First-order phase transitions in the heavy quark region of lattice QCD at high temperatures and high densities

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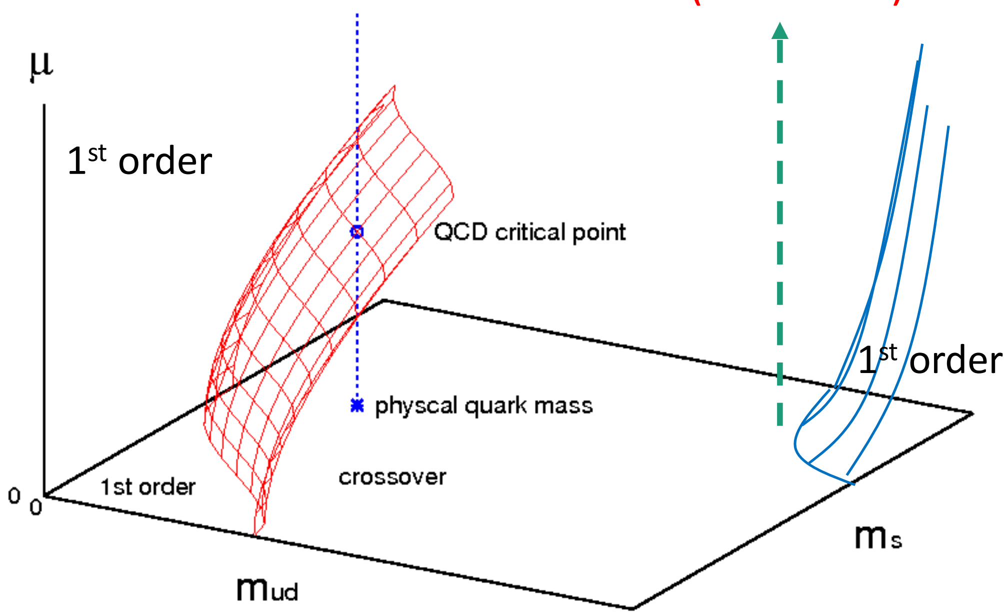
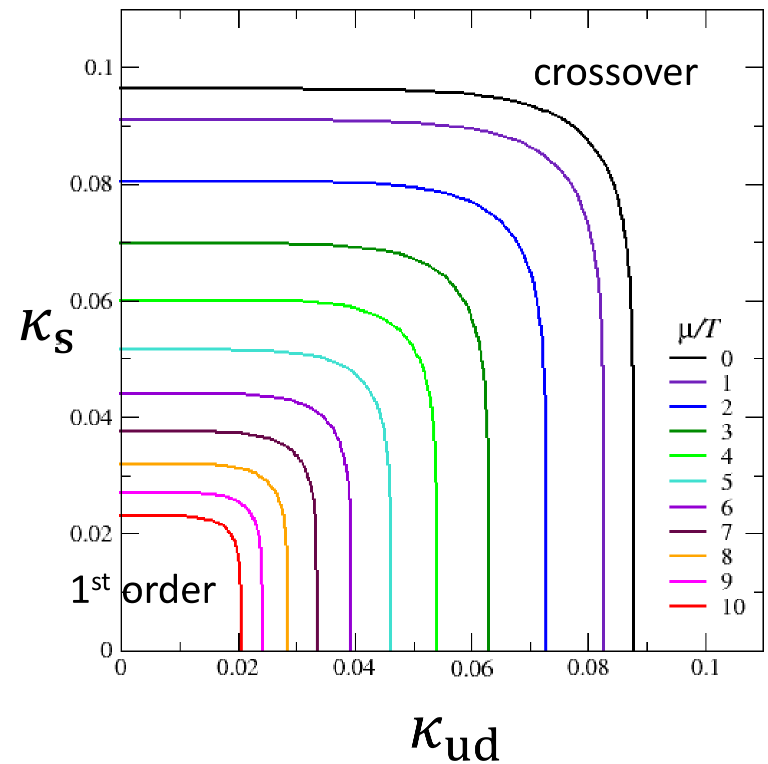
Lattice 2024, Liverpool, 2024/7/30

Boundaries of first-order transitions in QCD phase diagram



Boundaries of First-order transition in the heavy quark region (arXiv:2311.11508) $N_t = 6$



- We expect that the first-order phase transition in the light mass region expands with increasing density.
- The first-order phase transition region may expand into the heavy quark region.
- We discuss the appearance of first-order phase transitions in the heavy and dense region.

- Effective theory based on the hopping parameter expansion
- Sign problem is mild
- First-order region: narrower as μ

Effective theory based on the hopping parameter expansion

- We expand the quark determinant in terms of the hopping parameter κ.
- The term that winds around the periodic boundary in the time direction is important.

$$L(N_t,n) = \sum_m L_m(N_t,n) = \sum_m \left(L_m^+(N_t,n) + L_m^-(N_t,n)\right)$$

- Higher order expansion terms $L(N_t,n)$ is very strongly correlated with the leading term: Polyakov loop Ω. $m=1$ is dominant: $L_1(N_t,n) \approx L(N_t,n)$.

$$L(N_t,n) = L^0(N_t,n)c_n \,\mathrm{Re}\Omega$$

(PTEP 2022, 033B05)

$$\mathrm{Arg}\, L_1^+(N_t,n) \approx \mathrm{Arg}\,\Omega$$

(arXiv:2311.11508)

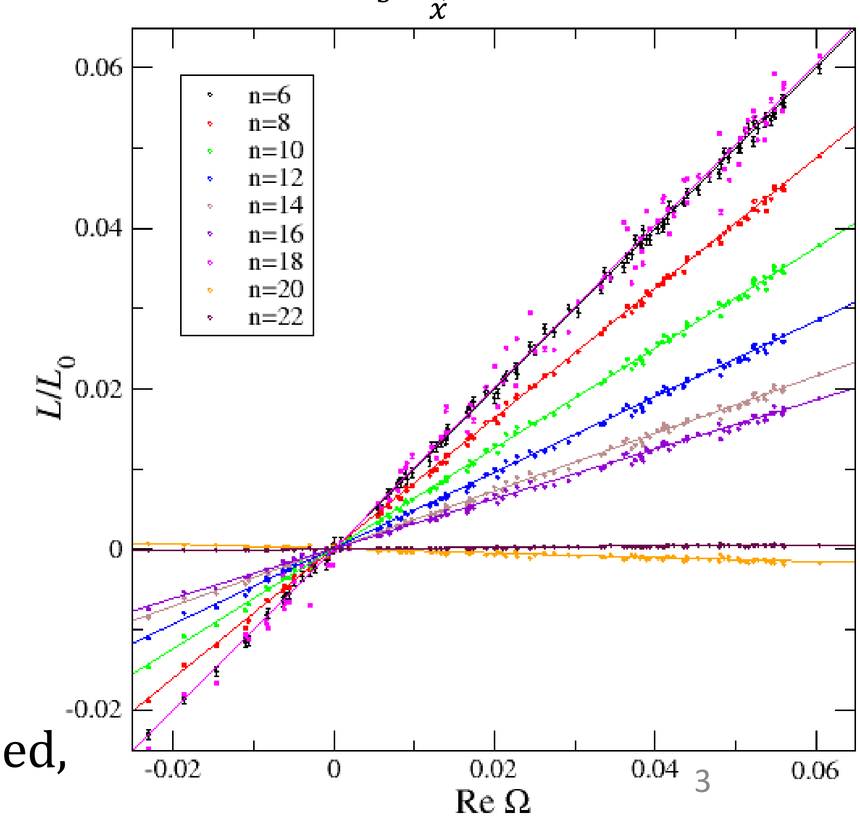
$$\Omega = \frac{1}{3N_s^3}\sum_{\vec{x}} \mathrm{tr}\,[U_4 U_4 U_4 \cdots U_4]$$

