QCD constraints on isospindense matter and the nuclear equation of state

Based on 2307.15014, 2406.09273 with Ryan Abbott, Fernando Romero-López, Zohreh Davoudi, Marc Illa, Assumpta Parreño, Phiala Shanahan, Mike Wagman [NPLQCD collaboration]

Will Detmold (MIT)

Dense QCD Matter

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Three approaches Isospin chemical potential

1. Path integral formulation (new MC calculation for each μ_I)

$$
Z(\beta,\mu_I) = \int_{\beta} [d\phi] e^{-(S[\phi] + \mu_I N_I[\phi])}
$$

2. Grand canonical partition function

$$
Z(\beta,\mu_I) = \sum_s e^{-\beta(E(s) - \mu_I I_z(s))}
$$

• Low temperature limit dominated by $I = I_z = n$ ground states

$$
Z(\beta \to \infty, \mu_I) \sim \sum_n e^{-\beta (E_n^{(0)} - \mu_I n)}
$$

3. Canonical partition function

$$
Z_n(\beta) = \sum_s \delta_{N_s,n} e^{-\beta E(s)} \qquad \Longrightarrow \qquad \mu_I = \frac{dE}{dn}\bigg|_V
$$

(Grand) canonical approach **Isospin chemical potential** based in the set of th ρ In obecasion is otopici suitable correlations must be constructed and the correlations must be constructed and the construction of the

- Need ground state energies of system as isospin charge changes cess the *n* of the presents increases. The points in the points in the present of the present of the present of the present $f(x)$ and atote energies of everyone as isospin shores large *z*-component of isospin, *Iz*, with vanishing total
- Correlation functions with quantum numbers of many charged pions ϵ \int culture $\sum_{n=1}^{\infty}$ tions not only vary by many orders of magnitude across ϵ and ϵ momentum . See Figure correlation correlation correlation ϵ correlation consider ϵ functions with **f**

$$
C_n(t) = \left\langle \left(\sum_x \pi^-(\mathbf{x}, 0)\right)^n \prod_{i=1}^n \pi^+(\mathbf{y}_i, t) \right\rangle
$$

Late time behaviour quently, any attainable statistical sample of a many-pion correlate where *n I***z** *I***z** *I***z** *I***z** *I***z** *I***z** *I***z** *I***z** *I***z** *I***z** *I*z *I***z** *I*z *I***z** *I*z *I***z** *I***z** *I*z *I***z** *I***z** *I*z *I***z** *I***z** *I*z *I***z** *I*z *I***z** *I***z** *I***z** *I***z** *I***z**

$$
C_n(t \to \infty) \to Z e^{-E_n^{(0)}t}
$$

- Large number of Wick contractions $\frac{n}{\sqrt{2}}$
- E.g. $\sim 10^{40,000}$ for $n = 6144$ \bullet Fe figurations in a range of contexts have been found to fect the spectrum of states that propagate over a Eu-separate over a Eu-separate over a Eu-separate over a Eu-
The spectrum of states that propagate over a Eu-separate over a Eu-separate over a Eu-separate over a Eu-separ

Pion blocks
and the approximation of the approximation **Many pion correlation functions** rany pront consideration rande stability make it possible to increase the increase the isospin charge that \mathcal{L} cient than previous algorithms and consider some general some general some general some general some gener--
The consider some general some g an correlation functions stability make it possible to increase the increase the increase the increase the increase the increase the is
In the increase the increase the increase the increase the increase the increase of the increase the increase

- Previous studies used: \cdots
	- Traces, Recursion relations, Vandermonde matrices & FFTs FTs
	- Limited in *n* by cost (best algorithm $\sim \mathcal{O}(n^4)$) and numerical precision demands A in B by east (best algorithm $B(x^4)$ As in Refs. [11, 17], a *zero-momentum pion block* can rical **interestigates** γ cost (best algorithm $\sim \mathcal{O}(n^+)$) and numerical
- Made use of zero-momentum pion block $(12L^3 \times 12L^3$ matrix) ⇧(*i,*↵)(*j,*)(x*,* y;*t*)

$$
\Pi_{(i,\alpha)(j,\beta)}(\mathbf{x},\mathbf{y};t) = \sum_{k,\gamma,\mathbf{z}} S_{(i,\alpha)(k,\gamma)}(\mathbf{x},0;\mathbf{z},t) S_{(k,\gamma)(j,\beta)}^{\dagger}(\mathbf{y},0;\mathbf{z},t)
$$

