The NRQCD Υ spectrum at non-zero temperature using Backus-Gilbert regularisations

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on behalf of the FASTSUM collaboration
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Motivation

- The dissociation of heavy quarkonia in a deconfined medium is important as it may serve as a termometer for relativistic heavy-ion collisions

- The suppression pattern of these states is complicated by the statistical recombination of heavy quarks in the Quark-Gluon Plasma (QGP). However, competing effects of suppression and recombination are expected to be less pronounced for bottomonium than for charmonium

- On the lattice we can perform an *ab initio* study of such systems at high temperatures employing the NRQCD formalism which allows for a robust simulation of the heavy quarks.

Ensembles

For our calculations we use the Gen 2L ensembles produced by the FASTSUM collaboration.

N_τ	128	64	56	48	40	36	32	28	24	20	16
T [MeV]	47	95	109	127	152	169	190	217	253	304	380

- We use Wilson-clover fermion action, tree-level tadpole improved with stout links.
- Same parameters as the HadSpec collaboration
- Approximately 1000 configurations for each temperature
- $m_\pi \sim 236$ MeV, $\xi \sim 3.5$, $T_c \sim 167$ MeV

Spectral functions

In order to study the spectrum of bottomonium at high temperatures the best way forward is to employ methods that do not make any assumption on the spectral structure. For this reason, the observable of interest here is the spectral function $\rho(\omega)$.

In the relativistic formulation of lattice QCD, one can relate the Euclidean correlator to $\rho(\omega)$ through the following integral equation

$$G(\tau) = \int_{\omega_{\text{min}}}^{\infty} d\omega \;\rho_L(\omega)\; K(\omega, T).$$

In the NRQCD formalism instead we have a much simpler equation

$$G(\tau) = \int_{\omega_{\text{min}}}^{\infty} d\omega \;\rho_L(\omega)\; e^{-\omega\tau}.$$

Inverse problem

Extracting the spectral function from lattice Euclidean correlator is a notorious ill-posed inverse problem and as such it needs to be regularised

$$G(\tau) = \int_{\omega_{\mathrm{min}}}^{\infty} d\omega \; \rho_L(\omega) \; e^{-\omega\tau}.$$

There are many methods which tackle such a problem in different ways, each resorting to different strategies and assumptions

- Maximum Entropy Method
- Bayesian Reconstruction
- Machine Learning
- Backus-Gilbert (Tikhonov) $\star$
- Chebyshev Polynomials

Spectral reconstruction

The central idea of any linear method for spectral reconstruction is to search for a plynomial reconstruction of a certain $smearing$ function

$$\overline{\Delta}(\omega, \omega_n) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\omega_n) e^{-\omega\tau}$$

Then, one can reconstruct the spectral function as

$$\begin{aligned}\widehat{\rho}(\omega_n) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\omega_n) G(\tau) &= \int_{\omega_{\min}}^{\infty} d\omega \ \rho_L(\omega) g_\tau(\omega_n) e^{-\omega\tau} \\ &= \int_{\omega_{\min}}^{\infty} d\omega \ \rho_L(\omega) \overline{\Delta}(\omega_n, \omega)\end{aligned}$$

In the limit $N_\tau \to \infty$, then $\overline{\Delta}(\omega_n, \omega) \to \delta(\omega - \omega_n)$, such that

$$\rho(\omega_n) = \int_{\omega_n}^{\infty} d\omega \ \rho_L(\omega) \delta(\omega - \omega_n)$$

Spectral reconstruction - The HLT way

Introduced in [Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

$$\overline{\Delta}_\sigma(\omega, \omega_n) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\omega_n) e^{-\omega\tau}, \qquad \Delta_\sigma(\omega, \omega_n) = a\ e^{-\frac{(\omega-\omega_n)^2}{2\sigma^2}}$$

Then, one can reconstruct the spectral function as

$$\begin{aligned}\widehat{\rho}_\sigma(\omega_n) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\omega_n) G(\tau) &= \int_{\omega_{\min}}^{\infty} d\omega\ \rho_L(\omega) g_\tau(\omega_n) e^{-\omega\tau} \\ &= \int_{\omega_{\min}}^{\infty} d\omega\ \rho_L(\omega) \overline{\Delta}_\sigma(\omega_n, \omega)\end{aligned}$$

