# Complex Langevin simulations of QCD: the effects of dynamical stabilization

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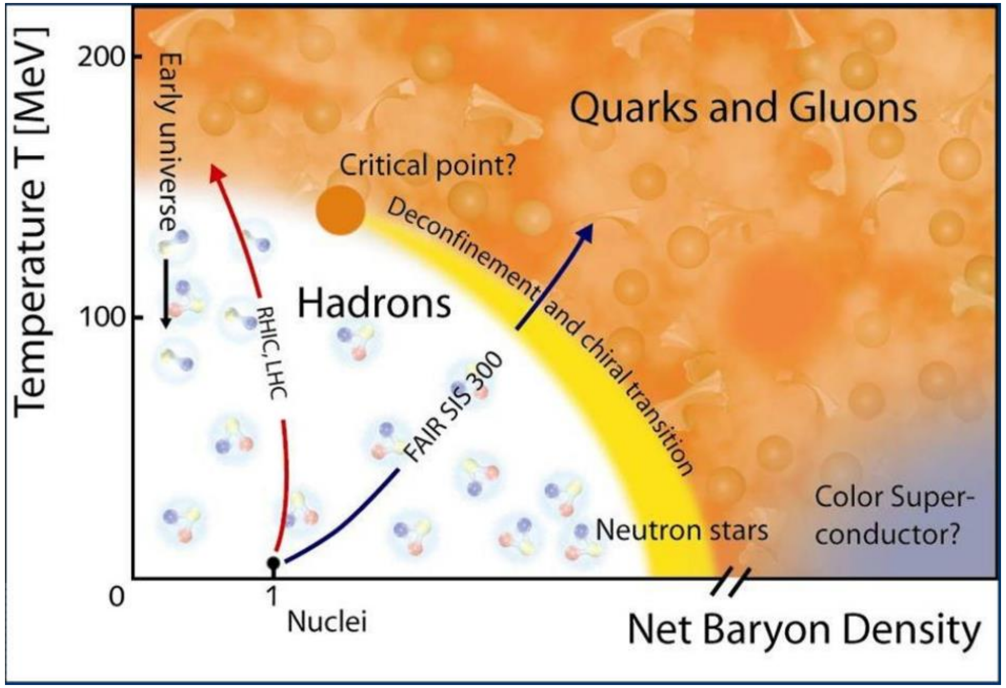
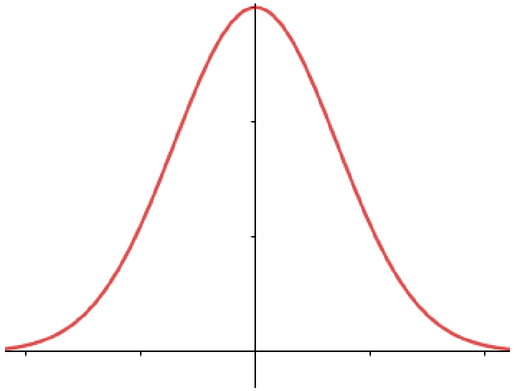
# Motivation

- The sign-problem
  - Non-zero chemical potential, implies complex action, which breaks the importance sampling algorithms
- "Solutions"
  - Reweighting HMC data
  - Complex Langevin



# Complex Langevin

- Doesn't use acceptance probabilities, as it is a stochastic process.
- $dx = \mu(x)dt + \sigma(x)dW$
  - With $\mu(x)$ being the drift, and $\sigma(x)$ the diffusion
- From Fokker-Planck we have
  - $\partial_t \rho(x,t) = \partial_x\big(\partial_x - \mu(x)\big)\rho(x,t)$
  - $dx = -\partial_x S(x)\, dt + \sqrt{2} dW$
- Can be simulated as an SDE, using Euler-Maruyama



# Updating Gauge-links using CLE

- Using an exponential Euler-Maruyama scheme, to keep the determinants of the links 1.

$$U_\mu^{\tau+\epsilon}(x) = \exp\left[i\lambda_a\left(\epsilon K_{\mu a}(x) + \sqrt{\epsilon}\,\eta_{\mu a}(x)\right)\right] U_\mu^{\tau}(x)$$

- When the drift-term becomes complex, rather than real, it shifts the links from SU to SL
- To prevent this drift, we introduce gauge invariant forces, such as Gauge-Cooling
- But it is often not enough, so to further prevent this, we introduce an extra force

