Anisotropic excited bottomonia from a basis of smeared operators

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QCD at non-zero temperature
Lattice 2024, the University of Liverpool

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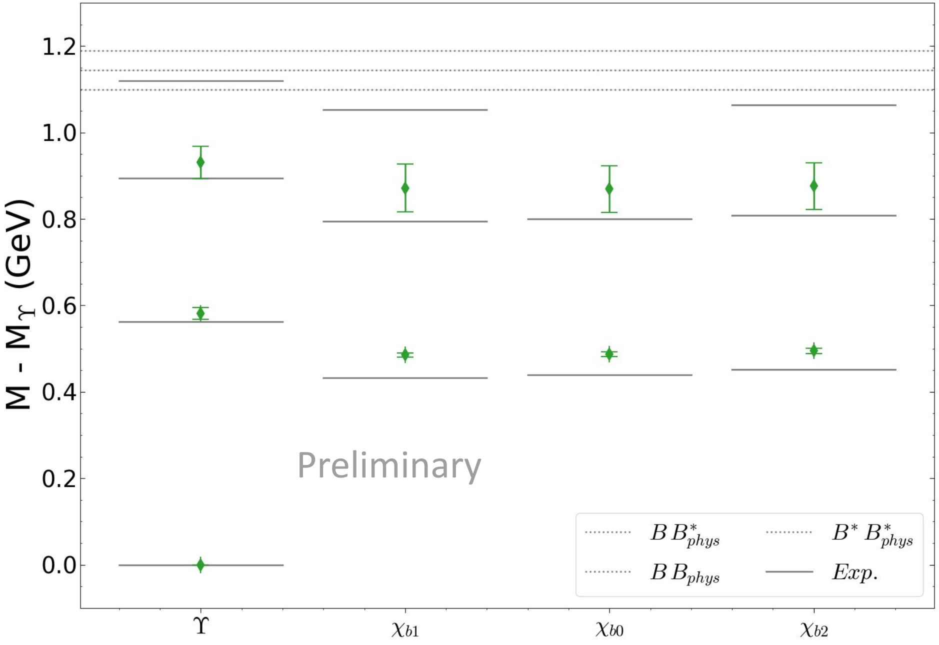
Bottomonium spectrum

- Experimentally accessible
- Phenomenologically relevant to
  - heavy-ion experiments
  - Quark gluon plasma
  - Early universe
- Easily computable
  - via lattice NRQCD
  - statistically well-behaved due to scale separation



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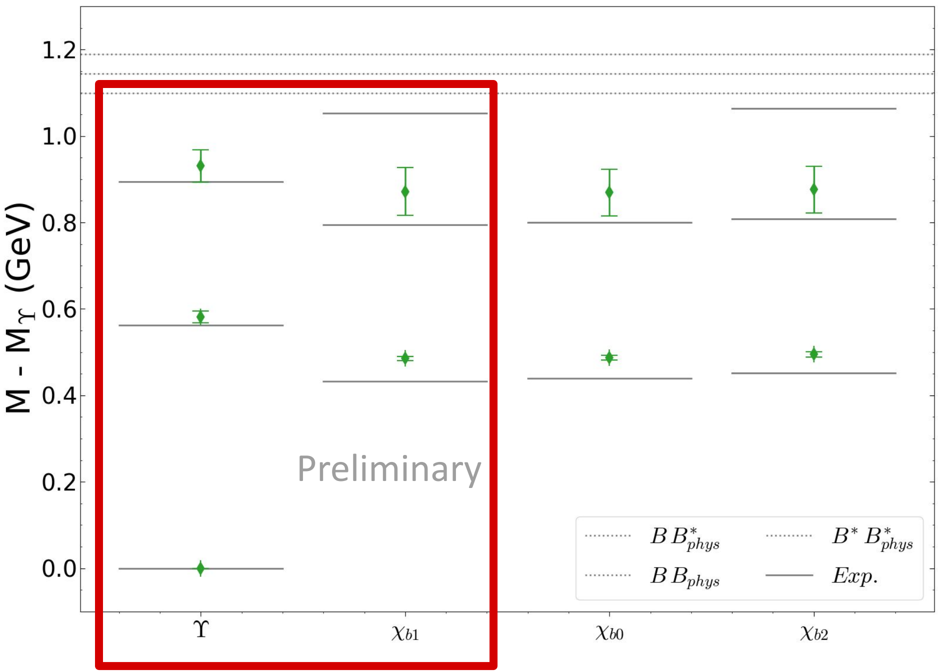

Ensemble Details

Generation 2L FASTSUM

N_T	128	64	56	48	40	36	32	28	24	20	16
Temperature (MeV)	47	95	109	127	152	169	190	217	253	304	380
# Wall Sources	16	16	16	20	24	24	32	28	24	20	16

Action details:

- **Gauge: Symanzik-improved, tree-level tadpole**
- **Fermion: Wilson-clover, tree-level tadpole, stout-links**
- **Same parameters as HadSpec Collaboration**
- **Approx. 1000 configurations at each temperature**

- **NRQCD action for bottom quarks**
  - Incorporating $O(v^4)$ corrections
  - Tree-level matching coefficients

$$m_\pi \sim 236 \text{ MeV}, \xi \sim 3.5, T_c \sim 167 \text{ MeV}$$

Excited State spectroscopy

Generalised EigenValue Problem - GEVP

- Build correlation matrix of two point functions

$$G_{ij}(\tau) = \left\langle \Omega \middle| \mathcal{O}_i \mathcal{O}_j{}^{\dagger} \middle| \Omega \right\rangle = \sum_\alpha \frac{Z_i^\alpha \, Z_j^{\alpha\,\dagger}}{2\,E_\alpha} \, \mathrm{e}^{-E_\alpha\,\tau}$$

- Solve generalised eigenvalue problems

$$G_{ij}(\tau_0 + \delta_\tau)\, u_j^\alpha = \mathrm{e}^{-E_\alpha \delta_\tau}\, G_{ij}(\tau_0)\, u_j^\alpha$$
$$v_i^\alpha\, G_{ij}(\tau_0 + \delta_\tau) = \mathrm{e}^{-E_\alpha \delta_\tau}\, v_i^\alpha\, G_{ij}(\tau_0)$$

- Construct Projected Correlator

$$G_\alpha(\tau) = v_i^\alpha\, G_{ij}(\tau)\, u_j^\alpha$$

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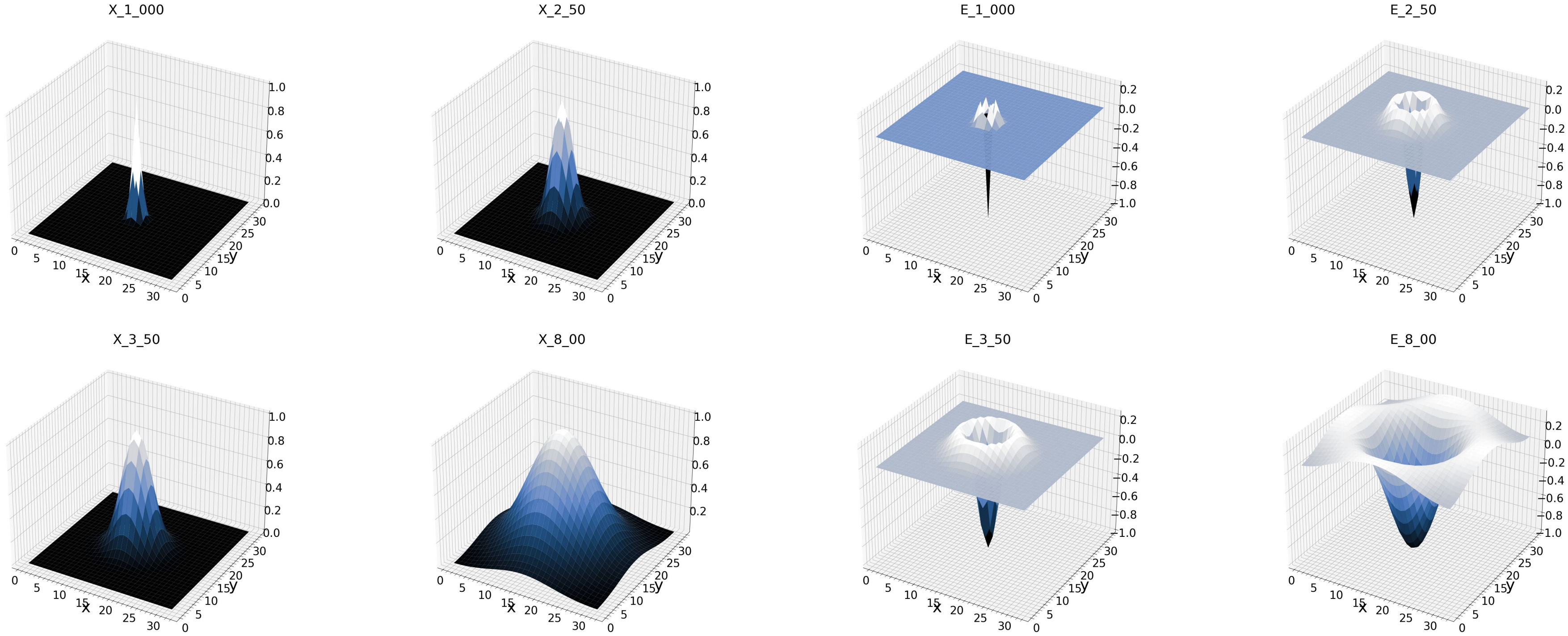
GEVP - Operator Basis

