Phase and equation of state of finite density QC_2D at lower temperature

Etsuko Itou (YITP, Kyoto U./ RIKEN iTHEMS)

K.Iida, EI, D.Suenaga, K.Murakami; arXiv:2405.20566 (Phase, EoS, 32^4 lattice, T=40MeV)

K.Iida and EI; PTEP 2022 (2022) 11, 111B01 (EoS, 16^4 lattice, T= 80MeV)

K.Iida, EI, and T.-G. Lee; JHEP01(2020)181 (Phase, 16^4 lattice T=80MeV and $32^3 \times 8$ T=160MeV)

The 41th International Conference on Lattice field thoery (LATTICE 2024), University of Liverpool, UK. 2024/08/01

Introduction

- 2color QCD action + finite density term + diquark source term (j)

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} - \underbrace{\frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

Kogut et al. NPB642 (2002)18

avoids the sign and onset problems

allows HMC simulation to be performed in whole T-μ regime

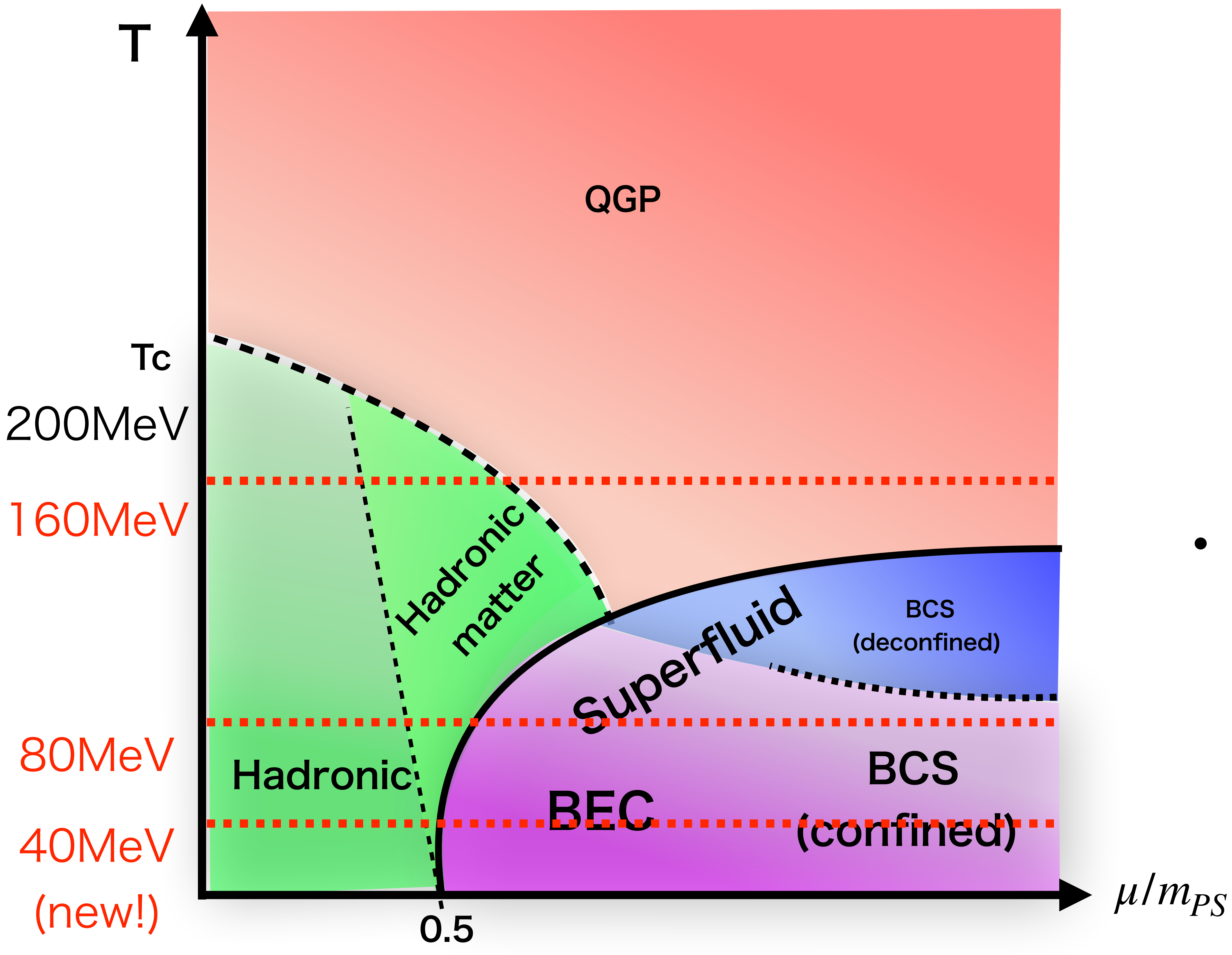
- Emergence of superfluidity, $\langle qq \rangle \neq 0$, has been confirmed by several independent groups (S.Hands et al. Russian group, von Smekal et al.)

- A rich phase structure below Tc as a function of μ has been revealed, but finite volume effects in a high-density regime sometimes cause a wrong understanding

- We investigate the T-dependence down to zero temperature

QC2D phase diagram

K.Iida, EI, T.-G. Lee:
JHEP2001(2020)181



- T=80MeV, there are 4 phases:

  Hadronic phase : $\langle n_q \rangle = 0,\ \langle qq \rangle = 0$

  Hadronic-matter : $\langle n_q \rangle > 0,\ \langle qq \rangle = 0$

  BEC phase: $\langle n_q \rangle > 0,\ \langle qq \rangle > 0$

  BCS phase: $\langle n_q \rangle \approx n_q^{\mathrm{tree}},\ \langle qq \rangle > 0$

- We newly found at T=40MeV:

  (1) $\langle qq \rangle \propto \mu^2$ scaling of BCS phase in lower-T

  (2) hadronic-matter phase shrinks in lower-T

  (3) non-zero topological susceptibility in BCS phase

Lattice setup

- beta=0.80 (Iwasaki gauge), Nf=2 Wilson fermion
  In previous works, T=160MeV ($32^3 \times 8$) and T=80MeV(16^4)

- New data: T=40MeV(32^4)

- diquark source parameter (j)
  j=0.010, 0.015, 0.020
  (linear or constant extrapolation to take the j=0 limit)

- 15 (μ) x 3 (j) ~ more than 40 parameters, generated 100 conf. for each parameter

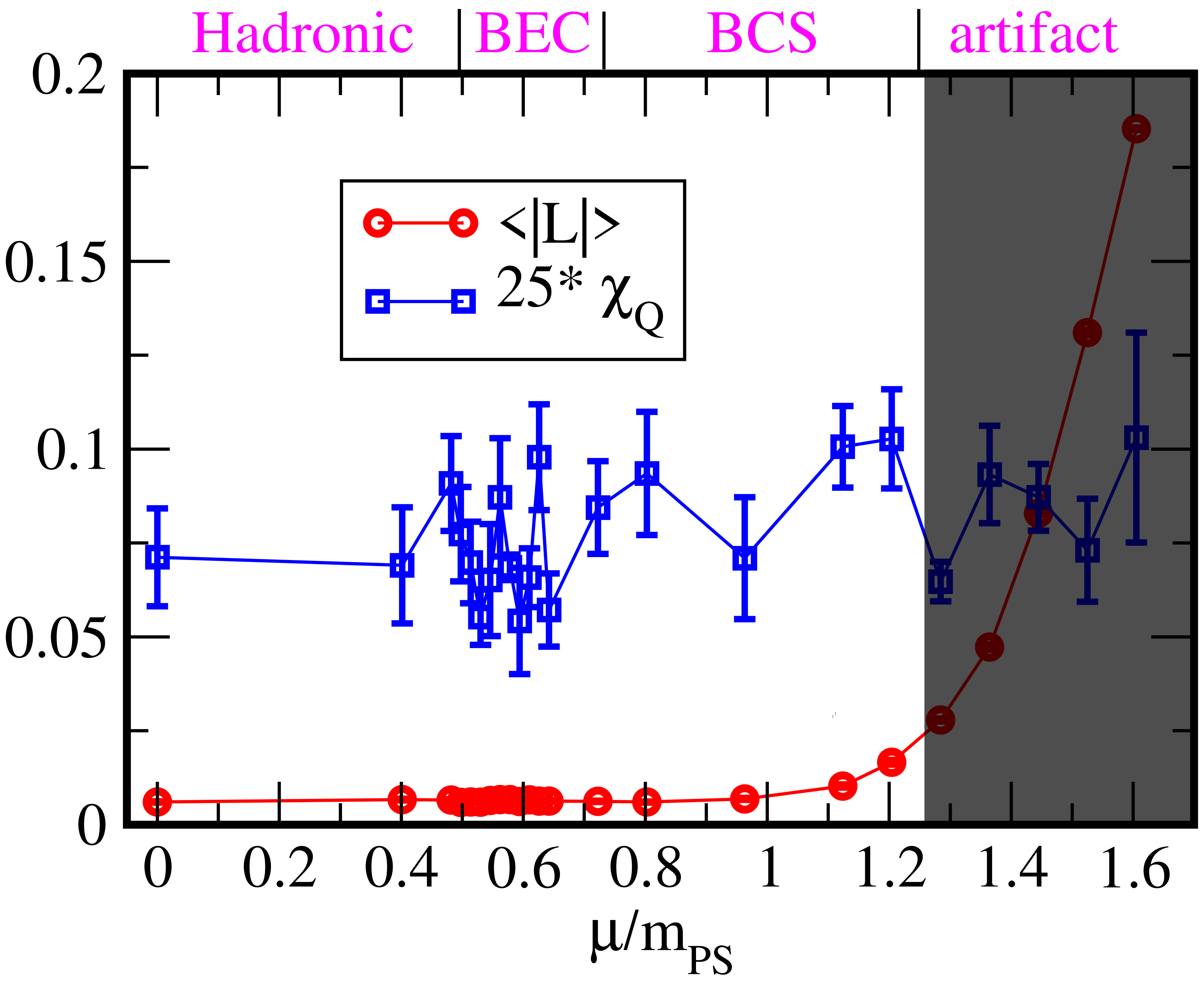
- Around μ_c, reweighing of j up to j=0.001 works well to perform a reliable j=0 extrapolation

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

(3) Topological susceptibility and confinement

Previous our work

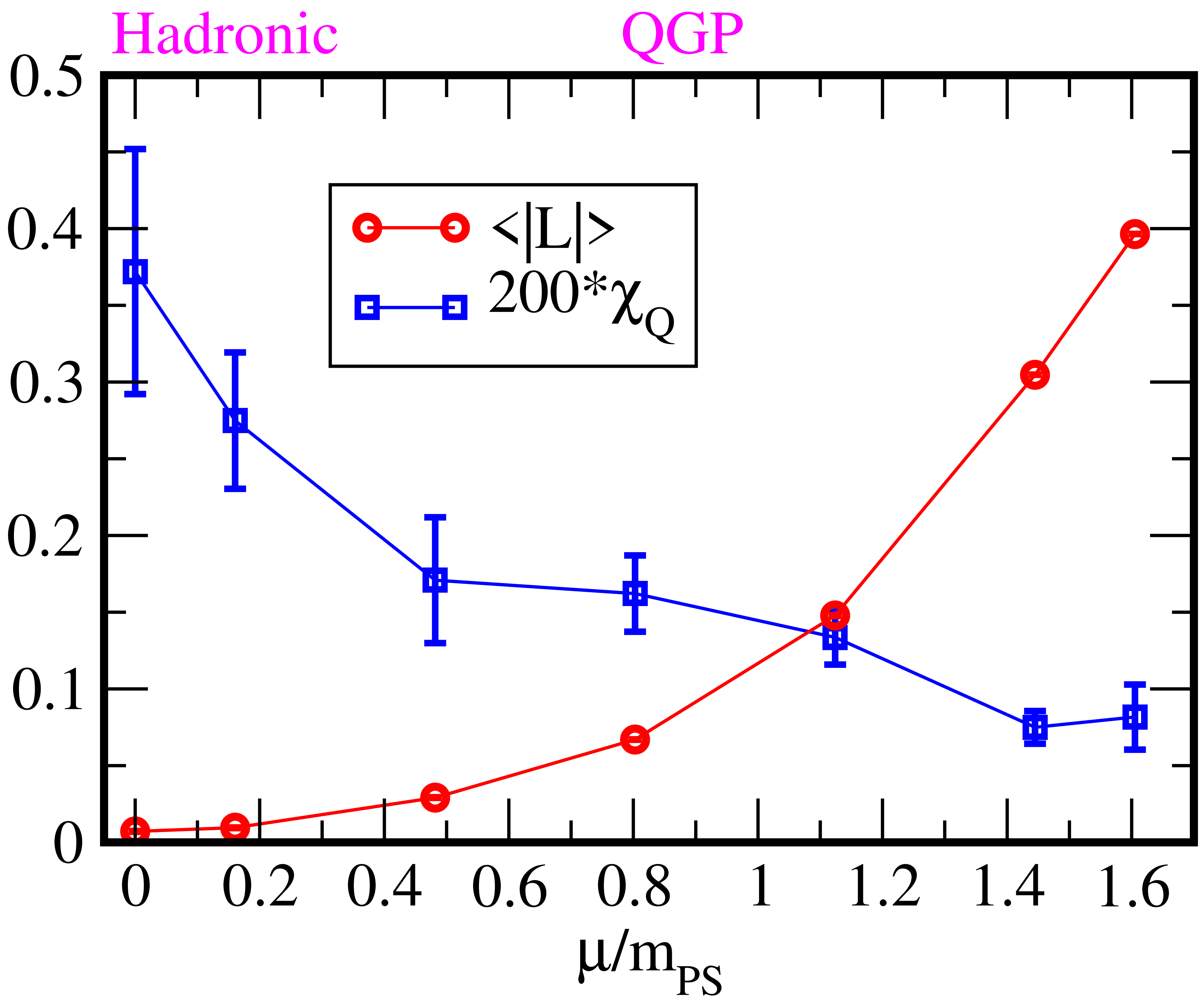
T=80MeV



X_Q does not change even in BCS phase

w/ confinement

T=160MeV

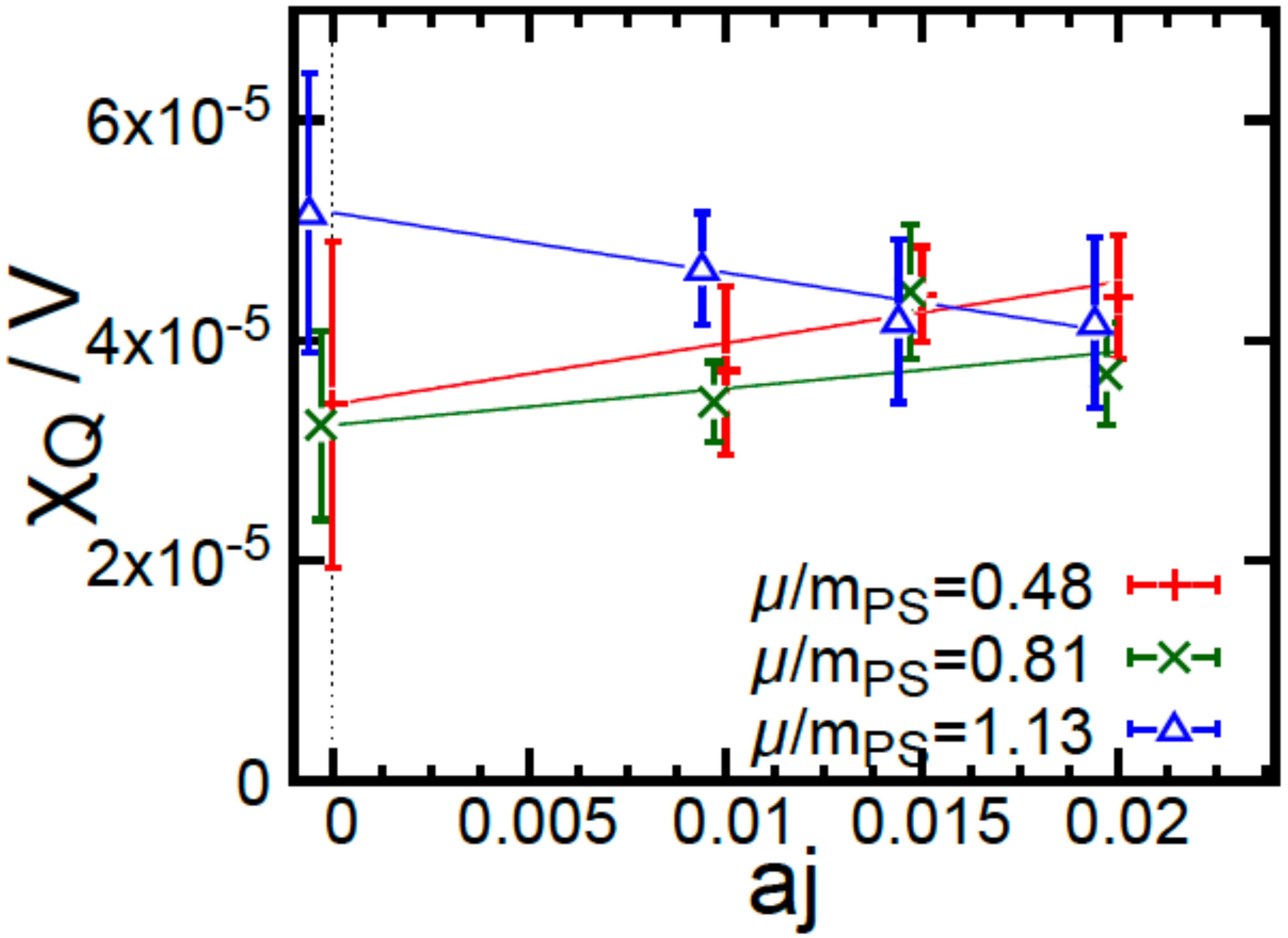


Polyakov loop increases <=> X_Q decreases

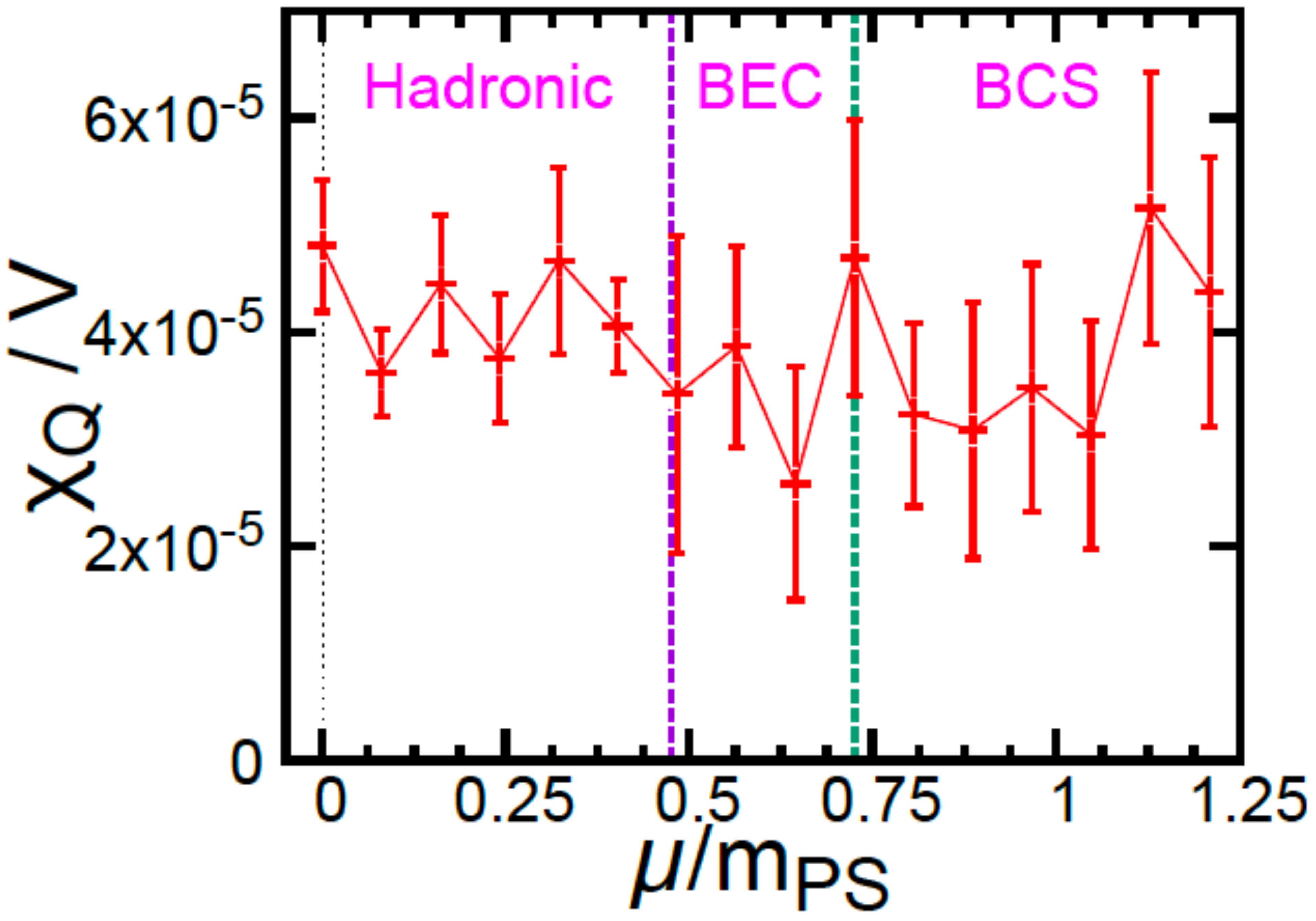
(3) Topological susceptibility and confinement

New data

j->0 extrapolation



T=40MeV



Even in high density, X_Q does not decrease

Is it related with confinement?

In $T \lesssim 100$MeV, the confinement remains even in high-μ ($\mu \sim 1$GeV)

T. Boz et al. (2019)

A.Begun et al. (2022)

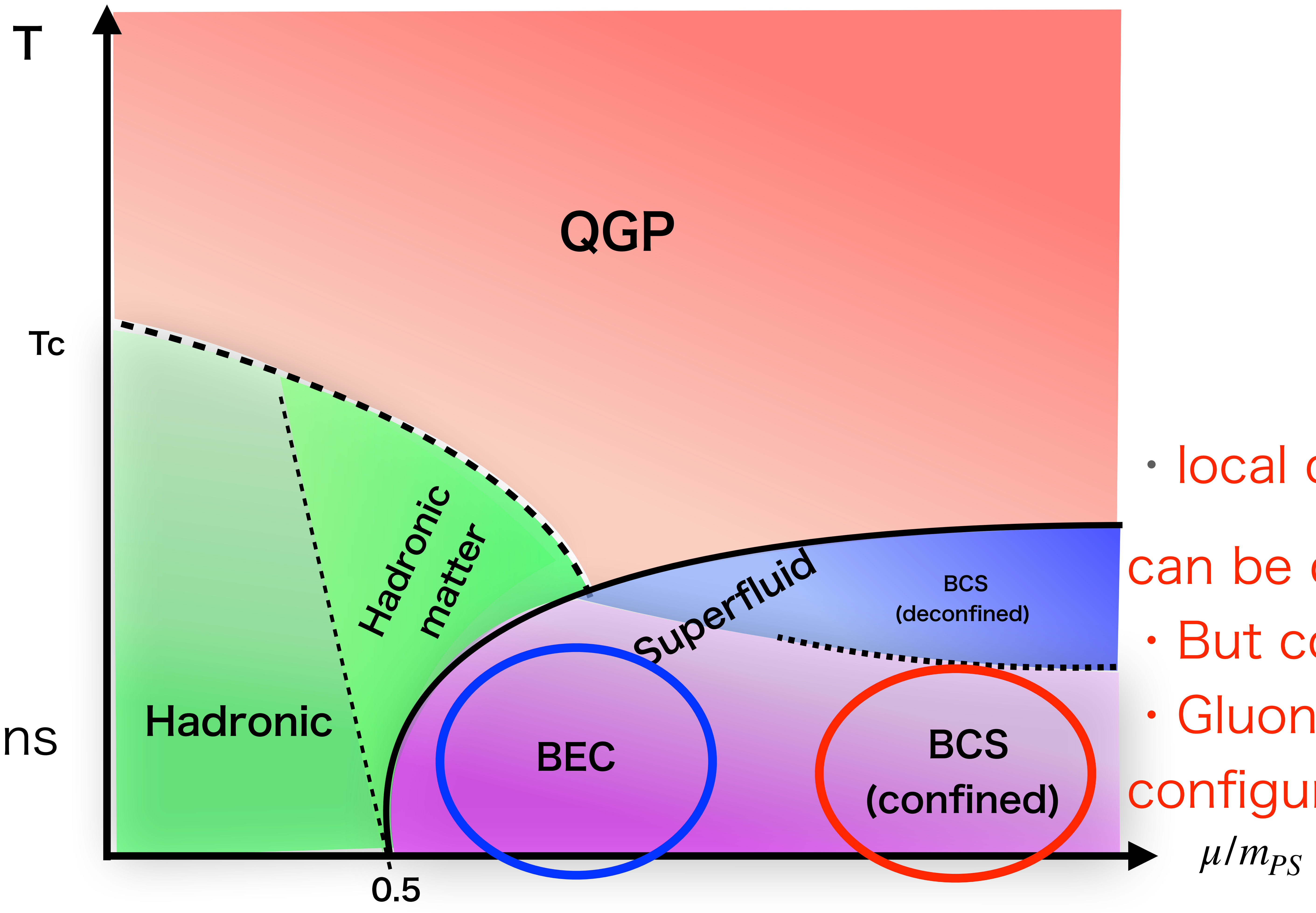
K.Iida, K.Ishiguro , EI, arXiv: 2111.13067

If quark mass is heavy then the decreasing is very gentle?