- **Effective action in the heavy quark region**

$$S_{\mathrm{eff}} = -6N_{\mathrm{site}}\beta^* P - \frac{N_s^3\lambda}{2}\left(e^{\frac{\mu}{T}}\Omega + e^{-\frac{\mu}{T}}\Omega^*\right)$$

$$\lambda = N_t \sum_{f=1}^{N_f}\sum_{n=N_t}^{n_{\max}} L^0(N_t,n)\, c_n \kappa_f^n$$

- Even if the number of expansion terms increases significantly, the effects of higher-order terms can be incorporated.
- Since the calculation cost can be dramatically reduced, calculations with high accuracy are possible.



Finite density Effect

$$S_{\text{eff}} = -6N_{\text{site}}\beta^* P - N_s^3\lambda\left(\cosh\frac{\mu}{T}\,\Omega_{\text{R}} + i\sinh\frac{\mu}{T}\,\Omega_{\text{I}}\right) \qquad (\Omega = \Omega_{\text{R}} + i\Omega_{\text{I}})$$

- Partition function:

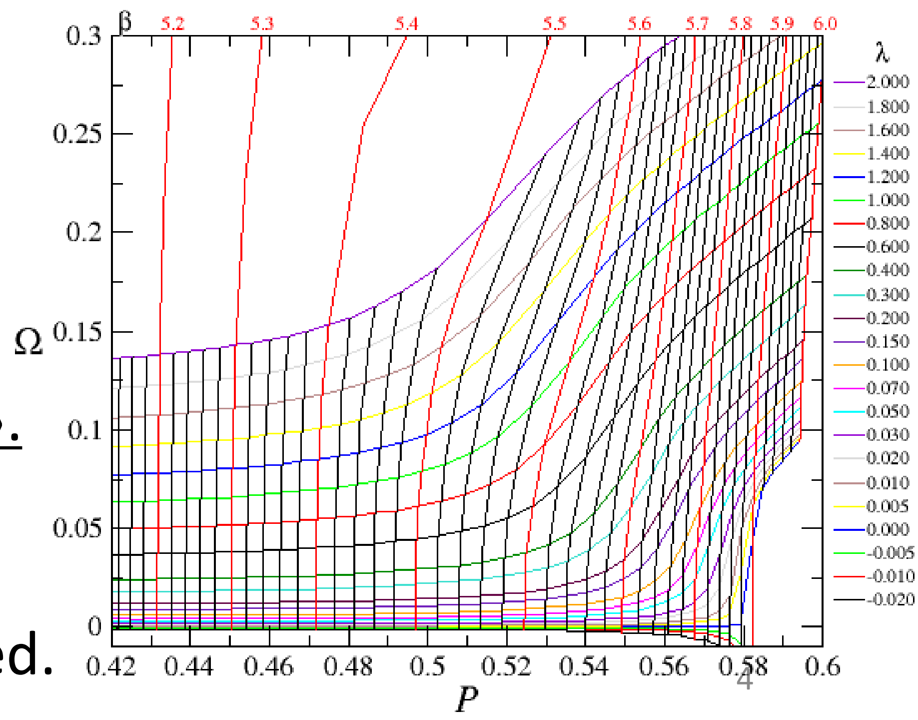
$$Z = \int DU\ e^{-S} = \int \underbrace{W(P,\Omega_{\text{R}})\, e^{6N_S^3N_t\beta P}\ e^{N_S^3\lambda\cosh\frac{\mu}{T}\Omega_{\text{R}}}\left\langle\cos\left(N_S^3\lambda\sinh\frac{\mu}{T}\,\Omega_{\text{I}}\right)\right\rangle_{P,\Omega_{\text{R}}}}_{=F(P,\Omega_{\text{R}})}\ dPd\Omega_{\text{R}}$$

- $W(P,\Omega_{\text{R}})$ is the provability distribution function in terms of (P,Ω_{R}).
- For the case of $\mu = 0$, when configurations are generated at β and λ, the following equations are satisfied at (P,Ω_{R}) where the configuration generation probability is maximized.

$$\frac{\partial F(P,\Omega_{\text{R}})}{\partial P} = 6N_S^3N_t\beta + \frac{\partial \ln W(P,\Omega_{\text{R}})}{\partial P} = 0$$

$$\frac{\partial F(P,\Omega_{\text{R}})}{\partial \Omega_{\text{R}}} = N_S^3\lambda + \frac{\partial \ln W(P,\Omega_{\text{R}})}{\partial \Omega_{\text{R}}} = 0$$

- The peak position of $F(P,\Omega_{\text{R}})$: $\approx \langle P\rangle, \langle\Omega_{\text{R}}\rangle$.
- Right figure: $\langle P\rangle, \langle\Omega_{\text{R}}\rangle$ as functions of β, λ.
- $\frac{\partial \ln W(P,\Omega_{\text{R}})}{\partial P}$ and $\frac{\partial \ln W(P,\Omega_{\text{R}})}{\partial \Omega_{\text{R}}}$ can be measured.



