The temperature of QCDs chiral transition at its tricritical point

About the type of phase transition in massless many-flavour QCD

in collaboration with
Owe Philipsen & Reinhold Kaiser

Jan Philipp Klinger
Lattice 2024
Liverpool 30.07.2024

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211 Strong-interaction matter under extreme conditions

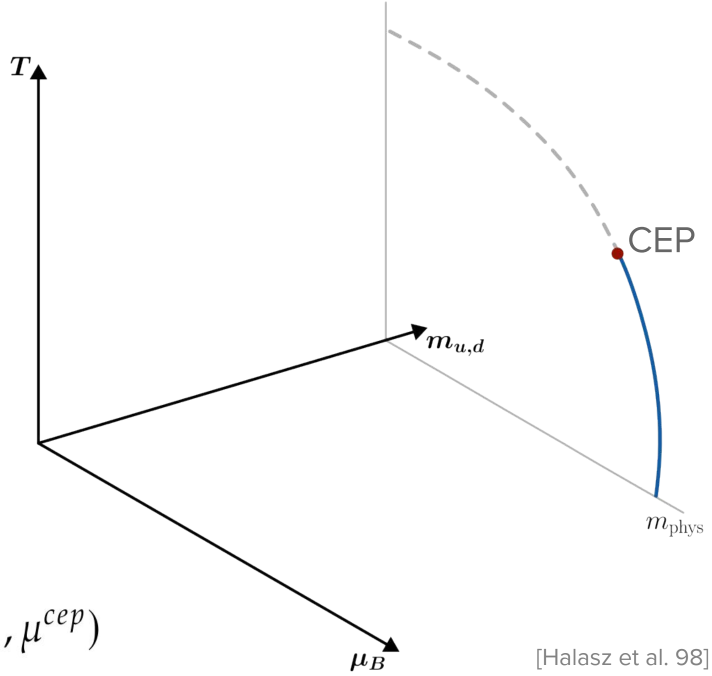
Why study massless QCD?

- Phase diagram at physical point is conjectured for $\mu \gtrsim 4T$
  - Existence of a 1st order transition?

Constraints from chiral limit:

- Physical QCD could fall into scaling region of chiral limit
- Ordering of temperatures:

$$T_c\,(m_{u,d} = 0, \mu_B = 0) > T_{tric}(m_{u,d} = 0, \mu_B = \mu_{tric}) > T_{cep}(m_{u,d}^{phys}, \mu^{cep})$$



[Halasz et al. 98]

Take step back to $\mu = 0$:

Understand chiral phase transition in the chiral limit $m = 0$

➡ Columbia plot

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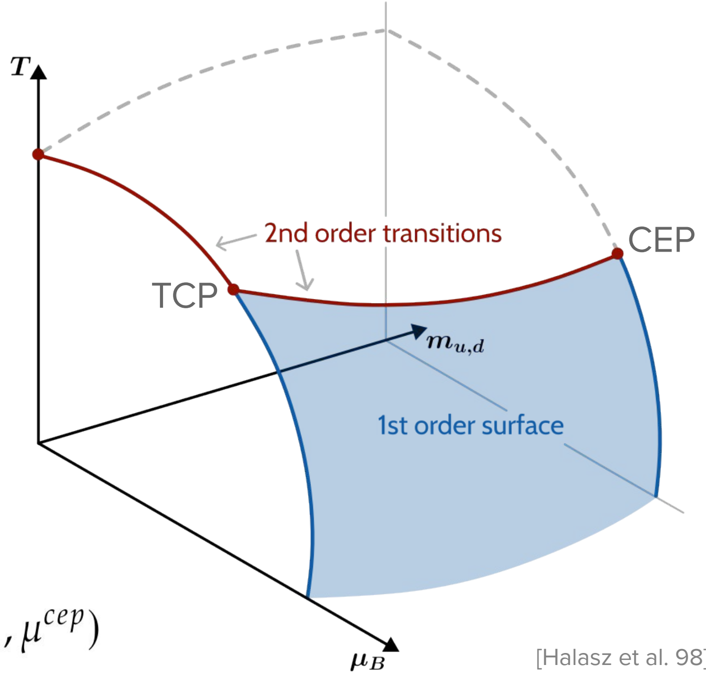

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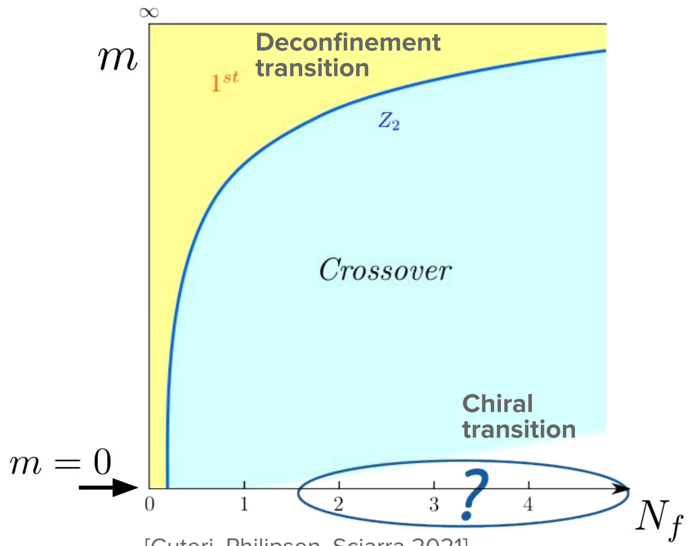
Type of phase transition in chiral QCD

- Open Question: order of chiral transition for massless quarks $m = 0$ for different N_f
  - Problem: $m = 0$ cannot be simulated on the lattice
- **[Pisarski, Wilczek 83]:** $N_f = 3$ is 1st order, $N_f = 2$ depends on axial anomaly
  - ➡ If 1st order region exists, there has to be a tricritical point



[Cuteri, Philipsen, Sciarra 2021]

<u>But: Evidence of 2nd order chiral limit</u>

- $\forall N_f \lesssim 6:$ 2nd order **[Cuteri, Philipsen, Sciarra 21]**

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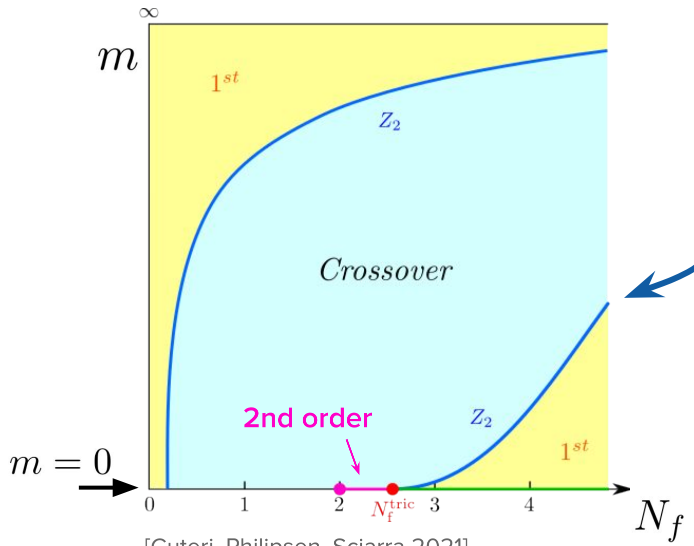

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$$N_f^c(m) = N_f^{tric} + A \cdot m^{2/5} + \mathcal{O}(m^{4/5})$$

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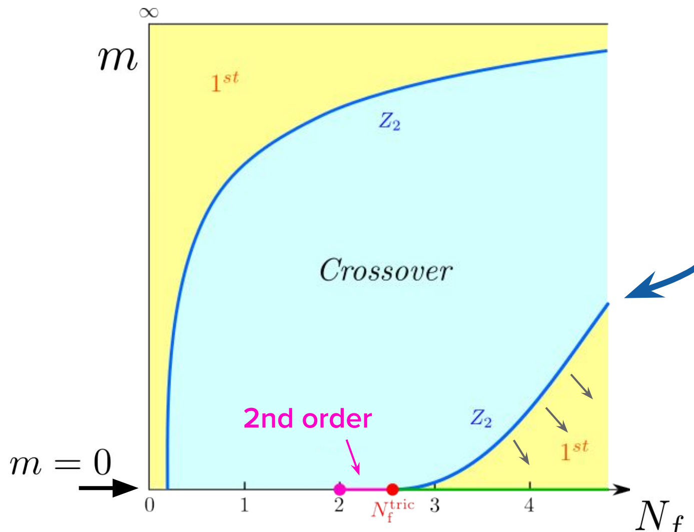

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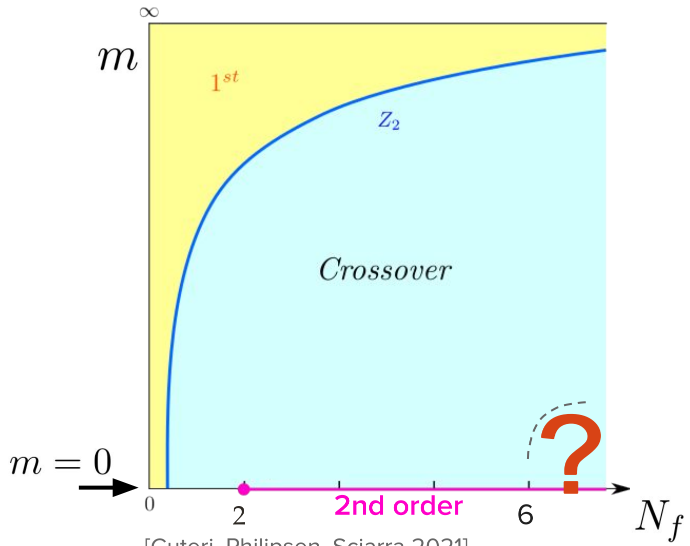

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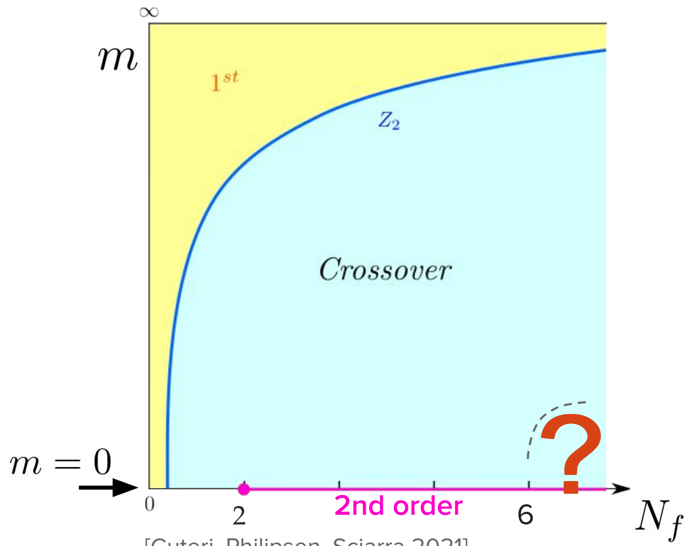

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But: Evidence of 2nd order chiral limit

- $\forall N_f \lesssim 6$: 2nd order **[Cuteri, Philipsen, Sciarra 21]**
- FRG
  - $N_f = 2$: 2nd order **[Braun et al. 23]:**
  - $\forall N_f$: 2nd order (possible) **[Fejos 22], [Fejos, Hatsuda 24]**
- DSE
  - $N_f = 2, 3$: 2nd order **[Bernhardt, Fischer 23]**
- Conformal Bootstrap
  - $N_f = 3$: 2nd order **[Kousvos, Stergiou 23]**

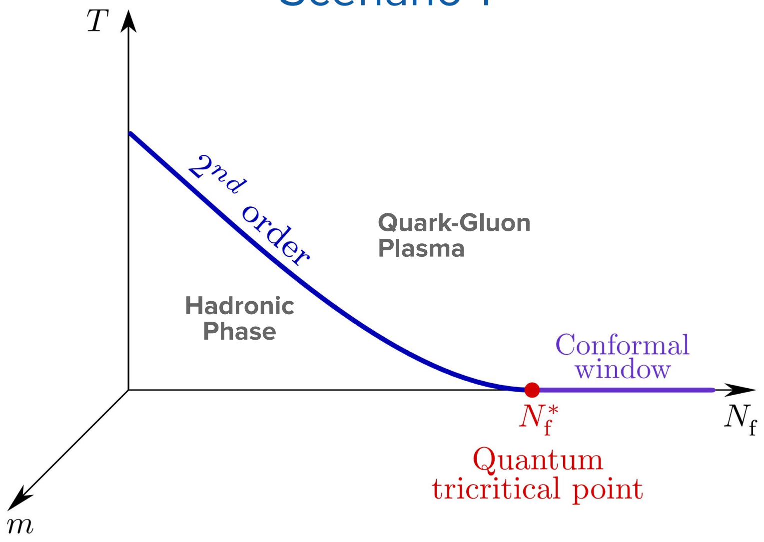
What happens for Nf > 6?