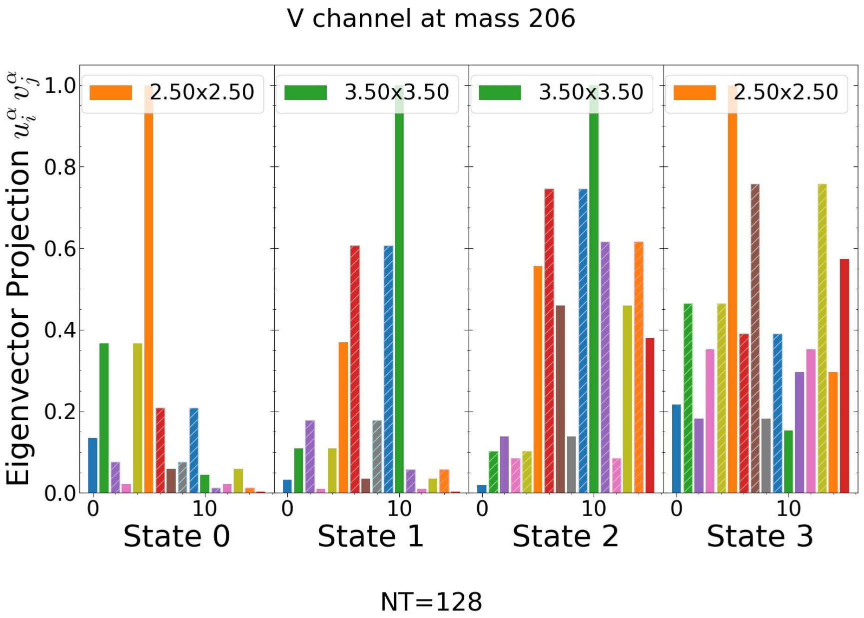
Four widths of Gaussian and `excited` operator



Eigenvectors

- Related to overlap of each ‘operator’ with each state
- Examine eigenvectors to see how they change as temperature increases

  – Plots have the largest contribution is normalised to one, and negative contributions are ‘hashed’



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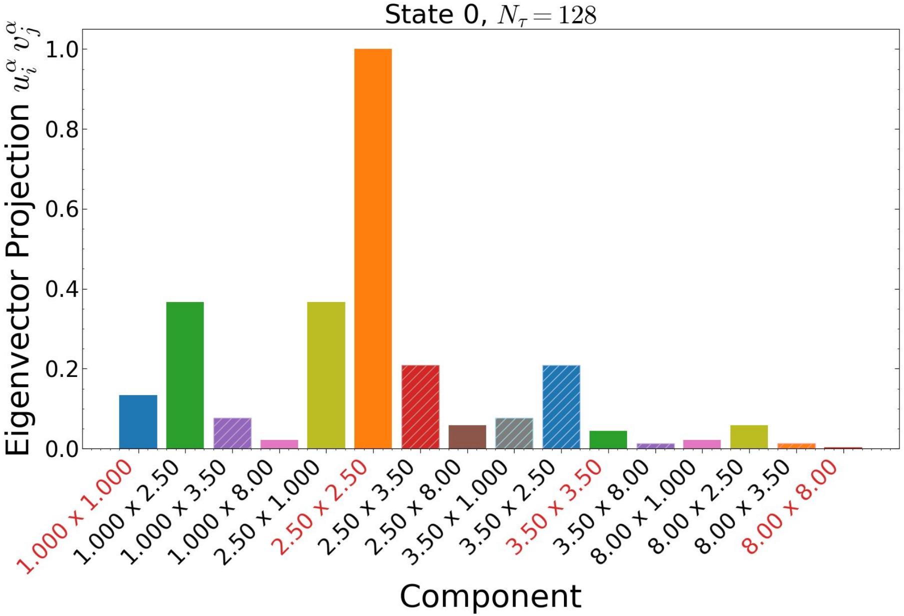

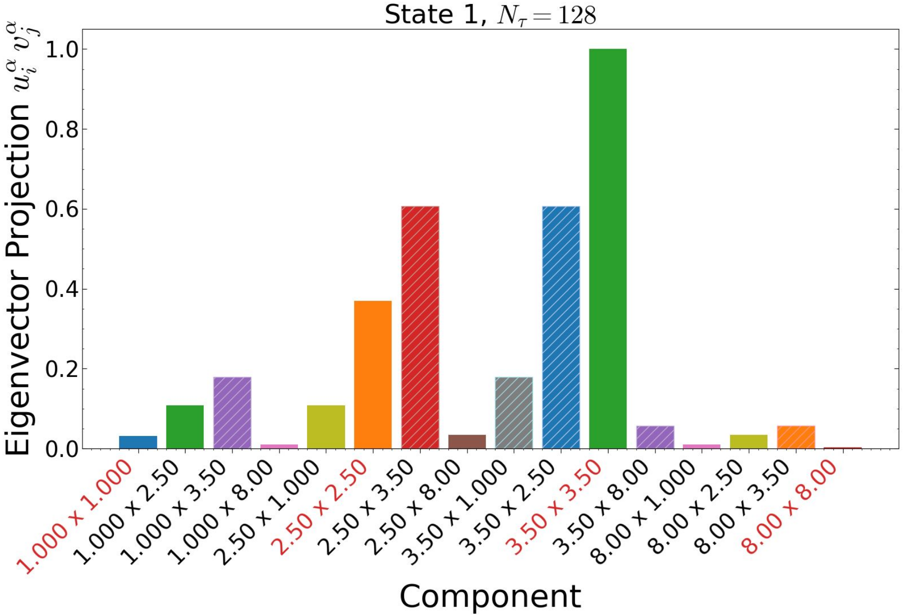

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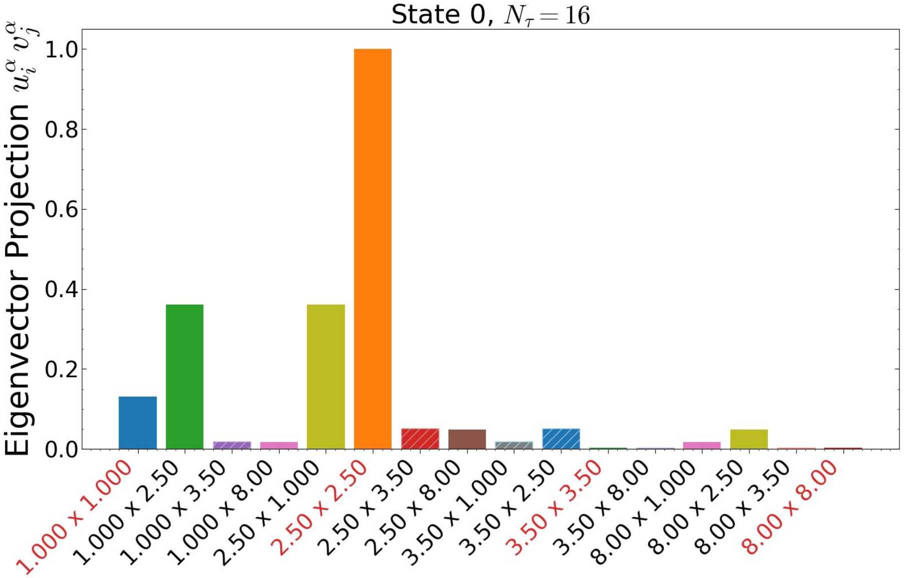
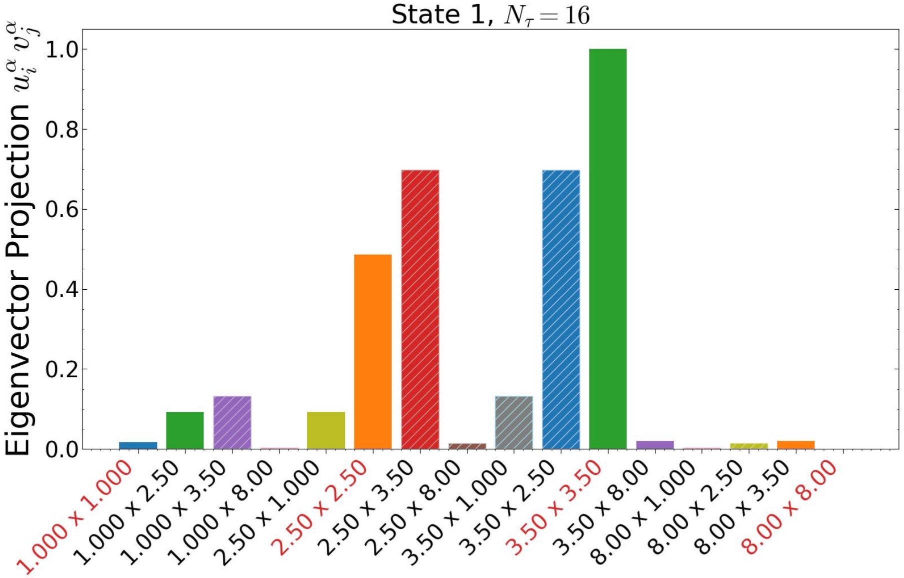
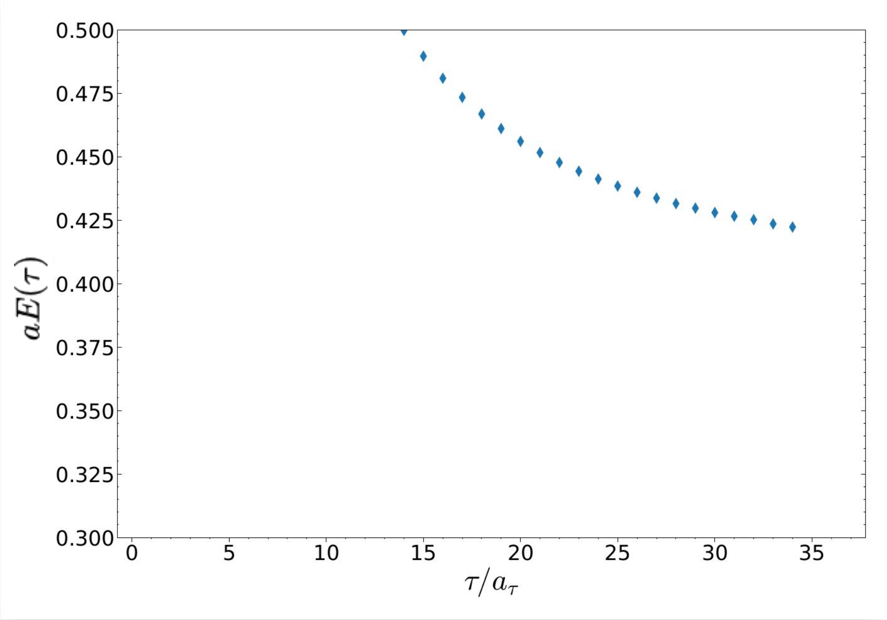
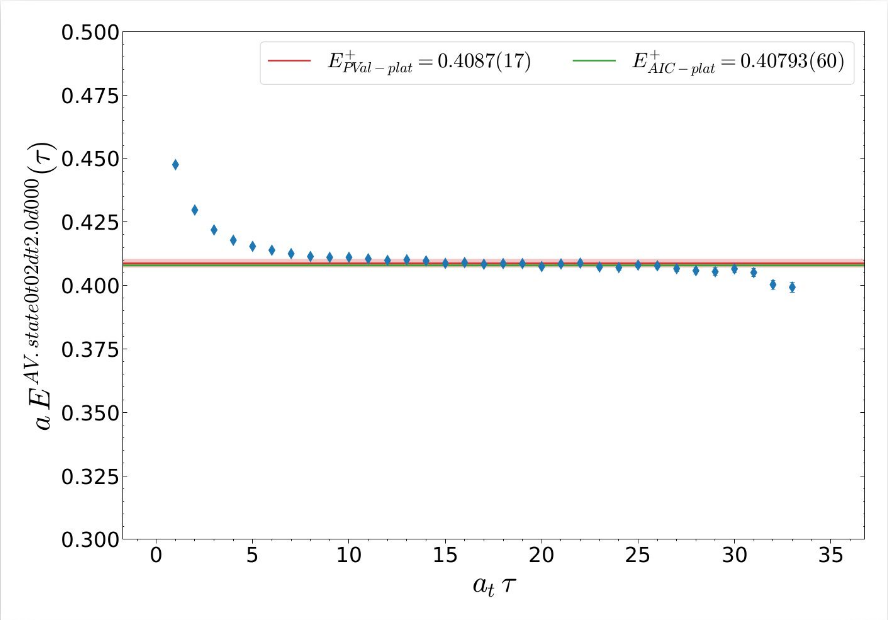