Effect of the complex phase at finite μ

- For finite μ, the peak position is

$$\frac{\partial F(P,\Omega_{\mathrm{R}})}{\partial P} = 6N_S^3 N_t \beta + \frac{\partial \ln W}{\partial P} + \frac{\partial \ln \left\langle \cos\left(N_s^3 \lambda \sinh \frac{\mu}{T} \Omega_{\mathrm{I}}\right)\right\rangle_{P,\Omega_{\mathrm{R}}}}{\partial P} = 0$$

$$\frac{\partial F(P,\Omega_{\mathrm{R}})}{\partial \Omega_{\mathrm{R}}} = N_s^3 \lambda \cosh \frac{\mu}{T} + \frac{\partial \ln W}{\partial \Omega_{\mathrm{R}}} + \frac{\partial \ln \left\langle \cos\left(N_s^3 \lambda \sinh \frac{\mu}{T} \Omega_{\mathrm{I}}\right)\right\rangle_{P,\Omega_{\mathrm{R}}}}{\partial \Omega_{\mathrm{R}}} = 0$$

- Avoiding the sign problem, cumulant expansion:

$$\ln \left\langle \cos\left(N_s^3 \lambda \sinh \frac{\mu}{T} \Omega_{\mathrm{I}}\right)\right\rangle_{P,\Omega_{\mathrm{R}}} = -\frac{1}{2}\left\langle \left(N_s^3 \lambda \sinh \frac{\mu}{T} \Omega_{\mathrm{I}}\right)^2 \right\rangle_c + \frac{1}{4!}\left\langle \left(N_s^3 \lambda \sinh \frac{\mu}{T} \Omega_{\mathrm{I}}\right)^4 \right\rangle_c - \frac{1}{6!}\left\langle \left(N_s^3 \lambda \sinh \frac{\mu}{T} \Omega_{\mathrm{I}}\right)^6 \right\rangle_c + \cdots$$

$$\langle \theta^2 \rangle_c = \langle \theta^2 \rangle, \qquad \langle \theta^4 \rangle_c = \langle \theta^4 \rangle - 3\langle \theta^2 \rangle^2, \qquad \langle \theta^6 \rangle_c = \langle \theta^6 \rangle - 15\langle \theta^4 \rangle\langle \theta^2 \rangle + 30\langle \theta^2 \rangle^3, \cdots$$

- We approximate $\ln \left\langle \cos\left(N_s^3 \lambda \sinh \frac{\mu}{T} \Omega_{\mathrm{I}}\right)\right\rangle_{P,\Omega_{\mathrm{R}}} \approx -\frac{1}{2}\left(N_s^3 \lambda \sinh \frac{\mu}{T}\right)^2 \langle \Omega_{\mathrm{I}}^2 \rangle$

  for qualitative estimation. (Gaussian approximation)

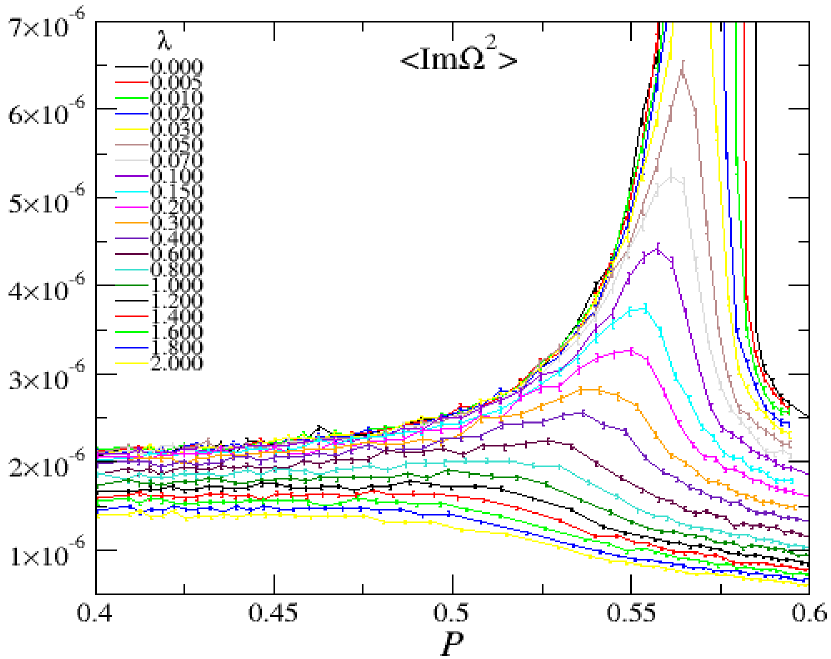
- If we write (β, λ) that are generated by configuration generation at $\mu = 0$ to be $(\langle P \rangle, \langle \Omega_{\mathrm{R}} \rangle)$ as (β_0, λ_0), then (β, λ) that are $(\langle P \rangle, \langle \Omega_{\mathrm{R}} \rangle)$ at finite μ are

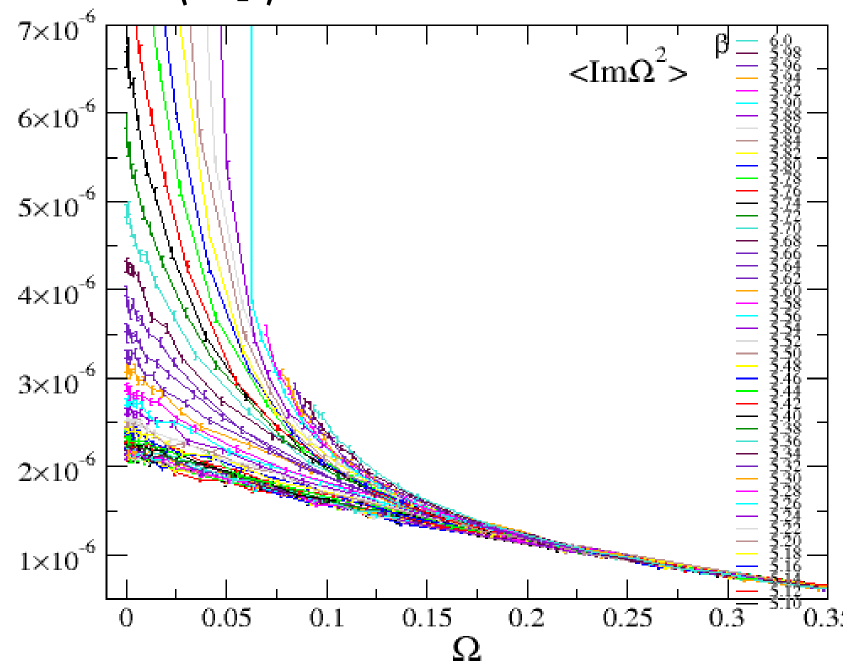
$$\beta = \beta_0 + \frac{N_s^3}{12 N_t}\left(\lambda \sinh \frac{\mu}{T}\right)^2 \frac{\partial \langle \Omega_{\mathrm{I}}^2 \rangle_c}{\partial P}, \qquad \lambda \cosh \frac{\mu}{T} = \lambda_0 + \frac{N_s^3}{2}\left(\lambda \sinh \frac{\mu}{T}\right)^2 \frac{\partial \langle \Omega_{\mathrm{I}}^2 \rangle_c}{\partial \Omega_R}$$