In the limit $\sigma \to 0$, then $\overline{\Delta}_\sigma(\omega_n, \omega) \to \delta(\omega - \omega_n)$, such that

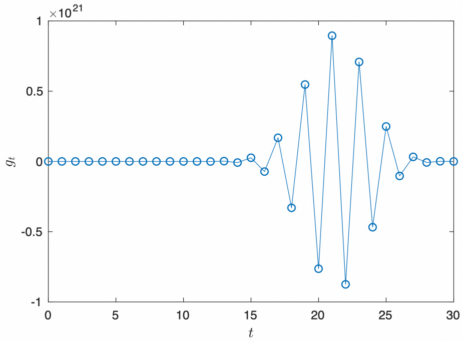
$$\rho(\omega_n) = \int_{\omega_n}^{\infty} d\omega\ \rho_L(\omega) \delta(\omega - \omega_n)$$

Regularising the problem

In order to find the coefficients g_τ, one needs to minimise the following functional

$$A^{BG,Tikh}[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega\ (\omega-\omega_n)^2 \{\overline{\Delta}^{BG,Tikh}(\omega,\omega_n)\}^2$$

$$A_n^{HLT}[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega\ e^{\alpha\omega} |\overline{\Delta}_\sigma^{HLT} - \Delta_\sigma^{HLT}|^2$$



[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

Regularising the problem

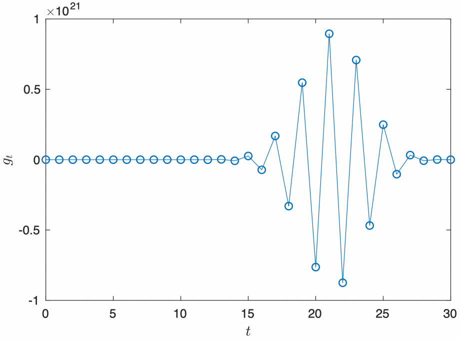
$$W[g_\tau] = A[g_\tau] + \lambda B[g_\tau]$$

$$A^{BG,Tikh}[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega \; (\omega - \omega_n)^2 \{\overline{\Delta}^{BG,Tikh}(\omega, \omega_n)\}^2$$

$$A_n^{HLT}[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega \; e^{\alpha\omega} |\overline{\Delta}_\sigma^{HLT} - \Delta_\sigma^{HLT}|^2$$

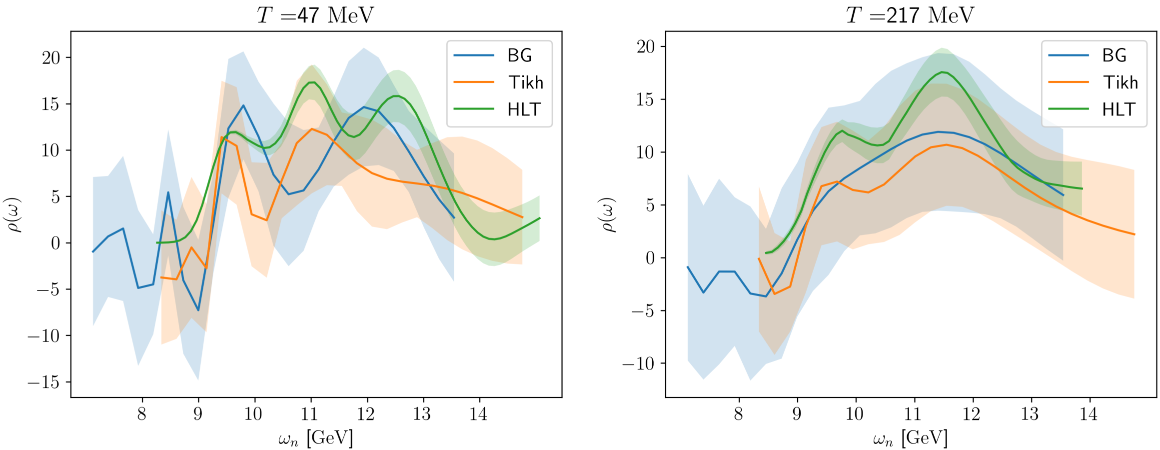
$$B^{Tikh}[g_\tau] = \sum_{\tau_1,\tau_2}^{\tau_{\max}} g_{\tau_1} g_{\tau_2} I(\tau_1, \tau_2)$$

