Continuum extrapolated high order baryon fluctuations

Phys.Rev.D 110 (2024) 1 [Borsanyi:2023wno]

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August 2nd 2024

BERGISCHE UNIVERSITÄT WUPPERTAL

WB collaboration

BMW collaboration

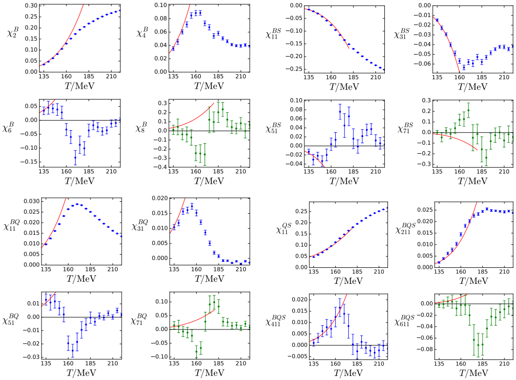
NRW-FAIR

Fluctuations on the lattice

$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}\,, \quad \hat{\mu}_i = \frac{\mu}{T}$$

Today:

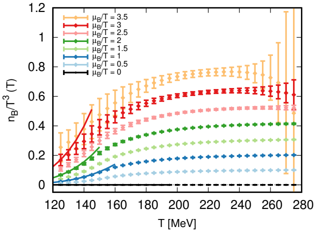
$$\chi^B_i = \frac{\partial^i(p/T^4)}{(\partial\hat{\mu}_B)^i}$$



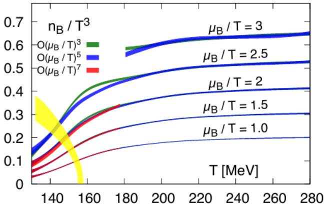
[Borsanyi:2018grb]

Why fluctuations?

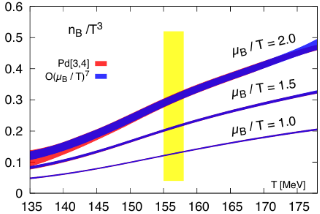
The Equation of State from fluctuations



[Borsanyi:2021sxv], [Borsanyi:2022qlh]
From determining higher order corrections from the shift with $\mathrm{i}\hat{\mu}_B$



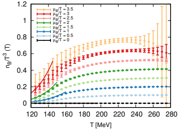
[Bollweg:2022fqq]
From Taylorexpansion



[Bollweg:2022rps]
From Padé approximants

Why fluctuations?

1. Extrapolation of Equation of State to finite $\mu \longrightarrow$ important for heavy ion collision phenomenology



Looking for criticallity with fluctuations

Experimental signature for critical end-point: non-monotonic behavior of $\frac{\chi_4^B}{\chi_2^B}(\mu_B)$ ([Stephanov:2011pb], [Mroczek:2020rpm])

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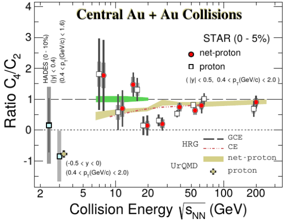

[STAR:2021fge]

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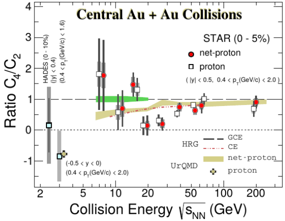

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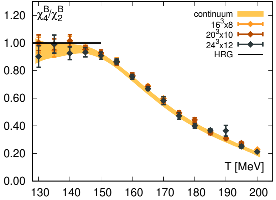

[Borsanyi:2023wno]

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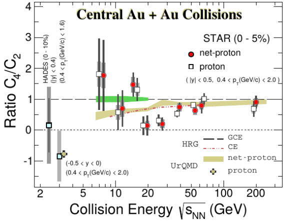

