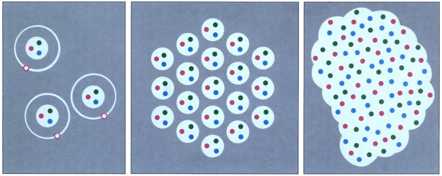
The chiral critical point from the strong coupling expansion

Wolfgang Unger, Bielefeld University
with Jangho Kim, Pratitee Pattanaik

Lattice 2024, Liverpool
July 31, 2024



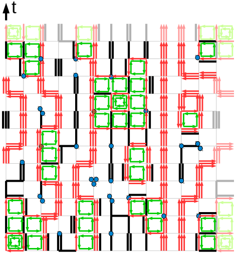
UNIVERSITÄT BIELEFELD
Faculty of Physics

CRC-TR 211
Strong-interaction matter under extreme conditions

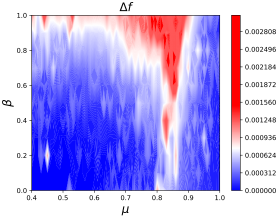
LATTICE 2024 LIVERPOOL

Dual Representation of Lattice QCD

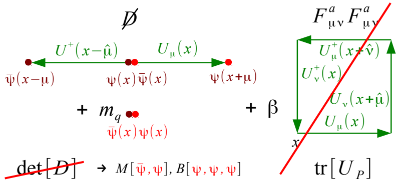
- Use “standard” QCD lattice action (staggered fermions, Wilson gauge action)
- But: **change order of integration:**
  - gauge links $\{U_\mu(x)\}$ first
  - quarks afterwards (Grassmann integration)
- At $\beta = 0$: link states are **mesons** and **baryons**
  [Rossi, Wolff, NPB 248 (1984)]
- For $\beta > 0$: use **strong coupling expansion**
  - $\mathcal{O}(\beta)$: via studied via reweighting
    [Langelage *et al.* PRL **113** (2014)],
  - for any order: has been mapped to a tensor network [Gagliardi, U, PRD 101 (2020)]
- Sign problem in regime $\beta = \frac{6}{g^2} \lesssim 1$
  mild enough to study full phase diagram:
  - baryons are heavy (almost static)
  - color singlets closer to physical states
    $\Rightarrow$ **sign reweighting** feasible: $\Delta f \simeq 10^{-5}$



2-dim. example of configuration in terms of dual variables



Strong Coupling Limit



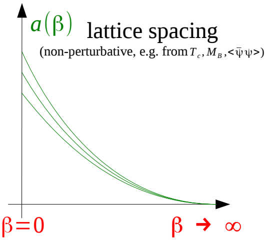
Regime where sign problem is mild:

Limit of strong coupling: $\beta = \frac{6}{g^2} \to 0$

- change order of integration: $\{U_\mu(x)\}$ first!
- **"dual" representation:** via color singlets on the links!
- at strong coupling: **mesons** and **baryons**

Interesting regime, because:

- exhibits **chiral symmetry** breaking and confinement
- (almost) no sign problem
- fast simulations (no supercomputers necessary)
  $\Rightarrow$ **complete phase diagram** can be calculated



Caveat:

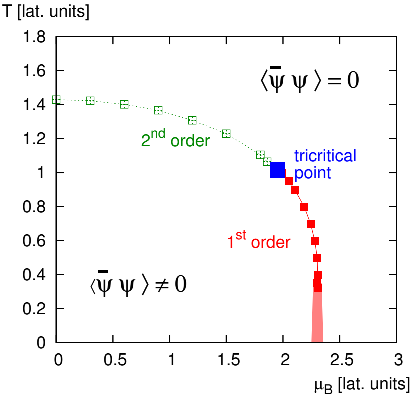
- **infinitely strong coupling** $\rightarrow$ **coarse lattices**

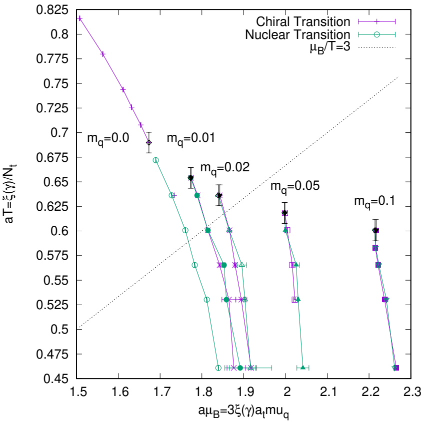
Gauge corrections for $\beta > 0$ **needed!**