$$B^{BG,HLT}[g_\tau] = \sum_{\tau_1,\tau_2}^{\tau_{\max}} g_{\tau_1} g_{\tau_2} \operatorname{Cov}(\tau_1, \tau_2)$$

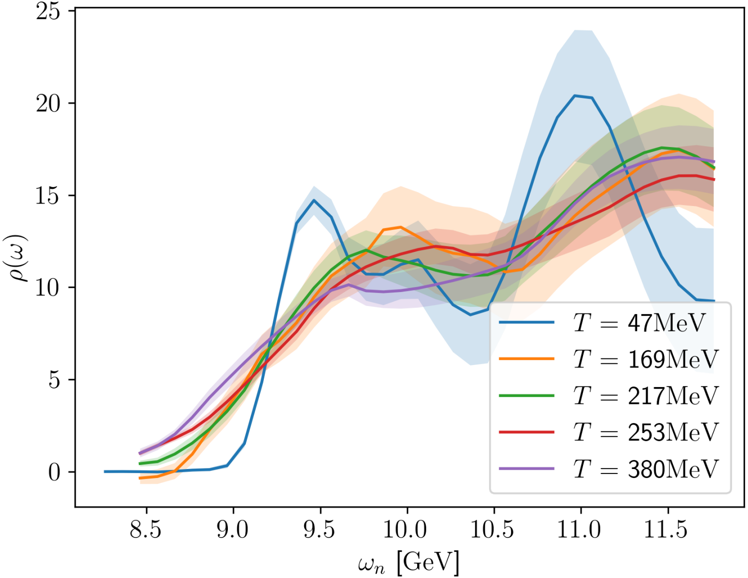


[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

Results



HLT results

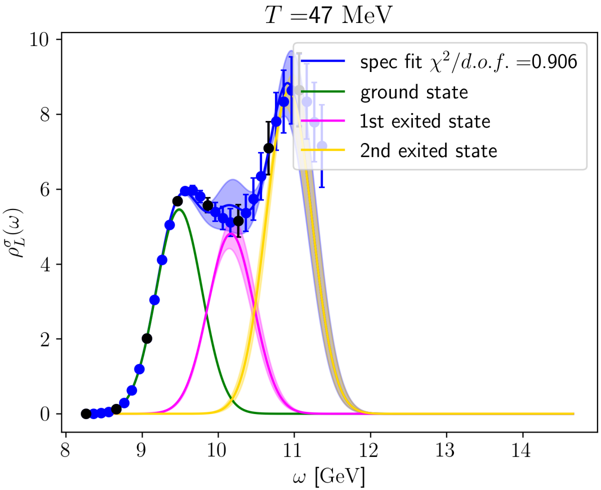


25
20
15
10
5
0
$\rho(\omega)$
8.5
9.0
9.5
10.0
10.5
11.0
11.5
ω_n [GeV]
T = 47MeV
T = 169MeV
T = 217MeV
T = 253MeV
T = 380MeV

Fitting the HLT results

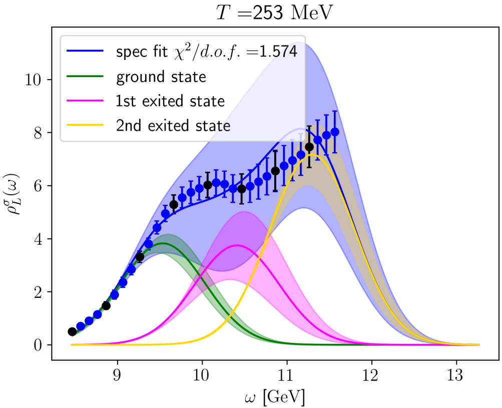
- In HLT σ is an input, hence we can only quote upper estimates of Γ
- We can use this extra information in fitting the spectrum as suggested in [Del Debbio *et al.*, Eur.Phys.J.C, 2211.09581]
- Minimise χ^2 defined in terms of $\mathrm{Cov}[\rho_\sigma]$

