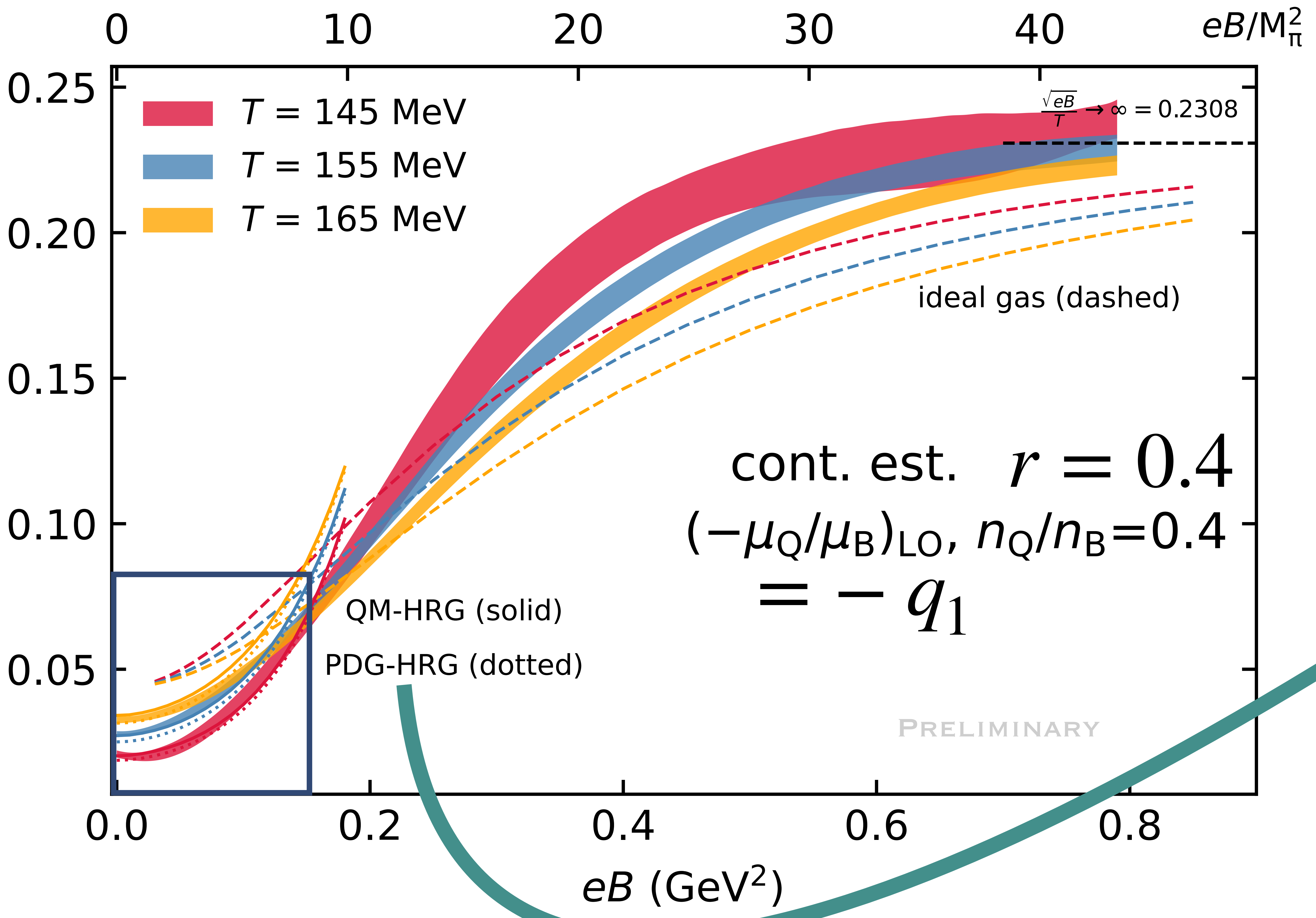
$$q_1 = \frac{r\left(\chi_2^{B}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^{Q}\chi_2^{S} - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$



A. q_1 is negative!
Grows/more -ve with eB!

μ_Q/μ_B in the presence of eB

$$q_1 = \frac{r\left(\chi_2^{B}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^{Q}\chi_2^{S} - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$



A. q_1 is negative!
Grows/more -ve with eB!

B. Good agreement with PDG-HRG and QM-HRG for smaller eB and low T

$$\frac{p^c_{\mathrm{HRG}}}{T^4} = \frac{|q_i|B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \epsilon_0 \times \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{e^{k\mu_i/T}}{k} \mathrm{K}_1\left(\frac{k\epsilon_0}{T}\right)$$

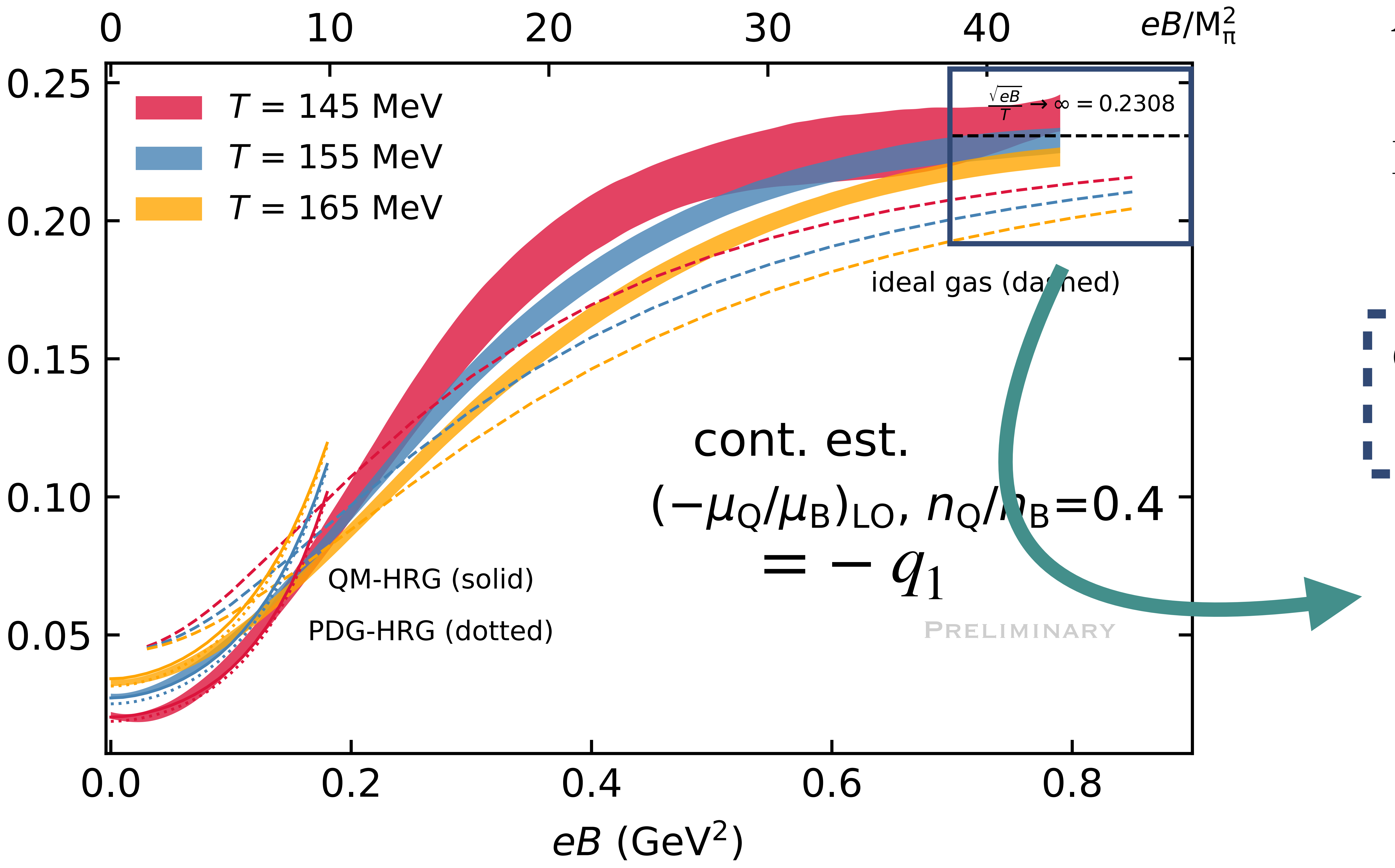
where $\epsilon_0 = \sqrt{m_i^2 + 2|q_i|B(l + 1/2 - s_z)}$

