Conserved charge fluctuations in (2+1)-flavor QCD with Möbius Domain Wall Fermions

Jishnu Goswami,
In collaboration with
Yasumichi Aoki, Hidenori Fukaya, Shoji Hashimoto, Issaku Kanamori, Takashi Kaneko, Yoshifumi Nakamura, Yu Zhang (JLQCD Collaboration)

02/08/2024

RIKEN Center for Computational Science

LATTICE 2024 LIVERPOOL

Acknowledgments

1. **Computational resource:**
    - Supercomputer Fugaku (hp230207, hp220174, hp210165, hp200130, ra000001).
2. **Funding sources :**
    - MEXT as "Program for Promoting Researches on the Supercomputer Fugaku",*Simulation for basic science: from fundamental laws of particles to creation of nuclei*, JPMXP1020200105; "Simulation for basic science: approaching the quantum era" (JPMXP1020230411).
    - JICFuS.
    - JPS KAKENHI(JP20K0396, I. Kanamori).

And to all the JLQCD members for regular meetings and discussions.

Code bases

Configuration generation: Grid (https://github.com/paboyle/Grid)

Measurements : (i) Hadrons (https://github.com/aportelli/Hadrons)

(ii) Bridge++ (https://bridge.kek.jp/Lattice-code/)

Data Analysis : https://github.com/LatticeQCD/AnalysisToolbox

Motivation : Electric charge fluctuations

Electric charge fluctuations :

- Directly accessible in both the theory and experiment!!
- Sensitive probe for freeze out parameter determination.

L. Adamczyk *et al.* (STAR Collaboration)
Phys. Rev. Lett. 113, 092301, (2014)

A. Adare *et al.* (PHENIX Collaboration)
Phys. Rev. C 93, 011901(R) (2016)

- Pions, being the pseudo-Goldstone bosons of spontaneous chiral symmetry breaking, control a large part of the low-energy dynamics.
- Electric charge fluctuations are sensitive to the pion spectrum in the hadronic phase in the QCD phase diagram.
- We chose Möbius Domain Wall Fermions for these calculations.
- Better Symmetry Control: Domain Wall Fermions (DWF) has a better control on chiral symmetry —> Better control on the pion spectrum at finite lattice spacing.

Lattice setup and Outline

- Quark number susceptibilities for $m_l = 0.1 m_s (m_\pi \sim 220\ \mathrm{MeV})$ for $24^3 \times 12,\ 32^3 \times 16$.

- Quark number susceptibilities and conserved charge fluctuations for $m_l = 0.0036 m_s (m_\pi \sim 135\ \mathrm{MeV})$ for $36^3 \times 12$.

- Sensitivity of the fluctuations on the pion masses.

- Fourth order conserved charge fluctuations for physical quark masses.

Quark number susceptibility and conserved charge fluctuations in (2+1)-flavor QCD

In QCD with two light (u, d) and one strange flavor (s), pressure is expressed via a Taylor expansion in quark chemical potentials (μ_f).

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \vec{\mu}) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{uds}}{i!j!k!} \hat{\mu}_u^i \hat{\mu}_d^j \hat{\mu}_s^k$$

$$\chi_{ijk}^{uds} = \frac{1}{VT^3} \left. \frac{\partial^{i+j+k} \ln Z(T, V, \vec{\mu})}{\partial \hat{\mu}_u^i \partial \hat{\mu}_d^j \partial \hat{\mu}_s^k} \right|_{\vec{\mu}=0} ; \quad i+j+k \text{ is even.}$$

$$= 0; \quad i+j+k \text{ is odd}$$

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q, \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q, \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S .$$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{uds}}{i!j!k!} \hat{\mu}_u^i \hat{\mu}_d^j \hat{\mu}_s^k = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k .$$

Quark number susceptibility with Domain wall fermions

The QCD partition function can be written as,

$$Z = \int DU \prod_{f=u,d,s} detM(m_f)\exp[-S_g], \qquad \det M(m_f,\hat{\mu}_f) = \left[\frac{\det D(m_f,\hat{\mu}_f)^{DWF}}{\det D(m_{PV},\hat{\mu}_f)^{DWF}}\right]$$

$$U_4(x) \to \exp(\hat{\mu}_f)U_4(x),\; U_4^\dagger(x) \to \exp(-\hat{\mu}_f)U_4^\dagger(x),$$

J. Bloch and T. Wettig, Phys. Rev. Lett. 97, 012003 (2006)

$\hat{\mu}_f = \mu_f/T$, where , μ_f is the quark chemical potential for flavor f.

The diagonal and off-diagonal quark number susceptibilities can be written as,

$$\chi_2^f = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial \hat{\mu}_f^2}\Bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\left[\left\langle \frac{\partial^2}{\partial \hat{\mu}_f^2}\ln\det M\right\rangle + \left\langle\left(\frac{\partial}{\partial\hat{\mu}_f}\ln\det M\right)^2\right\rangle\right]$$

$$= \frac{N_\tau}{N_\sigma^3}\langle D_2^f\rangle + \langle (D_1^f)^2\rangle,\; f=\{u,d,s\}$$

M. Cheng et al, Phys.Rev.D81:054510,2010 ; P. Hegde et al, PoS LATTICE2008:187,2008

$$\chi_{11}^{fg} = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial\hat{\mu}_f\partial\hat{\mu}_g}\Bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\langle D_1^f D_1^g\rangle,\; f\neq g,\; f,g=\{u,d,s\}$$

