Search for a Lee-Yang edge singularity in high-statistics Wuppertal-Budapest data

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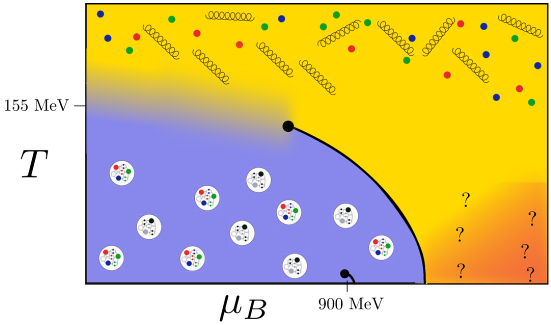
Introduction



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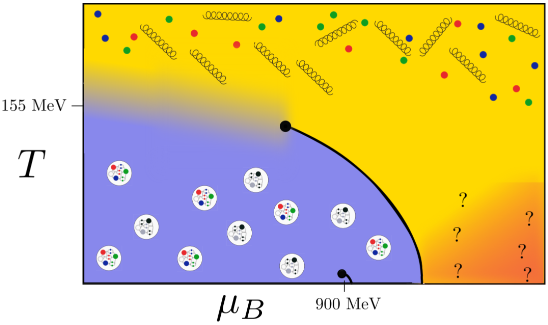

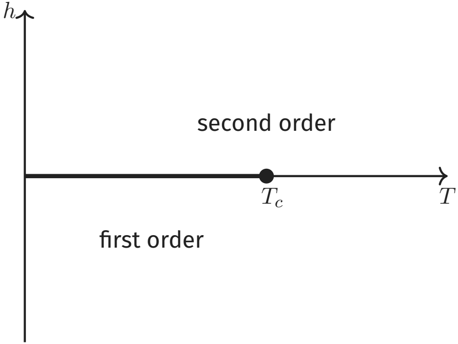

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- Lee-Yang zeros (LYZ) and Lee-Yang edge singularity [Lee,Yang'59]

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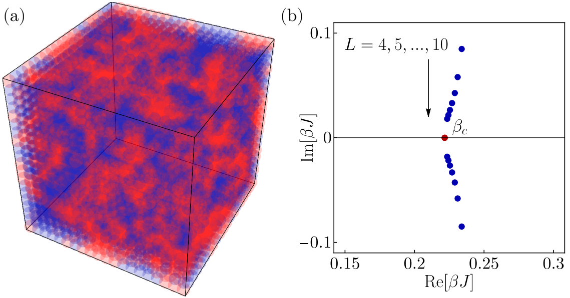

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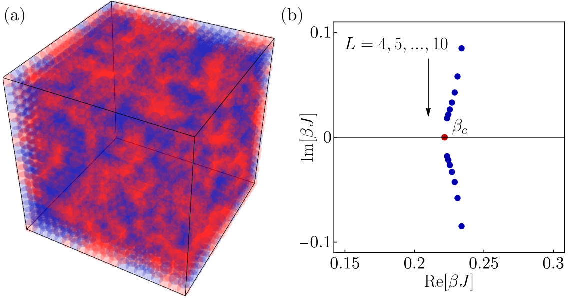

- a small excerpt of other work by others:
    - Parma Bielefeld [2405.10196]
    - Simran Singh PhD Thesis
    - Gökçe Başar [2312.06952]
    - Giordano, Pásztor [1904.01974]
    - Mukherjee, Skokov [1909.04639]
    - Wakayama et al. [1802.02014]

What we work with

- To access zeros of Z, we can look at $\log(Z) = p$
- Written as a Tayor series

$$\Delta p = \frac{p(T,\mu_B) - p(T,0)}{T^4} = \sum_{n=0} \frac{\chi_{2n}(T)}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_n = \frac{\partial^n (p/T^4)}{(\partial \mu/T)^2}$$

