First-order phase transition in dynamical QCD at imaginary isospin

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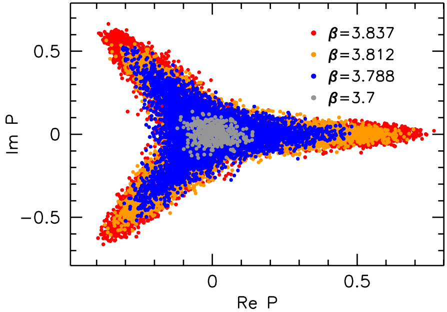
CRC-TR 211 Strong-interaction matter under extreme conditions

Lattice'24 @ Liverpool, July 30, 2024

Appetizer

- Polyakov loop histogram in a QCD simulation with dynamical fermions



Outline

- theory: center-symmetric three-flavor QCD at imaginary isospin
- finite size scaling analysis of observables
- mass dependence and phase diagram
- summary

Center symmetry

Center symmetry

- pure gauge theory

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-\beta S_g}$$

- center transformation

$$\forall \boldsymbol{n}: \qquad U_4(\boldsymbol{n}, n_4) \to z\, U_4(\boldsymbol{n}, n_4) \qquad z \in \mathrm{Z}(3) = \{\mathbb{1}, \mathbb{1}\, e^{i2\pi/3}, \mathbb{1}\, e^{-i2\pi/3}\} \subset \mathrm{SU}(3)$$

- invariance

$$S_g \to S_g$$

- with Polyakov loop as order parameter

$$P = \frac{1}{V} \sum_{\boldsymbol{n}} \prod_{n_4=1}^{N_t} U_4(\boldsymbol{n}, n_4) \to z\, P$$

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- invariance

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- with Polyakov loop as order parameter, revealing SSB at high T

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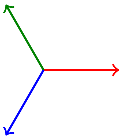

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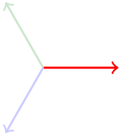
$$\log\det(\not{D}(0) + m) \overset{\text{large } m \text{ expansion}}{\approx} P\text{-independent} + \frac{1}{2m^{N_t}} \mathrm{Re}\left(P \qquad \right) + \mathcal{O}(m^{-2N_t})$$



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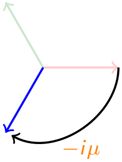

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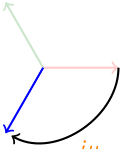

- including $\mu = i2\pi T/3$ equivalent to center trafo $z = \mathbb{1} e^{-i2\pi/3}$

Center-symmetric QCD

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- three degenerate quarks with imaginary isospin

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- amounts to imaginary isospin chemical potential $i\mu_I = 4\pi T/3$

$$\mu_u = \mu_I/2, \qquad \mu_d = -\mu_I/2, \qquad \mu_s = 0$$

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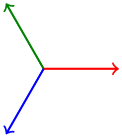

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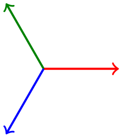
- first observed in PNJL model Kouno et al. '12
  general discussion on symmetries Cherman et al. '17
  hints for first-order transition on the lattice Iritani et al. '15
  scenarios for deconfinement, chiral symm. restoration Yonekura '19 Tanizaki et al. '17



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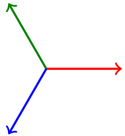

- goal: demonstrate first-order deconfinement and chiral transition

Results

Lattice setup and rotated Polyakov loop

- rooted stout-smeared staggered quarks

  $m_{u,d,s} = m_{ud}^{\rm phys}, \quad m_s^{\rm phys}, \quad 3 \cdot m_s^{\rm phys}$

  $N_s^3 \times N_t = 16^3 \times 6 \ldots 32^3 \times 6$

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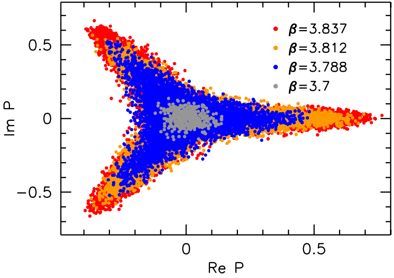