Onset of conformal window Nf*:

$10 \lesssim N_f^* \lesssim 12$ { [Braun, Gies 11] [Lombardo, Pallante, Deuzeman 13]

$8 \lesssim N_f^* \lesssim 9$ [Hasenfratz et al. 23]

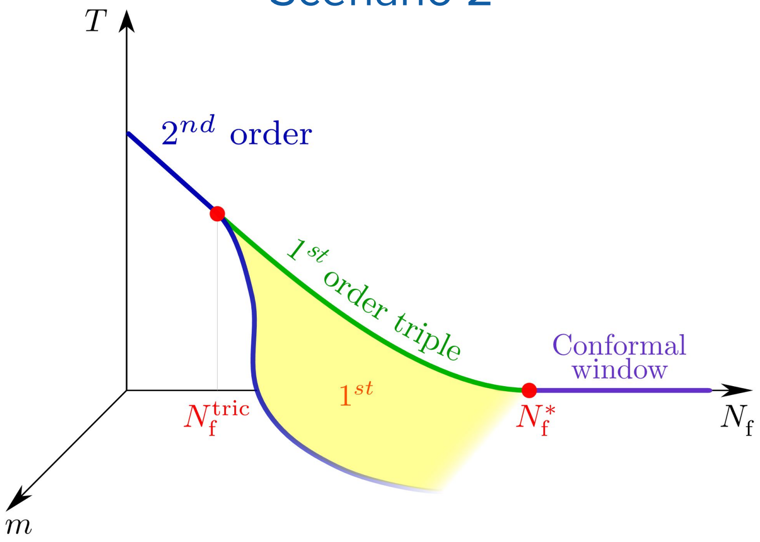
Scenario 1



- 2nd order for all N_f
- $N_f^{tric} = N_f^*$

N_f^{tric} at $T = 0$

Scenario 2



- 2nd order turns into 1st order at N_f^{tric}
- $6 < N_f^{tric} < N_f^*$

N_f^{tric} at $T > 0$

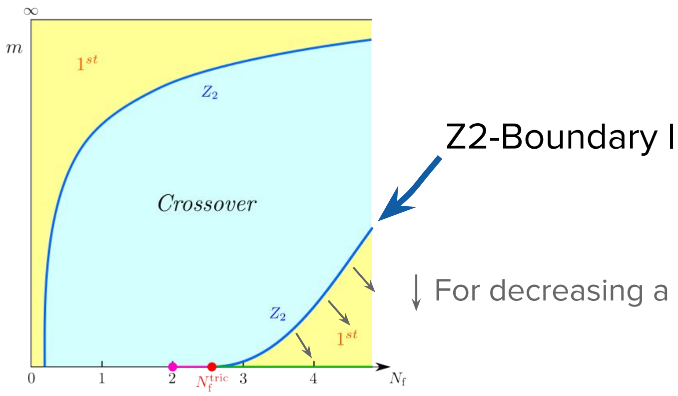
Lattice Setup

QCD with N_f degenerate quarks with mass m :

$$Z(m,g,N_f) = \int \mathcal{D}A_\mu \left[\det D(m,A_\mu)\right]^{N_f} e^{-S_G(g,A_\mu)}$$

Methodology:

Locate Z2-Boundary
in bare lattice parameter space $N_\tau,\ N_f,\ \beta,\ am$:



- unimproved Wilson gauge action S_G
- unimproved staggered fermions D
- bare parameters $\beta = 6/g^2,\ am,\ N_f$
- continuum limit $N_\tau = 1/aT \to \infty$

Kurtosis finite size scaling:

- order parameter $\mathcal{O}$: $\langle\bar\psi\psi\rangle$
- standardized moments:

$$B_n = \frac{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^n\rangle}{\left\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^2\right\rangle^{n/2}}$$

- phase boundary: $B_3(\beta_{pc}, am, N_s) = 0$
- order of transition: $B_4(\beta_{pc}, am, N_s \to \infty)$

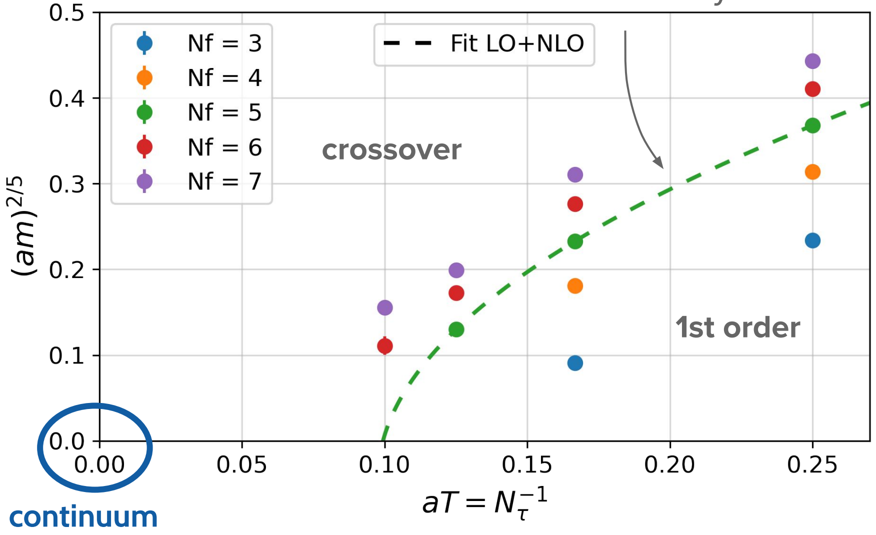
1st order	Z(2) 2nd order	crossover
1	1.604	3

Phase boundary for different flavours

- Continuum limit $\Leftrightarrow$ origin of plot:
  $a \to 0 \quad \Leftrightarrow \quad N_\tau = 1/aT \to \infty$
- Massless limit $\Leftrightarrow$ x-axis
- Z2 phase boundary:
  $aT_c(am) = aT_{tric} + A \cdot (am)^{2/5} + B \cdot (am)^{4/5}$
- 1st order ends at lattice spacing $N_\tau^{tric}(N_f)$

➡ 1st order is a lattice cutoff effect

Z2-boundary for Nf5



QCD in chiral limit

The chiral transition is of 2nd order for $N_f \leq 7$

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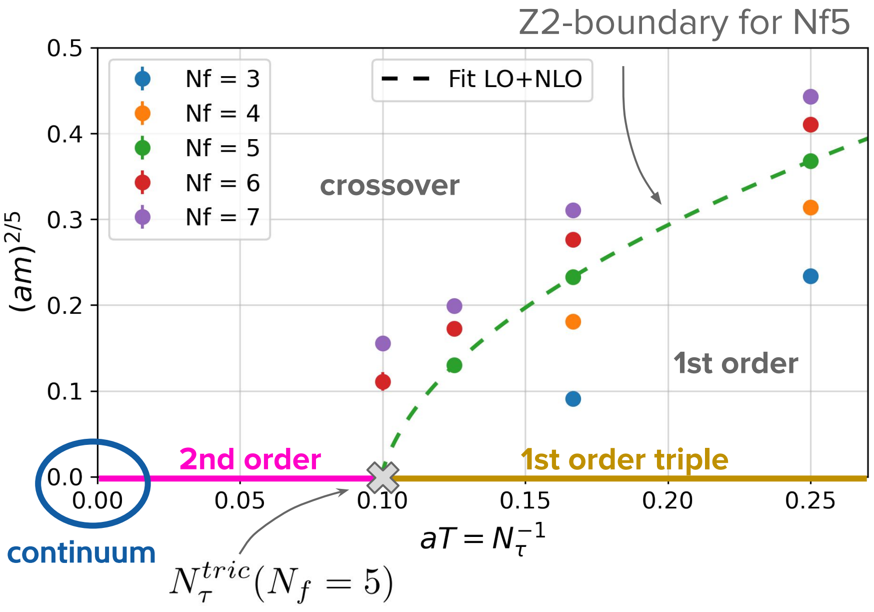

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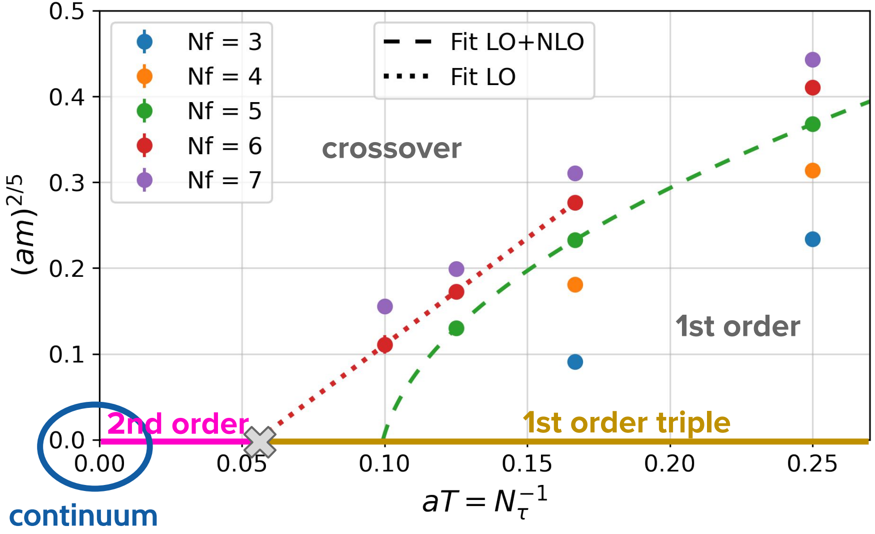

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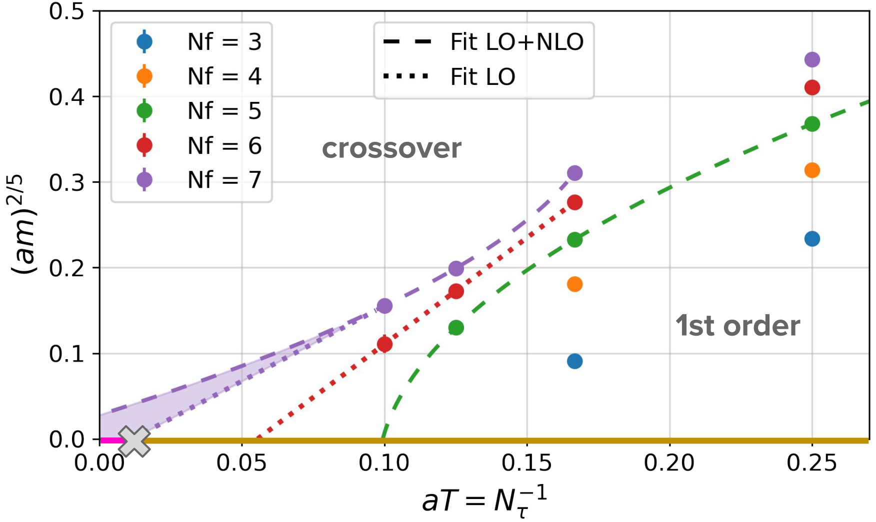

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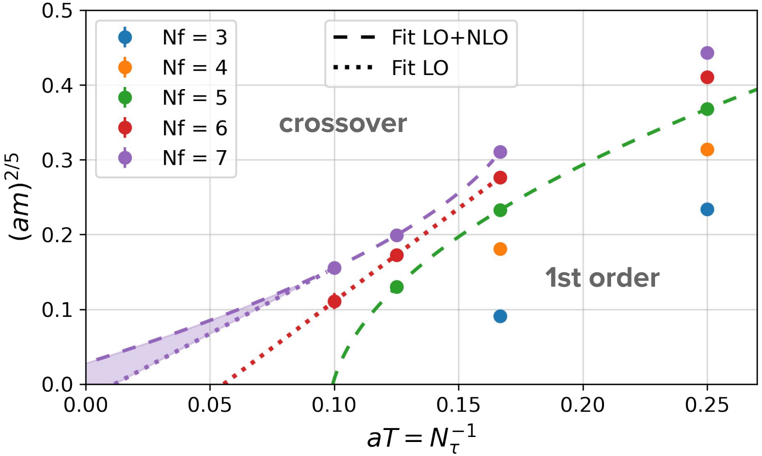
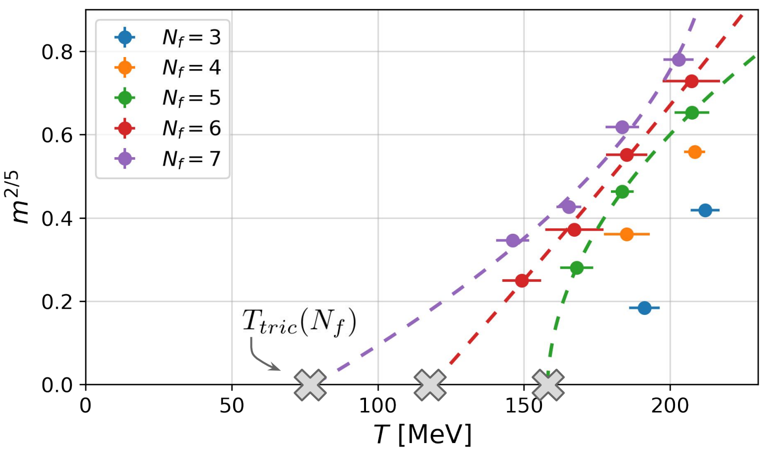
Tricritical temperatures I