Improved state isolation

N_t = 36, $\chi_{b1}(1P)$





Spectral Representation

of NRQCD correlator

$$G(\tau) = \int_0^\infty \frac{d\omega}{2\pi} e^{-\omega\tau} \rho(\omega)$$

Model spectral function $\rho(\omega)$ using a delta-function of the ground state.

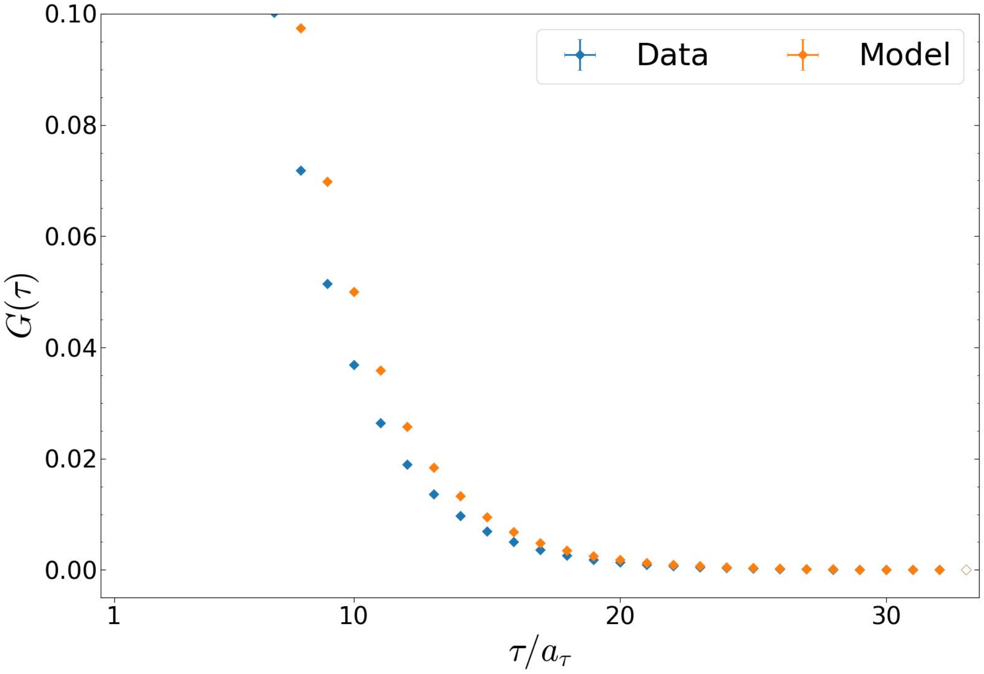
Construct single ratio

$$r(\tau; T, T_0) = \frac{G(\tau;T)}{G_{\mathrm{model}}(\tau;T,T_0)}$$

And hence double ratio

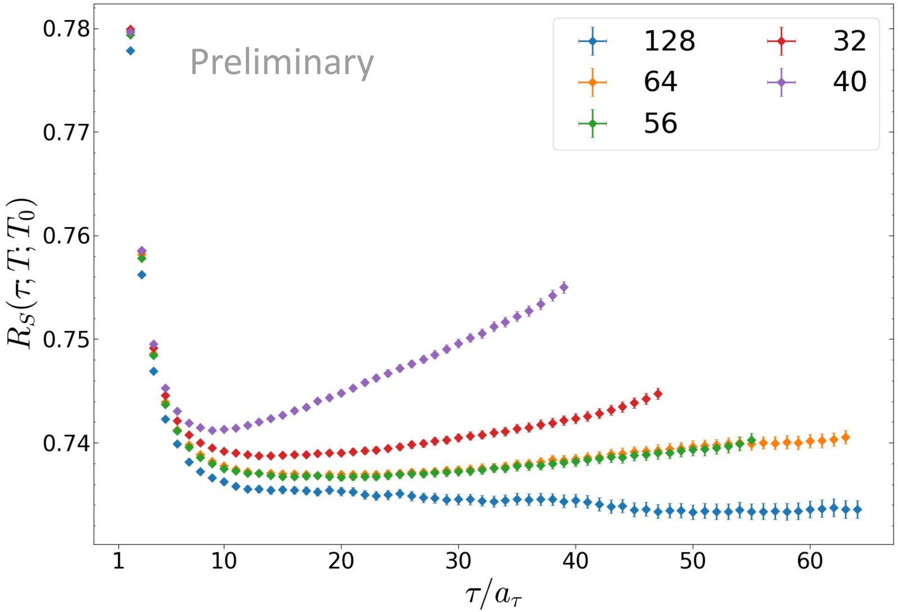
$$R(\tau; T, T_0) = \frac{r(\tau;T,T_0)}{r(\tau;T_0,T_0)}$$

Describes the `change` in spectral function $\rho(\omega)$



Single & Double Ratio

- Single Ratio shows how similar to zero-temperature
  - Excited states still present
  - Constant if $\rho(\omega)$ is a delta-function
- Double Ratio
  - Removes excited state effect
  - Differences from one show difference in correlator



