Pion condensation at non-zero isospin chemical potential with Wilson fermions

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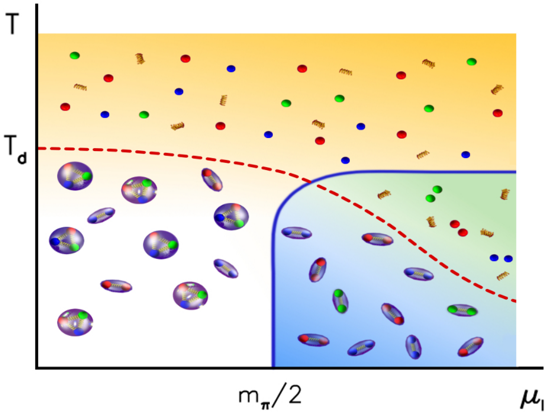
Setup

$$\mu_I \equiv (\mu_u - \mu_d)/2$$



Brandt, Endrődi, Schmalzbauer (2018)

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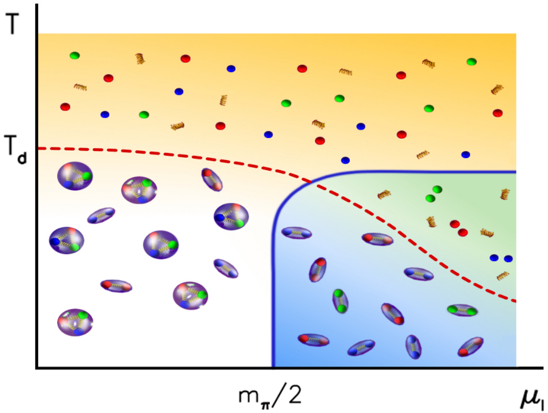
$$S = S_W + \overline{\psi}\mathcal{M}_{ud}\psi \qquad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\mathcal{M}_{ud} = \begin{pmatrix} D(\mu_I) & \lambda\gamma_5 \\ -\lambda\gamma_5 & D(-\mu_I) \end{pmatrix}$$

$$D(\mu_I) = \not{D}(\mu_I) + m_0$$



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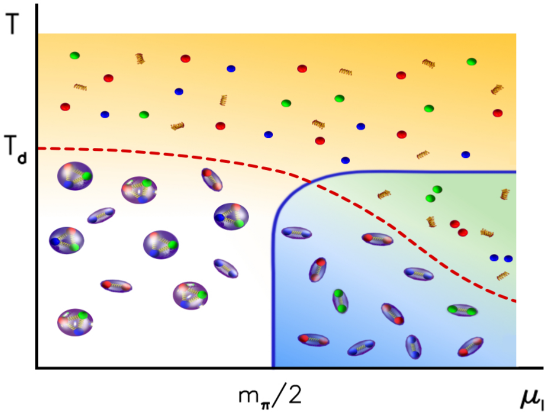
- Pionic source term $\lambda > 0$ for SB and IR regulator (*unphysical!*)



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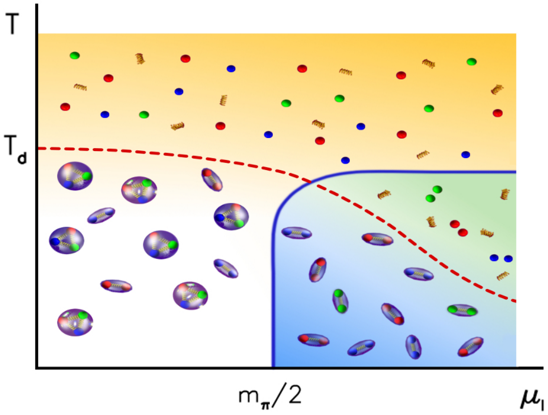
- Pionic source term $\lambda > 0$ for SB and IR regulator (*unphysical!*)
- No sign problem
  $\det \mathcal{M}_{ud} = \det(D^\dagger(\mu_I)D(\mu_I) + \lambda^2) \geq 0$

$$\mathcal{Z} = \int dU e^{-S_W[U]} \det \mathcal{M}_{ud}$$

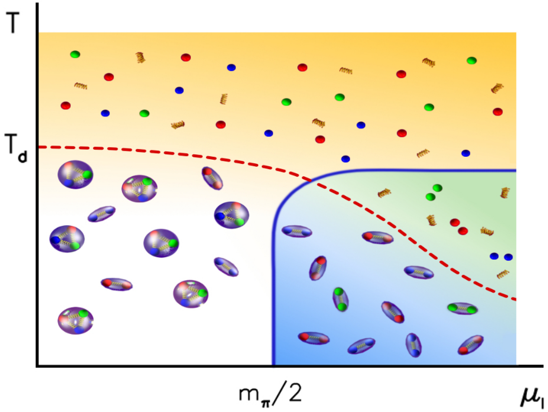


Brandt, Endrődi, Schmalzbauer (2018)

Conjectured phase diagram

- Low T, μ_I: Hadronic phase
- High T: Quark-Gluon plasma
- Low T, $\mu_I > m_\pi/2$: Pion condensation (BEC) (spontaneous breaking of $U_{\tau_3}(1)$)
- Large μ_I: BCS phase?