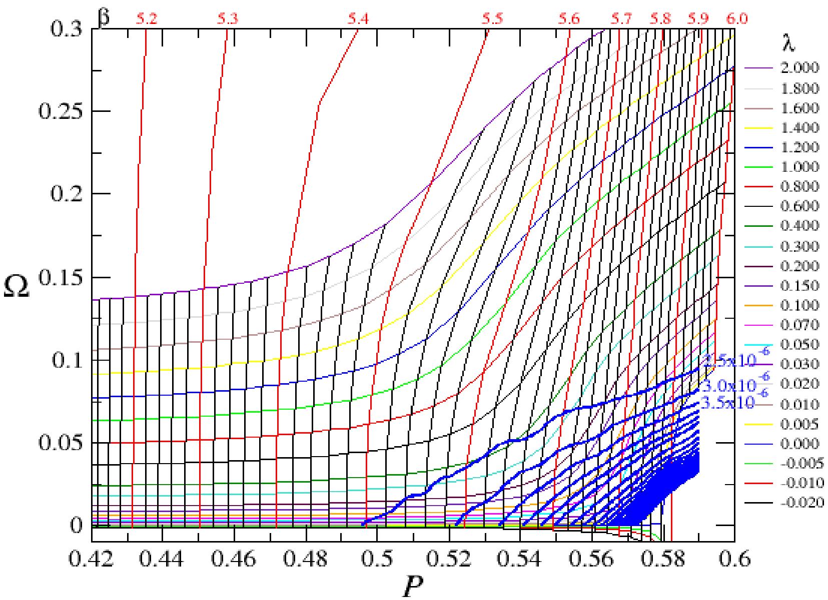
Fluctuation of the complex phase

- Variance of Imaginary part of Polyakov loop $\langle \Omega_{\mathrm{I}}^2 \rangle$

Lattice size: $30^3 \times 6$







$$\beta = \beta_0 + \frac{N_s^3}{12N_t}\left(\lambda \sinh\frac{\mu}{T}\right)^2 \frac{\partial\langle \Omega_{\mathrm{I}}^2 \rangle}{\partial P},$$

$$\lambda \cosh\frac{\mu}{T} = \lambda_0 + \frac{N_s^3}{2}\left(\lambda \sinh\frac{\mu}{T}\right)^2 \frac{\partial\langle \Omega_{\mathrm{I}}^2 \rangle}{\partial \Omega_R}$$

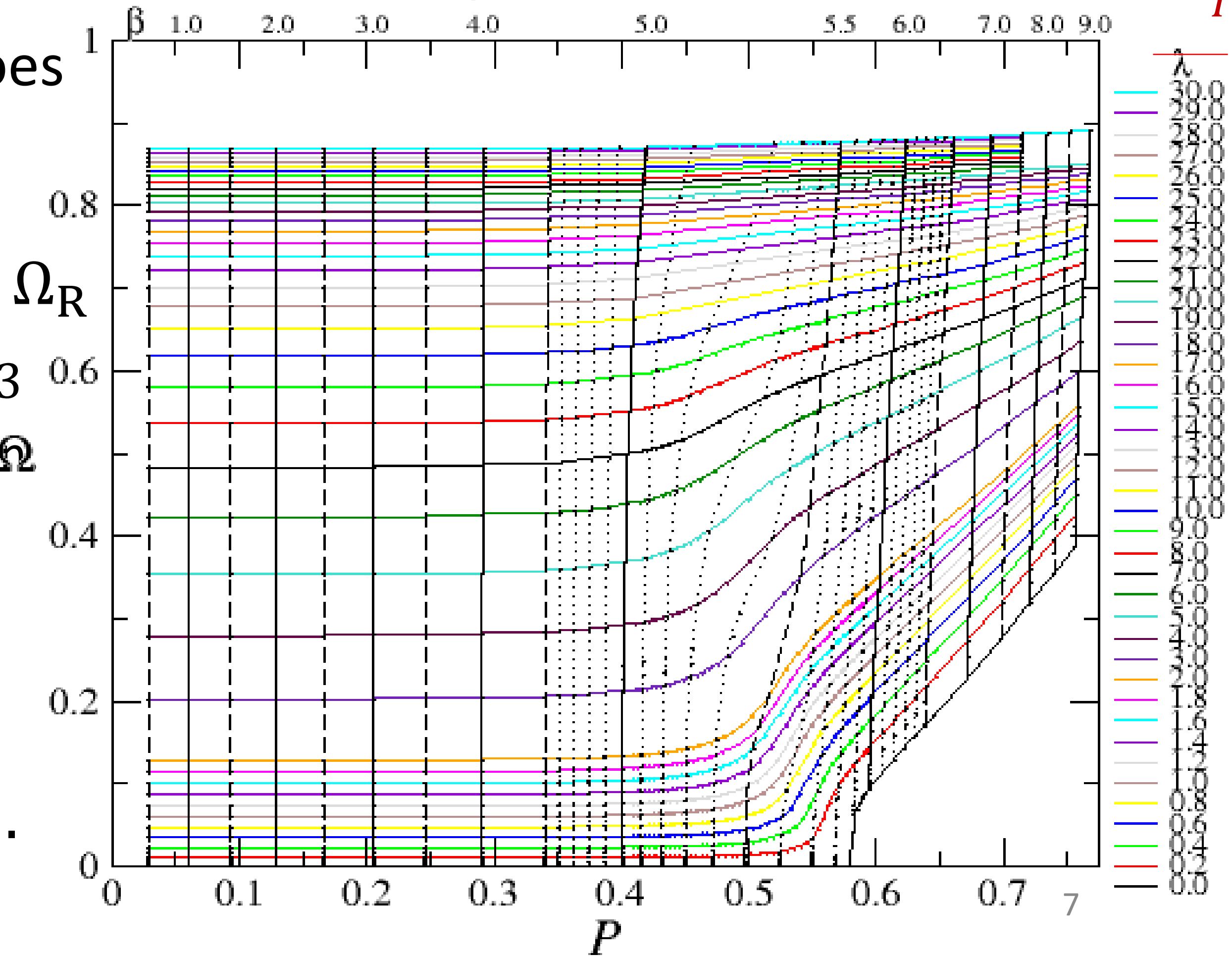
$\langle \Omega_{\mathrm{I}}^2 \rangle$ is large only near the first-order transition point.

This does not contribute to changing the nature of the phase transition at large μ.

Phase quenched QCD (Ignore complex phase)

- If we ignore the complex phase, $S_{\text{eff}} = -6N_{\text{site}}\beta^* P - N_s^3\lambda\left(\cosh\frac{\mu}{T}\,\Omega_{\text{R}} ~~+ i\sinh\frac{\mu}{T}\,\Omega_{\text{I}}~~\right)$
- Simply replace λ at $\mu = 0$ with $\lambda\cosh\frac{\mu}{T}$ to investigate phase quenched QCD.
- Increasing λ means increasing κ, so increasing λ makes the approximation of the hopping parameter expansion worse.
- Therefore, we fix λ at a small value and increase μ.
- If we fix λ, the convergence does not worsen.
- For example, $\lambda = 0.005$
  - For $N_{\text{f}} = 2,\ N_t = 6$
  - Critical point at $\mu = 0$: $\lambda_{\text{c}} = 0.0013$
  - $\lambda = 0.005$ is in the crossover region.
  - The convergence of the hopping parameter expansion is good.