Fukushima & Hidaka, *Phys. Rev. Lett.* ***117***, *102301*

Ding, Li, Shi & Wang, *Eur. Phys. J.A* ***57*** *(2021) 6, 202*

μ_Q/μ_B IN THE PRESENCE OF eB

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$



A. q_1 is negative!
   Grows/more -ve with eB!

B. Good agreement with PDG-HRG and QM-HRG for smaller eB and low T

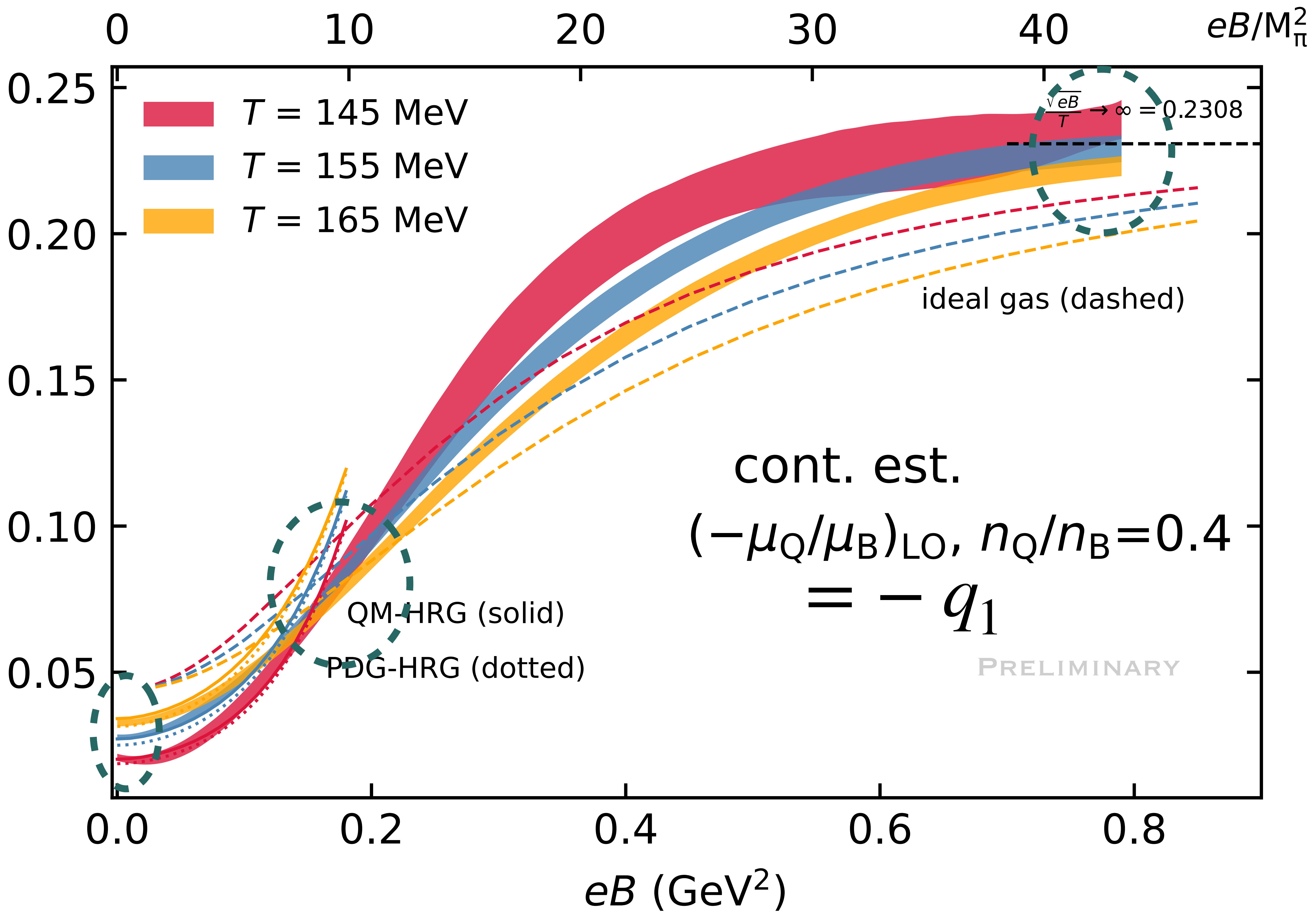
C. At very strong eB saturation to free limit

$$\frac{p_{\rm ideal}}{T^4} = \frac{8\pi^2}{45} + \sum_f \frac{3\,|q_f|\,B}{\pi^2 T^2}\left[\frac{\pi^2}{12} + \frac{\hat{\mu}_f^2}{4} + p_f^{ll}(B)\right]$$

Ding, Li, Shi & Wang, *Eur. Phys. J.A* **57** (2021) 6, 202

μ_Q/μ_B IN THE PRESENCE OF eB

$$q_1 = \frac{r\left(\chi_2^B \chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q \chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$



A. q_1 is negative!
   Grows/more -ve with eB!

B. Good agreement with PDG-HRG and QM-HRG for smaller eB and low T

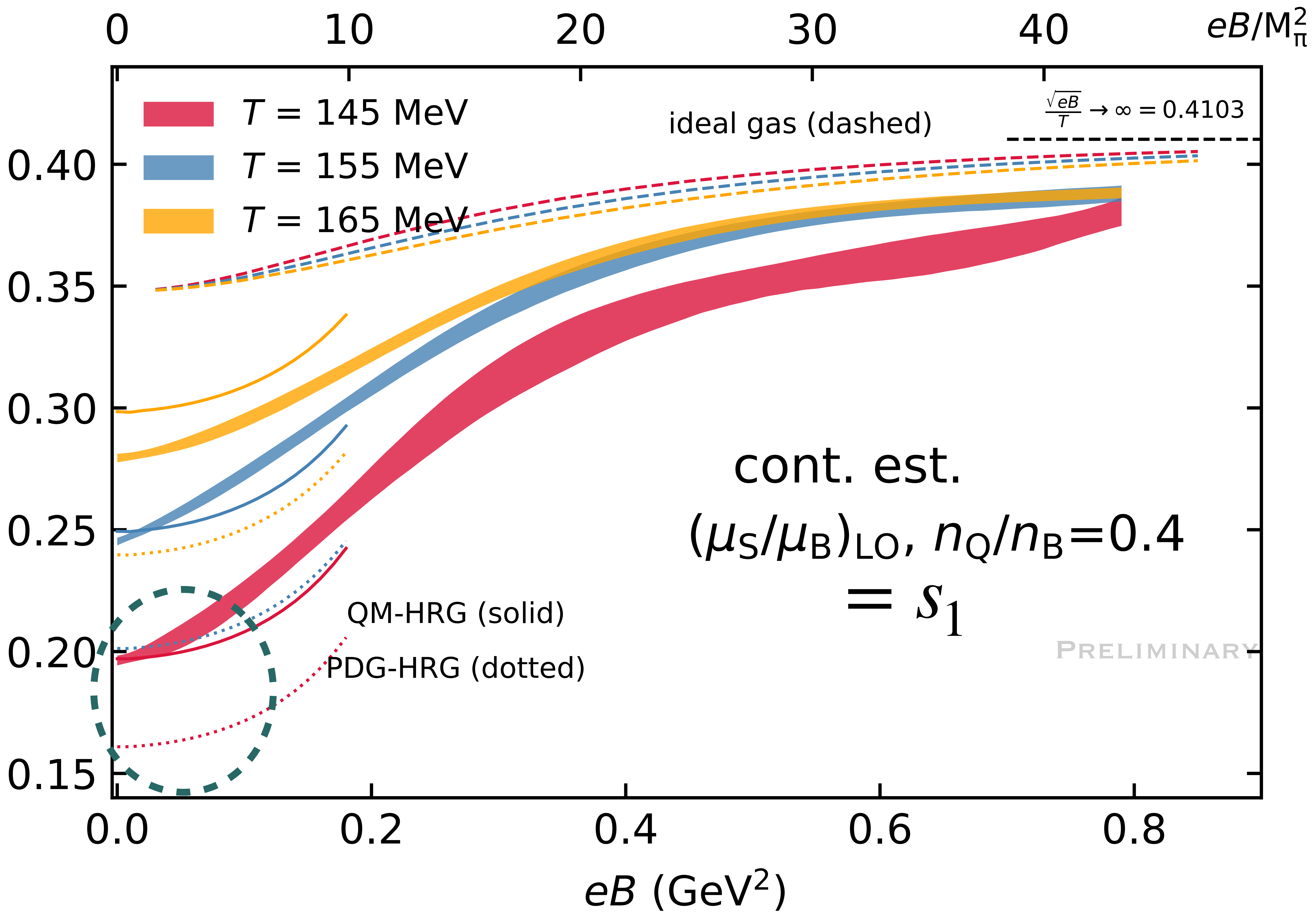
C. At very strong eB saturation to free limit