Symmetric polynomial algorithm Many pion correlation functions M_{\odot} \sim \sim $\frac{1}{2}$, \sim \sim \sim $\frac{1}{2}$, \sim \sim $\frac{1}{2}$, \sim \sim viany pion correiauon iuncuon dinate, the time-dependence of ⇧(*t*) will be suppressed. Let ~ *x* = *{x*1*,...,x^N }* denote the set of eigenvalues of ⇧. lations below will be independent of the temporal coorly pion correlation function etric polynomial algorithm Guil polynomiai algoriumi

• New algorithm based on symmetric polynomials over eigenvalues of Π (denoted $\vec{x} = \{x_1, \ldots x_N\}$ with $N = 12L^3$) As we will show in Sec. II E, the correlation function in Eq. (1) can be written for 1 *n N* as 3 refers to the strange quark propagator. Additionally, $t_{\rm max}$ species correlation functions, such as mixed systems of $\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ ($\mathbf{F} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and *i* allowing developed in Ref. \vec{r} $S₁$ sustainable to be significant were signi $\mathbf{r}_1 \times \mathbf{r}_2$, with $N = 10$ 773 λ_1,\ldots

$$
C_n(t) = n! E_n(\vec{x})
$$

where for $1 \leq n \leq N$ where for the **E** p where for $1 \leq n \leq N$ in LQCD, they su↵er from numerical instabilities when *n* in the sum in Eq. (4) is computationally in Eq. (4) is computationally intractable for the sum in Eq. (4) is c
In Eq. (4) is computationally intractable for the sum in Eq. (4) is computationally interactable for the sum i

Write for

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$$
E_n(\vec{x}) \equiv E_n(\{x_1, \ldots, x_N\}) \equiv \sum_{i_1 < \cdots < i_n}^N x_{i_1} \ldots x_{i_n}
$$
\nRecurrence relation for

• Recurrence relation for • Recurrence relat

(numerically stable and cost is $\mathcal{O}(N^2)$ for all $n \in \{1,...,N\})$ ecurrence relation for
 $r_{M}(\{r_{1} \ldots r_{M}\}) = r_{M}E_{L-1}(\{r_{1} \ldots r_{M-1}\}) + E_{L}(\{r_{1} \ldots r_{M-1}\})$ $\frac{1}{2}$
 $\frac{1}{2}$ where the individual $\sum_{i=1}^n x_i^2$ range from 1 to $\sum_{i=1}^n x_i^2$ range from 1 to $\sum_{i=1}^n x_i^2$ $(x_1, \ldots, x_M) = x_M E_{k-1}(\{x_1, \ldots, x_{M-1}\}) + E_k(\{x_1, \ldots, x_{M-1}\})$ $\mathcal{E}^{\text{rically stable and cost is }} \mathcal{O}(N^2) \text{ for all } n \in \{1,...,N\})$ $E_k({x_1, \ldots, x_M}) = x_M E_{k-1}({x_1, \ldots, x_{M-1}})$ $\sqrt{2}$ $E_{\mu}(\{x_1, \ldots, x_N\})$ $\lceil n \rceil$ (consider some generality) (numerically stat $+ E_k({x_1, \ldots, x_{M-1}}),$ U COST IS $U(N)$ for all $N \in \{1,...,N\}$

- Overall cost dominated by finding the eigenvalues: $\mathcal{O}(N^3)$ s Overall cost dominated by infuilig the eigenvalue **• Overall cost dom** sively computing $E = \frac{1}{2}$ ted by finding the eigenvalues: $\mathcal{O}(N^{\circ})$
	- Discussed last year see 2307.15014 cial cases *E*

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	2 Note that the spatial location $\frac{220746044}{\pi}$ culia range over 2007. I JULI4 \overline{C} the correlation function function function function function \mathbf{r}_i to **P** \sim 000 \pm 000