Scale Setting



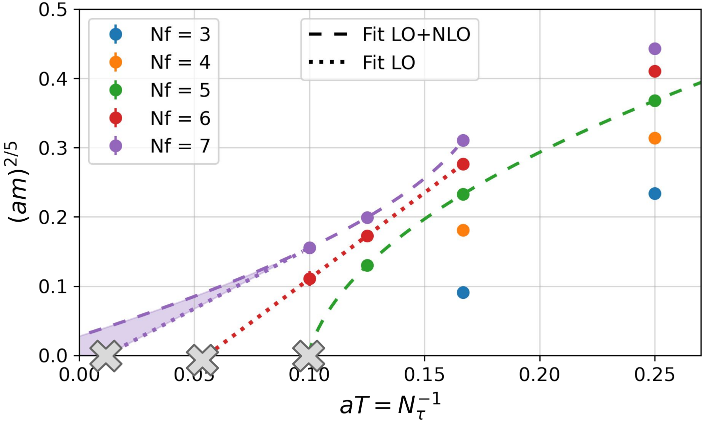
Scale setting:

- measure common UV-Scale: Sommer-scale r_1
  - debatable what “MeV” means away from physical point
- Find $T_{tric}(N_f)$ by extrapolating to $m = 0$ for each N_f :

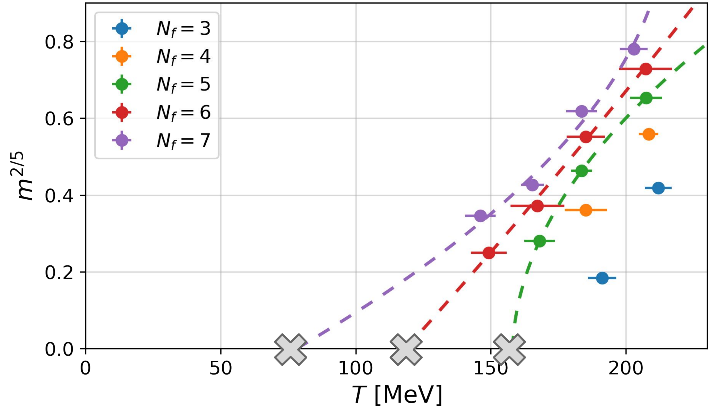
$$T_c(m) = T_{tric} + A \cdot m^{2/5} + B \cdot m^{4/5}$$

➡ Tricritical temperature $T_{tric}(N_f)$ decreases with N_f

Tricritical temperatures II



$$a_{tric}(N_f) \to 0$$



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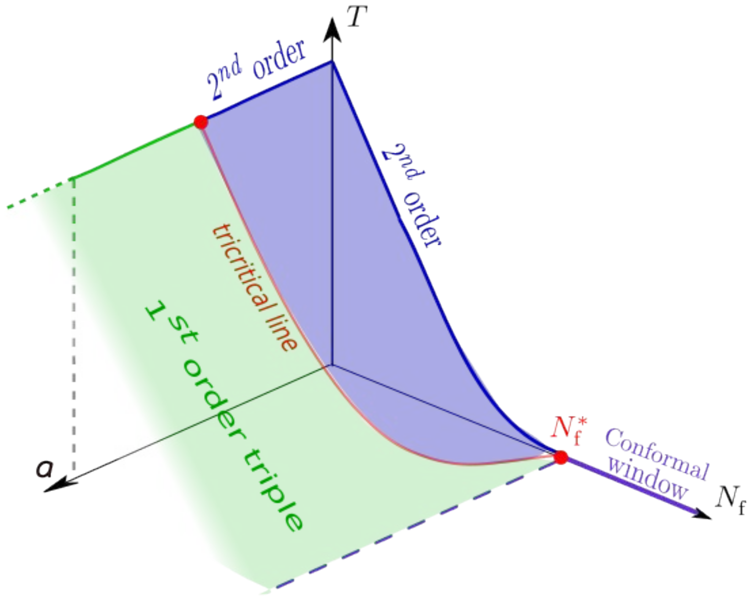
Qualitative picture for massless lattice QCD:



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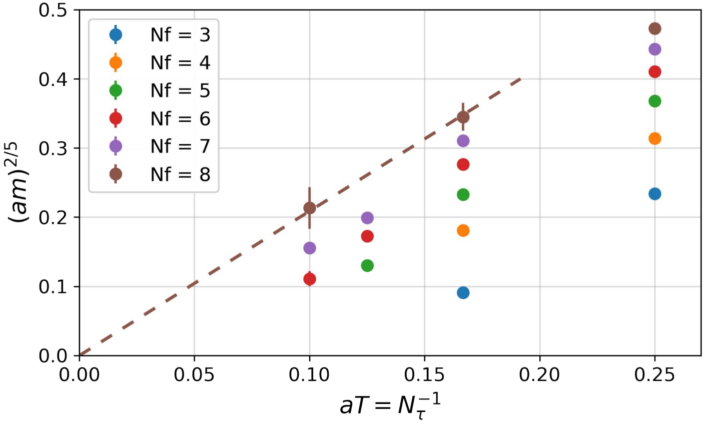

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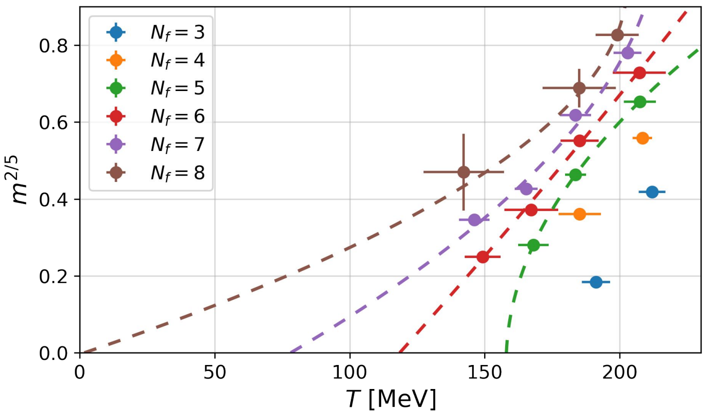

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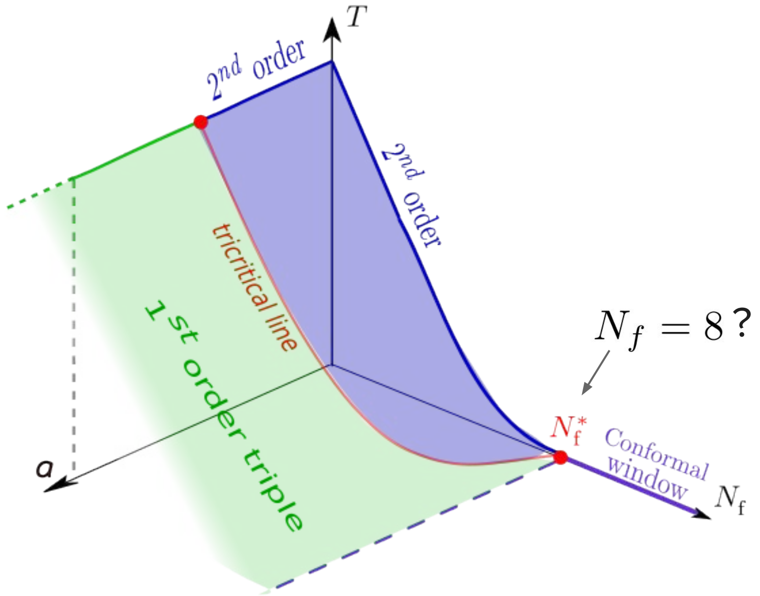


- Preliminary data consistent with: $N_f^{tric}(a=0) = 8$ at $T_{tric} = 0$

➡ $N_f = 8$ is onset of conformal window ?

➡ No 1st order transition in chiral continuum QCD for any N_f

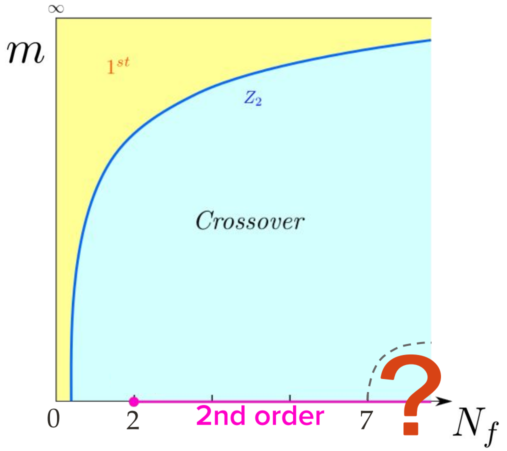
Conclusion

QCD in chiral limit

The chiral transition is of 2nd order for $N_f \leq 7$

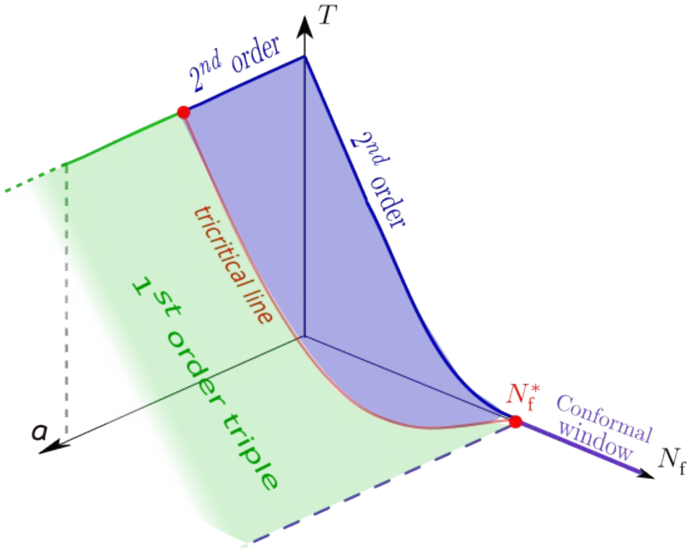
- Via tricritical scaling: 1st order is a lattice artefact

➡ No continuum extrapolation needed!



Outlook

- If $T_{tric}\left(N_f^{tric}(a=0)\right) = 0$, then:
  - ➡ Chiral transition is 2nd order $\forall N_f$
  - ➡ Possible to pinpoint onset of conformal window



- Similar analysis at imaginary chemical potential
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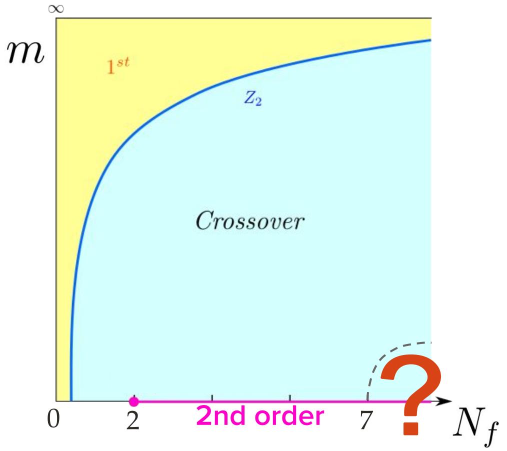

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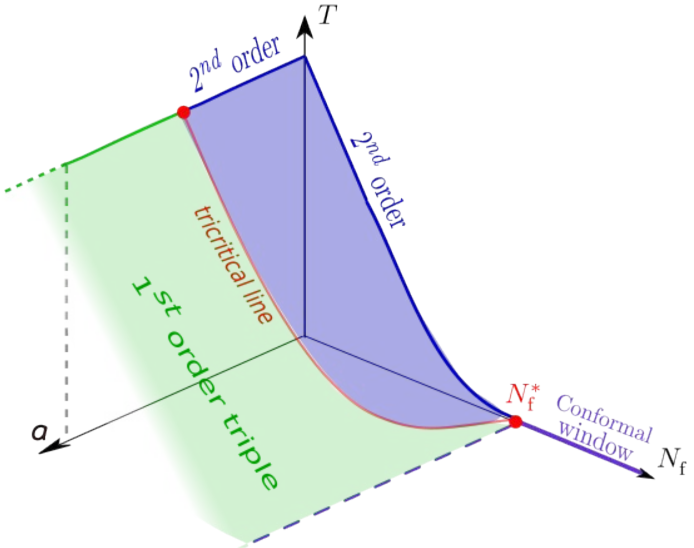

