DENSE QC_2D
What's up with that ?!?

DALE LAWLOR

National University of Ireland, Maynooth

JON IVAR SKULLERUD

National University of Ireland, Maynooth

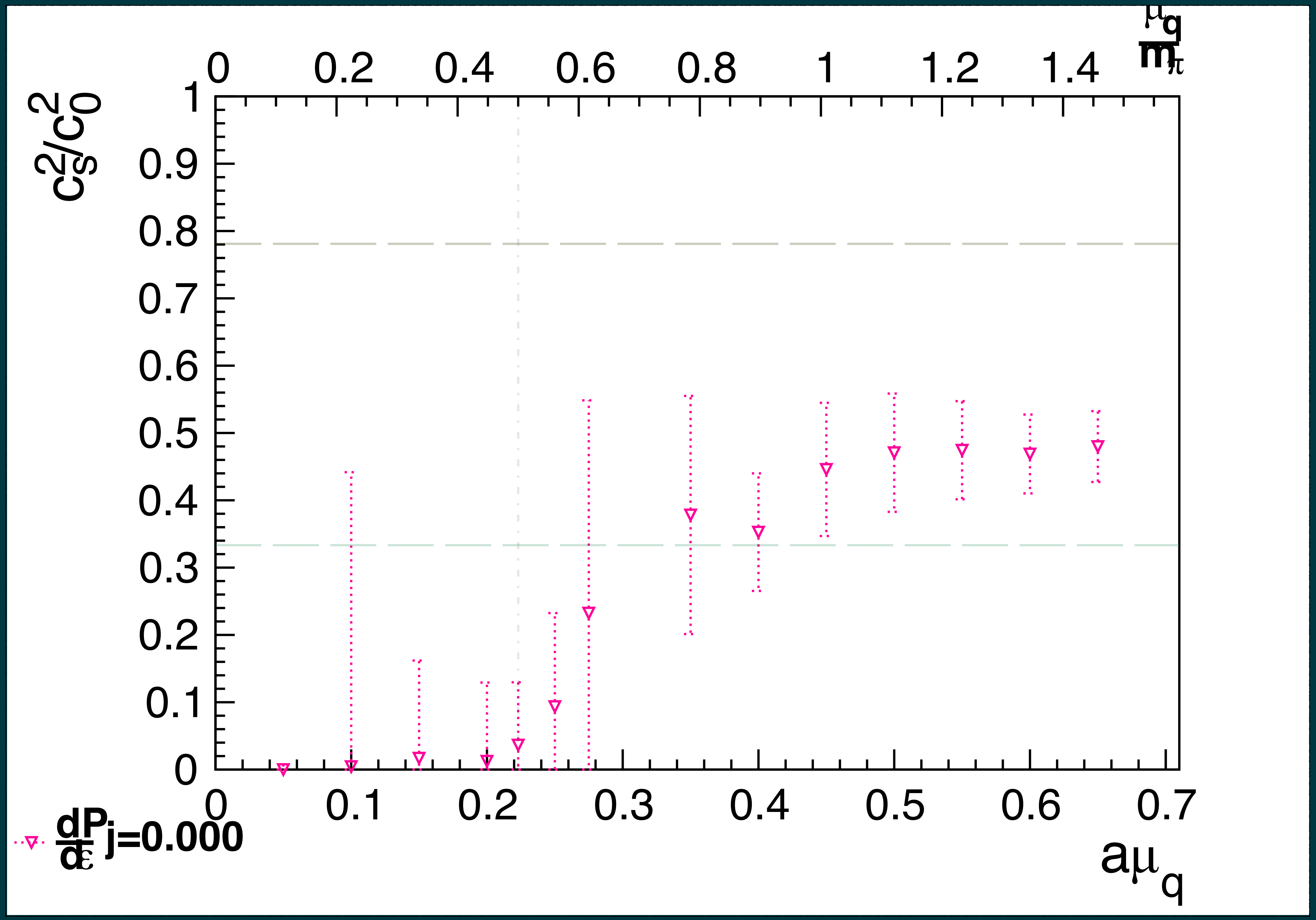
SIMON HANDS

Local Lad

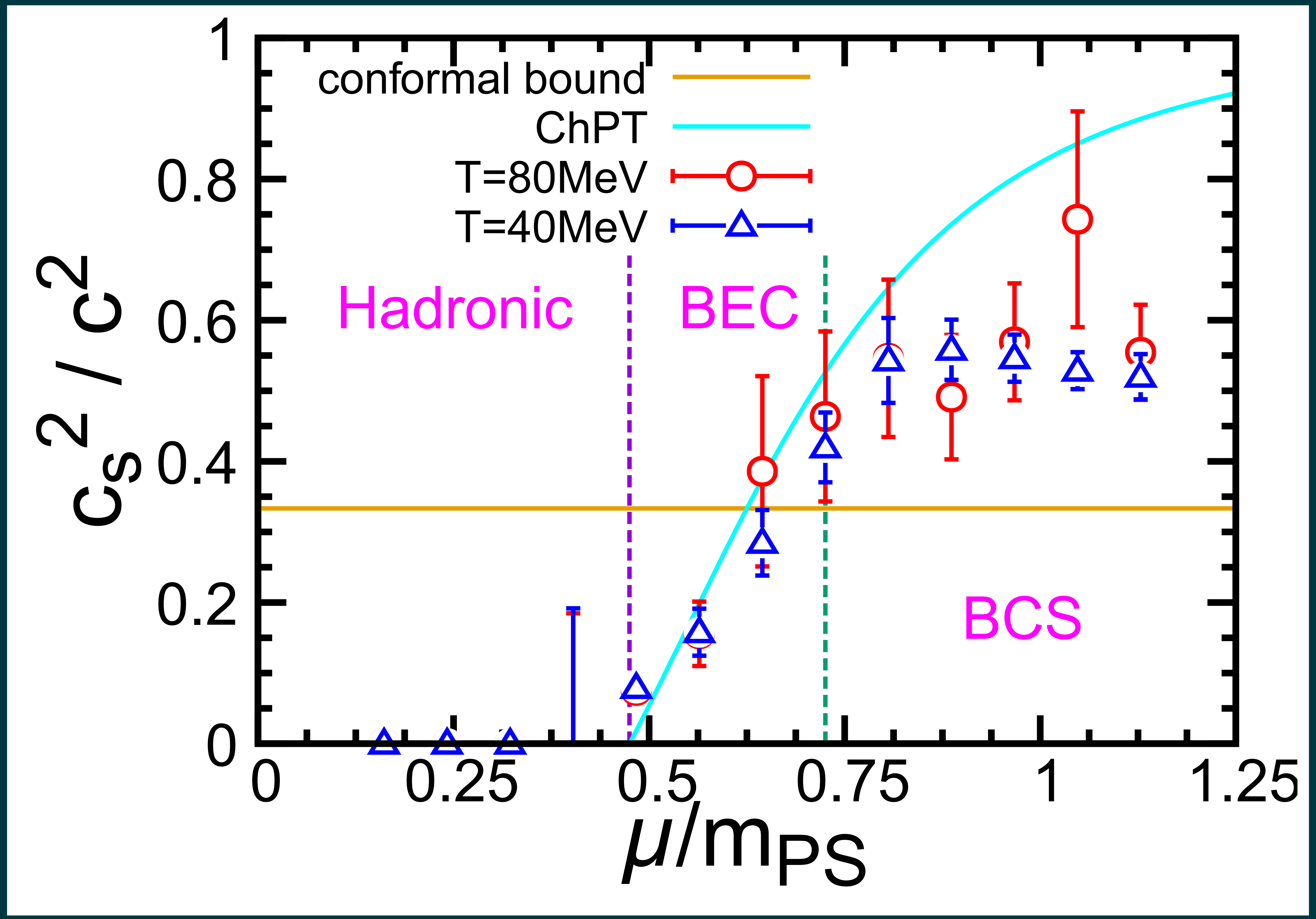
SEYONG KIM

Sejong University

Conclusion



This Work
(these Islands + Korea)



[2405.25006]
(Japanese Group)

Conclusion

Either we're both right

or

we're both wrong

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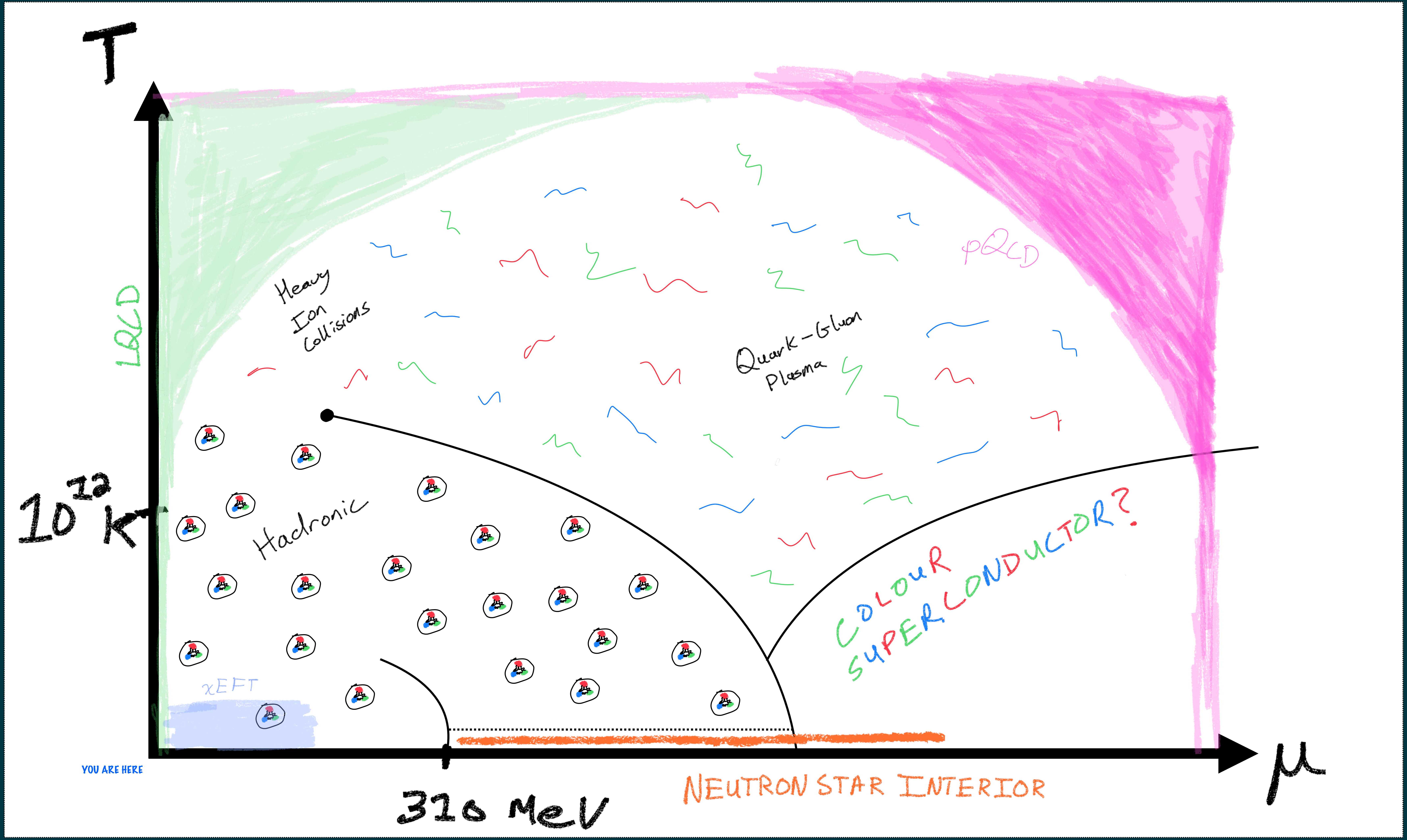
Conclusion

Questions?

Why QC₂D?



T
LQCD
Heavy Ion Collisions
Quark-Gluon Plasma
pQCD
10^{12} K
Hadronic
COLOUR SUPERCONDUCTOR?
χEFT
YOU ARE HERE
310 MeV
NEUTRON STAR INTERIOR
μ

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Why QC_2D?

- Neutron star cores fairly dense ($n_q \sim 5-10 n_0$)
- QCD inherently non-perturbative at finite density/temperature
- χEFT breaks down at $n_q \sim 2-3\, n_0$ and $T \lesssim 30$ MeV [2101.01709]
- Lattice also has flaws

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Why QC_2D?

Lattice QCD in a really tiny nutshell

1) Discretise space-time

2) Perform Monte Carlo update

3) Accept new configuration with weight

$$\rho[U] = e^{-S[U]} = \det(M) e^{-S_g[U]}$$

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Why QC_2D?

- $\mu = 0 \Rightarrow \rho(u) \in \mathbb{R}_0^+$
- $\mu > 0 \Rightarrow \rho(u) \in \mathbb{C}$

$\Rightarrow$ *!#?

Why QC_2D?

- $\mu = 0 \Rightarrow \rho(u) \in \mathbb{R}_0^+$
- $\mu > 0 \Rightarrow \rho(u) \in \mathbb{C}$

$\Rightarrow$ *!#?

If instead consider SU(2) gauge theory with even number of flavours

$$\rho(u) \in \mathbb{R}_0^+ \Rightarrow$$

Why QC_2D?

- Massless Goldstone mode at low T, high μ
- Makes inverting fermion matrix expensive
- Introduce diquark source

$$j\left[\psi_2^{Tr} C \gamma_5 \tau_2 \psi_1 - \bar{\psi}_1 C \gamma_5 \tau_2 \bar{\psi}_2^{Tr}\right]$$

- Extrapolate $j \to 0$

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Why QC_2D?

Other methods are available

- Isospin QCD [Rest of this session]
- Complex Langevin [M. Hansen, Yesterday]
- Imaginary Chemical Potential [Tues. Afternoon]
  [Fri. Afternoon]

Why QC_2D?

Other methods are available

- Isospin QCD [Rest of this session]
- Complex Langevin [M. Hansen, Yesterday]
- Imaginary Chemical Potential [Tues. Afternoon]
  [Fri. Afternoon]
- Schwinger–Dyson Equations
- pQCD $(n_B \gtrsim 20 n_0)$
- χEFT $(n_B \leq 2 n_0)$

Simulation Details

- Unimproved Wilson Fermion / Gauge Action
- HMC with Leapfrog integrator
- $a = 0.130$ fm
- $N_\tau = 32 \Rightarrow T \sim 48$ MeV
- $\frac{m_\pi}{m_\rho} = 0.81$
- $a m_\pi = 0.446$

Simulation Details

Click here for code!

- RIP FORTRAN, long live C!
- Mixed precision conjugate gradient
- Hybrid OpenMP / MPI on CPUs
- Ranlux PRNG

Simulation Details

- This work produced by CUDA port of the code

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- 1 A100 ~ 20x128 core AMD Rome nodes.
- There are 4 A100s per node ...