- χ_{2n} is given by simulations at $\mu_B = 0$
- These coefficients can be used in conjunction with a scaling relation for extrapolation
- In addition we can model:

$$\chi_1(T,\mu_B) = \sum_{n=1} \frac{\chi_{2n}(T)}{(2n-1)!} \left(\frac{\mu_B}{T}\right)^{2n+1} \qquad \chi_2(T,\mu_B) = \sum_{n=0} \frac{\chi_{2n+2}(T)}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

Padé

$$T_l(x) = \sum_{i=0}^{l} c_i x^i \qquad \text{Pade}[m,n] : \frac{P_m(x)}{1+Q_n(x)} = \frac{\sum_{i=0}^{m} a_i x^i}{1+\sum_{i=1}^{n} b_i x^i}$$

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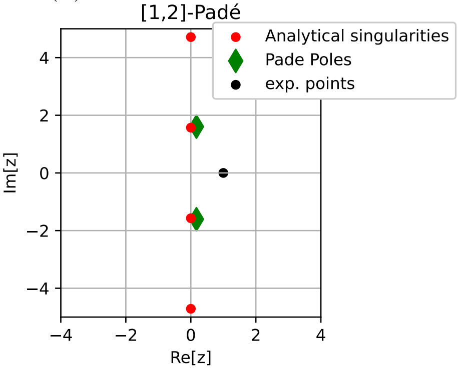

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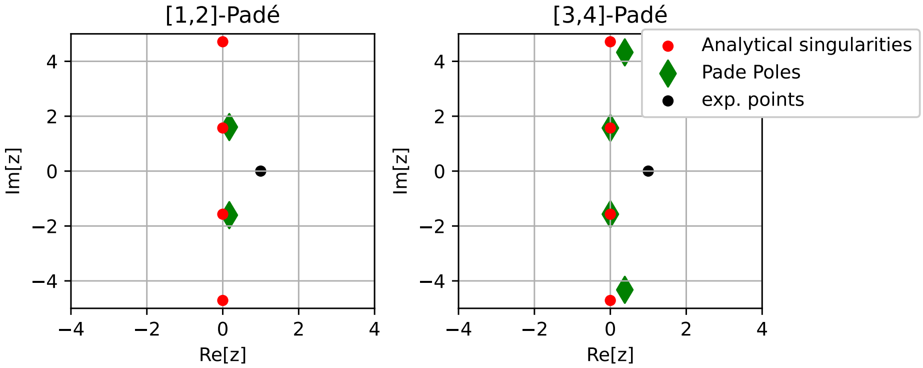
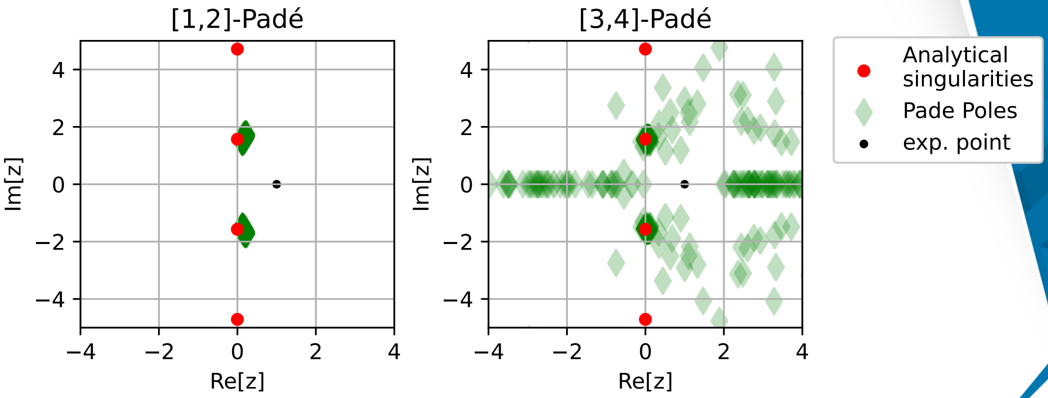