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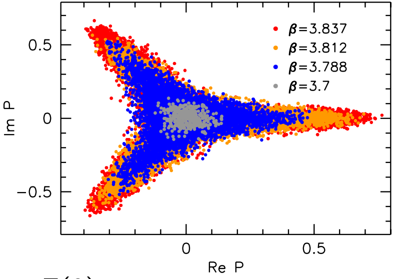
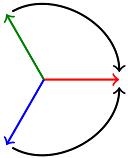


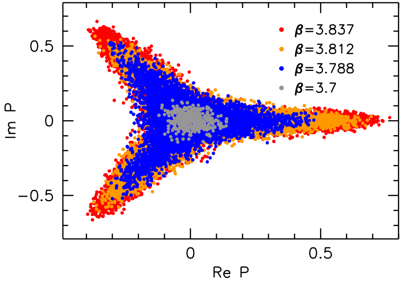
- rotated Polyakov loop $z_U \in \mathbb{Z}(3)$

$$\bar{P} = \mathrm{Re}\,(z_U P), \qquad -\pi/3 \leq \arg(z_U P) \leq \pi/3$$



Histograms and MC history

- for $m_{u,d,s} = m_s^{\mathrm{phys}}$, $16^3 \times 6$



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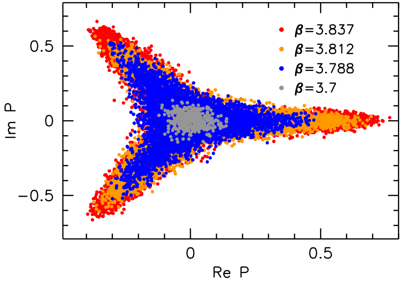

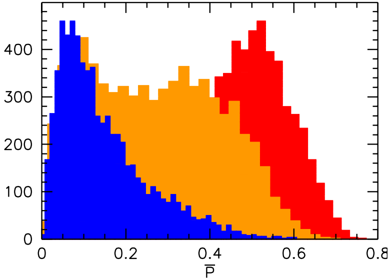

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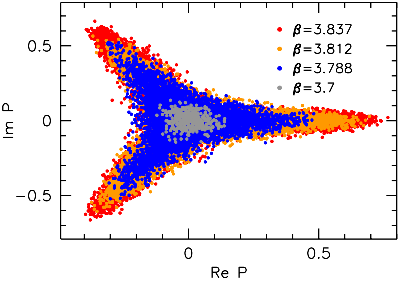
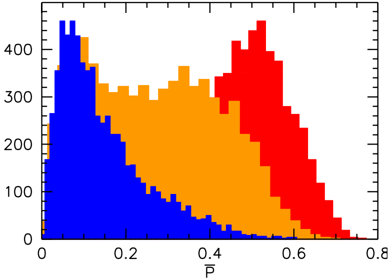
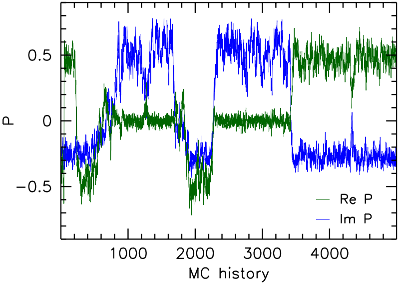
- Monte-Carlo history of P around β_c

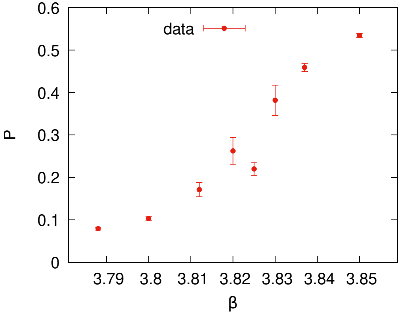






Multi-histogram method

- raw data for $\bar{P}$ on $16^3 \times 6$, $m_{u,d,s} = m_s^{\text{phys}}$



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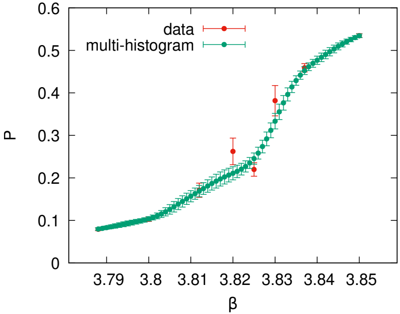