Double Ratio

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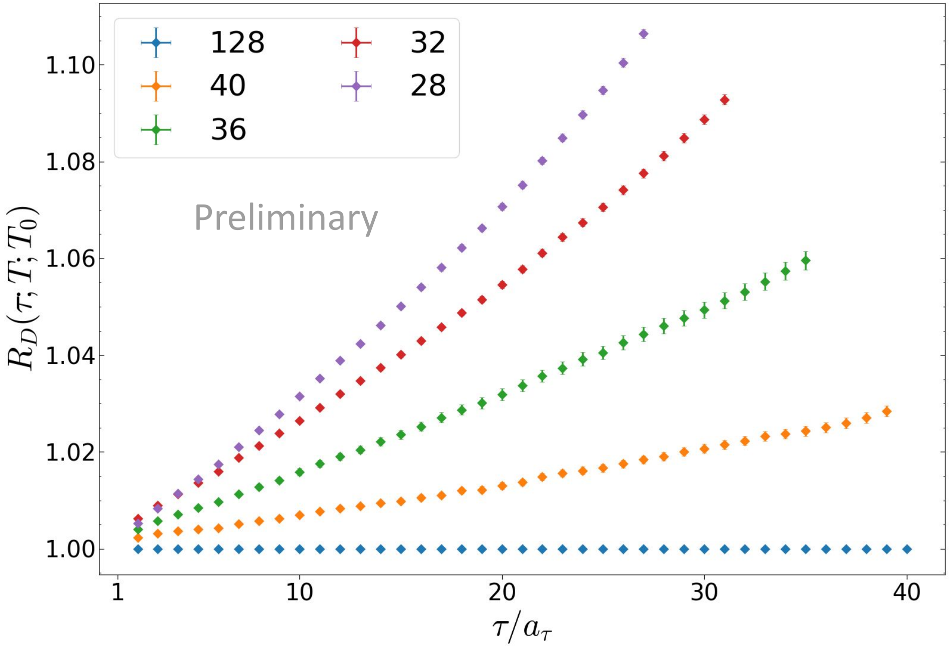
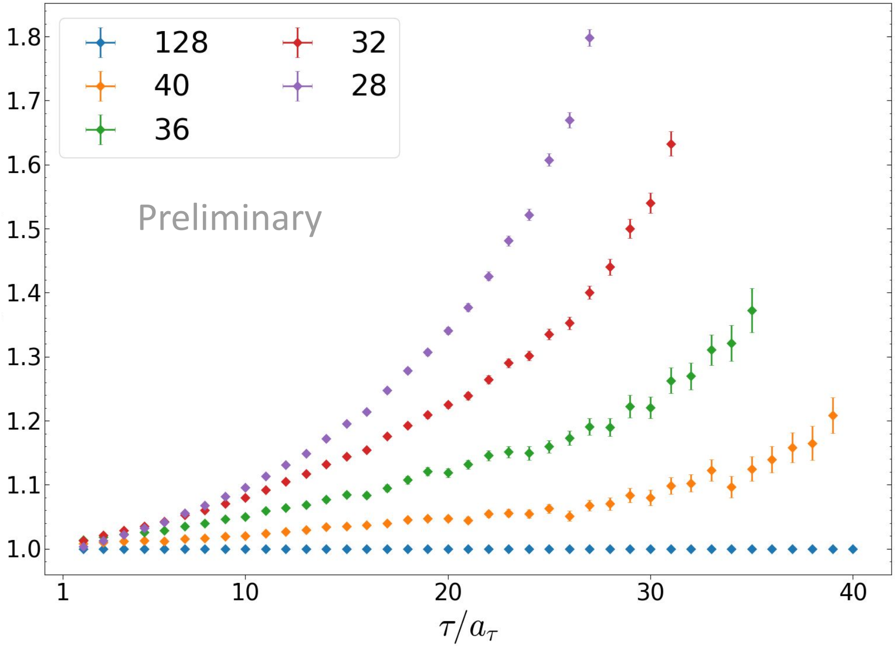


$\Upsilon(1S)$

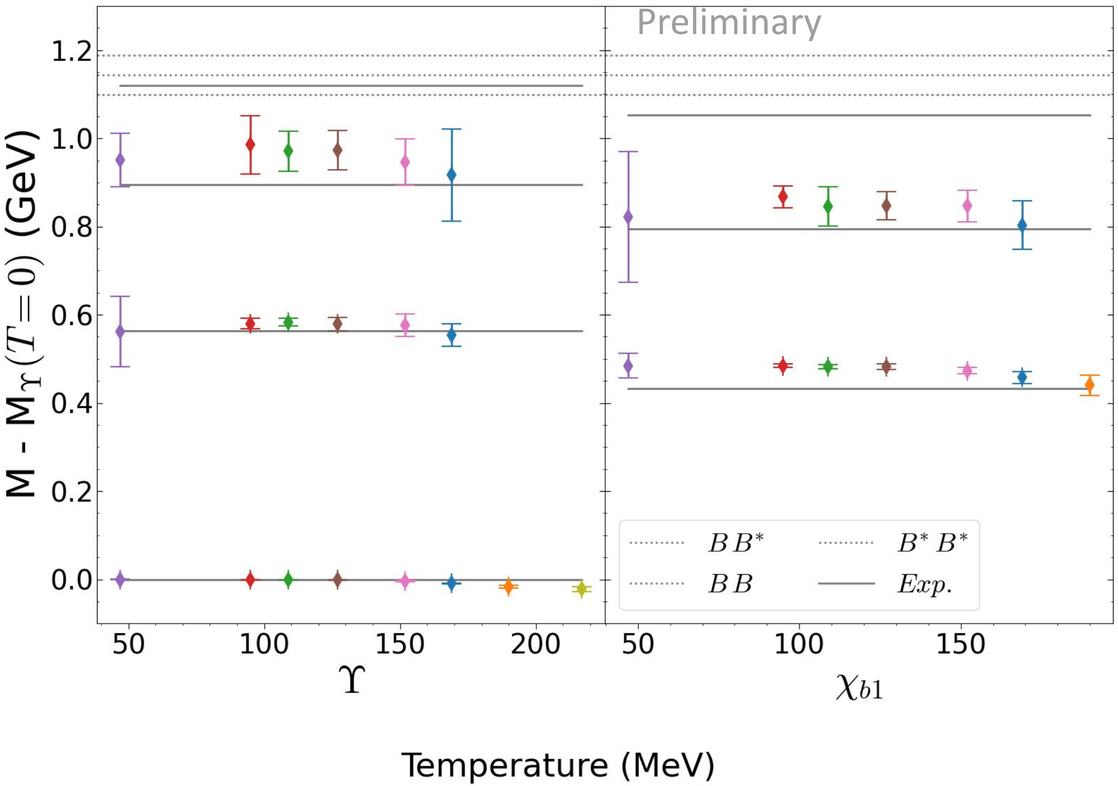


$\Upsilon(2S)$

Mass Spectrum Results

Subtract zero-temperature $\Upsilon(1S)$

- Double Ratio informs trust in standard (multi-) exponential fits $\sum_i A_i e^{-E_i \tau}$
- Model averaging techniques used to give robust determination of energy.



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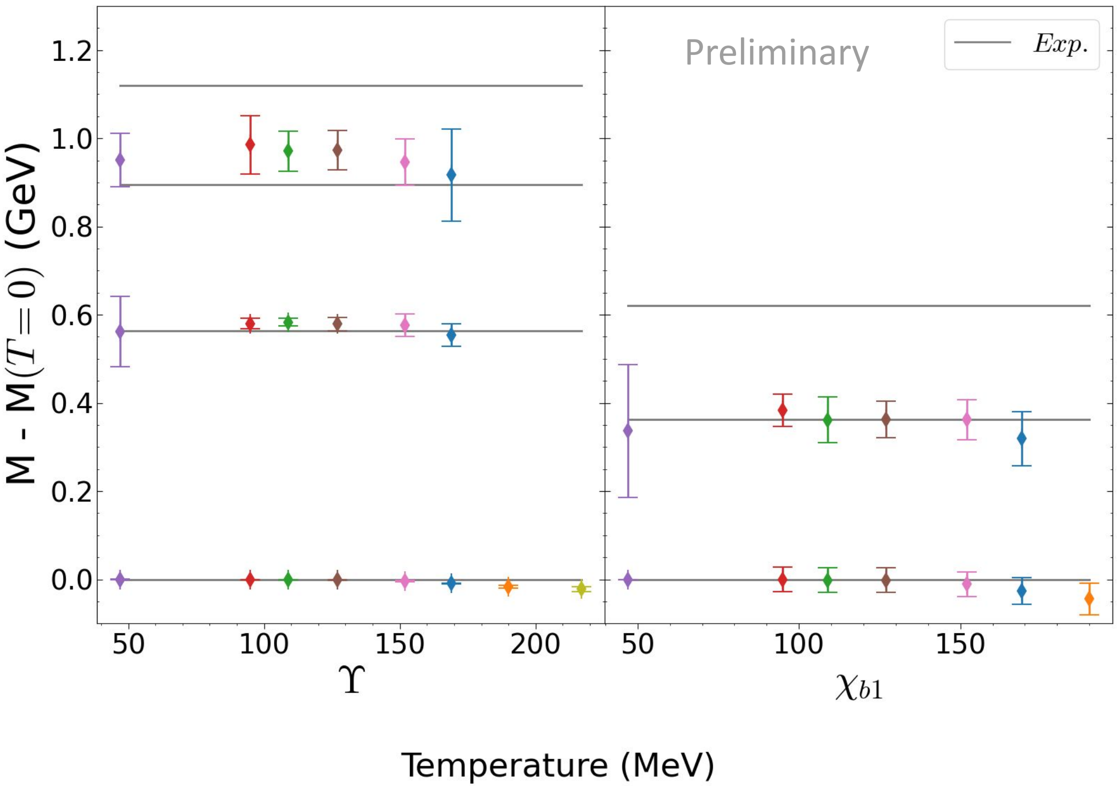

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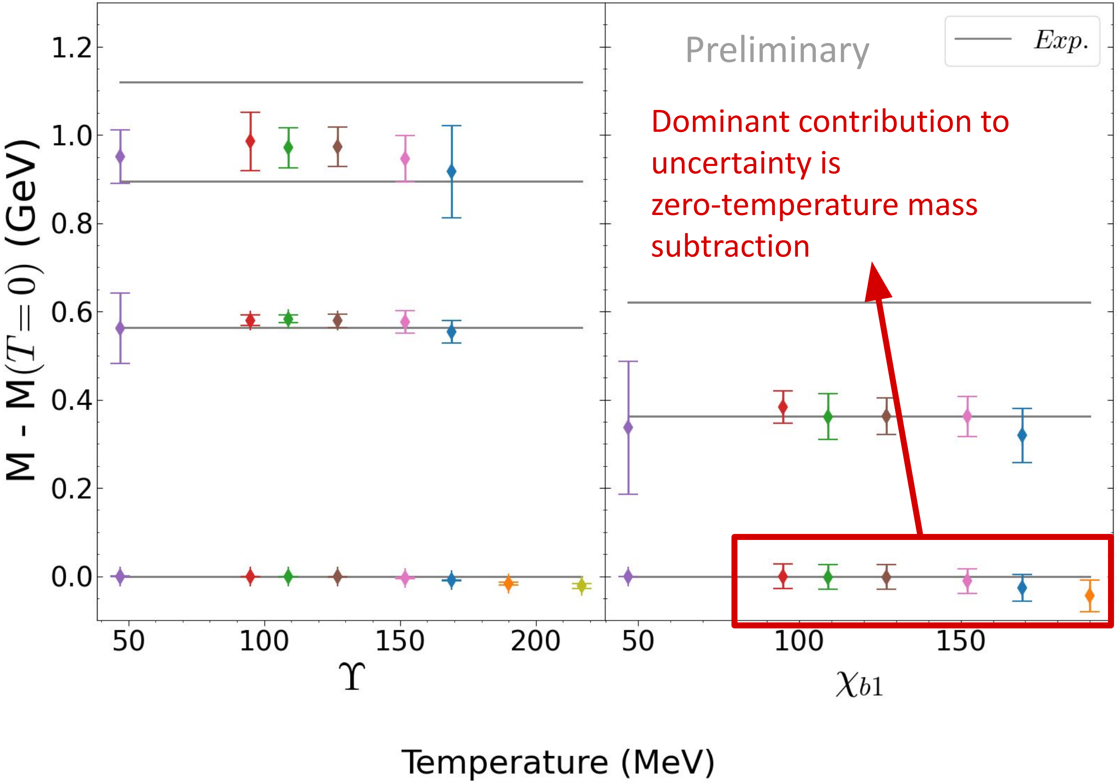