# Dynamical stabilization **arxiv: 1808.04400 (B. Jäger and F. Attanasio)**

- Adding an extra force to slow/stop the drift from SU to SL.

$$K_{a\mu}(x) \to K_{a\mu}(x) + i\alpha_{DS} M_{a\mu}(x)$$

$$M_{a\mu}(x) = b_{a\mu}\left(\sum_c b_{c\mu}(x) b_{c\mu}(x)\right)^3$$

$$b_{a\mu}(x) = Tr\left[\lambda_a U_\mu(x) U_\mu^\dagger(x)\right]$$

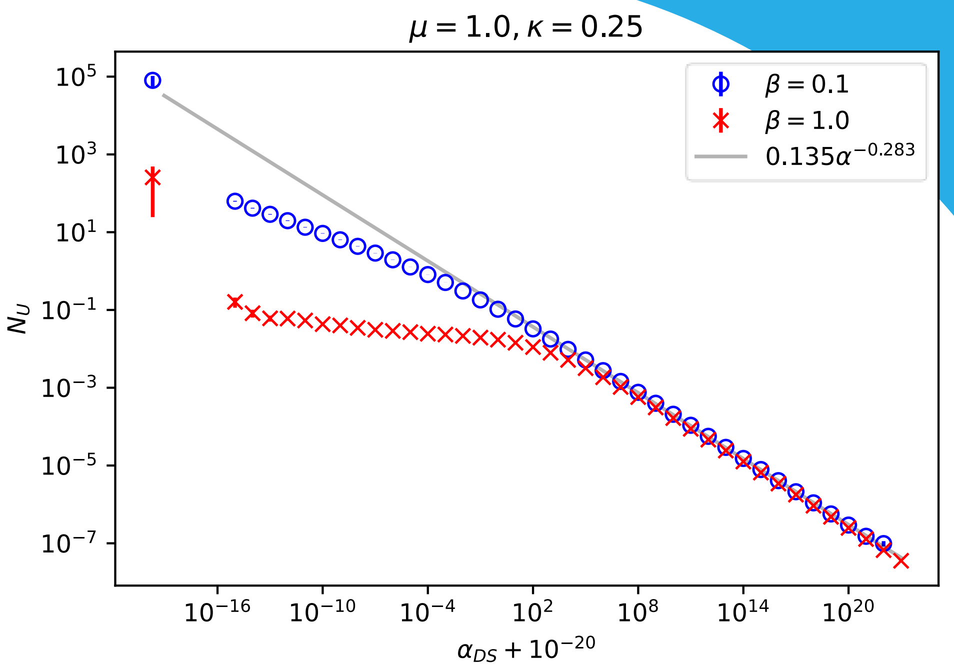
- Introduces non-holomorphic force to action, which induces incorrectness to simulations
- But this incorrectness is "predictable"