(cf.)Kawaguchi-Suenaga(2023)

Short summary for phase diagram

quarks and gluons
perturbative



quarks and gluons
are confined

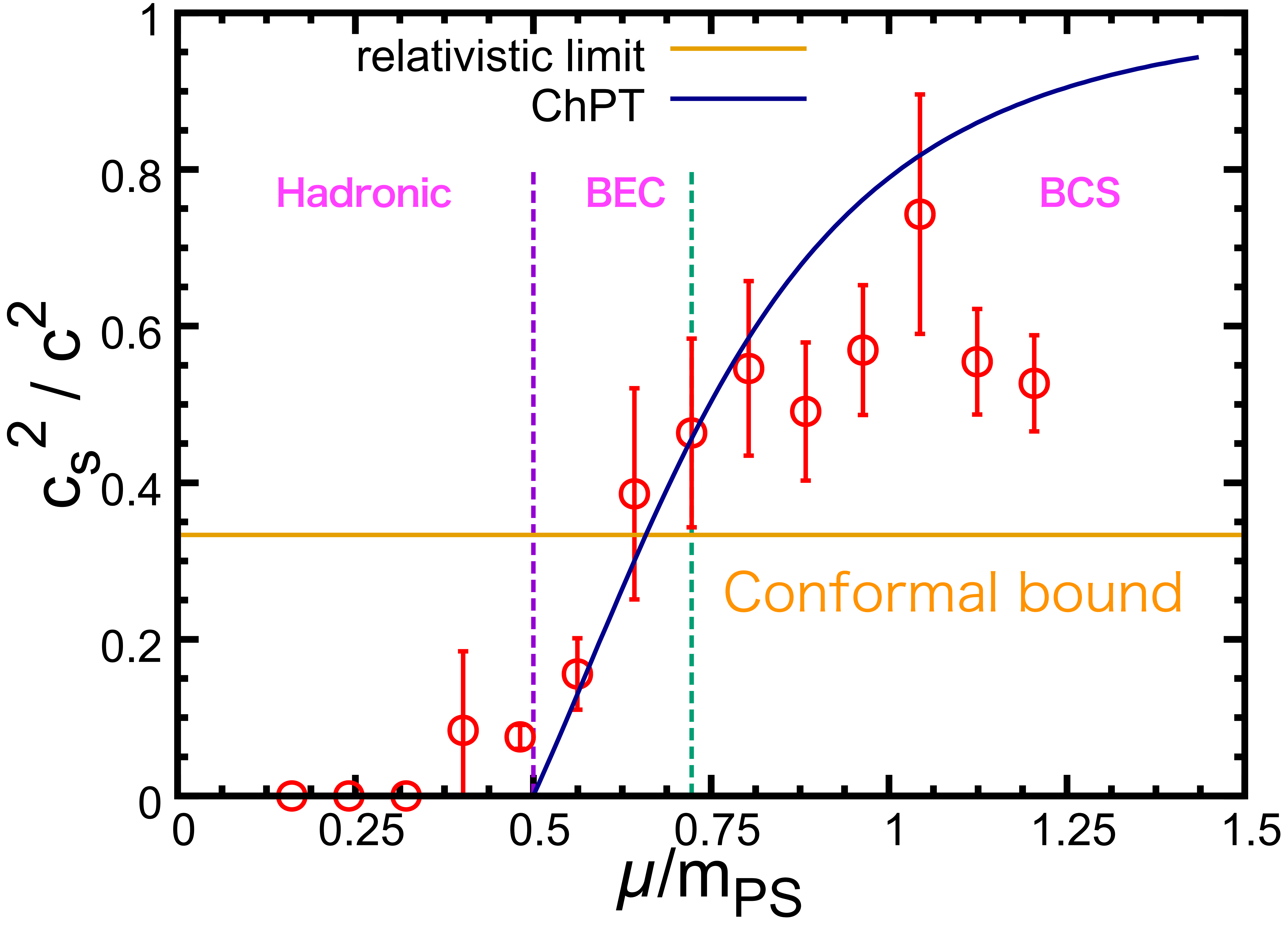
- local quantities, $\langle n_q \rangle$, $\langle qq \rangle$, can be described by free theory
- But confinement remains.
- Gluon has nontrivial instanton configuration

Predictions by ChPT works very well!

Eq. of state

Sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$), T=80MeV ($16^4$ lattices)

K.Iida and EI, PTEP 2022 (2022) 11, 111B01



Chiral Perturbation Theory (ChPT)

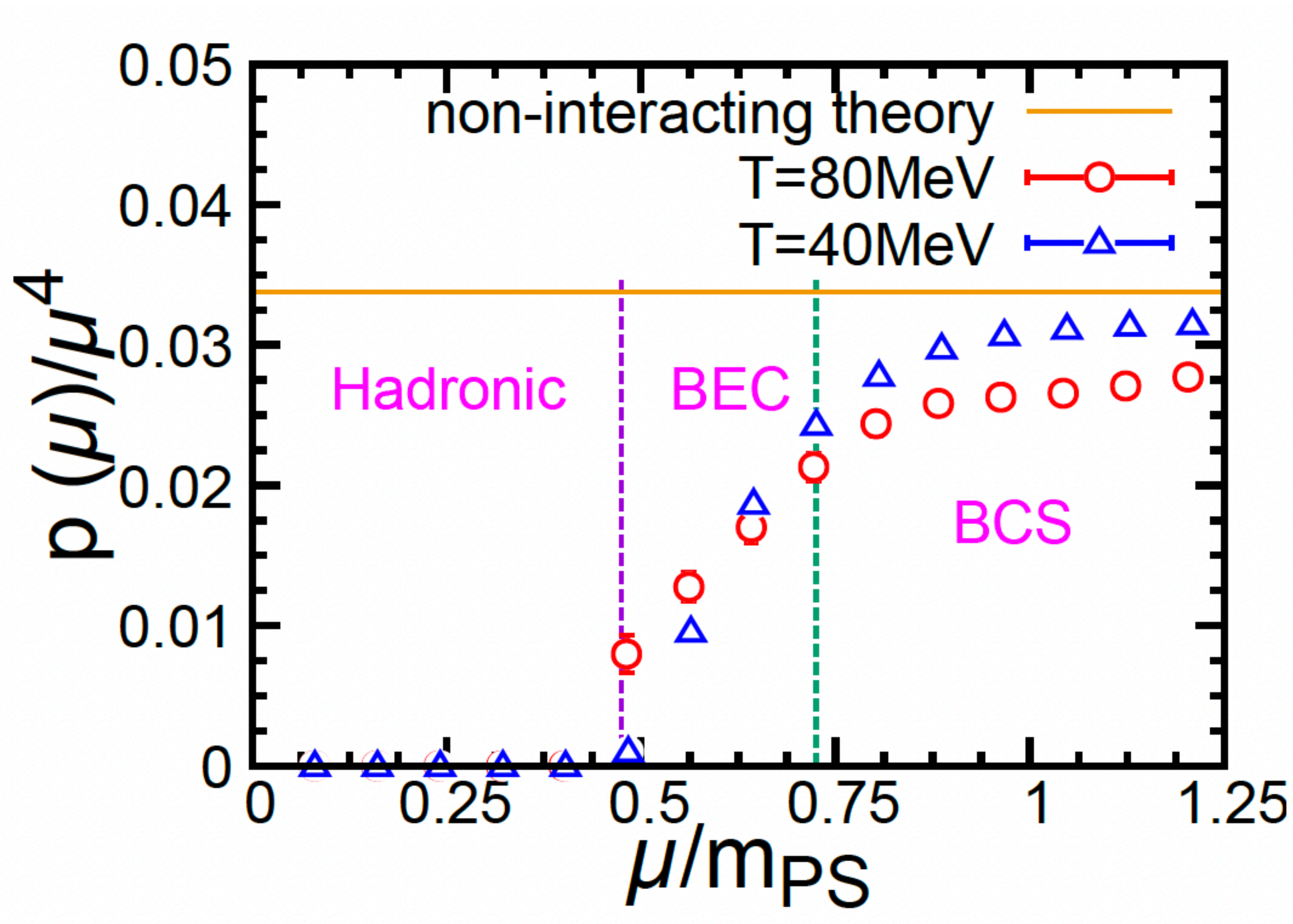
$$c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$$: no free parameter!!

Son and Stephanov (2001) : 3color QCD with isospin μ

Hands, Kim, Skullerud (2006) : 2color QCD with real μ

- In BEC phase, our result is consistent with ChPT.
- c_s^2/c^2 exceeds the conformal bound

T dependence of EoS



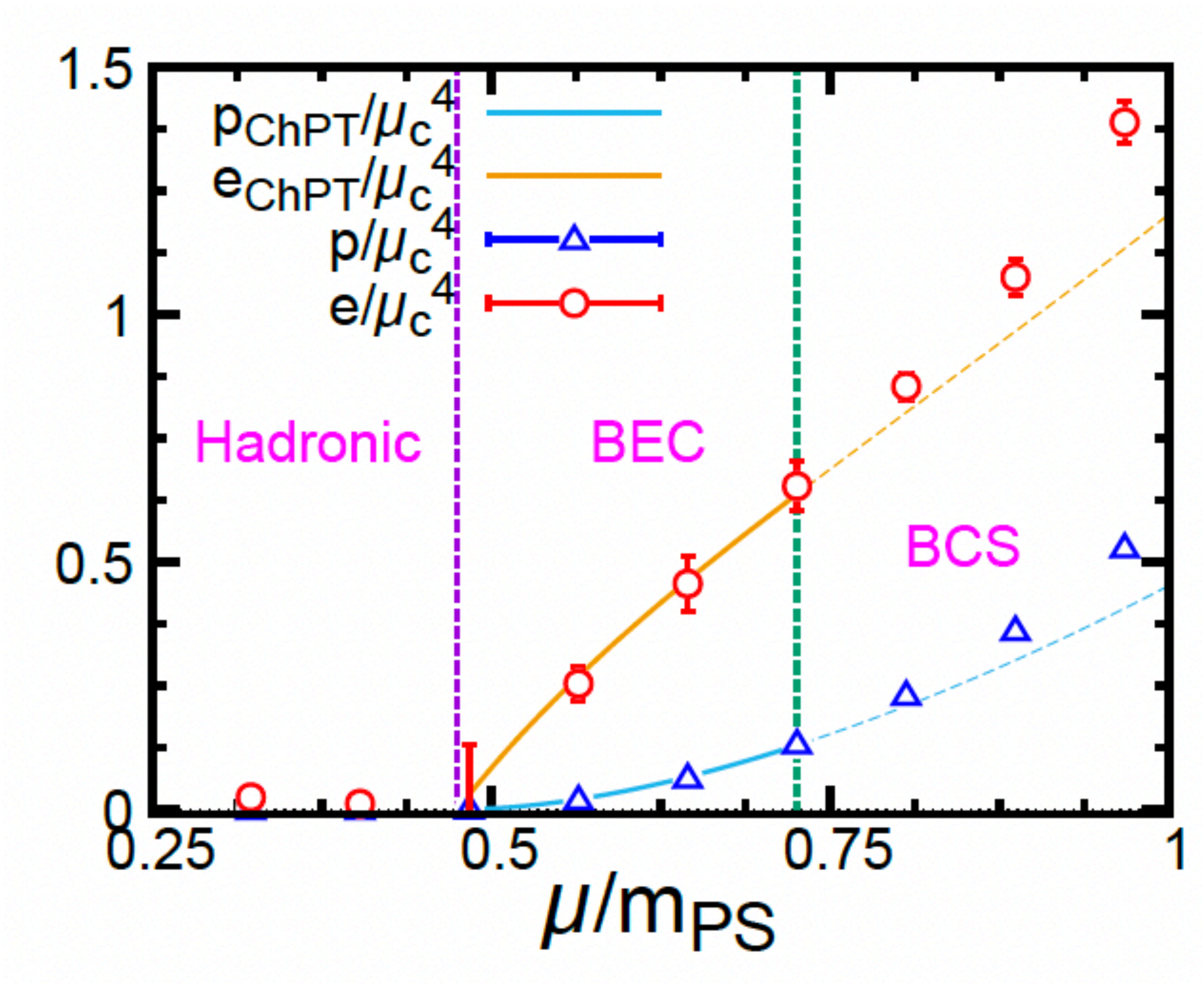
- p increases more rapidly near the critical point at lower-T

- In high-μ, the data approaches the Stefan-Boltzmann limit (=non-interacting theory)

$$p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$$

- Our largest data of p at T=40MeV reaches at 93% of p_{SB}

EoS and consistency with ChPT result in BEC



- ChPT prediction (valid for near μ_c)

$$p_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)^2$$

$$e_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)\left(1 + 3\frac{\mu_c^2}{\mu^2}\right)$$

- We obtain the pion decay constant(F)

  from fit of p : F=51.1(5) MeV

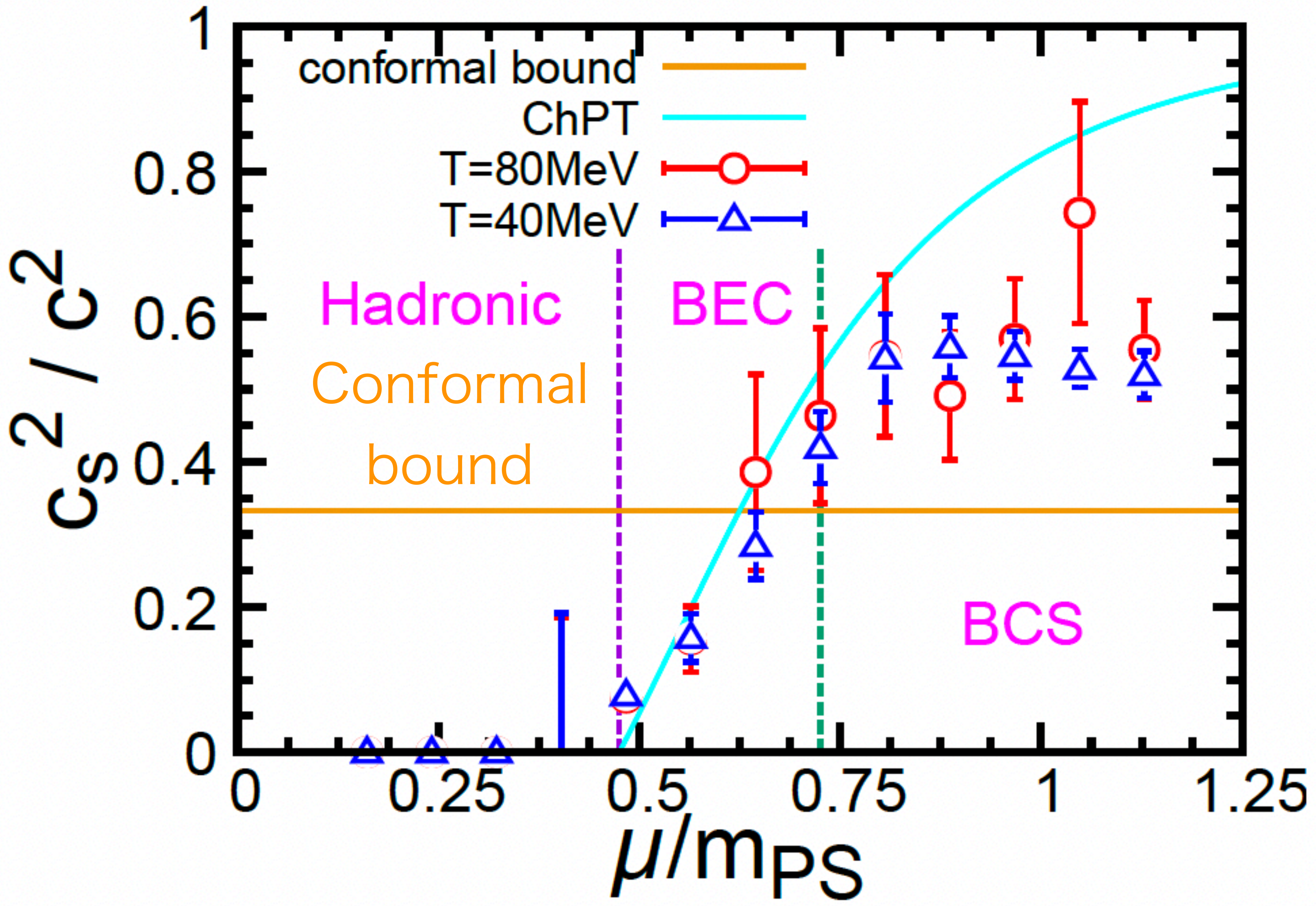
  from fit of e : F=56.7(7) MeV

  cf.) F=60.8(1.6) by fitting of $\langle n_q \rangle$ at 140MeV

  (different mass, staggered fermion)

N. Astrakhantsev et al. (2020)

Square of sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



- T-dependence of the sound velocity is negligible!
- In BEC phase, our result is consistent with ChPT
- It exceeds the conformal bound
- Confirmed by the data with small statistical errors!!

Summary

- Phase structure:

(1) $\langle qq \rangle \propto \mu^2$ scaling of BCS phase in lower-T

(2) hadronic-matter phase shrinks in lower-T, it comes from thermal exitation

(3) non-zero topological susceptibility exists even in BCS phase

In high-μ (μ~1GeV), local quantities can be explained by a perturbative analysis, but confinement and topology still show a non-perturbative properties

(Lattice study must be important!)

- EoS:

(1) pressure (also energy density) shows a T-dependence

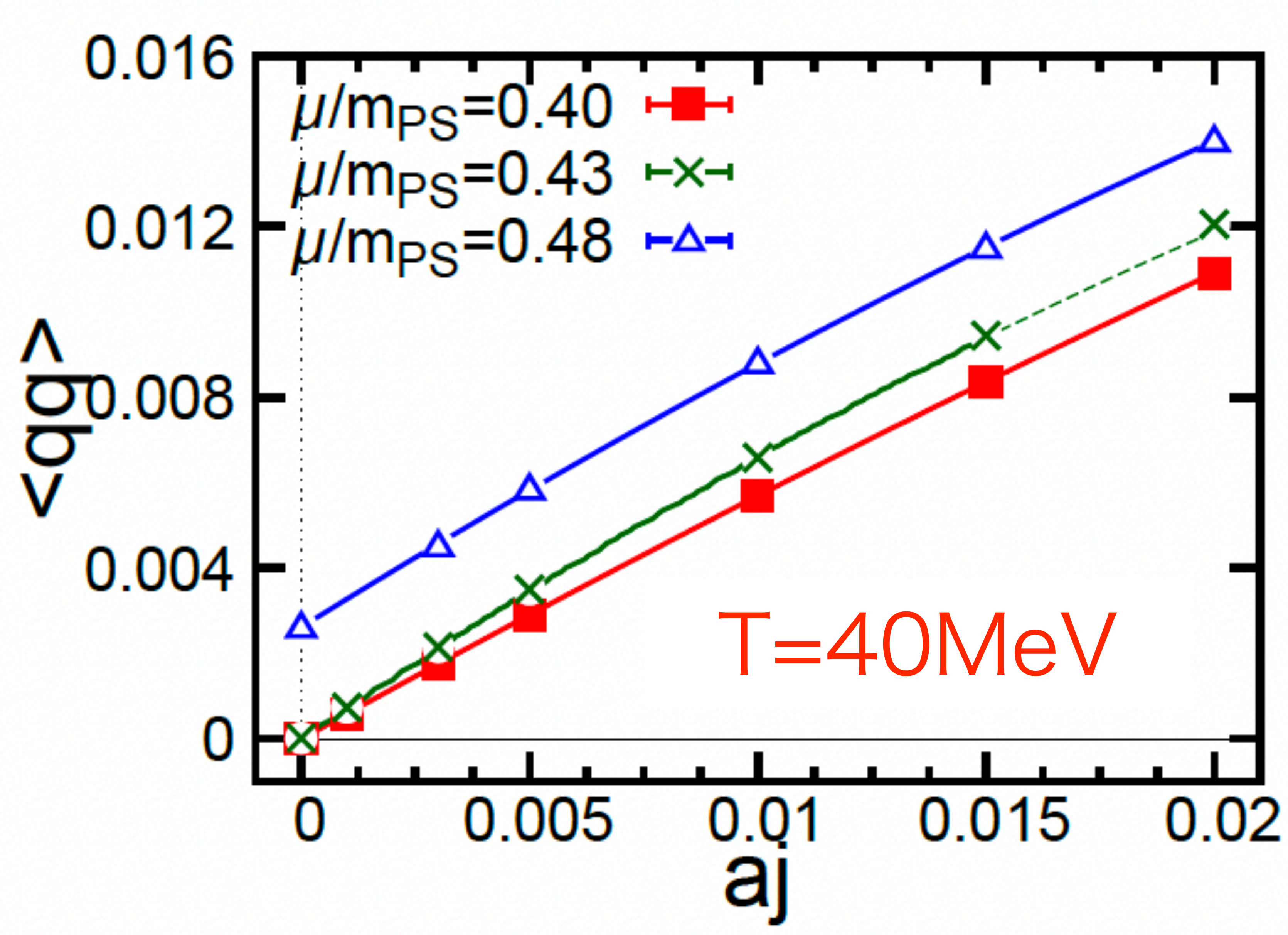
p increases more rapidly near the critical point at lower-T

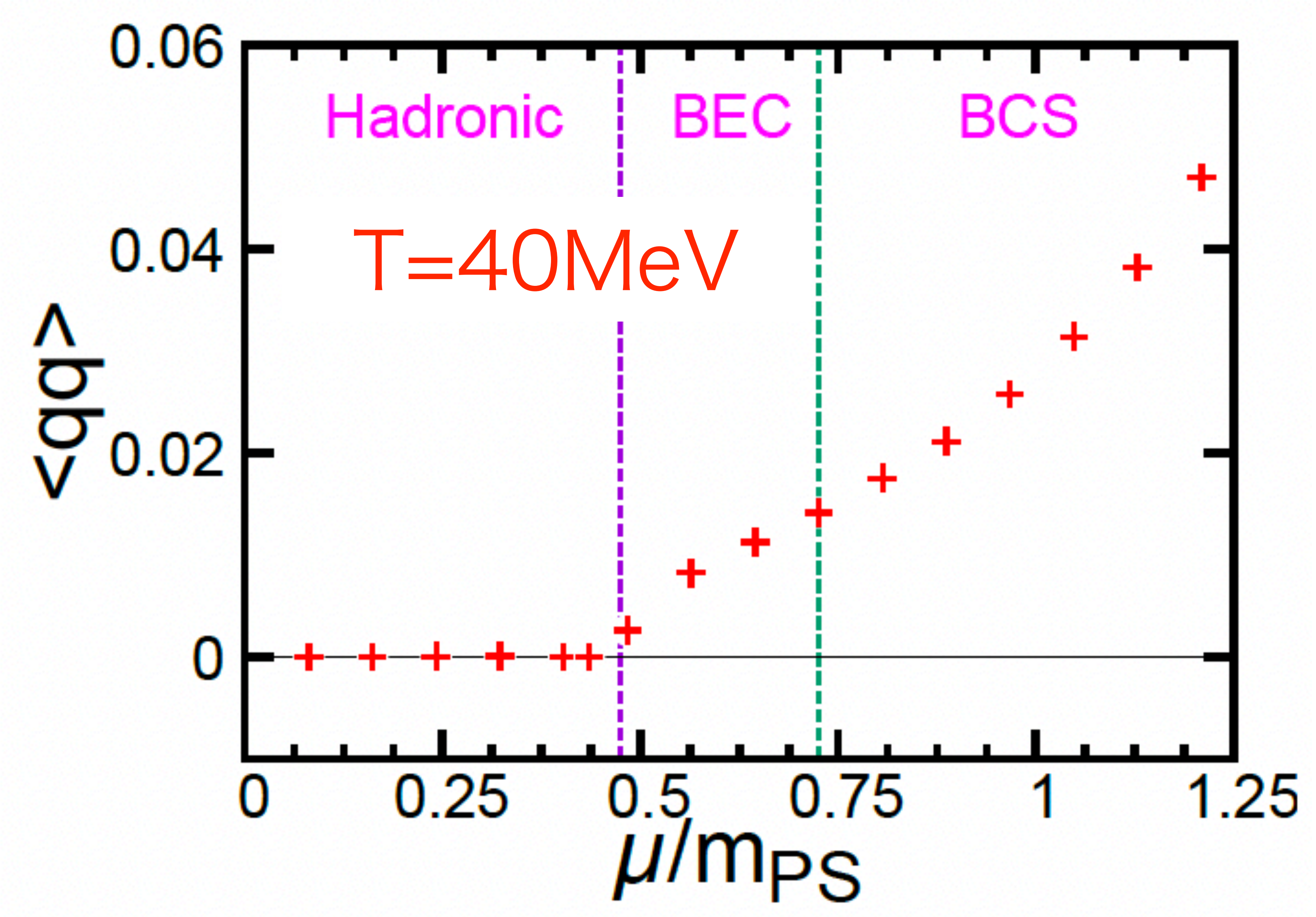
(2) Sound velocity does not show a clear T-dependence

our new data confirms the breaking of the conformal bound with small statistical error

backup

(1) Scaling of diquark condensate





- Around $\mu/m_{PS} = 0.5$,

  $\langle qq \rangle$ becomes non-zero!