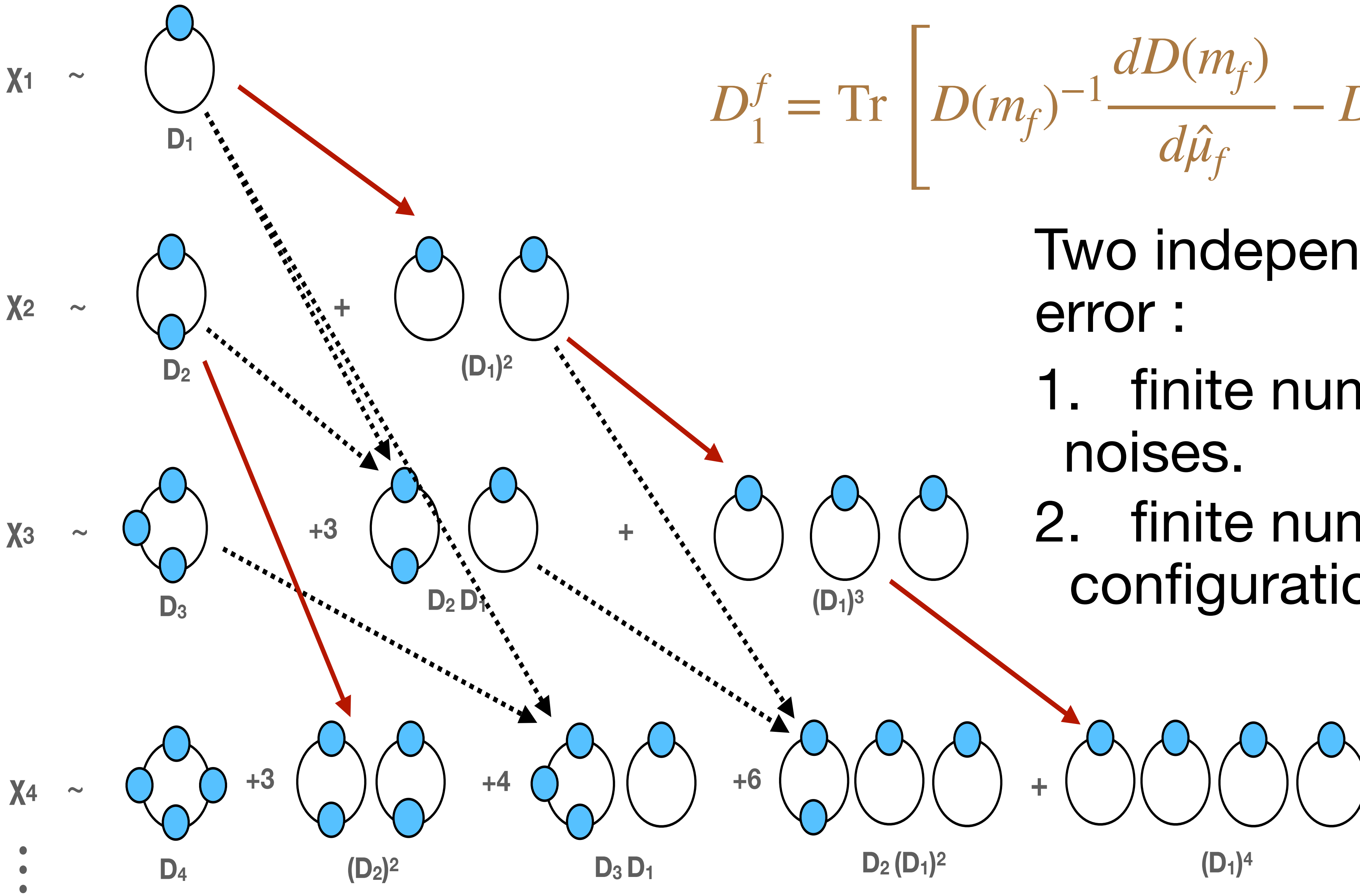
$(D_1^f)^2$ and $D_1^f D_1^g$ are the most noisy part in our calculation

Stochastic estimation of traces

One can express all the quark number fluctuations in terms of the ,

$$D_n^f = \frac{\partial^n}{\partial \hat{\mu}_f^n} \ln \det M(m_f, \hat{\mu}_f)\,|_{\vec{\mu}=0}$$

$$D_1^f = \mathrm{Tr}\left[D(m_f)^{-1} \frac{dD(m_f)}{d\hat{\mu}_f} - D(m_{pv})^{-1} \frac{dD(m_{pv})}{d\hat{\mu}_f} \right]$$



Two independent source of error :

1. finite number of random noises.
2. finite number of gauge configurations.

We will focus on stochastic error reduction for D_1^f .

Sourav Mondal et al, 2106.03165 [hep-lat]

Stochastic trace estimation

$$D_1^f = \mathrm{Tr}\left[D(m_f)^{-1}\frac{dD(m_f)}{d\hat{\mu}_f} - D(m_{pv})^{-1}\frac{dD(m_{pv})}{d\hat{\mu}_f}\right]$$

$$D_1^f = \frac{1}{N_n}\sum_j^{N_n}\left[\eta_j^\dagger D(m_f)^{-1}\frac{dD(m_f)}{d\hat{\mu}_f}\eta_j - \eta_j^\dagger D(m_{pv})^{-1}\frac{dD(m_{pv})}{d\hat{\mu}_f}\eta_j\right]$$

η_j is the gaussian random noise.

Stochastic error reduction using dilution vectors :

$$D_1^f = \frac{1}{N_n}\sum_j^{N_n}\left[\sum_{a=1}^{N_p}\eta_{aj}^\dagger D(m_f)^{-1}\frac{dD(m_f)}{d\mu_f}\eta_{aj} - \sum_{a=1}^{N_p}\eta_{aj}^\dagger D(m_{pv})^{-1}\frac{dD(m_{pv})}{d\mu_f}\eta_{aj}\right]$$

η_{aj} is the diluted gaussian random noise.