Thank You

Backup Slides

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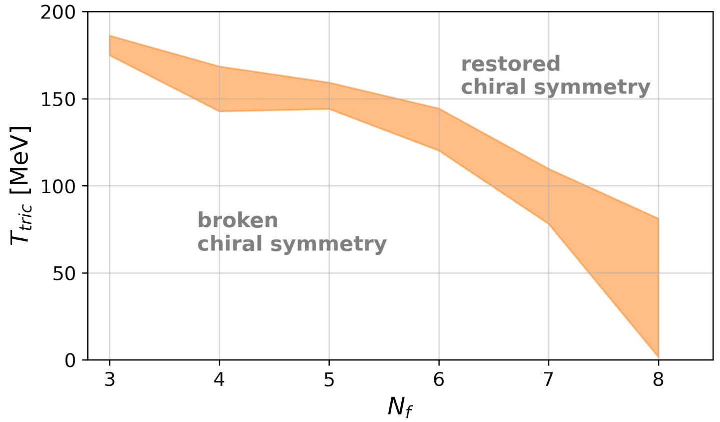
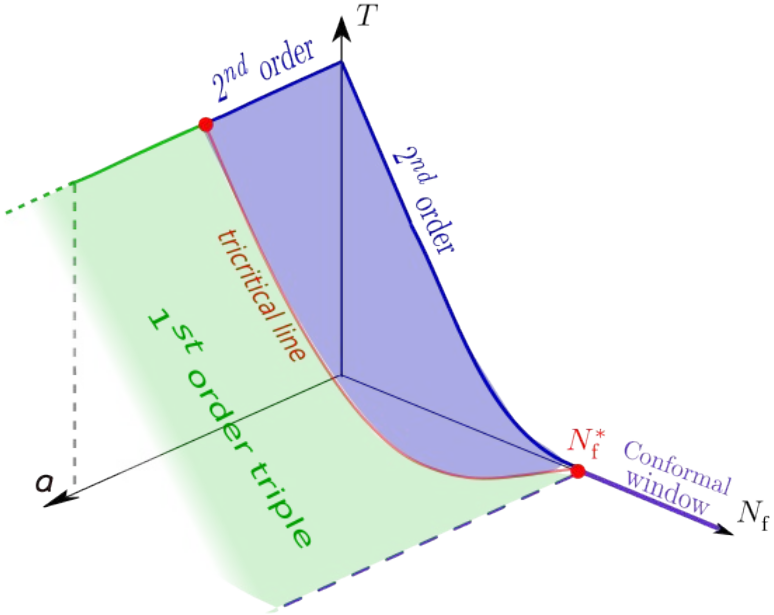
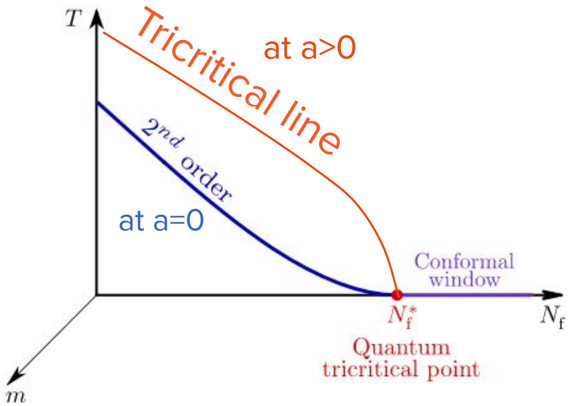
T on tricritical Line







Columbia Plot I

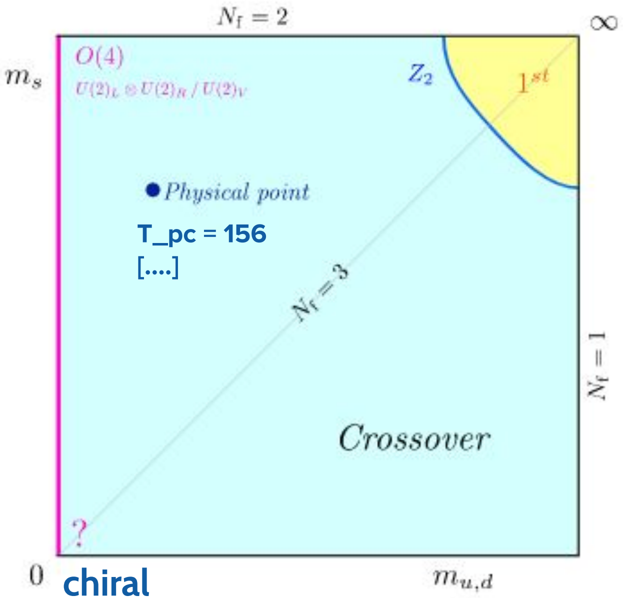
- order of thermal phase transition as a function of quark masses m_s and m_ud
- Recent finding [Philipsen, Cuteri, Sciara 21]: 2nd order in chiral limit for Nf=2 and Nf=3
  - contrary to theory [Pisarski, Wilzek]: Nf=3 is 1st order and Nf=2 depends on U(1)_A

T_c = 143
[Braun et al.]

T_c = 132(6)
[HotQCD]

T_c = 90
[HotQCD]

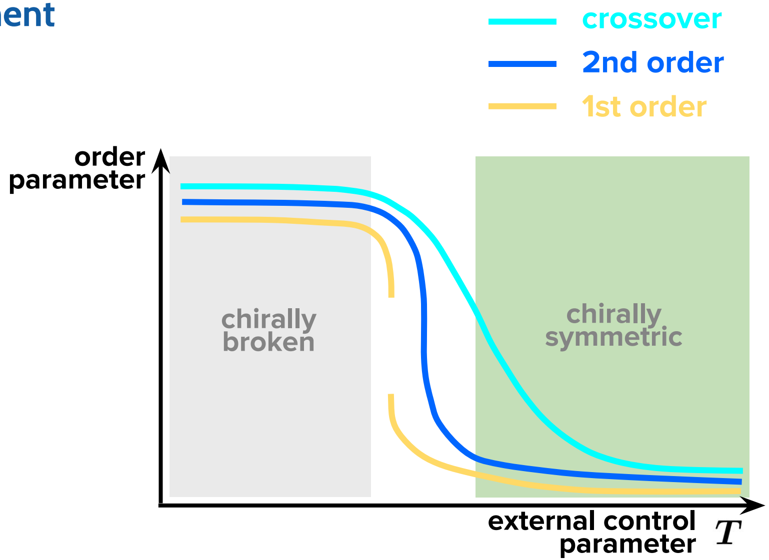
[Fischer]



T_pc = 156
[....]

Deconfinement transition

chiral transition



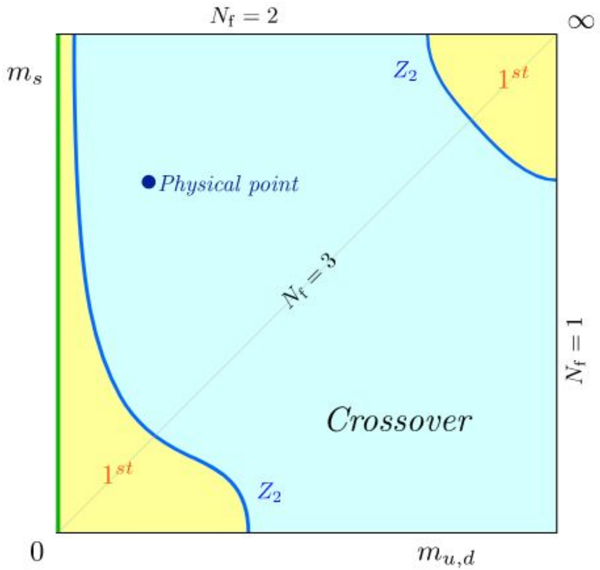
Columbia Plot II

Problem: Chiral limit not symulatable on the lattice

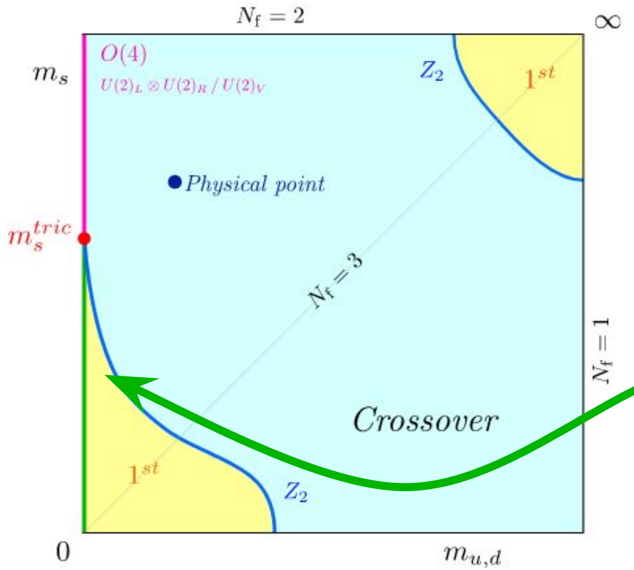
Theoretical prediction by Pisarski and Wilzek 1984:

- epsilon expansion in linear sigma model
- Nf >= 3 first order
- Nf = 2 depends on axial anomaly U(1)_A

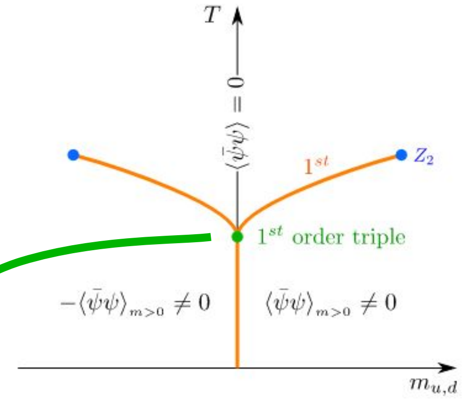
U(1)_A restored



U(1)_A broken



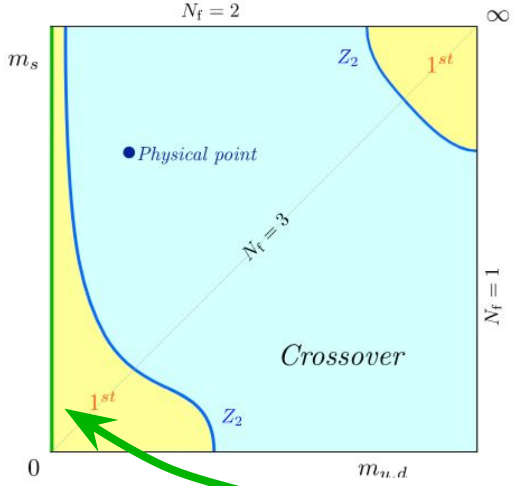
3-state coexistence for all ms<ms_tric



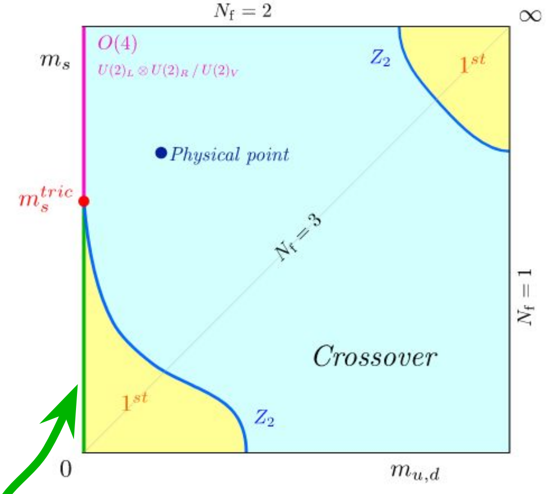
Columbia Plot III

Lattice:

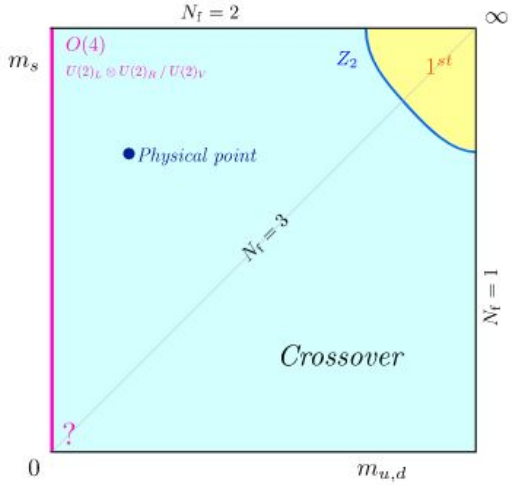
a1>0



a1>a2



Continuum limit



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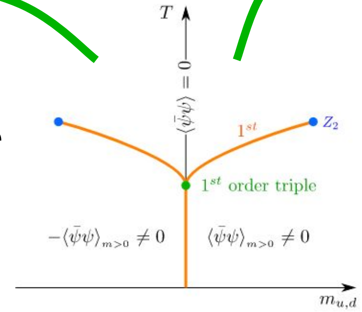
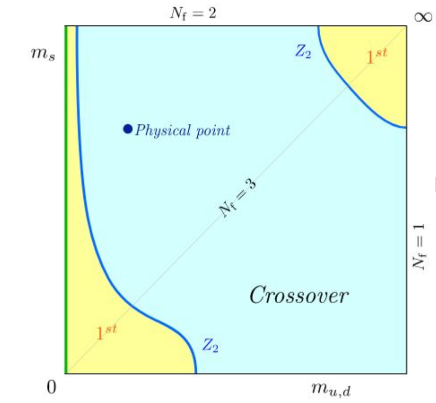


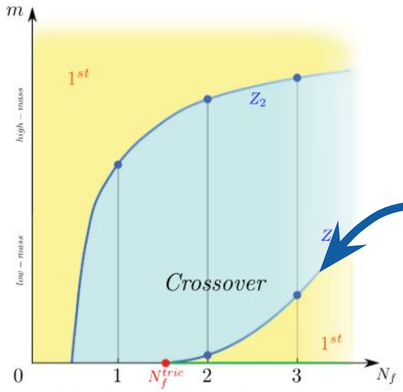
Tricritical scaling:

Known exponents for critical line entering tric. point

$$m_s^c(m_{u,d}) = m_s^{\mathrm{tric}} + \mathcal{A}_1 \cdot m_{u,d}^{2/5} + \mathcal{O}(m_{u,d}^{4/5}) \,,$$