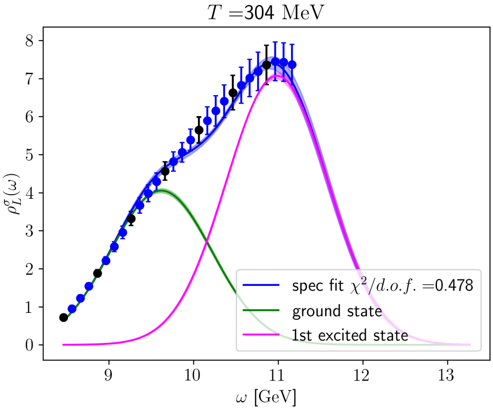
$$f_k^\sigma(\omega_n) = \sum_k a_k e^{-\frac{(\omega-\omega_n)^2}{2\sigma^2}}$$



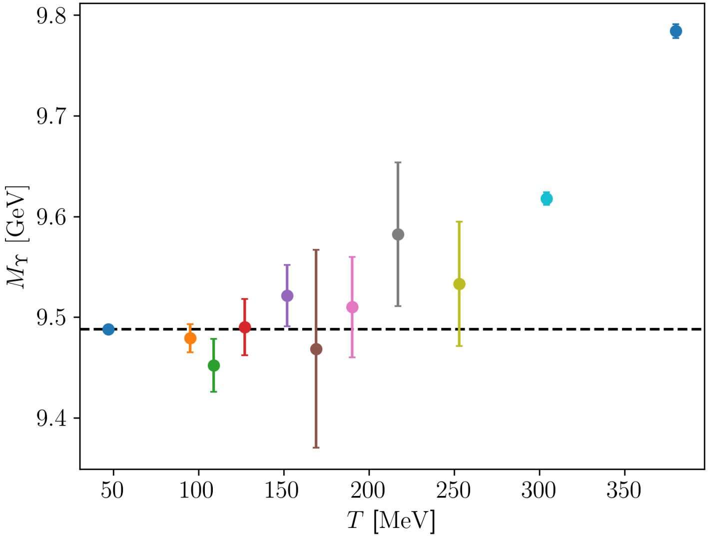
Fits at high T

Naturally at high T the spectral reconstruction is more challenging and one needs larger σ





M_Υ results



Preliminary results summary, $M_\Upsilon^{exp} = 9.46040(1)$ GeV

T (MeV)	47	95	109	127	152	169
M_Υ^{HLT} (GeV)	9.4880(6)	9.479(14)	9.452(26)	9.490(28)	9.521(30)	9.47(10)
M_Υ^{BG} (GeV)	9.7(2)	9.62(37)	9.65(54)	9.65(54)	9.63(60)	9.66(75)
M_Υ^{Tikh} (GeV)	9.54(12)	9.6(2)	9.81(54)	9.63(25)	9.64(26)	9.84(14)

T (MeV)	190	217	253	304	380
M_Υ^{HLT} (GeV)	9.51(5)	9.582(71)	9.53(6)	9.618(6)	9.784(7)
M_Υ^{BG} (GeV)	9.69(94)	9.7(1.4)	9.7(1.3)	9.8(1.8)	9.8(2.1)
M_Υ^{Tikh} (GeV)	9.67(30)	9.67(36)	9.67(36)	9.76(35)	9.83(87)

Preliminary results summary Γ

T (MeV)	47	95	109	127	152	169
M_{Υ}^{HLT} (GeV)	0.3	0.3	0.3	0.3	0.3	0.3
M_{Υ}^{BG} (GeV)	0.20(27)	0.30(46)	0.31(57)	0.32(60)	0.32(67)	0.33(78)
M_{Υ}^{Tikh} (GeV)	0.17(25)	0.25(20)	0.36(50)	0.27(25)	0.30(31)	0.40(16)