- Theoretical predictions:

  (a) ChPT (near μ_c, T=0)

  $$\langle qq \rangle \propto (\mu - \mu_c)^{1/2}$$

  we fit 4data, and obtain μ_c

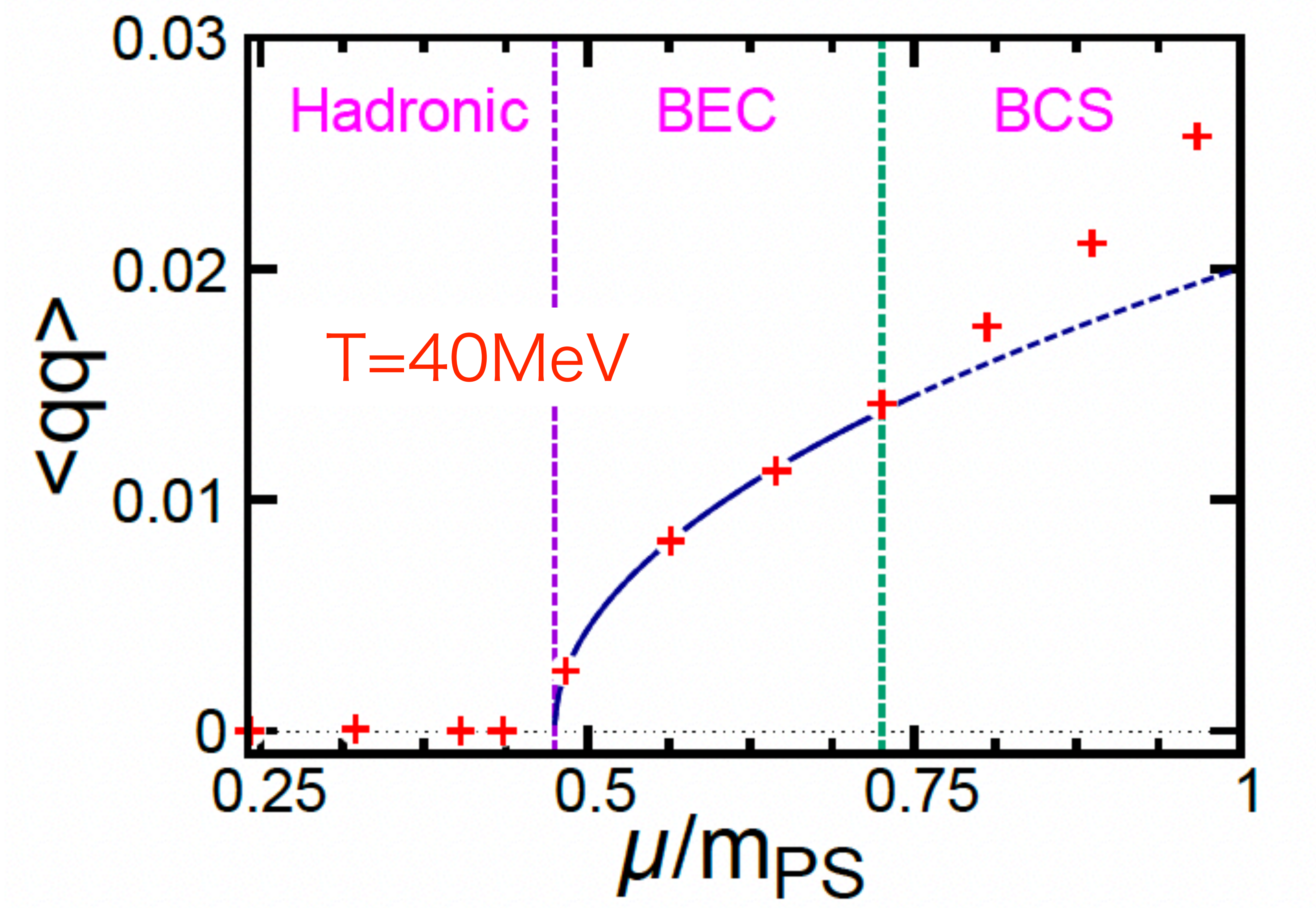
  (b) weak coupling analysis (high μ, T=0)

  $$\langle qq \rangle \propto \mu^2$$

(1) Scaling of diquark condensate

Fitting function in BEC phase

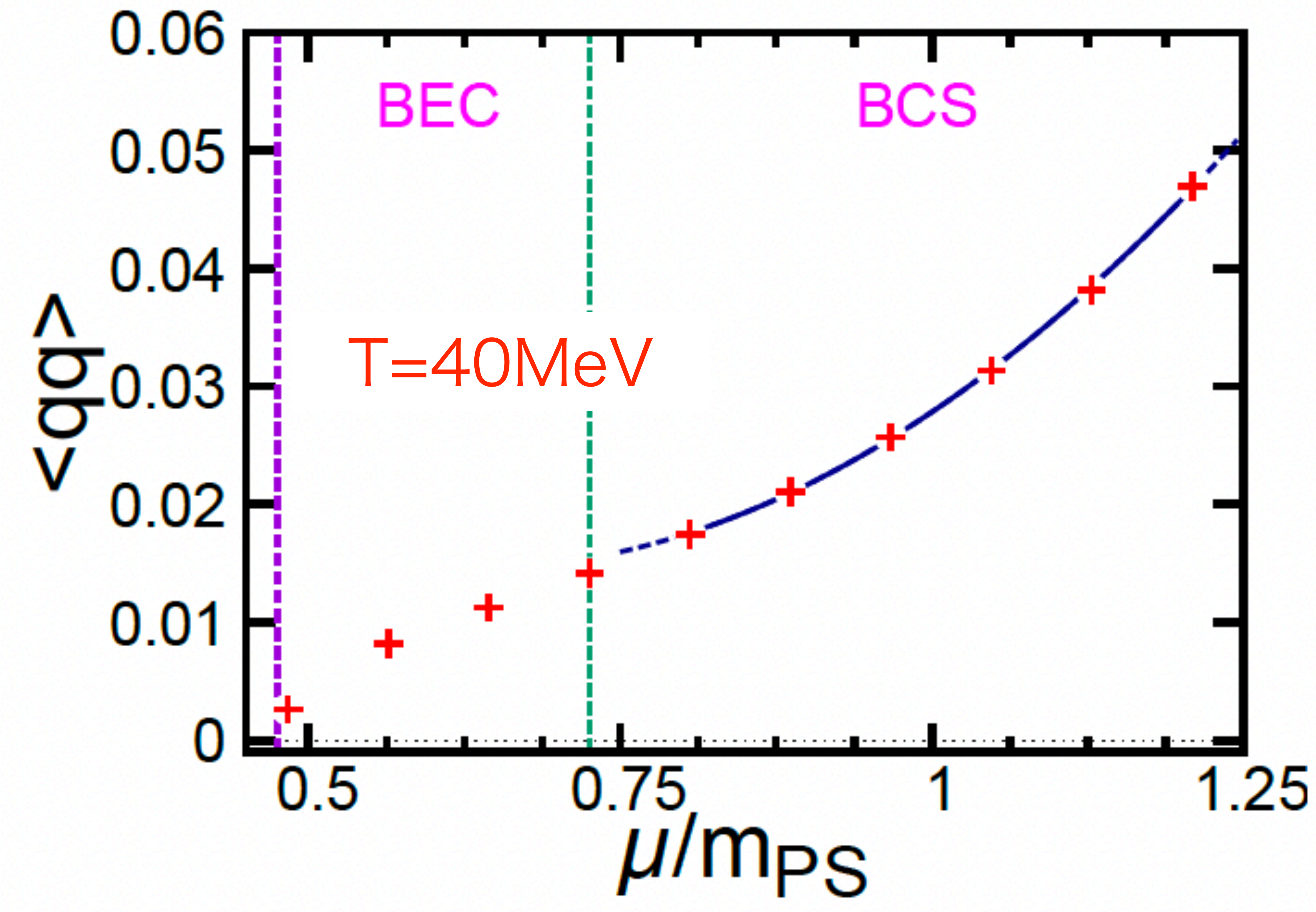
$$\langle qq \rangle = A(\mu - \mu_c)^{1/2}$$



We obtain $\mu_c = 0.47 m_{PS}$

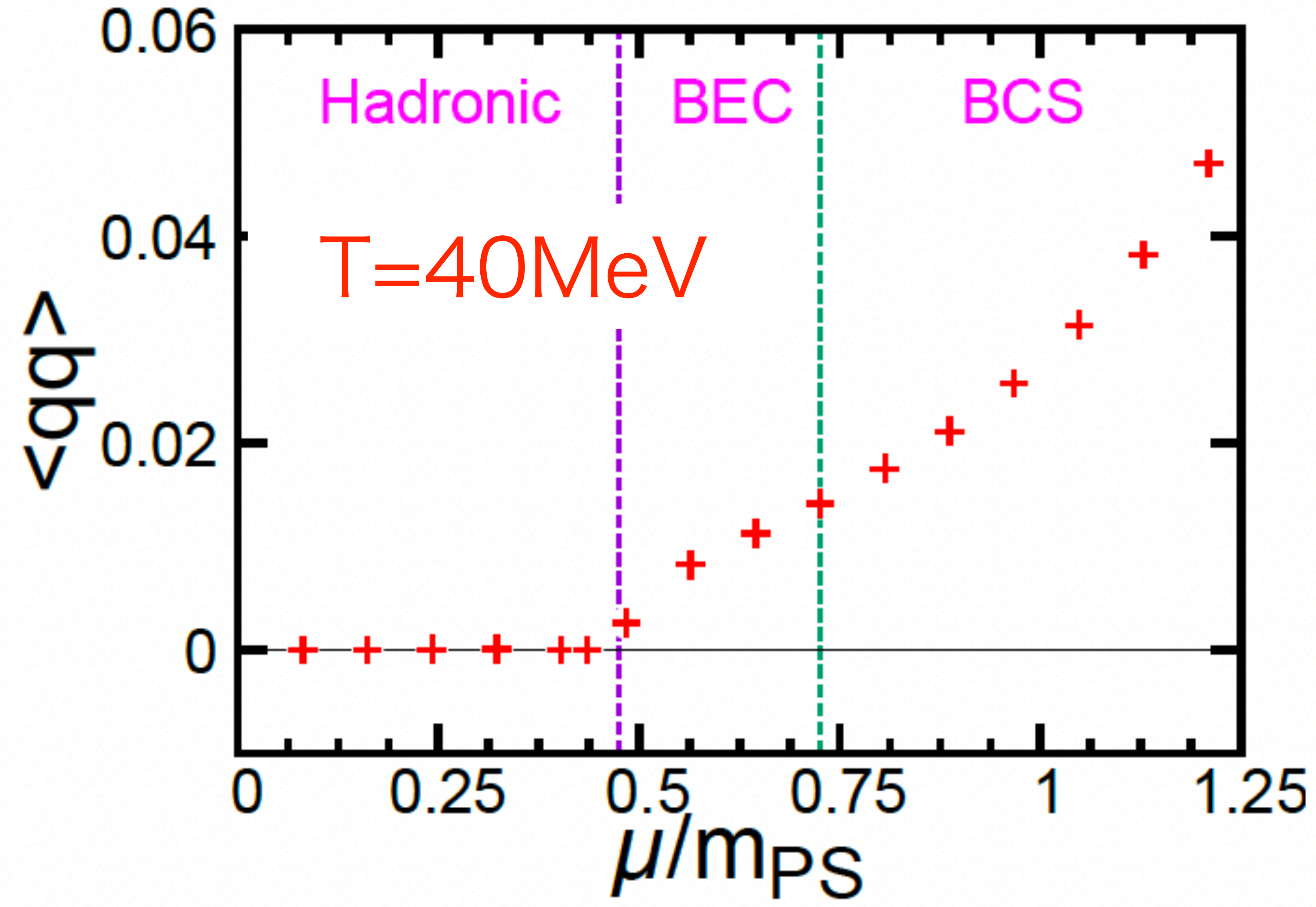
Fitting function in BCS phase

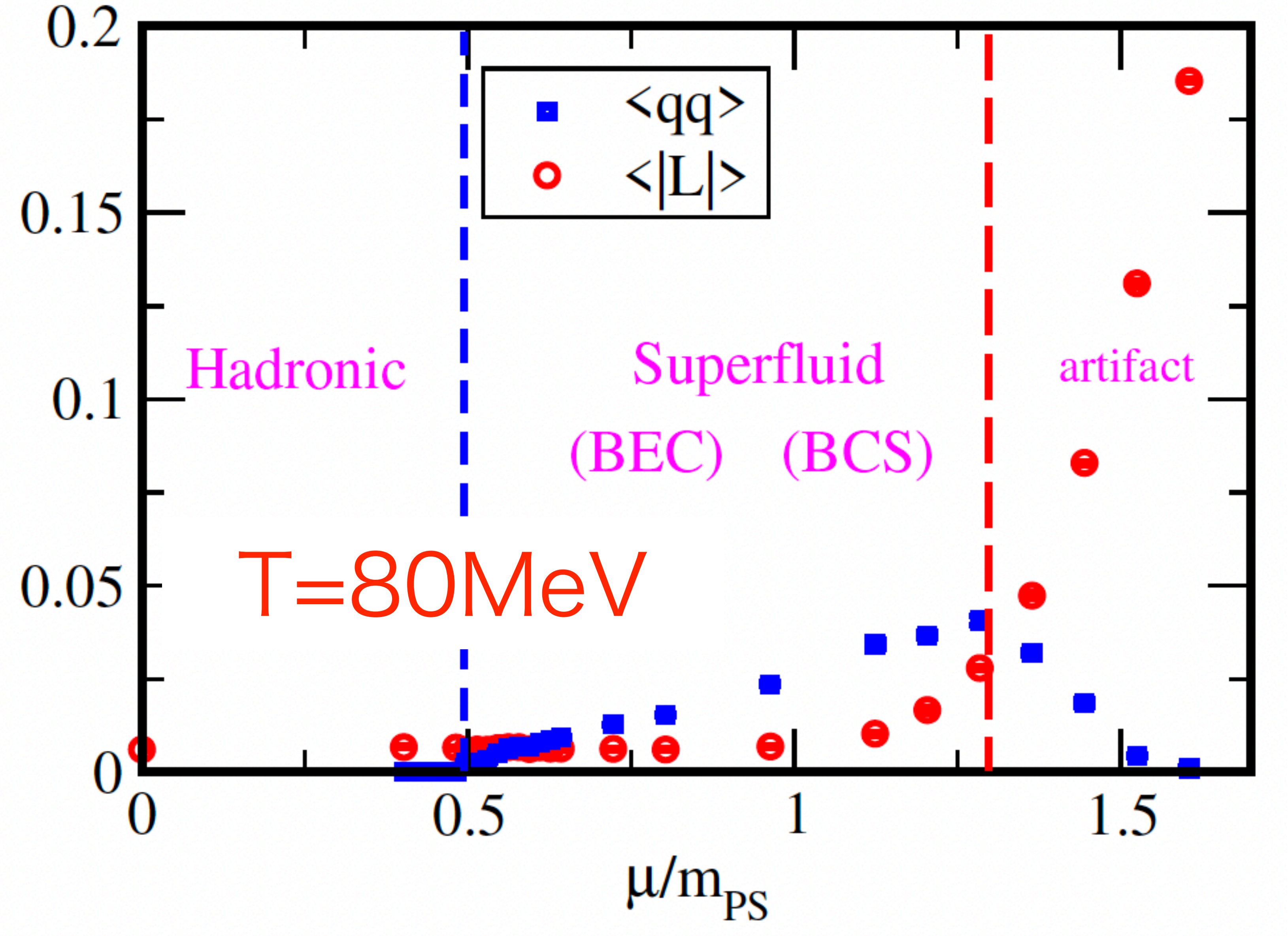
$$\langle qq \rangle = c_2\mu^2 + c_1\mu + c_0$$



μ^2 term exists!

(1) Scaling of diquark condensate





- Theoretical predictions at T=0

  (a) ChPT (near μ_c) $\langle qq \rangle \propto (\mu - \mu_c)^{1/2}$

  (b) weak coupling analysis (high μ) $\langle qq \rangle \propto \mu^2$

- T=80MeV:

  (a) was found, but (b) was unclear

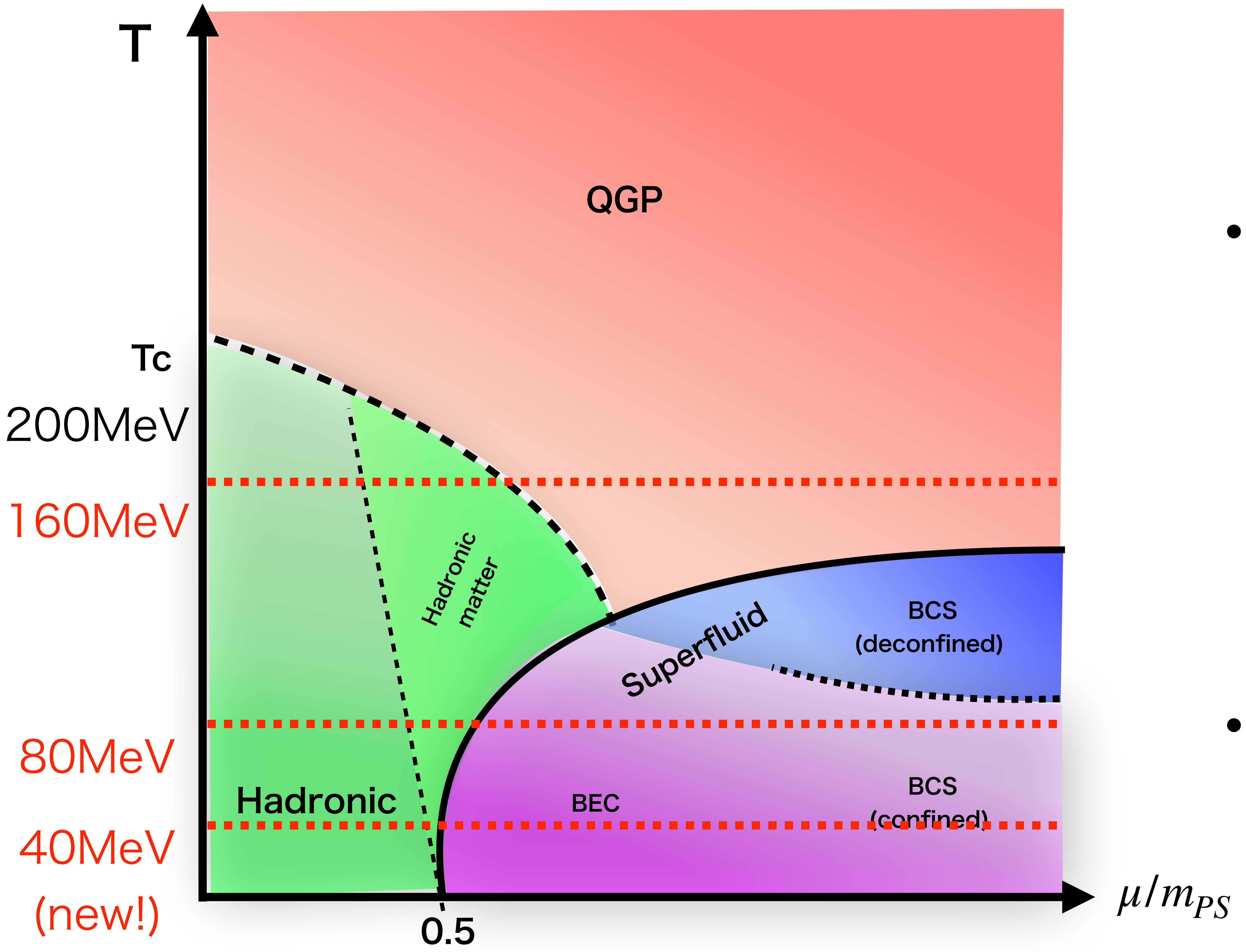
  <qq> seems to be rather linear in μ even in BCS phase

- T=40MeV:

  quadratic behavior emerges though a linear term still remains

- At T=0, both (a) and (b) will be observed.

(2) Fate of Hadronic-matter phase $\langle n_q \rangle > 0$ and $\langle qq \rangle = 0$

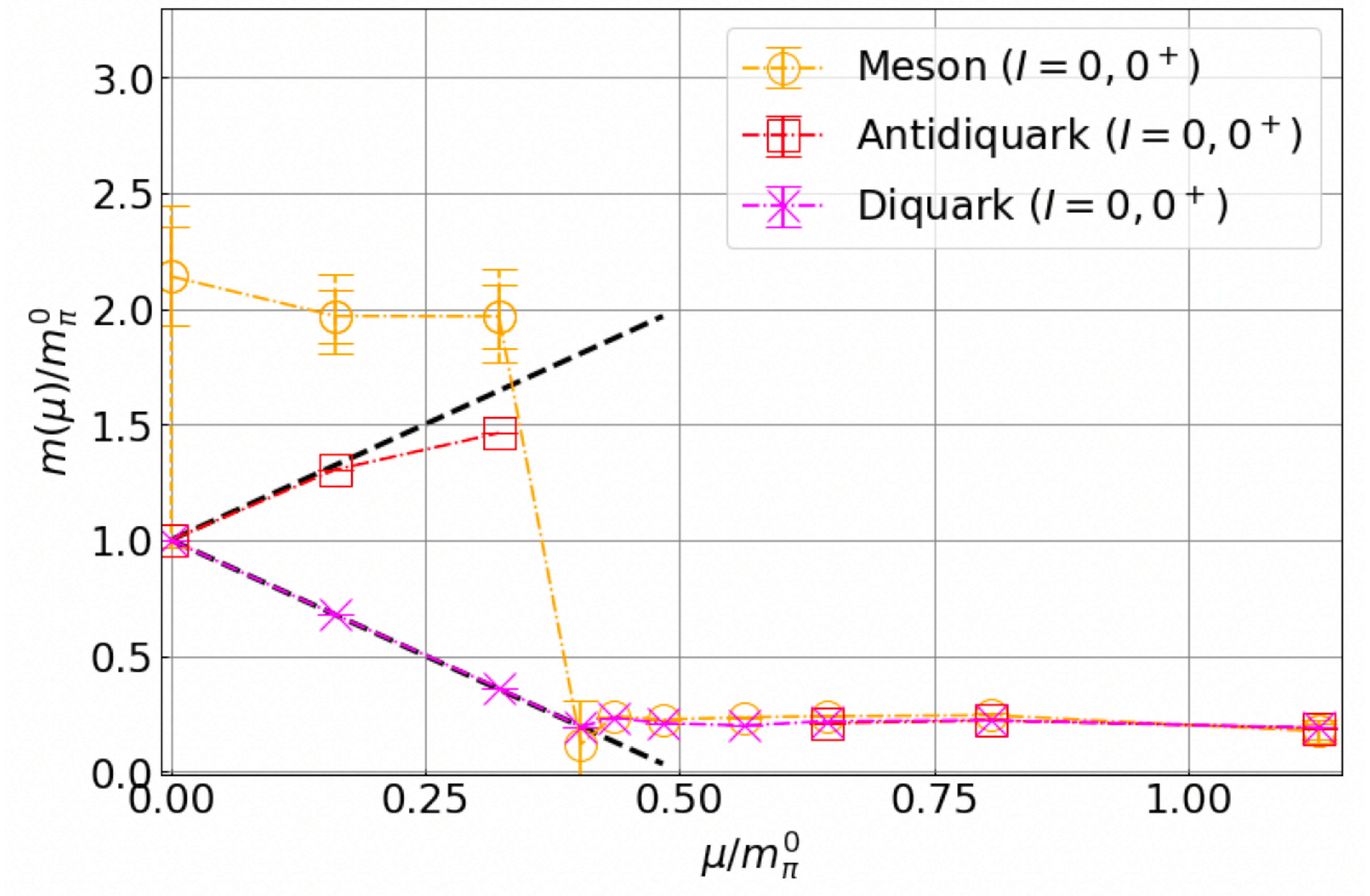


- At T=0, $\langle n_q \rangle > 0$ occurs simultaneously with $\langle qq \rangle > 0$ (superfluid transition)
- In the previous work for T=80MeV, we found a subtle phase: hadronic-matter phase just before the superfluid transition
- We consider that at finite-T, the lightest hadron (scalar diquark) can be excited by the temperature