Padé with Noise

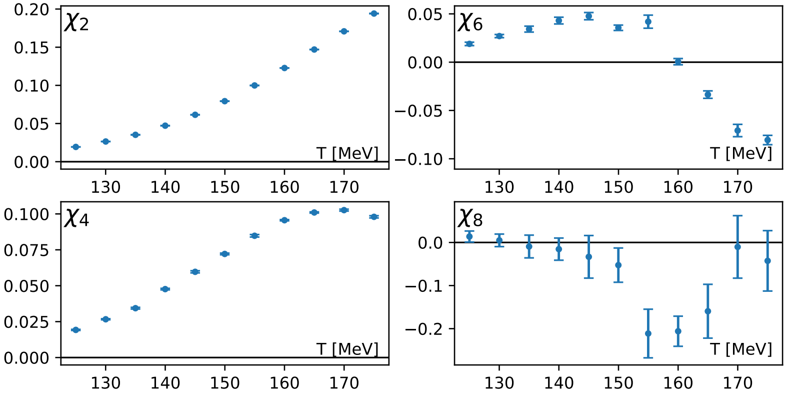
- Adding 3% of noise to each of the derivatives of $1/\cosh(z)$:



Results

Lattice Setup

- Volume : $16^3 \times 8$
- $\mathcal{O}(5 \cdot 10^5)$ configurations per T
- 4 HEX smearing
- Simulated at physical quark mass



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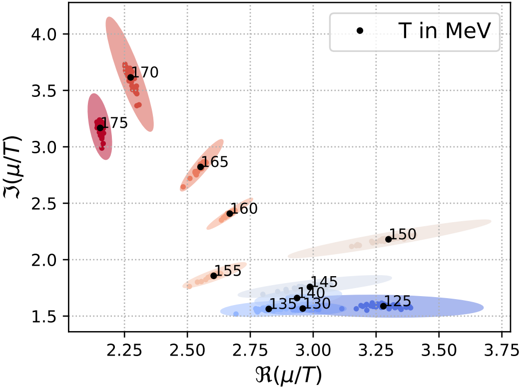

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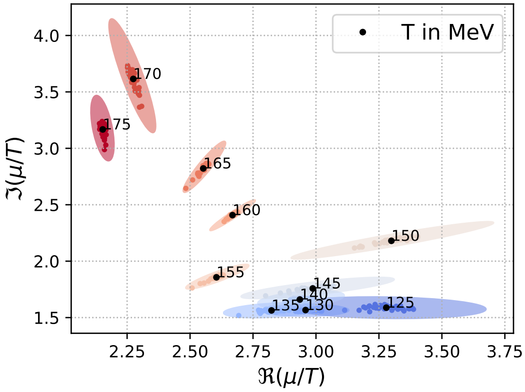

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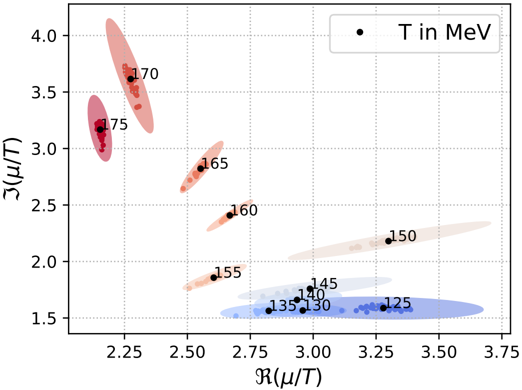
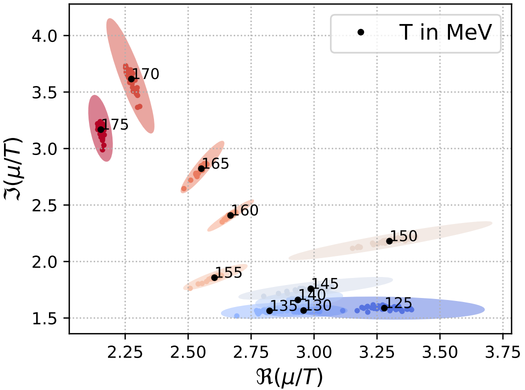
- Estimate systematic effects
  - Use Δp or χ_1 or χ_2
  - Vary fit range in temperature
  - Use different scaling ansatz