$\lambda\cosh\frac{\mu}{T}$

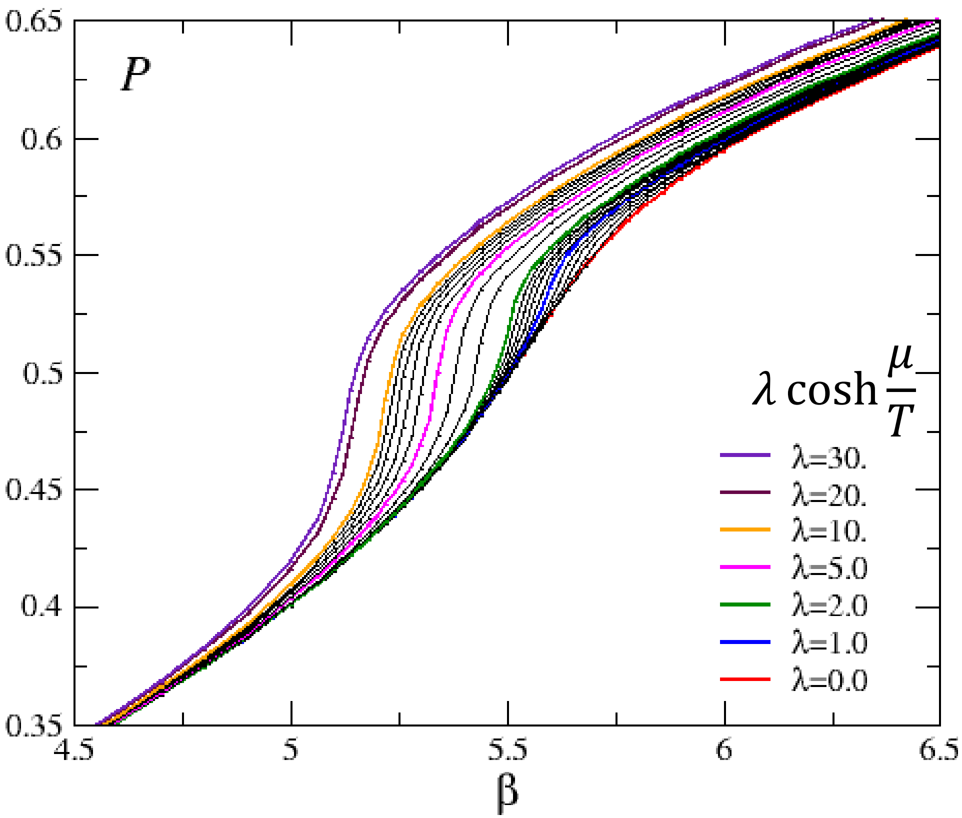


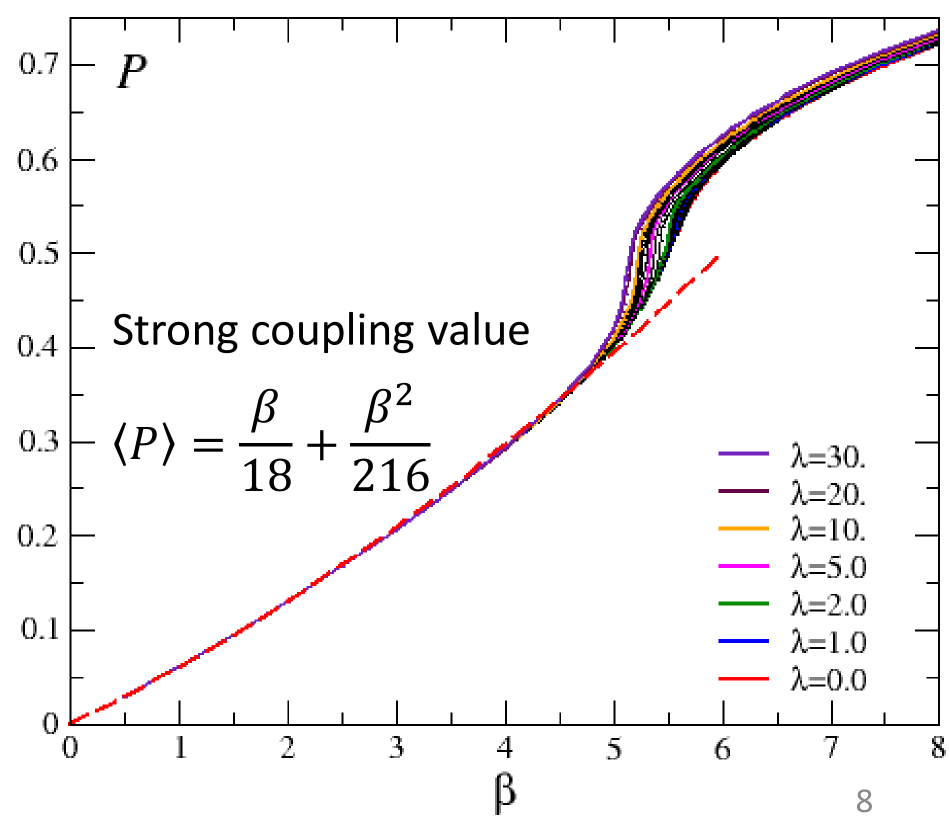
Simulations were performed over a wide range of $\beta, \lambda\cosh\frac{\mu}{T}$.

Plaquette in phase quenched QCD

- As $\cosh\frac{\mu}{T}$ increases, the change in plaquette becomes steeper.
- The strong coupling expansion of $\langle P \rangle$ does not depend on $\lambda \cosh\frac{\mu}{T}$.
- In the confinement phase, $\langle P \rangle$ is consistent with the strong coupling expansion.
- The $\lambda \cosh\frac{\mu}{T}$ term forces the deconfinement phase.