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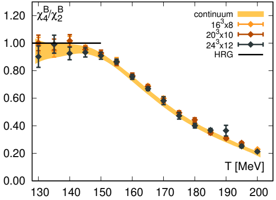
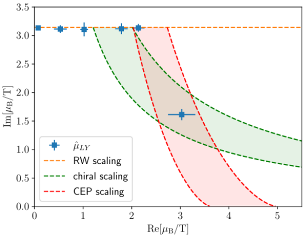
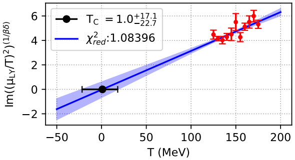


[Borsanyi:2023wno]

Search for the critical endpoint with lattice QCD by looking at Lee-Yang-Zeros (for example [Giordano:2019slo], [Mukherjee:2019eou], [Giordano:2019gev], [Basar:2021hdf])



[Dimopoulos:2021vrk]

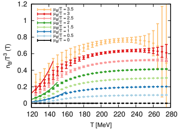


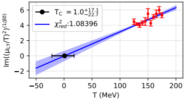
Talk at 12:15 by Tatsuya Wada and 12:35 by Alexander Adam

Why fluctuations?

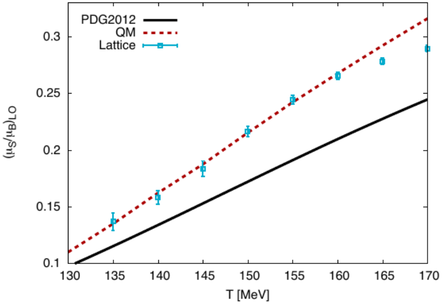
1. Extrapolation of Equation of State to finite $\mu \longrightarrow$ important for heavy ion collision phenomenology

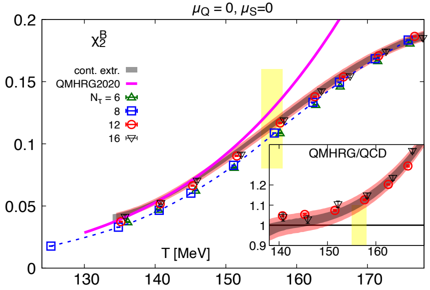
2. Sensitive to criticallity both in experiment and theory





Resonances from fluctuations





[Alba:2017mqu]
see also: [Majumder:2010ik], [Bazavov:2014xya]

[Bollweg:2022rps]

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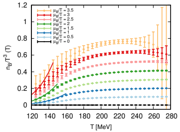

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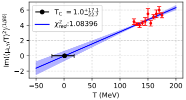
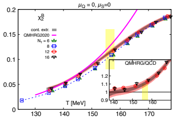
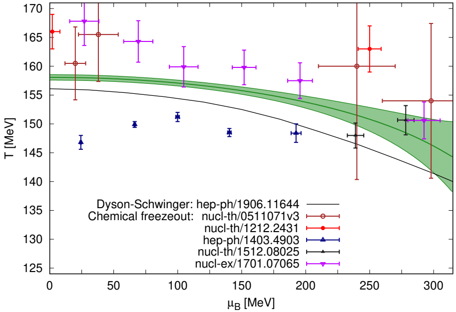


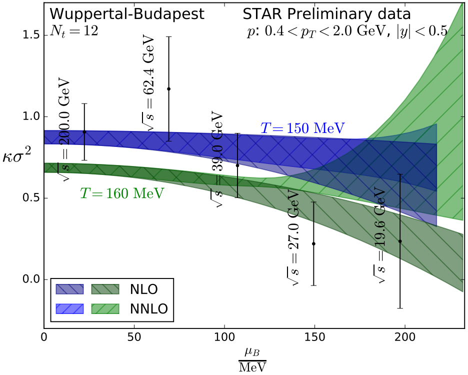
3. Can be used to search for new resonances in the Hadron spectrum



Fluctuations and Heavy Ion collision experiments



[Borsanyi:2020fev]



[Borsanyi:2018grb]

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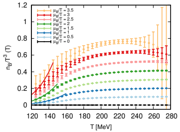
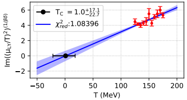
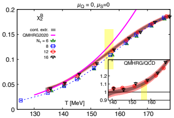
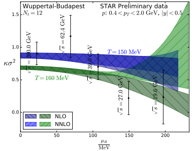
4. Comparison to freeze-out physics in heavy ion collisions