modeling χ_2

Varying the approximated function



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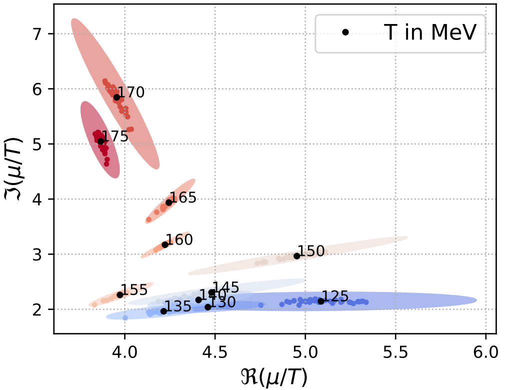
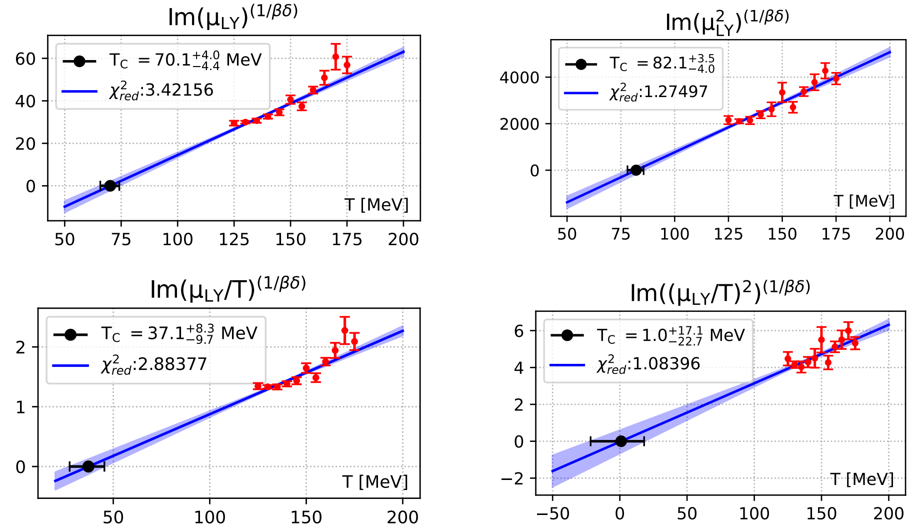
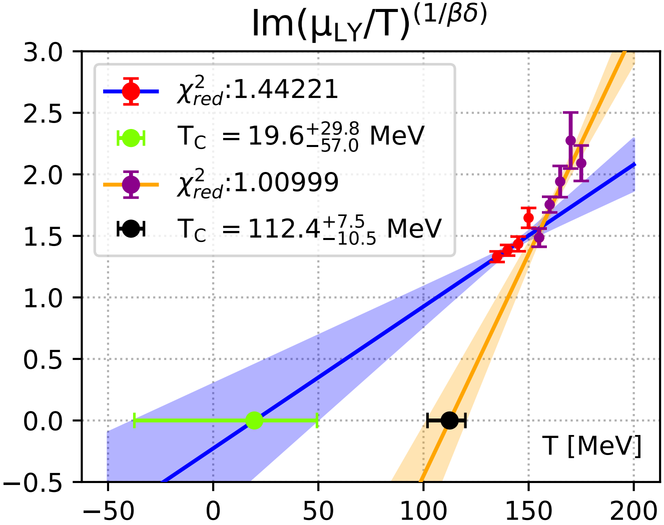


modeling $p - p_0$

Varying the scaling variable for χ_2 : $\kappa \Delta T = \text{Im}(x)^{1/\beta\delta}$