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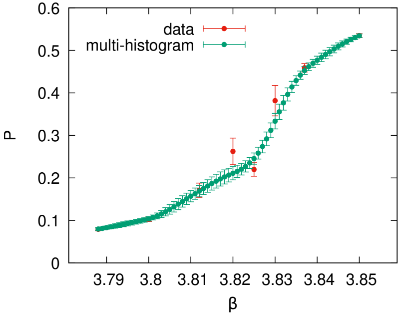
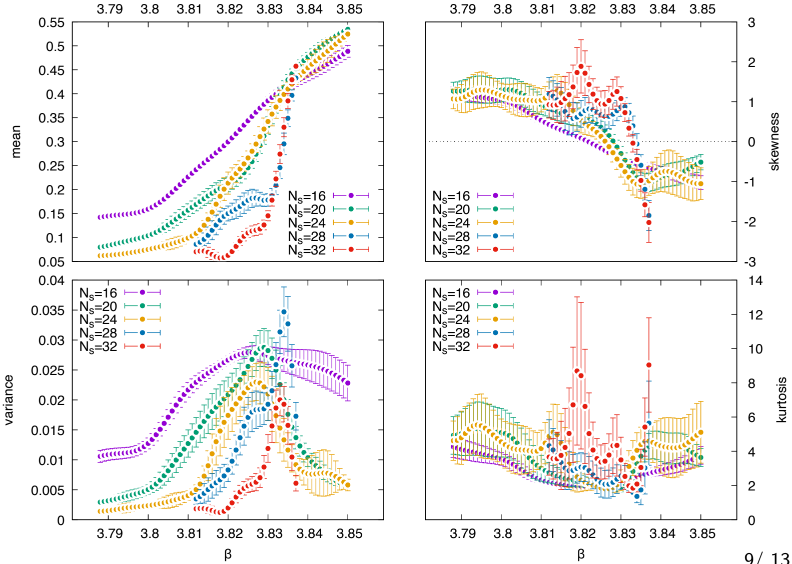


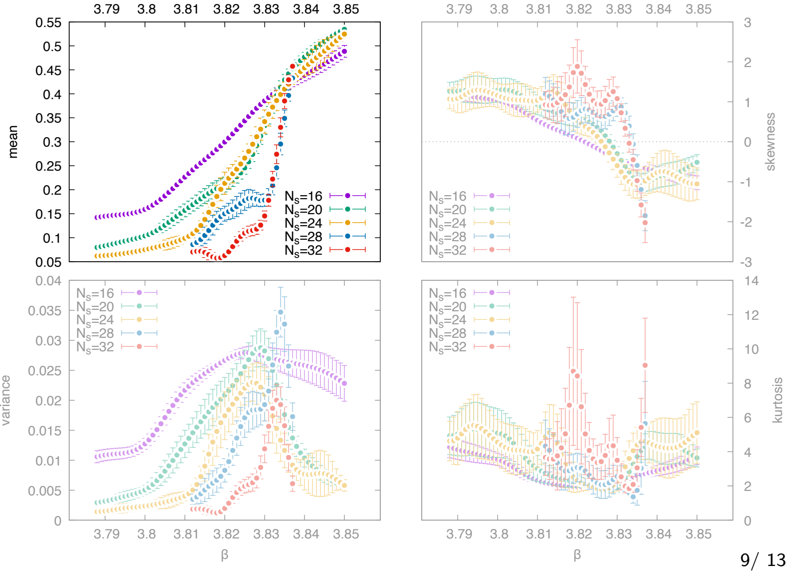
- allows interpolation of higher moments