(2) Fate of Hadronic-matter phase $\langle n_q \rangle > 0$ and $\langle qq \rangle = 0$

In small μ

$$\begin{aligned} m_{PS}(\mu) &= m_{PS}(\mu = 0) \\ m_{qq}(\mu) &= m_{PS}(\mu = 0) - 2\mu \\ m_{\bar{q}\bar{q}}(\mu) &= m_{PS}(\mu = 0) + 2\mu \end{aligned}$$



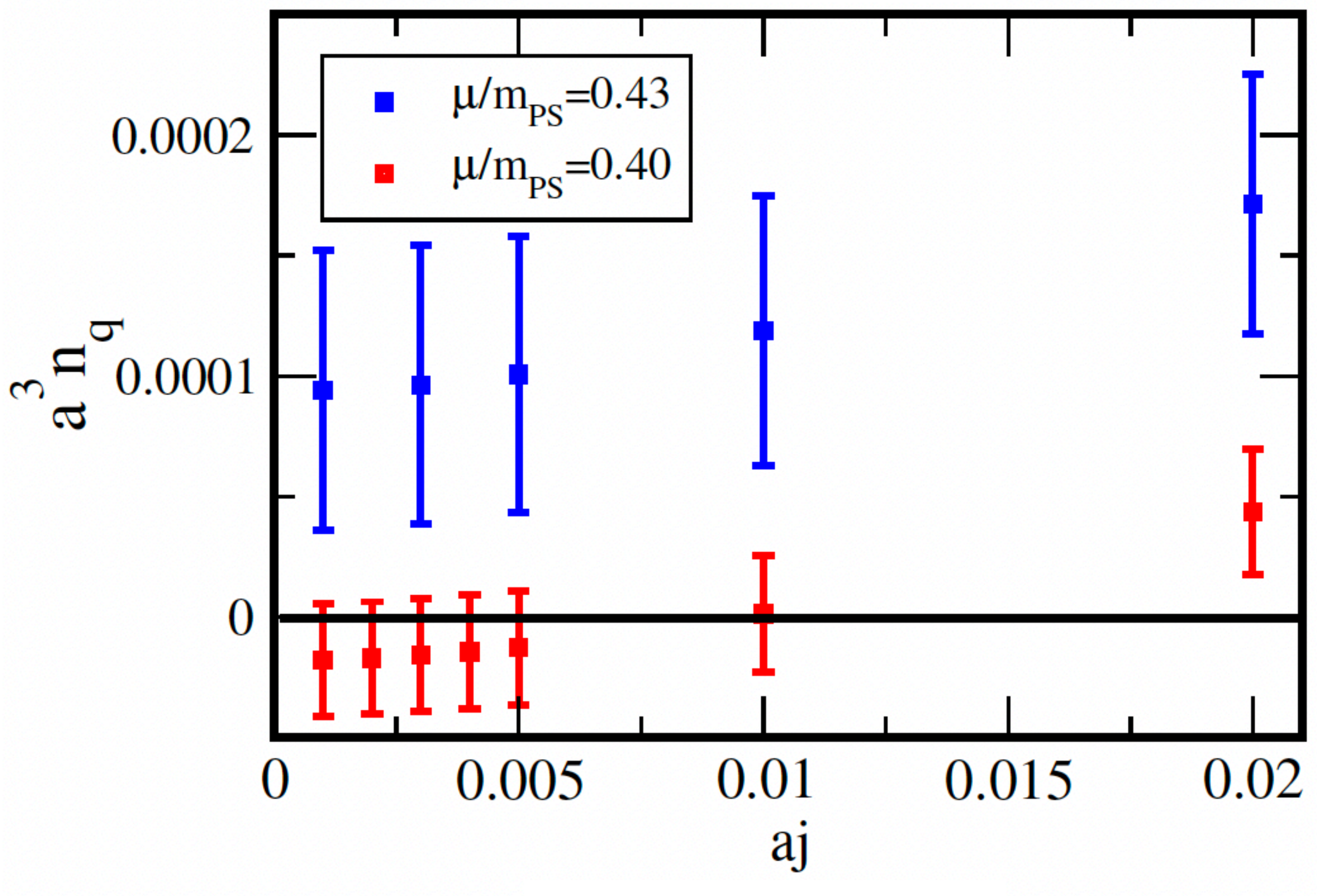
K.Murakami et al. (2022)

- The lightest hadron mass can be estimated by $m_{qq} = m_{PS} - 2\mu$ by ChPT
- If T > m_{qq} then, the diquark con be excited, but the anti-diquark cannot
- Force on $\mu/m_{PS} = 0.43$
- Measured value of the diquark mass in j=0 limit : $am_{qq} = 0.0692(2)$ -> $m_{qq} \approx 80$ MeV

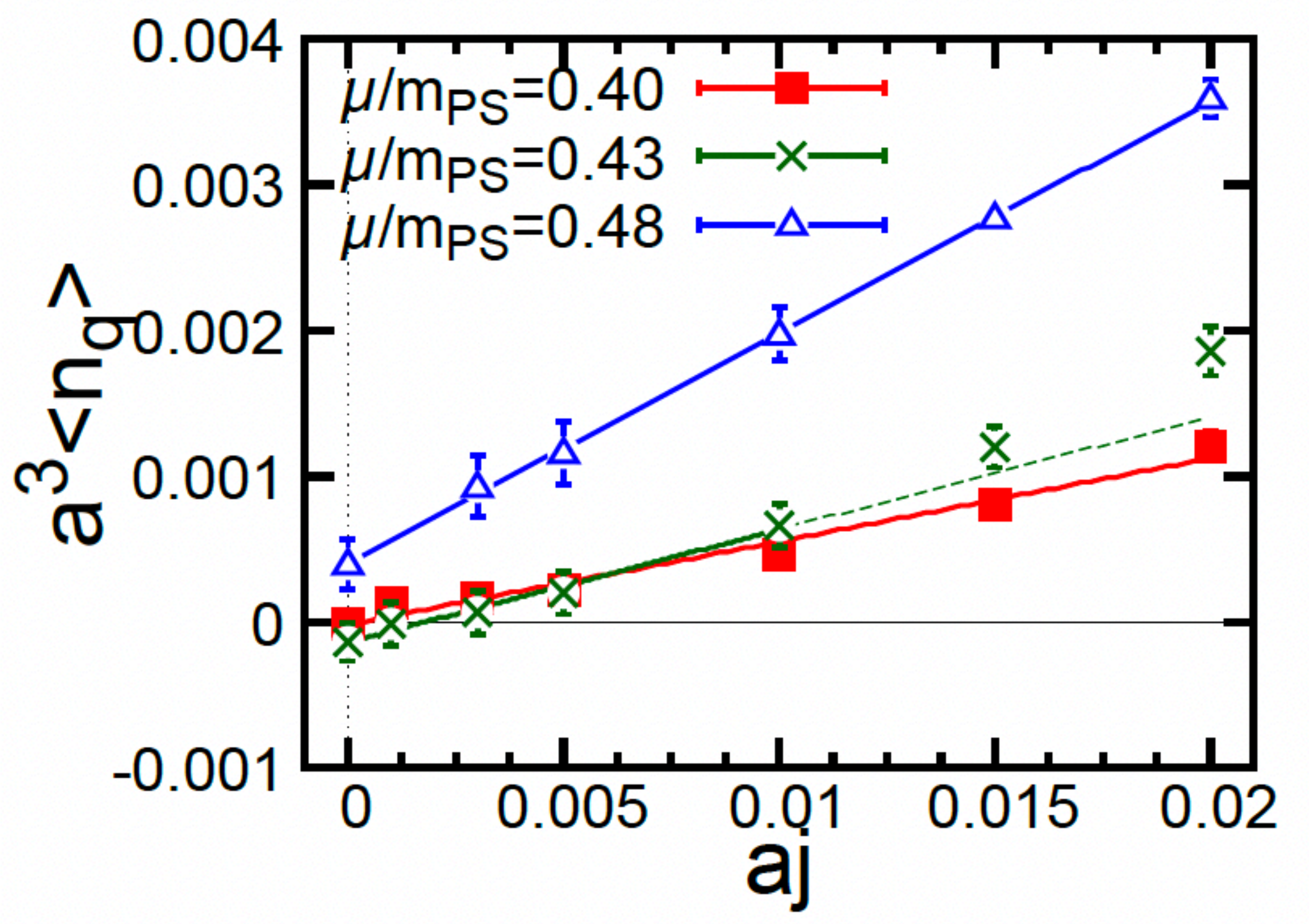
  ($m_{qq} = m_{PS} - 2\mu = 0.14 m_{PS} \approx 100$TeV)

(2) Fate of Hadronic-matter phase $\langle n_q \rangle > 0$ and $\langle qq \rangle = 0$

T=80MeV



T=40MeV



- Indeed, at $\mu/m_{PS} = 0.43$, T=80MeV ,

  $\langle n_q \rangle > 0$ in the j=0 limit

- At $\mu/m_{PS} = 0.43$, T=40MeV

  $\langle n_q \rangle = 0$ in the j=0 limit

- At lower-T, the hadronic-matter phase shrinks

- We expect it disappears at T=0

(3) Topological susceptibility and confinement

Nf=4, T=0 (12^3x24)

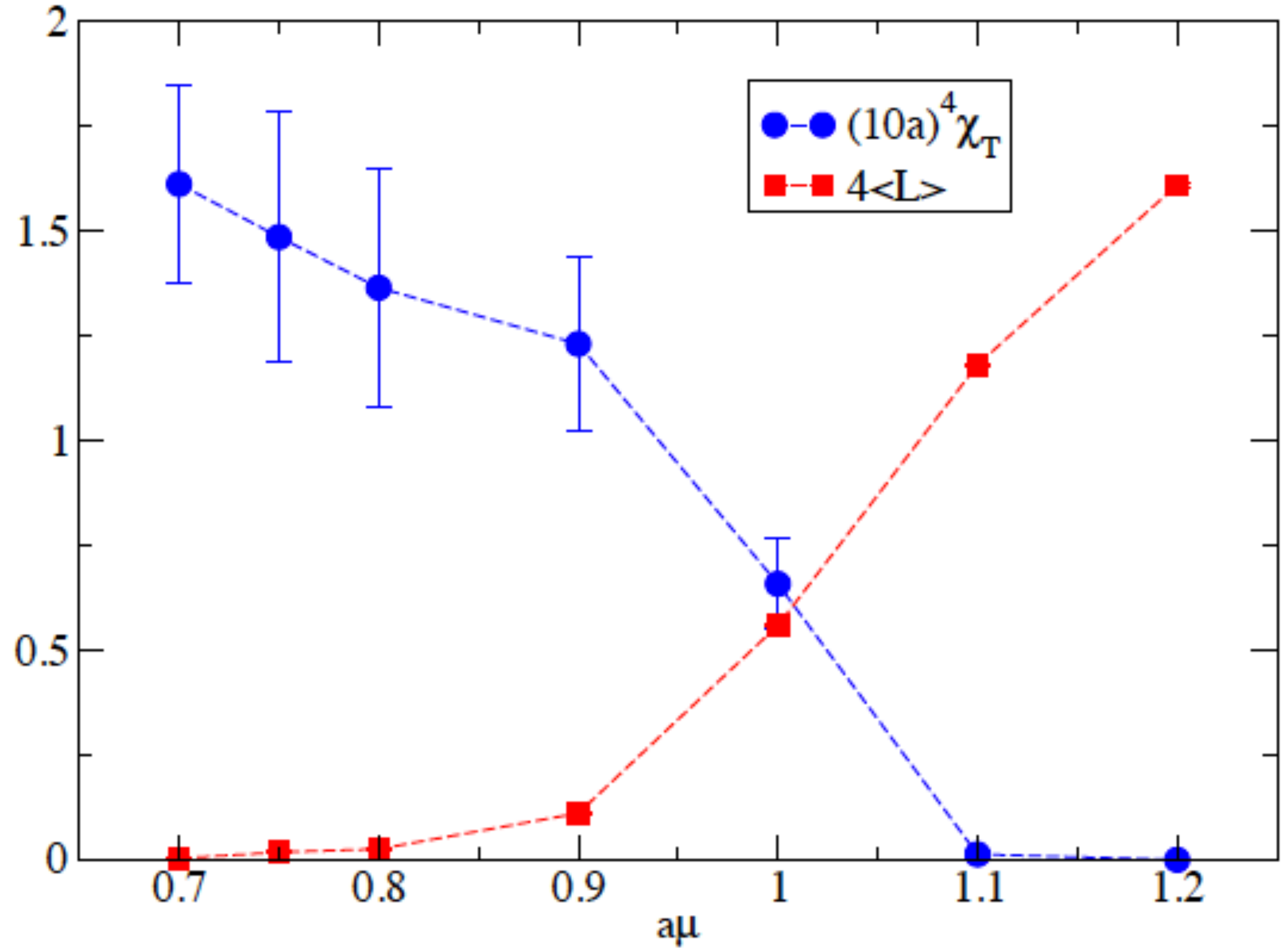


Figure 2: *The suppression of* χ_T *coinciding with the rise in* $\langle L \rangle$ *for* $N_f = 4$. *Note* $\langle L \rangle$ *has been rescaled for clarity.*

Hands et.al. (arXiv:1104.0522)

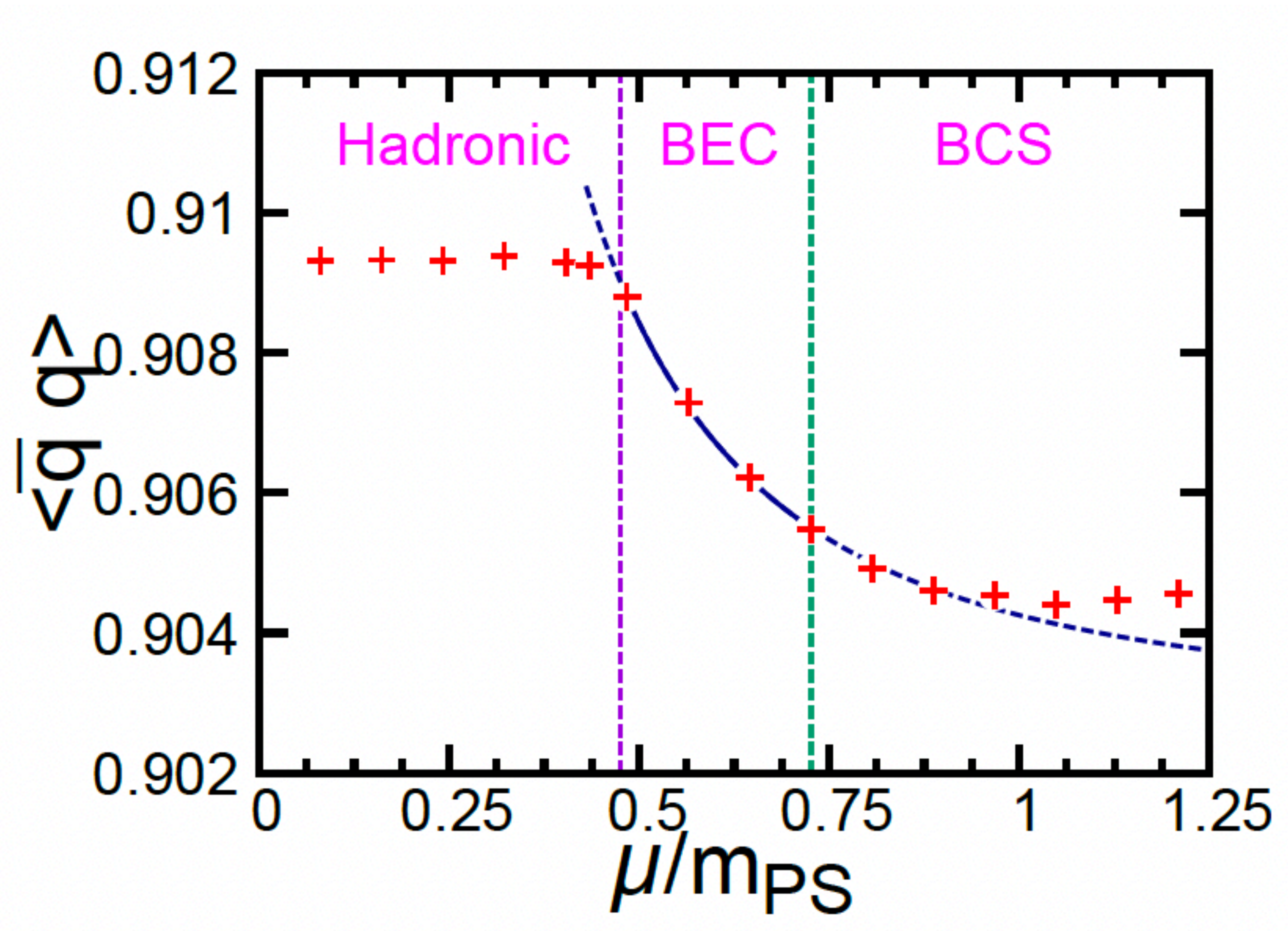
- Early works show the χ_Q decreasing in high-μ, and simultaneously the Polyakov loop is decreasing
- Recently, in $T \lesssim 100$MeV, the confinement remains even in high-μ ($\mu \sim 1$GeV)

T. Boz et al. (2019)

A.Begun et al. (2022)

K.Iida, K.Ishiguro , EI, arXiv: 2111.13067

Chiral condensate



- Naive data of $\langle\bar{q}q\rangle$
- We use the Wilson fermion, so additive renormalization is needed
- ChPT predicts $\langle\bar{q}q\rangle \propto 1/\mu^2$ scaling at T=0 near μ_c
- Our data can be fitted well using $f(\mu) = c_1/\mu^2 + c_0$

Our projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

  Phase diagram by Lattice simulation
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0

  Scale setting of Lattice simulation
- K.Iida, K.Ishiguro, EI, arXiv: 2111.13067 (PoS, Lattice 2021)

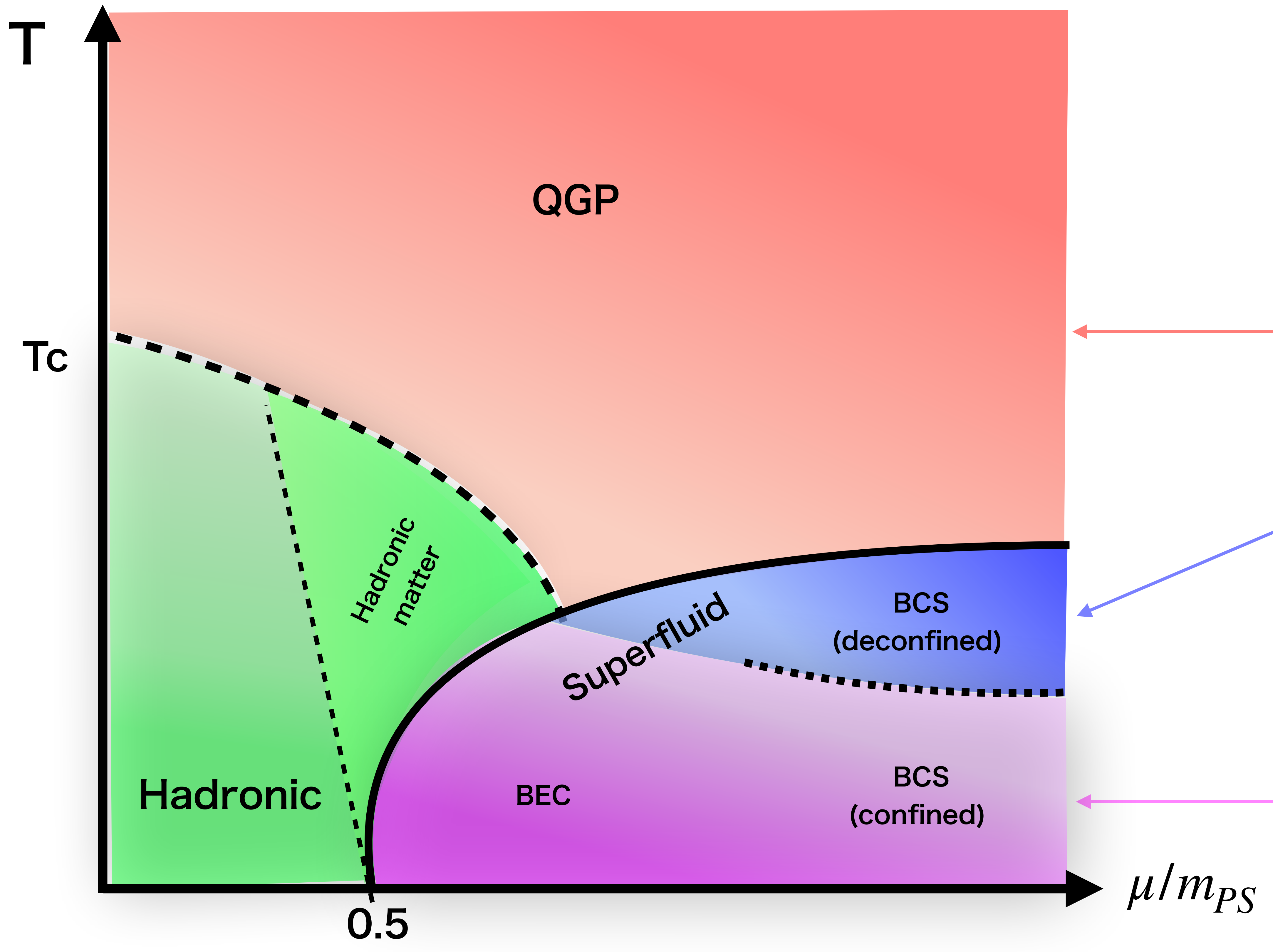
  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01

  Velocity of sound by Lattice simulation
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023)

  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472 (PoS, Lattice 2022)

  Mass spectrum by Lattice simulation

Current status on 2color QCD phase diagram



At least three independent group studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)

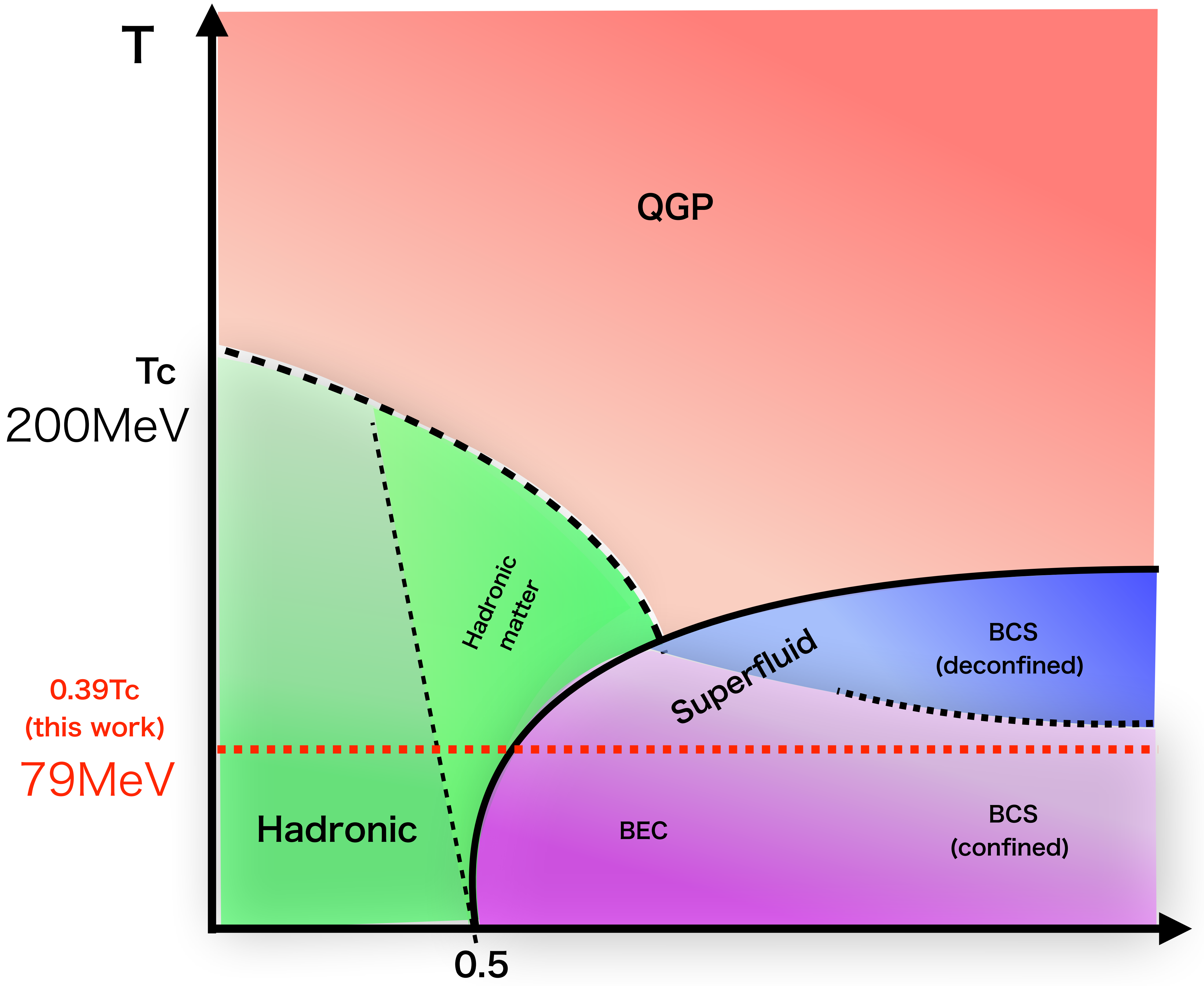
- Even $T \approx 100\text{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)$\leq T_{SF}$ (superfluid/QGP) : constraint from 't Hooft anomaly matching