COLUMBIA PLOT FOR DEGENERATE MASSES



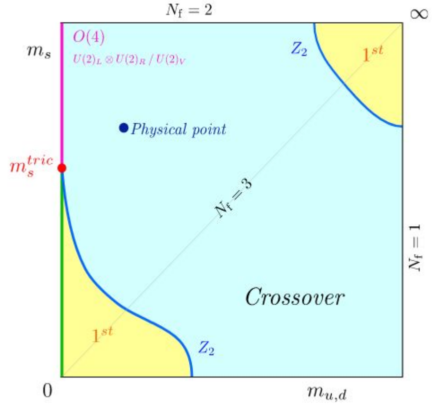


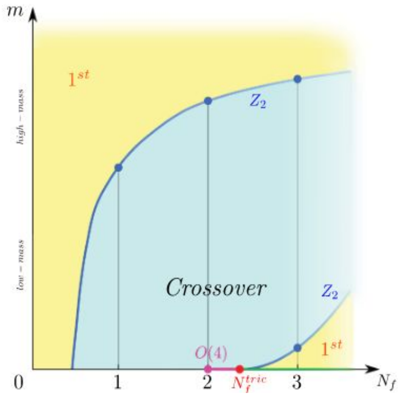
Tricritical scaling:

Nf_tric marks onset of 1st order transition

$$N_f^c(am) = N_f^{tric} + A \cdot am^{2/5} + \mathcal{O}(am^{4/5}).$$

If there is somewhere first order:
Nf_tric needs to exist





Our Work:

Map out Z2 phase boundary in m and Nf plane for several lattice spacings

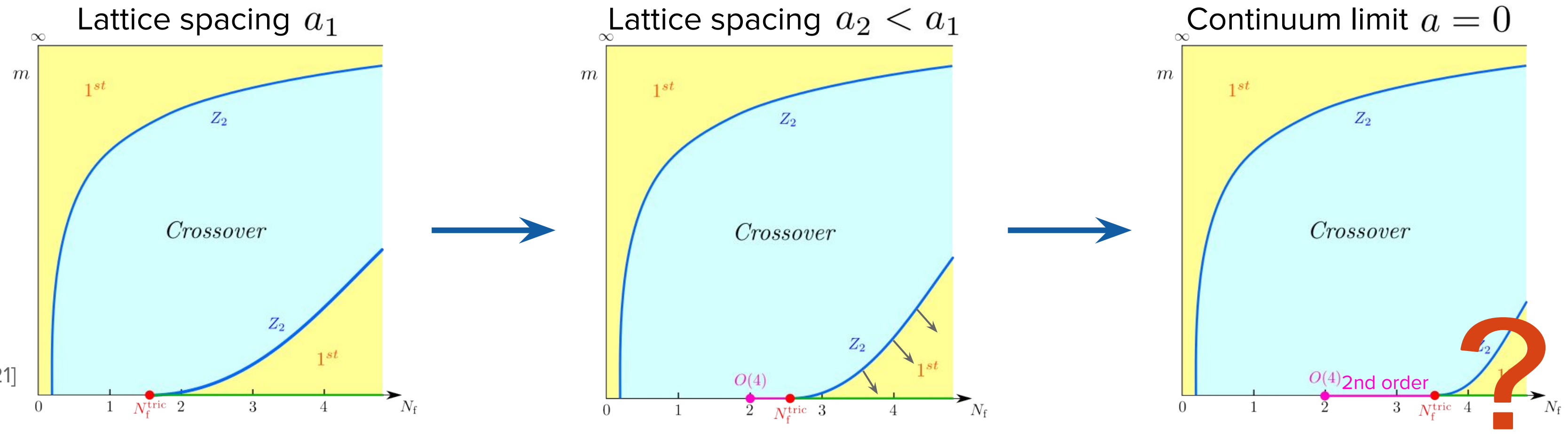
$$Z(N_\mathrm{f}, g, m) = \int \mathcal{D}A_\mu \,(\det M[A_\mu, m])^{N_\mathrm{f}}\, e^{-\mathcal{S}_{\mathrm{YM}}[A_\mu]}.$$

Find Nf_tric by fitting

1st order region is a cutoff effect

Strategy of our group:

- Map out Z2 phase boundary in (m, N_f)-plane
- Observation: 1st order region shrinks for decreasing lattice spacing



[Cuteri, Philipsen, Sciarra 2021]

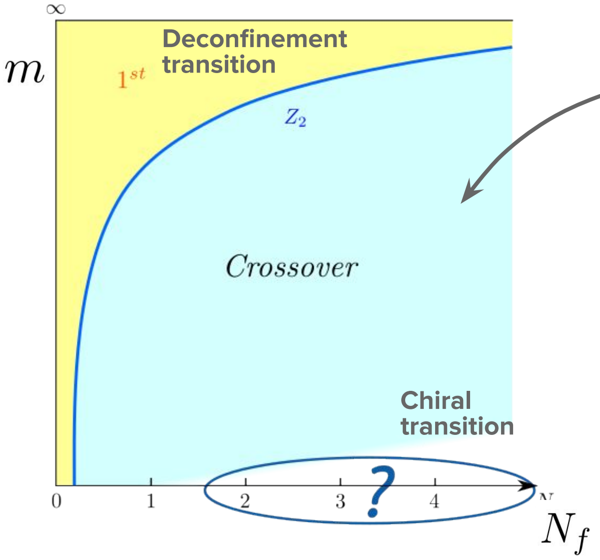
Key Question:

Does a 1st order transition remain for some N_f at $m = 0$?

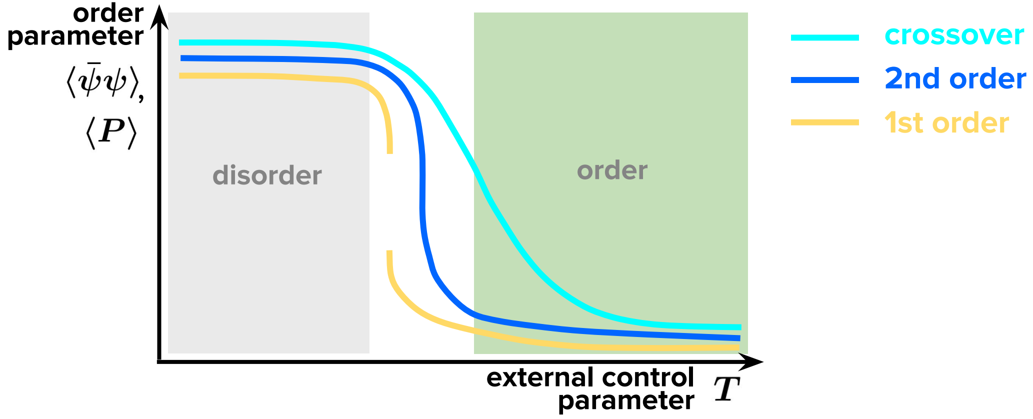
Learning by DefOrmING

Leave physical QCD:

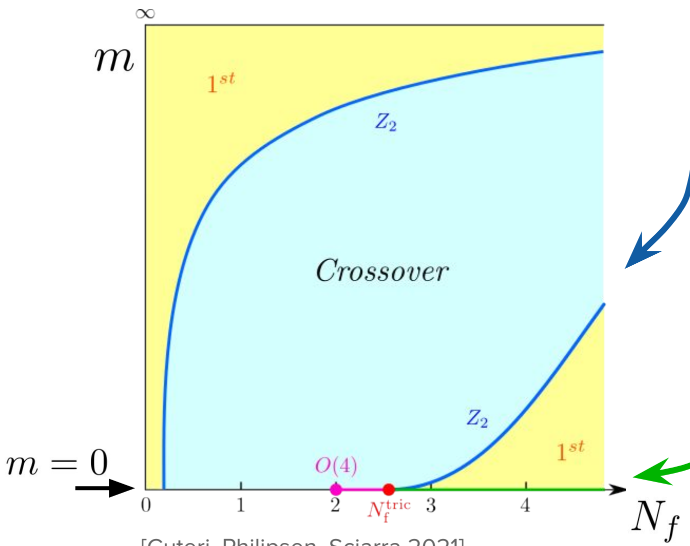
- Chiral symmetry & center symmetry are only approximate at physical point ($m_{u/d},\ m_s$)
  - mass is an interesting parameter to vary: chiral QCD (m=0) ↔ quenched QCD (m->infty)
- Columbia Plot: Study QCD with N_f degenerate quarks with mass m at $\mu = 0$
  - shows order of deconfinement and chiral thermal transition



Every point has its own (pseudo-) critical T_c :



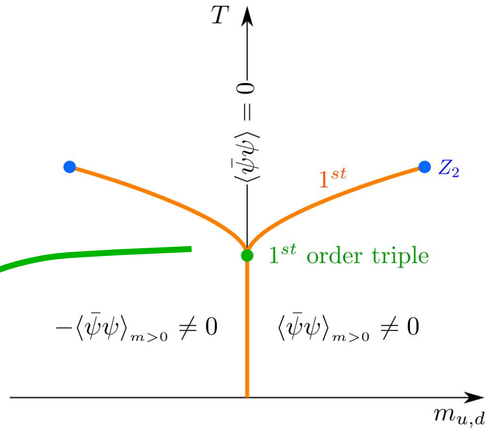
Type of phase transition in chiral QCD



[Cuteri, Philipsen, Sciarra 2021]

Tricritical scaling:

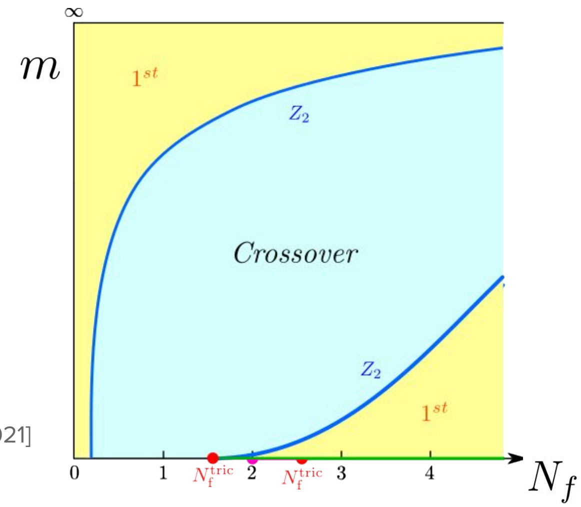
$$N_f^c(m) = N_f^{tric} + A \cdot m^{2/5} + \mathcal{O}(m^{4/5})$$



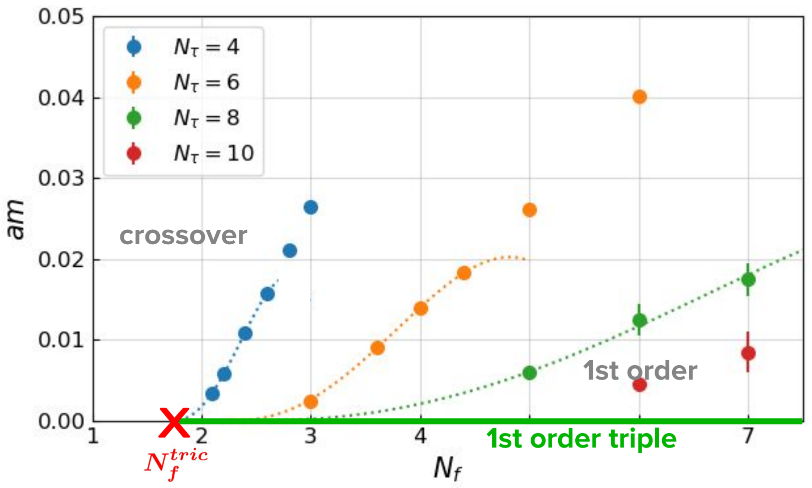
[Cuteri, Philipsen, Sciarra 2021]

- Triple points at $m = 0$: 3-state coexistence
- End of triple line: Tricritical point N_f^{tric}

Phase boundary for different lattice spacings



[Cuteri, Philipsen, Sciarra 2021]



- Z2-boundary $(\beta_c,\ am_c)$ was mapped out for 4 lattice spacings $N_\tau = 4,\ 6,\ 8,\ 10$ and $2 \leq N_f \leq 7$
- LO + NLO tricritical scaling fits describe data for small am :
  $N_f^c(am) = N_f^{tric} + A \cdot (am)^{2/5} + B \cdot (am)^{4/5} + \mathcal{O}((am)^{6/5})$
- 1st order region shrinks for decreasing lattice spacing (increasing N_τ)

➡ But: No statement about continuum limit and high N_f possible

What happens for Nf>6?