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THE FIRST FEW TIMES EINSTEIN IMAGINED FLYING ALONGSIDE A BEAM OF LIGHT, HE DIDN'T HAVE ANY PARTICULAR INSIGHTS.

Simulation Details

Have been producing for ~6 weeks on DiRAC's Tursa and Luxprovide's MeluXina machines

Existing Ensembles ($N_x = 16$)

$j = 0.20 / 0.30$

$a\mu = 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7$

$j = 0.10$

$a\mu = 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4$

New ensembles (54 total)

$j = 0.30, N_x = 24$

$a\mu = 0.05, 0.1, 0.15, 0.2, 0.21, 0.22, 0.223, 0.23, 0.25, 0.275, 0.3, 0.325, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7$

$j = 0.15, N_x = 16$

$a\mu = 0.05, 0.1, 0.15, 0.2, 0.21, 0.22, 0.223, 0.23, 0.25, 0.275, 0.3, 0.325, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7$

$j = 0.1, N_x = 16$

$a\mu = 0.05, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7$

$j = 0.2, N_x = 16$

$a\mu = 0.05, 0.21, 0.22, 0.223, 0.23, 0.275, 0.325$

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Lowering Expectations

- ~6 weeks of running. Only 200–300 traj. per parameter choice.
  - Similar to old $aj = 0.10$ data
- Limited error analysis
- Measurement every trajectory ⇒ autocorrelation risk
- Small volumes ⇒ Finite volume effects

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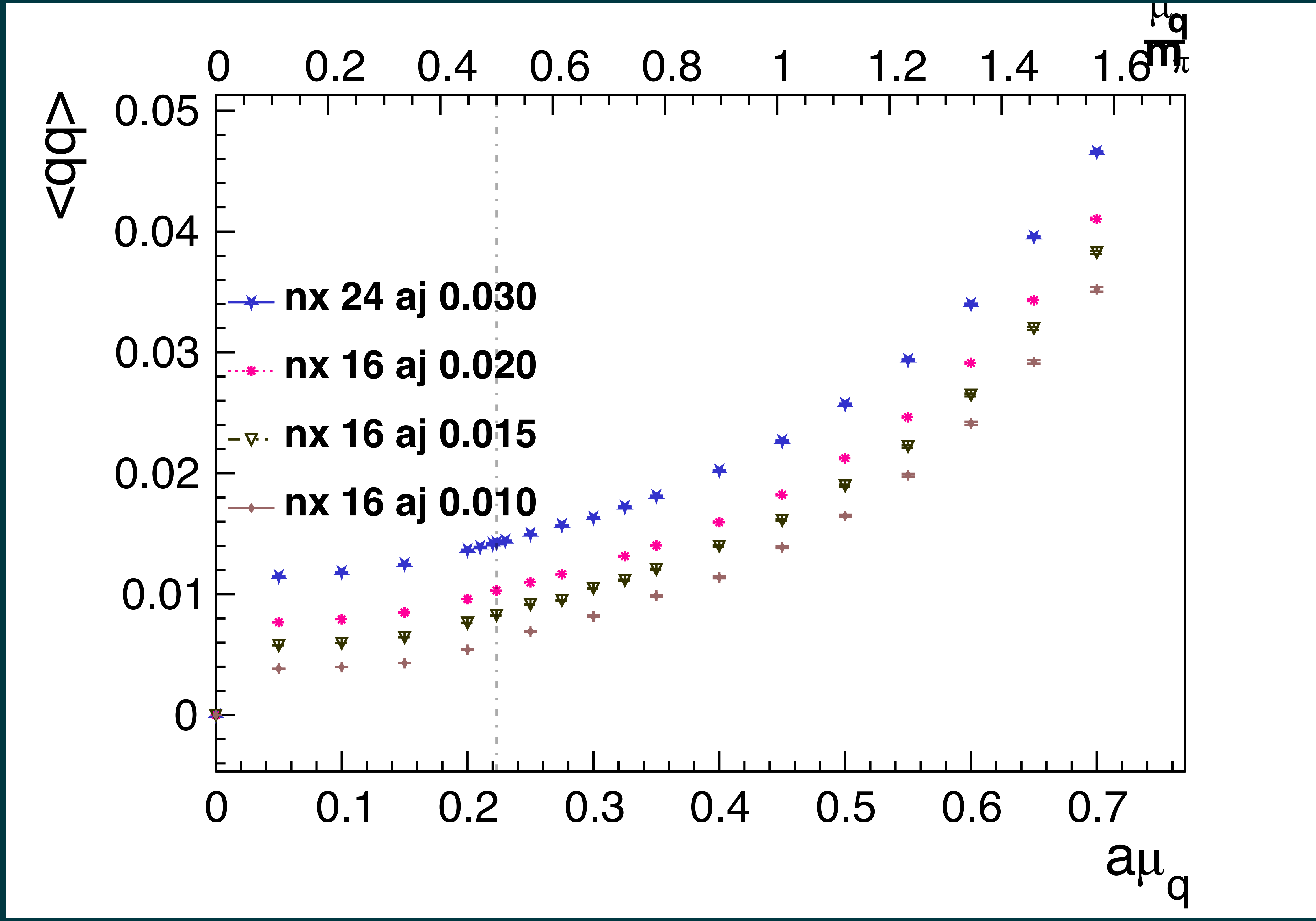
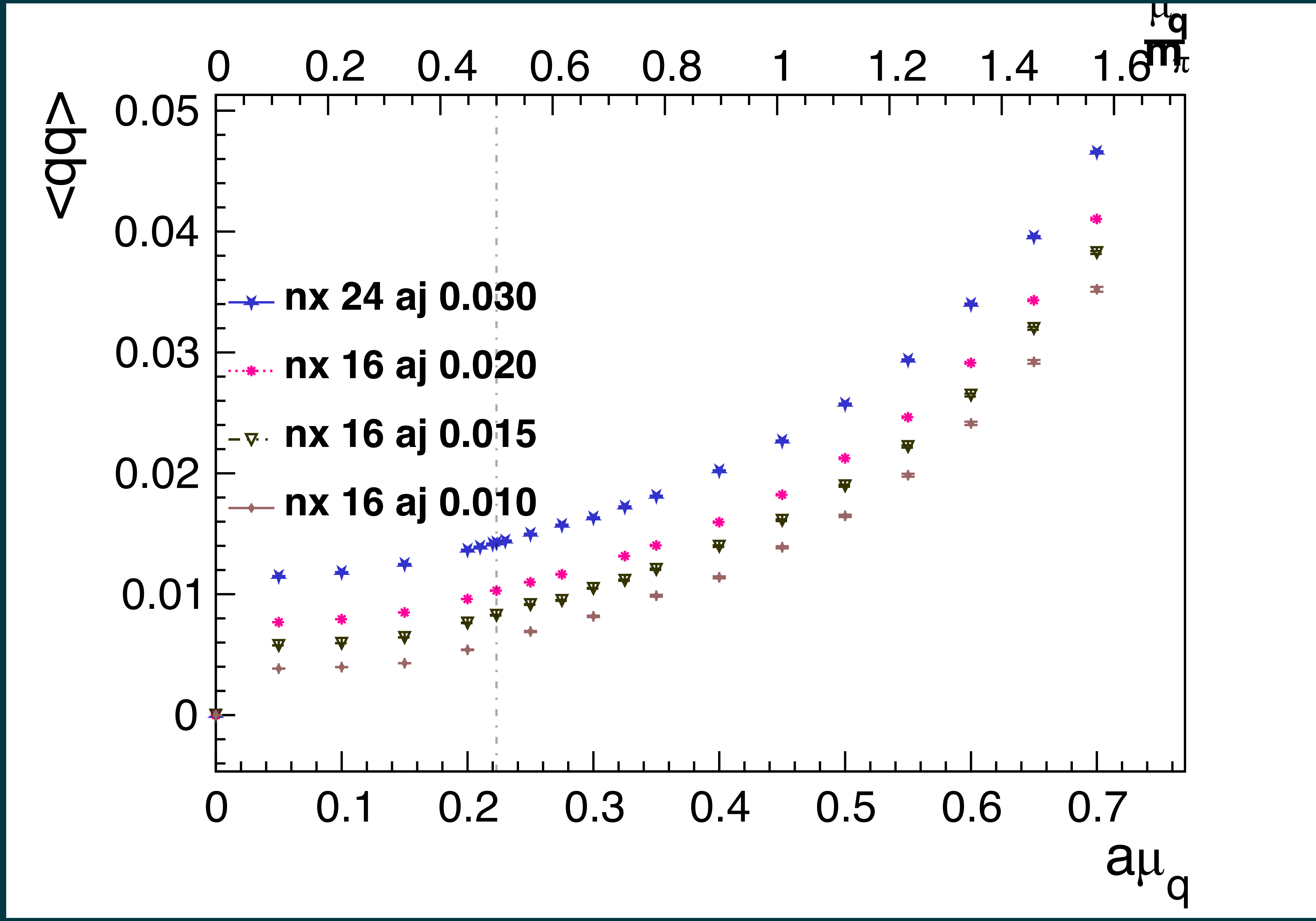
Diquark Condensate



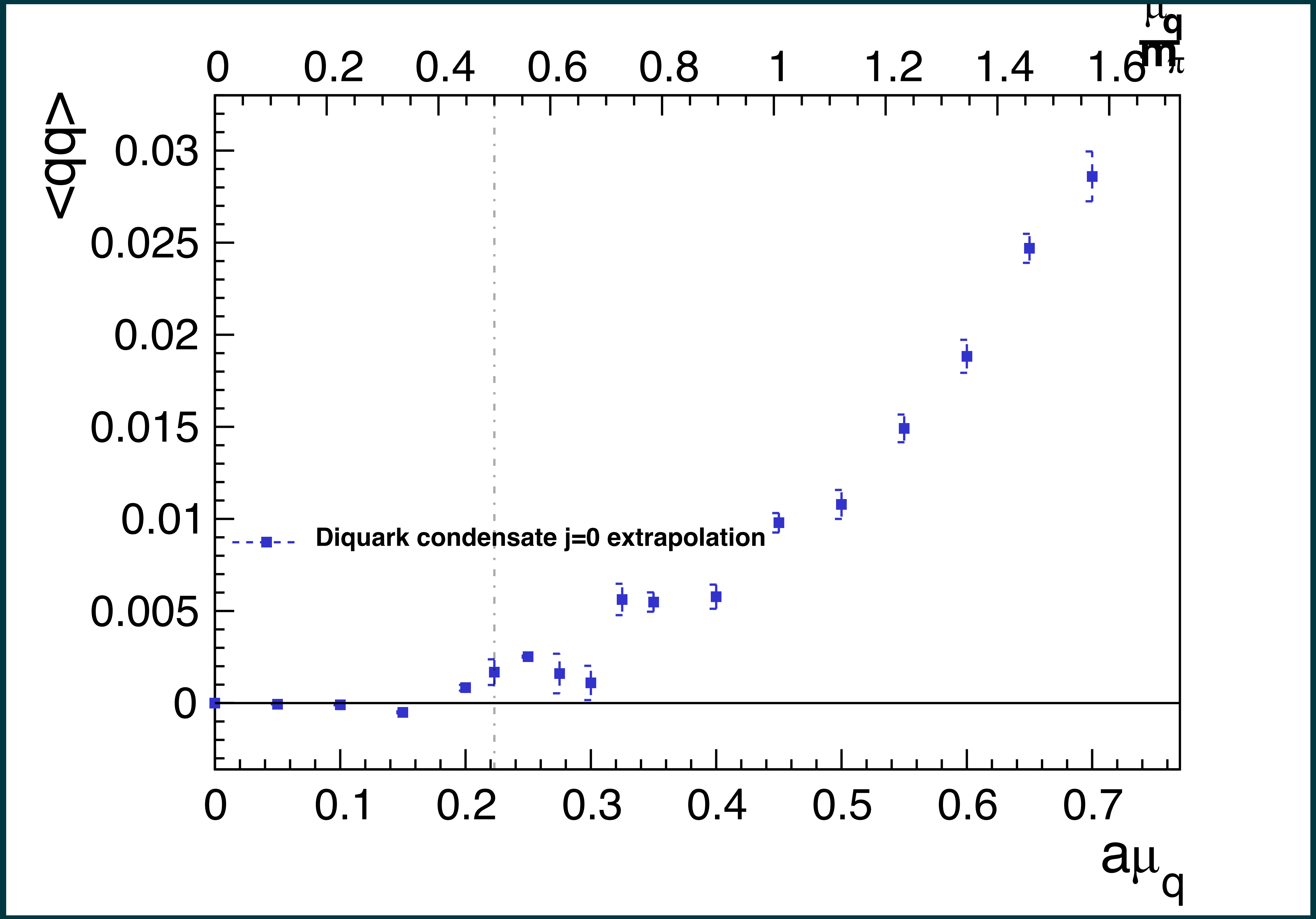
- $\langle qq \rangle$ form "baryons" of theory
- Integer spin $\Rightarrow$ bosonic!