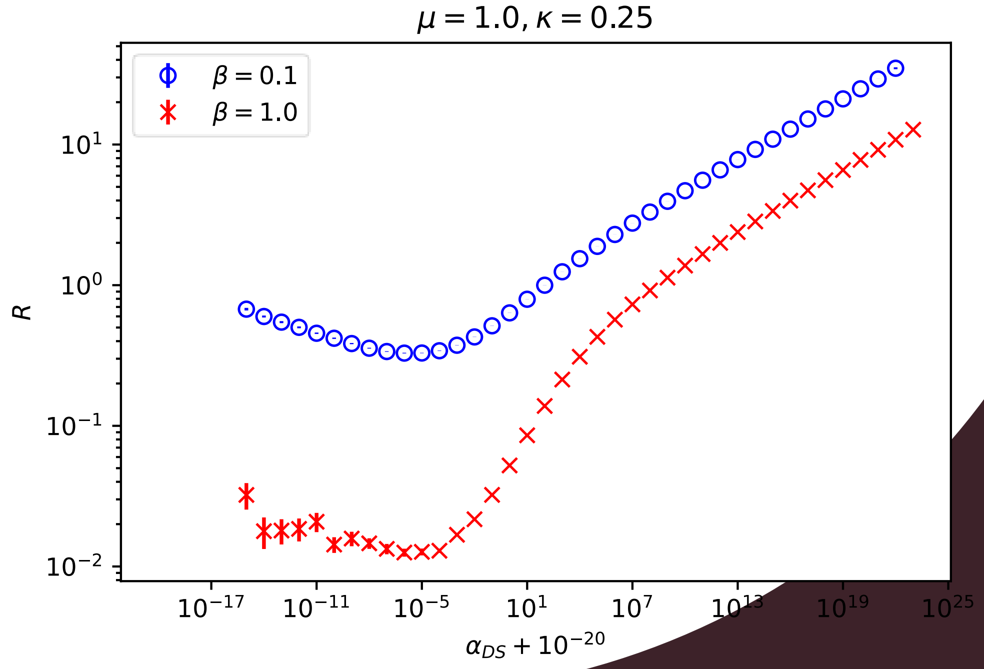
# 1- Link test model

- $-S(U) = \beta_1 Tr(U) + \beta_2 Tr(U^{-1}),\ U \in SU(3)$
- With $\beta_1 = \beta + \kappa e^{\mu},\ \ \beta_2 = \beta + \kappa e^{-\mu}$
- Unitarity norm

$$N_U = Tr\left(U^{\dagger}U\ -1\right)^2$$

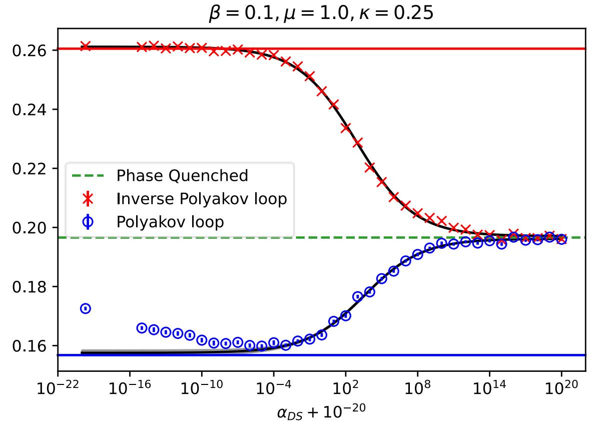
- As the force is increased, the Links gets closer to unitarity, as we wanted

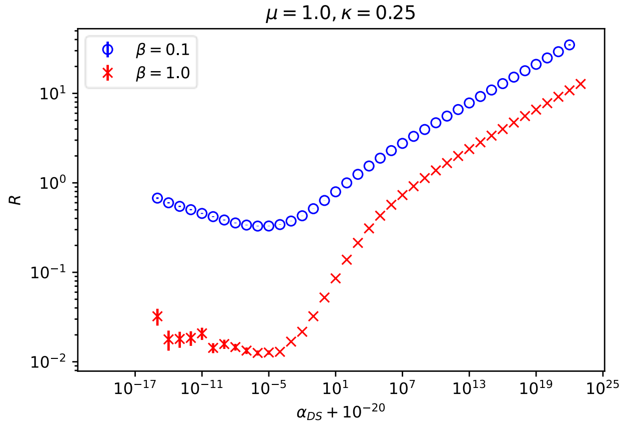


# Fitting the data

- “Predictable” behavior with DS
  - The sigmoidal function fits well
  - $f(\alpha) = A + \dfrac{B - A}{1 + C\,\alpha^D}$
- Very similar results to QCD simulations
- DS limit goes to Phase Quenched(Only real part of action remains $e^{-Re(S(x))}$)
- PQ simulations are “easy” and one fit-parameter can essentially be replaced