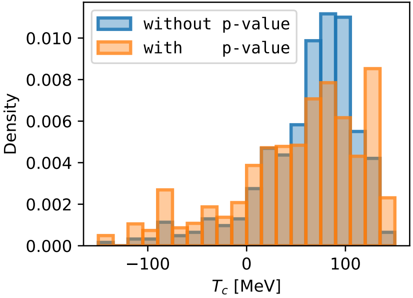


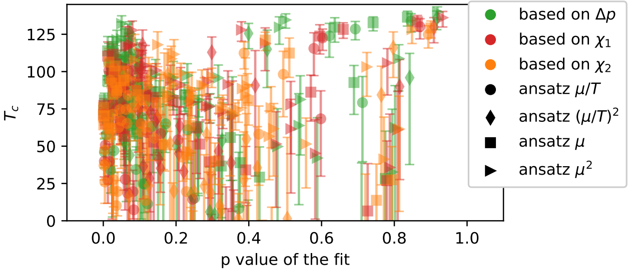
Varying the fit range for χ_2



Combination

Based on $3[\Delta p, \chi_1, \chi_2] \times 4\left[\mu, \mu^2, \frac{\mu}{T}, \frac{\mu^2}{T^2}\right] \times 36$ Temp. ranges $=\ 432$ Fits





Conclusion

- We used a high statistics campaign to look for the Lang Yee Zeros
- We can estimate T_c with a reasonable statistical error but a high systematic error
- No systematic control of the Padé order
- Approximation from a great distance requires strong assumptions

- Reliable prediction of the CEP with LYZ from the lattice data requires great care and consideration

Backup

Example for the susceptibility

$$(a_1 + a_2\mu^2) = (\chi_2 + \chi_4\mu^2 + \chi_6\mu^4 + \chi_8\mu^6)(1 + b_1\mu^2 + b_2\mu^4)$$

$$\left.\frac{\mathrm{d}^0}{\mathrm{d}\mu^0}\right|_{\mu=0} : \qquad a_1 = \chi_2$$

$$\left.\frac{\mathrm{d}^2}{\mathrm{d}\mu^2}\right|_{\mu=0} : \qquad 2a_2 = 2b_1\chi_2 + 2\chi_4$$

$$\left.\frac{\mathrm{d}^4}{\mathrm{d}\mu^4}\right|_{\mu=0} : \qquad 0 = 24(b_2\chi_2 + b_1\chi_4 + \chi_6) \iff \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \chi_2 & 0 \\ 0 & 0 & \chi_4 & \chi_2 \\ 0 & 0 & \chi_6 & \chi_4 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \chi_2 \\ \chi_4 \\ \chi_6 \\ \chi_8 \end{pmatrix}$$

$$\left.\frac{\mathrm{d}^6}{\mathrm{d}\mu^6}\right|_{\mu=0} : \qquad 0 = 720(b_2\chi_4 + b_1\chi_6 + \chi_8)$$

pressure vs the density vs the susceptibility

$$\Delta p(T): \qquad \frac{a_1\mu^2 + a_2\mu^4}{1 + b_1\mu^2 + b_2\mu^4} \stackrel{!}{=} \sum_{i=1} \chi_{2i}\mu^{2i}$$

$$\chi_1(T): \qquad \frac{a_1\mu^1 + a_2\mu^3}{1 + b_1\mu^2 + b_2\mu^4} \stackrel{!}{=} \sum_{i=0} \chi_{2i+1}\mu^{2i+1}$$

$$\chi_2(T): \qquad \frac{a_1\mu^0 + a_2\mu^2}{1 + b_1\mu^2 + b_2\mu^4} \stackrel{!}{=} \sum_{i=0} \chi_{2i+2}\mu^{2i+2}$$

$$\left.\frac{\mathrm{d}^{2n}}{\mathrm{d}\mu^{2n}}\right|_{\mu=0} (a_1 + a_2\mu^2) = \left.\frac{\mathrm{d}^{2n}}{\mathrm{d}\mu^{2n}}\right|_{\mu=0} (\chi_2 + \chi_4\mu^2 + \chi_6\mu^4 + \chi_8\mu^6)(1 + b_1\mu^2 + b_2\mu^4)$$

Taylor and Pade for $f(x) = 1/\cosh(x)$

$$T_l(x) = \sum_{i=0}^{l} c_i x^i \quad R_n^m(x) = \frac{P_m(x)}{1 + Q_n(x)} = \frac{\sum_{i=0}^{m} a_i x^i}{1 + \sum_{i=1}^{n} b_i x^i}$$