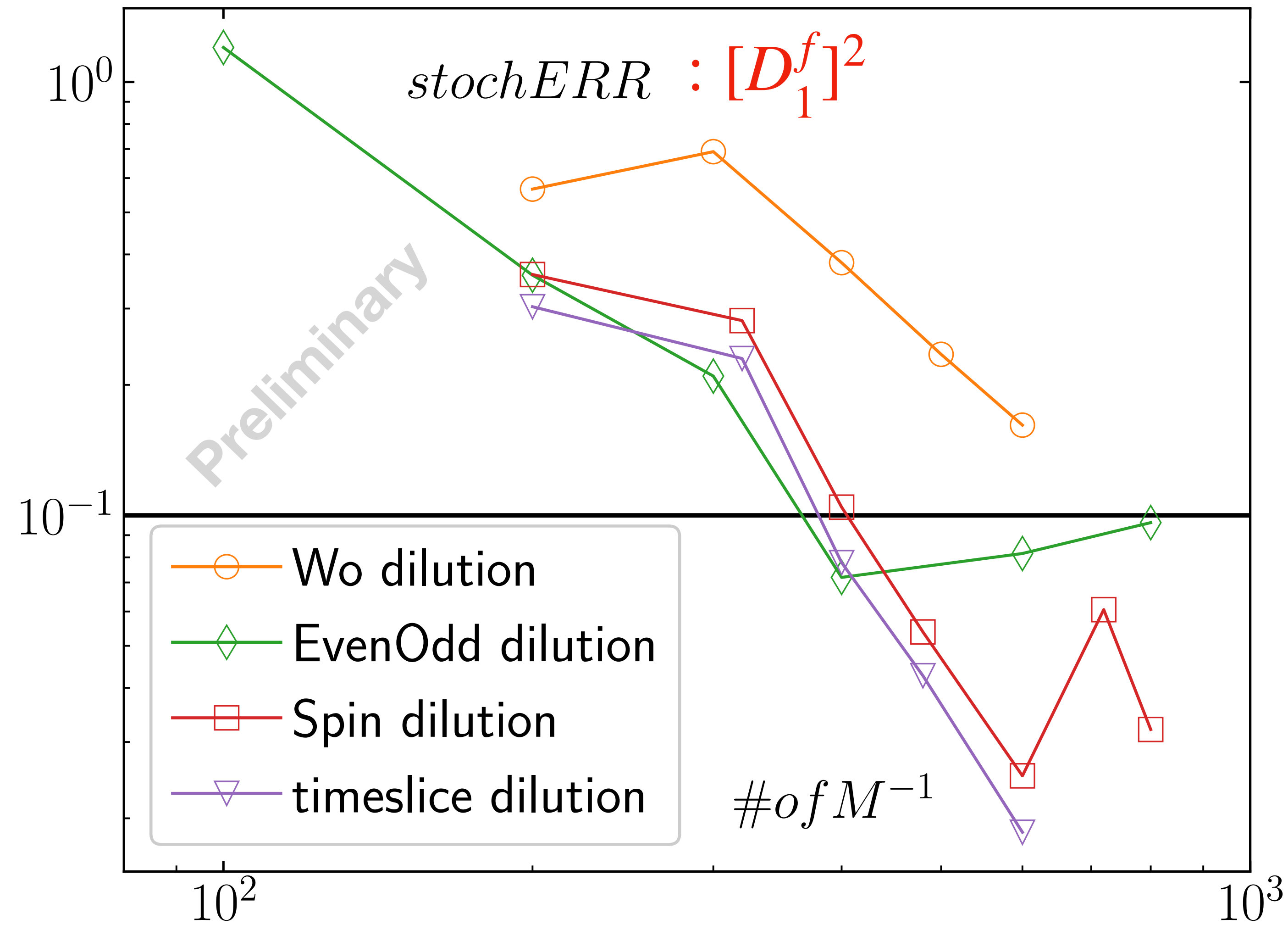
Timeslice dilution : splitting the η_j into four parts, using $(N_\tau \mod 4)$.

The product of the traces are done with the unbiased estimator method.

We use 500 Gaussian random noise for estimating $(D_1^f)^2$ in each configuration for the physical quark masses.

Stochastic trace estimation

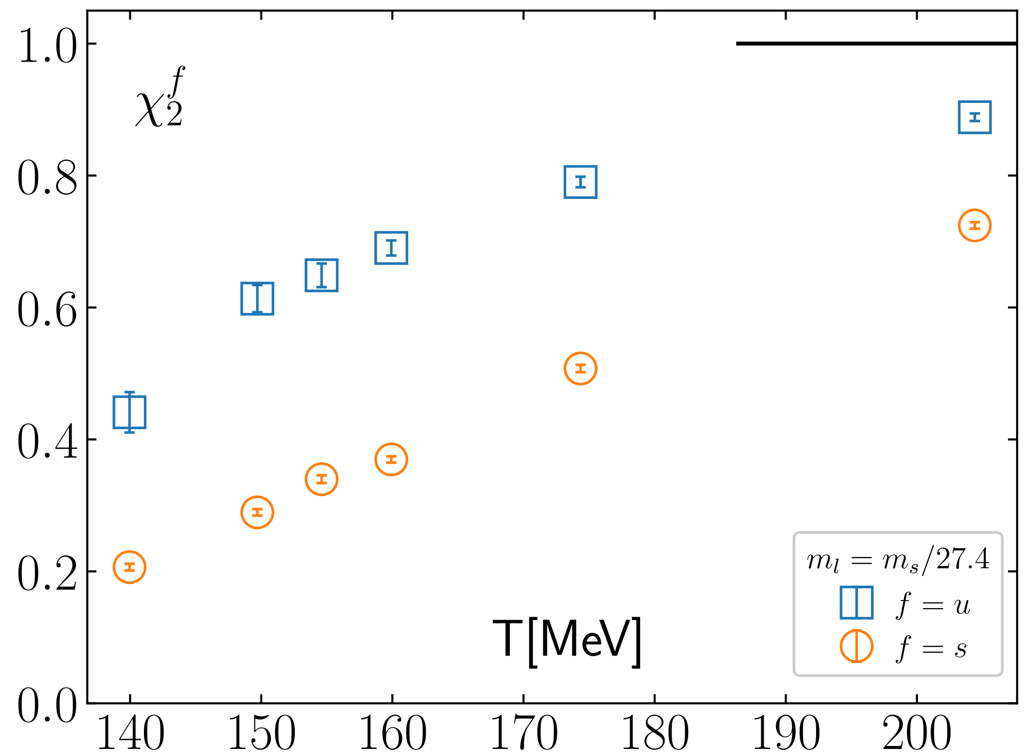
$$D_1^f \simeq \frac{1}{N_n}\sum_j^{N_n}\left[\sum_{a=1}^{N_p}\eta_{aj}^\dagger D(m_f)^{-1}\frac{dD(m_f)}{d\mu_f}\eta_{aj} - \sum_{a=1}^{N_p}\eta_{aj}^\dagger D(m_{pv})^{-1}\frac{dD(m_{pv})}{d\mu_f}\eta_{aj}\right]$$