Lattice size: $30^3 \times 6$



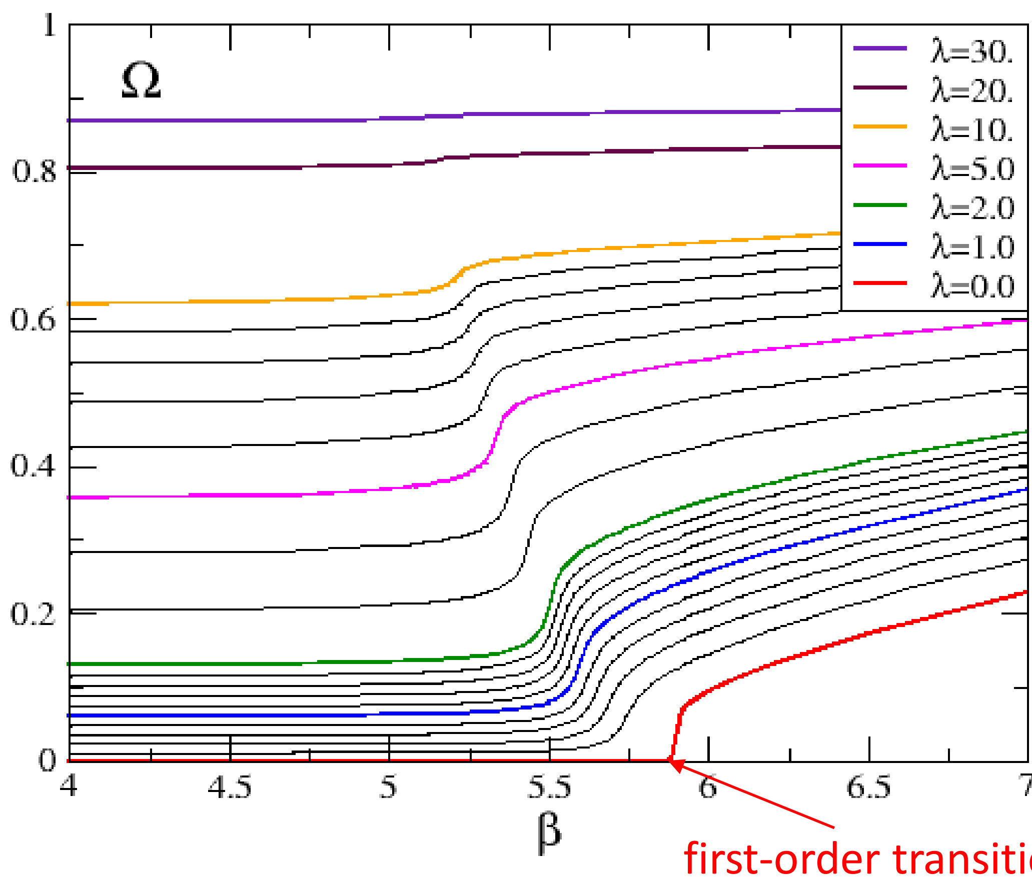


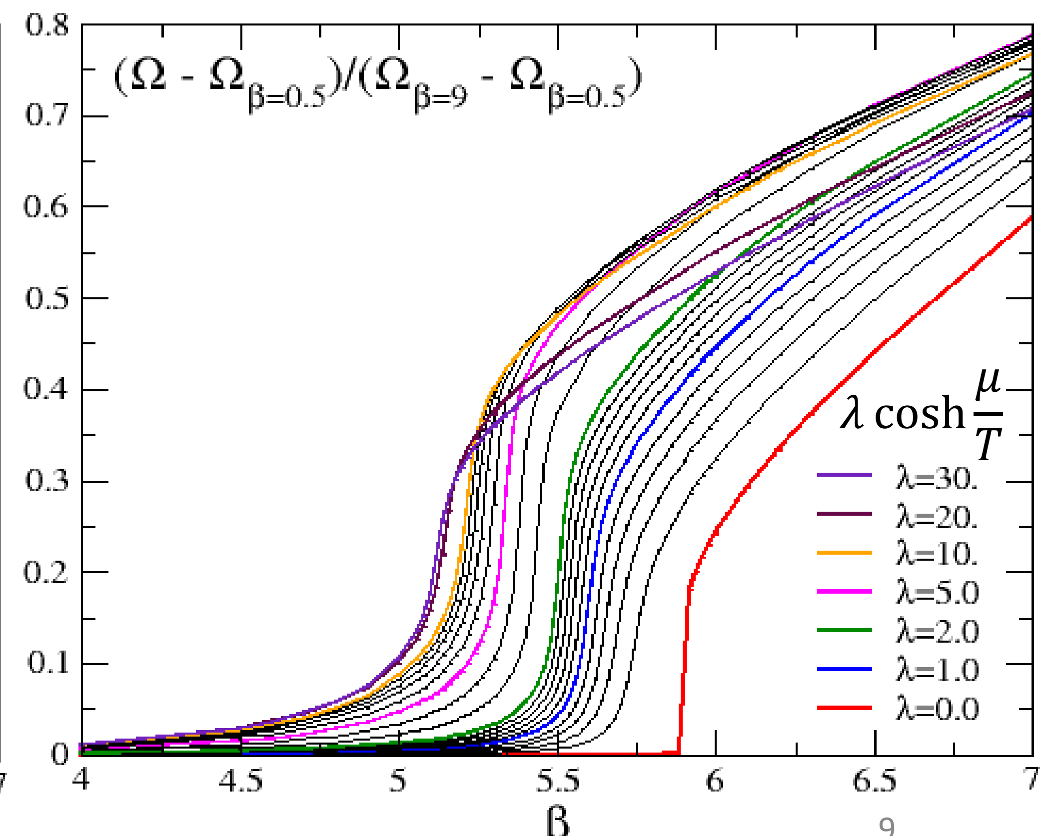
Polyakov loop in phase quenched QCD

- First order phase transition at $\lambda = 0$
- Regarding the Polyakov loop, if we look closely at the changing part, the change becomes steeper as μ increases.
- $\langle \Omega_{\mathrm{R}} \rangle$ changes almost perpendicular to the horizontal axis
- Shift of β by the complex phase may be important.

Discontinuity at large μ? → first-order transition?

$$\frac{\langle \Omega_{\mathrm{R}} \rangle - \langle \Omega_{\mathrm{R}} \rangle_{\beta=0.5}}{\langle \Omega_{\mathrm{R}} \rangle_{\beta=9} - \langle \Omega_{\mathrm{R}} \rangle_{\beta=0.5}}$$