- Previous literature on the topic:
  Analytical: Son, Stephanov (2001)
  Grand canonical approach (Staggered): Kogut, Sinclair (2002), , (2004) , Endrődi (2014) , Brandt et al. (2018) , Brandt et al. (2023)
  Canonical approach: de Forcrand et al. (2007) , Detmold et al. (2012) , NPLQCD (2023)



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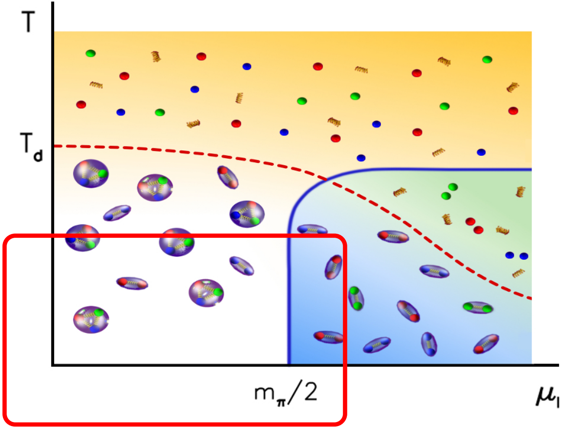

Brandt, Endrődi, Schmalzbauer (2018)

Simulation setup

$$\mathcal{Z} = \int dU e^{-S_W[U]} \det \mathcal{M}_{ud}$$

- Wilson fermions + plaquette action
- $N_f = 2$ degenerate quarks
- 8×24^3 lattice, $m_\pi \simeq 560$ MeV, $T \simeq 79$ MeV, $a \simeq 0.31$ fm
- SMD update algorithm Francis, Fritzsch, Lüscher, Rago (2020)

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$$\det \mathcal{M}_{ud} = W \det R^2[\lambda_0] \prod_{k=0}^{n} \frac{\det R^2[\lambda_{k+1}]}{\det R^2[\lambda_k]}$$

- Hasenbusch preconditioning in λ necessary for stability of MD integration

Improved pion condensate from Banks-Casher

$$\left\langle \pi^{\pm} \right\rangle = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial \lambda} = 2\lambda \frac{T}{V} \mathrm{Tr} \frac{1}{D(\mu_I)^{\dagger} D(\mu_I) + \lambda^2}$$

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The pion condensate can be written in terms of the singular values ξ of $D(\mu_I)$

$$D(\mu_I)^{\dagger} D(\mu_I) \psi_n = \xi_n^2 \psi_n \qquad D(\mu_I) \equiv \not{D}(\mu_I) + m_0$$

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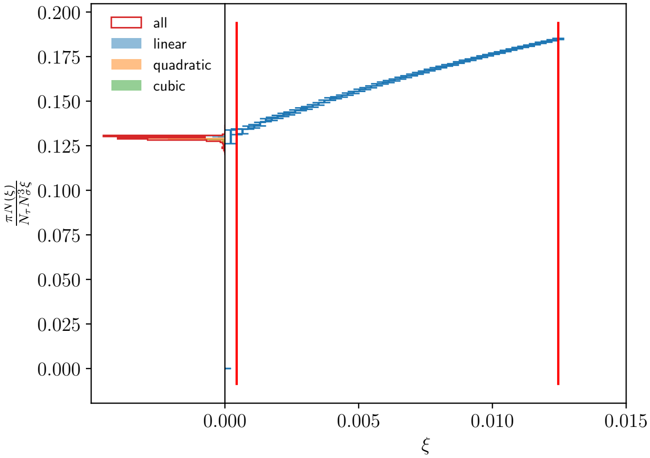
$$\begin{aligned} \left\langle \pi^{\pm} \right\rangle &= 2\lambda \frac{T}{V} \mathrm{Tr} \frac{1}{D(\mu_I)^{\dagger} D(\mu_I) + \lambda^2} \\ &= 2\lambda \frac{T}{V} \left\langle \sum_n \frac{1}{\xi_n^2 + \lambda^2} \right\rangle \xrightarrow[V \to \infty]{} 2\lambda \left\langle \int d\xi \rho(\xi) \frac{1}{\xi^2 + \lambda^2} \right\rangle \xrightarrow[\lambda \to 0]{} \pi \left\langle \rho(0) \right\rangle \end{aligned}$$