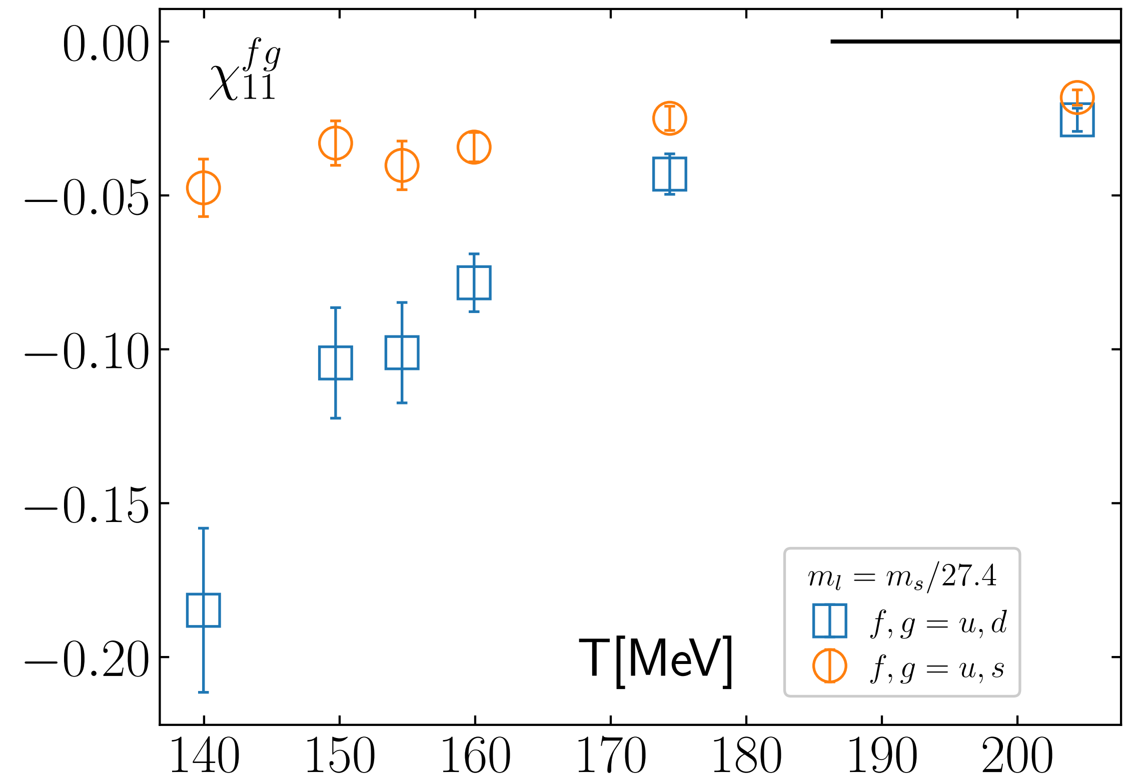


Timeslice dilution : splitting the η_j into four parts, using $(N_\tau \mod 4)$.

We see 2-3 times error reduction using Spin and time slice dilution.

Quark number susceptibility with Möbius Domain Wall Fermions in (2+1)-flavor QCD





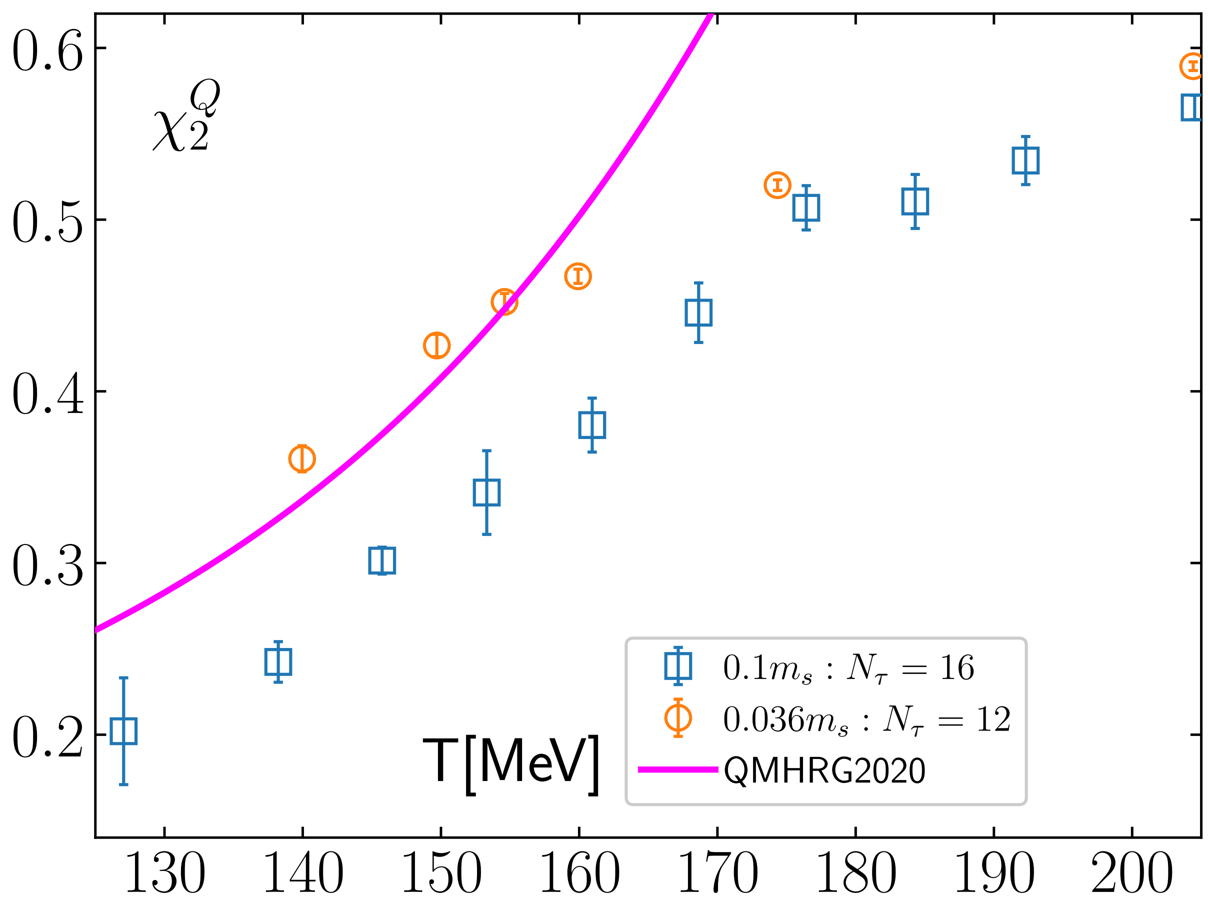
χ_2^f's rise rapidly in the vicinity of the T_{pc}.

At high T: χ_2^f's are smaller than the Ideal gas limit.

χ_{11}^{fg} reaches closer to Ideal gas limit.

In high T PT : $\chi_2^f \sim \chi_2^{f,ideal} + O(g^2)$, $\chi_{11}^{fg} \sim O(g^6 lng)$ **A. Vuorinen, PRD68, 054017 (2003)**