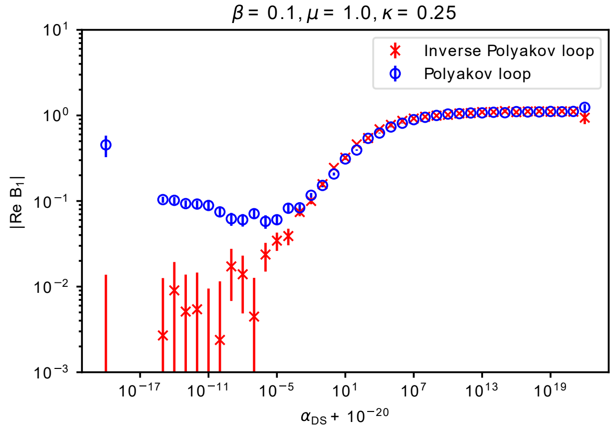


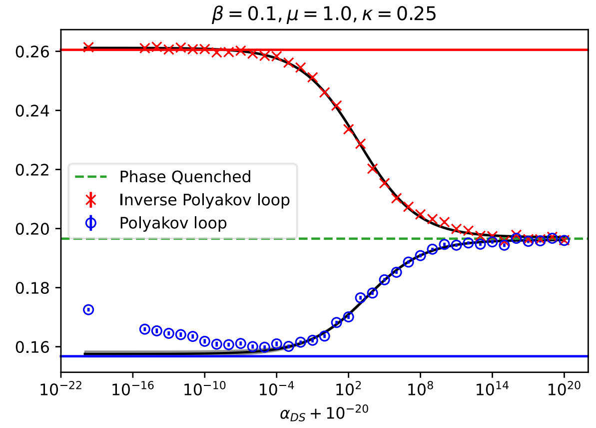



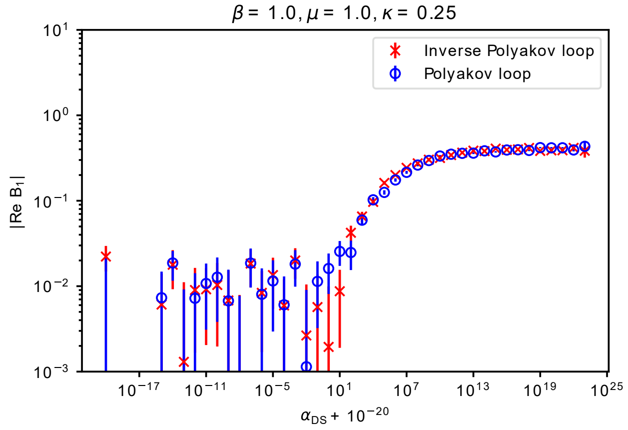
# Results in toy-model

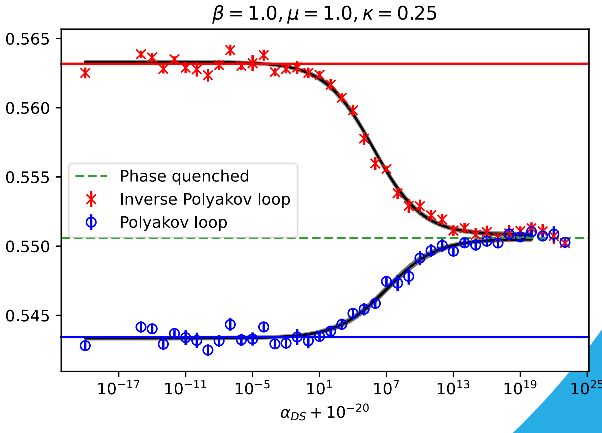
- Results are fitted nicely, with $\frac{\chi^2}{N_{dof}}$ between 1 and 2
- Boundary terms behaves nicely and shows incorrect results for large DS
- Low temperature also has “slight” incorrectness at low DS

| | $\langle U \rangle$ | $\langle U^{-1} \rangle$ |
|---|---|---|
| exact | 0.15744528 | 0.26051165 |
| extrapolated | 0.15756(94) | 0.26113(25) |
| CLE $\alpha_{\mathrm{DS}} = 0$ | 0.17254(61) | 0.26139(62) |