Onset of conformal window Nf*:

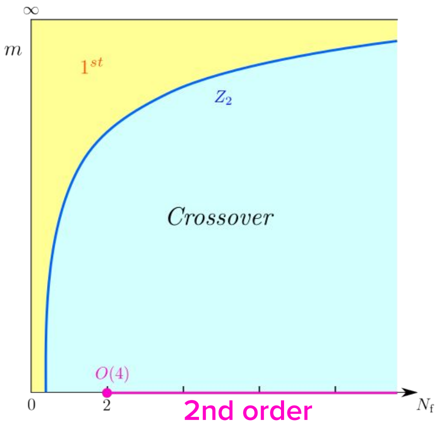
10<= Nf* <= 12 **[Braun, Gies]**

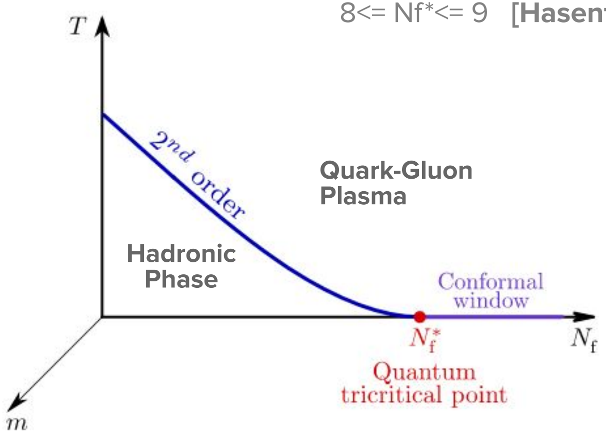
10<= Nf* <= 12 **[Lombarda, Pallante, Deuzeman]**

8<= Nf*<= 9 **[Hasenfratz et al.]**

Scenario 1

- 2nd order for all N_f
- Nftric = Nf*

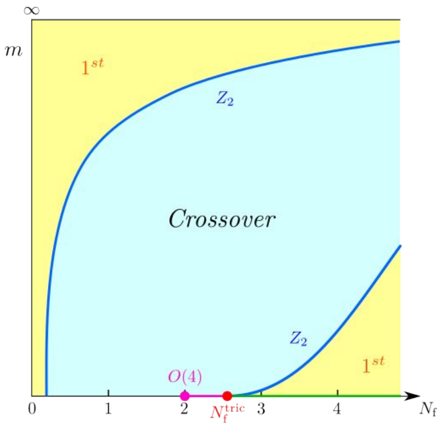


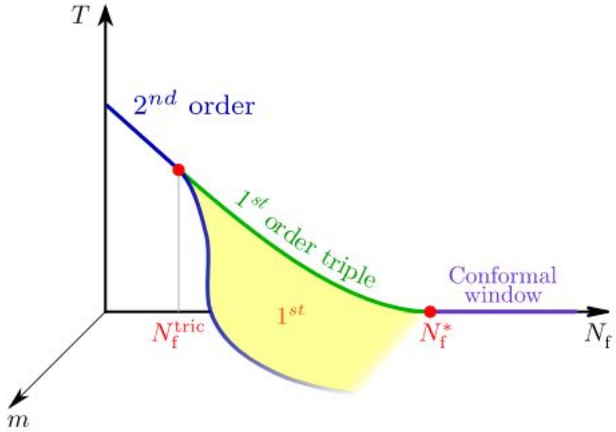


N_f^{tric} at $T = 0$

Scenario 2

- 2nd order turns into 1st order at N_f^{tric}
- Nf_tric < Nf*





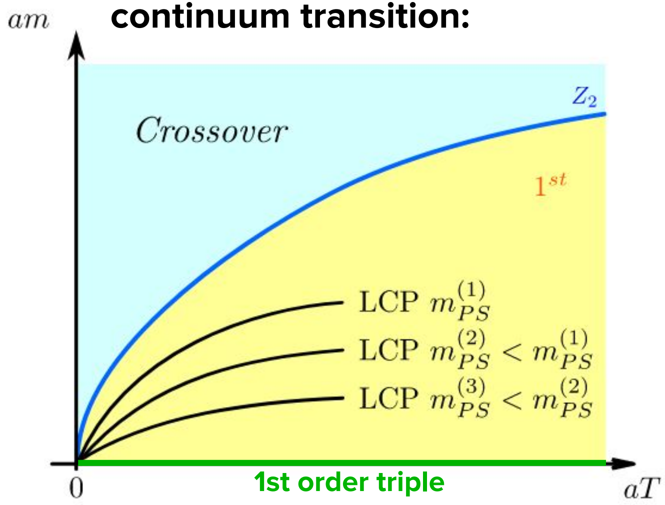
N_f^{tric} at $T > 0$

[Philipsen, Cuteri, Sciarra 21]

Chiral limit and the continuum limit

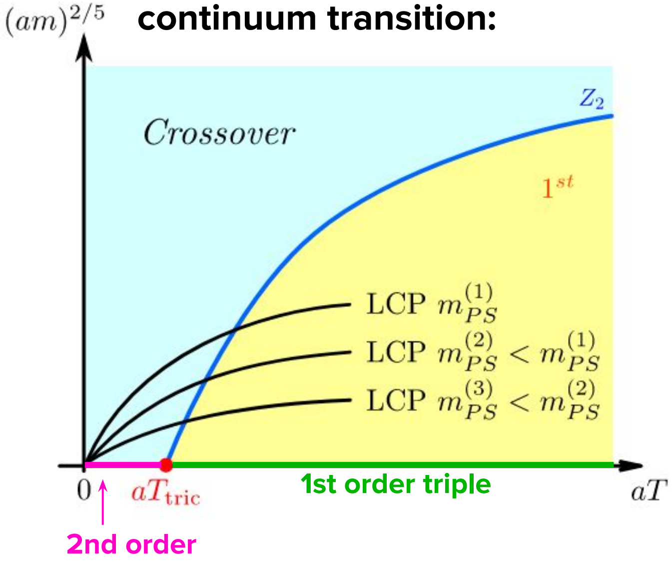
- Demand: First continuum limit ($a \to 0$), then chiral limit ($m \to 0$)
- We do neither: we only map out phase boundary

For 1st order continuum transition:

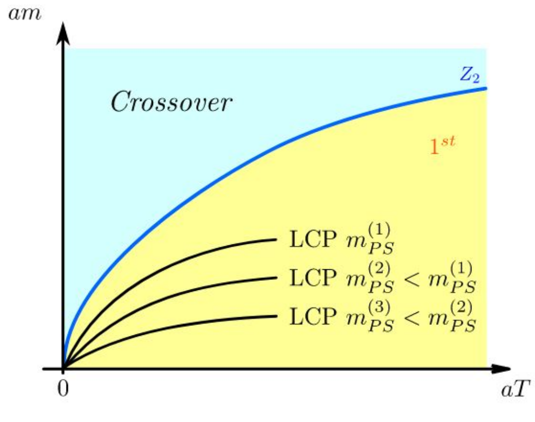


no tricritical scaling

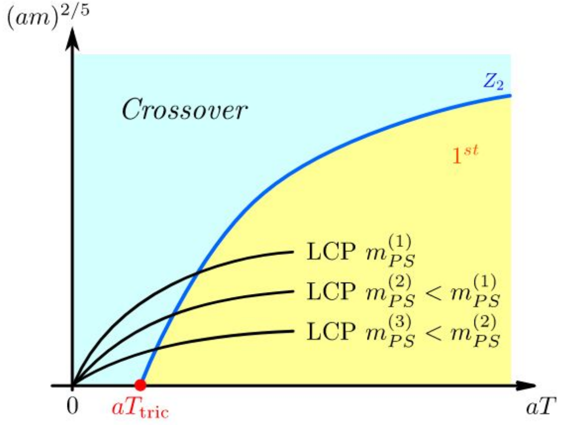
For 2nd order continuum transition:



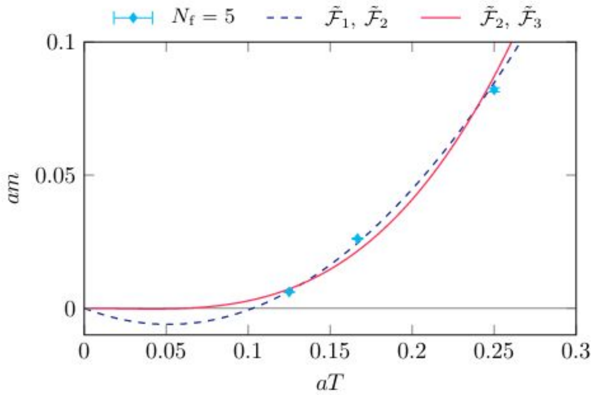
1st order alternative does not describe data

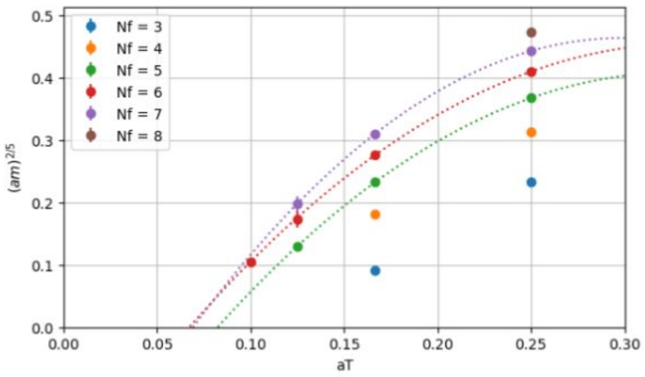


(a) First-order continuum transition.



(b) Second-order continuum transition.





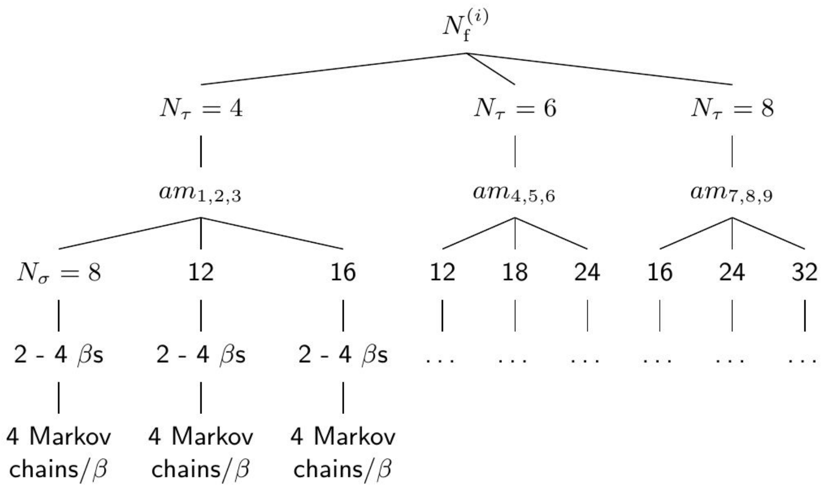
Computational Strategy

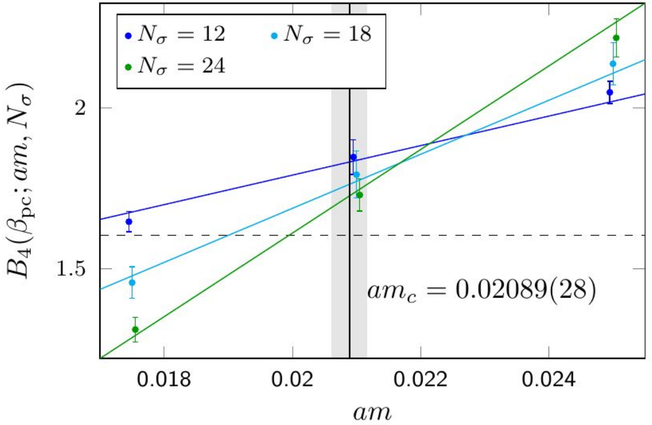
Finite size scaling formula of B_4

$$B_4(\beta_{\text{pc}}; am, N_\sigma) = (1.604 + Bx + \dots)\left(1 + C N_\sigma^{y_t - y_h} + \dots\right)$$

$y_t = 1/\nu$, y_h: Ising 3D critical exponents,
$x = (am - am_c) N_\sigma^{1/\nu}$: scaling variable

- fit finite size scaling formula to $B_4(\beta_{\text{pc}}; am, N_\sigma)$ values
- determine critical mass am_c as fit parameter

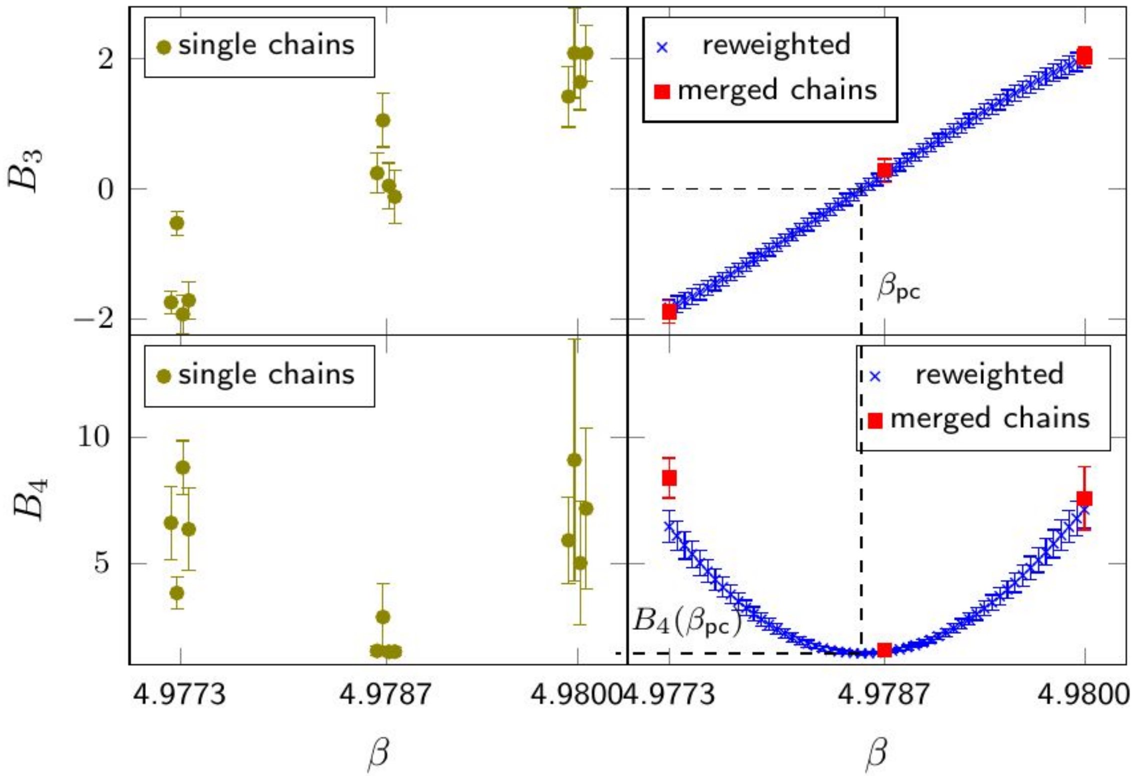




[Kaiser]