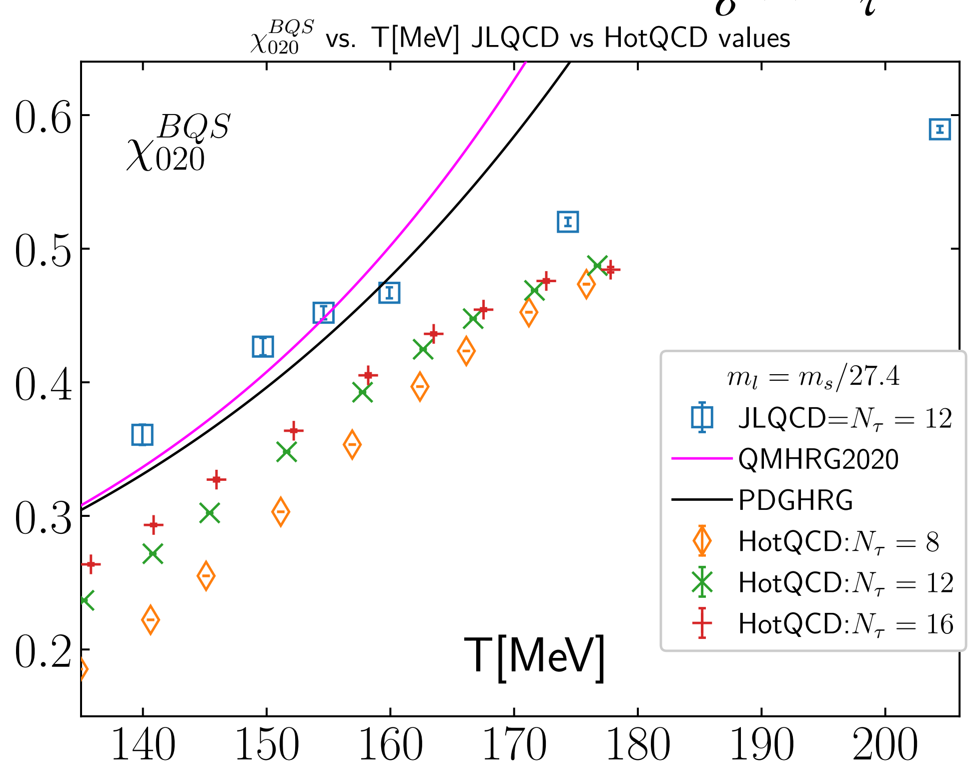
Comparison of χ_2^Q calculations with different light quark masses

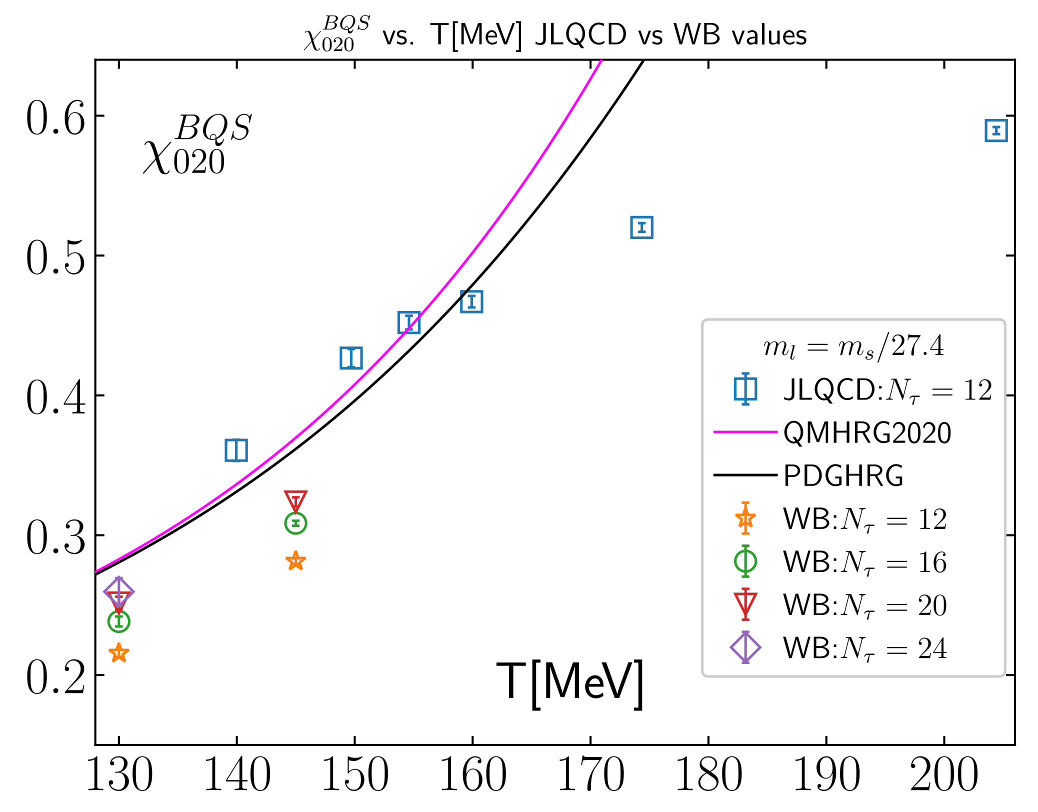


- In a non interacting HRG, χ_2^Q is dominated by pions.
- We see that χ_2^Q is sensitive to the pion mass in the temperature, $T_{pc} \leq 160$ MeV.
- $m_\pi \sim 220$ MeV for $m_l = 0.1 m_s$ and $m_\pi \sim 135$ Mev for $m_l = 0.036 m_s$.

Comparison of χ_2^Q calculations with Möbius Domain Wall Fermions and Staggered fermions