Lattice QCD calculations Many pion correlati⁵⁰ is the state if the state of

 6.5 1.17008 -0.2095 -0.1793

 $\log C_{\rm 450}$ B(3) $\qquad \qquad 14.7$ New in

2406.09273

- Study on four ensembles of $N_f = 2 + \frac{1}{50\sqrt{t/a = 18}}$ configurations with (close-to/below) physical matrix masses $\frac{-1167(2)^{12}}{166(2)}$ $\frac{-3182-108.08-106}{5.048}$ $\frac{106}{3}$ $\frac{17104}{14.7}$ $\frac{-102}{160}$ $\frac{100}{160}$ $\frac{0.091(1)}{0.091(10)}$ $a^2(f_m)$ $t/a = 18$ $t/a = 17$ $t/a = 16$ $t/a = 15$ Label N_{conf} β_g C_{SW} am_{ud} am_s $(L/a)^3 \times (L_4/a)$ $\frac{(a)^3 \times (L_4/a) - a^2(\hat{f}m)}{48^3 \times 96}$ $\frac{(M_4 \times (MeV) - L_4 \times 7)}{166(2)}$ $\frac{(fm) - M_{\pi}L}{4.37}$ $\frac{1}{3.75}$ $\frac{(MeV)}{22.8}$ A 665 6.3 1.20537 -0.2416 -0.2050 $48^3 \times 96$
B 1262 6.3 1.20537 -0.2416 -0.2050 $64^3 \times 128$ B 1262 6.3 1.20537 -0.2416 -0.2050 $64^3 \times 128$ $0.091(10)$ $-1067(2)12$ -5182 -108.08 106 1710
C 846 6.5 1.17008 -0.2091 -0.1778 $72^3 \times 192$ $0.070(1)$ $166(2)$ 5.0498 C_{40} B(3) 14.7
- Sparsened quark propagators computed from grid of $8³$ sites on **ONE timeslice:** $N = 12 \times L^3 = 12 \times 8^3 = 6144$ Eigenvalues on a single configuration $\overline{\mathcal{A}}$ red in \bullet Snarsened quark propagators computed from grid of 2^3 sites on quarks. The first column lists the label used to refer to the ensemble, *N*conf is the number of configurations, and *^g* and *C*SW

C 846 6.5 1.17008 -0.2091 -0.1778 $72^3 \times 192$ 0.070(1) $166(2)$ 5.0 $48 \text{ }^\circ \text{ }^\circ$

- Eigenvalues computed by SVD of time sliced quark-propagator $(Since \Pi = S^{\dagger}S)$ from analysis of the e \overline{t}
	- Calculations performed in double, 2-double and 3-double *aE*(*n*) ^e↵ (*t*) = log *^Cn*(*t*)

Many pion correlation functions Lattice QCD calculations

- Correlation functions vary rapidly in Euclidean time
	- $C_{6144}(t)$ varies by $> 10^5$ orders of magnitude
- Correlation functions vary between samples by many orders of magnitude
	- Central Limit Theorem only valid at unachievable sample size
	- Correlation function distributions are approximately log-normal

()⁰ () = *,*⁰ (14)

Many pion energies Many pion correlation functions

Effective energy from log-normality

$$
E_{\text{eff}}^{(n)}(t) = \mu_n(t) - \mu_n(t-1) + \frac{\sigma_n^2(t)}{2} - \frac{\sigma_n^2(t-1)}{2}
$$

- CLT: χ^2 -fitting makes no sense
- Bootstrap analysis takes value of $E_{\text{eff}}^{(n)}$ for random timeslice in plateau region eff
- Entire bootstrap histogram propagated into subsequent analysis
- Energy significantly larger than that of *n* free pions

Many pion energies Many pion correlation functions

Many pion energies Many pion correlation functions

QCD at $\mu_I \neq 0$ & $\mu_R = 0$

Thermodynamic observables

• Observables defined as

QCD at $\mu_I \neq 0$ & $\mu_R = 0$

Chiral interpolation & continuum extrapolation

- Action is perturbatively $improved: \mathcal{O}(a^2, g^2(a^{-1})a)$
- Masses range over *m*^π ∈ [125,165] MeV
- Extrapolation/interpolation form

 $X^{(j)}(a, \mu_I, m_{\pi}) = X_0^{(j)}(\mu_I) + X_1^{(j)}a^2 + X_2^{(j)}a^2\mu_I$ $+X_3^{(j)}a^2\mu_I^2 + X_4^{(j)}(m_\pi^2 - \overline{m}_\pi^2)$

for $X \in \{P, \epsilon, c_s^2, ...\}$

• Systematic uncertainty from model-averaging all sub-models

Superconducting gap QCD at $\mu_I \neq 0$ & $\mu_R = 0$

- Asymptotic freedom for $\mu_I \to \infty$: attractive interaction \Longrightarrow superconductivity
- Superconducting gap