Finite size scaling



Finite size scaling

- $\bar{P}$ gets steeper as $V \to \infty$



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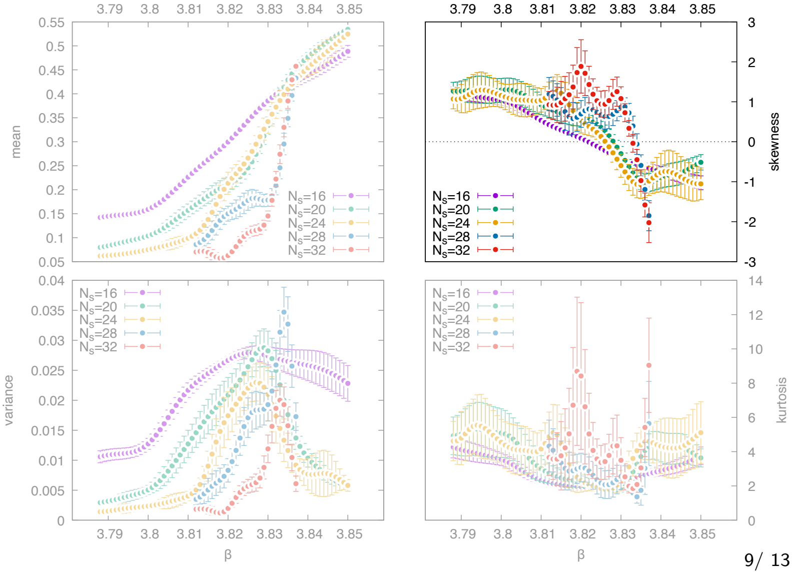
- $\bar{P}$ gets steeper as $V \to \infty$
- skewness $= 0$ to define T_c



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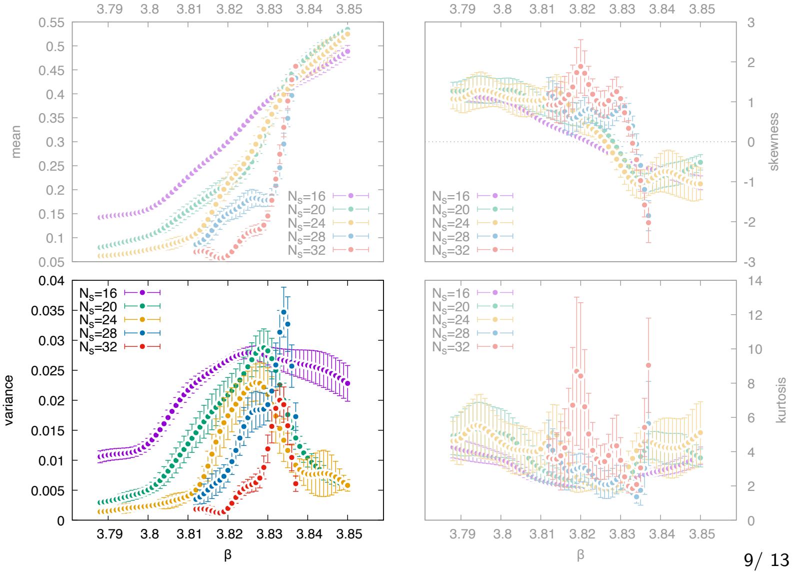

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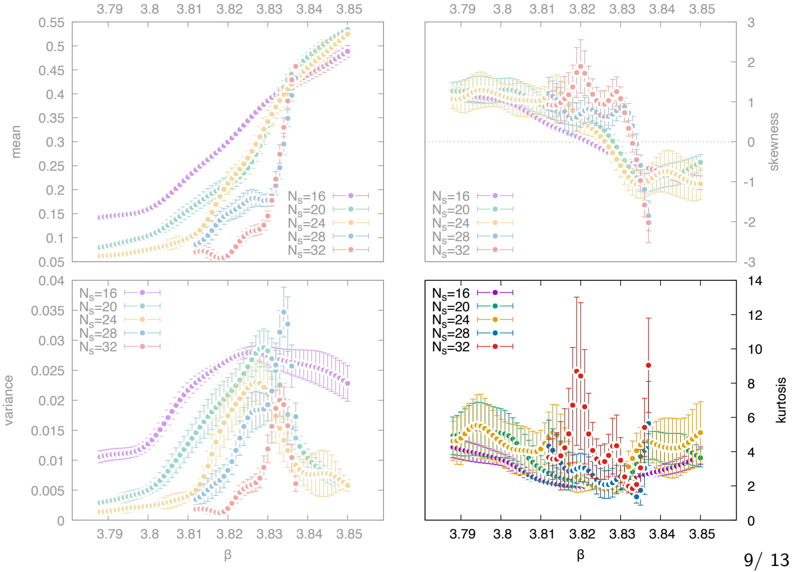
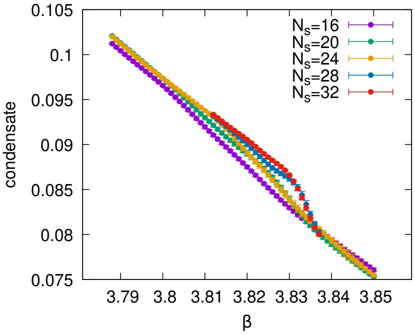