The phase diagram in the strong coupling limit

Chiral and nuclear phase boundary obtained via Monte Carlo:

- at finite quark mass, the **tri-critical point** turns into Z_2 critical end point
- chiral and nuclear first order lines also match at finite quark mass





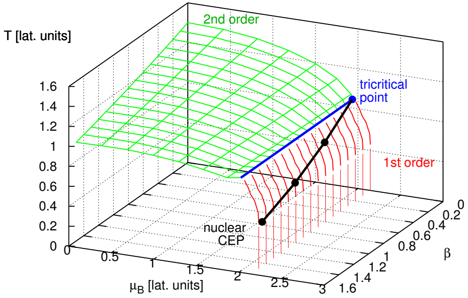
$\mu_q - T$ phase diagram in renormalized parameters [Kim & U. PoS Lattice 2016]

Goal: What does the Phase Diagram including β look like?

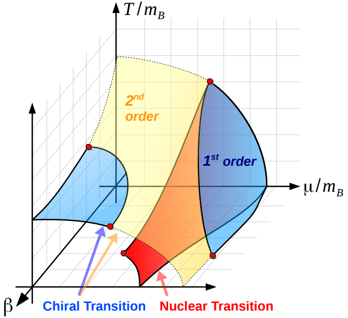
Phase Diagram in the Strong Coupling Regime and **Chiral Limit:**

- Via **reweighting** in β from $\beta = 0$: $\mathcal{O}(\beta)$ **corrections for SU(3)**

[Langelage, de Forcrand, Philipsen & U., *PRL* **113** (2014)]



[measured via Worm algorithm]



[one of serval possilbe scenarios]

Questions we want to address:

- Do the **nuclear and chiral transition split?**
- Does the **tri-critical point** move to smaller or larger μ as β is increased?

Dualization of full lattice QCD

1. combined **Taylor expansion** in the reduced gauge coupling $\tilde{\beta} \equiv \frac{\beta}{2N} = \frac{1}{g^2}$ and quark mass $\hat{m}_q$, giving rise to **dual variables**:
n_p, $\bar{n}_p$, d_ℓ, $\bar{d}_\ell$ and m_x:
$$\mathcal{Z}(\beta, \mu_q, \hat{m}_q) = \sum_{\substack{\{n_p, \bar{n}_p\} \\ \{d_\ell, \bar{d}_\ell, m_x\}}} \prod_p \frac{\tilde{\beta}^{n_p+\bar{n}_p}}{n_p! \bar{n}_p!} \prod_\ell \frac{1}{d_\ell! \bar{d}_\ell!} \prod_x \frac{(2\hat{m}_q)^{m_x}}{m_x!} \mathcal{G}_{n_p, \bar{n}_p, d_\ell, \bar{d}_\ell, m_x}$$

2. Evaluate 1-link integrals in $\mathcal{G}$ in terms of **generalized Weingarten functions**

3. decouple those integrals via a choice of orthogonal projectors

4. collect operators into a **local tensor** $T_x^{\rho^x_{-d} \cdots \rho^x_d}$ that depends on participating dual degrees of freedom $\mathcal{D}_x = \{m_x, d_{x,\pm\mu}, n_{x,\mu\nu}, \bar{n}_{x,\mu\nu}\}$

5. Final **dual partition function**:
$$\mathcal{Z}(\beta, \mu_q, \hat{m}_q) = \sum_{\substack{\{n_p, \bar{n}_p\} \\ \{k_\ell, f_\ell, m_x\}}} \sigma_f \sum_{\{\rho^x_{\pm\mu}\}} \prod_p \frac{\tilde{\beta}^{n_p+\bar{n}_p}}{n_p! \bar{n}_p!} \prod_{\ell=(x,\mu)} \frac{e^{\mu_q \delta_{\mu,0} f_{x,\mu}}}{k_\ell! (k_\ell + |f_\ell|)!} \prod_x \frac{(2\hat{m}_q)^{m_x}}{m_x!} T_x^{\rho^x_{-d}, \ldots, \rho^x_d}(\mathcal{D}_x)$$

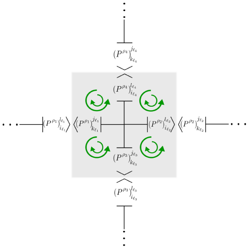
[G. Gagliardi & W. U. PRD 101, (2020) 034509]