`Moments`

`Time-Derivative Moments`

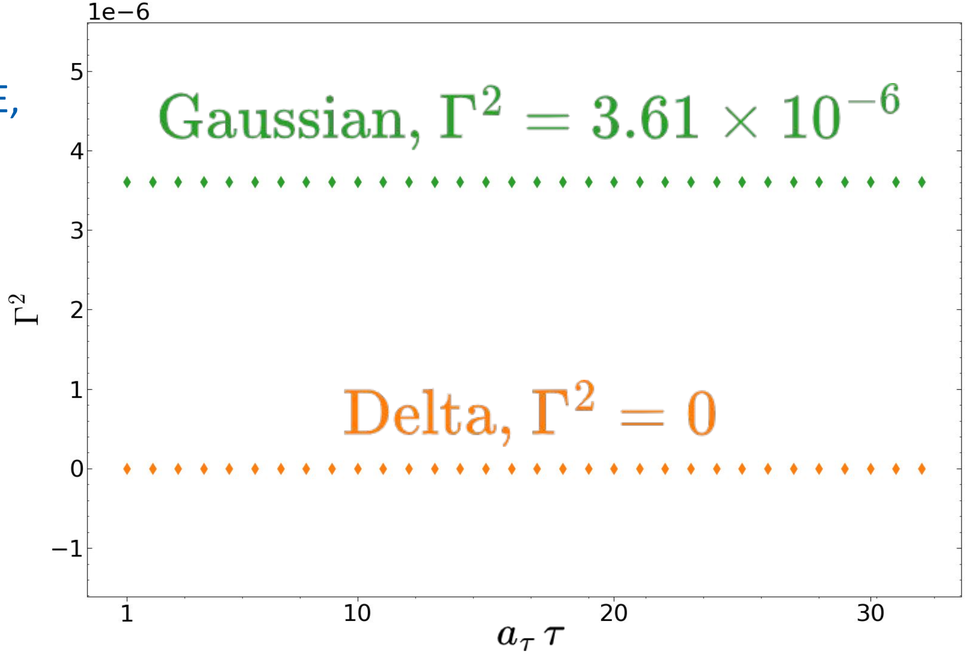
$$G(\tau) = \int_0^\infty \frac{d\omega}{2\,\pi}\, \mathrm{e}^{-\omega\,\tau}\, \rho(\omega)$$

If $\rho(\omega)$ is Gaussian with width Γ and mean E, second log-derivative is

$$\frac{d^2\ \log(G(\tau))}{d\,\tau^2} = \frac{G''(\tau)}{G(\tau)} - \left(\frac{G'(\tau)}{G(\tau)}\right)^2$$
$$= \cancel{E}^2 + \Gamma^2 - (\cancel{E})^2$$
$$= \Gamma^2$$

This is the difference between 2nd and 1st non-central moments of a Gaussian

Model/Mock Correlator Data



`Moments`

Point-Point

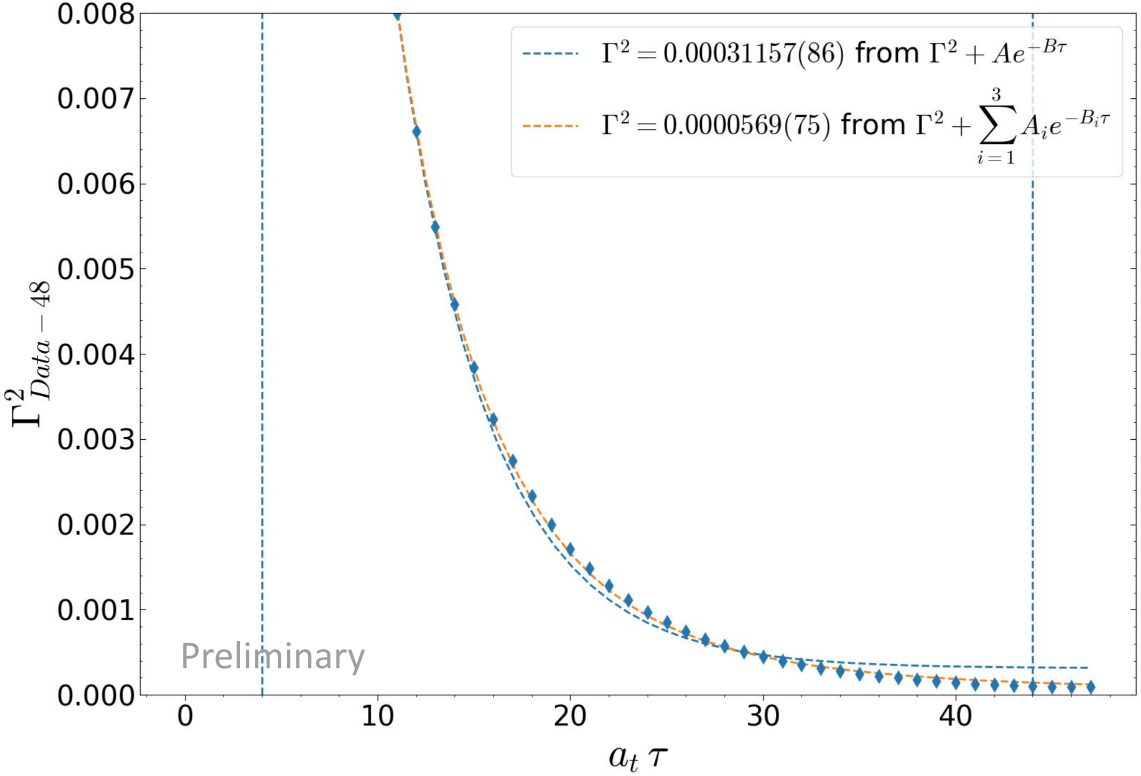
- Excited states shift form
- Fit with function

$$\Gamma^2 + \sum_{i=1}^{N} A_i e^{-B_i\,\tau}$$

- Easier at higher temperatures as Γ^2 becomes larger

- This is an upper bound only

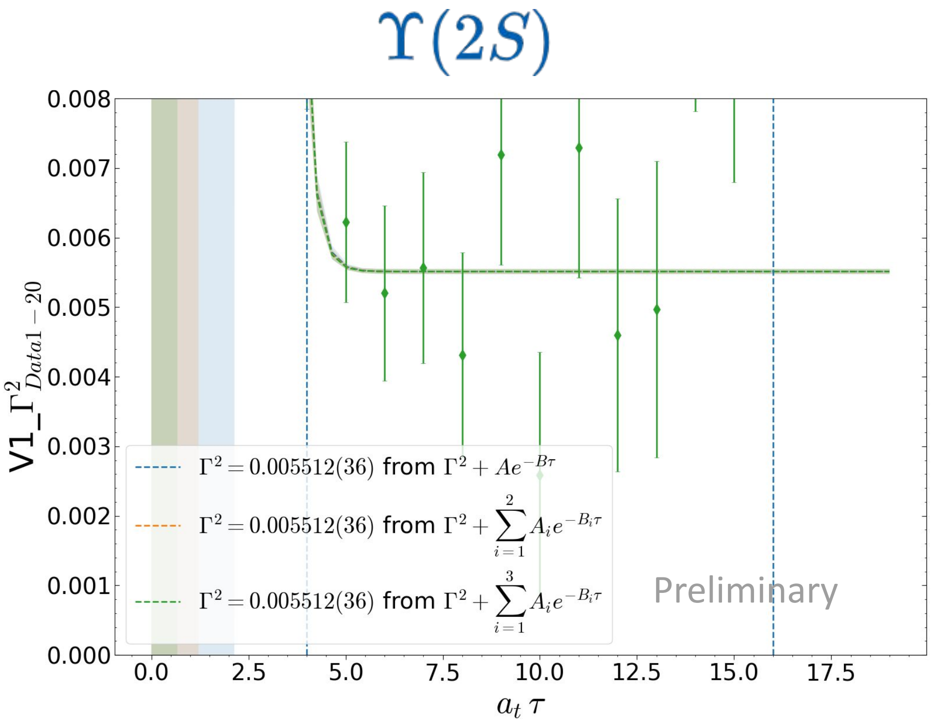
$\Upsilon(1S)$ Point-Point Correlator



`Moments`

GEVP

- Apply `moments` method to GEVP projected correlators
- GEVP essential for access to excited states for moments
- Method is fairly robust against noise
    - Constant Γ^2 term helps
    - Exponential terms not well constrained
    - More statistics ongoing