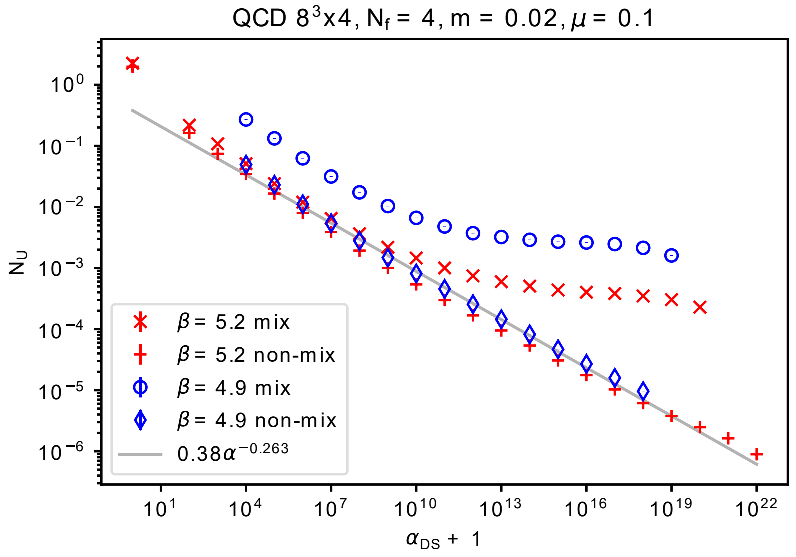


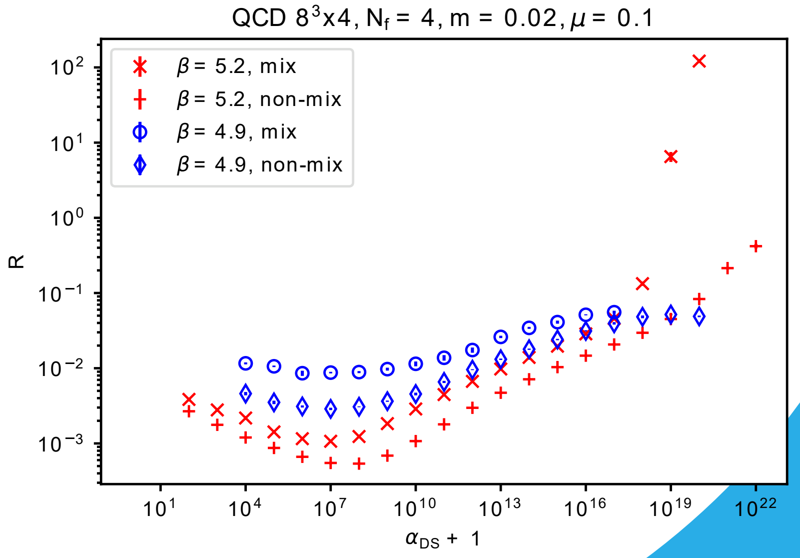



# For full QCD simulations




- Two types of DS tested

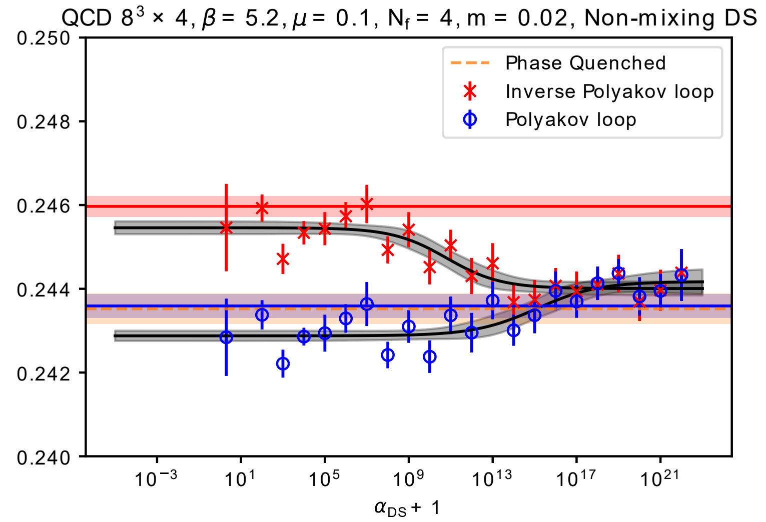
  **Sum over directions**

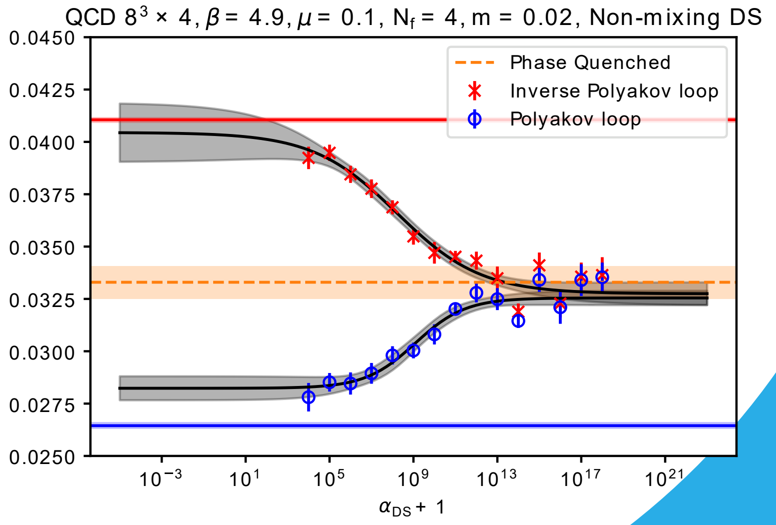
  **Keeping directions separate**

- First checking behavior of the unitarity norm
- Minimum in $R$

# For full QCD simulations

- Checking the Polyakov
- Good comparison to the test model
- Good fit to data
- Low DS simulations are unstable, but would (most likely) improve extrapolation, as seen by the toy model