T (MeV)	190	217	253	304	380
M_{Υ}^{HLT} (GeV)	0.4	0.4	0.5	0.6	0.6
M_{Υ}^{BG} (GeV)	0.35(91)	0.40(1.3)	0.36(1.2)	0.40(1.4)	0.40(1.3)
M_{Υ}^{Tikh} (GeV)	0.31(37)	0.31(41)	0.37(37)	0.38(72)	0.40(80)

Conclusions

- We investigated almost all the available linear methods for spectral reconstruction in order to investigate the Υ spectrum at finite temperatures.
- We find that the introduction of the smearing kernel in the HLT method gives a more stable reconstruction. The drawback giving Γ as the input is not so severe considering that we find comparable widths with the other two methods.
- It is clear that with linear methods one can only quote upper estimates for the decay width.
- The HLT method allows for a better control over the systematic errors via the stability plot analiyisis compared to the other methods discussed here.
- We're in the process of extending this analysis to η_b and χ_b systems.

BACKUP SLIDES

Laplace shift

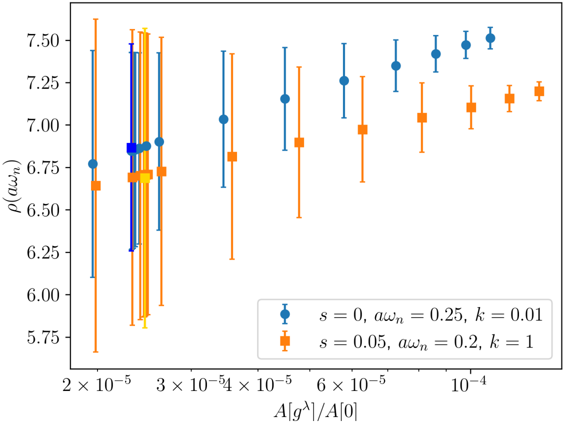
Introduced in [Page *et al.*, PoS, 2112.02075]
Uses the fact that linear methods work best for low energies

$$G'(\tau) = e^{s\cdot\tau} G(\tau) \rightarrow \widehat{\rho}'(\omega_n + s)$$

However,

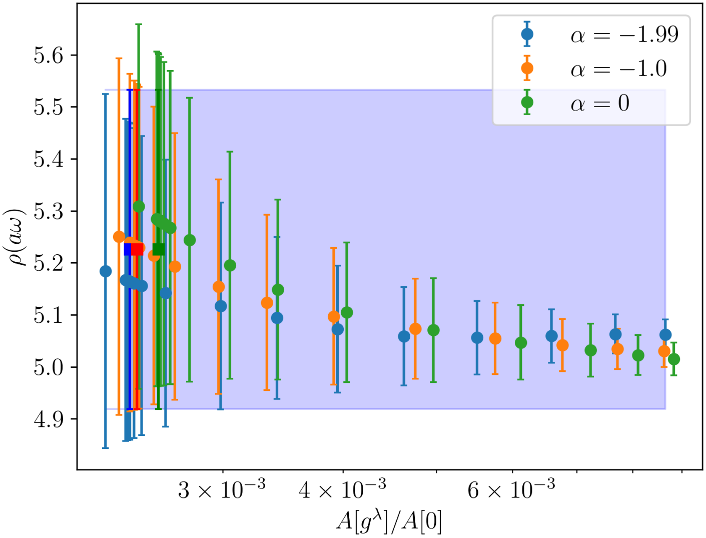
$$\widehat{\rho}'(\omega_n + s) = \sum_{\tau} g_\tau G'(\tau) = \sum_{\tau} g'_\tau G(\tau)$$

- For the same value of λ we get a better result (at a price)
- In BG using of LS equivalent of reducing λ
- In practice convenient for Tikhonov