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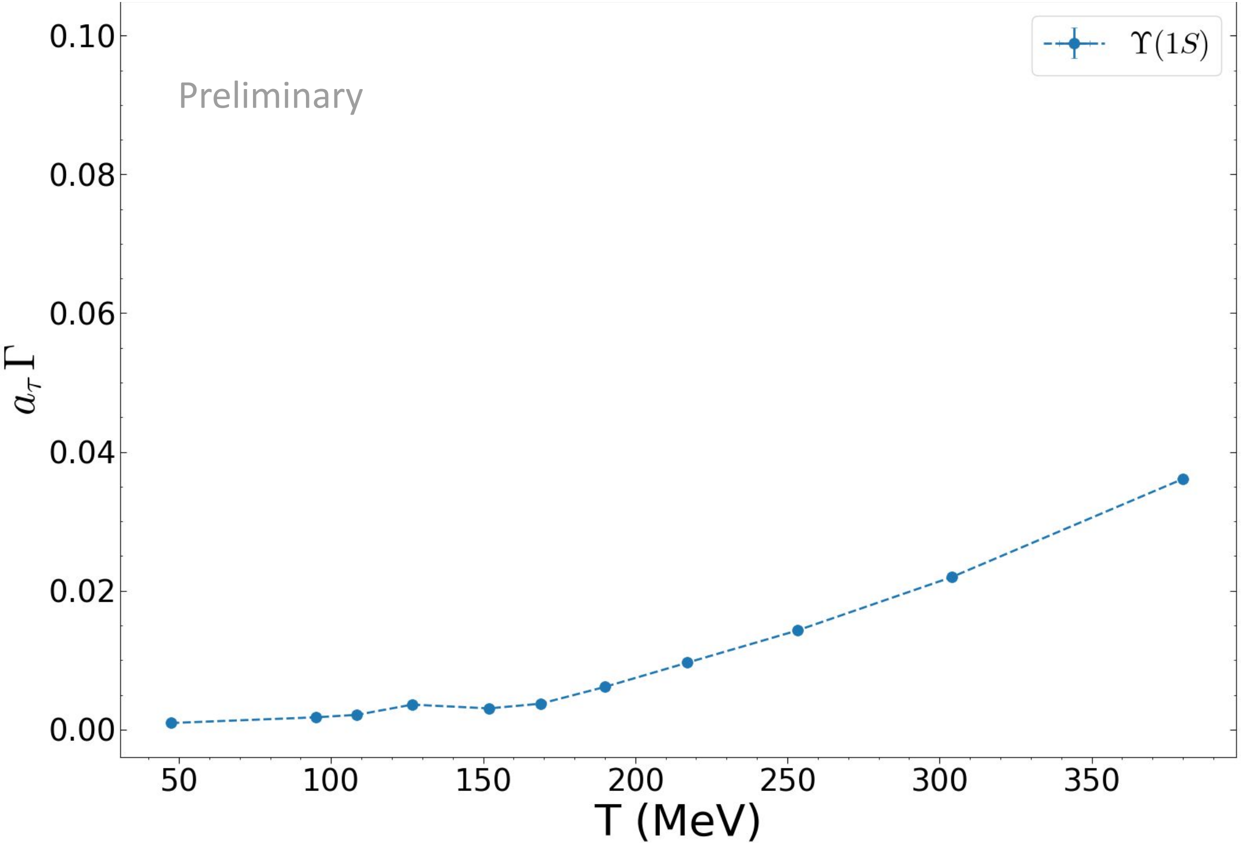

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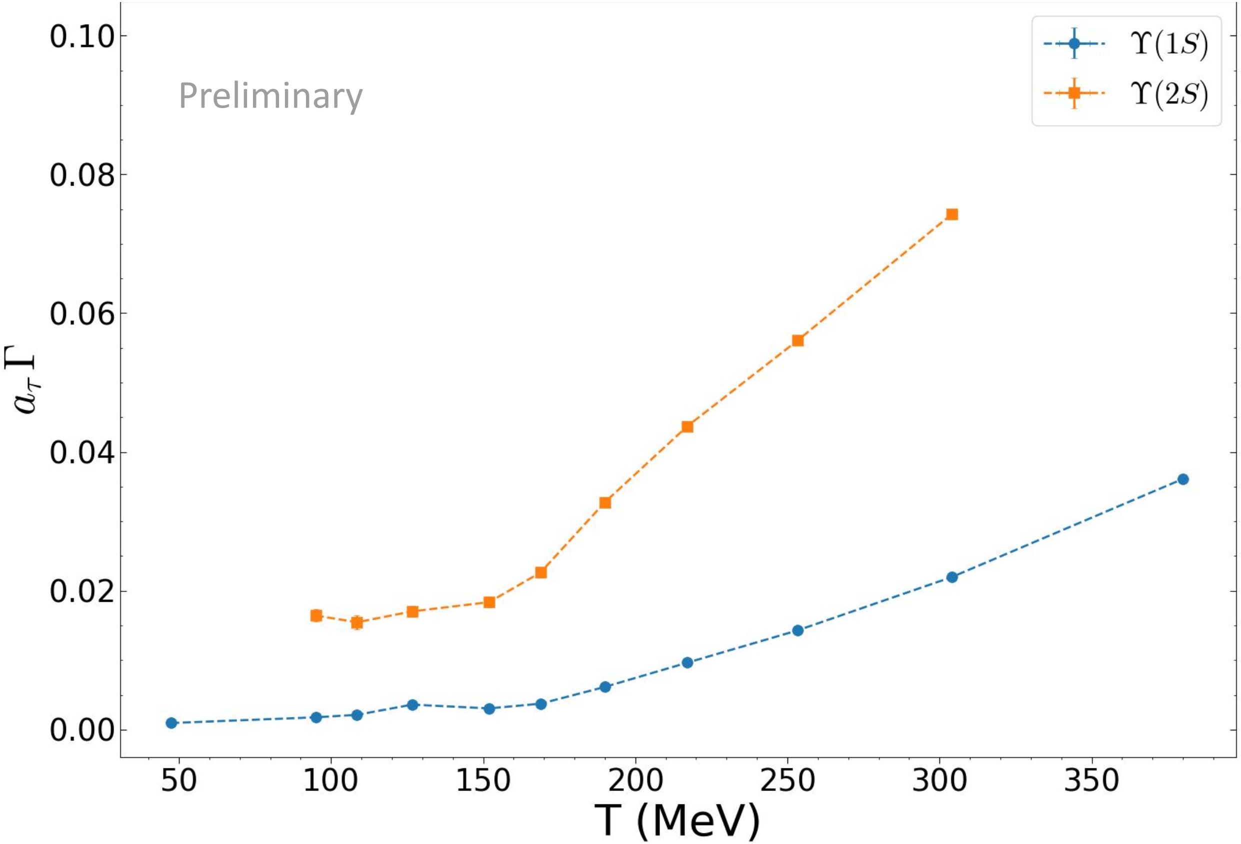

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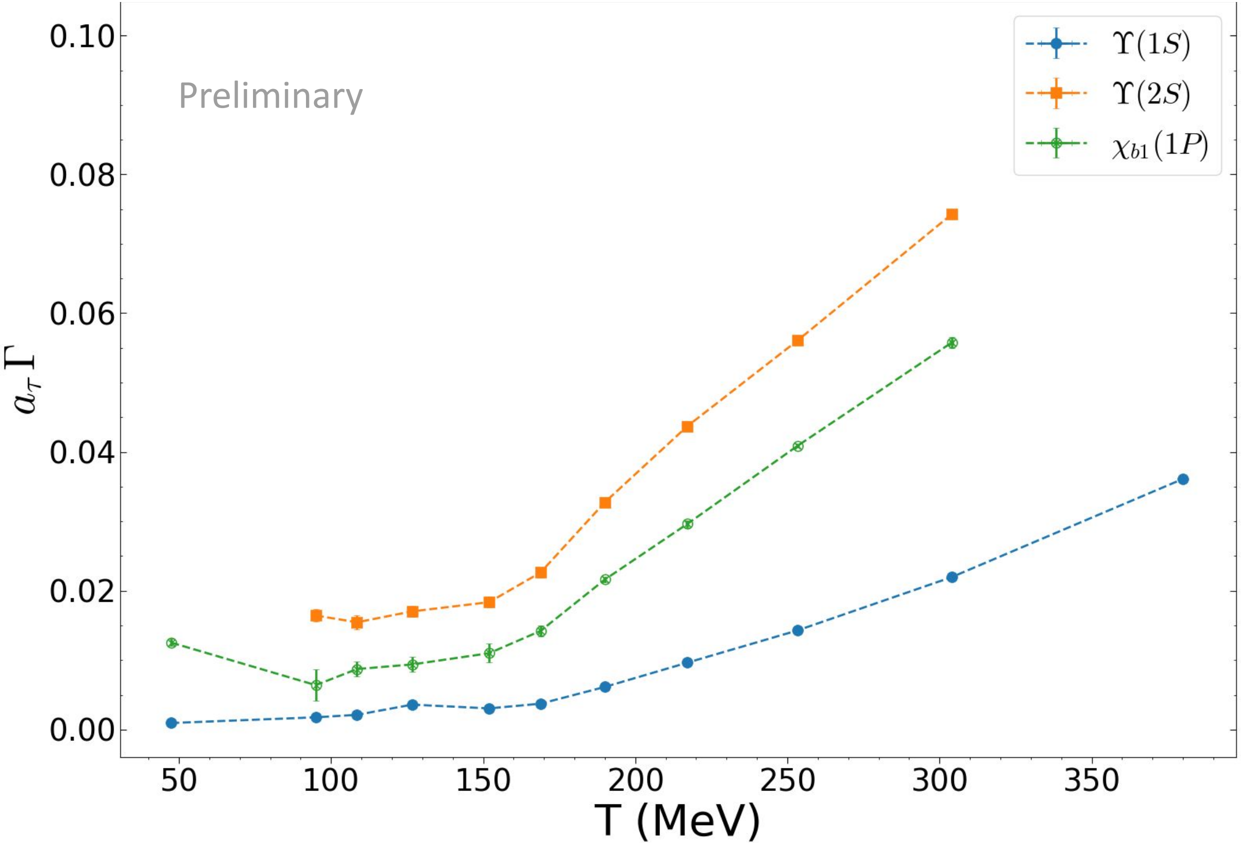