Diquark Condensate



- $\langle qq \rangle$ form "baryons" of theory
- Integer spin ⟹ bosonic!
- INDIVIDUAL QUARKS STILL FERMIONS

Diquark Condensate



- Fit to $\langle qq \rangle = c_0 + c_1 j^{c_2}$
- Did NOT obtain χPT result $c_2 = 1/3$

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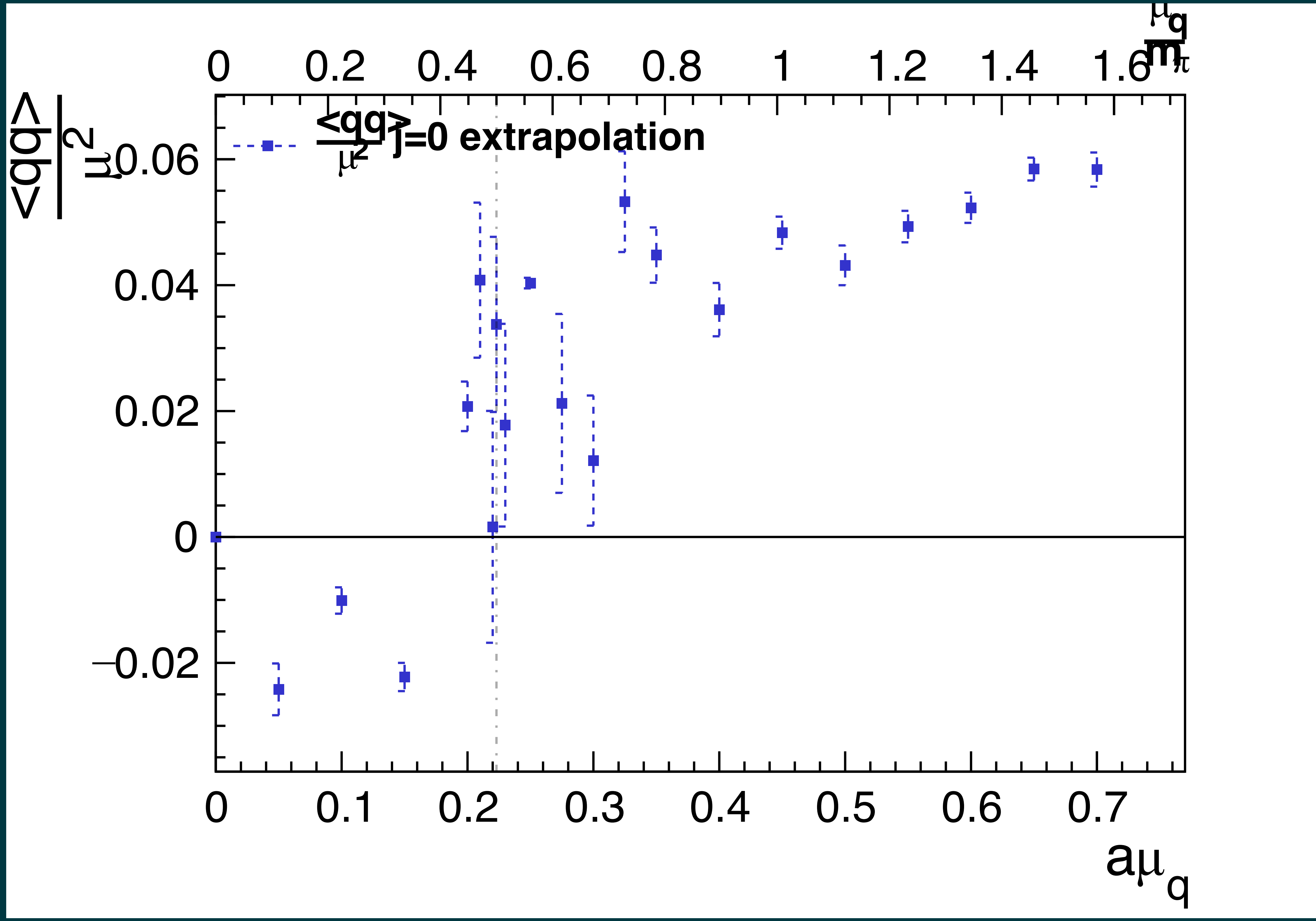
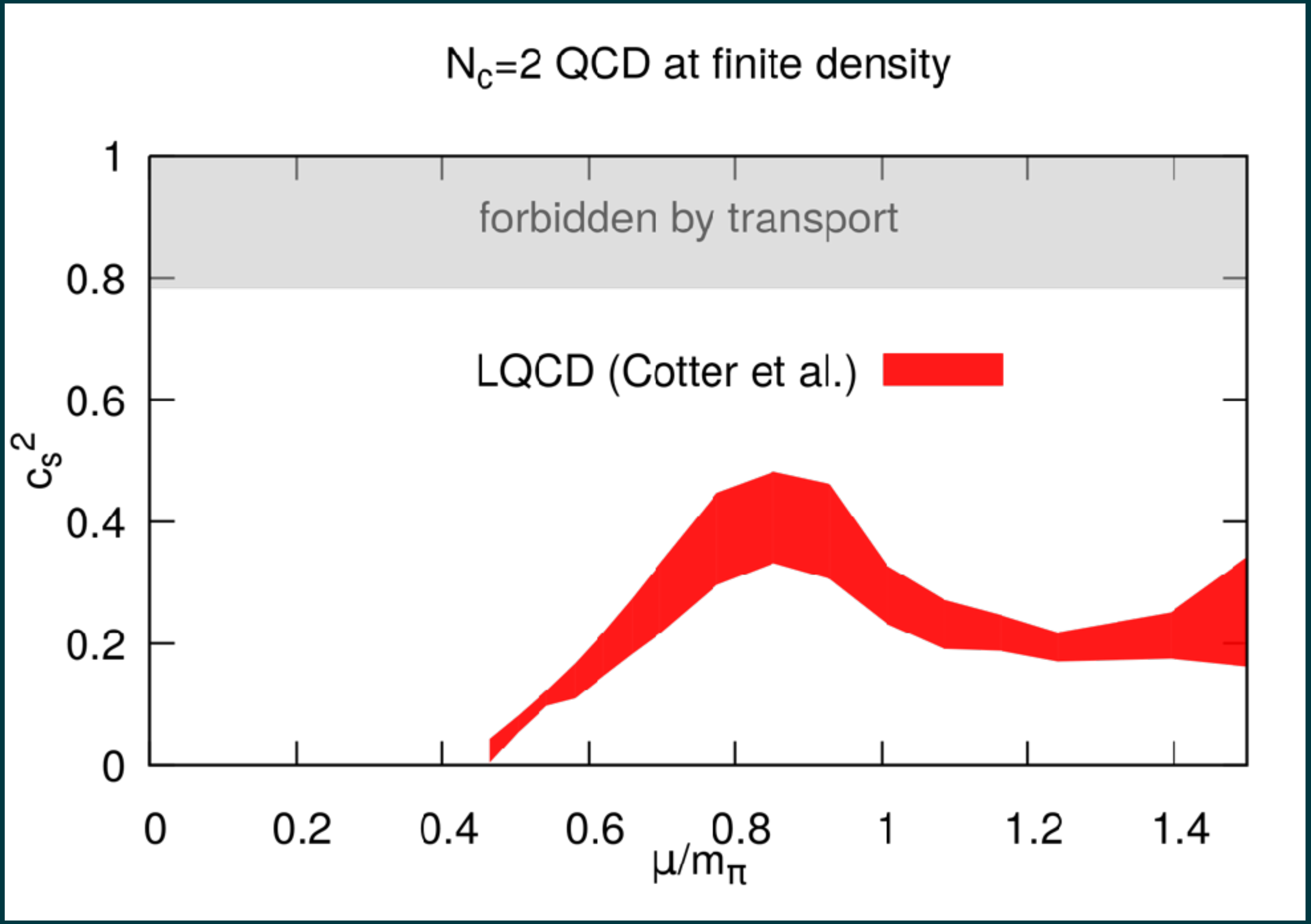


- Fit to $\langle qq \rangle = c_0 + c_1 j^{c_2}$
- Did NOT obtain χPT result $c_2 = 1/3$
- More work needed to explore BEC and BCS transition

Speed of Sound

- $c_s^2 = \frac{\partial P}{\partial \varepsilon}$
- Conformal limit
  $\Rightarrow c_s^2 \to 1/3$
- Transport properties
  $\Rightarrow c_s^2 < 0.781$

[2402.14085]



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Energy Density / Trace Anomaly

- [1210.4496] got ε from Karsch Coefficients
- Can also use $T_{\mu\mu} = \varepsilon - 3p$ to find ε

$$T_{\mu\mu} = T^{q}_{\mu\mu} + T^{g}_{\mu\mu}$$

Energy Density / Trace Anomaly

- [1210.4496] got $\mathcal{E}$ from Karsch Coefficients
- Can also use $T_{\mu\mu} = \mathcal{E} - 3p$ to find $\mathcal{E}$

$$T_{\mu\mu} = T^{q}_{\mu\mu} + T^{g}_{\mu\mu}$$

$$T^{g}_{\mu\mu} = -\frac{3a}{N_c} \frac{\partial \beta}{\partial a} \mathrm{Re}\left(\mathrm{Tr}\, U_{ij} + \mathrm{Tr}\, U_{i0} \right)$$

$$T^{q}_{\mu\mu} = -a \frac{\partial \kappa}{\partial a} \kappa^{-1} \left(4 N_f N_c - \langle \bar{\psi} \psi \rangle \right)$$

Beta Functions

- $T^{g}_{\mu\mu}$ and $T^{q}_{\mu\mu}$ need renormalisation
- Beta functions $\frac{\partial \beta}{\partial a}$ and $\frac{\partial \kappa}{\partial a}$ evaluated numerically
- Requires several rounds of scale setting to get a on line of constant physics. (educated trial and error)

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Scale Setting — Mass

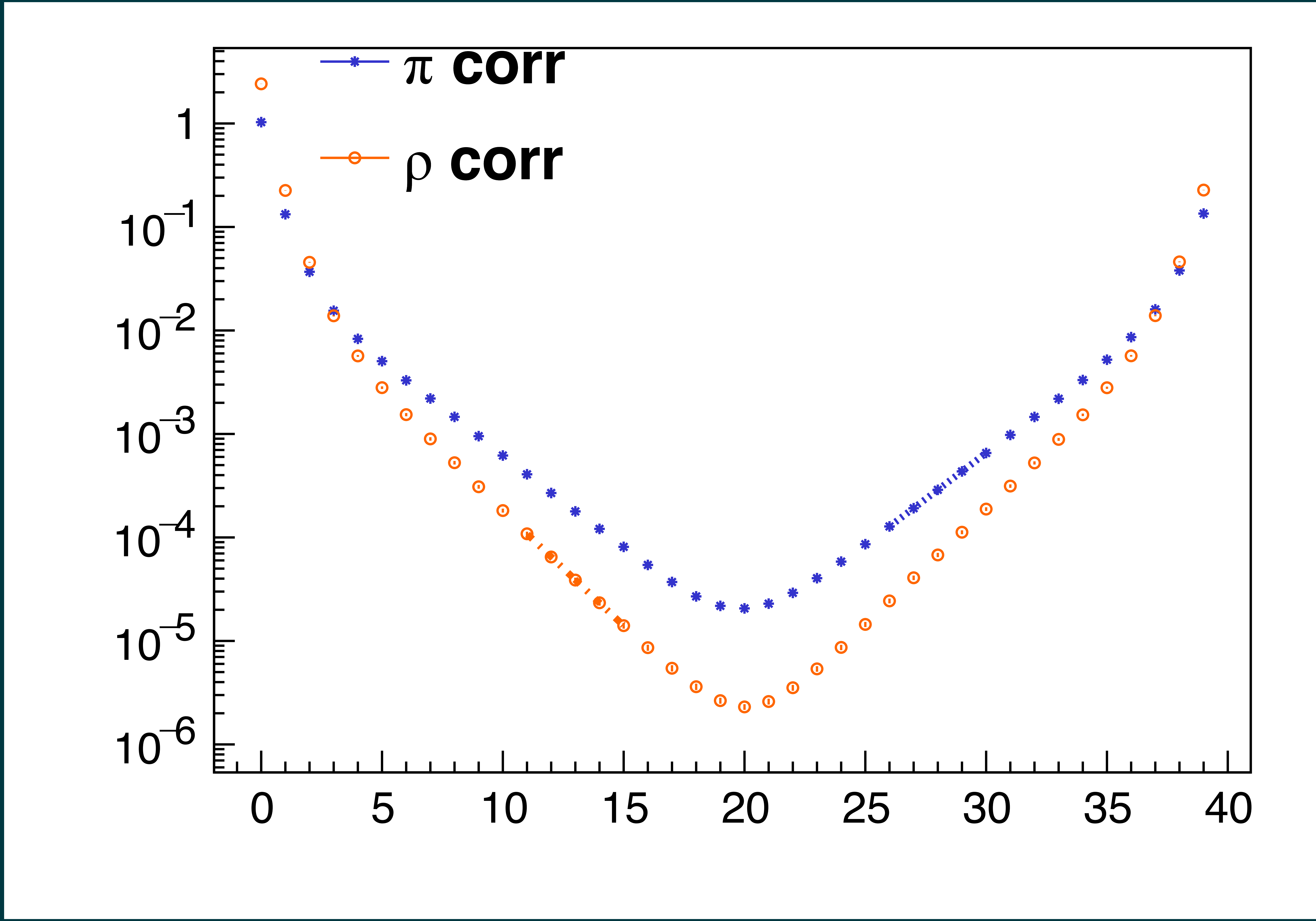
- Can't use physical masses to set scale
- Instead use ratio of two colour pseudoscalar (pion) and vector (rhon) mesons
- Can extract masses from correlators as in real QCD

$$aM_{eff} = -\ln\left(\frac{C(\tau+1)}{C(\tau)}\right)$$

$$C(\tau) = C_0 \cosh\left[M\left(\tau - \frac{N_\tau}{2}\right)\right]$$

$$am = |M|$$

Obligatory Correlator Plot



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Scale Setting – Spacing

- Can extract lattice spacing a from Lattice Static Quark Potential

$$a V(r) = -\frac{N_c^2-1}{N_c}\frac{\alpha}{r} + a^2\sigma r + a V_0$$

- Assume $\sigma = (440\ \text{MeV})^2$

- [2405.25006] sets scale using T_c instead.

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Scale Setting - Spacing

- Extract Static Quark Potential from

$$W(T) = A_0 e^{-V_0(r)T}$$

- Previous works used Wilson Loops
  - Path finding computationaly expensive
- This work uses Wilson Lines instead
- Wilson line implemented by a second year electronic engineering undergrad as a SPUR (Summer Programme for Undergraduate Research) project.

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Calculation of Wilson Lines

- Firstly, the gauge was fixed to Coulomb gauge fixing

  $\nabla \cdot \vec{A} = 0$

- Temporal gauge link products were calculated at various lengths, l, from 1 to N, where N is half the temporal size of the lattice, at every spatial site n and every temporal site at that spatial site n_t [1]

  $$U_\nu = \prod_{j=n_t}^{n_t+l} U_4(n, j)$$

- To calculate the Wilson Line values, we have to multiply products of the same length and on the same temporal level

  $$W = \mathrm{tr}\left(U_\nu U_\nu^\dagger\right)$$

  $U^\dagger$ represents the conjugate transpose of the matrix U

Calculation of the static quark potential

- From the Wilson Lines, we can calculate the static quark potential
- The static quark potential has the form

  $$V(r) = V_0 - \frac{\alpha}{r} + \sigma r$$

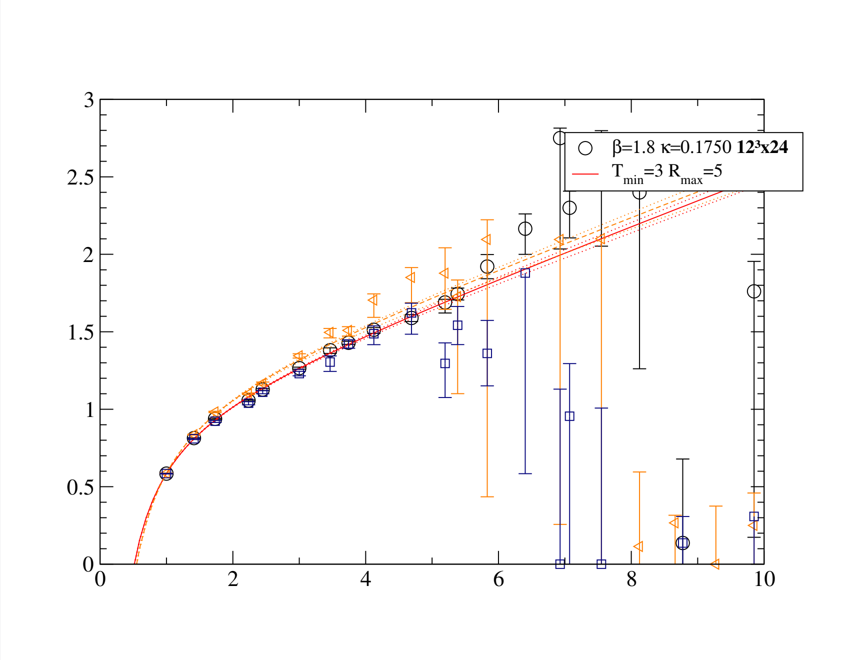
- On the lattice this becomes

  $$aV(r) = aV_0 - \frac{\alpha}{\left(\frac{r}{a}\right)} + a^2\sigma\left(\frac{r}{a}\right)$$

  with a being the lattice spacing.
- The value of the Wilson Line can be connected to the static quark potential via

  $$W = A_0 e^{-V_0(r)T}$$

- From this the static quark potential can be calculated



[TEMP IMAGE]Comparison of potential using Wilson Lines and Wilson Loops

What do quarks do when squeezed and heated?