D. Crossing in T & sign of slope change at strong enough eB

near HRG: low $T \to$ small q_1
near ideal: low $T \to$ large q_1

μ_S/μ_B IN THE PRESENCE OF eB

$$s_1 = -\frac{\left(\chi_{11}^{BS} + q_1\chi_{11}^{QS}\right)}{\chi_2^S}$$



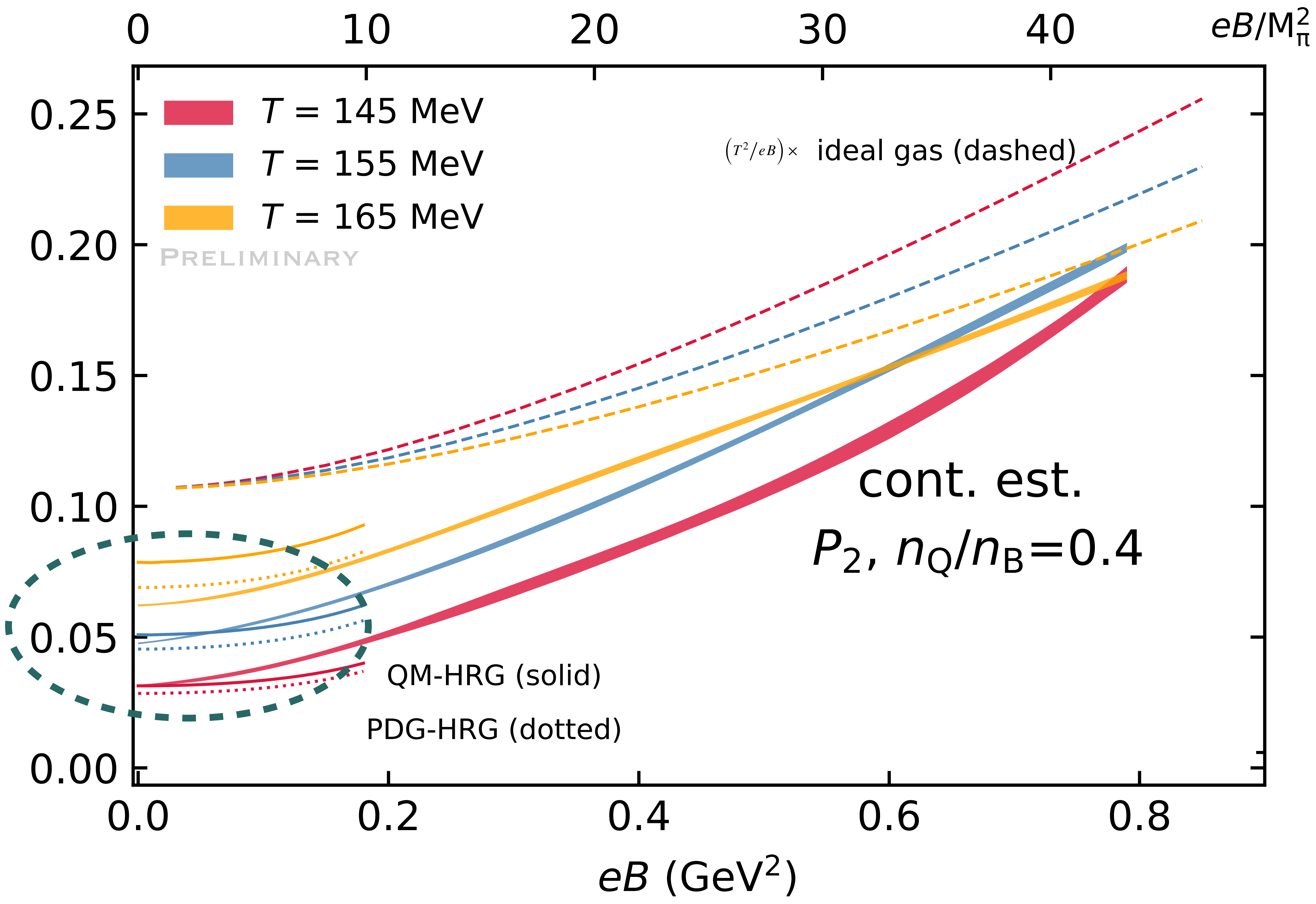
★ Lattice results better agreement with QM-HRG than PDG-HRG

Magnetic EoS: Pressure P_2

$$\Delta\hat{p} \equiv \hat{p}(T, eB, \mu_B) - \hat{p}(T, eB, 0) = \sum_{k=1}^{\infty} P_{2k}(T, eB)\ \hat{\mu}_B^{2k}$$

$$P_2(T, eB) = \frac{1}{2}\left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2\right) + \chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1$$

A. HRG agreement? Subject to smaller eB and low T



Magnetic EoS: Pressure P_2

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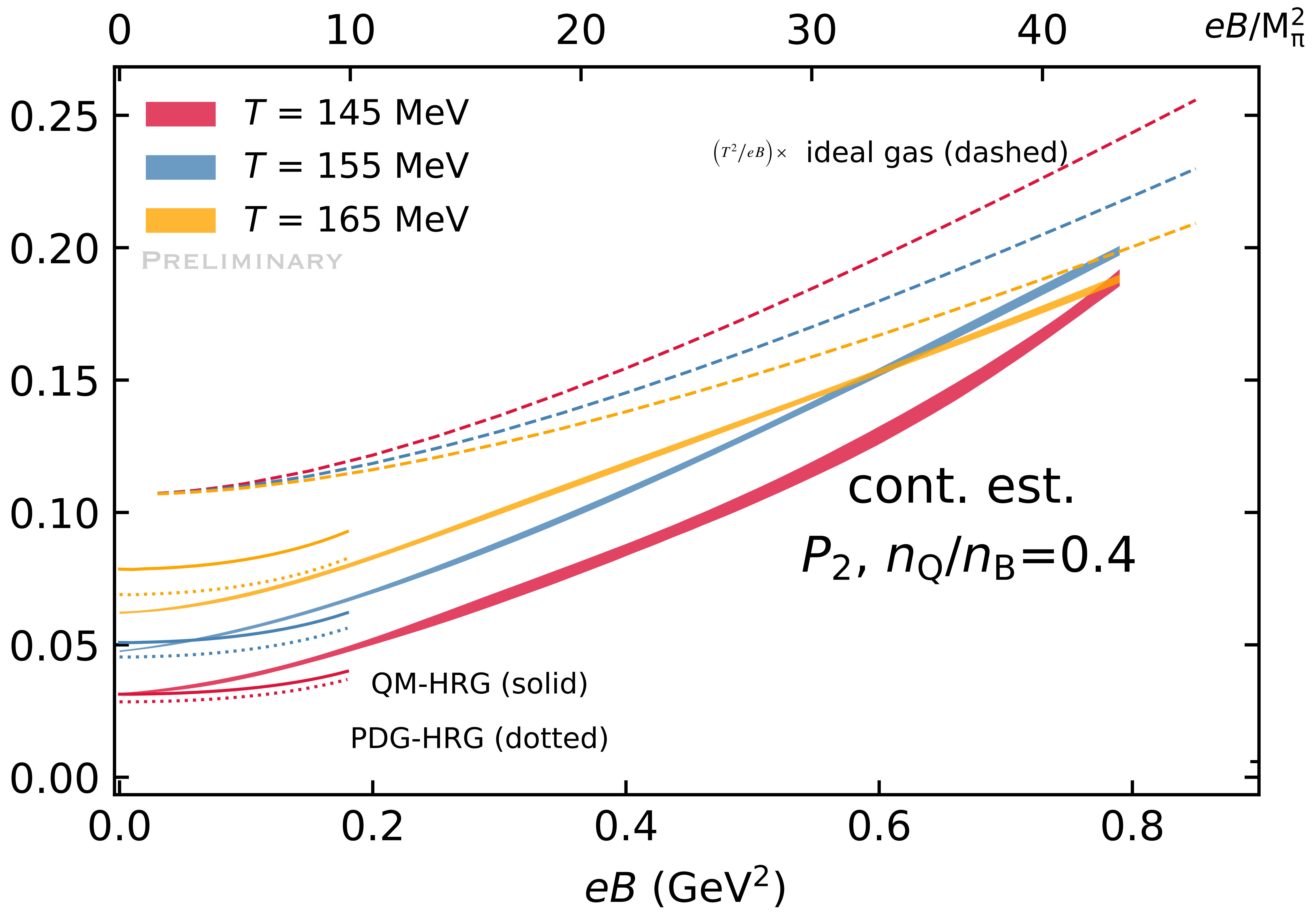
A. HRG agreement? Subject to smaller eB and low T

B. P_2 grows with eB, ideal gas saturation expected at high T



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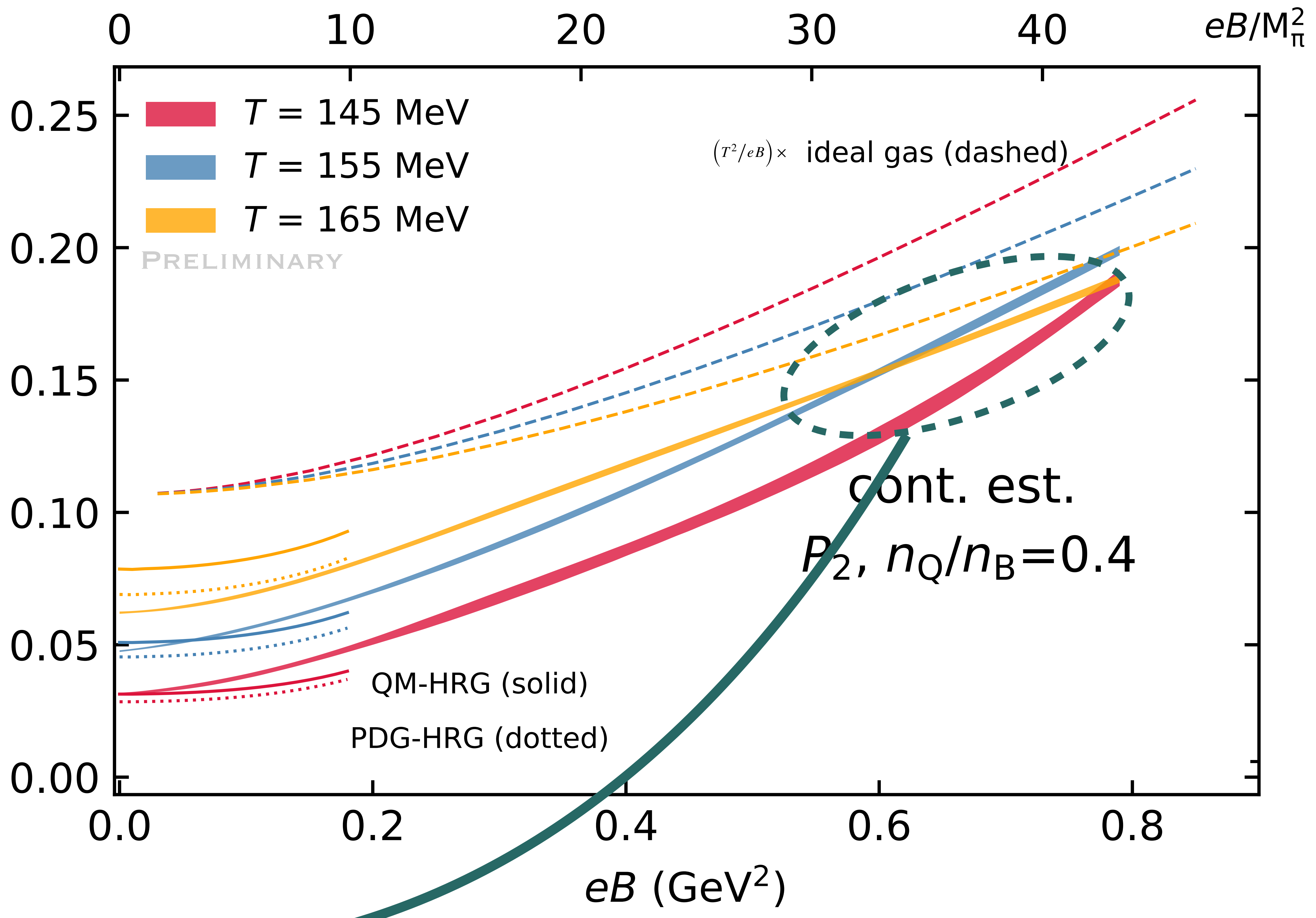
$$P_2(T, eB) = \frac{1}{2}\left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2\right) + \chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1$$