 \langle *ū*_{*a*γ₅*d*_{*b*}} Δ

• Solve gap equation [Fujimoto 2023]

$$
\Delta(\mu_I) = b' \exp\left(-\frac{3\pi^2}{2g(\mu_I)}\right)
$$

• Nontrivial background

$$
P(\Delta) - P(\Delta = 0) = \frac{N_c \mu_I^2}{8\pi^2} \Delta^2
$$

• Estimate of gap from

$$
P_{\text{LQCD}} - P_{\text{pQCD}}^{\text{NNLO}}(\Delta = 0)
$$

QCD at $\mu_I \neq 0$ & $\mu_B = 0$

Speed of sound

• Since temperature is $0 \sim T \leq 20$ MeV, isentropic speed-of-sound can be determined

$$
\frac{1}{c_s^2} = \frac{\partial \epsilon}{\partial P} = \frac{1}{\langle n \rangle_{\beta, \mu_I}} \frac{\partial}{\partial \mu_I} \langle E \rangle_{\beta, \mu_I}
$$

- Exceeds conformal bound $c_s^2 \leq 1/3c^2$ over wide range of *μI*
- Similar behaviour seen for small *μI* [Brandt, Cuteri, Endrodi 2022]
- Similar behaviour seen in $N_c=2$ QCD [E. Itou et al]
- Eventually relaxes to pQCD/ideal gas

QCD at $\mu_I \neq 0$ & $\mu_R = 0$

Isospin dense equation of state

- Combine information from LQCD, χ PT, pQCD
- Bayesian model mixing
	- $χ$ PT: NLO, errors from NLO-LO
	- pQCD: NNLO with pairing gap, errors from scale variation
	- LQCD: continuum, physical mass
	- Details follow nuclear EoS methods

Isospin dense equation of state QCD at $\mu_I \neq 0$ & $\mu_B = 0$

10^3 10^4 μ_I [MeV] 10^{-1} 10 $10¹$ ≤ */* ≤SB \cdots NLO χ PT pQCD+pairing LQCD Mixture model $--$ SB 10^3 10^4 μ_I [MeV] 10^{-1} 10^{0} 10^{1} $P/P_{\rm SB}$ \cdots NLO χ PT pQCD+pairing LQCD Mixture model $--$ SB Pressure Energy Density

QCD at $\mu_I \neq 0$ & $\mu_B =$ QCD at $\mu_I \neq 0$ & $\mu_B = 0$ **QCD at** $\mu_I \neq 0$ & $\mu_B = 0$
sospin dense equation of stat **is achieved without any phase transition by the inter**
Sospin dense equation of state CD at $\mu_I \neq 0$ & $\mu_B = 0$
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Combine information from
Bayesian model mixing Fig. 2: Simultaneous cooling of a bosonic cooling of a bosonic cooling of a bosonic quan-
Fig. p. p. q. cooling of and fermionic quantities and fermionic quan-
intervals of and fermionic quantities and fermionic quan- $\begin{array}{ccc} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array}$ $s \cup \alpha$, $u_n = 0$ \sim \sim \sim \sim \sim $U \cup U \text{ at } \mu_I \neq U \text{ or } \mu_B = 0$
Isosnin dense equation of sta $T = \frac{1}{2}$ and interacting for non-interacting for non-interacting for non-interacting for α This is supplemented by a discussion on boundstates and the contingencies of their appearence in The BEC paradigm, first developed for non-interacting bosons and later generalized to take into account repul-**ISO**
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combine information from LQCD

Bayesian model mixing
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The interaction strength via Feshbach resonances. This is supplement on boundstates and the continuum on boundstates and the continent of the continent o The BEC paradigm, first developed for non-interactions. $f \neq 0$ & \cup α μ _B
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information from LQCD, χ PT, pQCD

model mixing QCD at $\mu_I \neq 0$ & $\mu_B = 0$

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on the sense of bound-state formation in quantum systems, and how results from the same describe Cooper Pairing. and a weak attraction theory. A generalized by α and α and α and α are α and α are α to be, but rather are the two extrema of a continuum.
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The two extrema of a continuum of a continuum of a continuum. The difference between the pairs and the molecules is $\begin{split} \mathbf{L} & \mathbf{L} \mathbf{$ sive interactions, describes bosonic fluids like the set of the set Combine in
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Simultaneous cooling and fermionic and fermionic and fermionic and fermionic and fermionic and fermionic an of the Fermi gas, the Fermi pressure prohibits the atom cloud to show that is approximate a shareholder in space as $\mathsf{p}\mathsf{QCD}$
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QCD inequalities [T Cohen 2004] Bounding the nuclear equation of state