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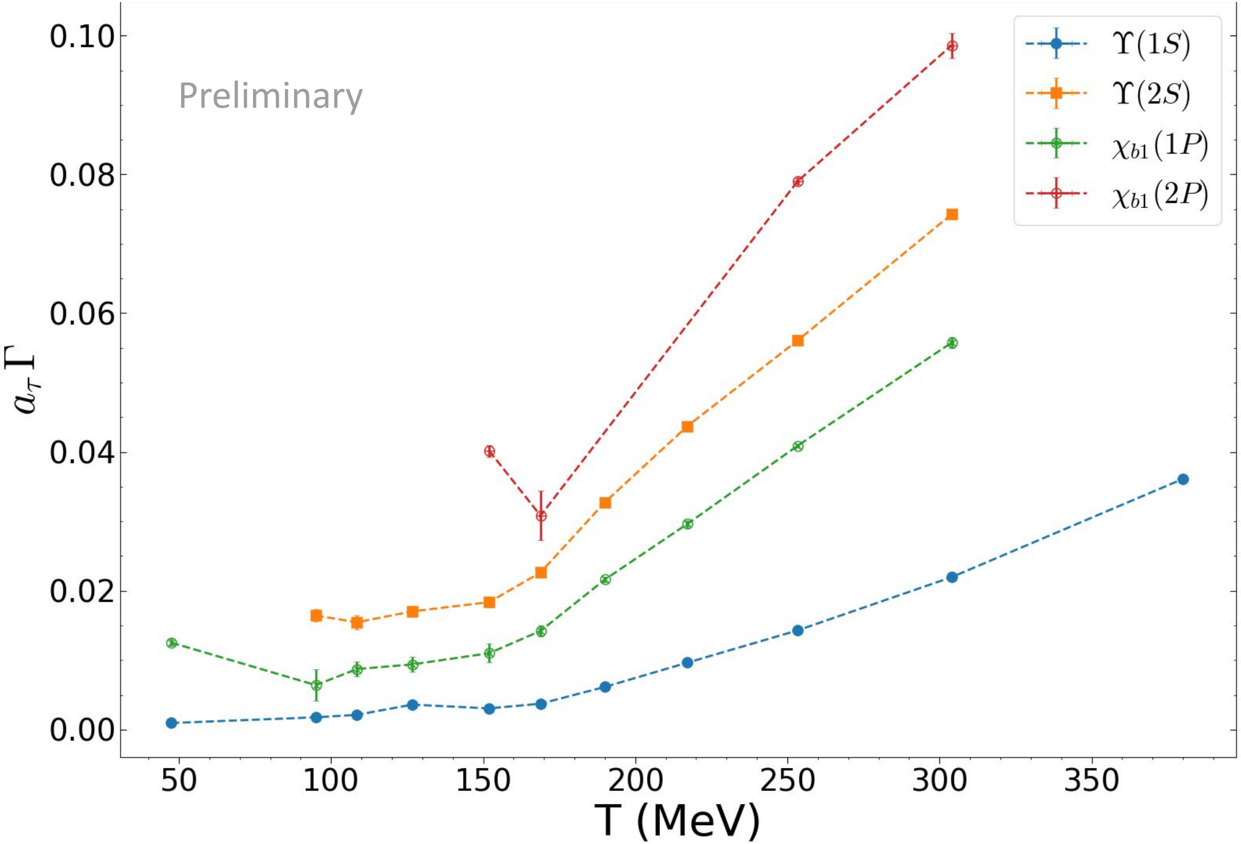
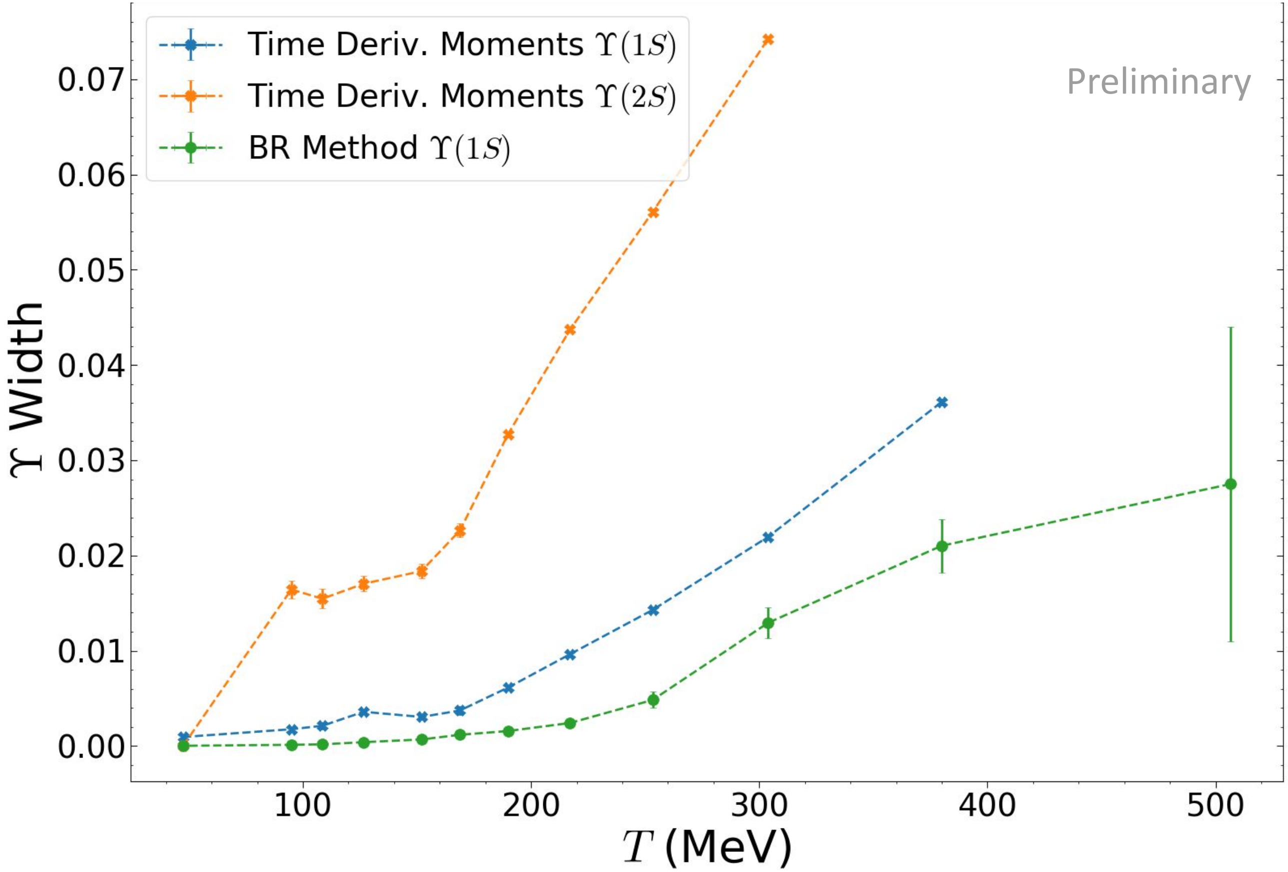


`Moments`

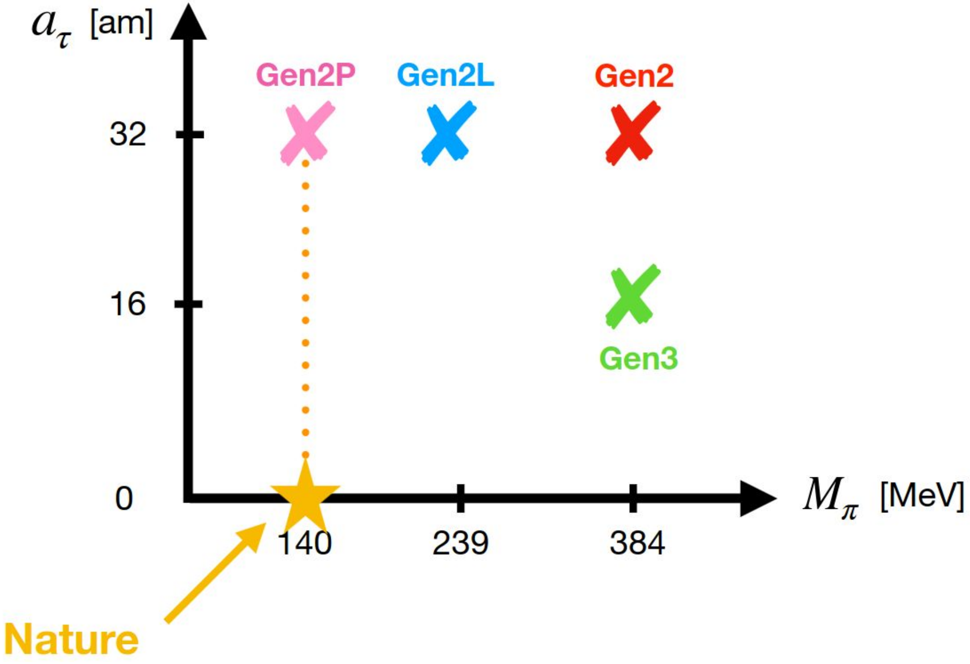
Comparison

- Bayesian Reconstruction method
- Moments method for ground & excited states
- Encouraging similarity between methods
- Excited state is broader than ground state