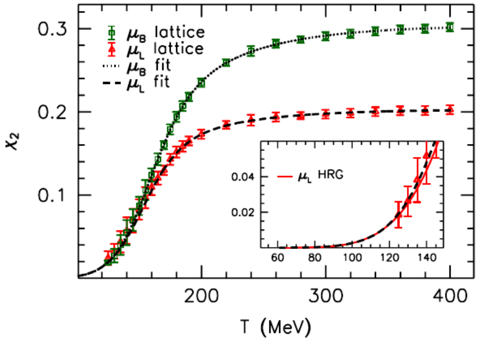




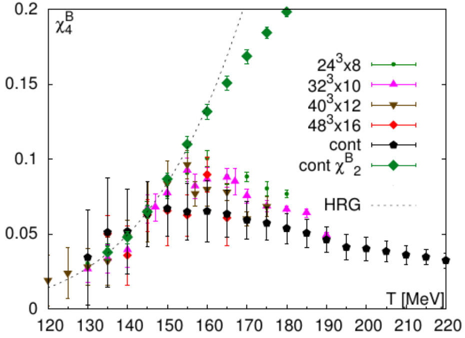




χ_2^B and χ_4^B in the continuum

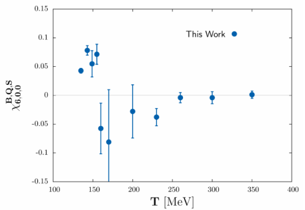


[Borsanyi:2012cr]

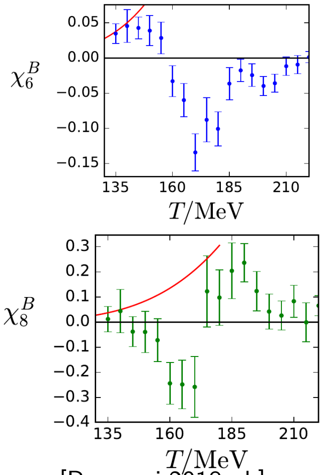


[Bellwied:2015lba]

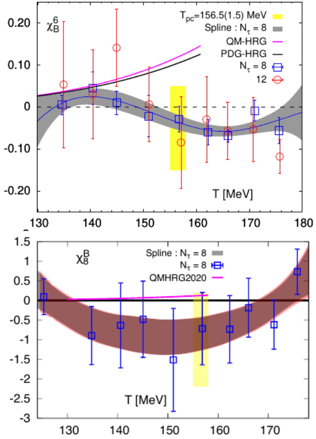
χ_6^B and χ_8^B on finite lattices



[DElia:2016jqh]
$N_t = 6$, 2stout

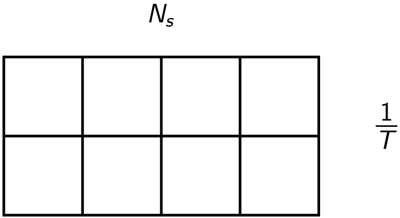


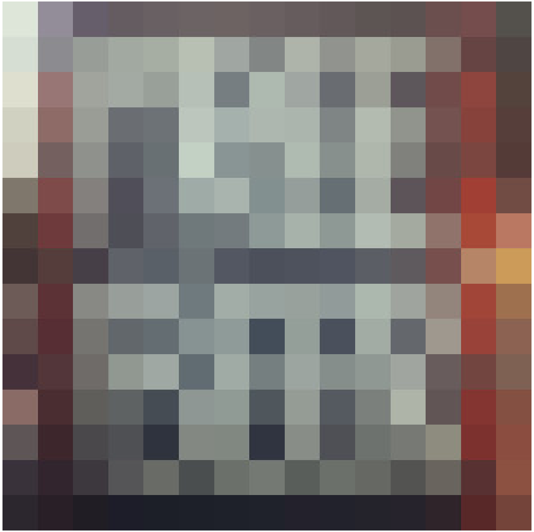
[Borsanyi:2018grb]
$N_t = 12$, 4stout



[Bazavov:2020bjn],
[Bollweg:2022rps]
$N_t = 8$, HISQ

The continuum limit





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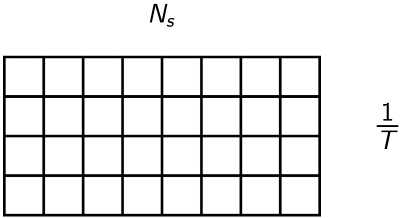