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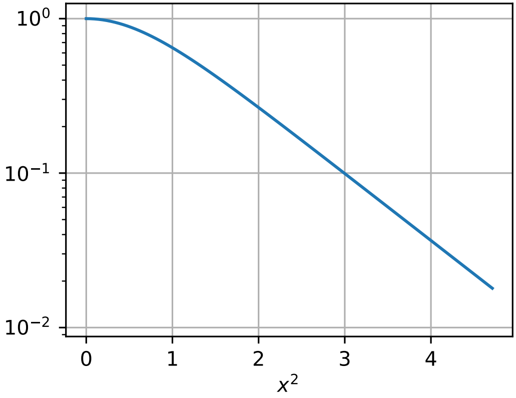

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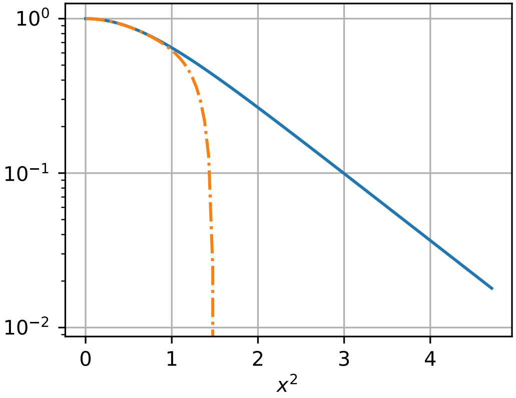
$$\frac{1}{\cosh(x)} \approx 1 - \frac{x^2}{2} + \frac{5x^4}{25} - \frac{61x^6}{720}$$



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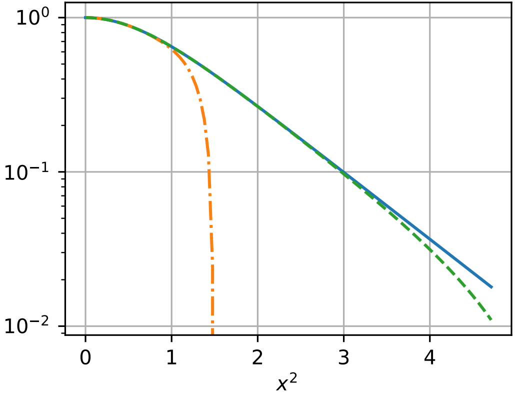
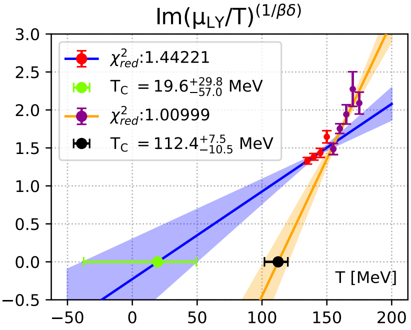
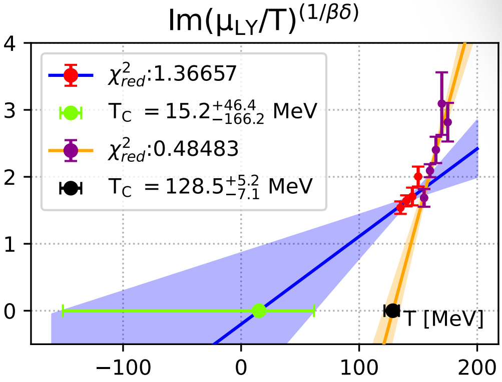
$$\frac{1}{\cosh(x)} \approx \frac{1 - \frac{1}{30}x^2}{1 + \frac{7}{15}x^2 + \frac{1}{40}x^4}$$