- order parameter $\mathcal{O}$: chiral condensate $\langle\bar{\psi}\psi\rangle$
- standardized moments: $B_n = \frac{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^n\rangle}{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^2\rangle^{n/2}}$
- phase boundary β_{pc}: $B_3(\beta_{\mathrm{pc}}; am, N_\sigma) = 0$
- order of the transition: $B_4(\beta_{\mathrm{pc}}; am, N_\sigma)$
- $B_4(N_\sigma \to \infty)$ values:

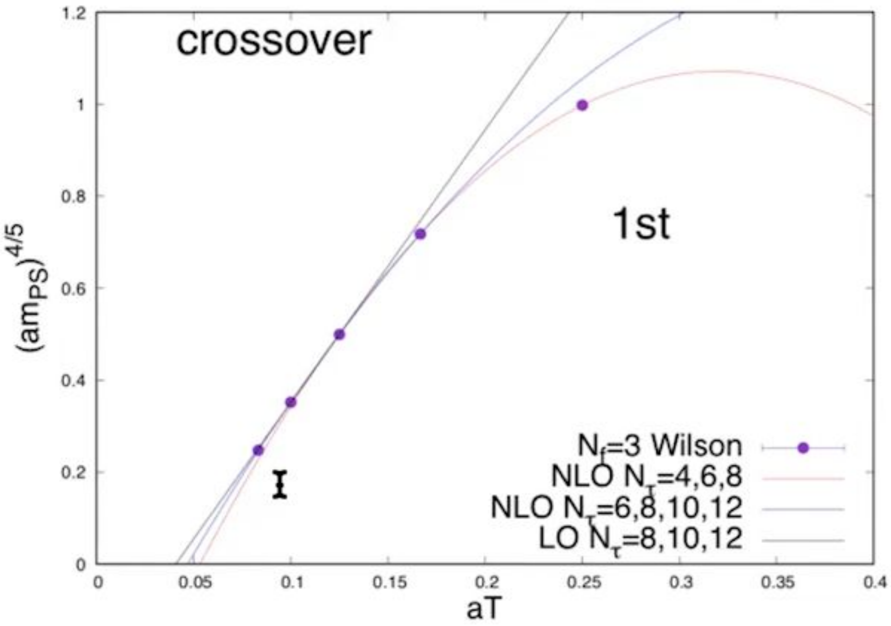
1. order	$Z(2)$ 2. order	crossover
1	1.604	3



Analysis for fixed μ_i, N_{f}, N_τ, am and N_σ.

O(a) improved Wilson fermions Nf=3

[Kuramashi et al. PRD 20] - consistent with tricritical scaling



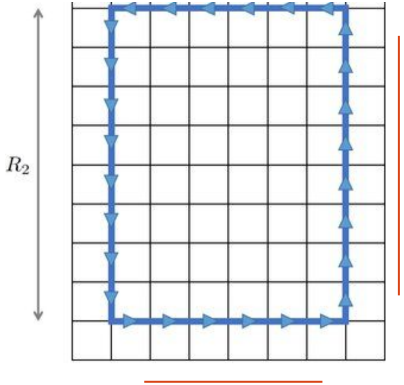
Determining the temperature

Scale Setting:

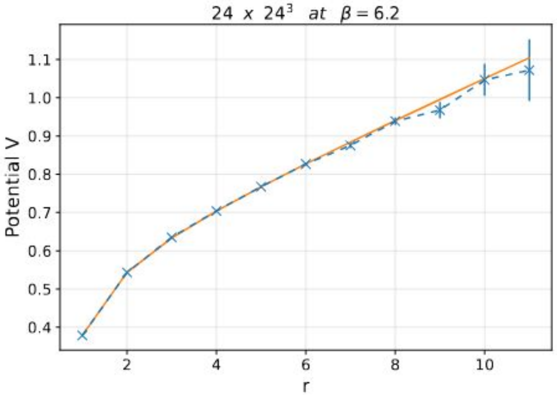
- T = 1/aNt ➔ need to determine lattice spacing a
- relate dimensionless observables on the lattice to physical quantities
- Sommer-scale: characteristic length-scale based on force F(r) between two static quarks

$$F(r_0)r_0^2 = 1.65 \text{ corresponds to } r_0 \simeq 0.5 \text{ fm}$$

- Get F(r) via static quark potential V(r): F(r)=d/dr V(r)
- Get V(r) via Wilson-Loops $\langle W_\mathcal{C} \rangle \sim e^{-F_{q\bar{q}}(\mathcal{C})} = e^{-V(r)n_t}$
    - $W_\mathcal{C} = \mathrm{Tr}\left[\prod_{(n,\mu)\in\mathcal{C}} U_\mu(n)\right]$



Wilson loop



static quark potential

$$V(r) = A + \frac{B}{r} + \sigma r$$

Symmetries of QCD

$$\mathcal{L} = \bar{\psi}\left(\gamma_\mu D_\mu + m\right)\psi + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$$

Chiral Symmetry

- $SU(N_f)_L \times SU(N_f)_R$ - Symmetry
- Projections: $\psi_{L/R} = \frac{1 \mp \gamma_5}{2}\psi$
  - $\mathcal{L}_D = \bar{\psi}\partial\!\!\!/\psi = \bar{\psi}_L\partial\!\!\!/\psi_L + \bar{\psi}_R\partial\!\!\!/\psi_R$
- Order parameter: chiral condensate
  - $\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\rangle = \begin{cases} 0, & \text{symmetric} \\ \neq 0, & \text{broken} \end{cases}$
- Mass term breaks symmetry explicitly
  - $m\bar{\psi}\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$

➡ Symmetry only exact for $m = 0$

Center Symmetry

- Global Z_3 - Symmetry
  - only for pure Yang-Mills
- Order parameter: polyakov loop
  - $\langle P\rangle = \begin{cases} 0, & \text{confined} \quad \text{(center symmetric)} \\ \neq 0, & \text{deconfined} \quad \text{(center broken)} \end{cases}$
- Dynamical quarks break symmetry explicitly
  - broken by $\det D$

➡ Symmetry only exact for $m \to \infty$
- since $\lim_{m\to\infty} \det D = 1$

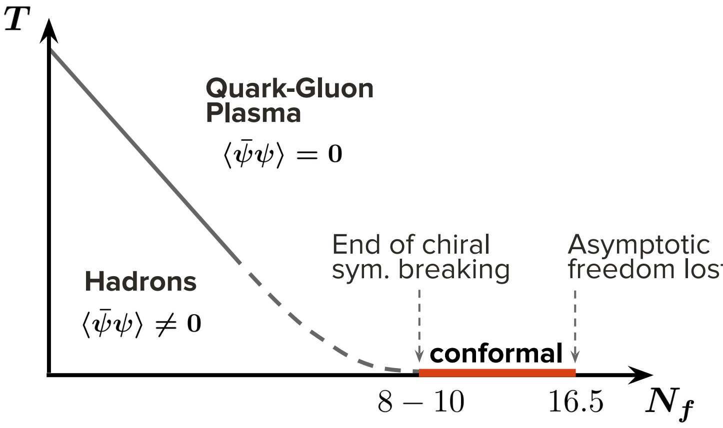
chiral QCD
$m = 0$

⟷ **quark mass** ⟷

quenched QCD
$m \to \infty$

Conformal Window

For continuum QCD at $m = 0$



Large Nf

- QCD is conformal/scale-invariant at $T = 0$
- Bank Saks IR fixed point: α^*

  ↳ $\beta(\alpha^*) = \mu \dfrac{\partial}{\partial\mu}\alpha^*(\mu) \overset{!}{=} 0$

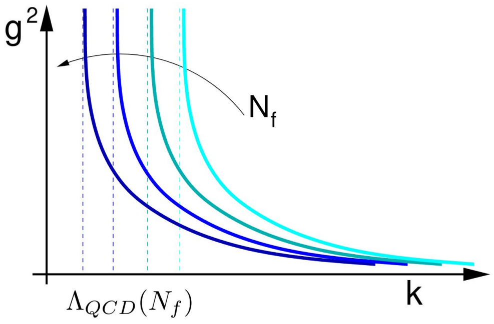
- No chiral symmetry breaking anymore
- Onset expected: $8 \lesssim N_f \lesssim 10$
  - [Braun, Gies 06], [Lombardo 10, 12]

Small Nf ($\lesssim 8$)

- No scale expect Λ_{QCD}

  ↳ $T_{\chi SB},\ f_\pi,\ |\langle\bar{\Psi}\Psi\rangle|^{1/3}, \ldots \ \sim \Lambda_{QCD}$

- From perturbation theory:



[Braun 12]

- Λ_{QCD} decreases linearly with N_f

  ↳ $T_{\chi SB} \sim \Lambda_{QCD} \approx 1 - \epsilon N_f + \mathcal{O}\left[(\epsilon N_f)^2\right]$

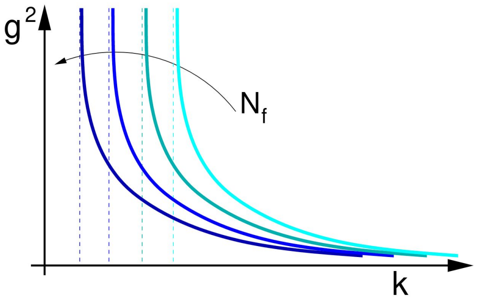
  ↳ $\Delta T_{\chi SB} = T(N_f) - T(N_f + 1) \approx 25\ MeV$

Coupling vs. Chiral Symmetry

$N_f < 8$: **Without IR fixpoint**

- Divergent interaction strength in the IR
- Λ_{QCD} decreases with increasing N_f

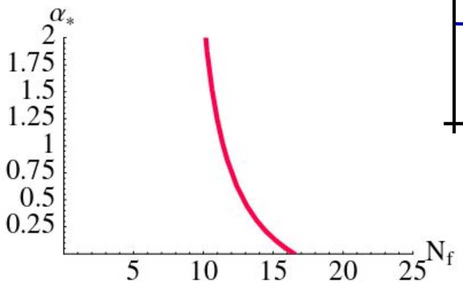
Strong interactions in the IR:
➡ spontaneous symmetry breaking



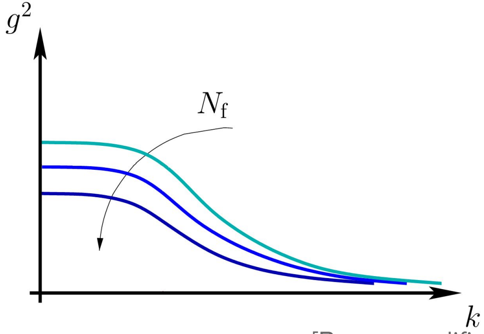
[Braun, modified]

$8.05 \leq N_f \leq 16.5$: **With IR fixed point**

- coupling saturates at fixed point $\alpha^*(N_f) = -\frac{b}{c}$
- fixed point α^* increases with decreasing N_f



[Braun]



[Braun, modified]