This definition facilitates $\lambda \to 0$ and $V \to \infty$

Improved pion condensate from Banks-Casher



$$\left\langle \pi^{\pm} \right\rangle \equiv \pi\rho(0)$$

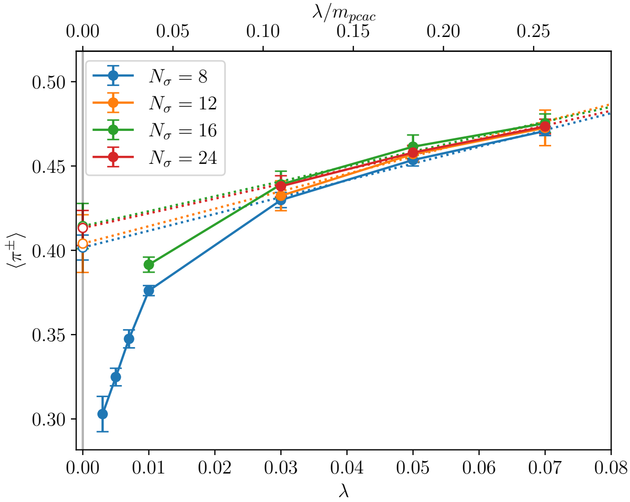
We extrapolate $\pi\rho(0)$ from

$$\lim_{\xi\to 0} \frac{\pi}{N_\tau N_\sigma^3} \frac{N(\xi)}{\xi} = \pi\rho(0)$$

where $N(\xi)$ is the integrated singular value density

$$N(\xi) \equiv \int_0^{\xi} \rho(\xi')d\xi'$$

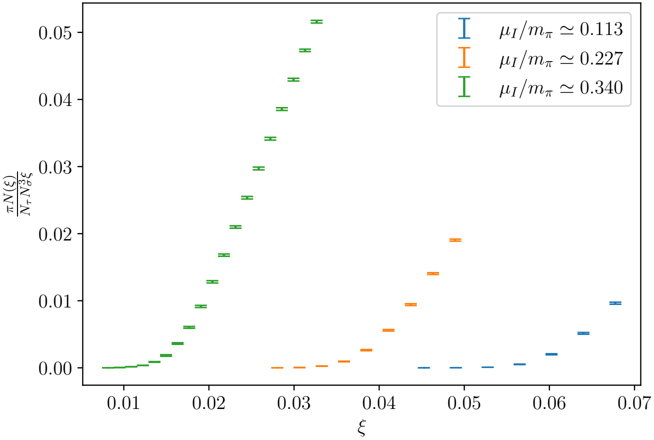
Finite volume effects at $\mu_I \simeq 0.7 m_\pi$



- $8 \times N_\sigma^3$
- Pion condensation phase
- $T \simeq 79$ MeV
- Results are fairly insensitive to N_σ for $\lambda/m_{pcac} \gtrsim 0.10$
- The pion mass in the condensed phase is $\propto \lambda^2 \implies \lambda/m_{pcac} < 0.10$ triggers significant finite-size effects

The integrated spectral density across the transition: $\mu_I \ll m_\pi/2$

8×24^3, $\lambda = 0.05$



- No positive extrapolation to $\xi \to 0$
- $\implies$ No pion condensate

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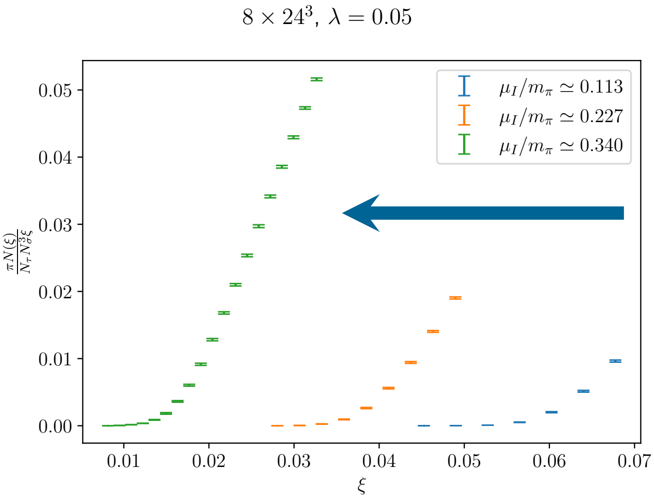