Computation of the static quark potential using Wilson Lines

Abstract

- Space-time is discretised into 4-dimensional lattices in lattice QCD (quantum chromodynamics)
- We are interested in calculating the static potential between two quarks
- Previously, this was done by calculating Wilson Loops, which have spatial and temporal aspects
- However, the spatial part requires path finding algorithms which can be slow as they may be used millions of times when processing data
- Wilson Lines are similar to Wilson Loops, but only contain temporal aspects
- No spatial aspect $\rightarrow$ no path finding algorithms $\rightarrow$ faster computation of the static quark potential
- Performed on SU(2) gauge link configurations

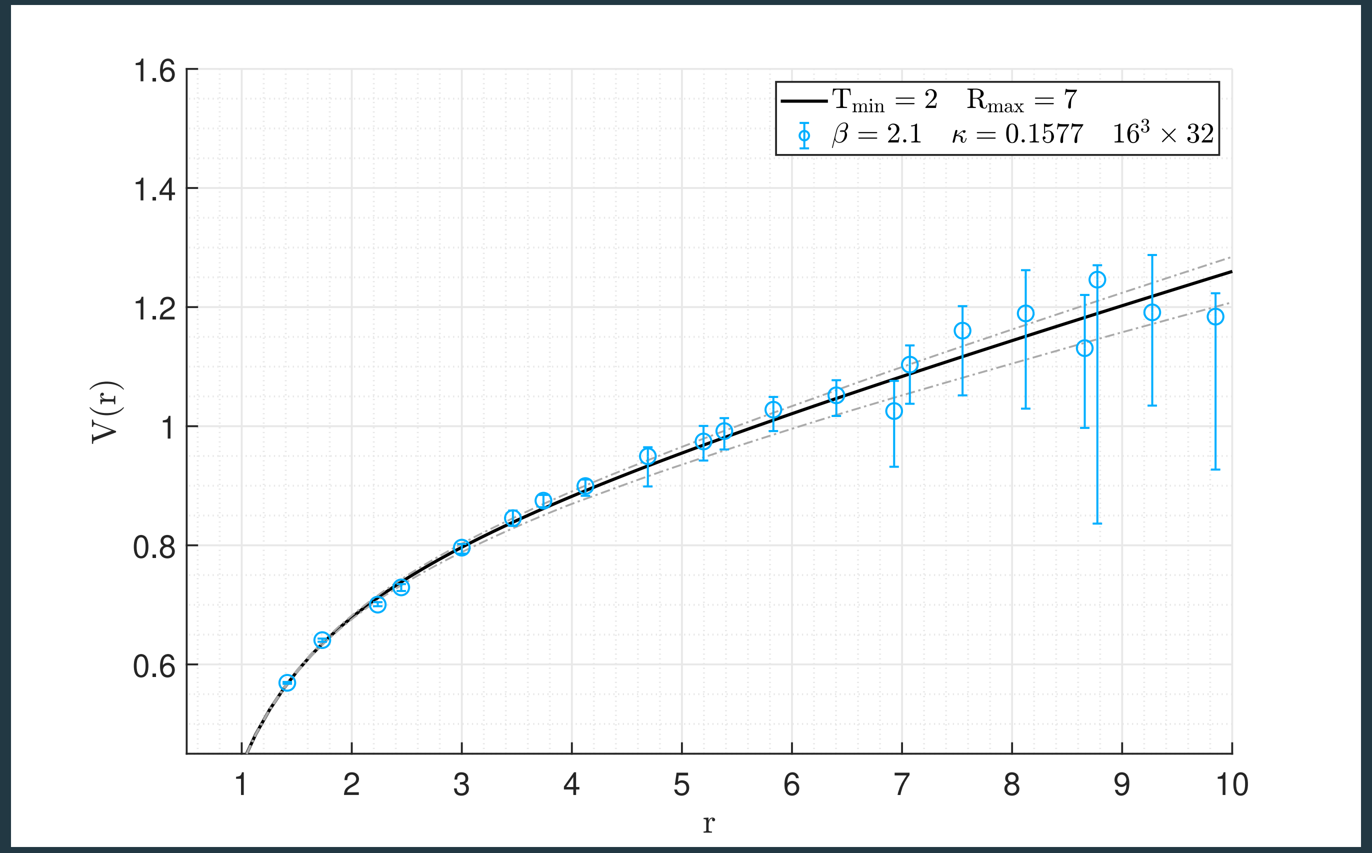


Figure 1. Static quark potential

Analysis of data in Figure 1

- 2060 configurations of data were run
- $\beta = 2.1$, $\kappa = 0.1577$
- The lattice size was $16^3 \times 32$
- Run-time was 18 hours and 20 minutes on an Intel i3-10100 processor
- The results are consistent with previous results using Wilson Loops

Uses and advantages

- The static quark potential can be used to calculate the lattice spacing which is important to know the 'fineness' of the lattice we are working on
- Calculating the potential can help with studying confinement or deconfinement of quarks at high temperatures or densities
- Faster calculation of this potential will assist both of these uses

Performance improvement

- Much faster than the Wilson Loop for the same size lattice
- Gets much faster as the lattice size gets larger
- Results for the potential remain nearly the same (within 2 standard deviations)

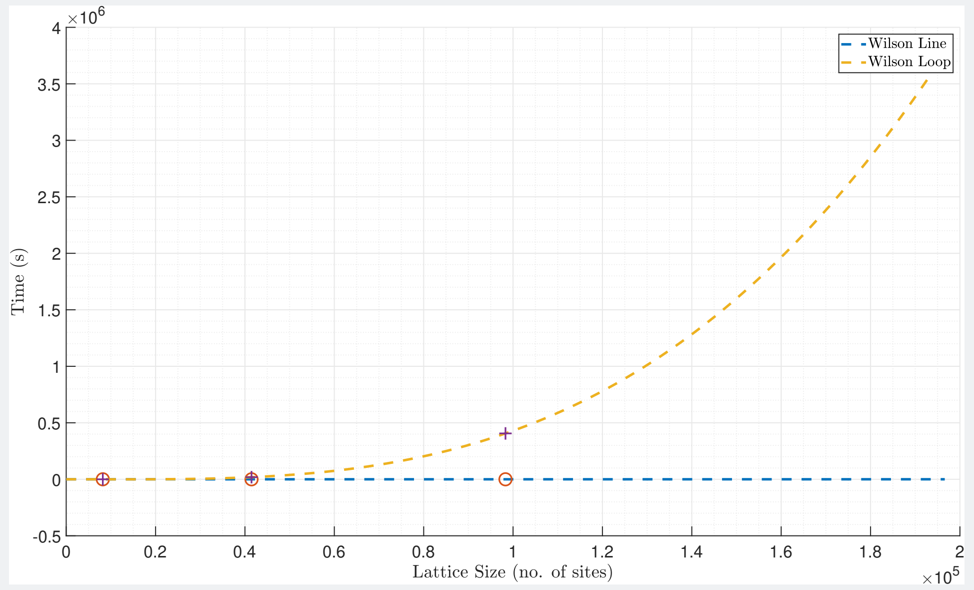


Figure 2. Performance of Wilson Loops versus Wilson Lines

References

[1] Christof Gattringer and Christian Lang. *Quantum chromodynamics on the lattice: an introductory presentation*. Vol. 788. Springer Science & Business Media, 2009

Acknowledgments

I would like to thank Jonivar Skullerud for giving me the opportunity to work on this project, as well as for his assistance whenever I needed it. I would also like to thank the PhD students I worked with, Dale and Jesuel.

Andy Lee Mitchell
Department of Theoretical Physics
Maynooth University
National University of Ireland Maynooth

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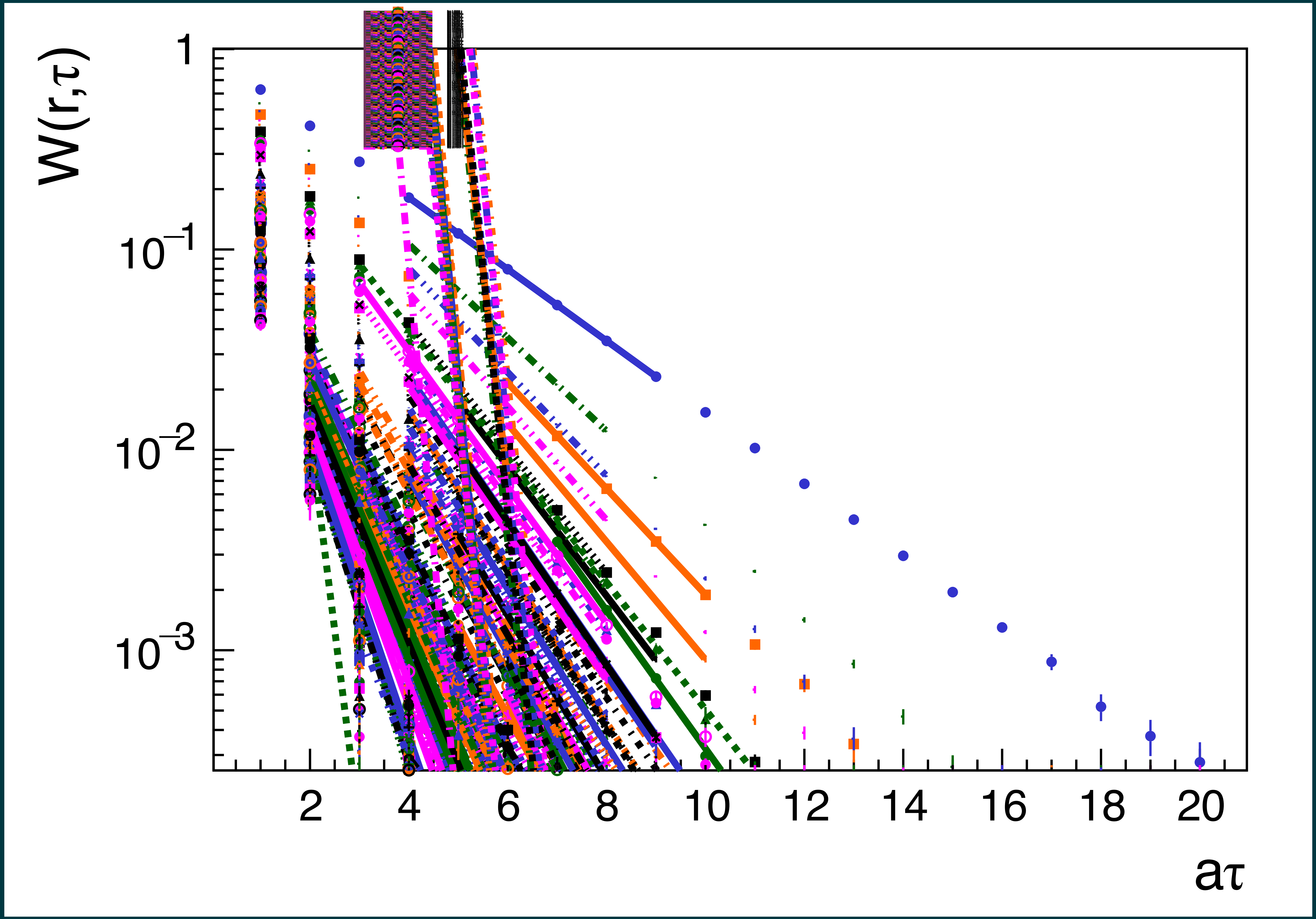

A few Wilson line fits

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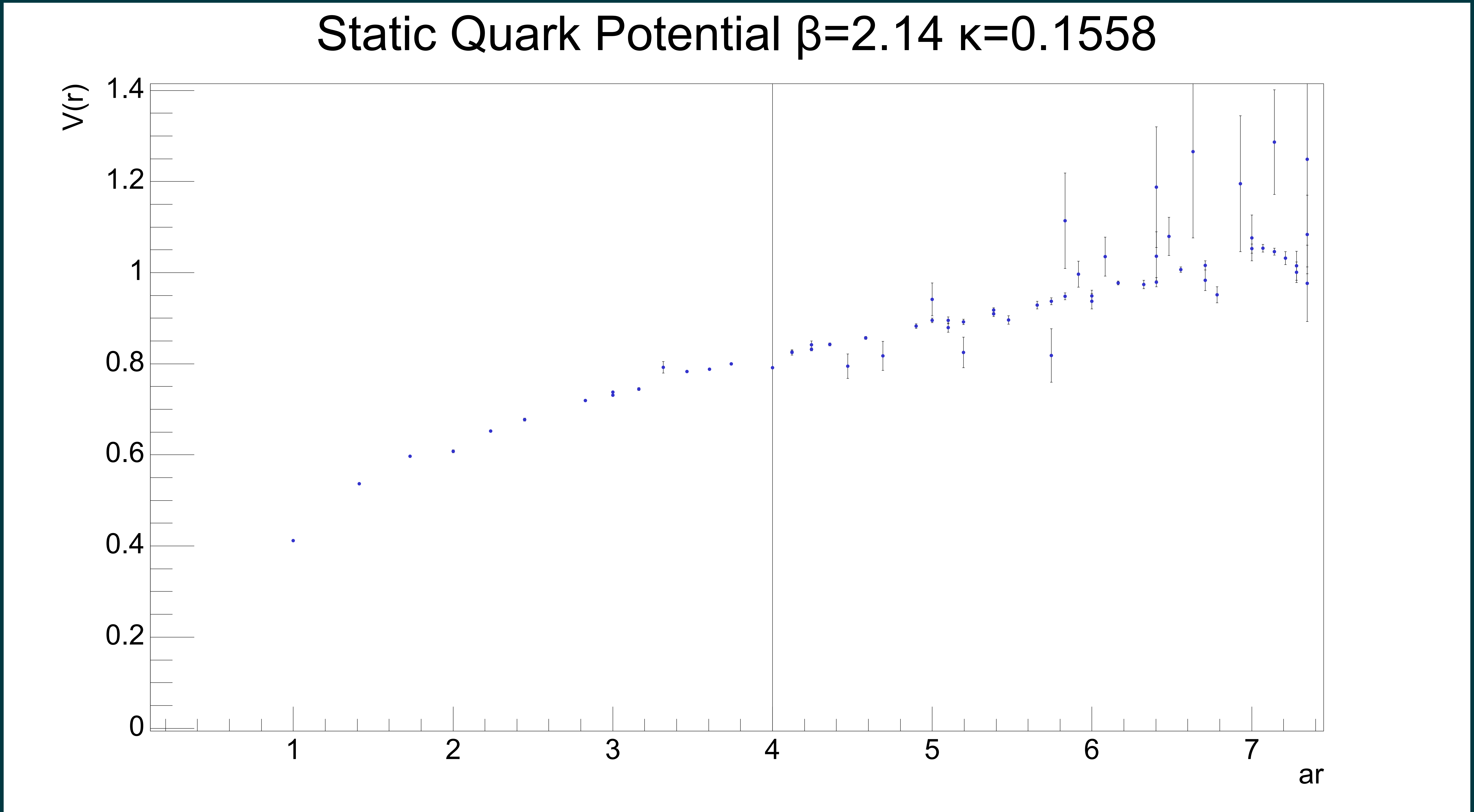