Chiral symmetry restoration

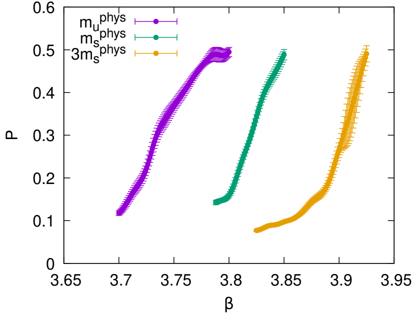
- how is chiral symmetry restoration affected by first-order deconfining phase transition?
- center-symmetric quark condensate Cherman et al. '17

$$\bar{\psi}\psi = \bar{\psi}_u\psi_u + \bar{\psi}_d\psi_d + \bar{\psi}_s\psi_s$$



Impact of quark mass and scale

- first-order nature of transition not expected to be affected by $m_{u,d,s}$
- scan for $16^3 \times 6$



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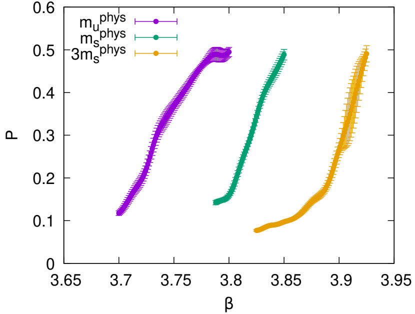

- calculate scale at $T = \mu = 0$ via w_0

$m_{u,d,s}$	m_{ud}^{phys}	m_s^{phys}	$3 \cdot m_s^{\mathrm{phys}}$
$T_c w_0$	0.239(1)	0.248(1)	0.2505(5)

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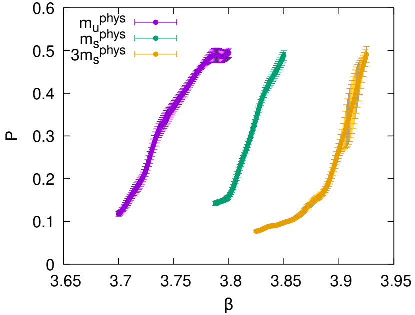
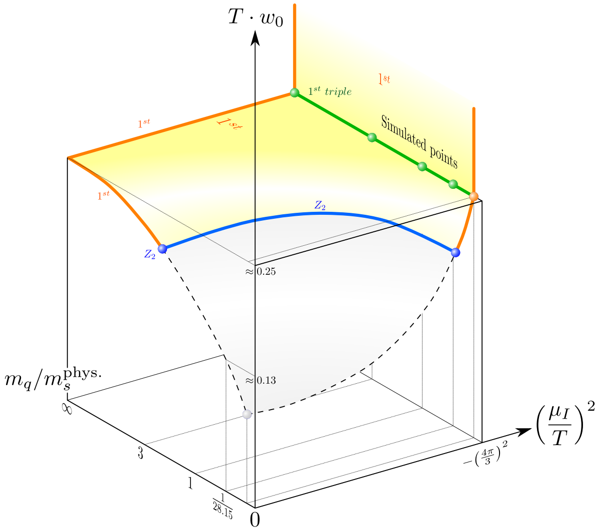


- calculate scale at $T = \mu = 0$ via w_0 comparison

$m_{u,d,s}$	m_{ud}^{phys}	m_s^{phys}	$3 \cdot m_s^{\text{phys}}$	pure gauge	m_{ud}^{phys}, m_s^{phys}
$T_c w_0$	0.239(1)	0.248(1)	0.2505(5)	0.2507(2) Borsányi et al. '22	$\approx$0.14 Borsányi et al. '12

Phase diagram



Summary

- QCD with $m_u = m_d = m_s$ at $\mu_I = 4\pi i T/3$ has exact $\mathbb{Z}(3)$ symmetry

- strong lattice evidence for a first-order phase transition

- T_c approximately m-independent and related to upper right corner of Columbia plot