A. HRG agreement? Subject to smaller eB and low T

B. P_2 grows with eB, ideal gas saturation expected at high T

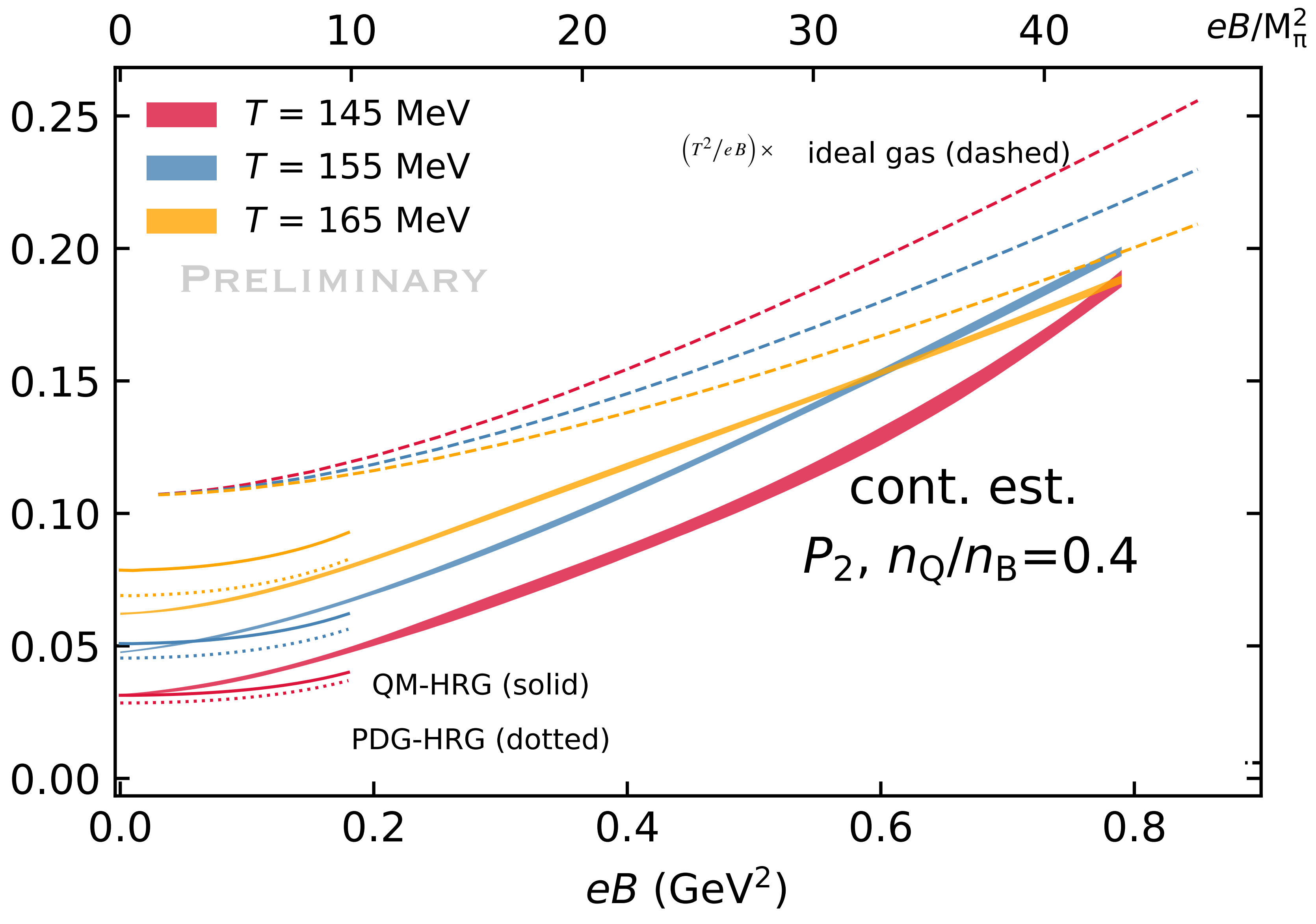
C. After $eB \sim 0.6$ GeV^2, signs of T crossing

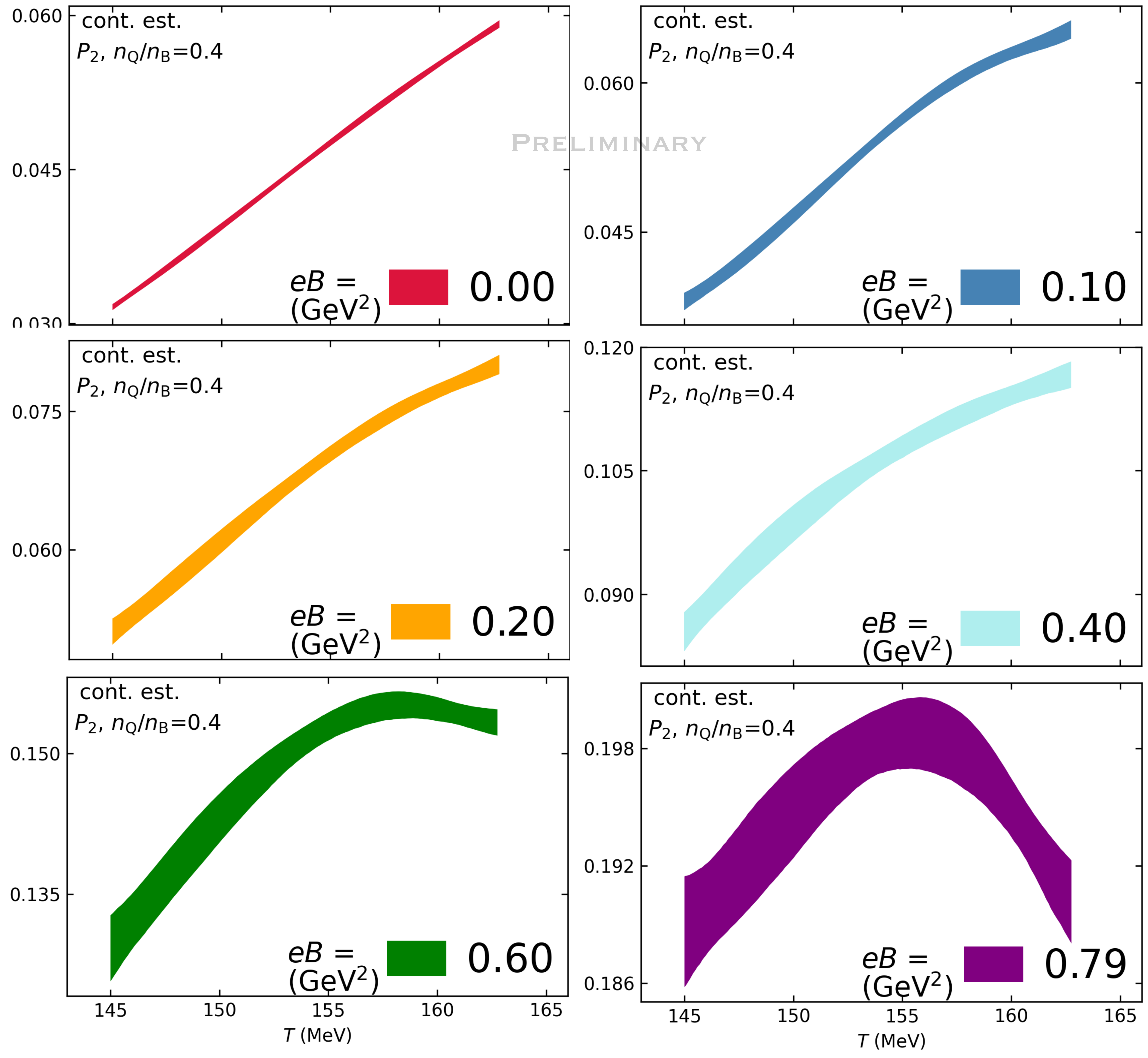


Magnetic EoS: Pressure P_2 vs T

★ Mild peak structure forms in P_2 and appears to have shifted towards low T as eB grows.

★ Interestingly, T_{pc} lowering consistent with chiral susceptibility!