Static Quark Potential β=2.14 κ=0.1558
V(r)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
ar

β	κ	ncfg	a (fm)	+/-	m_π/m_ρ	+/-	$a\,m_\pi$
1.7	0.1780		0.223	0.004	0.779	0.004	0.79
1.9	0.1680		0.178	0.004	0.805	0.009	0.645
2.1	0.1577		0.130	0.0004	0.810	0.007	0.446
1.80	0.1730	1019	0.2125	0.0019	0.811	0.008	0.731
1.85	0.1707	277	0.1846	0.0015	0.828	0.021	0.684
1.97	0.1646	175	0.1617	0.0008	0.829	0.027	0.546
2.00	0.1626	268	0.1528	0.0006	0.799	0.022	0.562
2.04	0.1606	293	0.1437	0.0004	0.799	0.018	0.520
2.14	0.1558	106	0.1004	0.0007	0.801	0.045	0.410
2.17	0.1544	100	0.0741	0.0016	0.813	0.009	0.381

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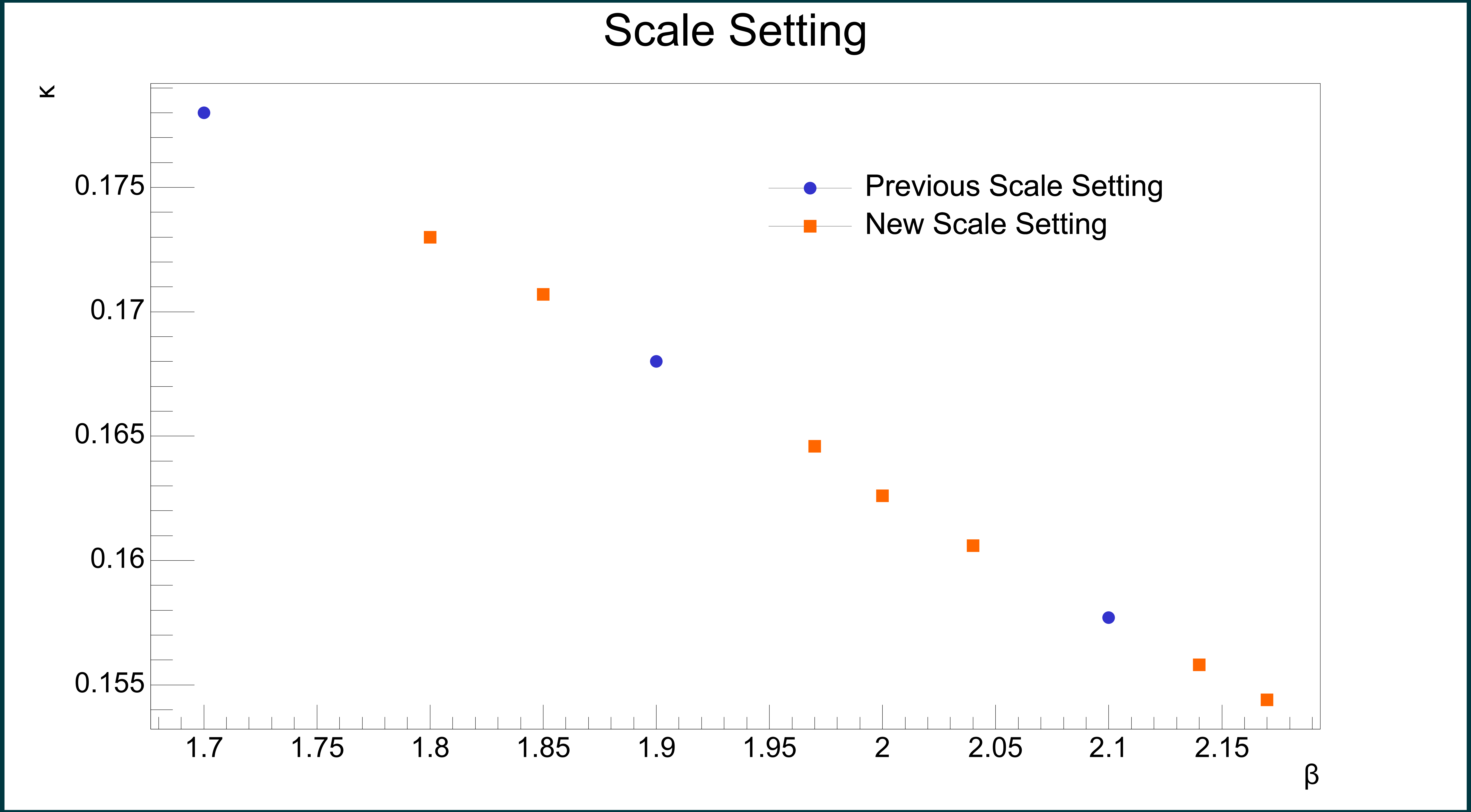

Line of constant physics $\frac{m_\pi}{m_\rho} = 0.81$

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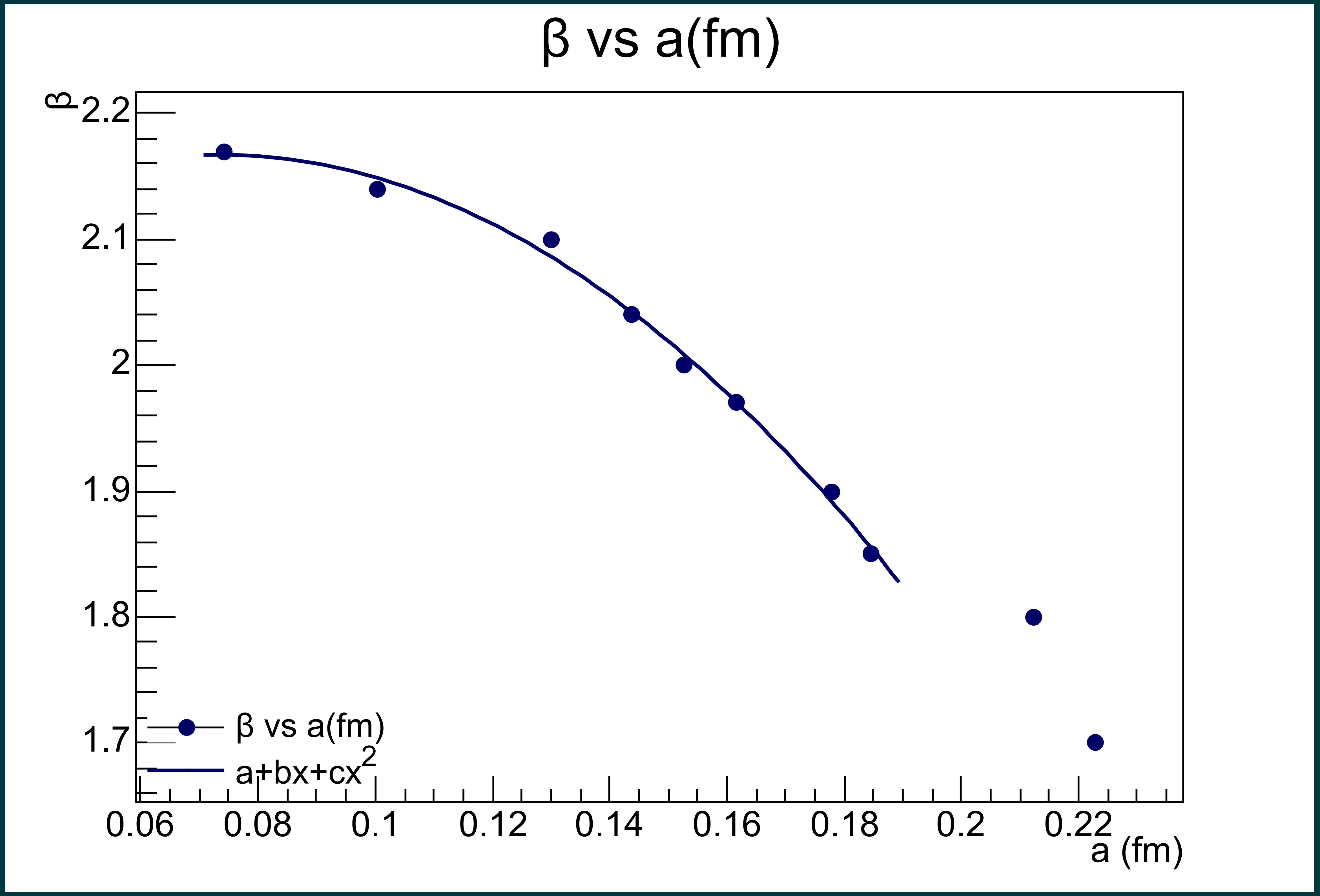
Beta Functions



- Fit to $\beta = C_0 + C_1 a + C_2 a^2$

$$\Rightarrow \frac{\partial \beta}{\partial a} = C_1 + C_2 a$$

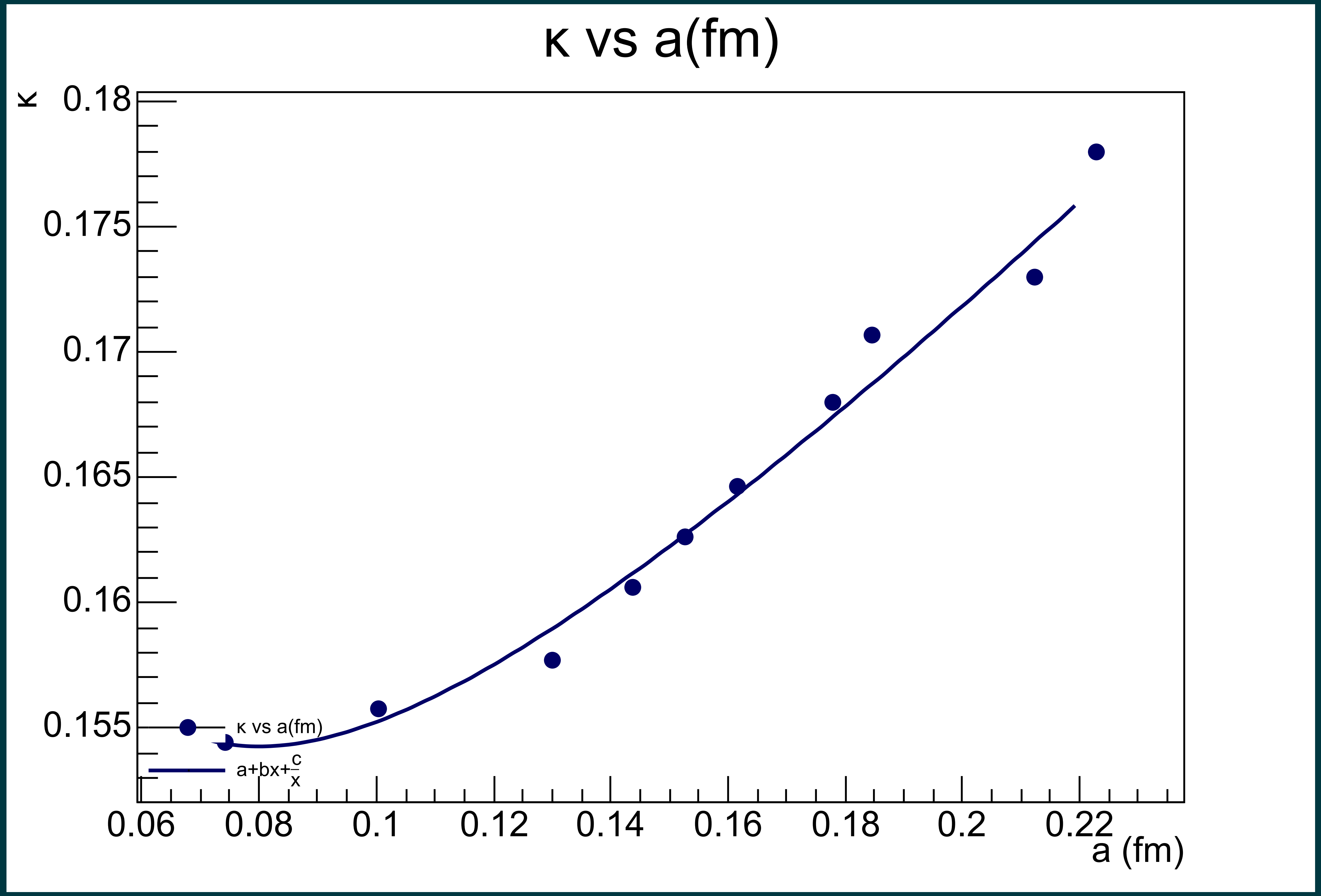
Fit using ROOT

$C_0 = 2.03 \pm 3.16 \times 10^{-2}$

$C_1 = 3.69 \pm 7.52 \times 10^{-1}$

$C_2 = -25.18 \pm 2.87$

Beta Functions



- Fit to $\kappa = c_0 + \frac{c_1}{a} + c_2 a$

$$\Rightarrow \frac{\partial \kappa}{\partial a} = c_1 \ln(a) + c_2$$

Fit using ROOT

$C_0 = 1.15 \times 10^{-2} \pm 8.26 \times 10^{-3}$

$C_1 = 1.56 \times 10^{-3} \pm 4.93 \times 10^{-4}$

$C_2 = 2.44 \times 10^{-1} \pm 3.16 \times 10^{-2}$

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β	κ	$\frac{\partial\beta}{\partial a}$	$\pm$	$\frac{\partial\kappa}{\partial a}$	$\pm$	Remarks
1.9	0.1680	−2.71	−0.16	0.197	0.16	Cotter Thesis
1.9	0.1680	−2.86	0.08	0.195	0.032	NEW
2.1	0.1577	−2.85	0.10	0.152	0.032	THIS WORK