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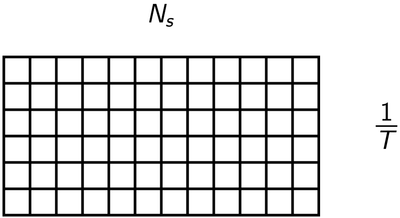
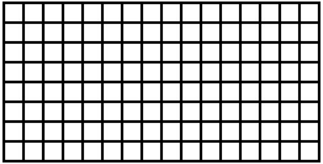


The continuum limit

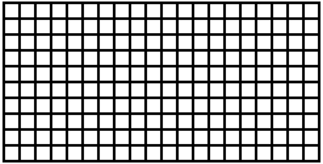
N_s



$\frac{1}{T}$

The continuum limit

N_s



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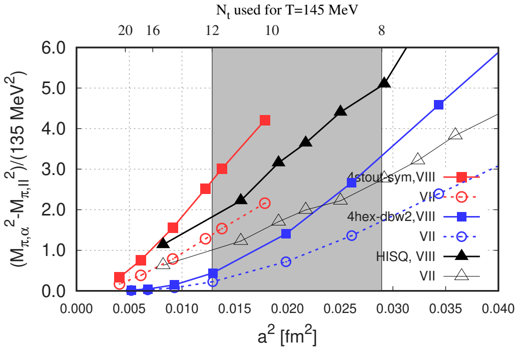
Lattice set-up



- 4Hex + dbw2 action
- lattices: $16^3 \times 8$, $20^3 \times 10$, $24^3 \times 12$
- $\mu_S = 0$
- scale setting with f_π and w_1
- Exponential definition of the chemical potential (introduced like a constant imaginary gauge field) $\rightarrow$ derivatives with respect to the chemical potential can be shown to be UV finite by virtue of a $U(1)$ symmetry [Hasenfratz:1983ba]

Systematic errors

We have to deal with systematic errors from the continuum extrapolation and the scale setting.

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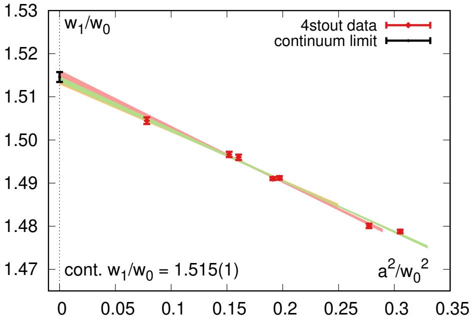
$$W(t)|_{t=w_1^2} = 0.7 \ , W(t) \equiv t\frac{d}{dt}\left\{t^2\langle E(t)\rangle\right\}$$



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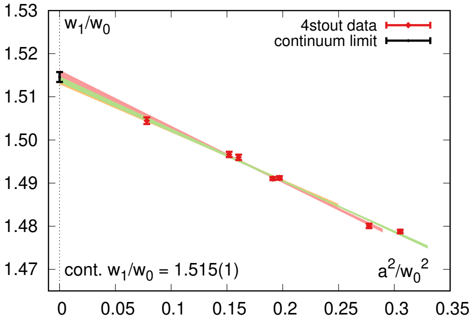

Continuum extrapolation:

$$\hat{O}(T, 1/N_\tau^2) = \sum_{i=1}^{M}\left(\alpha_i + \beta_i \frac{1}{N_\tau^2}\right) s_i(T) \ ,$$

s_i: set of basis spline function. We take three different sets of node points.