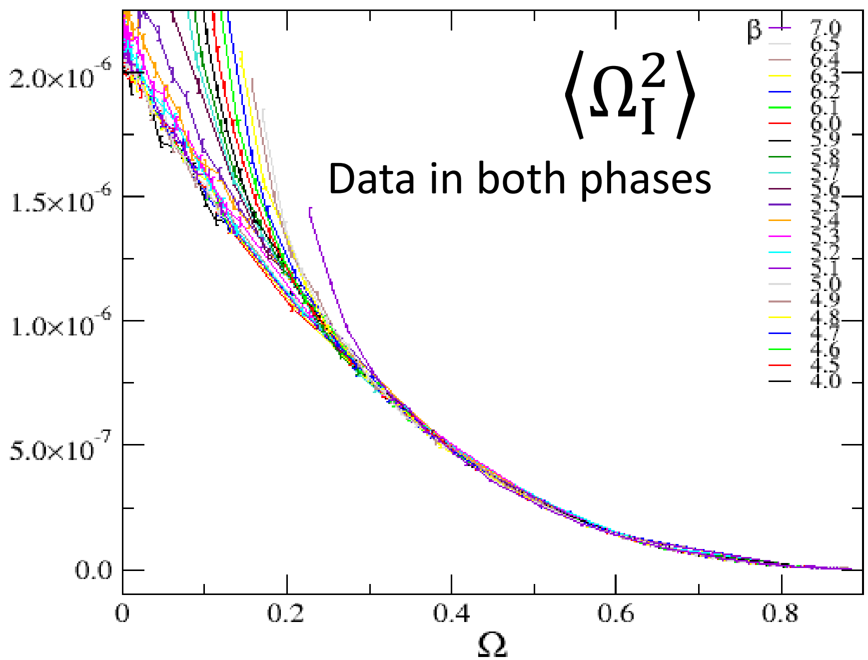


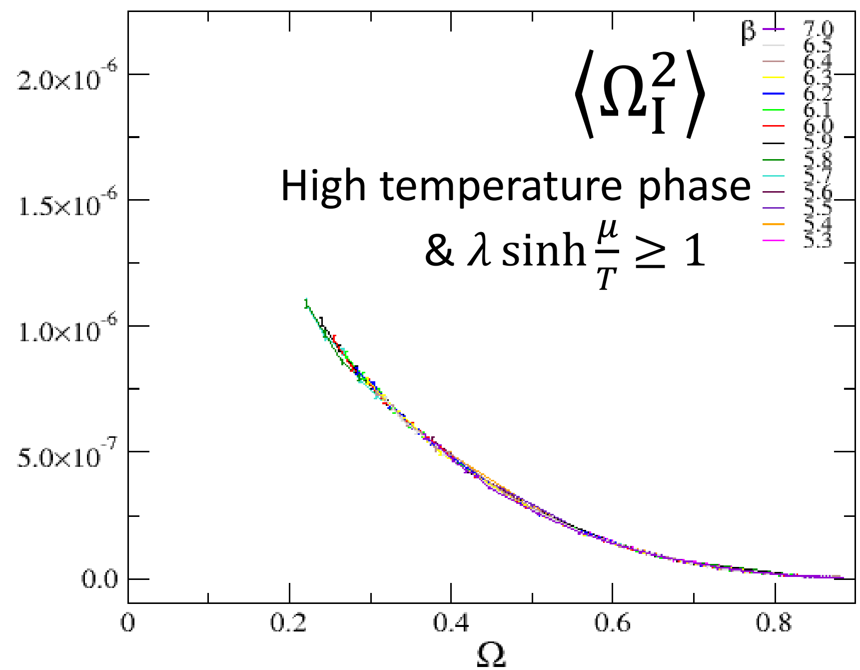
Effect of the complex phase

- We estimate the change in β by the complex phase.

$$\beta = \beta_0 + \frac{N_s^3}{12N_t}\left(\lambda \sinh\frac{\mu}{T}\right)^2 \frac{\partial\langle\Omega_{\rm I}^2\rangle}{\partial P}$$

Lattice size: $30^3 \times 6$





- P-dependence is much smaller than $\Omega_{\rm R}$-dependence
- No P- dependence in the high temperature phase.
- Hight temperature phase

$$\left.\frac{\partial\langle\Omega_{\rm I}^2\rangle}{\partial P}\right|_{\Omega_{\rm R}} = 0$$

Strong coupling limit (at low temperature)

$$\left.\frac{\partial\langle\Omega_{\rm I}^2\rangle}{\partial P}\right|_{\Omega_{\rm R}} > 0$$

Strong coupling limit (low temperature phase)

- Strong coupling expansion of $\langle \Omega_{\mathrm{I}}^2 \rangle$ at $\lambda = 0$

$$\langle \Omega_{\mathrm{I}}^2 \rangle = \left\langle \left(\frac{\Omega - \Omega^*}{2i} \right)^2 \right\rangle \approx \frac{1}{2} \langle \Omega \Omega^* \rangle \approx \frac{1}{2N_s^6 N_c^2} \sum_{x,y} \left\langle \mathrm{tr}(UU \cdots U)_x \mathrm{tr}\left(U^\dagger U^\dagger \cdots U^\dagger\right)_y \right\rangle$$

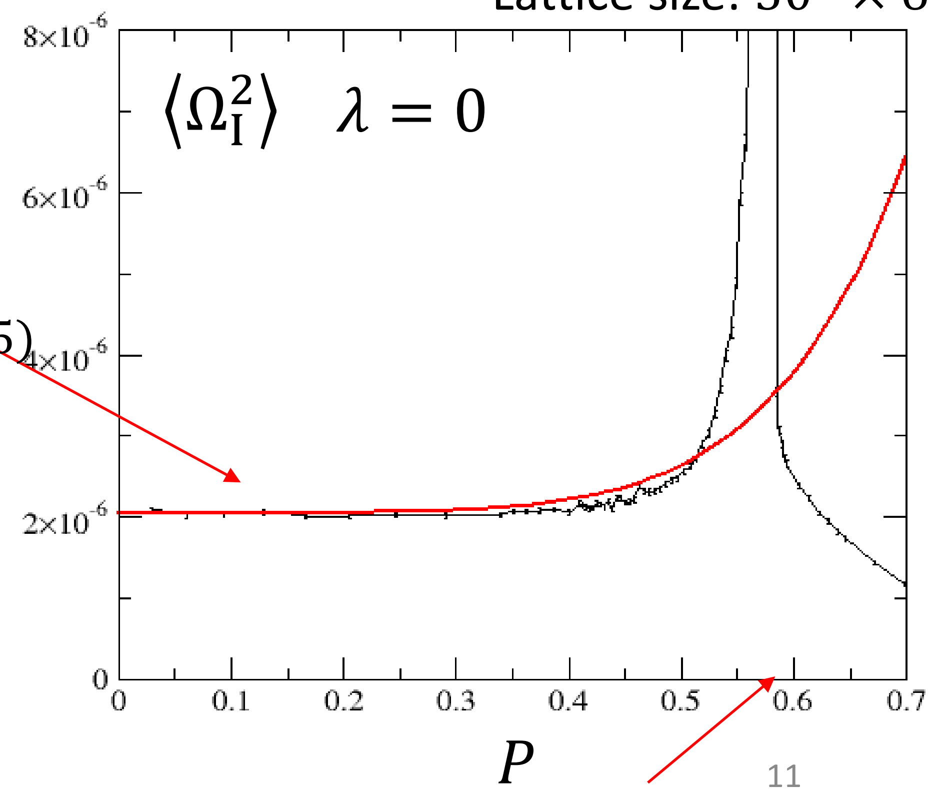
$$\approx \frac{N_s^3}{2N_s^6 N_c^2} \left\langle \mathrm{tr}(UU \cdots U)_x \mathrm{tr}\left(U^\dagger U^\dagger \cdots U^\dagger\right)_x \right\rangle + \frac{6N_s^3}{2N_s^6 N_c^2} \left\langle \mathrm{tr}(UU \cdots U)_x \mathrm{tr}\left(U^\dagger U^\dagger \cdots U^\dagger\right)_{x+1} \right\rangle + \cdots$$

$$\approx \frac{N_s^3}{2N_s^6 N_c^2} + \frac{6N_s^3}{2N_s^6 N_c^2} N_c \left(\frac{\beta}{2N_c^2} \right)^{N_t} = \frac{1}{2N_s^3 N_c^2} + \frac{3}{N_s^3 N_c} \left(\frac{\beta}{2N_c^2} \right)^{N_t}$$