$N_\sigma^3 \times N_\tau = 36^3 \times 12$

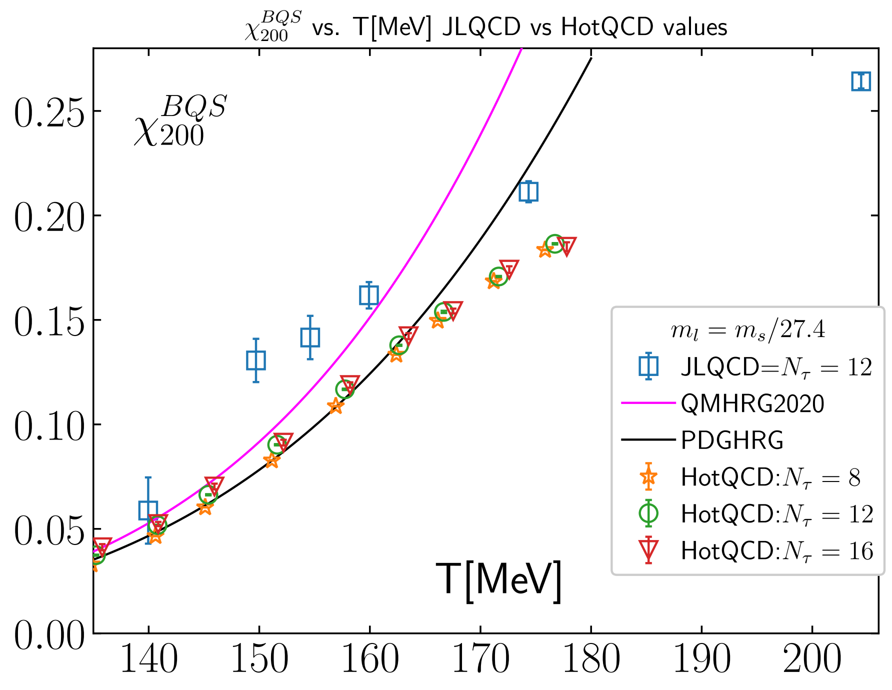


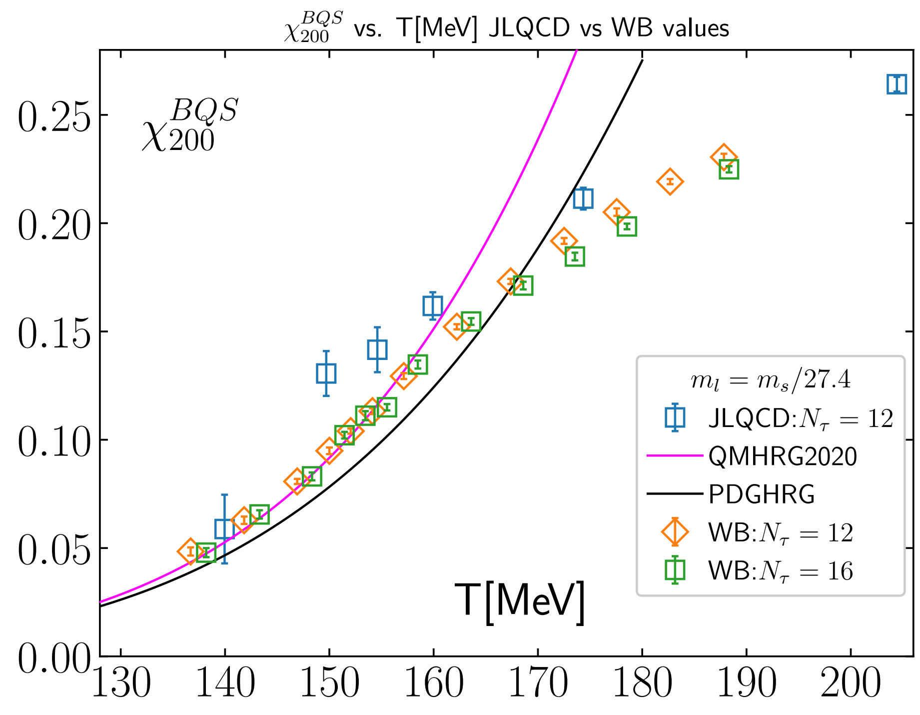


- We saw larger value in the χ_2^Q in the hadronic phase, compared to the HISQ and stout smeared staggered quarks calculations at finite lattice spacing.
- But our results at finite lattice spacing are closer to the Hadron Resonance Gas model calculations below $T \leq 160$ MeV.

Refs: HotQCD : D. Bollweg et al, arXiv:2107.10011 [hep-lat].
WB : R. Bellwied et al, arXiv:1507.04627 [hep-lat]

Comparison of χ_2^B calculations with Möbius Domain Wall Fermions and Staggered fermions

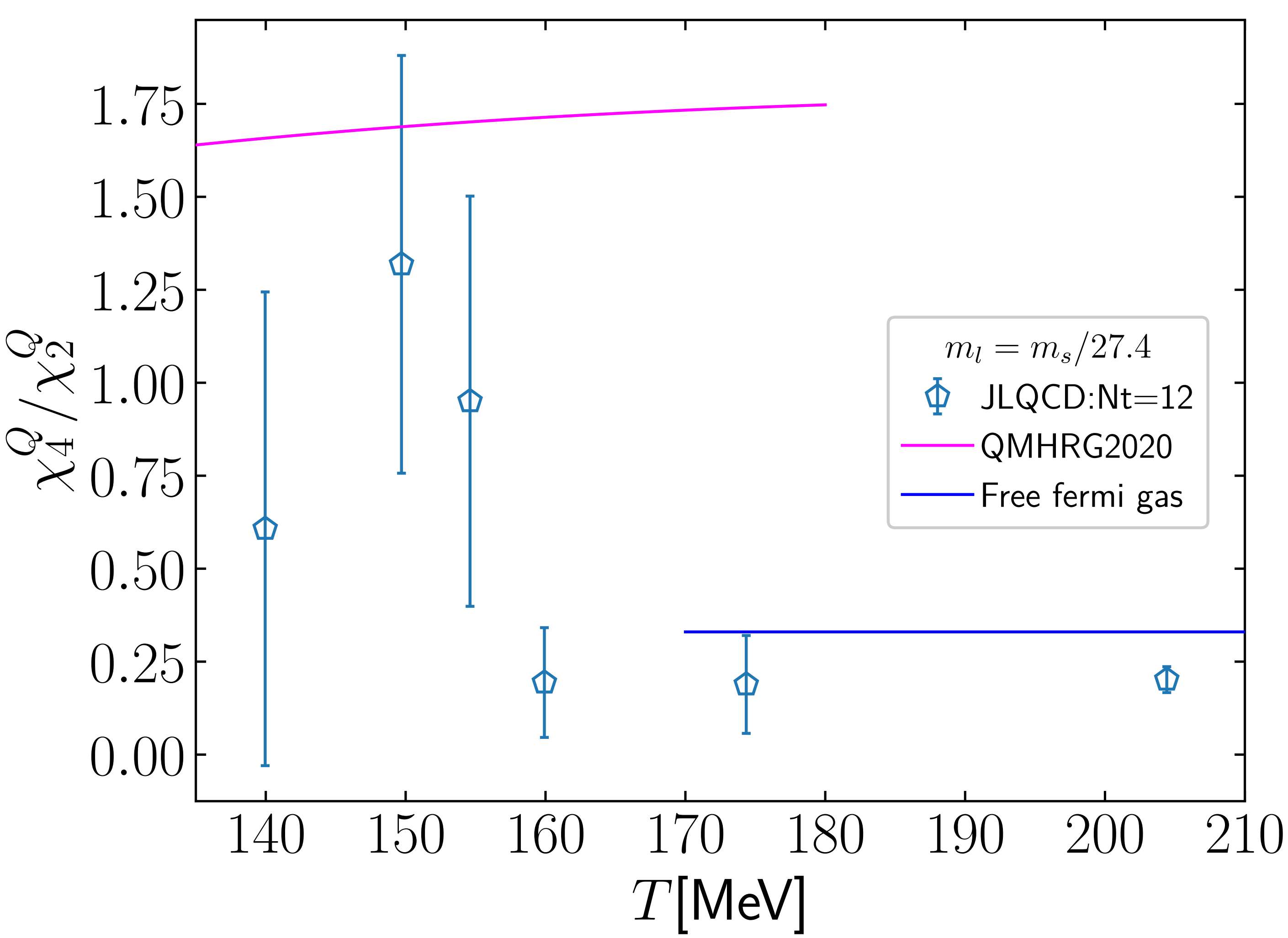




- **Data Comparison**: Our lattice data are systematically higher than those from HISQ and stout smeared staggered quarks near the pseudo-critical temperature. Although, as expected this observable is much more noisier than χ_2^Q.
- **Measurements**: Performed on 150 gauge configurations per temperature, with 100 trajectory separations.
- **Further Analysis**: Additional lattice spacing and more statistics may be required to better understand this discrepancy.