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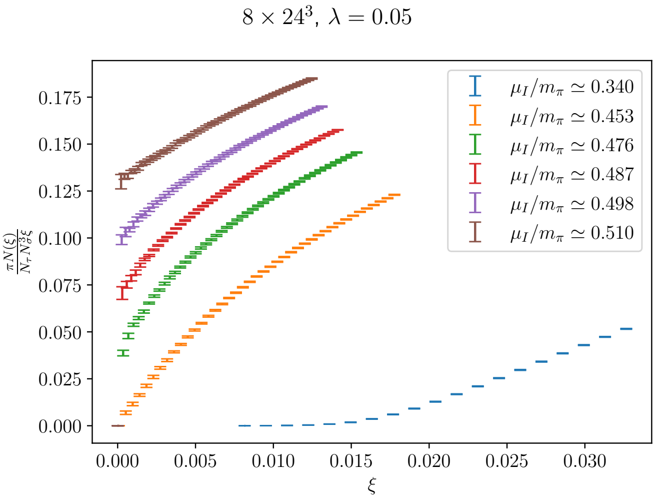
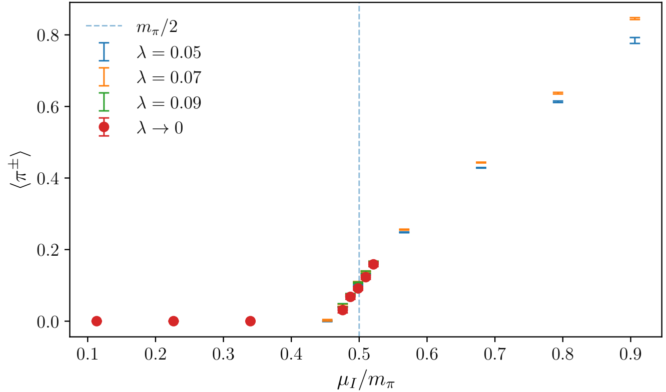
The integrated spectral density across the transition: $\mu_I \sim m_\pi/2$



- Low modes of $D^\dagger D$ accumulate when $\mu_I \sim m_\pi/2$
- $\implies$ Non-zero extrapolation of $\langle \pi^\pm \rangle (\mu_I, \lambda)$

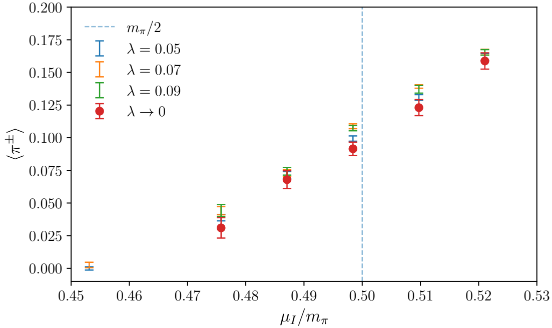
The pion condensate across the transition at $T \simeq 79$ MeV

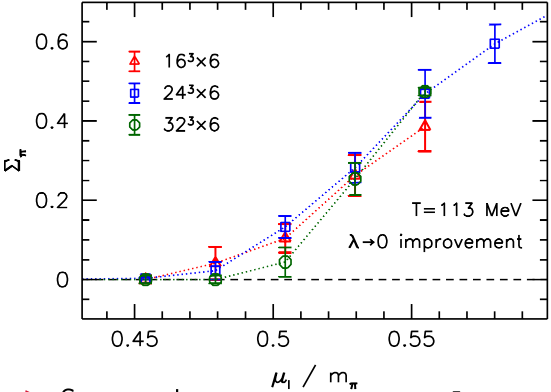


- $\lambda > 0$ is non-physical
- We need to extrapolate $\lambda \to 0$
- Linear ansatz (as in staggered simulations)

- We observe pion condensation for $\mu_I \gtrsim m_\pi/2$

The pion condensate across the transition at $T \simeq 79$ MeV





Brandt et al. (2018)

- Wilson
- $m_\pi \simeq 560$ MeV
- $T \simeq 79$ MeV

- Staggered
- Physical point
- $T \simeq 113$ MeV

Conclusions

- We simulate $N_f = 2$ Wilson fermions at non-zero μ_I at $m_\pi \simeq 560$ MeV and fixed lattice spacing $a \simeq 0.31$ fm
- Pion condensation is related to the low modes of $D^\dagger D$ through a Banks-Casher relation
- This definition facilitates $\lambda \to 0$ and $V \to \infty$
- We observe pion condensation for $\mu_I \gtrsim m_\pi/2$ at $T \simeq 79$ MeV
- Future outlook: First computation of the EoS with Wilson fermions at intermediate temperatures