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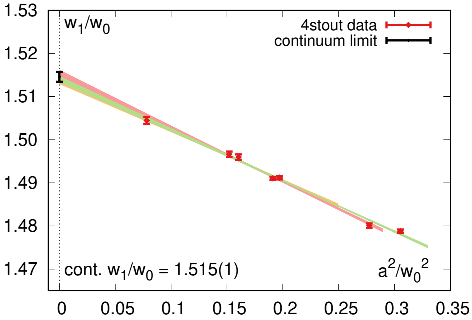

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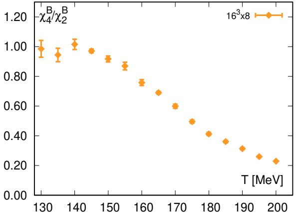
The final results are obtained by combining the $6 = 2 \times 3$ analyses to construct a histogram.

New continuum results



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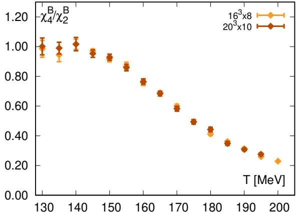

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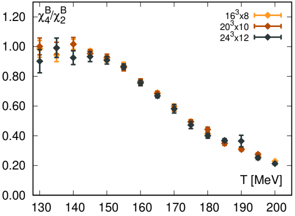

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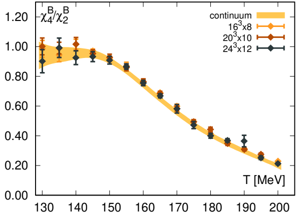

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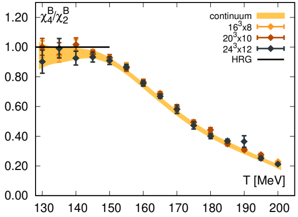

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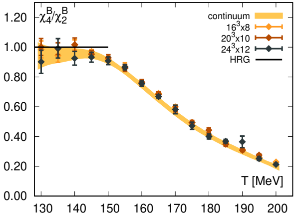

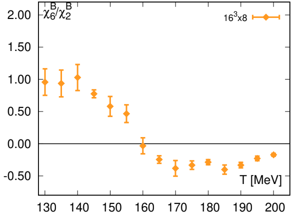

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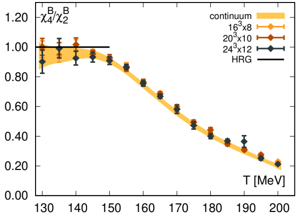

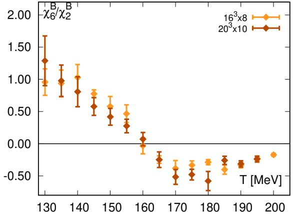

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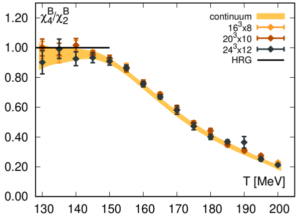

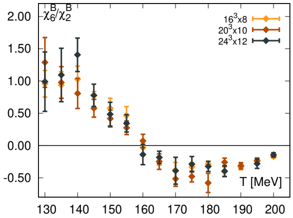

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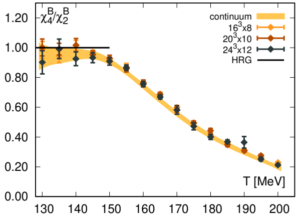

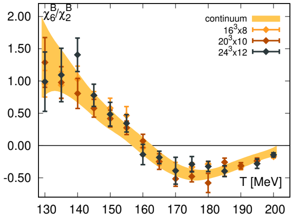

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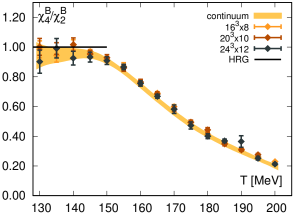

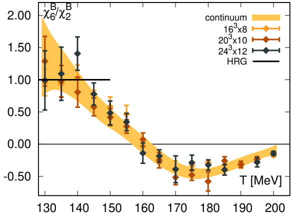

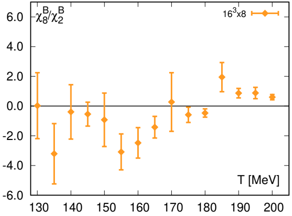

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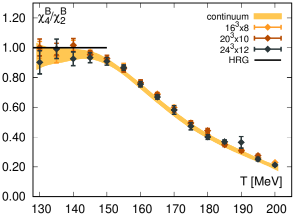