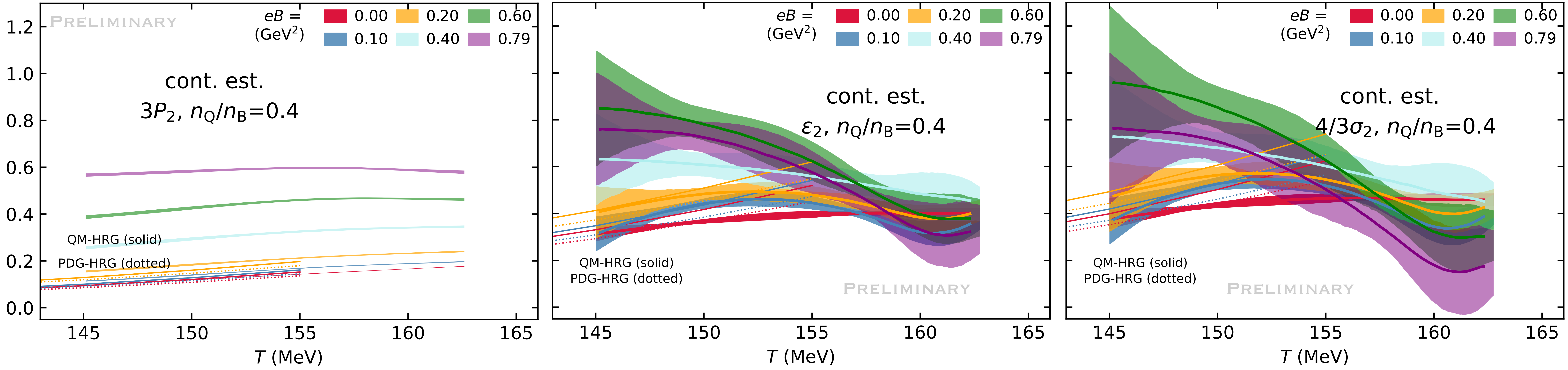
Energy and Entropy Density

$$\Delta\hat{\epsilon} \equiv \hat{\epsilon}(T,\mu_B) - \hat{\epsilon}(T,0) = \sum_{k=1}^{\infty} \epsilon_{2k}(T,eB)\,\hat{\mu}_B^{2k} \quad \& \quad \Delta\hat{\sigma} = \sum_{k=1}^{\infty} \sigma_{2k}(T,eB)\,\hat{\mu}_B^{2k}$$

$$\epsilon_2(T,eB) = 3P_2 + TP_2' - rTq_1'N_1^B$$

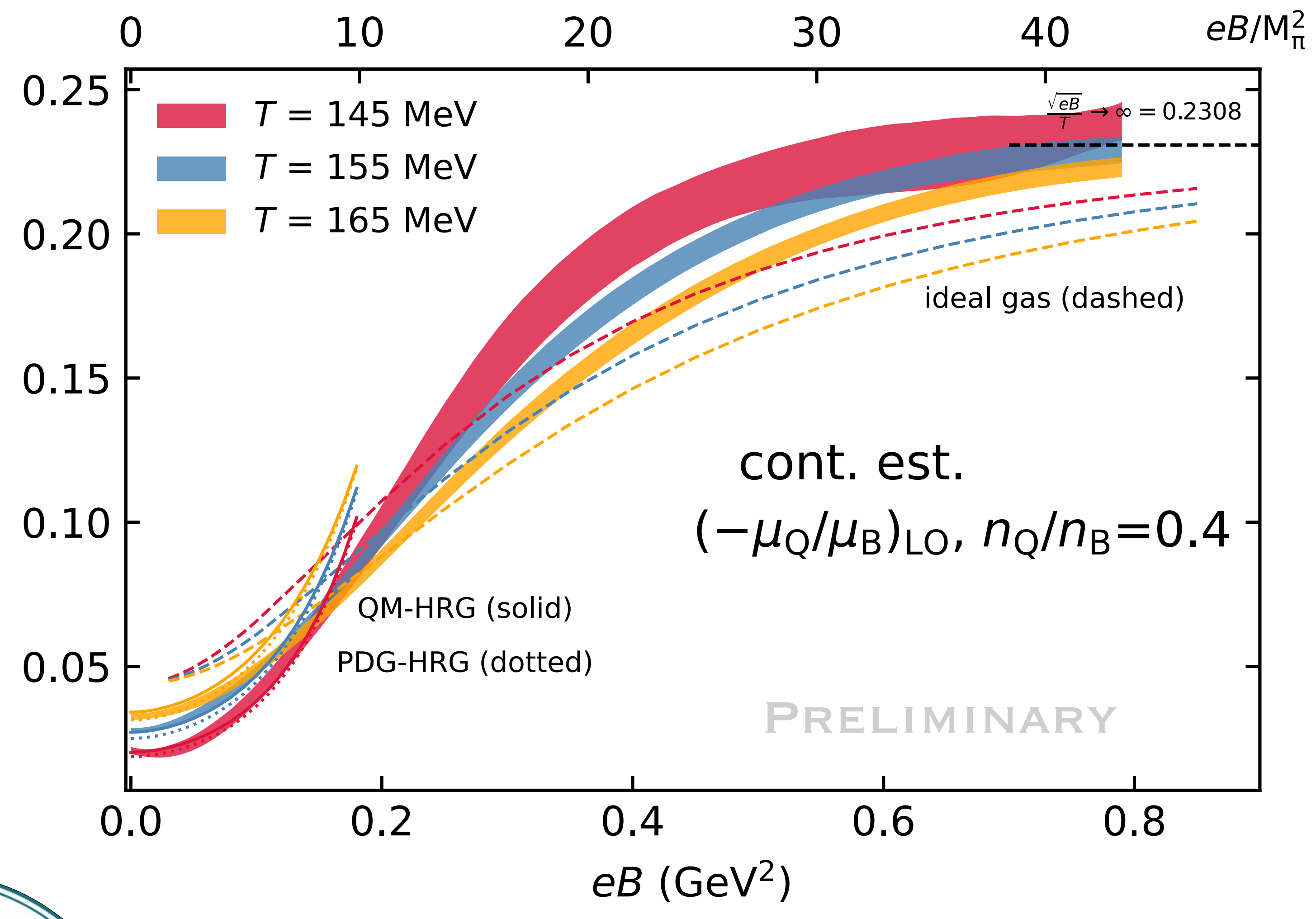
$$\sigma_2(T,eB) = \epsilon_2 + P_2 + TP_2' - (1+rq_1)N_1^B$$

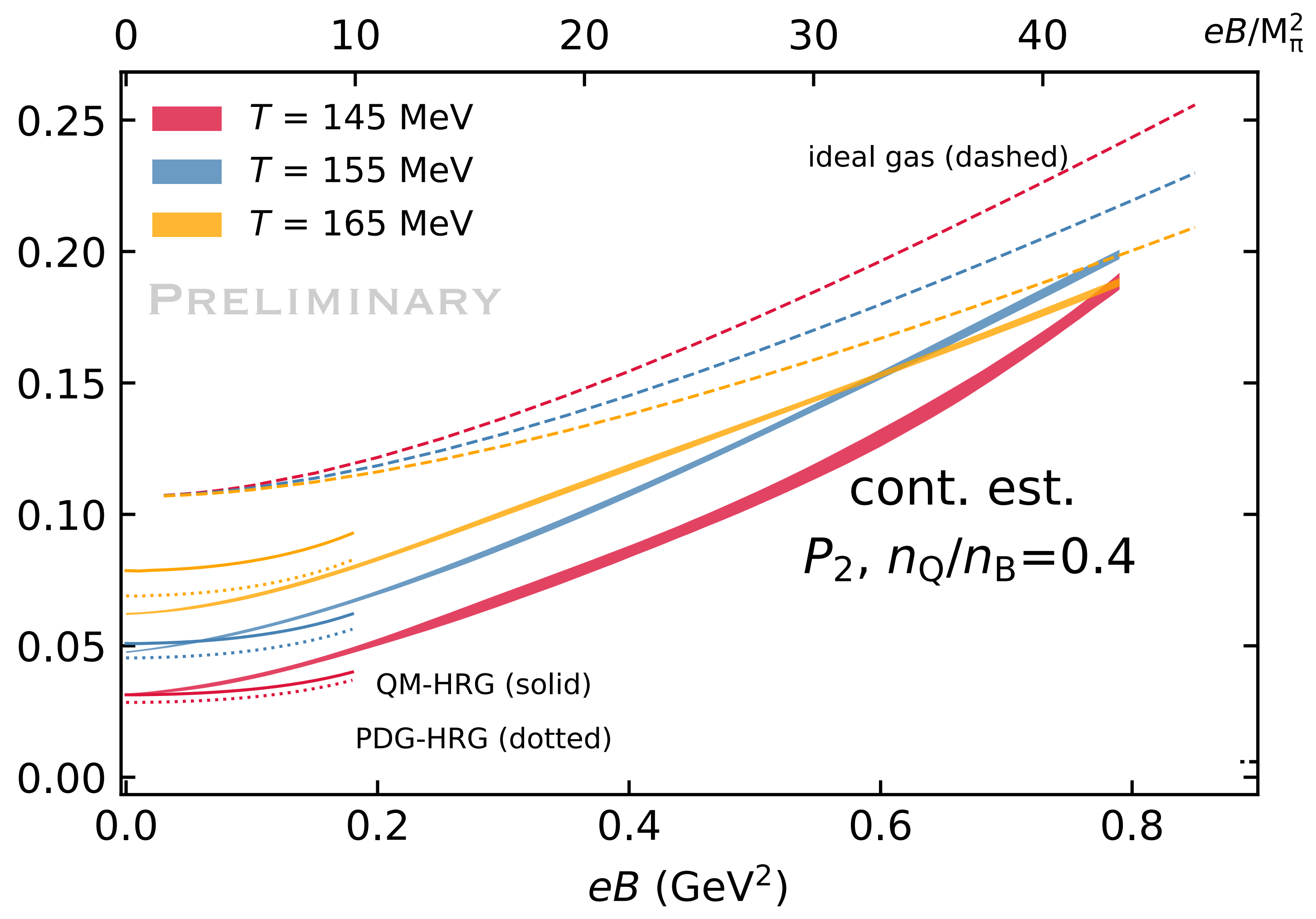
- ★ Clearly, very strong eB modifies the T dependence of P_2, ϵ_2 and σ_2
- ★ Peak structure developed in P_2, corresponds to decrease in magnitude of ϵ_2 and σ_2