Comparing χ_2 & Δp approaches in fit range

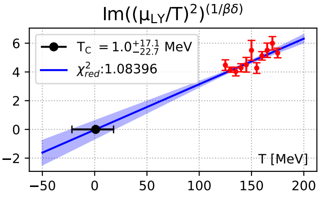


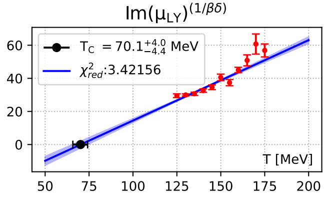
using χ_2



using Δp

Comparing χ_2 & Δp approaches in fit model





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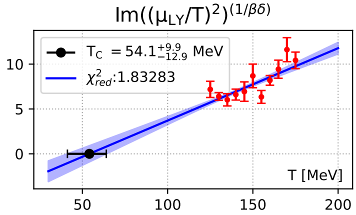

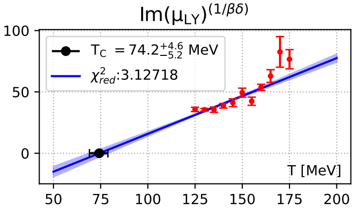

using Δp

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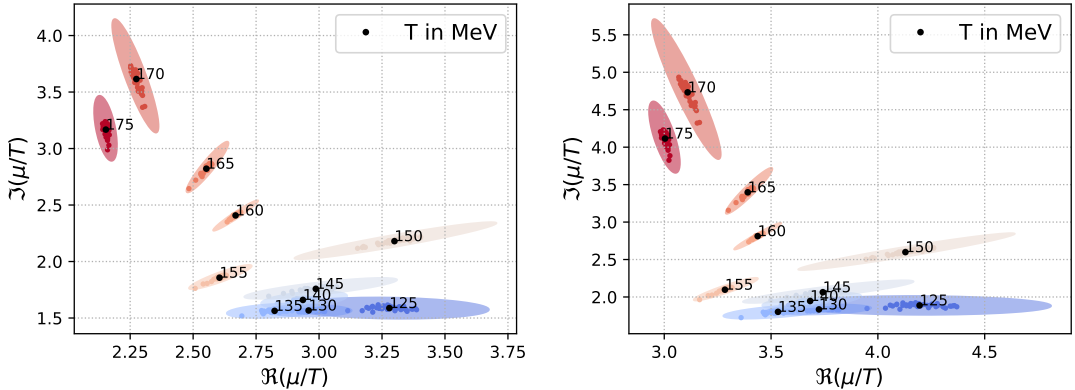
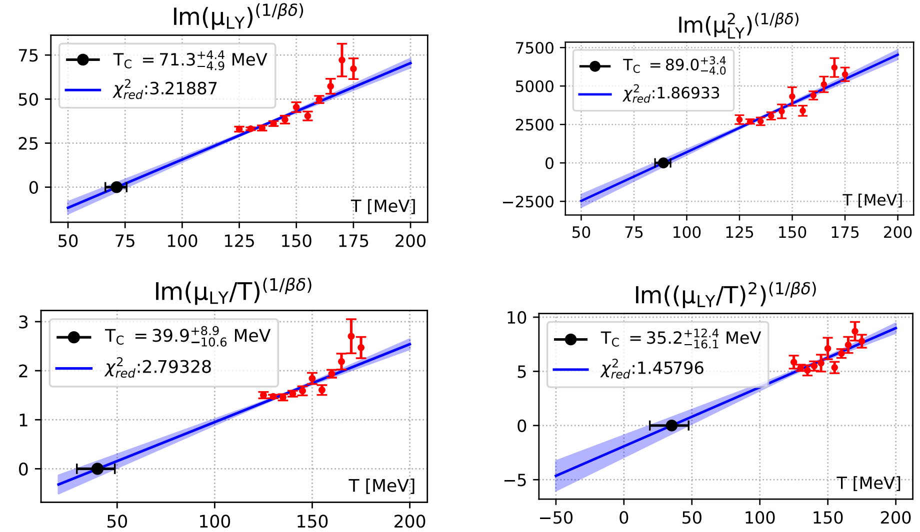


modeling χ_2 modeling χ_1

Varying the scaling variable for χ_1



Varying the fit range for χ_1