T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

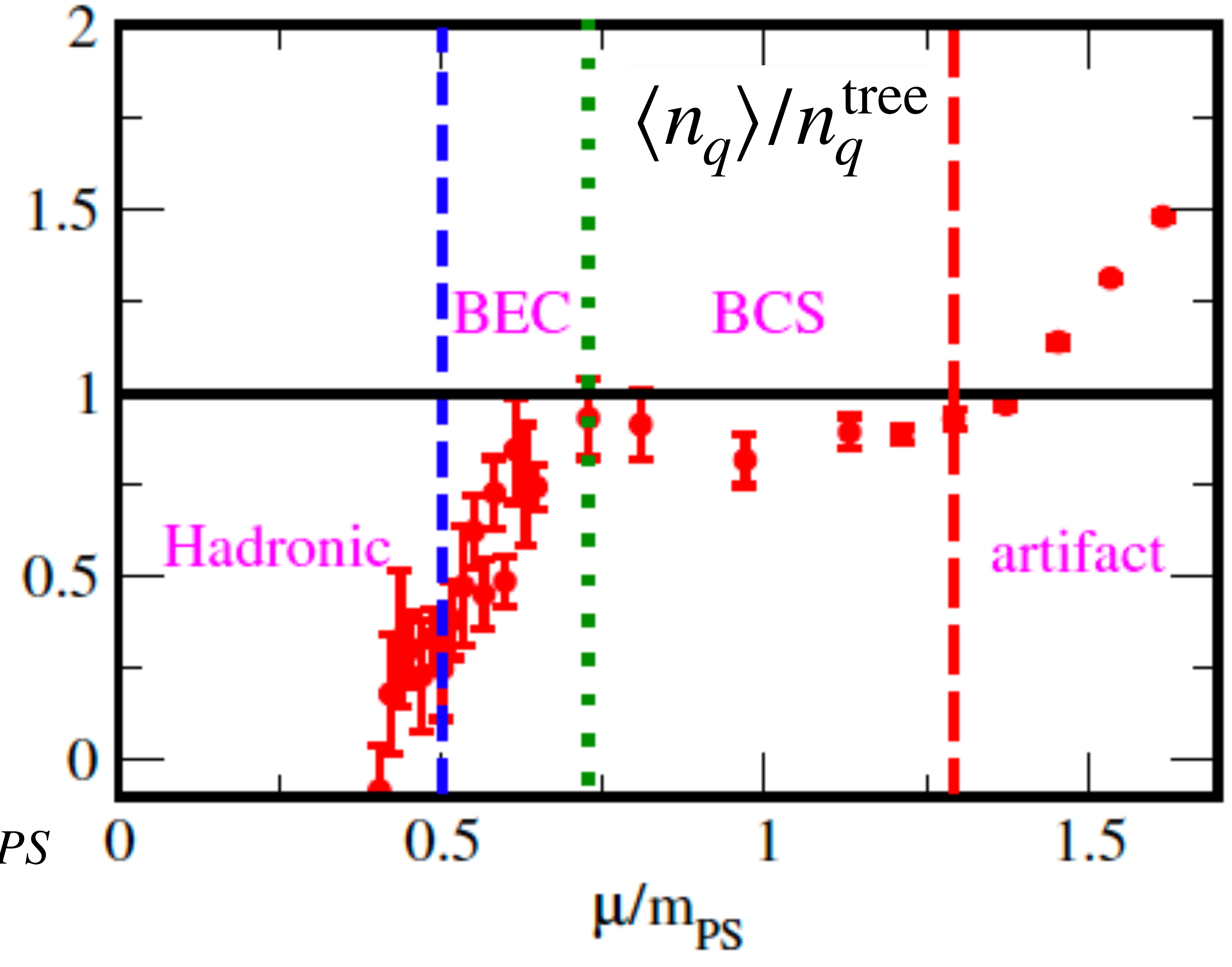
- 2color QCD phase diagram has been determined by independent works!

Phase diagram of 2color QCD

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



	Hadronic	Hadronic-matter	QGP	Superfluid	
				BEC	BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\rm tree} \approx 1$



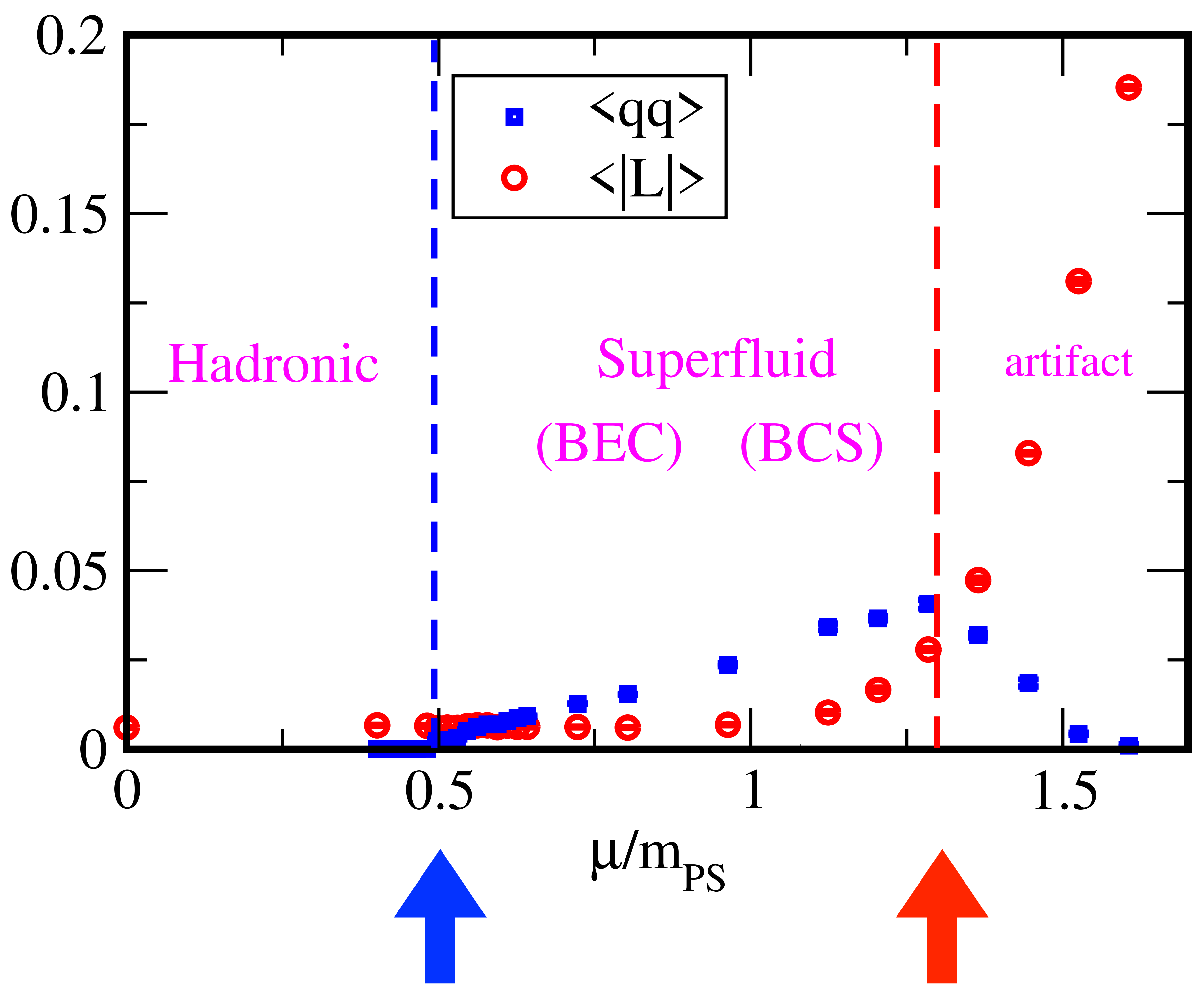
In high-μ, $\langle n_q \rangle \approx n_q^{\rm tree}$

number density of free particle

⬇

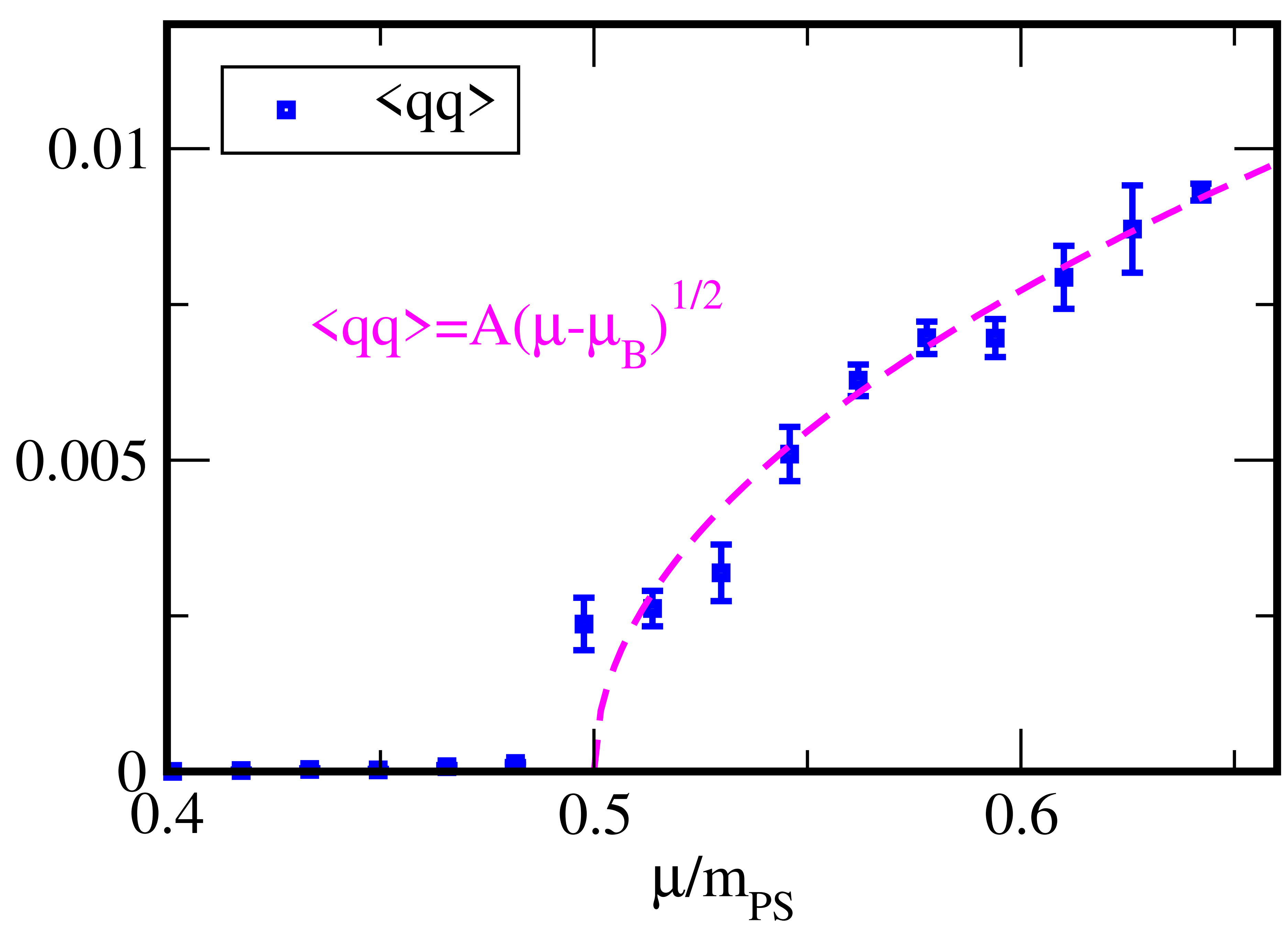
BEC-BCS crossover

Order parameters in j=0 limit



$\mu_B/m_{PS} \simeq 0.50$

$\mu/m_{PS} \simeq 1.28$
$(\mu_D/m_{PS} \simeq 1.44)$

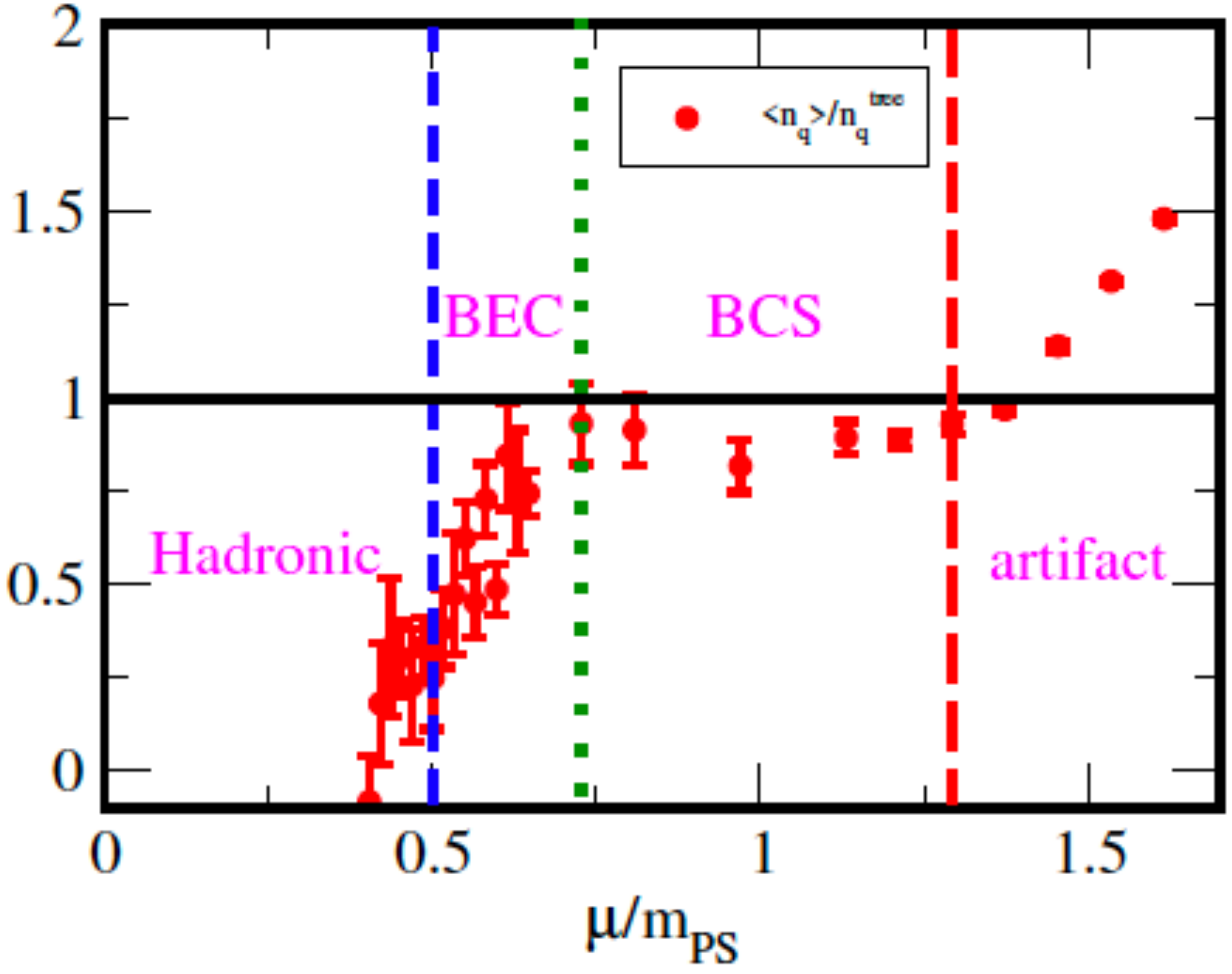


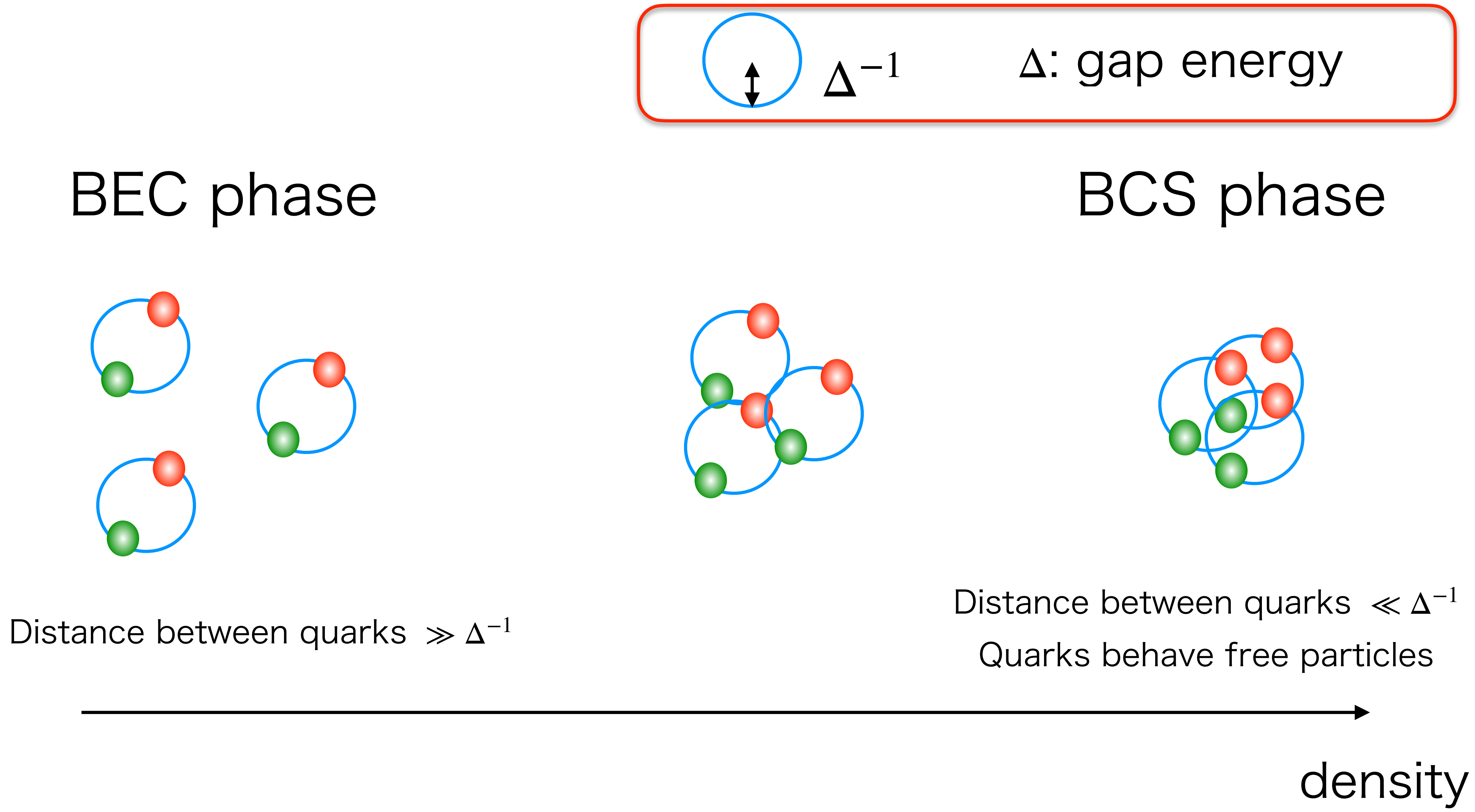
Scaling law of order param.
is consistent with ChPT.

Ref.) Kogut, Stephanov, Toublan, Verbaarshot, Zhitnitsly
NPB 582 (2000) 477

At T=0.39Tc, we find the BCS with confined phase until $\mu \lesssim 1152 MeV$.

BEC/BCS crossover





Number density of free particle

$$n_q^{\text{tree}}(\mu) = \frac{4N_cN_f}{N_s^3N_\tau}\sum_k \frac{i\sin\tilde{k}_0[\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos\tilde{k}_\nu]^2 + \sum_\nu \sin^2\tilde{k}_\nu}$$