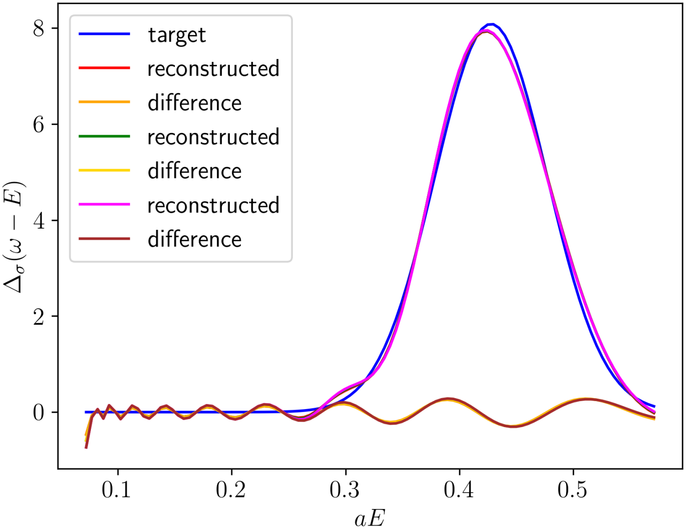


$$\frac{A[g^{\lambda^*}]}{A[0]} = k\ B[g^{\lambda^*}]$$

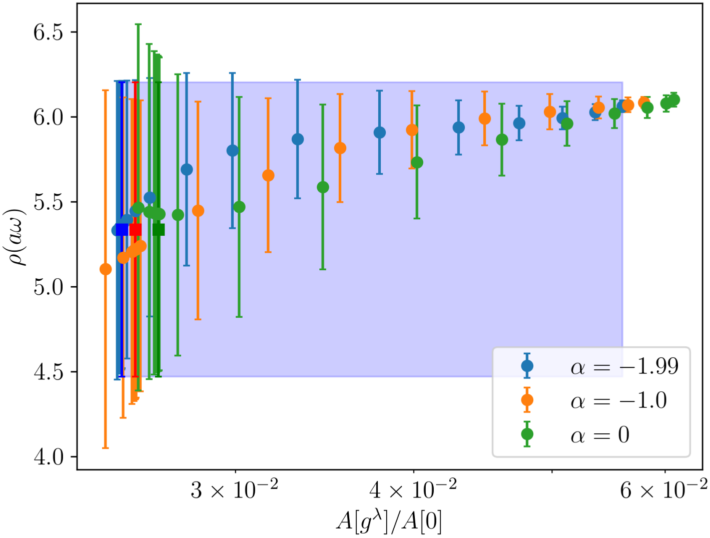
Stability plot $E \simeq 10$ GeV, $T = 47$ MeV



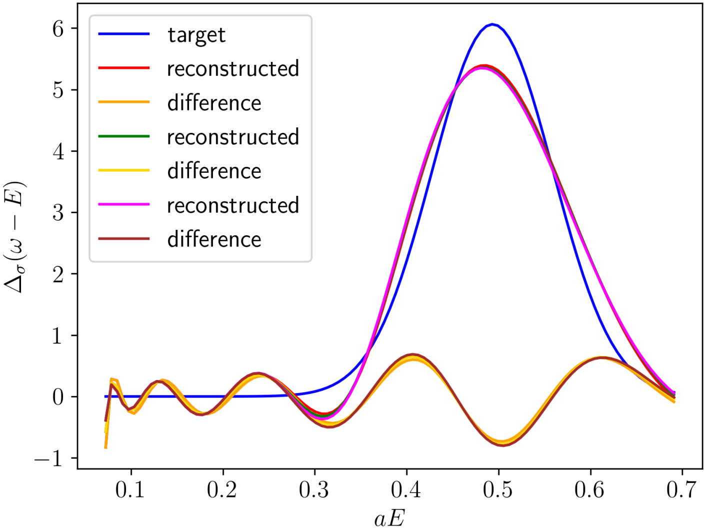
Kernel reconstruction $E \simeq 10$ GeV, $T = 47$ MeV



Stability plot $E \simeq 10.5$ GeV, $T = 217$ MeV



Kernel reconstruction $E \simeq 10.5$ GeV, $T = 217$ MeV



HLT Fit at $T = 253$ MeV