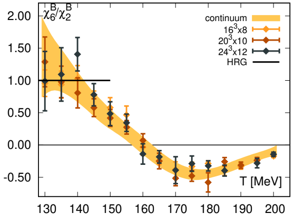

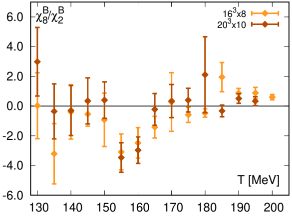

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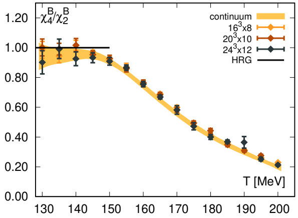
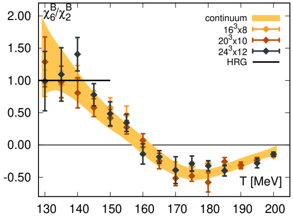
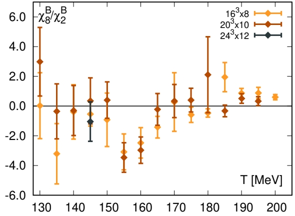
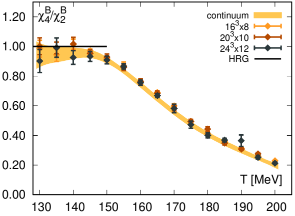
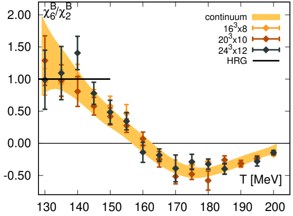


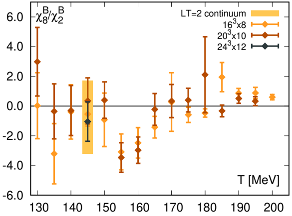




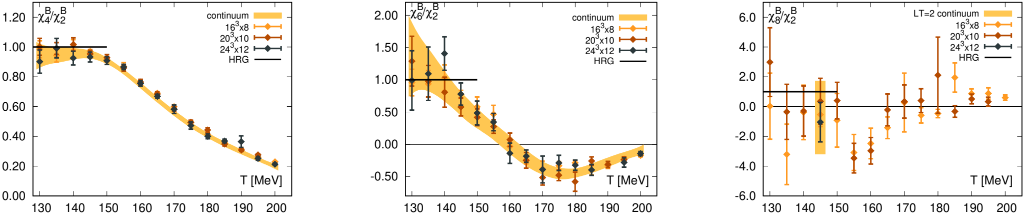
New continuum results



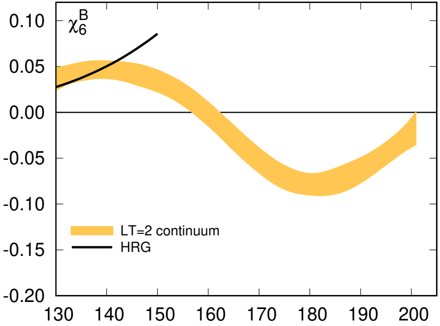




New continuum results



Comparision with different actions



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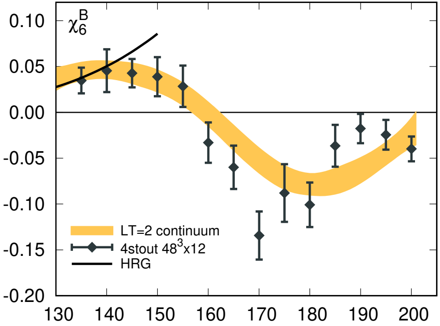

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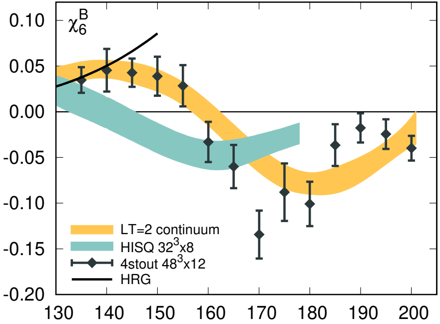