J->0 extrapolation

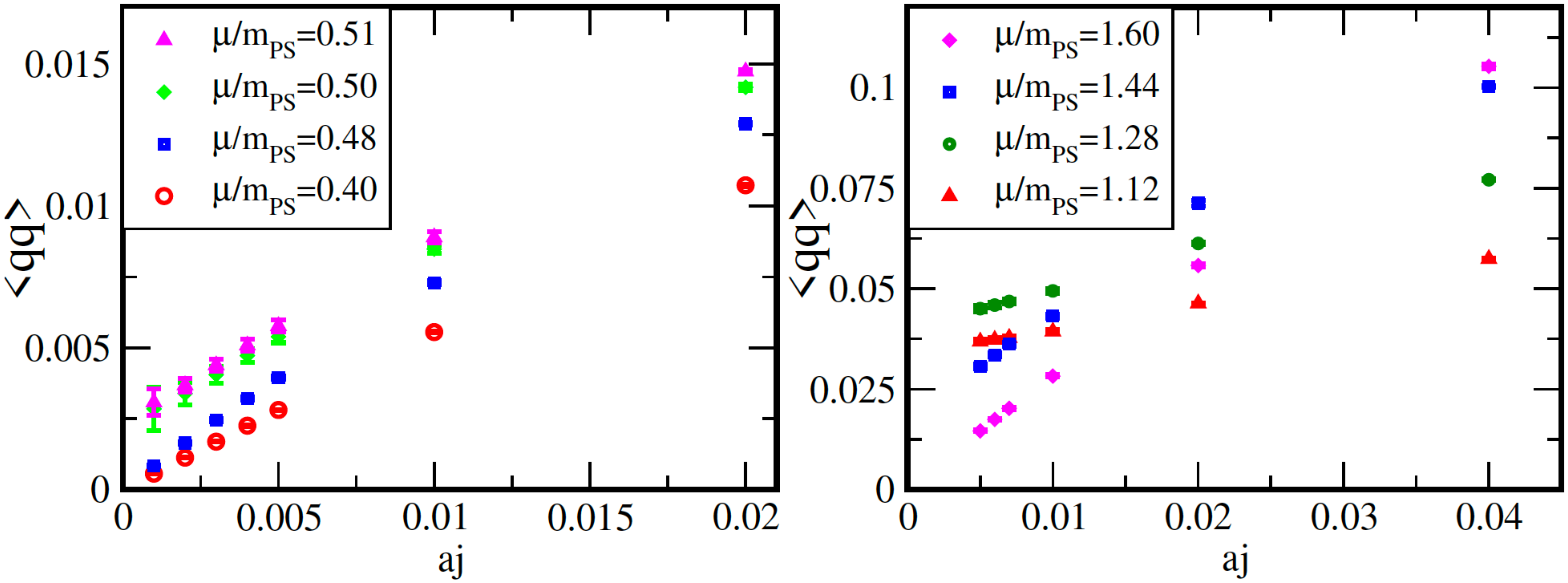
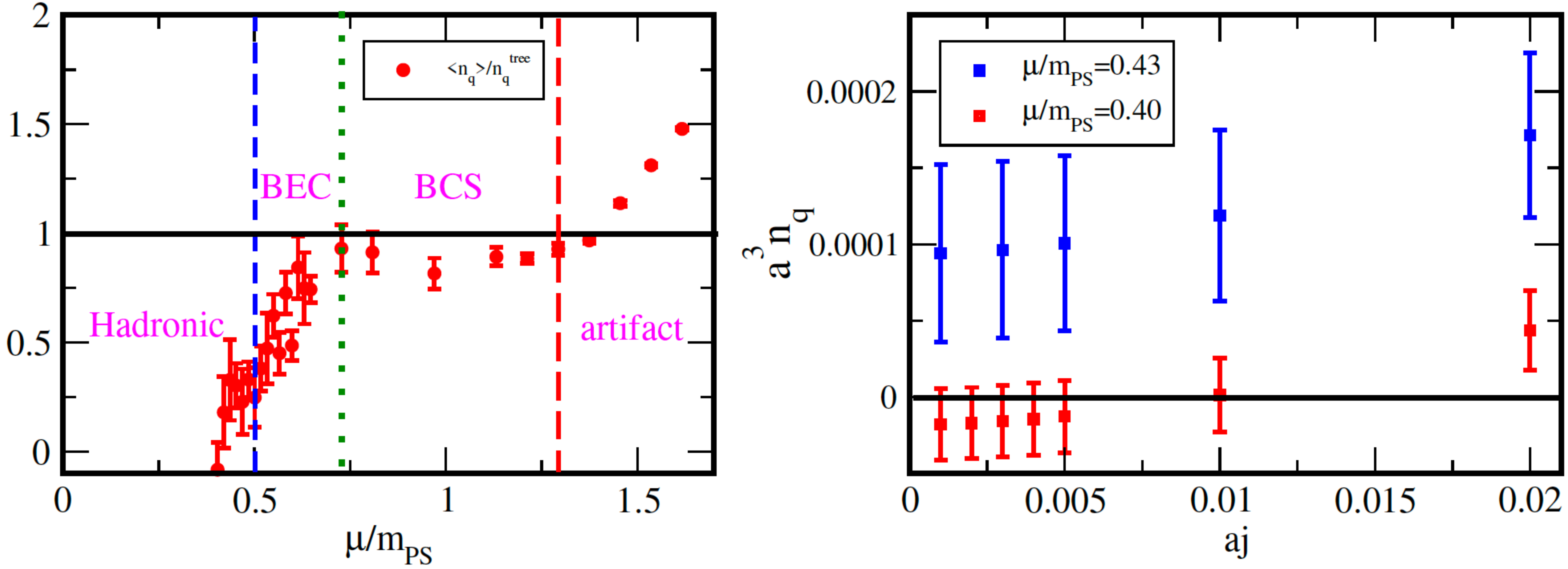
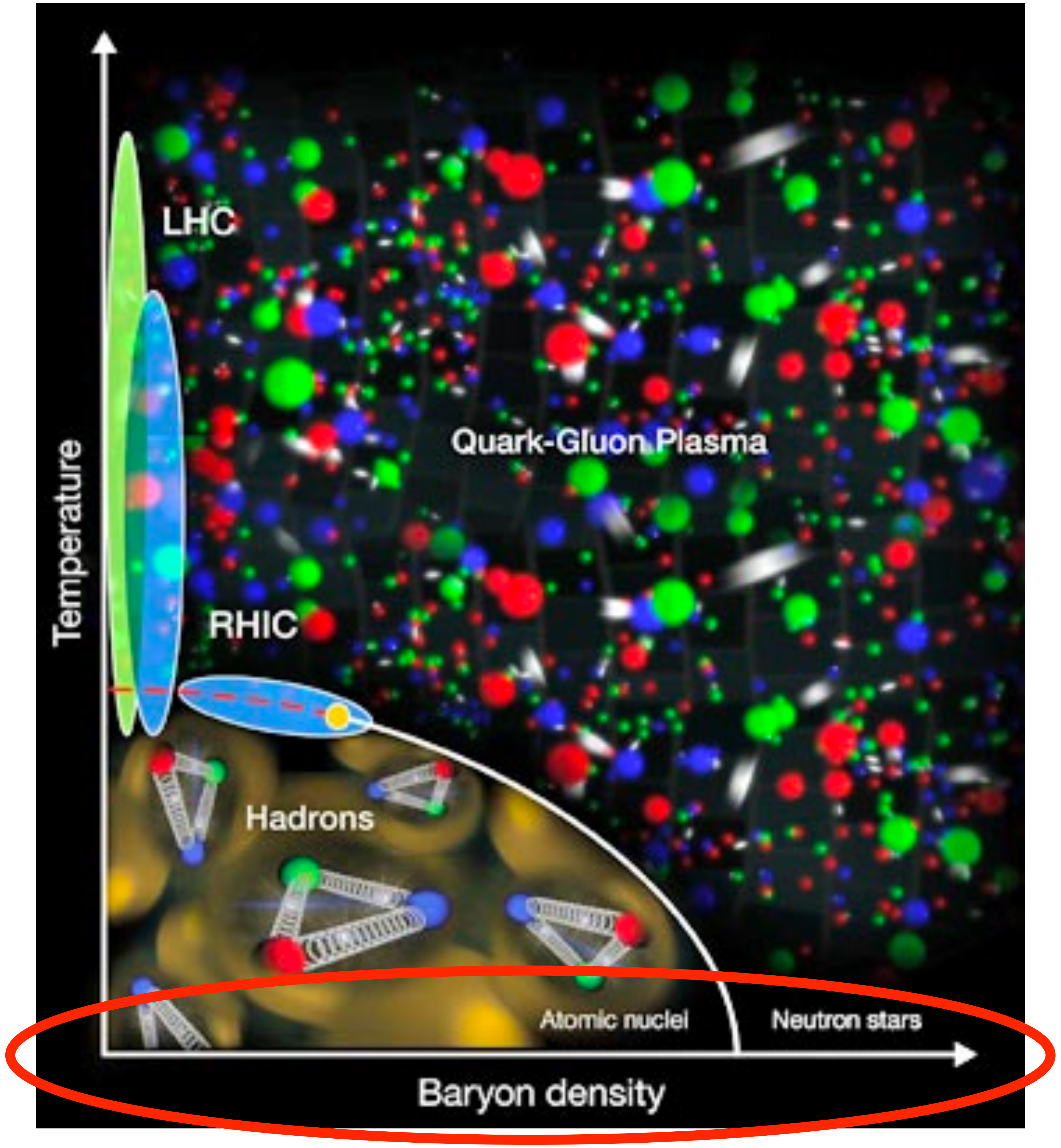


Figure 5. The j-dependence of the diquark condensate for several μ/m_{PS}.

J->0 extrapolation



Sound velocity: finite density regime



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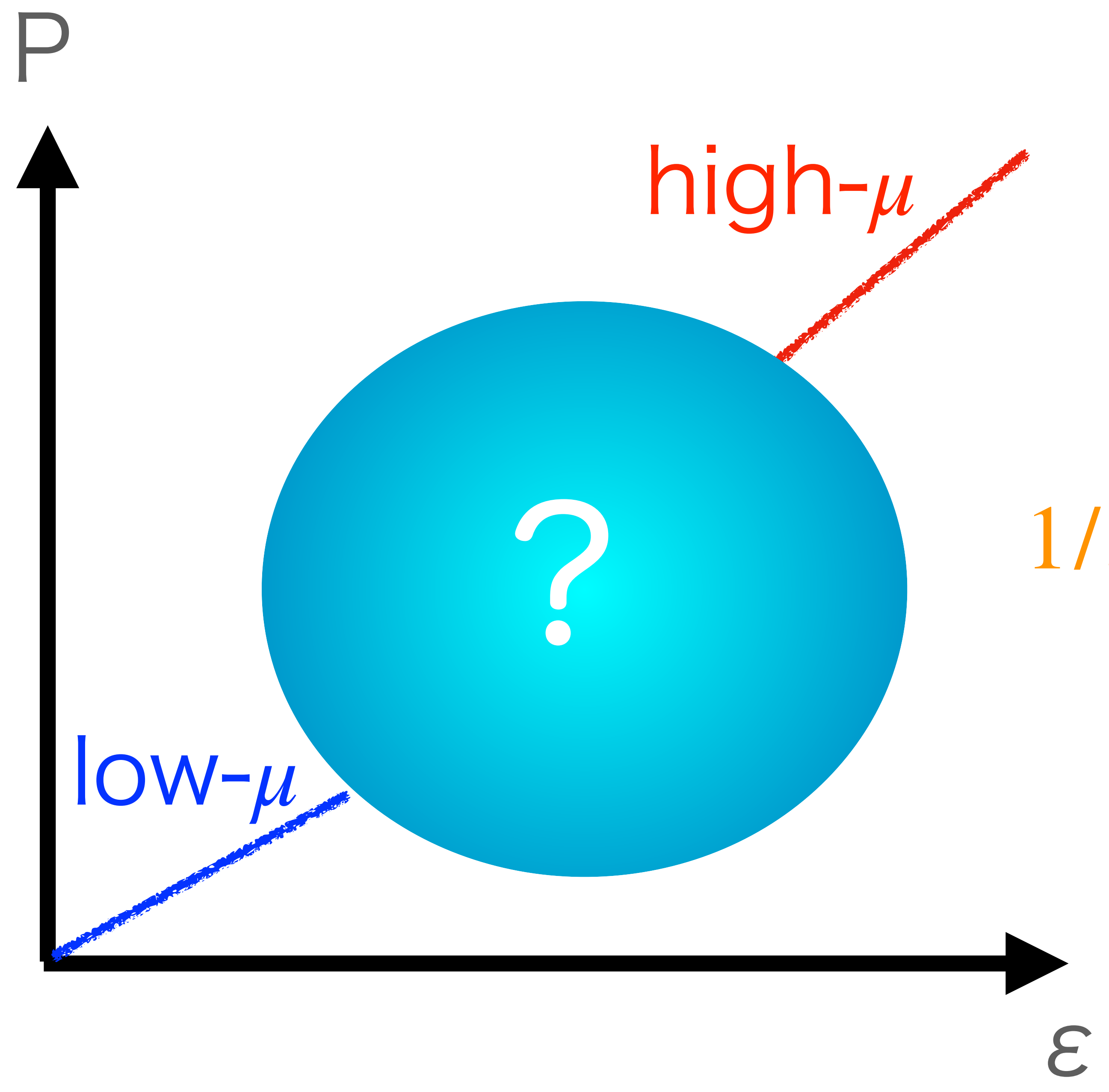
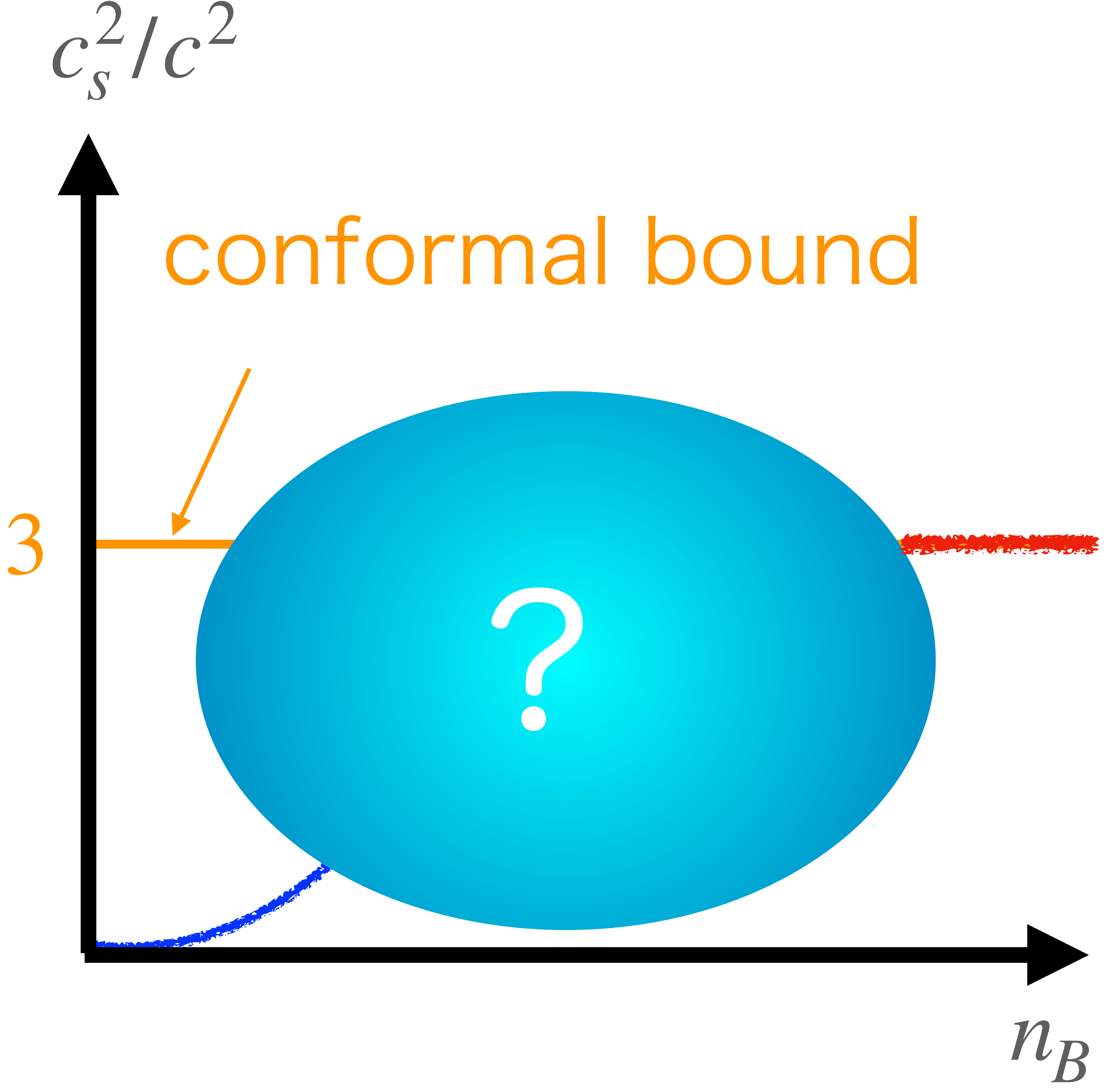
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p / \partial \epsilon$



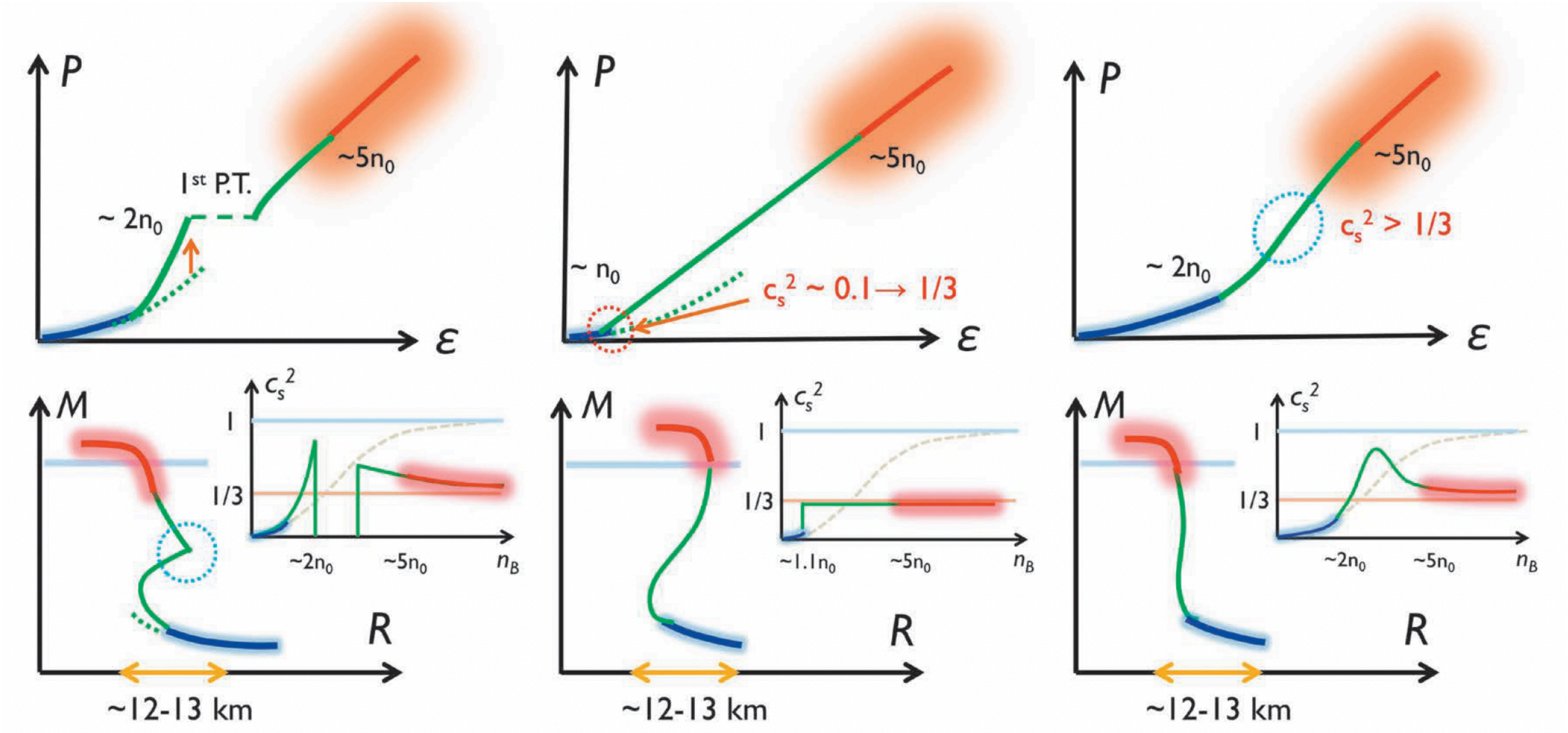
low -μ ($n_B \lesssim 2n_0$): Hadronic matter

high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

EoS (ε vs. p), c_s and neutron star

T. Kojo, arXiv:2011.10940

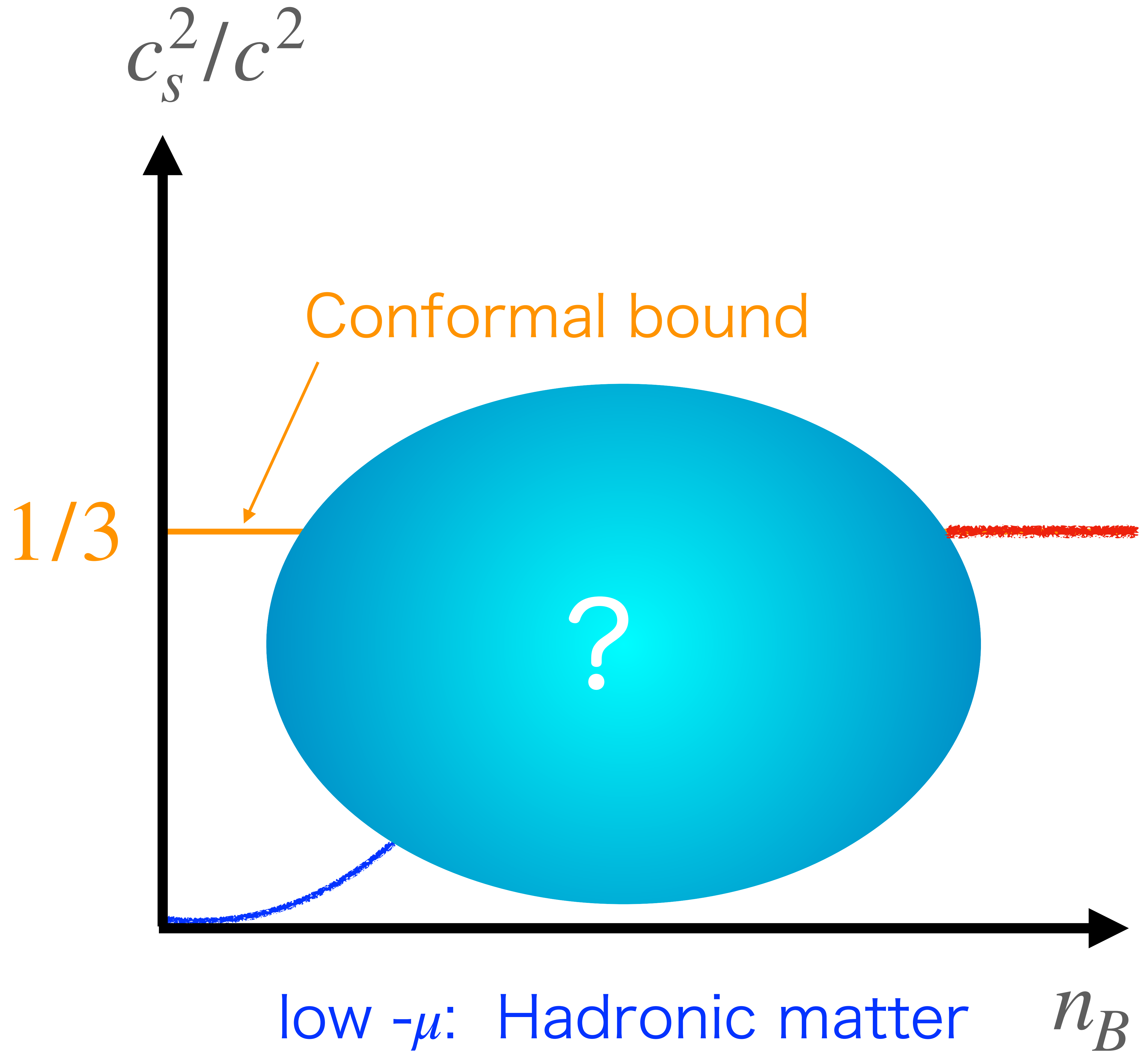


Sound velocity

$$c_s^2 = \partial p/\partial \epsilon$$

Mass-Radius of neutron star <=> EoS in dense QCD

Prediction by phenomenology and effective models



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests c_s^2 peaks at $n_B = 1 - 10n_0$

  Masuda,Hatsuda,Takatsuka (2013)
  Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model c_s^2 peaks at $n_B = 1 - 5n_0$

  McLerran and Reddy (2019)

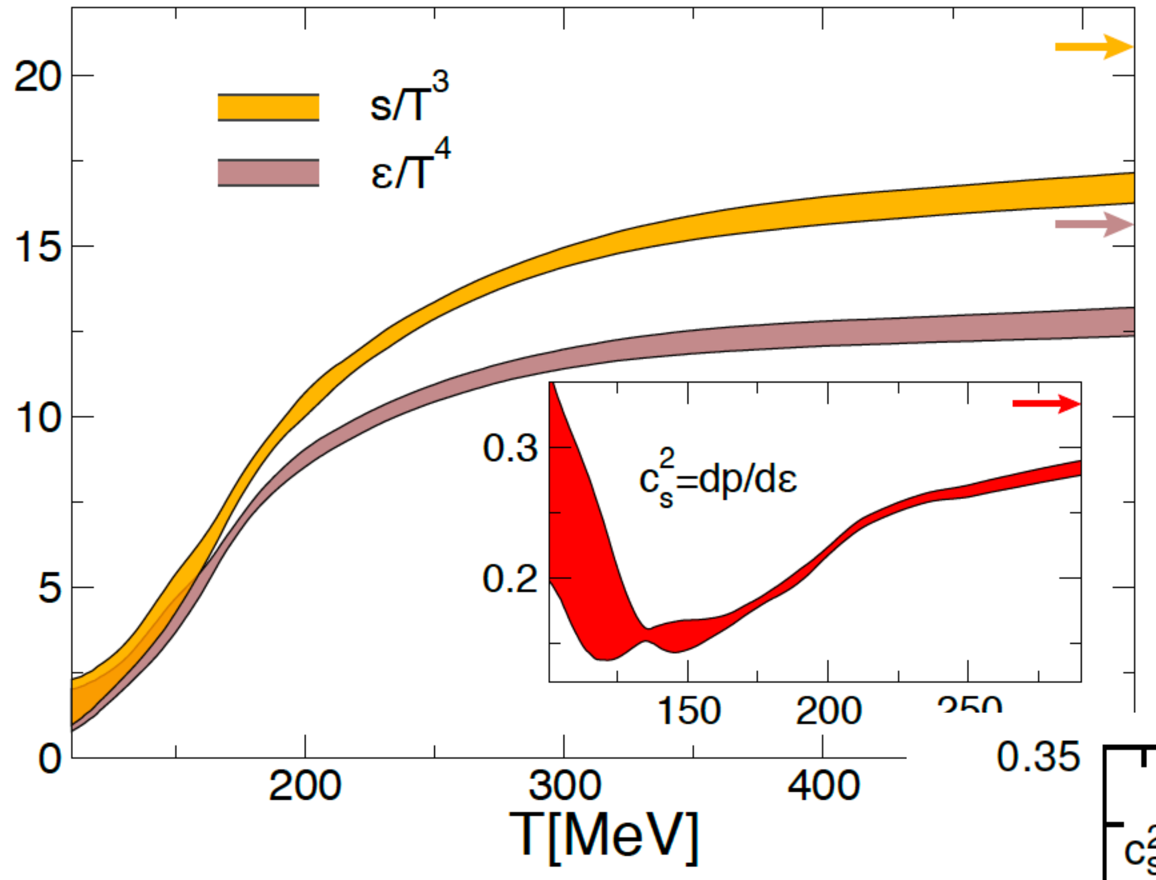
- Microscopic interpretation on the origin of the peak = quark saturation (work for any # of color)

  Kojo (2021), Kojo and Suenaga (2022)

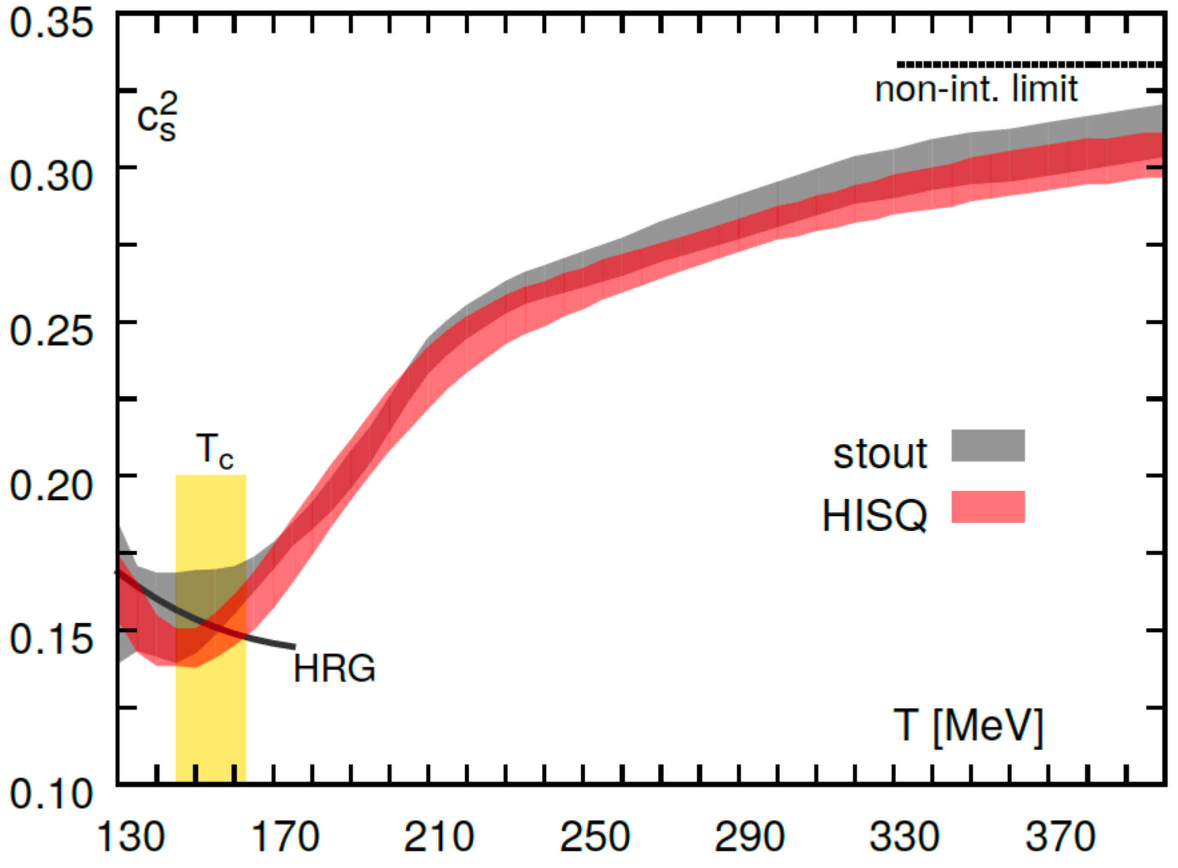
➡ Lattice study on 2color dense QCD the sign problem is absent!!

Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)



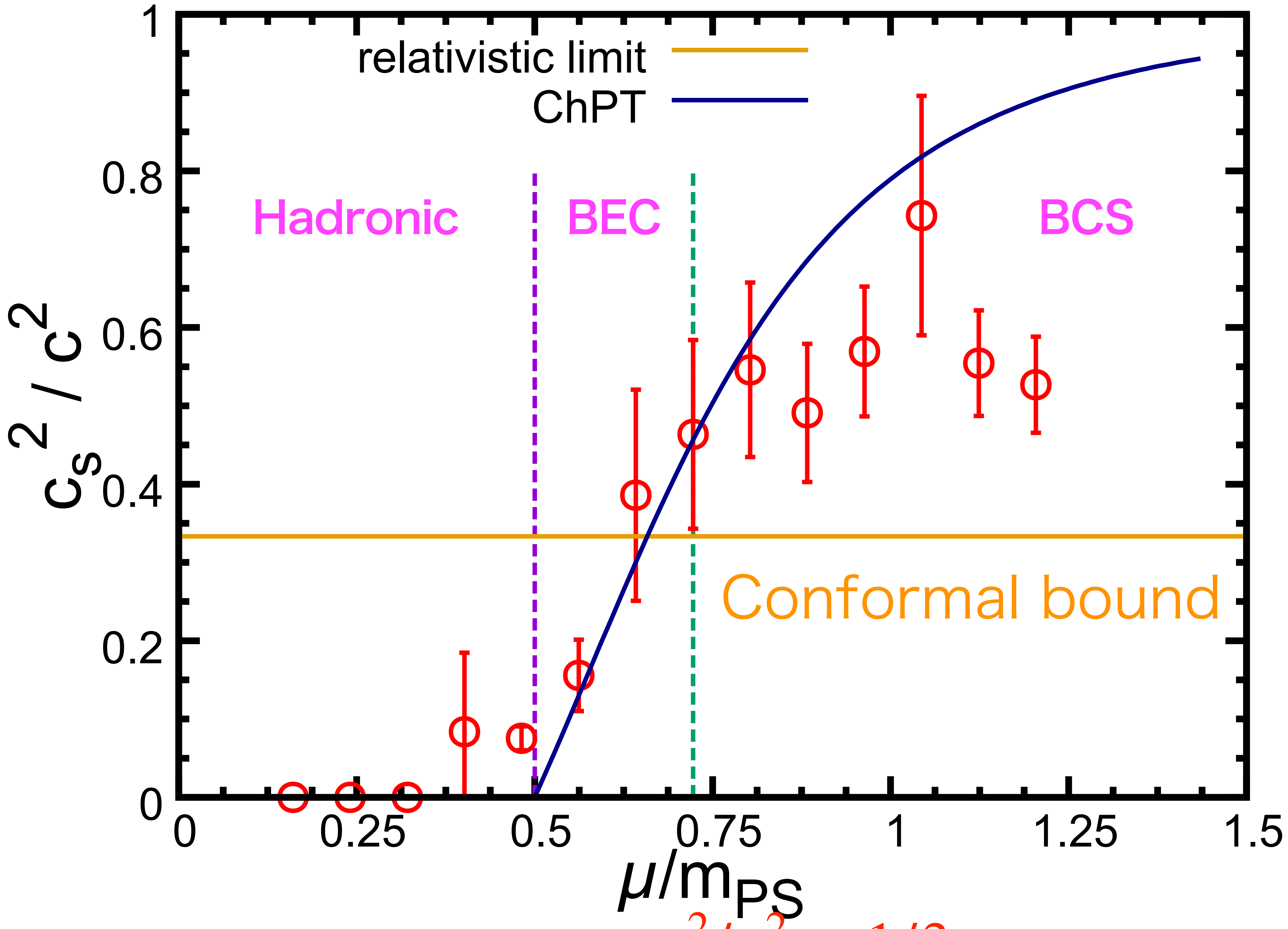
Borsanyi et al. (2013)



HotQCD (2014)

Finite Density transition (Nf=2 2color QCD)

Iida and EI arXiv: 2207.01253



- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories.

Method to see EoS at finite density regime

- Fixed scale approach ($\mu \neq 0$ version)

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle\frac{\partial S}{\partial \beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + a\xcancel{\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle}\right)$

No renormalization for μ

$\langle \cdot \rangle_{sub.} = \langle \cdot \rangle_{\mu} - \langle \cdot \rangle_{\mu=0}$

Zero at $j \to 0$

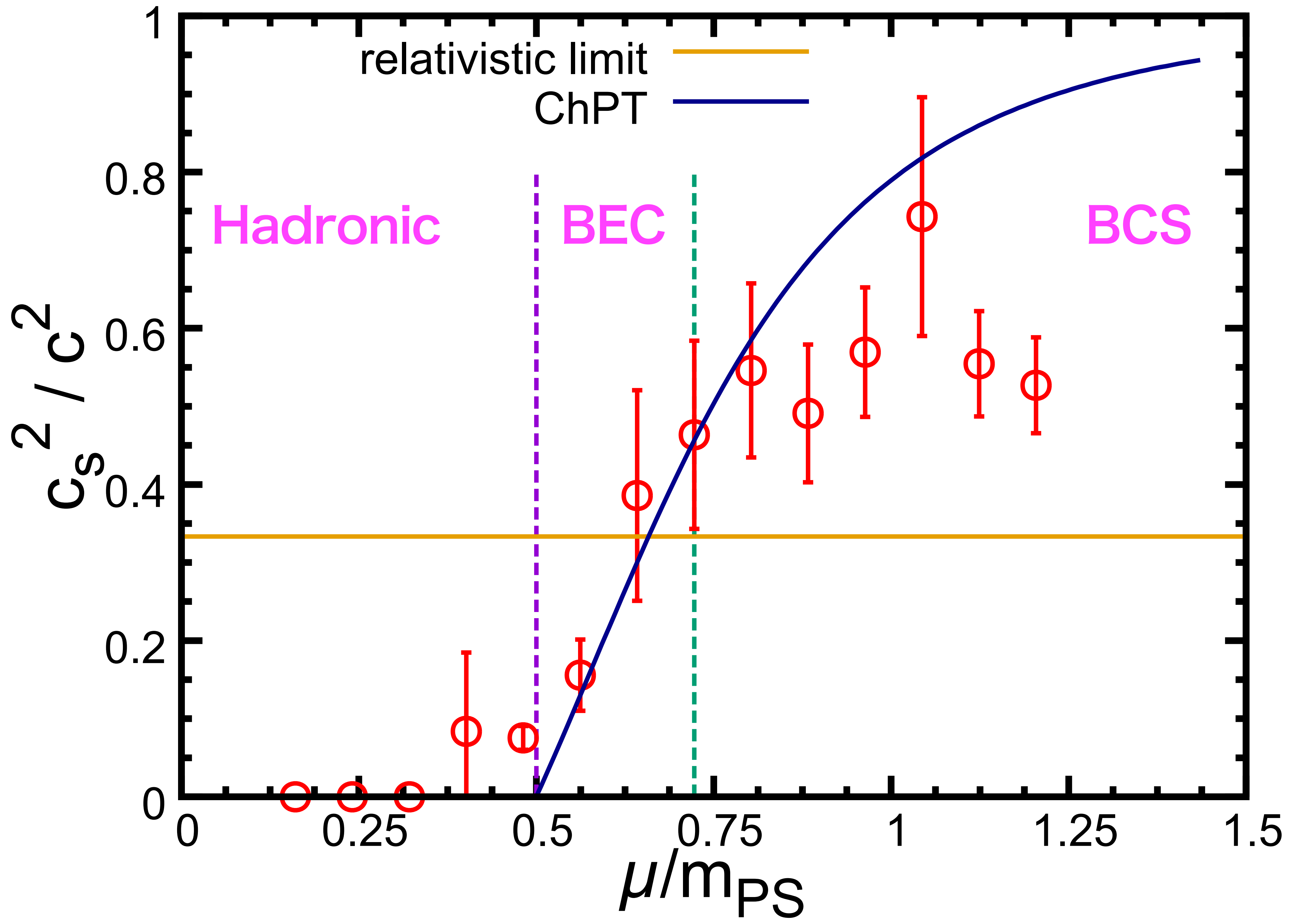
- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Nonperturbative beta-fn.

$$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

Further high density?

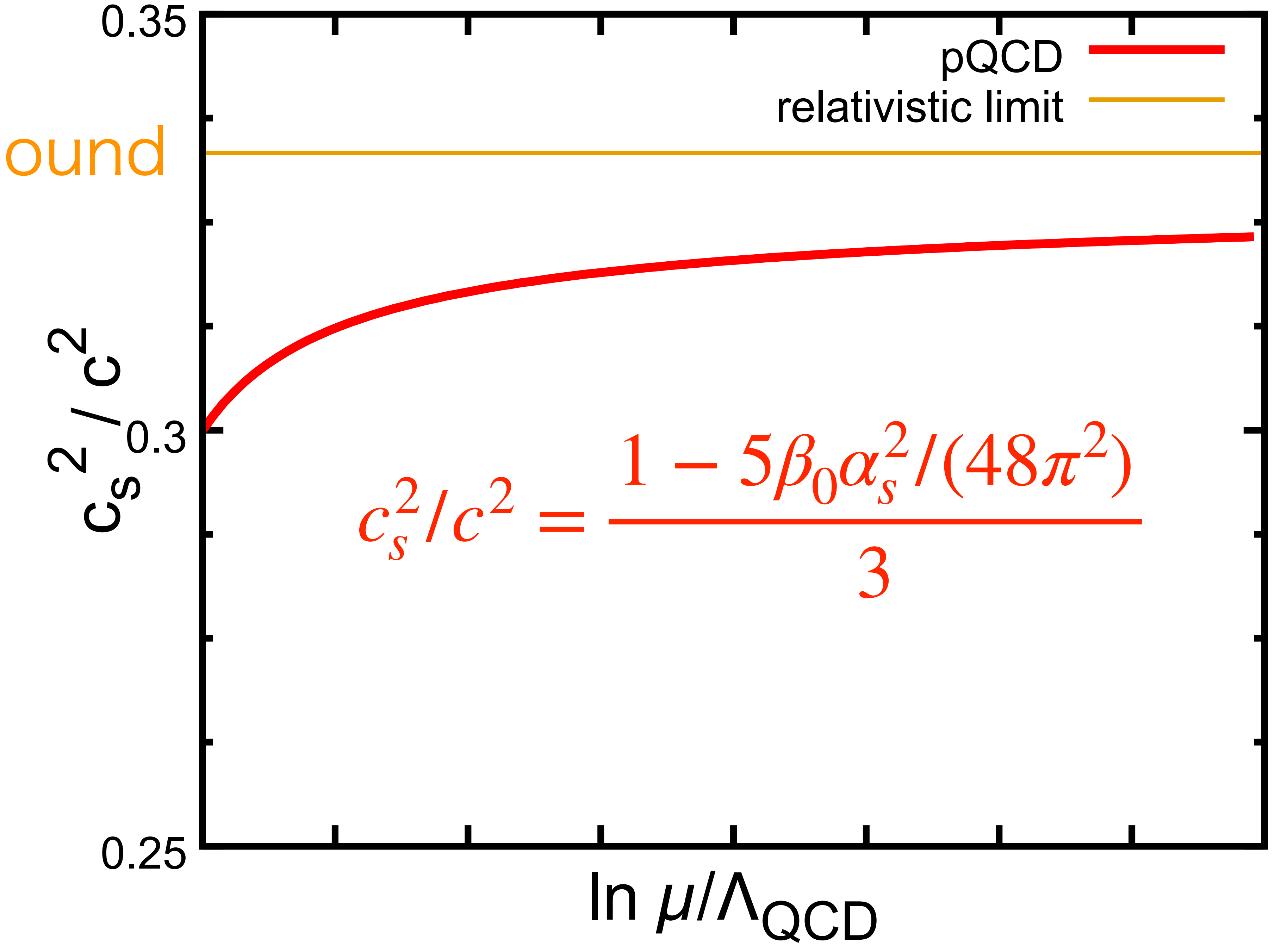


Kojo, Baym, Hatsuda (2021)

pQCD prediction

(Ultra high-density regime)

Conformal bound



$$c_s^2/c^2 = \frac{1 - 5\beta_0\alpha_s^2/(48\pi^2)}{3}$$

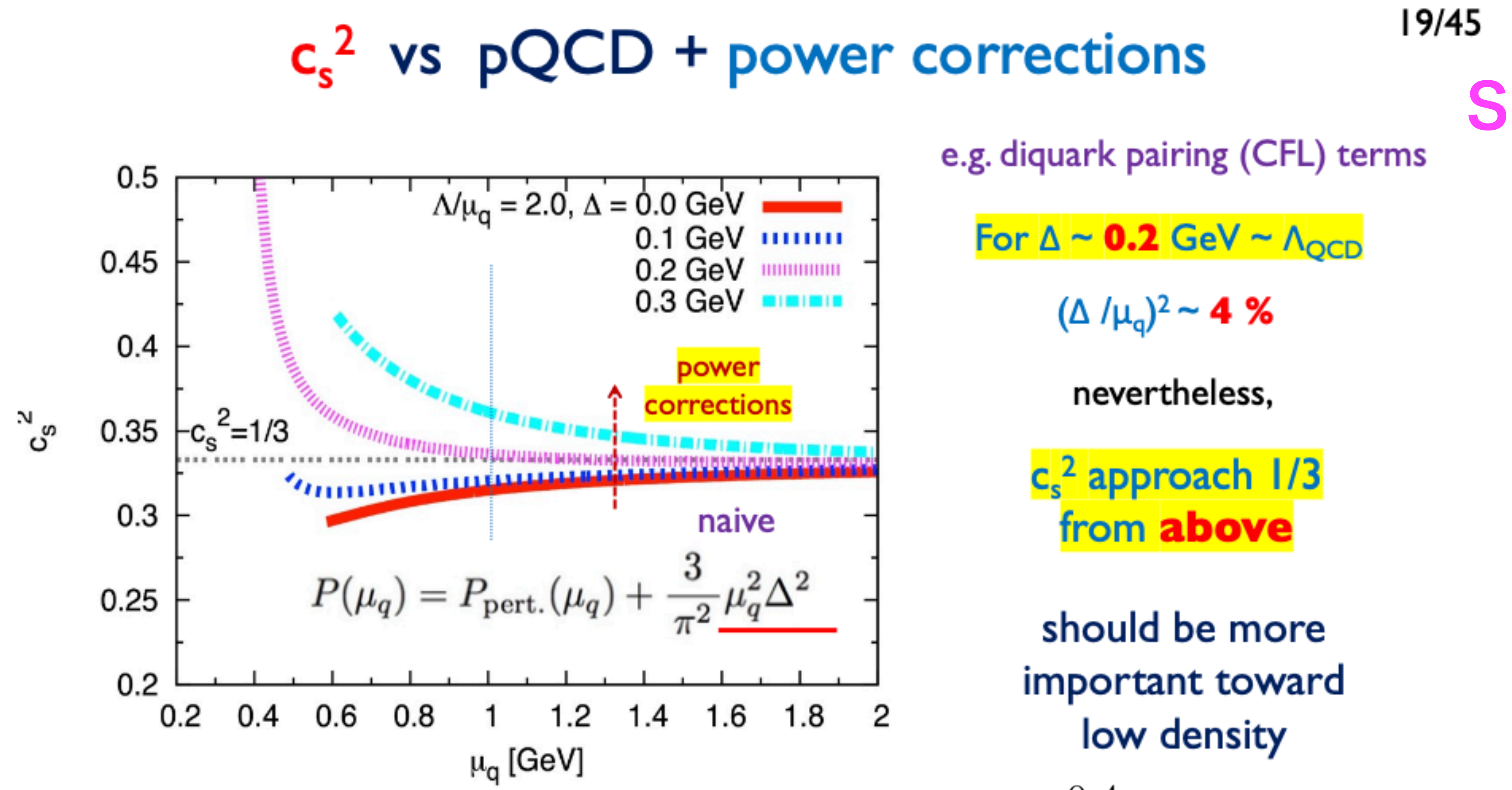
potential in lattice simulation comes from $a\mu \ll 1$

(Here, we take $a\mu \leq 0.8$)

e lighter mass / finer lattice spacing are needed

Further high density?

pQCD + power correction due to diquark gap



c_s^2 vs pQCD + power corrections

19/45

e.g. diquark pairing (CFL) terms

For $\Delta \sim$ **0.2** GeV ~ Λ_{QCD}

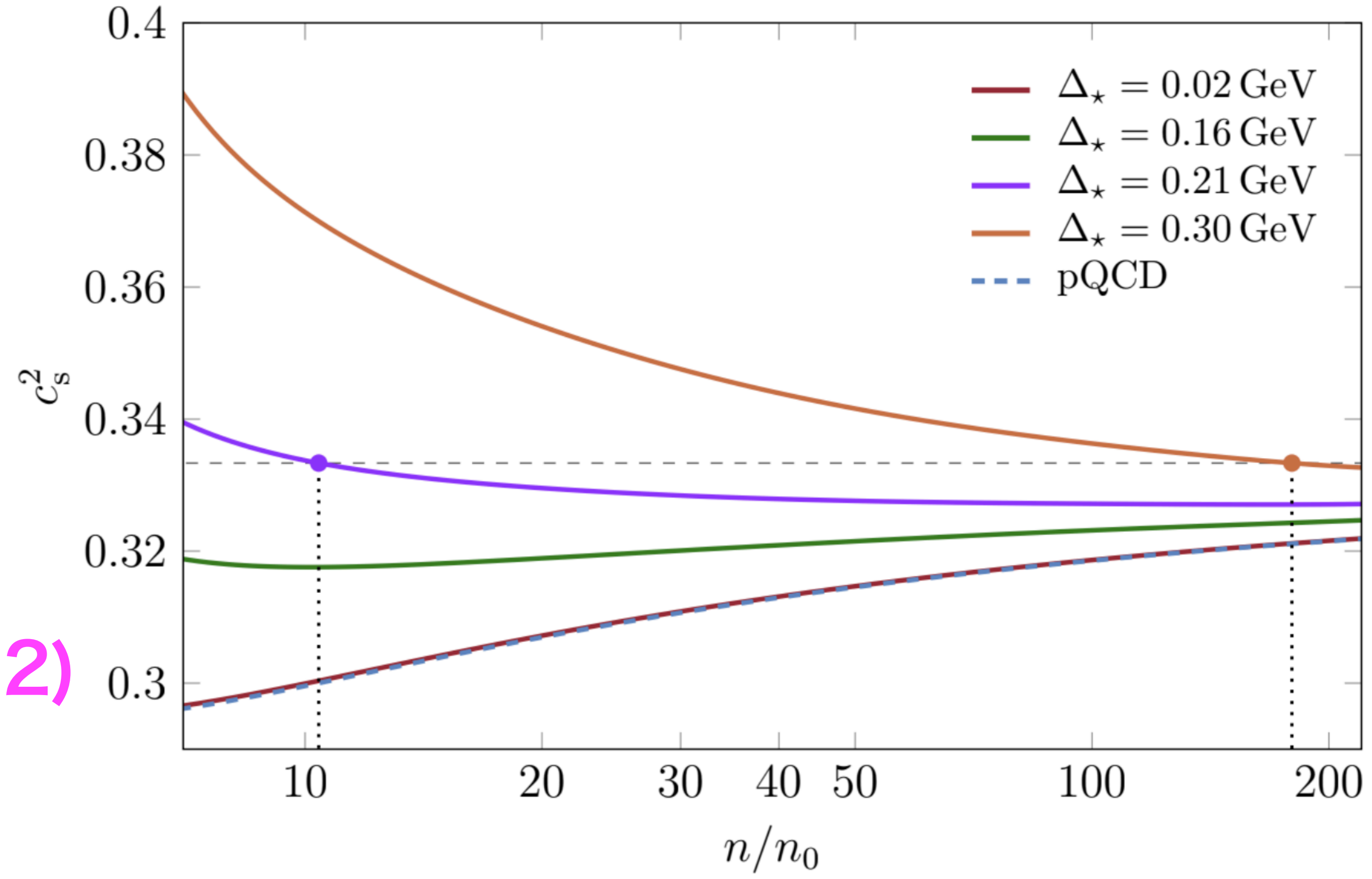
$(\Delta/\mu_q)^2 \sim$ **4 %**

nevertheless,

c_s^2 approach 1/3
from **above**

should be more
important toward
low density

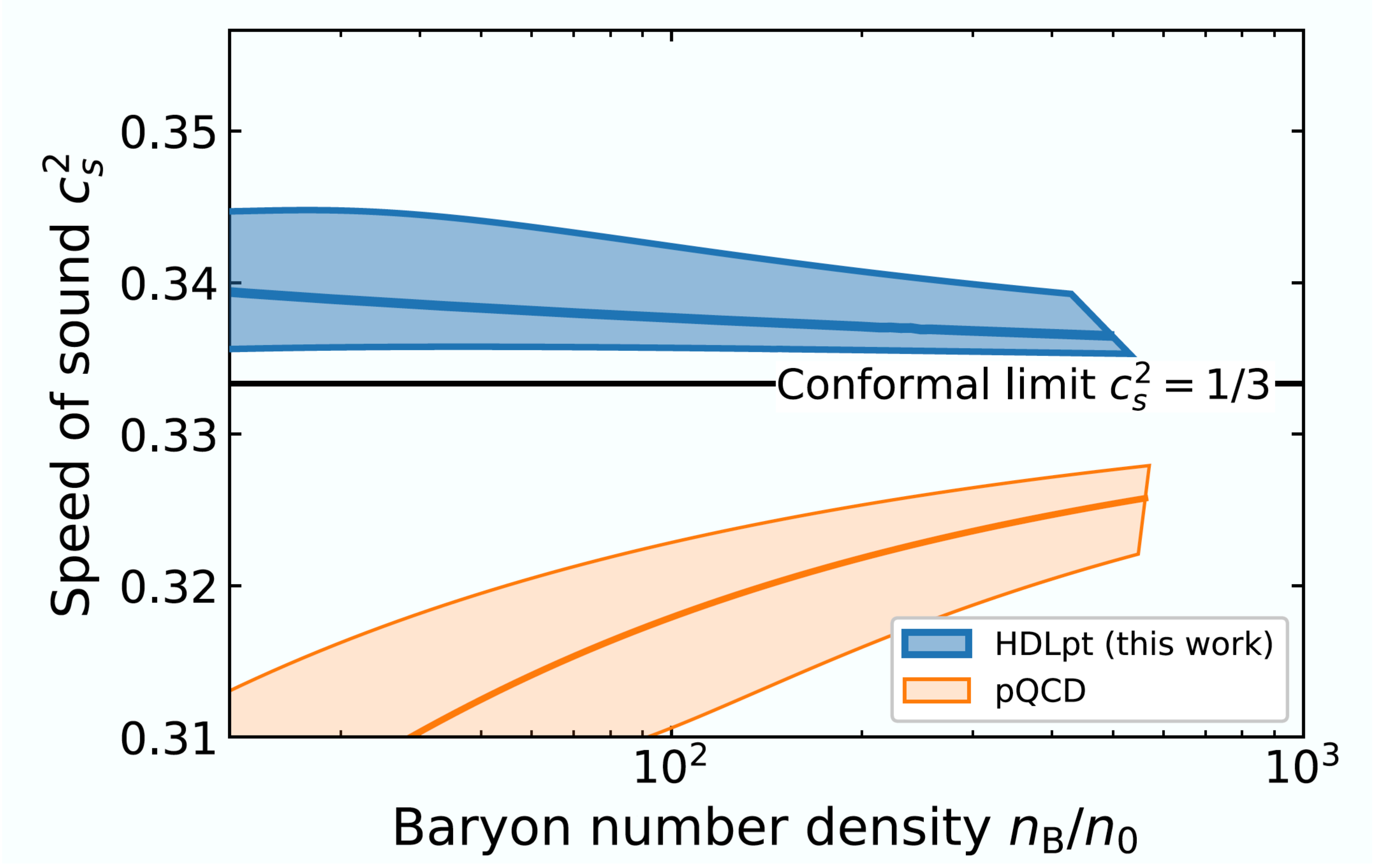
Slide by Kojo (2019)



fRG analysis
Braun, Geissel, Schallmo(2022)

Hard thermal loop resummation

Fujimoto and Fukushima(2021)



$^2/c^2$ approaches 1/3; from below or from above?