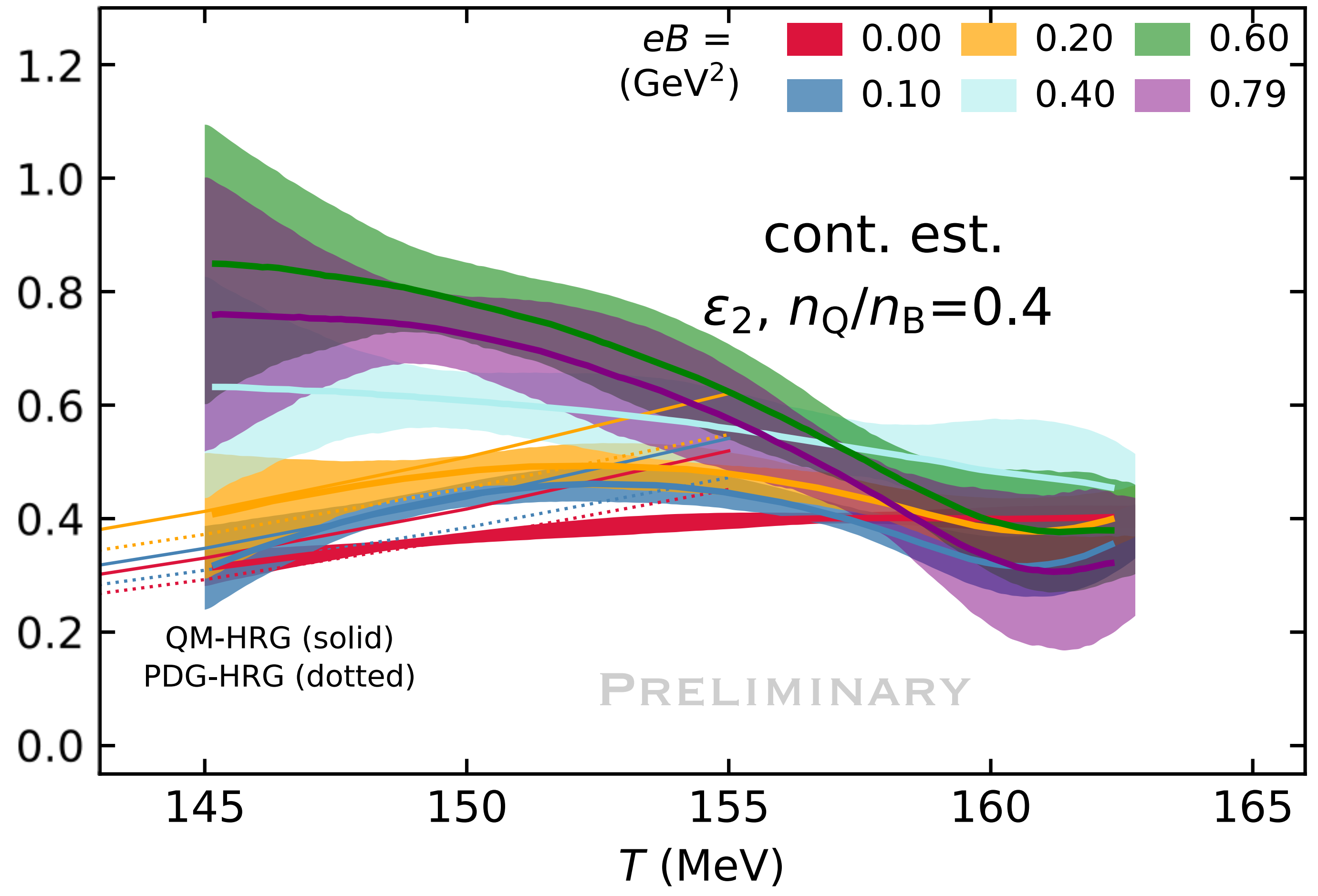


Take Home Message

- ★ Explored (2 + 1)-f QCD magnetic EoS at non-zero density, upto leading order, from first principle lattice calculation using Taylor expansion
- ★ HRG breaks down in strong eB regime. For smaller eB, good agreement with QM-HRG subject to lower T
- ★ Different growth rates of bulk observables with eB. Crossing in T, and mild peak shift of P_2 towards low T as eB grows; T_{pc} lowering







Thank You!

Central China Normal University

Nuclear Science Computing Center at CCNU

Some Backups!

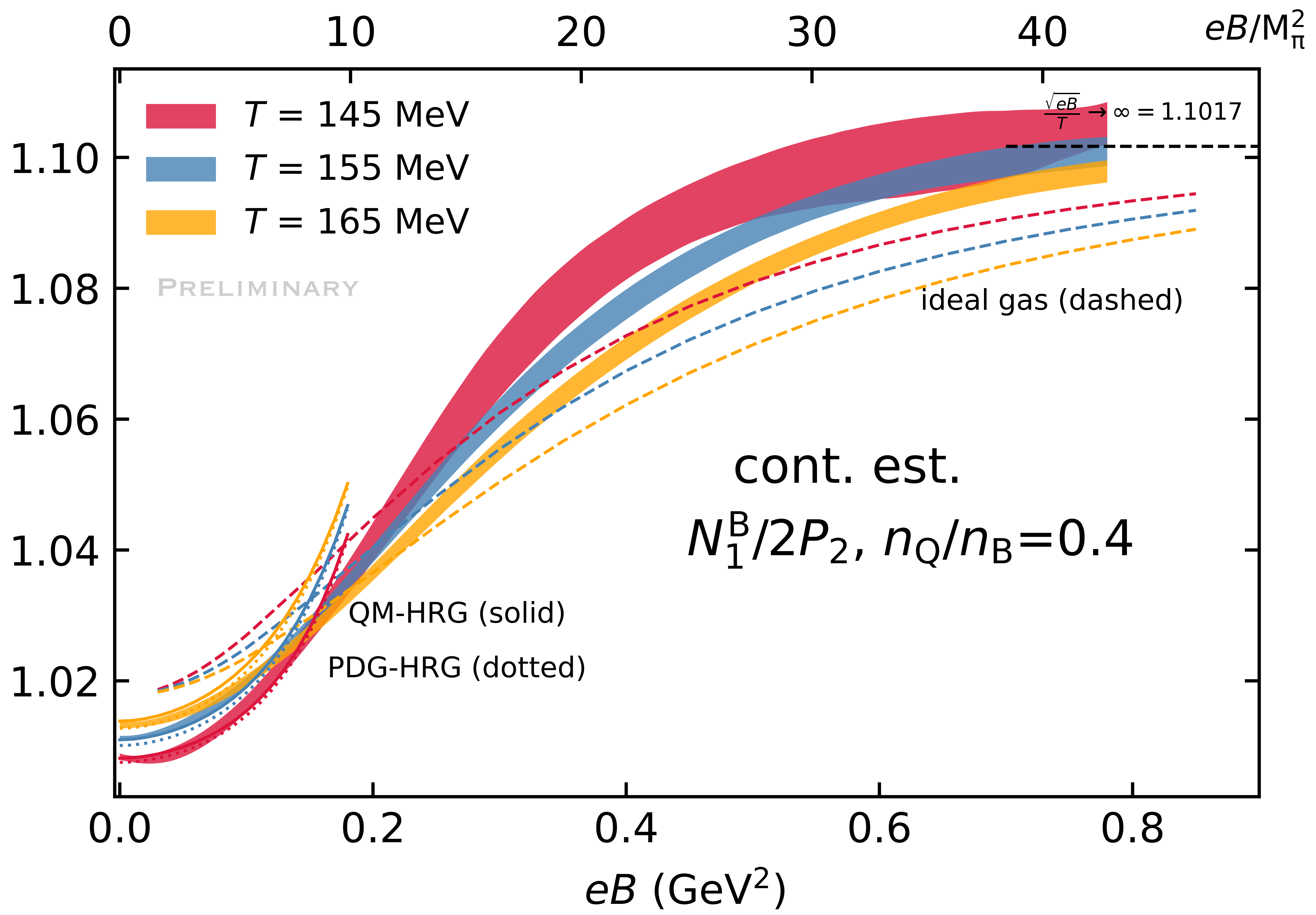
Baryon Density over Pressure

$$\hat{n}^{\mathscr{C}} \equiv \partial_{\hat{\mu}_{\mathscr{C}}} \hat{p} = \sum_{k=1}^{\infty} N^{\mathscr{C}}_{2k-1}(T, eB)\ \hat{\mu}_{\mathrm{B}}^{2k-1}$$

$$N_1^B(T, eB) = \chi_2^{\mathrm{B}} + q_1 \chi_{11}^{\mathrm{BQ}} + s_1 \chi_{11}^{\mathrm{BS}}$$