Truncation at $\mathcal{O}(\beta^2)$: allow for plaquette occupations $(n_p, \bar{n}_p) \in \{(1,1), (2,0), (0,2)\}$

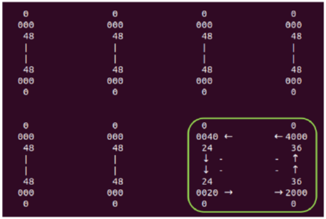
Monte Carlo for TN-Representation via Vertex Model

- Each tensor can be transformed into a vertex: weight depends on directions
- Number of distinct vertices:

limit	$\mathcal{O}(\beta^0)$	$\mathcal{O}(\beta^1)$	$\mathcal{O}(\beta^2)$	$\mathcal{O}(\beta^3)$
all	221	3485	51125	681013
chiral	176	2960	**44672**	607792
quenched	1	1	25	137



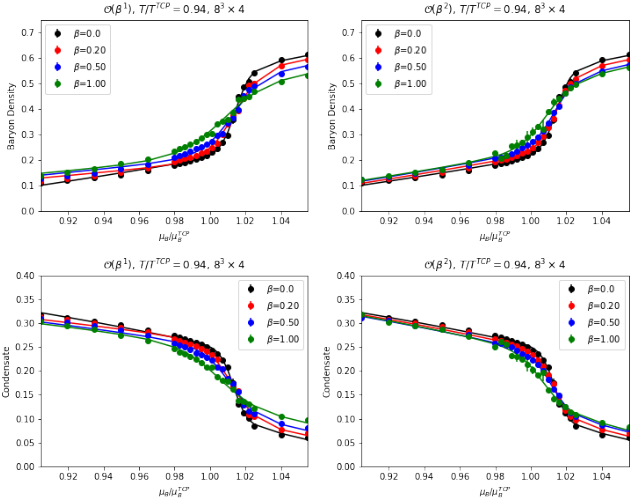
- Some vertices have negative weight, but most configurations are positive after contraction
- Use heatbath algorithm for to modfiy vertices along closed countours; has been parallelized; Worm algorithm not yet applicable beyond $\mathcal{O}(\beta)$
  [Pattanaik & U. PoS Lattice (2023)]



- At $\mu_B = 0$: crosschecked with HMC
- Lattice Setup: 8^3x4, 12^3x4 and $16^3 \times 4$, SC, and $\mathcal{O}(\beta)$, $\mathcal{O}(\beta^2)$ for $\beta = [0.0, \ldots, 1.0]$ and at $T = 0.8, 0.85, 0.9, 0.95, 1.0$, all for chiral limit

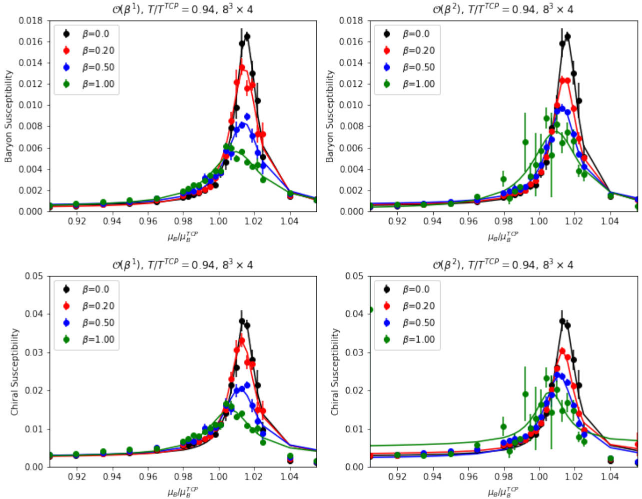
Results: Baryon Density and Chiral Condensate

- All results relative to the location of the strong coupling tricritical point: $T^{TCP}_{N_\tau=4} = 0.85$, $\mu_B{}^{TCP}_{N_\tau=4} = 1.99$



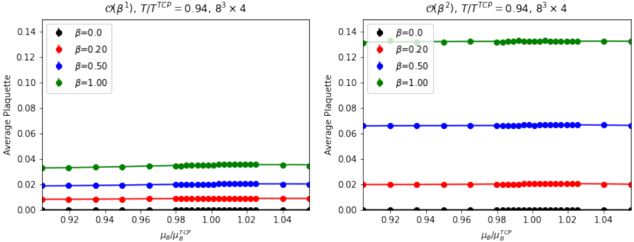
Results: Baryon Susceptibility, Chiral Susceptibiltiy