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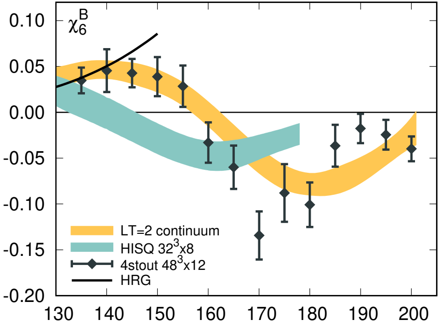

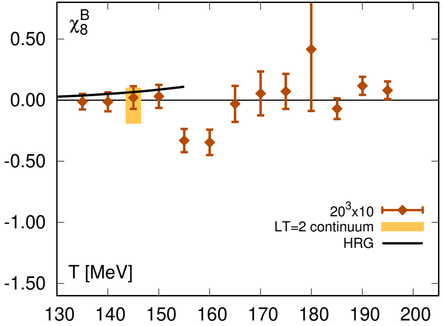

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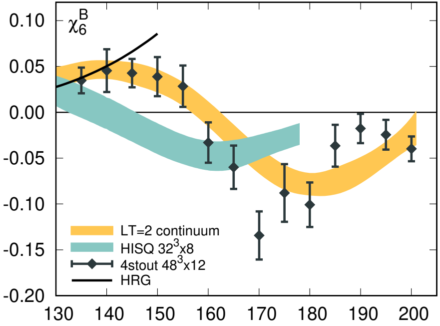
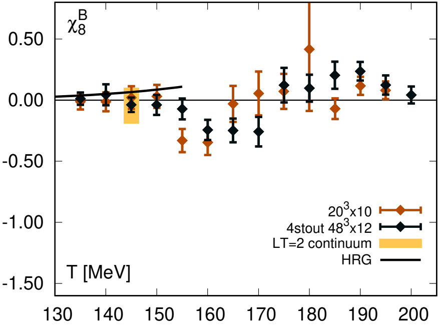
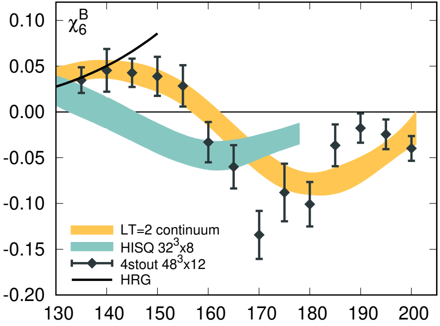
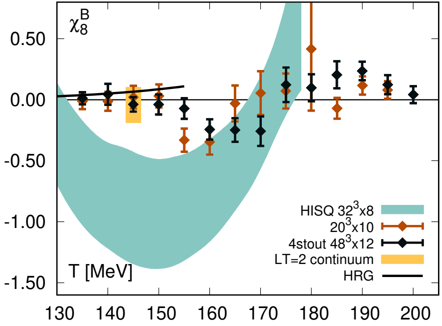