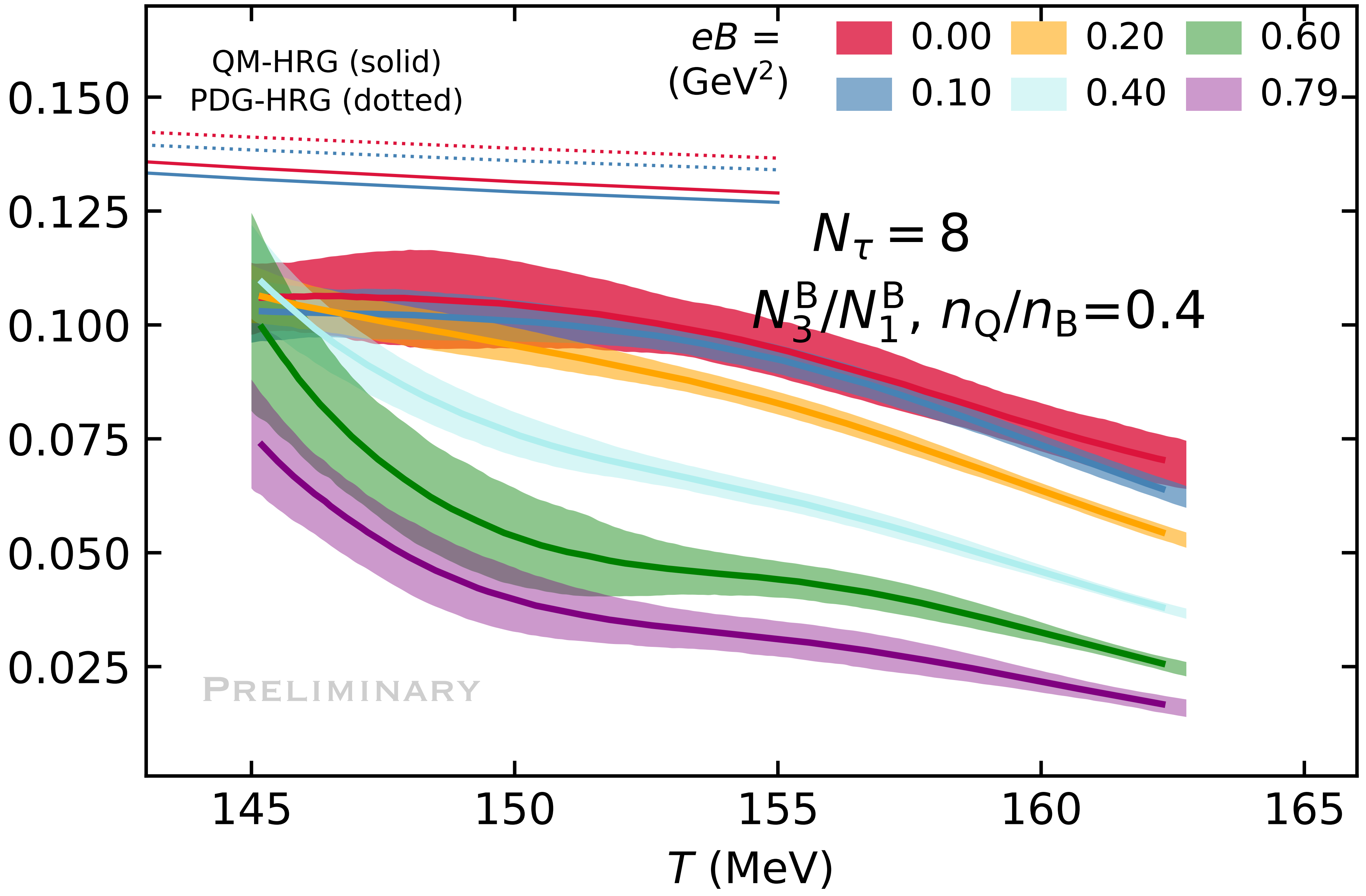
Consider $N_1^B / 2P_2$

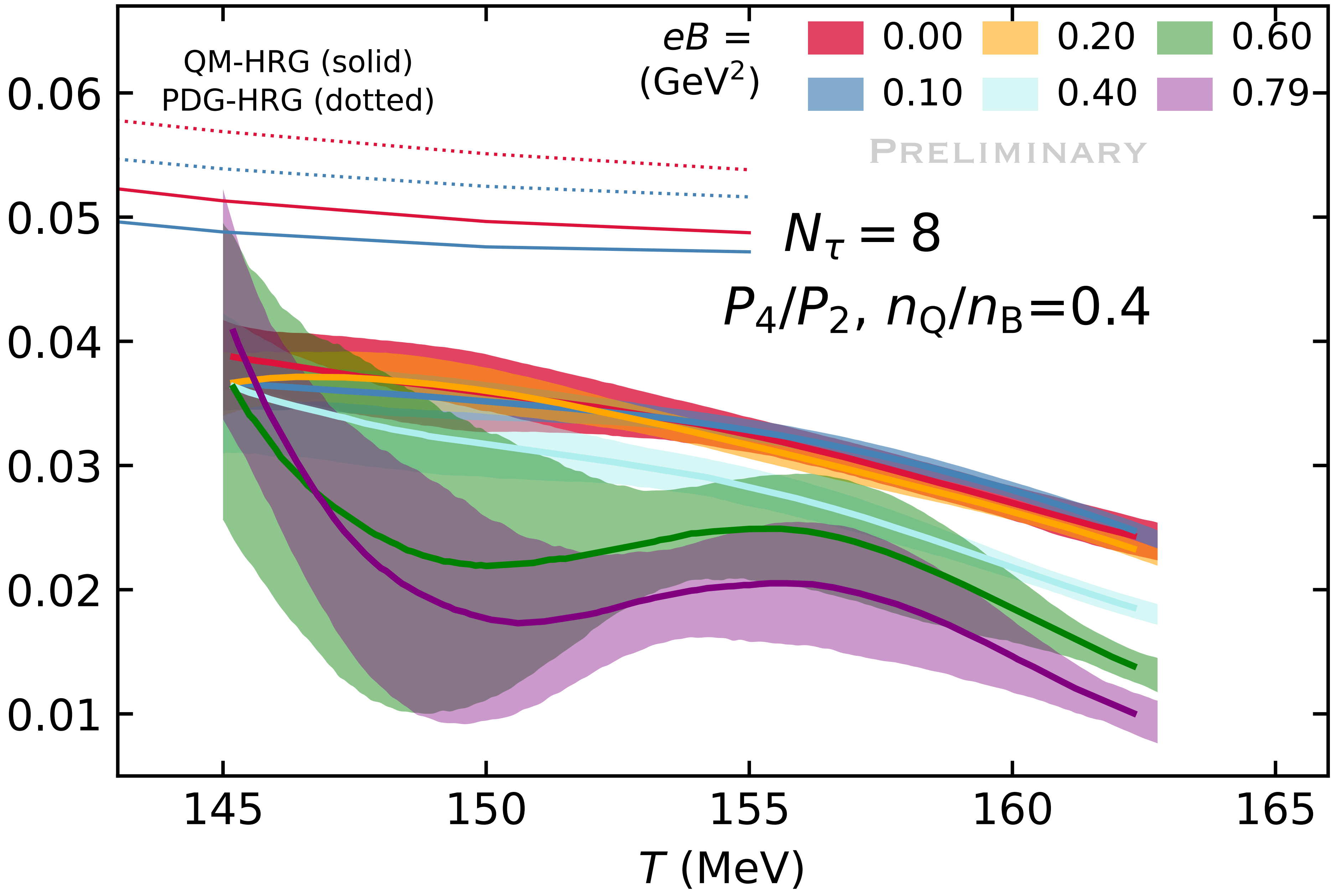
- ★ Deviation from unity, reflects isospin symmetry breaking by rq_1 factor
- ★ $N_1^{\mathrm{B}} / 2P_2$ saturates at very strong eB