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Energy Density / Trace Anomaly

- [1210.4496] got ε from Karsch Coefficients
- Can also use $T_{\mu\mu} = \varepsilon - 3p$ to find ε

$$T_{\mu\mu} = T^{q}_{\mu\mu} + T^{g}_{\mu\mu}$$

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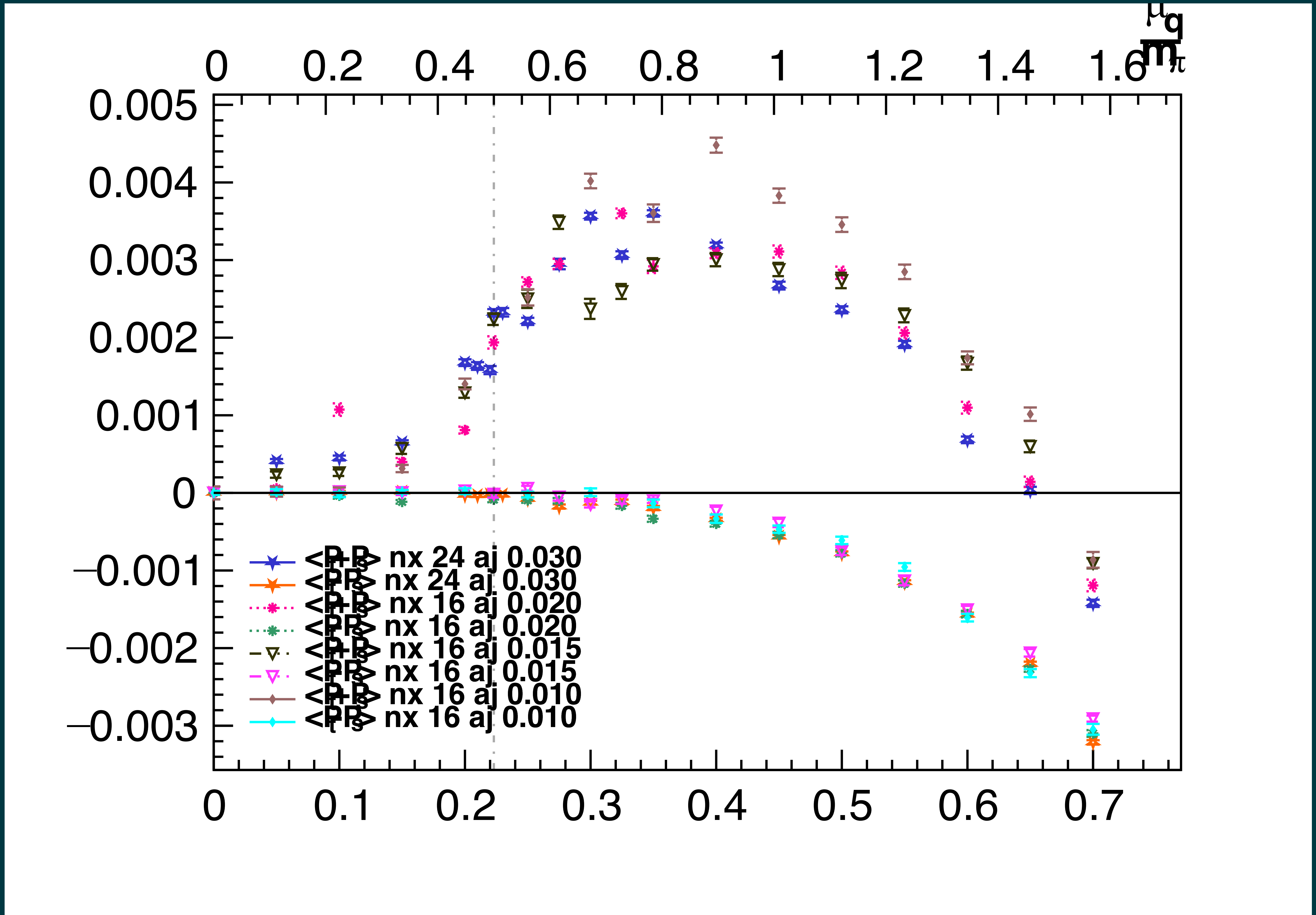
$$T^{q}_{\mu\mu} = -a\frac{\partial\kappa}{\partial a}\kappa^{-1}\left(4N_f N_c - \langle\bar{\psi}\psi\rangle\right)$$

Plaquettes



- $\langle P_s \pm P_t \rangle$ zero subtracted
- Linear diquark source extrapolation

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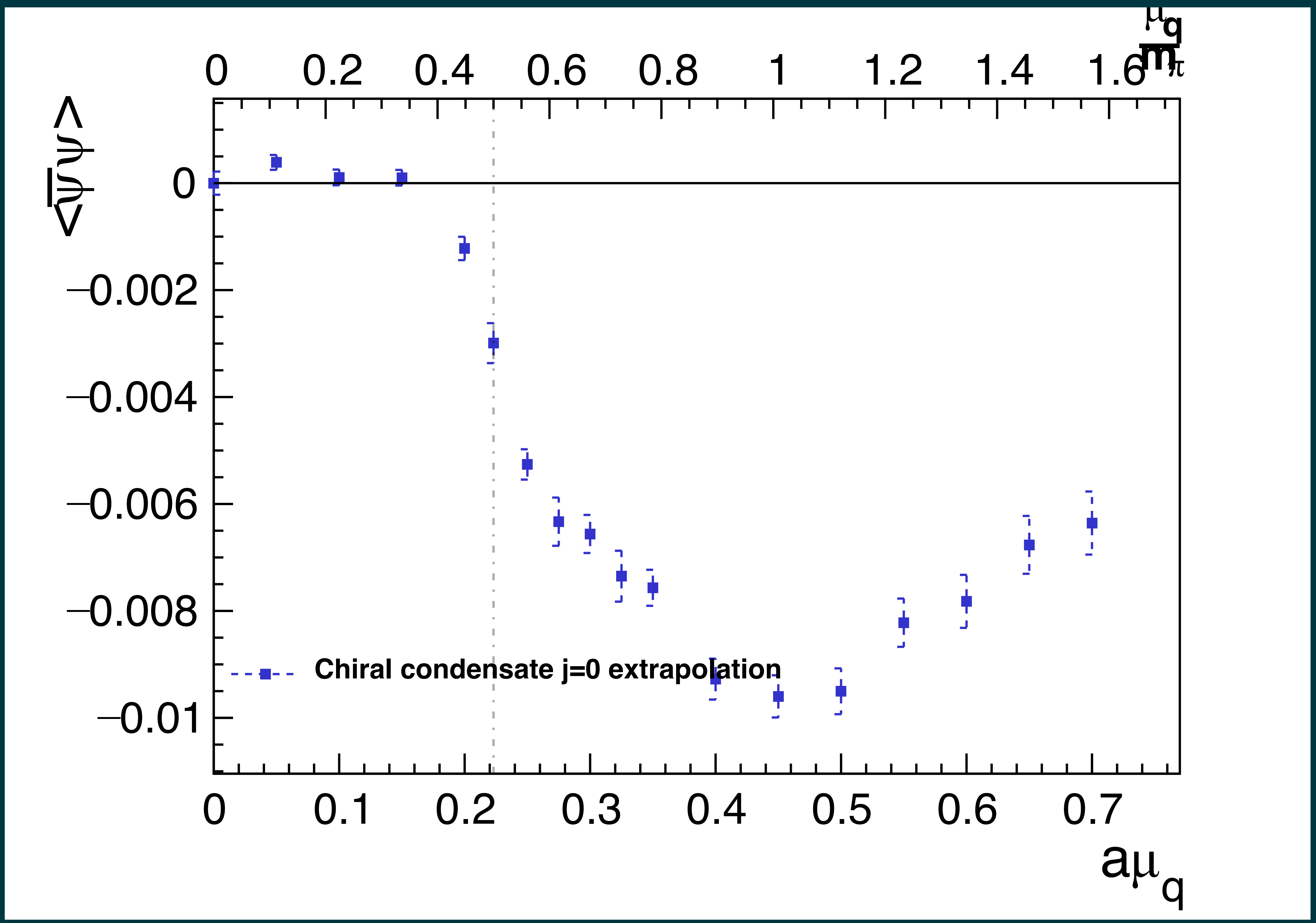
Chiral Condensate



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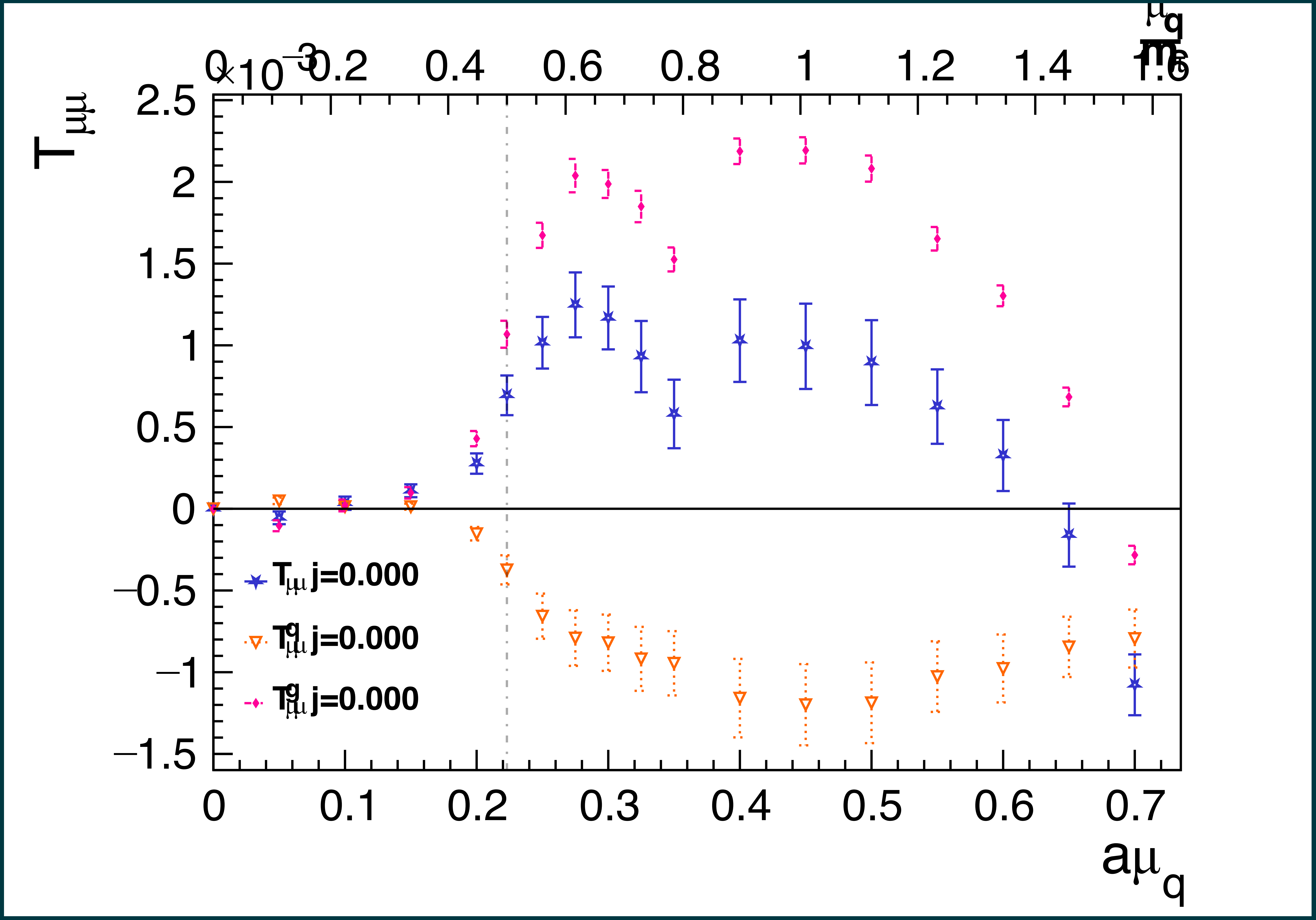
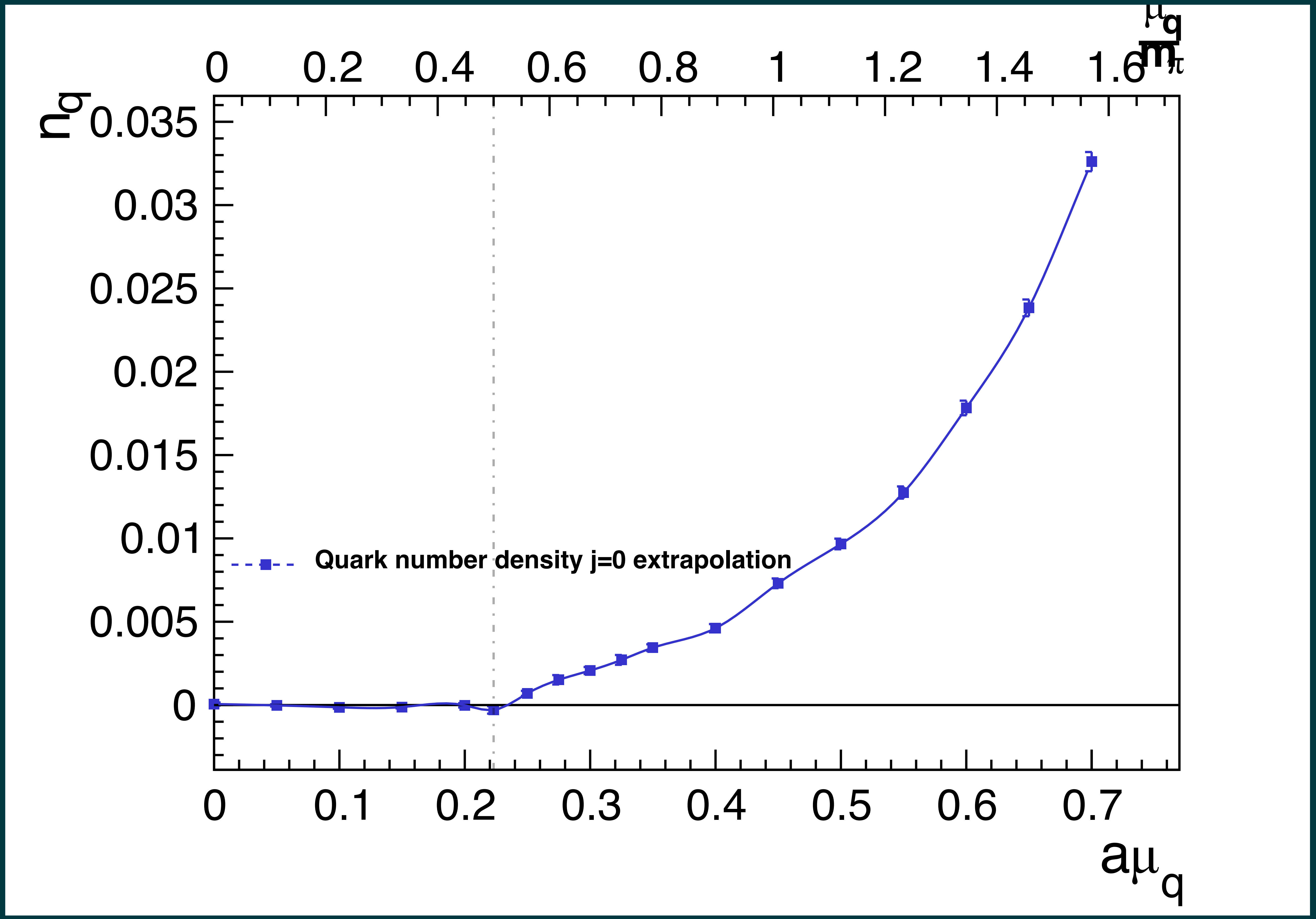
Trace Anomaly