Weak interactions in the IR:
➡ chirally symmetric

Recent Standing

Critical Flavour Number N_f^{cr}

fRG: rather 12

$$10 \lesssim N_f^{cr} \lesssim 12$$

Gies, Jäckel '05
Braun, Gies '05, '06
Braun, Fischer, Gies '11

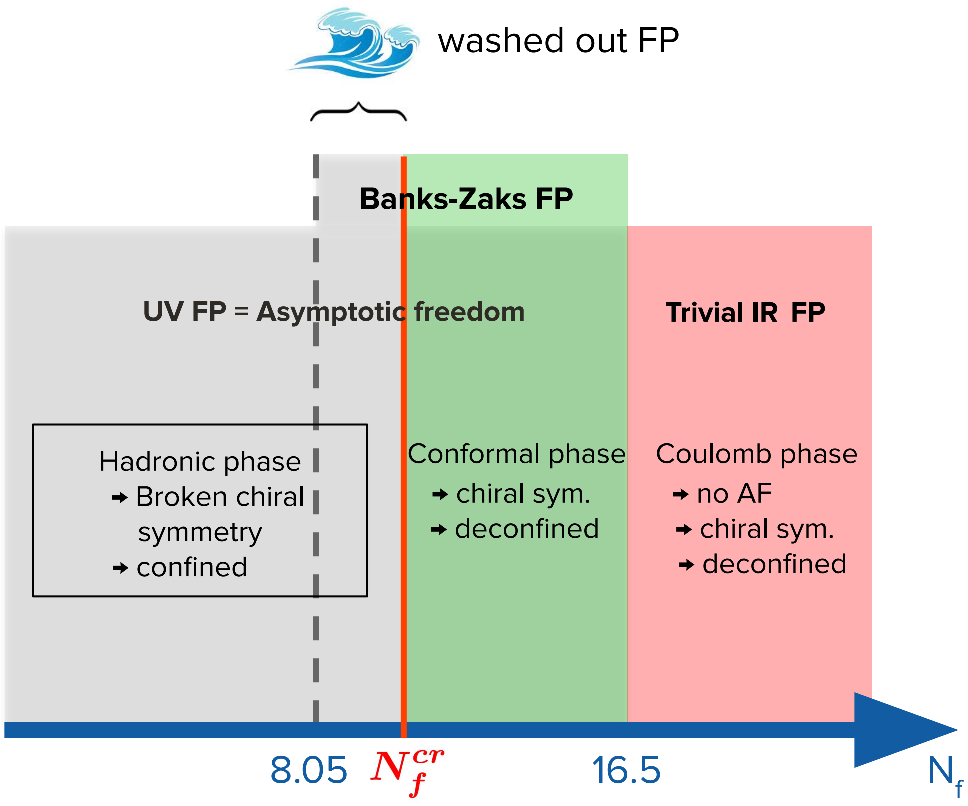
Lattice: rather 10

$$10 \lesssim N_f^{cr} \lesssim 12$$

Appelquist, Fleming, Neil '08, '09
Fodor et al. '08, '09
Fodor, Holland, Kuti, Nogradi, Schroeder '09
Jin, Mawhinney '09
Deuzeman, Lombardo, Pallante '08, '10, '12

$$8 \lesssim N_f^* \lesssim 9$$

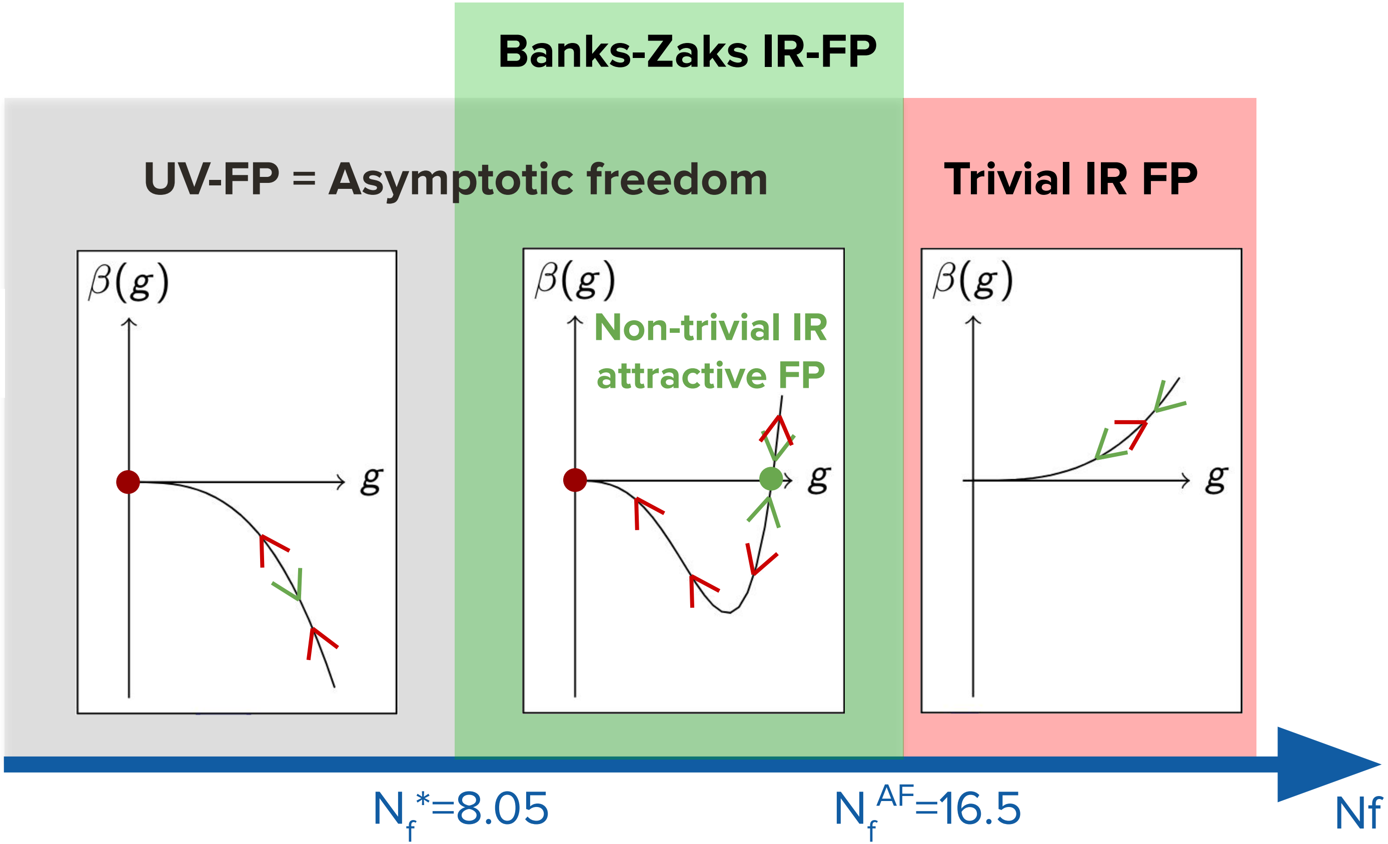
Hasenfratz et al. '15, '18, '23



Banks-Zaks Fixed Point

$$\beta(\alpha) = \mu \frac{\partial}{\partial \mu} \alpha(\mu)$$

Towards UV
Towards IR



Temperature Scaling I: small Nf

- All IR observables $T_{\chi SB},\ f_\pi,\ |\langle\bar\Psi\Psi\rangle|^{1/3},\ldots \sim \Lambda_{QCD}$
- Estimation of Λ_{QCD} :

$$\mu\frac{\partial}{\partial\mu}\alpha(\mu) = -b\alpha^2(\mu) \quad with \quad b = \frac{1}{6\pi}(11N_c - 2N_f)$$

Integrate

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\mu_0)} + b\ln\frac{\mu}{\mu_0}$$

$$\frac{1}{\alpha(\Lambda_{QCD})} = \frac{1}{\alpha(\mu_0)} + b\ln\frac{\Lambda_{QCD}}{\mu_0} \to 0$$

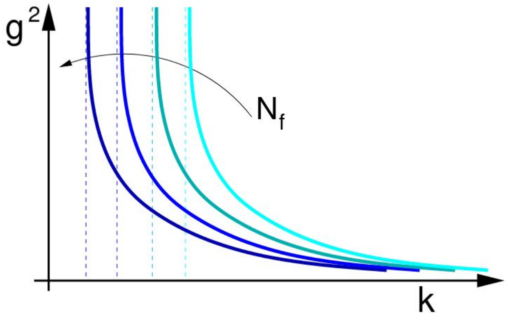
Linearity

$$T_{\chi SB} \sim \Lambda_{QCD} \approx \mu_0 \exp\left[-\frac{1}{b\alpha(\mu_0)}\right]$$

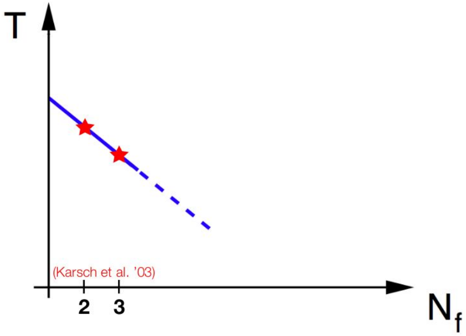
$$\approx \mu_0 \exp\left[-\frac{6\pi}{11N_c\alpha(\mu_0)}\right]\left(1 - \epsilon N_f + \mathcal{O}\left[(\epsilon N_f)^2\right]\right)$$

$$\epsilon = \frac{12\pi}{121N_c^2\alpha(\mu_0)} \approx 0.107 \quad for \quad N_c = 3 \quad and \quad \mu_0 = m_\tau$$

Λ_{QCD} decreases with increasing N_f



[Braun, modified]



[Braun]

$$\Delta T_{\chi SB} = T(N_f) - T(N_f + 1) \approx 25\ MeV$$

Temperature Scaling II: Power-law

- Assumption: onset of chiral symmetry breaking requires $g^2 > g^2_{cr}$
- k_{cr} is defined by $g^2_*(N_f) = g^2_{cr}$

- Estimation of k_{cr} by linearizing beta function:

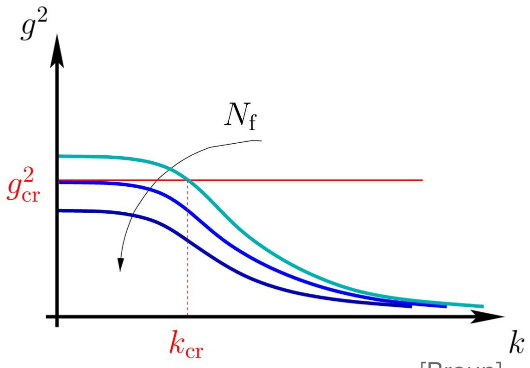
$$k\frac{\partial}{\partial k}g^2 = -\Theta(g^2 - g^2_*) + \mathcal{O}\left[(g^2 - g^2_*)^2\right]$$

Integrate

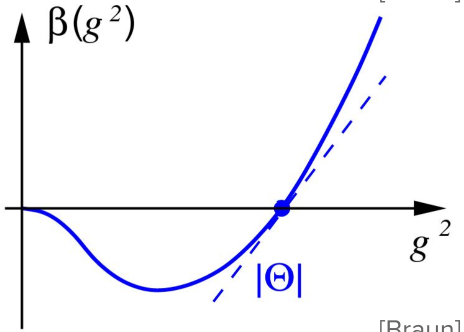
$$g^2(k) = g^2_* - \left(\frac{k}{k_0}\right)^{|\Theta|}$$

$g(k_{cr}) = g_{cr}$

$$k_{cr} \simeq k_0 \left(g^2_* - g^2_{cr}\right)^{\frac{1}{|\Theta|}}$$



[Braun]



[Braun]

- Linearize coupling in Nf: $g^2_*(N_f) - g^2_{cr}(N_f^{cr}) = \alpha(N_f - N_f^{cr}) + \mathcal{O}\left[(N_f - N_f^{cr})^2\right]$

$$T_{\chi SB} \sim k_{cr} \simeq k_0 \left|N_f - N_f^{cr}\right|^{\frac{1}{|\Theta|}}$$

Temperature Scaling: Total

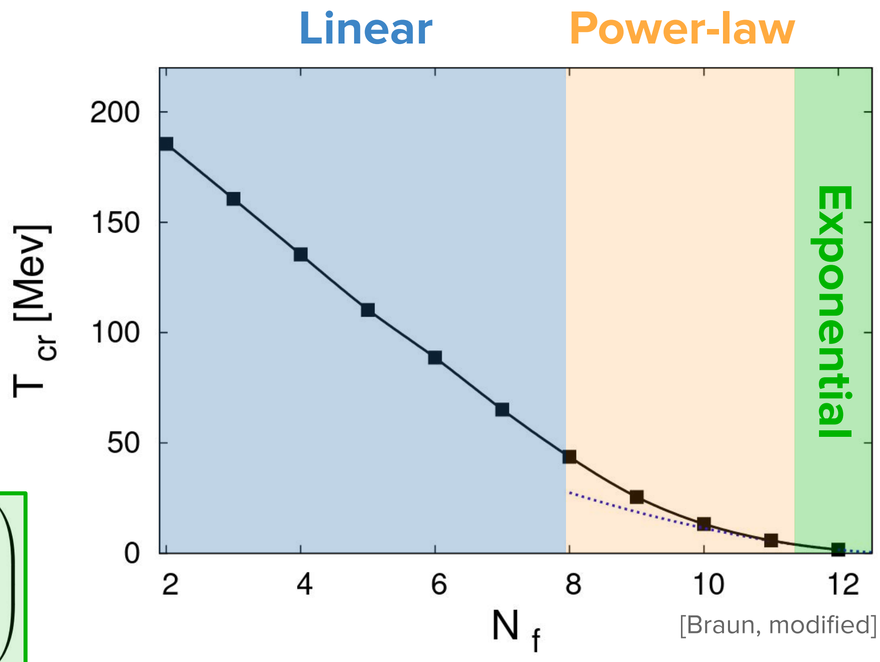
Small Nf: Linear Scaling

$$k_{\chi SB} \sim k_0 \exp\left[-\frac{6\pi}{11 N_c \alpha(k_0)}\right] \left(1 - \epsilon N_f + \mathcal{O}\left[(\epsilon N_f)^2\right]\right)$$

Large Nf: Power-Law and Miransky Scaling

$$k_{\chi SB} \sim k_0 \theta(N_f^{cr} - N_f) \left|N_f^{cr} - N_f\right|^{\frac{1}{|\Theta|}} \exp\left(-\frac{\text{const}}{\sqrt{\left|N_f^{cr} - N_f\right|}}\right)$$

$$\mathcal{O} \sim f_{\mathcal{O}}(N_f)\,(k_{\chi SB})^{d_{\mathcal{O}}}\,, \qquad \mathcal{O} = T_{\chi SB}, f_\pi, |\langle\bar{\Psi}\Psi\rangle|, m_{c.q.}\ \ldots$$



[Braun, modified]

Braun, Gies: $N_f^{cr} = 12 \ \&\ |\Theta| < 1$

Disclaimer: $k_{\chi SB}$ is upper limit for onset of chiral symmetry