Lattice MC for 3 color QCD with isospin chemical potential

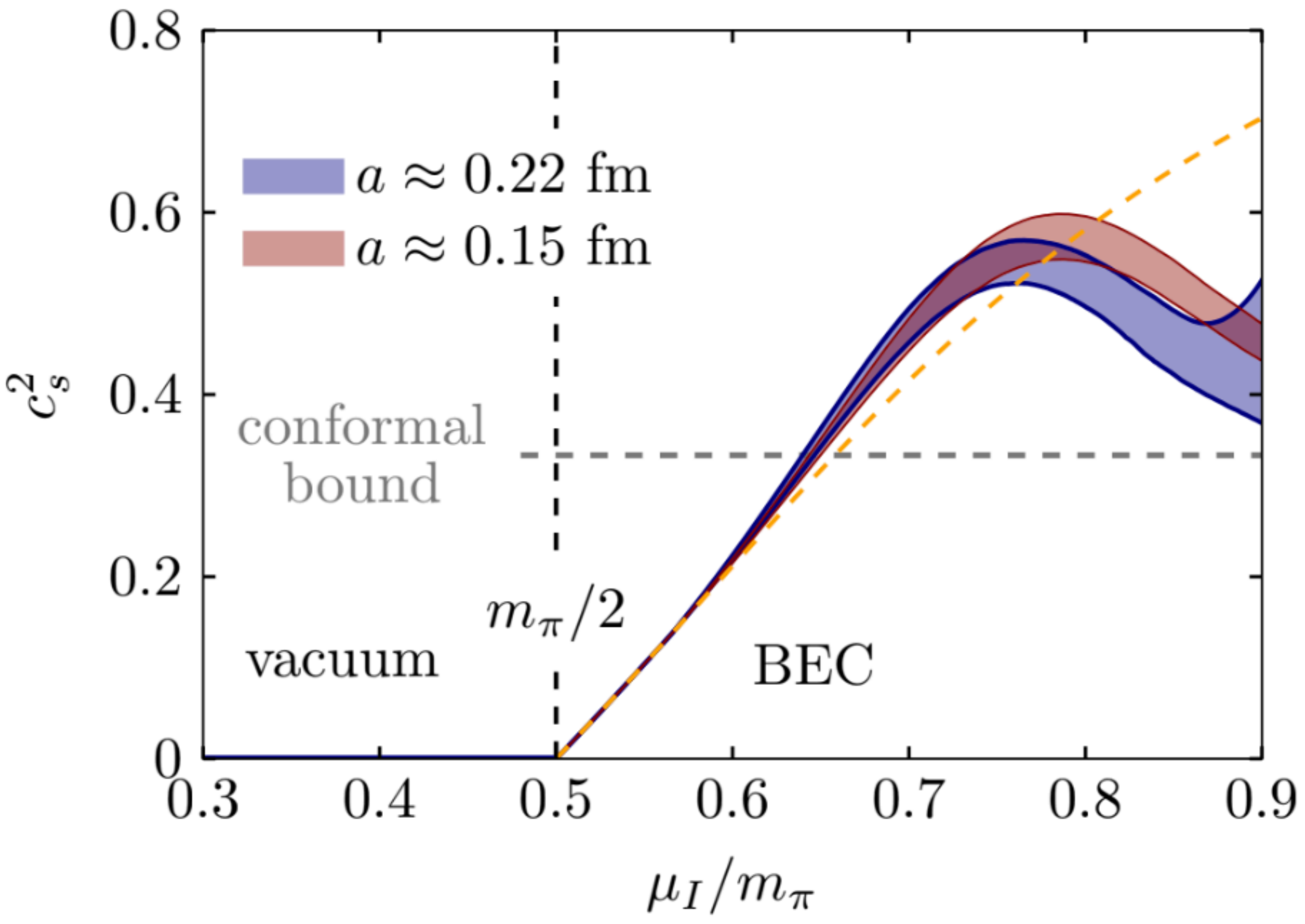
3 color QCD w/ Isospin-$\mu_I \approx$ 2color QCD w/ real μ

B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

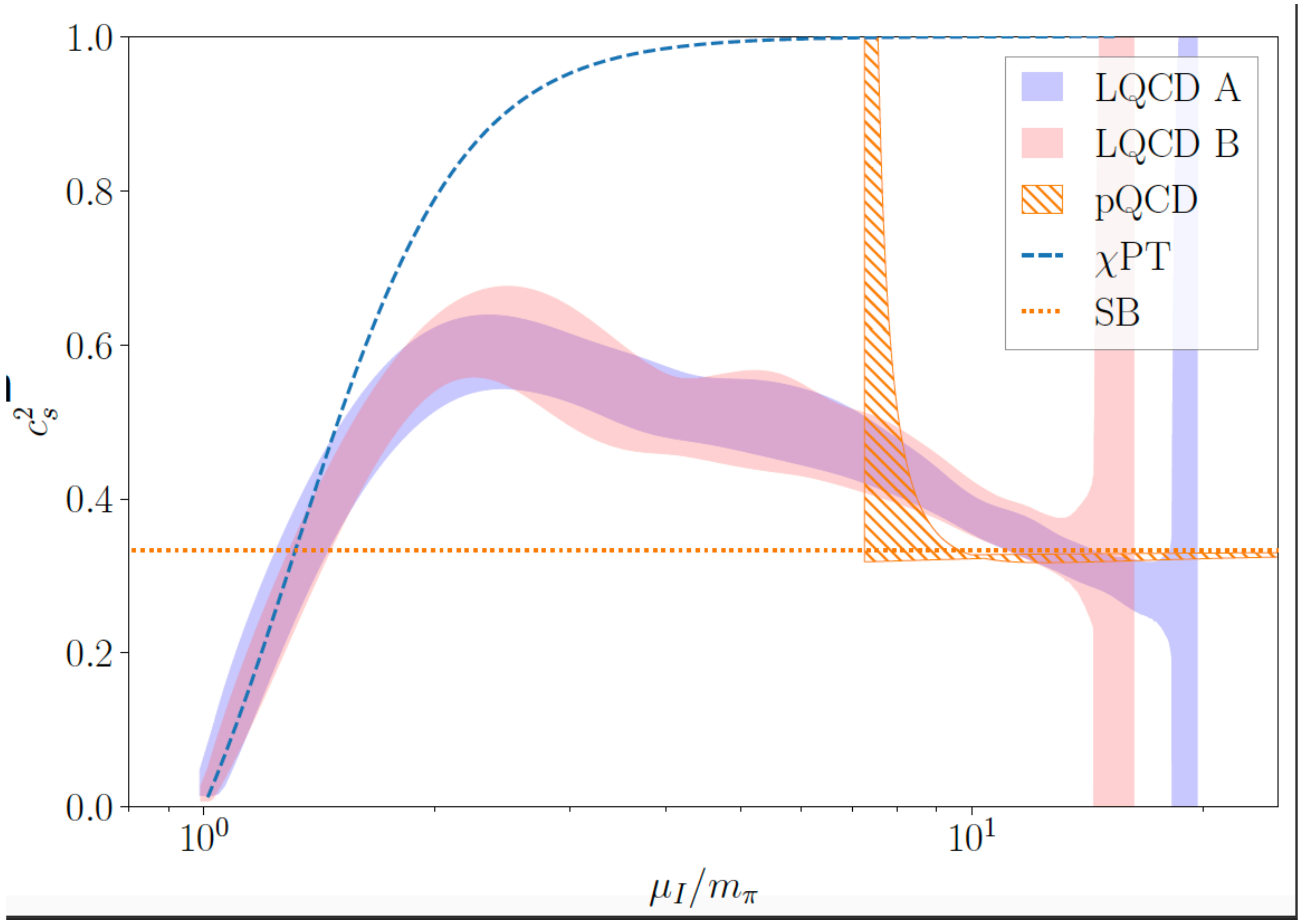
R. Abbott et al. arXiv:2307.15014

(W.Detmold's talk Monday)

Result with spline interpolation

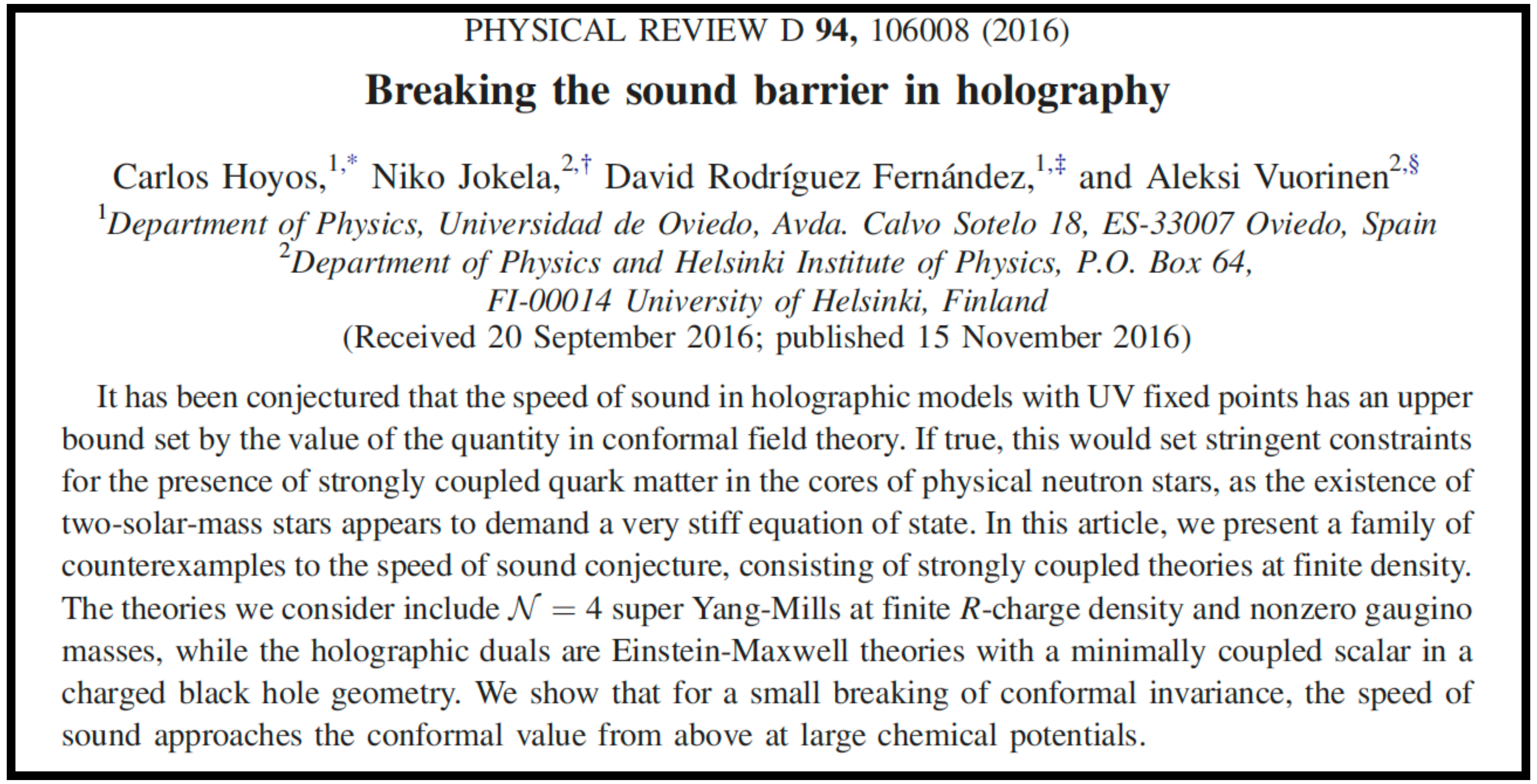


New algorithm for n-point fn. calc.



Counterexamples of conformal bound

N=4 SYM at finite density



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Breaking the sound barrier in holography

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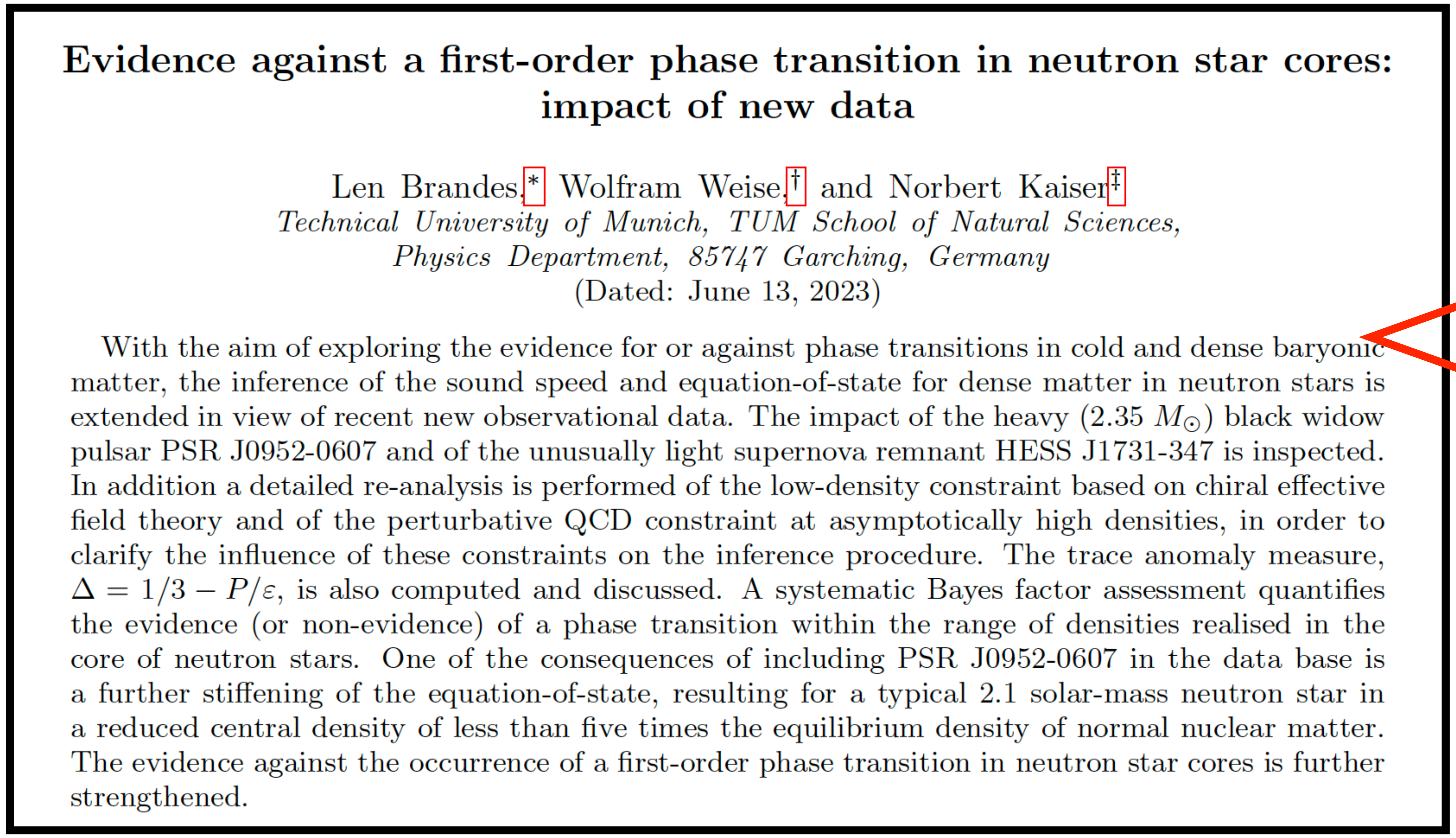
(Received 20 September 2016; published 15 November 2016)

It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite R-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.



Evidence against a first-order phase transition in neutron star cores: impact of new data

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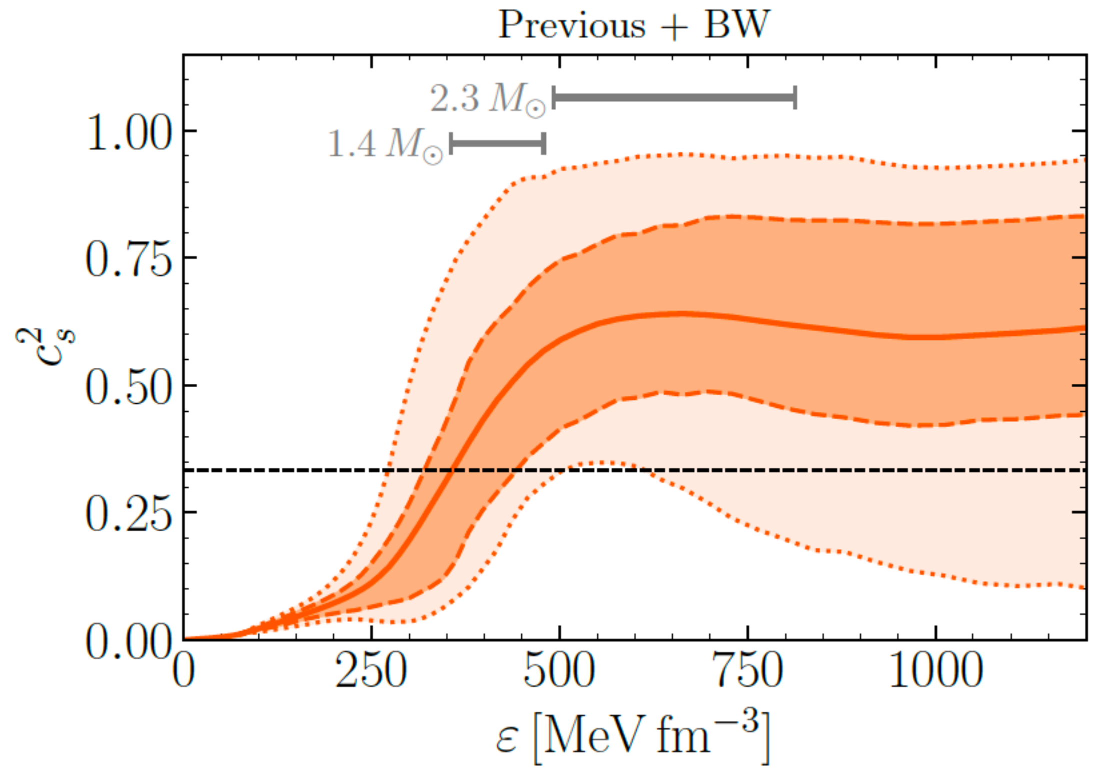
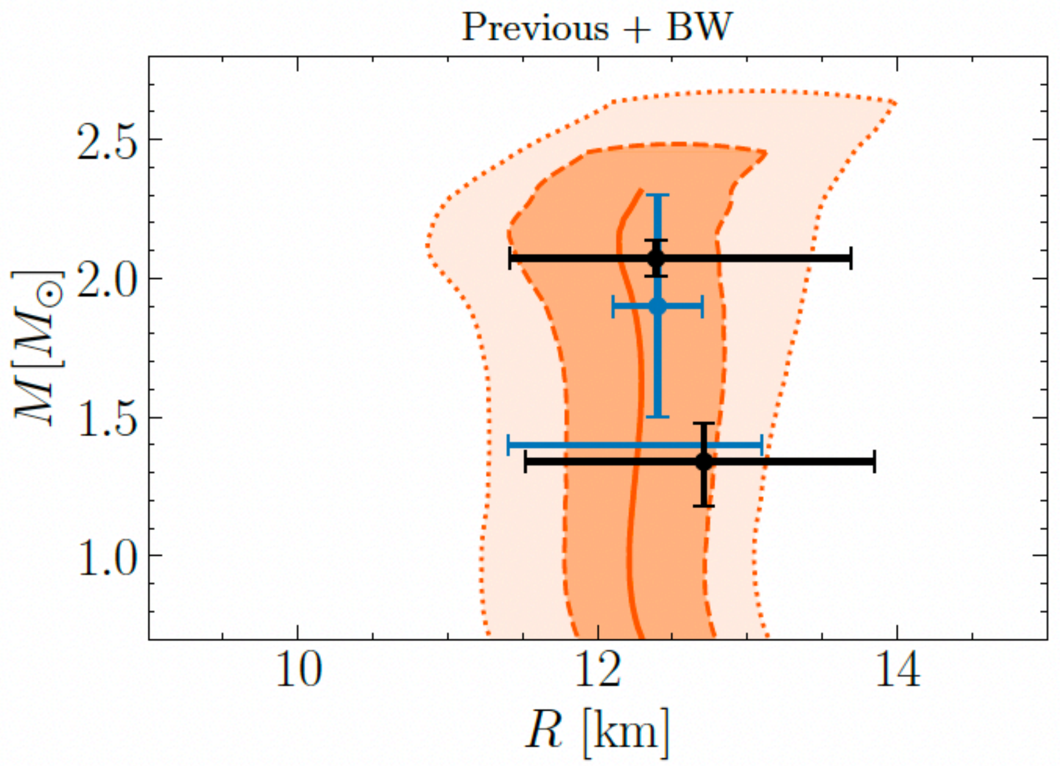
Physics Department, 85747 Garching, Germany
(Dated: June 13, 2023)

With the aim of exploring the evidence for or against phase transitions in cold and dense baryonic matter, the inference of the sound speed and equation-of-state for dense matter in neutron stars is extended in view of recent new observational data. The impact of the heavy (2.35 $M_\odot$) black widow pulsar PSR J0952-0607 and of the unusually light supernova remnant HESS J1731-347 is inspected. In addition a detailed re-analysis is performed of the low-density constraint based on chiral effective field theory and of the perturbative QCD constraint at asymptotically high densities, in order to clarify the influence of these constraints on the inference procedure. The trace anomaly measure, $\Delta = 1/3 - P/\varepsilon$, is also computed and discussed. A systematic Bayes factor assessment quantifies the evidence (or non-evidence) of a phase transition within the range of densities realised in the core of neutron stars. One of the consequences of including PSR J0952-0607 in the data base is a further stiffening of the equation-of-state, resulting for a typical 2.1 solar-mass neutron star in a reduced central density of less than five times the equilibrium density of normal nuclear matter. The evidence against the occurrence of a first-order phase transition in neutron star cores is further strengthened.

Bayian analyses of recent observation data of neutron star





arXiv:2306.06218