Pressure

- $P(\mu) = \int_0^{\mu} n_q(\mu')\, d\mu'$
- $n_q(\mu)$ interpolated using cubic spline
- Linear diquark extrapolation for number density



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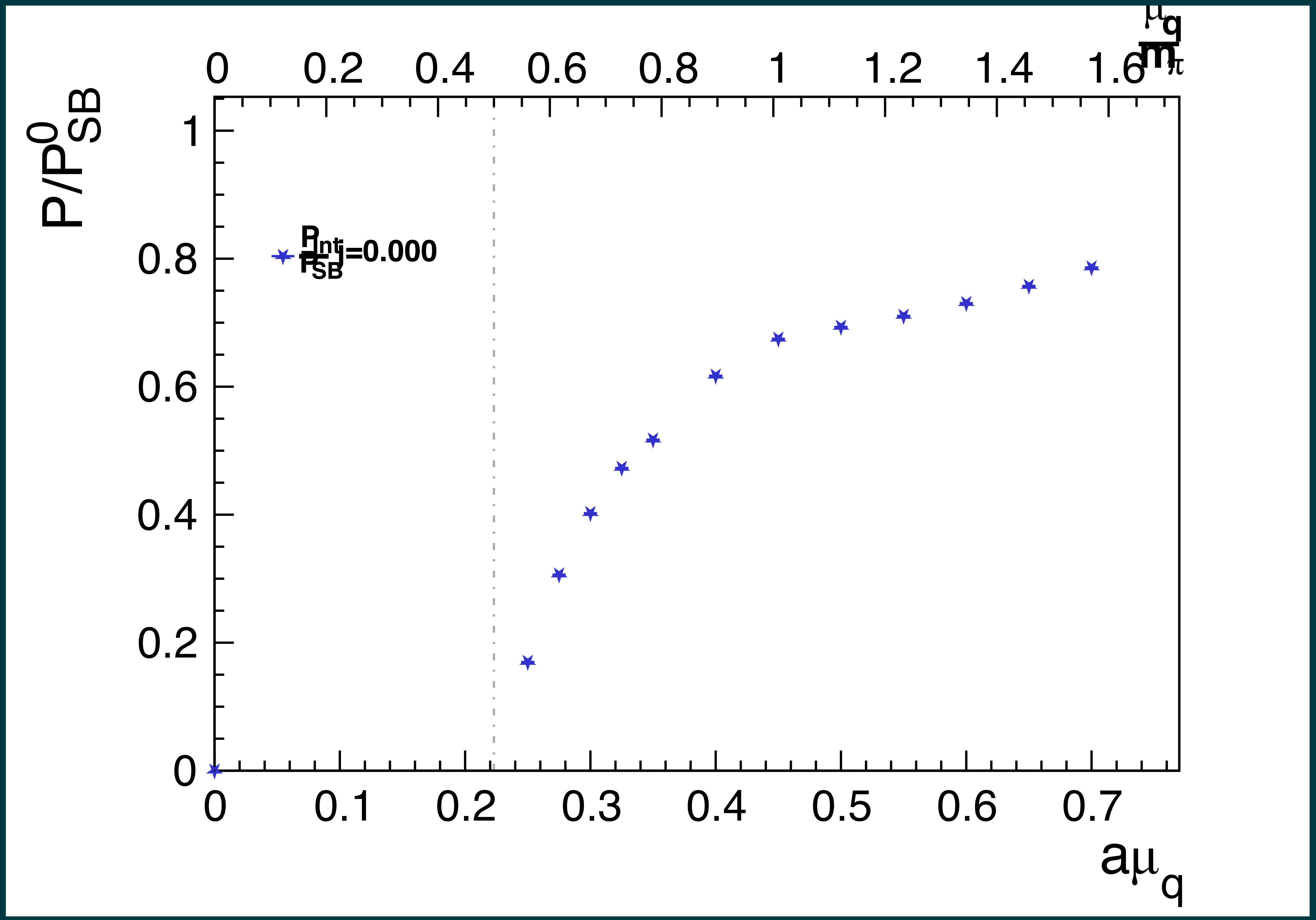
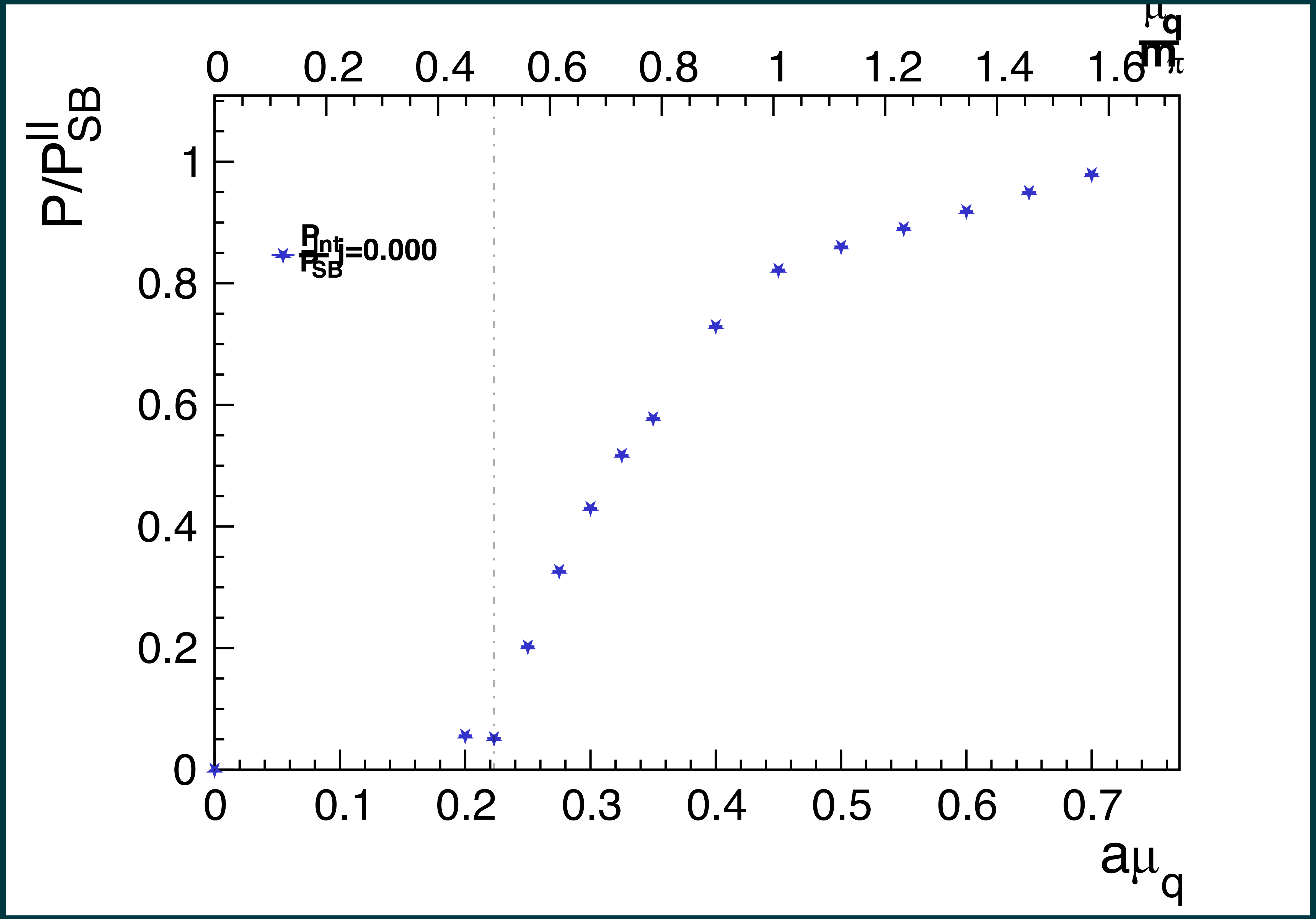
Pressure

- Two renormalisation schemes considered
- $\frac{P_0}{P_{SB}}(\mu) = \frac{1}{P_{cont}(\mu)} \int_0^{\mu} n_q(\mu')\, d\mu'$ $\qquad$ $\frac{P_{II}}{P_{SB}}(\mu) = \frac{1}{P_{cont}(\mu)} \int_0^{\mu} \frac{n_{cont}}{n_{Lat}}(\mu')\, n_q(\mu')\, d\mu'$
- Full details in [1912.10975] and [1210.4496]
- Scheme II preferred in this work