- All results relative to the location of the strong coupling tricritical point: $T_{N_\tau=4}^{TCP} = 0.85$, $\mu_B{}_{N_\tau=4}^{TCP} = 1.99$



Results: Average Plaquette

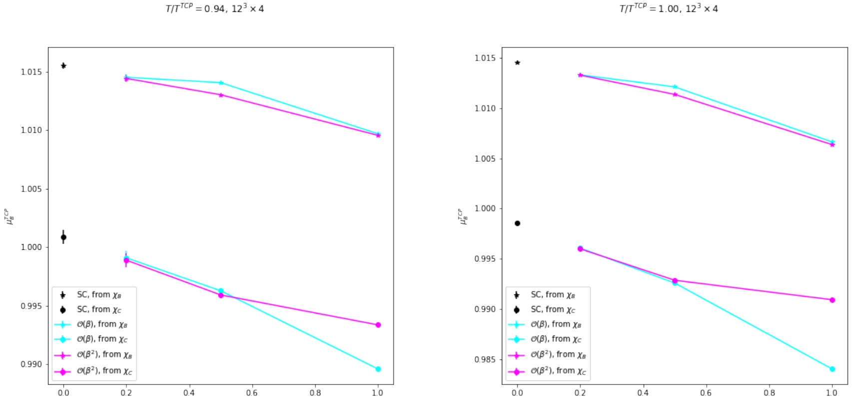
- All results relative to the location of the strong coupling tricritical point: $T^{TCP}_{N_\tau=4} = 0.85$, $\mu_B{}^{TCP}_{N_\tau=4} = 1.99$



- Average plaquette and its susceptibiltiy: no imprint of the chiral/nuclear transition

Preliminary Result on the chiral TCP/nculear CEP

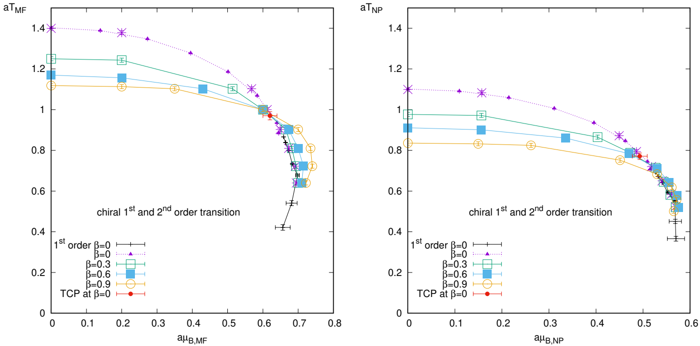
- To compare $\mathcal{O}(\beta)$ with $\mathcal{O}(\beta^2)$: restricted to rather small lattice volume



- Extrapolation to thermodynamic limit requires larger volumes: expensive for $\mathcal{O}(\beta^2)$
- Does $\mu_B^{TCP}(\beta)$ move to smaller values? Not quite...

Phase Diagram in the Strong Coupling Regime $\beta > 0$

- Still required for $\mathcal{O}(\beta^2)$: **anisotropy** $\frac{a_s}{a_t} \equiv \xi(\gamma, \beta)$ at finite β
- Taking into account the β-dependent renormalization of aT and $a\mu_B$:



- Back-bending vanished
- **Even weaker (no?) β-dependence** after renormalization: $aT \mapsto \xi(\gamma,\beta) aT$, $a\mu_B \mapsto \xi(\gamma,\beta) a\mu_B$

Conclusions

Results:

- Dual representation established that is in principle not truncated in β
- Caveat: it re-introduces the sign problem gradually with β
- Still at $\mathcal{O}(\beta^2)$: TCP has weak dependence on β up to $\beta \lesssim 1$
- If TCP remains invariant for higher orders in β: CEP also exists in the continuum!

Prospects:

- Character expansion with staggered fermion feasible??? SC regime up to $\beta \simeq 6$?
- Connect $\mathcal{O}(\beta^2)$ results to nuclear liquid gas transition at finite quark mass, low T
  [Kim, Pattanaik & U. PRD 107 (2023)]
- Strong Coupling LQCD on a quantum annealer allows very low T
  [Luu, Kim & U. PRD 108 (2023)]
- Extending the Hamiltonian formulation of LQCD to $\beta > 0$ and/or $N_{\mathrm{f}} = 2$,
  well suited for quantum simulations:
  $\rightarrow$ Required QC Resources: [talk by Michael Fromm (Tue 16:35)]