- Plaquette is given by $P \approx \frac{\beta}{2N_c^2}$
- Thus, $\langle \Omega_{\mathrm{I}}^2 \rangle \approx \frac{1}{2N_s^3 N_c^2} + \frac{3}{N_s^3 N_c} (P)^{N_t}$
- consistent with the simulation data $(P < 0.5)$
- but P-dependence is very small.
- A large increase near the transition point.
- The derivative:

$$\frac{d\langle \Omega_{\mathrm{I}}^2 \rangle}{dP} \approx \frac{3N_t}{N_s^3 N_c} (P)^{N_t - 1}$$

Lattice size: $30^3 \times 6$



First-order phase transition

Estimation of β shift by the complex phase

Fit function: $\langle \Omega_{\mathrm{I}}^2 \rangle \approx \langle \Omega_{\mathrm{I}}^2 \rangle_{\lambda=0}^{\text{strong}} (1 - \Omega_R) \exp(-3\Omega_R)$

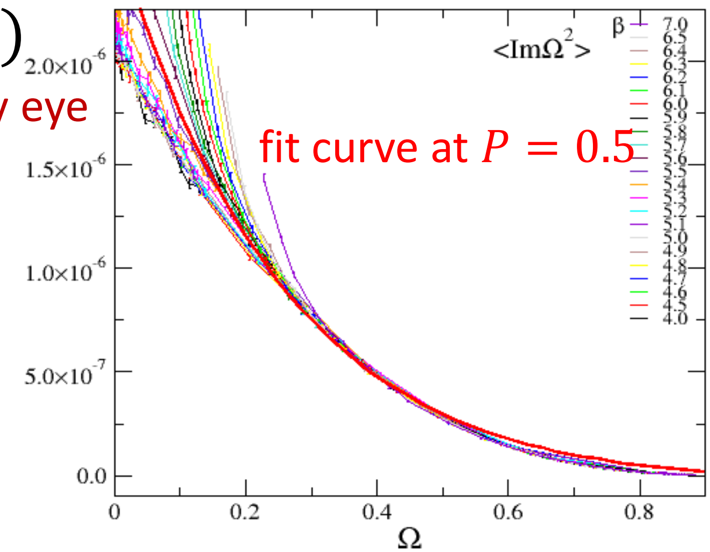
Strong coupling limit at $\lambda = 0$

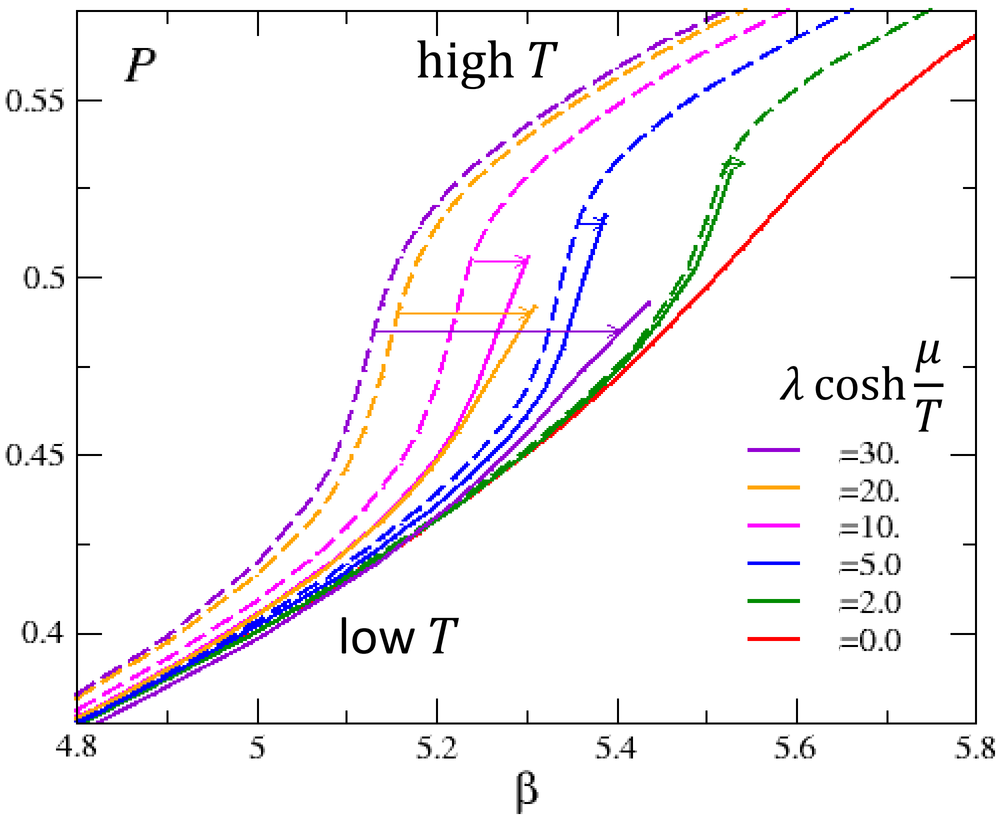
determined by eye

We assume

$$\begin{cases} \dfrac{d\langle \Omega_{\mathrm{I}}^2 \rangle}{dP} = 0 \quad (\text{high } T \text{ phase}) \\ \dfrac{d\langle \Omega_{\mathrm{I}}^2 \rangle}{dP} \approx \dfrac{3N_t}{N_s^3 N_c} (P)^{N_t - 1} (1 - \Omega_R) \exp(-3\Omega_R) \quad (\text{low } T \text{ pahse}) \end{cases}$$

$\langle \Omega_{\mathrm{I}}^2 \rangle = 0$ at $\Omega_R = 1$





- β shift: $$\beta = \beta_0 + \frac{N_s^3}{12N_t} \left(\lambda \sinh \frac{\mu}{T} \right)^2 \frac{\partial \langle \Omega_{\mathrm{I}}^2 \rangle}{\partial P}$$

ex.) $\lambda = 0.005$

- When $\lambda \cosh \frac{\mu}{T} = 5.0$ $(\mu/T = 7.60)$ or greater, the graph changes significantly.
- $\lambda \cosh \frac{\mu}{T} = 10, \ \ 20 \rightarrow (\mu/T = 8.29, \ \ 8.99)$
- This suggests the discontinuity in P at the transition point.
- A first-order phase transition is expected at large μ/T.

Summary

- First, we discussed the nature of the phase transition of phase-quenched finite-density QCD in the heavy quark region.
- The first-order transition at zero density turns into a crossover as μ is increased, but, when we increase μ further, the change in the plaquette value near the crossover point becomes much steeper.
- Then, we estimate the effect of the complex phase to discuss whether the QCD phase transition changes again to a first-order phase transition at very large μ.
- In the high-temperature phase, the effect of the complex phase is negligible.
- In the low-temperature phase, complex phase effects lead to a steeper change in the plaquette when estimated from the results in the strong coupling limit.
- This effect of the complex phase becomes larger as μ increases.
- This suggests the appearance of a first-order phase transition region at high density.