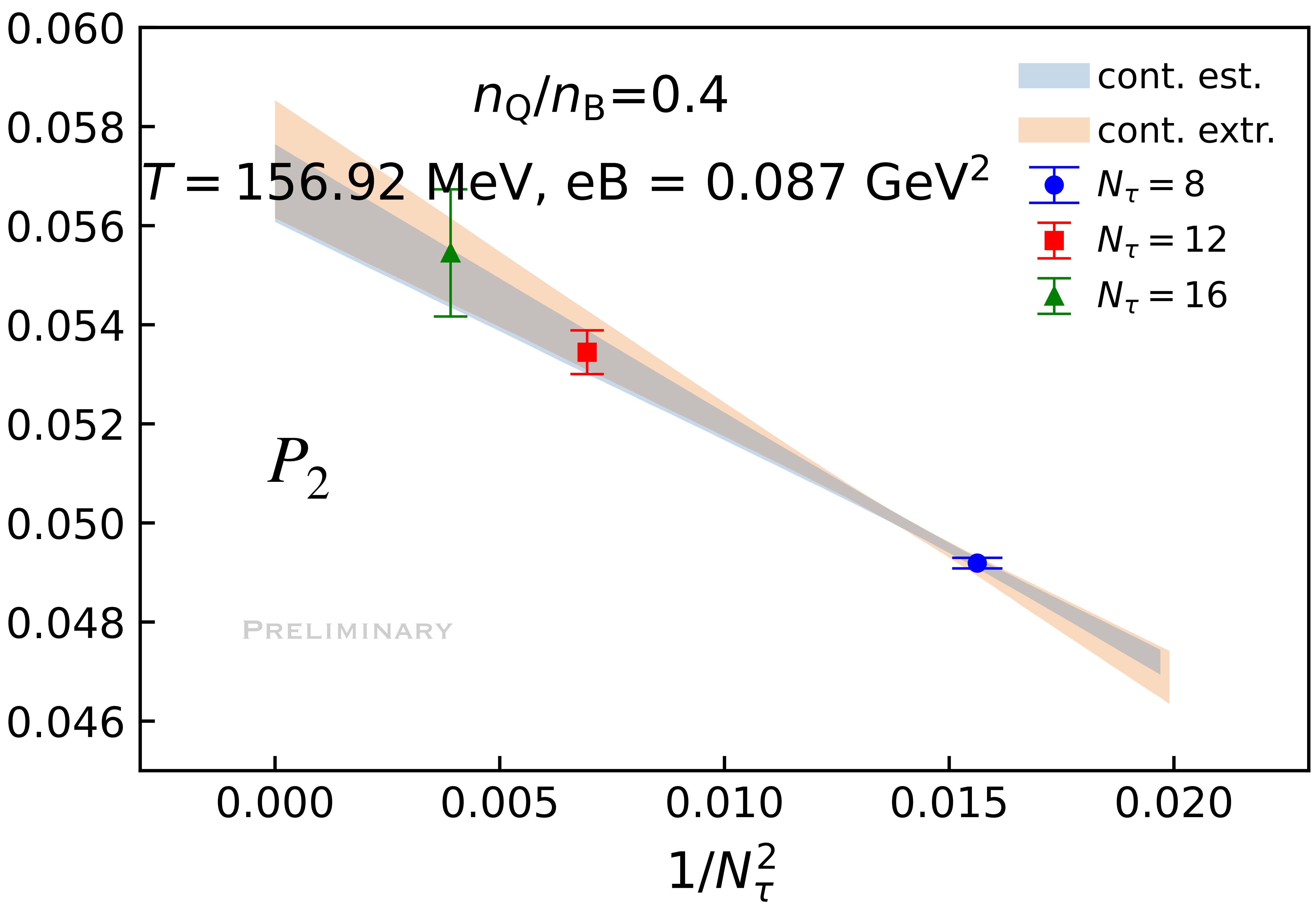
Next-to-leading order

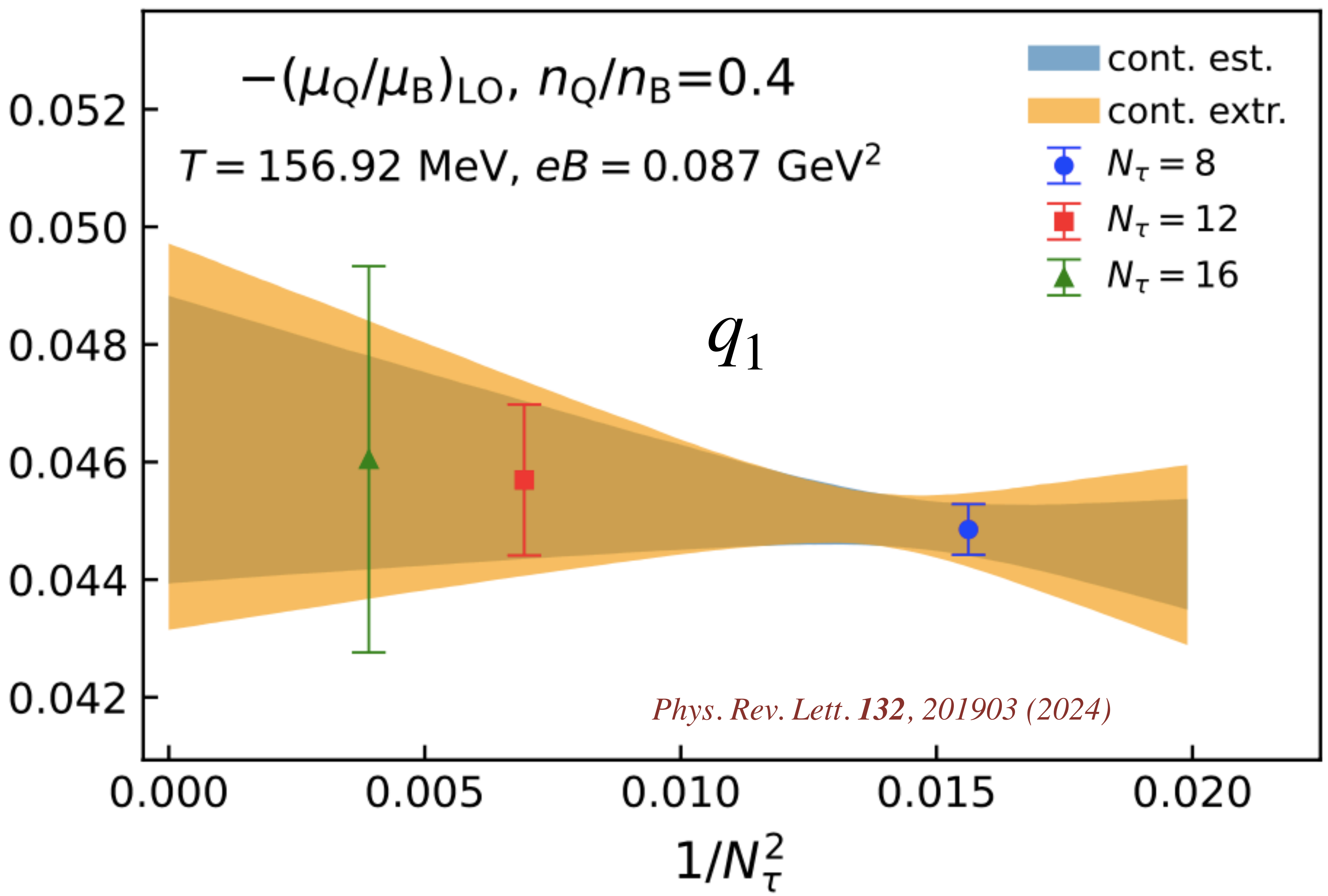
★ Ongoing work: insights on next-to-leading order contributions. n^{B} dominant to Δp (factor ~ 2), but interestingly as eB grows contributions reduce drastically.



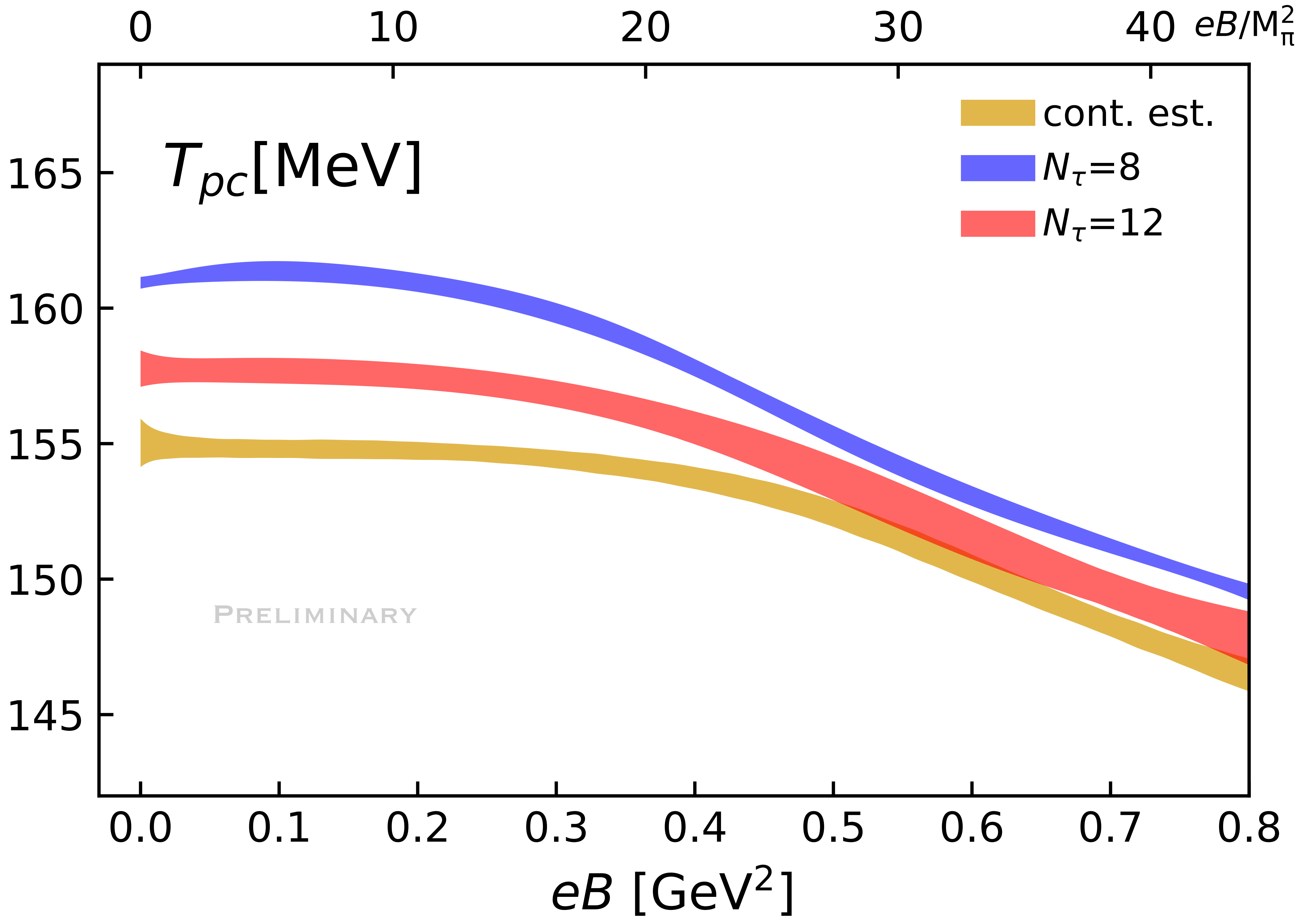


Continuum Estimates vs Extrapolations





Transition Line and Chiral Susceptibility



★ Finding the peak location of χ_M at each value of eB to determine $T_{pc}(eB)$

$$M = \frac{1}{f_K^4}\left[m_s\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) - \left(m_u + m_d\right)\langle\bar{\psi}\psi\rangle_s\right]$$

$$\chi_M(eB) = \frac{m_s}{f_K^4}\left[m_s\chi_l(eB) - 2\langle\bar{\psi}\psi\rangle_s(eB=0) - 4m_l\chi_{su}(eB=0)\right]$$

Thank you!