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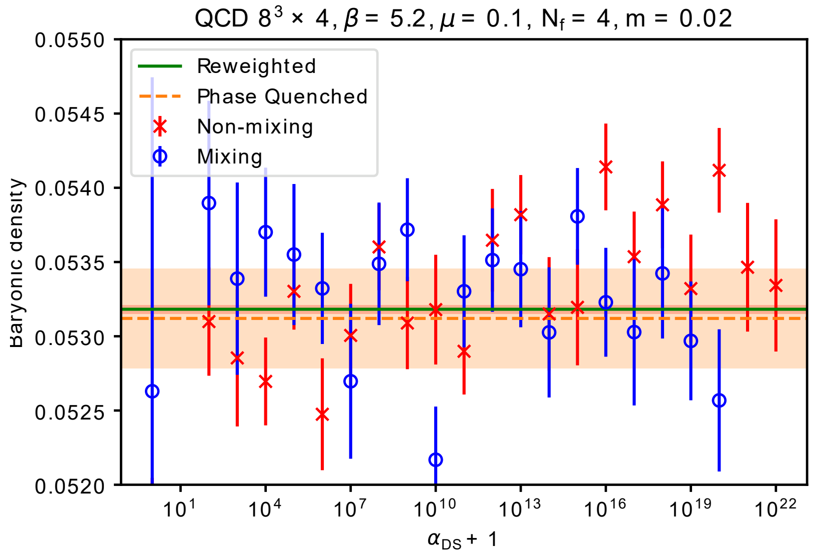


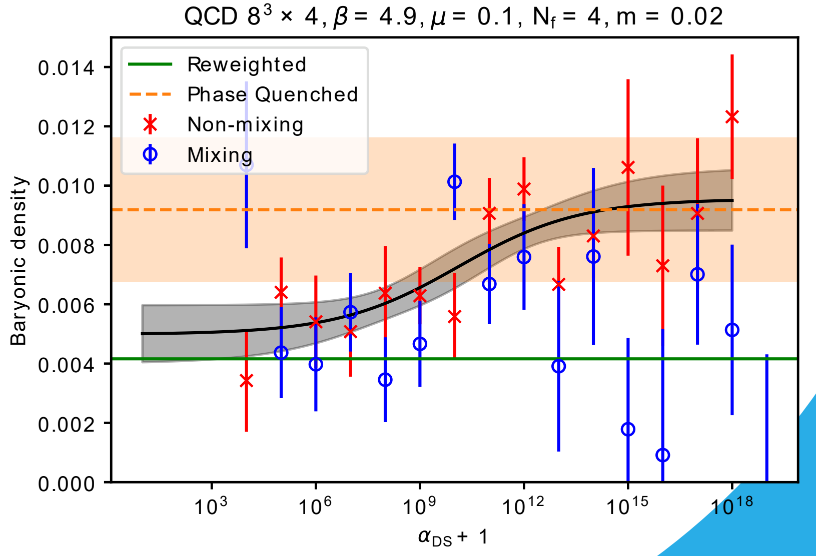


- Other observables
  - Correct results are very close to PQ-values, and therefore difficult to separate between
  - Low DS simulations are unstable, but would (most likely) improve extrapolation

# Conclusion

- CL gives very good results in the phase quenched region
- For low temperature simulations, Dynamical Stabilization helps stabilize simulations, and can be used to extrapolate to correct results (with enough data)
- For high temperature simulation, it is not needed to use dynamical stabilization, as the difference is within error
- For more info: arxiv2405.20709

# Reweighting for comparison

- Change the weights

$$\langle x\rangle_w = \frac{\sum w_i x_i}{\sum w_j} = \frac{\sum w_i x_i \frac{w_i'}{w_i'}}{\sum w_j \frac{w_j'}{w_j'}} =$$

$$\frac{\sum w_i' x_i \frac{w_i}{w_i'}}{\sum w_j' \frac{w_j}{w_j'}} = \frac{\left\langle x \frac{w}{w'}\right\rangle_{w'}}{\left\langle \frac{w}{w'}\right\rangle_{w'}}$$

- Used in HMC, to simulate non-zero chemical potential

$$\left\langle \frac{w}{w'} \right\rangle = \left\langle \frac{\det M(\mu)}{\det M(\mu = 0)} \right\rangle = \exp\left(-\frac{V}{T}\Delta F(\mu, t)\right)$$

- Large $\mu \Rightarrow \left\langle \frac{w}{w'} \right\rangle$ goes towards zero

# Full QCD boundary terms