Pressure

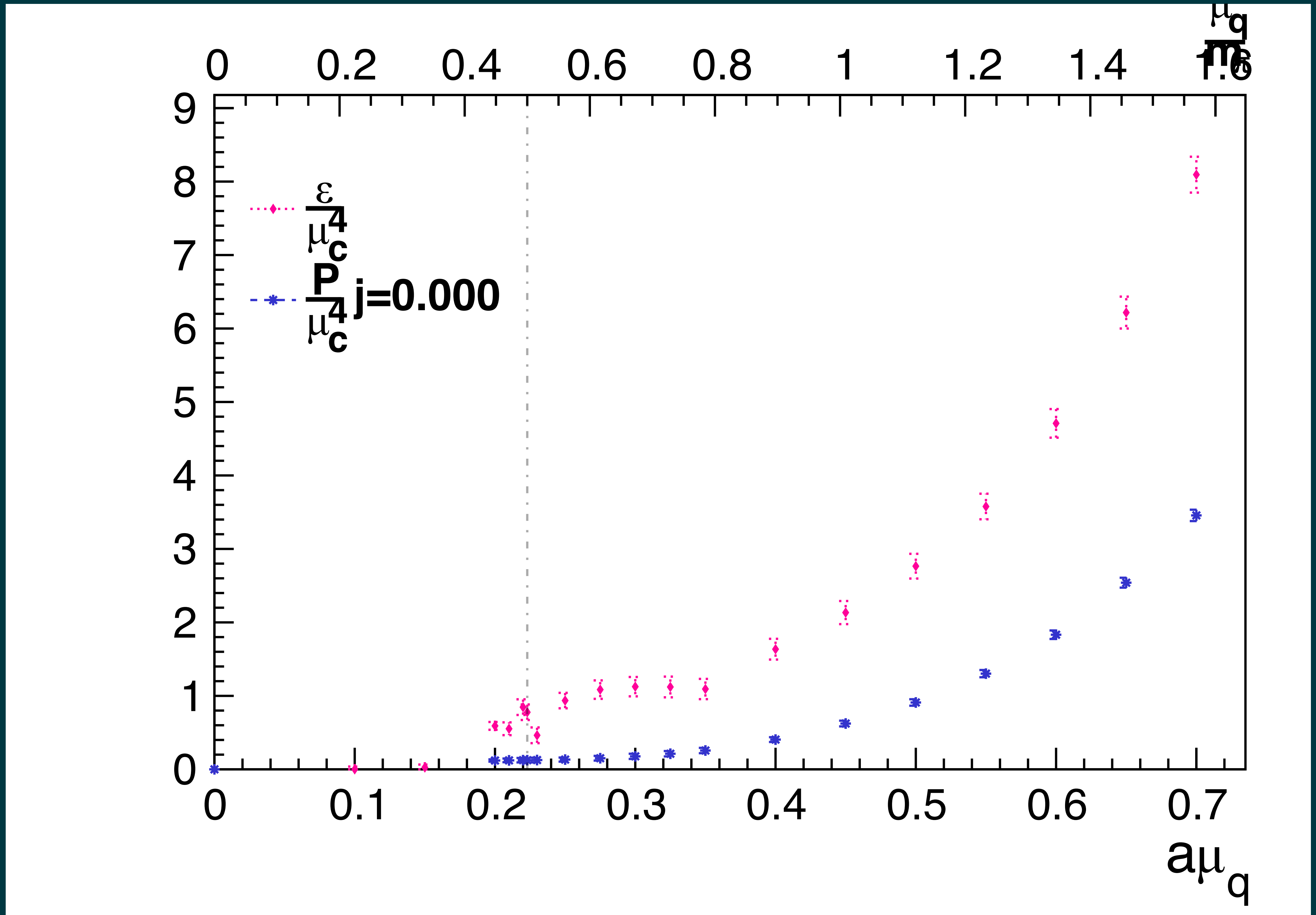


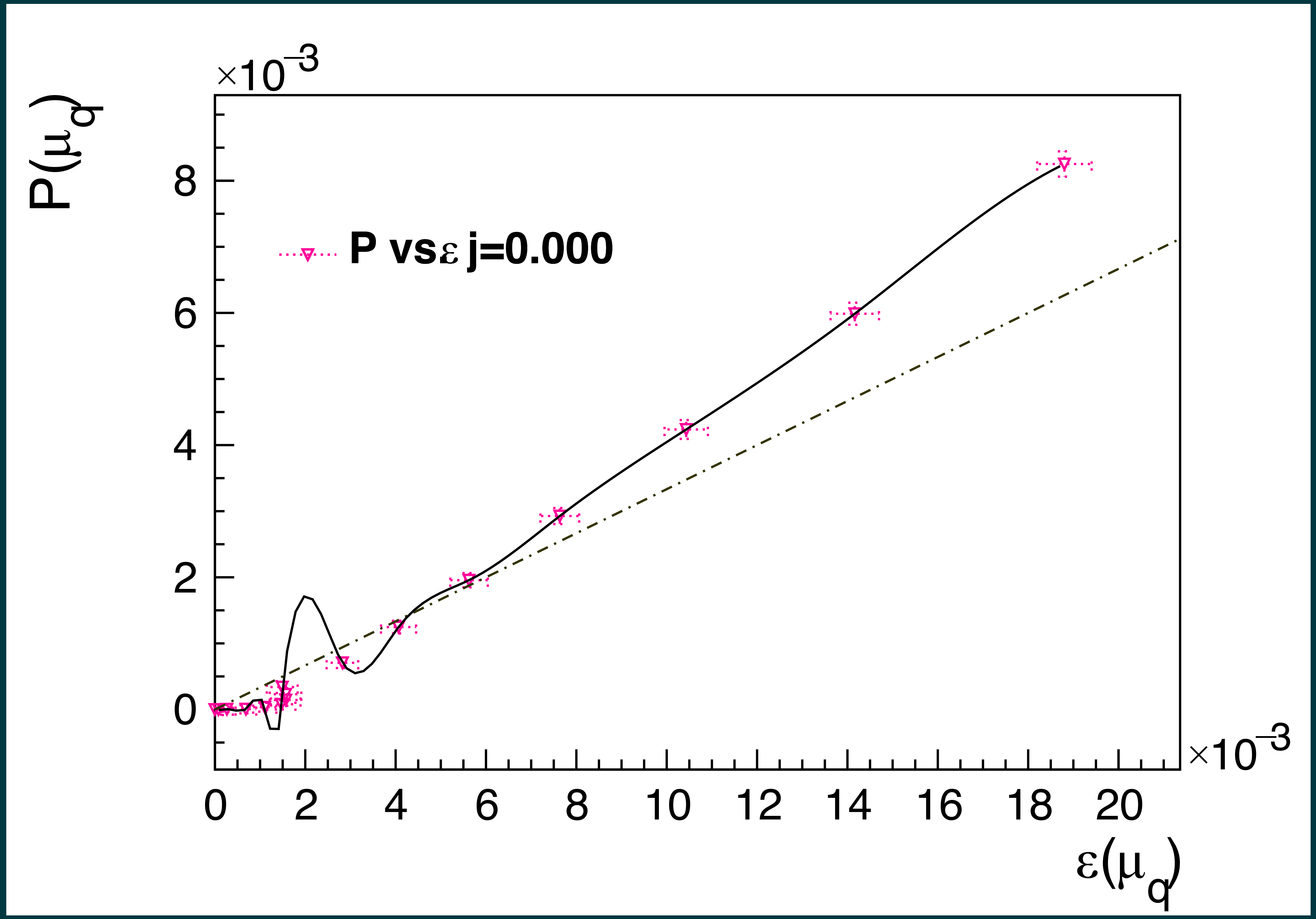
Scheme 0



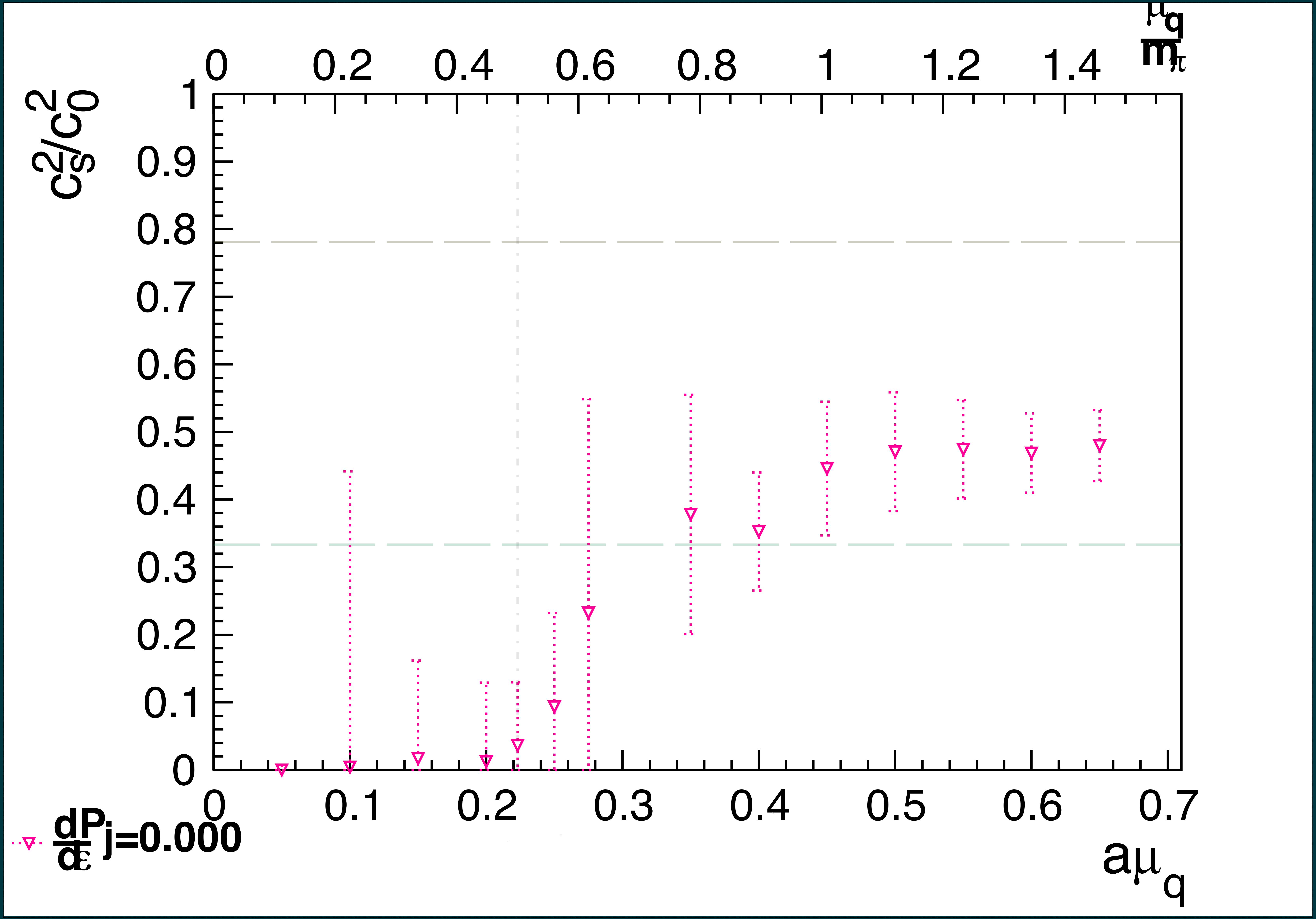
Scheme II

Pressure and Energy Density





Speed of Sound



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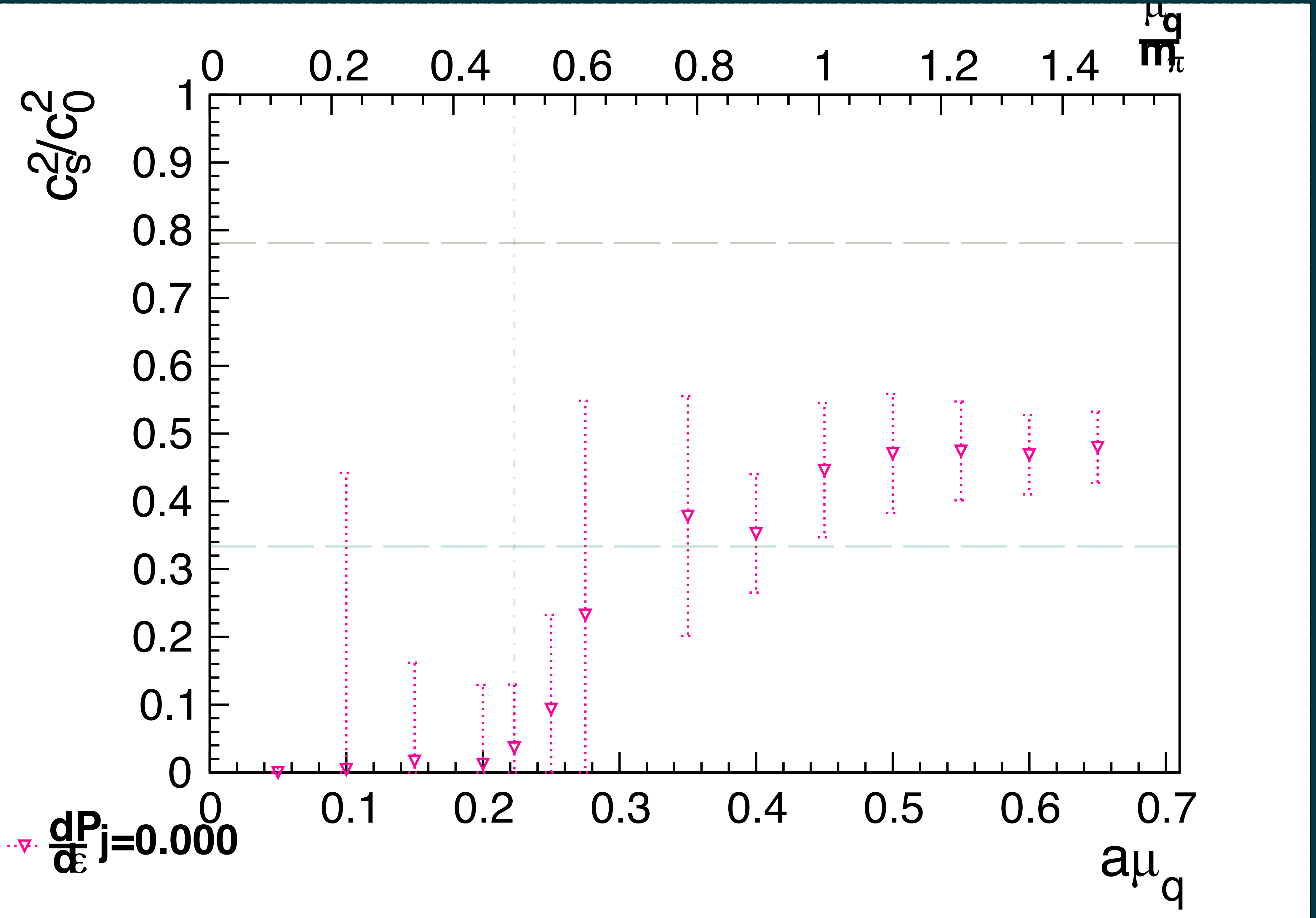
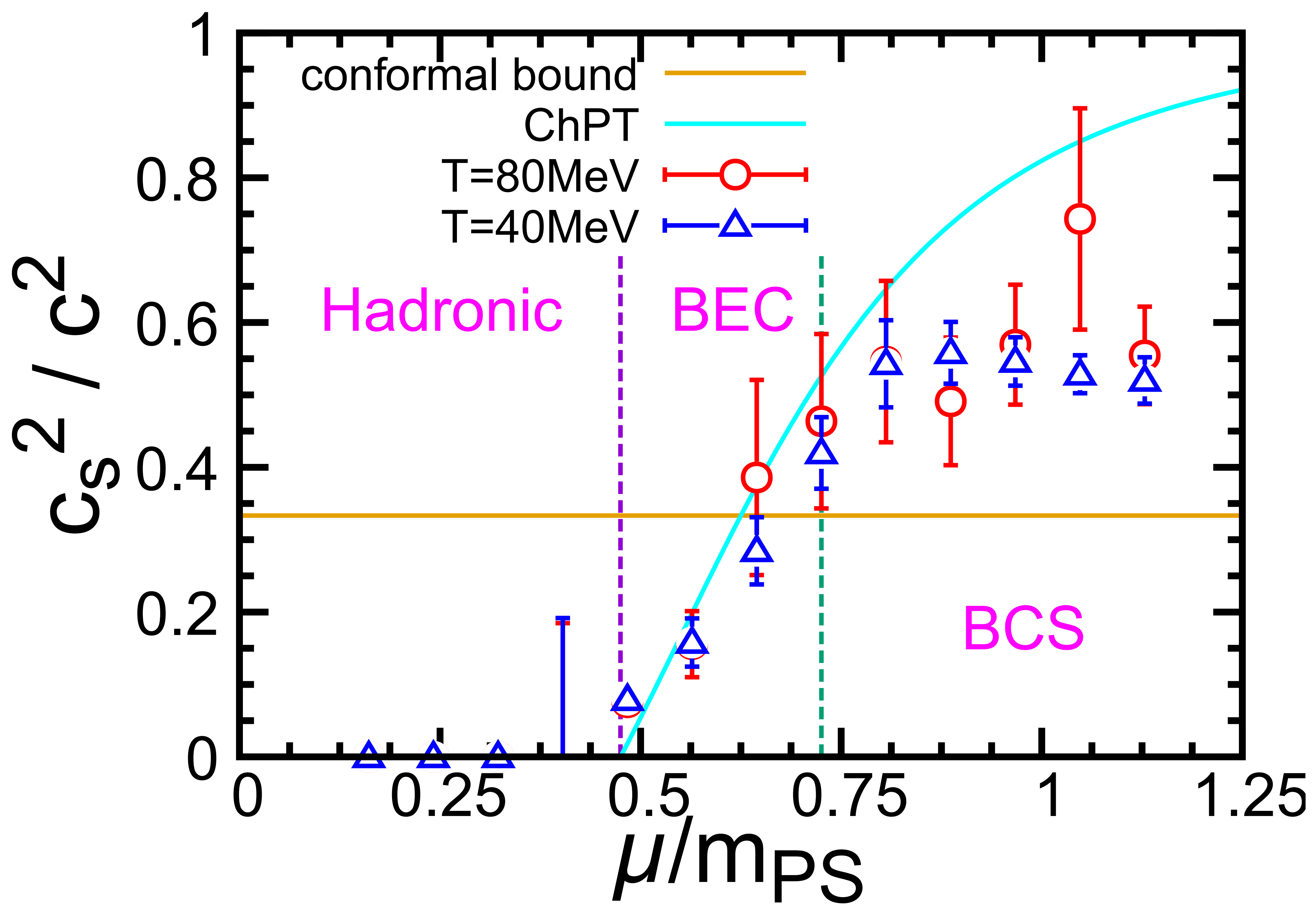
Conclusion





This work

(these Islands + Korea)

[2405.25006]

(Japanese Group)

Conclusion

Either we're both right

or

we're both wrong

Conclusion and Outlook

- Conformal limit exceeded again at finer spacing (i.e. larger $\frac{m_\pi}{\mu}$)
- Need a lot more work on error analysis
- Rerun all diquarks at larger N_s (24 or 32)?
- Run at $aj = 0.05$ also being considered
- (Correctly) upgrade integrator from LF to OMF
- Clover improved Wilson fermion action to catch up with the 1980s.

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Questions?
Comments?
Remarks?