• $N_f = 2$ partition function for baryon chemical potential

$$
Z_{\rm B}(\beta,\mu_{\rm B}) = \int_{\beta} [dA] \det \mathcal{D} \left(\frac{\mu_{\rm B}}{N_{\rm c}}\right)^2 e^{-S_{\rm G}}
$$

where $\mathcal{D}(\mu) = D + m - \mu \gamma_0$

- Fermion determinant complex for $\mu_B\neq 0$
- $N_f = 2$ partition function for isospin chemical potential $Z_I (\beta, \mu_I) = \int_{\beta}$ [*dA*]det $\mathscr{D}\left(-\frac{\mu_I}{2}\right)$ det $\mathscr{D}\left(\frac{\mu_I}{2}\right)$ μ _I $\left(\frac{a_1}{2}\right)e^{-S_G} = \int_{\beta} [dA] \det \mathcal{D}$ $\mu_{\rm I}$ $\frac{1}{2}$ 2 e^{-S_G}
- QCD inequality

$$
Z_{\rm B}(\beta,\mu_{\rm B}) \le \int_{\beta} [dA] \left| \det \mathcal{D} \left(\frac{\mu_{\rm B}}{N_{\rm c}} \right) \right|^2 e^{-S_{\rm G}} = Z_{I}(\beta,\mu_{I} = 2\mu_{B}/N_{c})
$$

• Translates to pressure bound $(PV = T \log(Z))$

$$
P_B(\mu_B) \le P_I(\mu_I = 2\mu_B/N_c)
$$

• Saturated up to in pQCD [Moore&Gorda 2023]

Bounding the nuclear equation of state

QCD inequalities [T Cohen 2004, Fujimoto & Reddy 2023]

• Using GP-model isospin-dense EoS gives bound on symmetric nuclear matter EoS

QCD at $\mu_I \neq 0$ & $\mu_R = 0$

A fascinating playground

- Complete determination of isospin-dense matter equation of state
	- LQCD+*χ*PT+pQCD
- Clear signal for transition to pion BEC and eventually to BCS superconducting state
	- Determination of superconducting gap from difference to pQCD
- Conformal bound $c_s^2 < c^2/3$ clearly exceeded as in $N_c = 2$ QCD
- Rigorous bound on EoS of symmetric nuclear matter (but not so practical \odot)

Many pion correlation functions Log-normality tests and cumulants applicability of the CLT.⁷ Many nion correlat that that correlation functions *C*[*U*] *ⁿ* (*t*) were log-normally **r** and **contained and can** $t_{\rm{max}}$ is neglectively ruling out the use of standard $t_{\rm{max}}$ statistical methods for *n* & 100 pion systems. Note that the value of *N*(min) g-normality tests and cumulants lonu nion ookkohi idity plui This argument can be made more precise. Suppose the made more precise $\mathcal{L}_\mathcal{F}$ \overline{z} i.e., i.e., \overline{z} across and \overline{z} \mathbf{c} \mathbf{c} \mathbf{c} \mathbf{a} \mathbf{a} \mathbf{b} \mathbf{c} \mathbf{c} **n** Tunctions tions is needed, e↵ectively ruling out the use of standard statistical methods for *n* & 100 pion systems. Note that ants **a**

Pressure QCD at $\mu_I \neq 0$ & $\mu_R = 0$

Energy Density QCD at *μ^I* ≠ 0 & *μ^B* = 0 4 **e** 8 *P/P*SB

10^2 10³ μ_I [MeV] $0\frac{1}{10^2}$ 5 10 15 20 25 */* SB $-$ LQCD A LQCD B LQCD C LQCD D LQCD cont pQCD $\frac{1}{\sqrt{2}}$ NLO χ PT $m_{\pi} = 170$ MeV NLO χ PT $m_{\pi} = 139$ MeV 10^0 10¹ $\mu_I/\rlap{/}m_\pi$ 0 5 10 15 20 25 */* SB LQCD A LQCD B LQCD C LQCD D LQCD cont **i** pQCD \leftarrow NLO χ PT $m_{\pi} = 170$ MeV $\cdot \cdot$ NLO χ PT $m_{\pi} = 139$ MeV NLO PT *m* = 170MeV NLO PT *m* = 139MeV NLO PT *m* = 170MeV NLO PT *m* = 139MeV

Details of mixture model QCD at $\mu_I \neq 0$ & $\mu_B = 0$

• Models without pairing

Details of mixture model QCD at $\mu_I \neq 0$ & $\mu_R = 0$

• Models without LQCD input (similar to baryon density case) 16