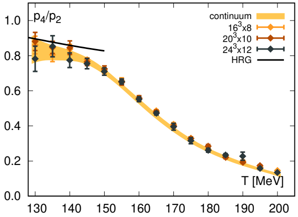


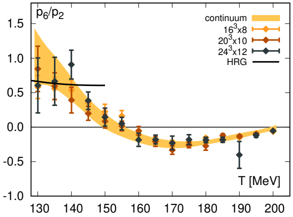
Comparision with different actions

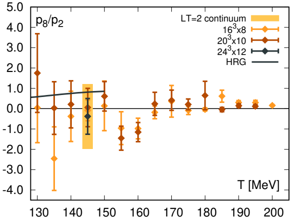




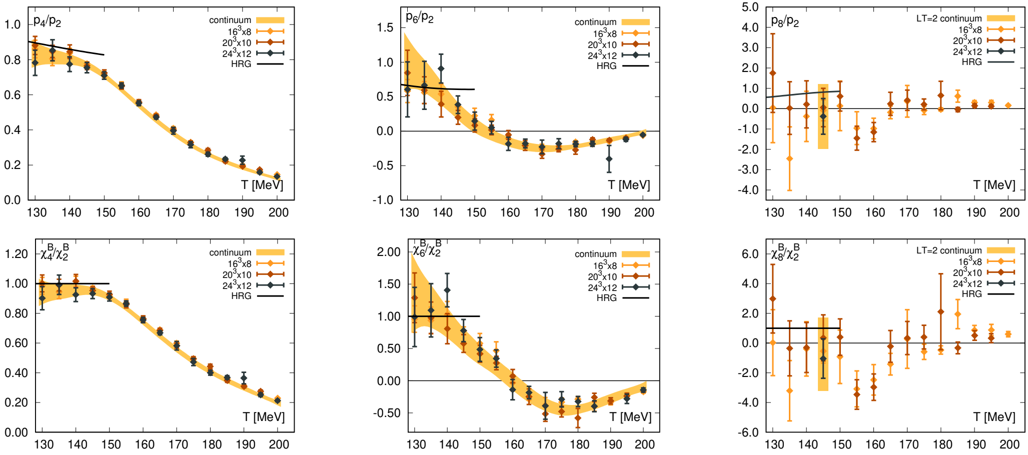
Strangeness neutrality: $\langle n_S \rangle = 0$ – Continuum results



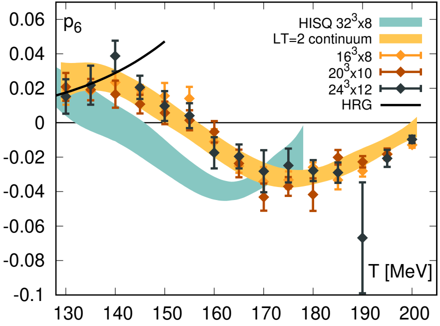


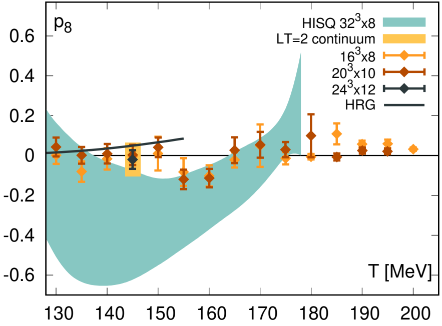


Strangness neutrality: $\langle n_S \rangle = 0$ – Continuum results

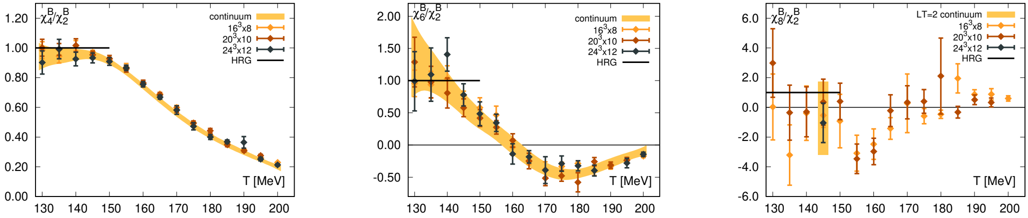


Strangness neutrality: $\langle n_S \rangle = 0$ –Comparision with different actions





Conclusion



- First continuum extrapolated results for high order baryon number fluctuations
- A 4Hex + dbw2 action allowed for a continuum limit from $N_t = 8, 10, 12$
- With $LT = 2$ the volume effects are under control in the low temperature region.

T [MeV]	$16^3 \times 8$	$20^3 \times 10$	$24^3 \times 12$
130	31741	71090	68689
135	33528	106403	66960
140	34977	69690	75229
145	336975	188571	111435
150	65374	108481	81590
155	34057	96985	89559
160	37145	68619	94053
165	156044	67668	98744
170	34397	42314	11831
175	34180	36522	12089
180	30594	25229	12727
185	30951	18396	13066
190	30293	18267	7141
195	31276	15008	7199
200	31919	13346	7390

Table: Number of configurations analyzed on our three lattice geometries.