Refs: HotQCD : D. Bollweg et al, arXiv:2107.10011 [hep-lat].
WB : R. Bellwied et al, arXiv:1910.14592 [hep-lat]

Leading order kurtosis of electric charge cumulants



$\overrightarrow{\mu} = \{\mu_B, \mu_Q, \mu_S\}$

$R_{42}^Q = \chi_4^Q/\chi_2^Q + O(\overrightarrow{\mu}^2)$

Leading order kurtosis value close to the Pseudo-critical temperature,

- $R_{42}^Q = 1.3(5)$, $T = 150$ MeV
- $R_{42}^Q = 0.9(5)$, $T = 155$ MeV

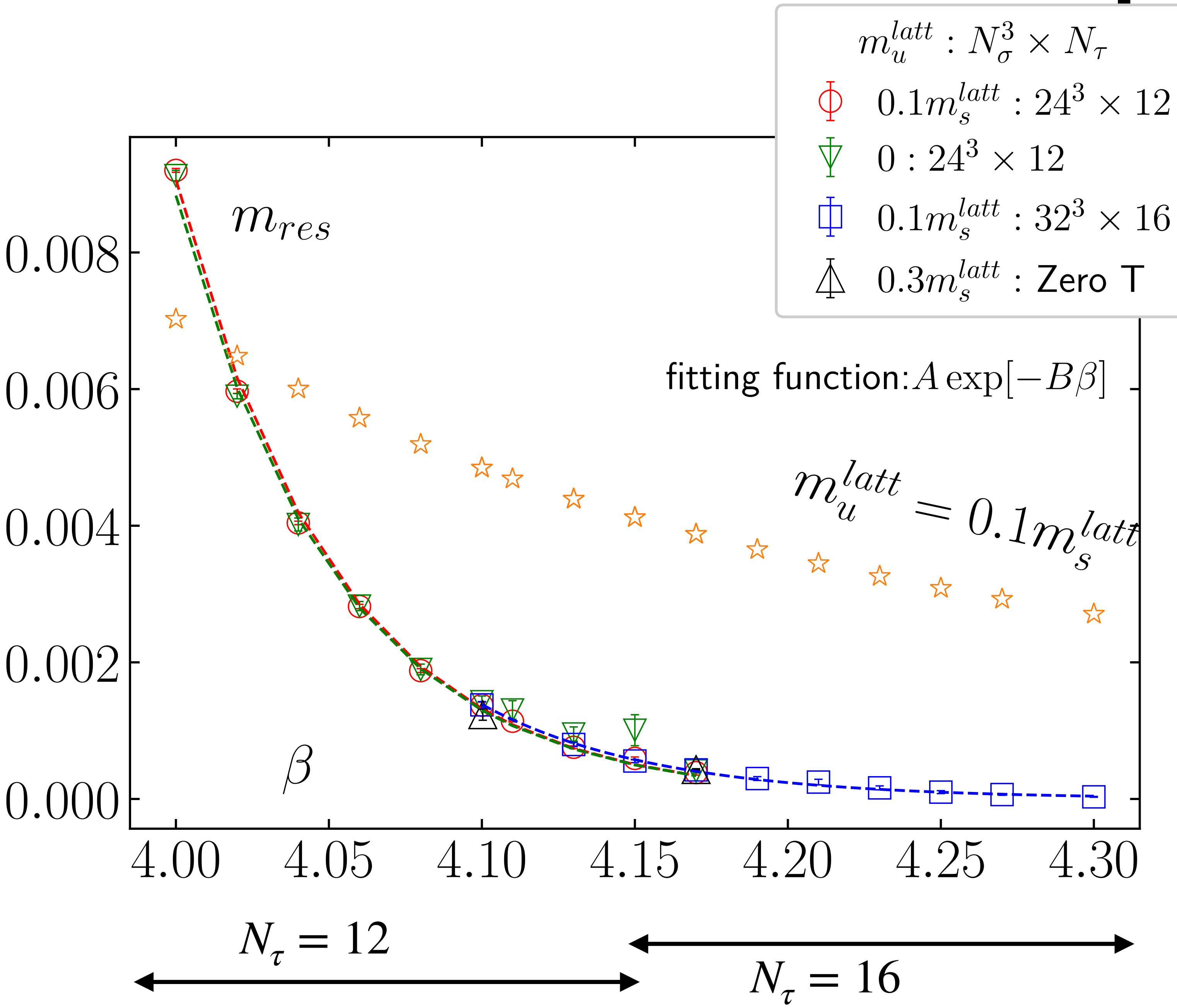
Summary and Conclusions

- We present results of conserved charge fluctuations using (2+1)-flavor QCD with a chiral fermion formalism, specifically Möbius Domain Wall Fermions.
- We compare our calculations of χ_2^B and χ_2^Q with the staggered fermion formalism calculations at finite lattice spacing.
- We also present fourth order conserved charge fluctuations for the physical value of the quark masses.
- In future, we will extend our calculations to smaller lattice spacings to study the cut-off dependence of conserved charge fluctuations.

Thank you for your attention !!

Back up slides

Tuning of the bare input quark masses on the line of constant physics (LCP)



Tuning of bare input quark masses (m_f^{input}) in the Domain Wall action:

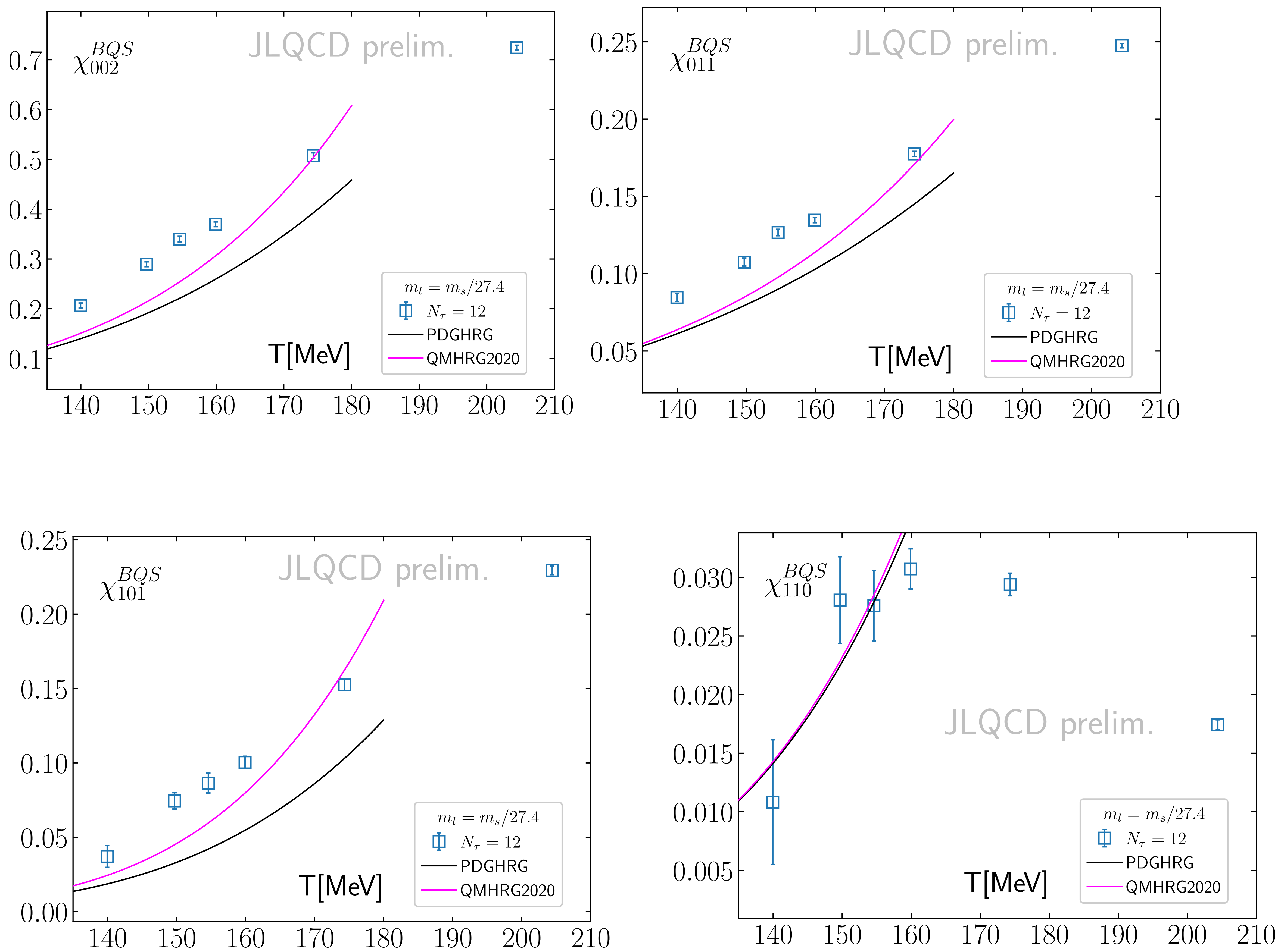
$$m_f^{latt} = m_f^{input} + m_{res}\ ,\ f = \{u, d, s\}$$

Y. Aoki et al, *PoS* LATTICE2021 (2022) 609

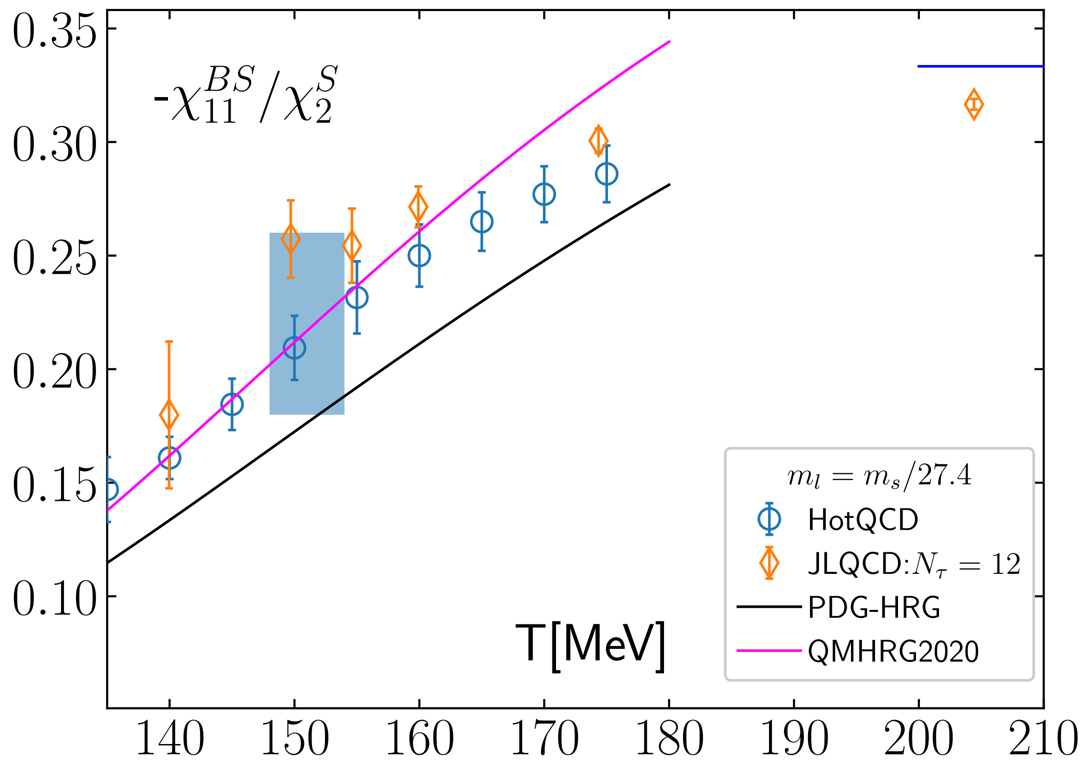
$$\frac{m_u^{latt}}{m_s^{latt}} = \frac{m_u^{input} + m_{res}}{m_s^{input} + m_{res}} = 0.1$$

For, $N_\tau = 12$, $m_u^{latt} \le m_{res}$. We tune the m_u^{input} in the LCP.

For, $N_\tau = 16$, $m_u^{latt} > m_{res}$, we perform mass reweighting.



χ_{002}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{011}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{101}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{110}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]



-χ_{11}^{BS}/χ_2^S
$m_l = m_s/27.4$
HotQCD
JLQCD:$N_\tau = 12$
PDG-HRG
QMHRG2020
T[MeV]
0.10
0.15
0.20
0.25
0.30
0.35
140
150
160
170
180
190
200
210