Summary

- Presented results for the mass of Υ and χ_{b1} excited states using a basis of 'smeared' operators
  - At zero and finite temperature
- (Re-)introduced `moments` method to examine 'widths' of ground state (Gaussian) spectral functions
- Applied `moments` to GEVP projected correlators
- GEVP of smeared operators was successful in allowing use of the `moments` method for excited states
- Systematics of method not fully explored for this study (GEVP correlators)



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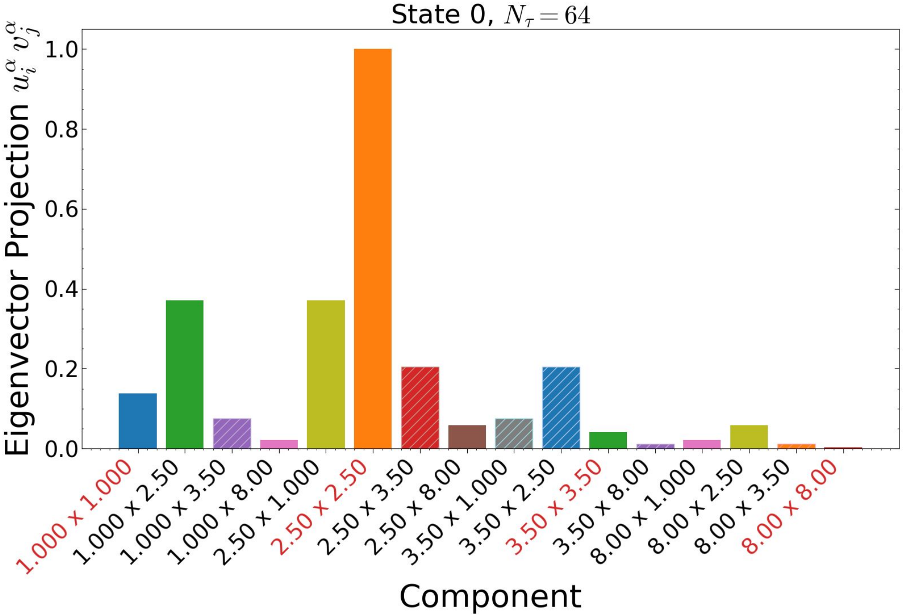
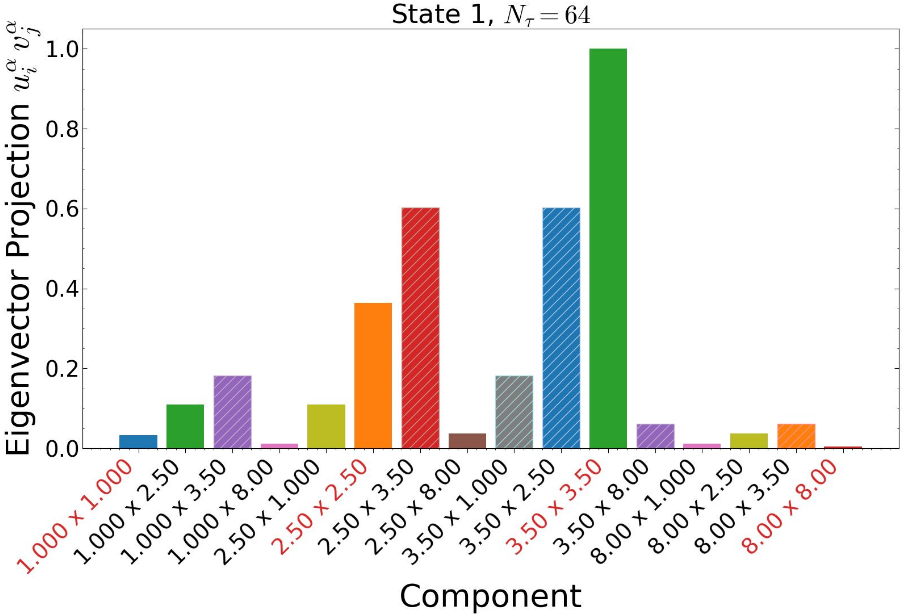
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Additional Slides

Eigenvectors





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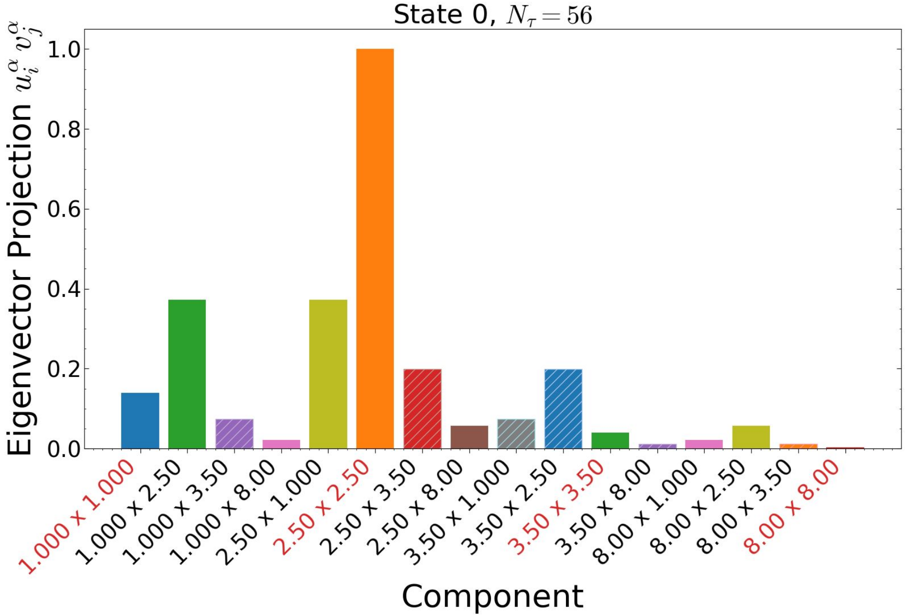

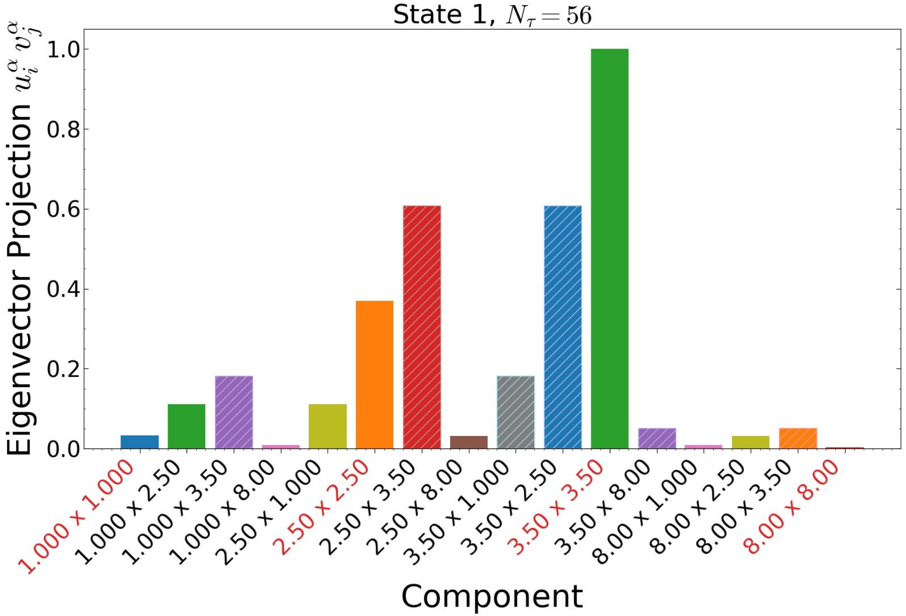

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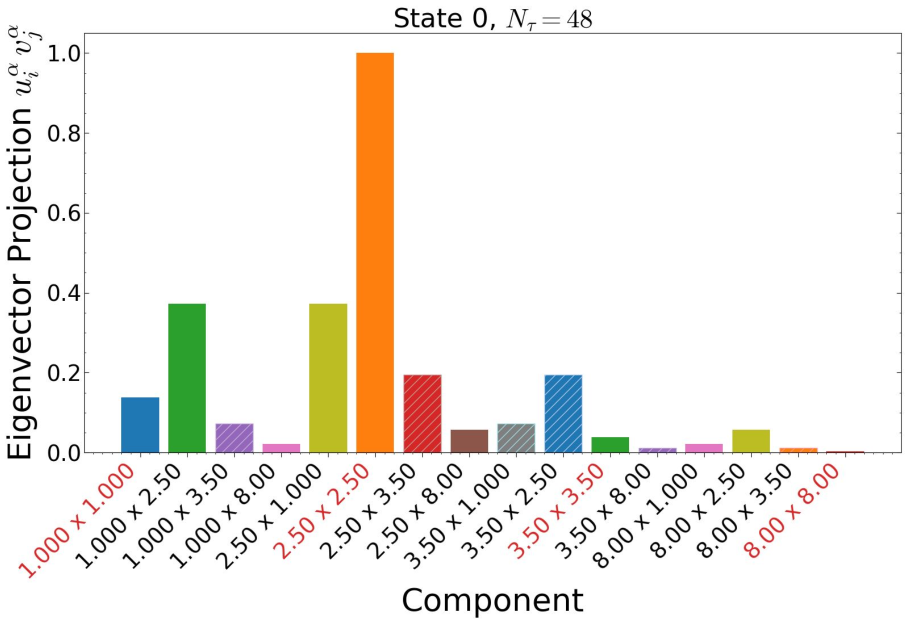

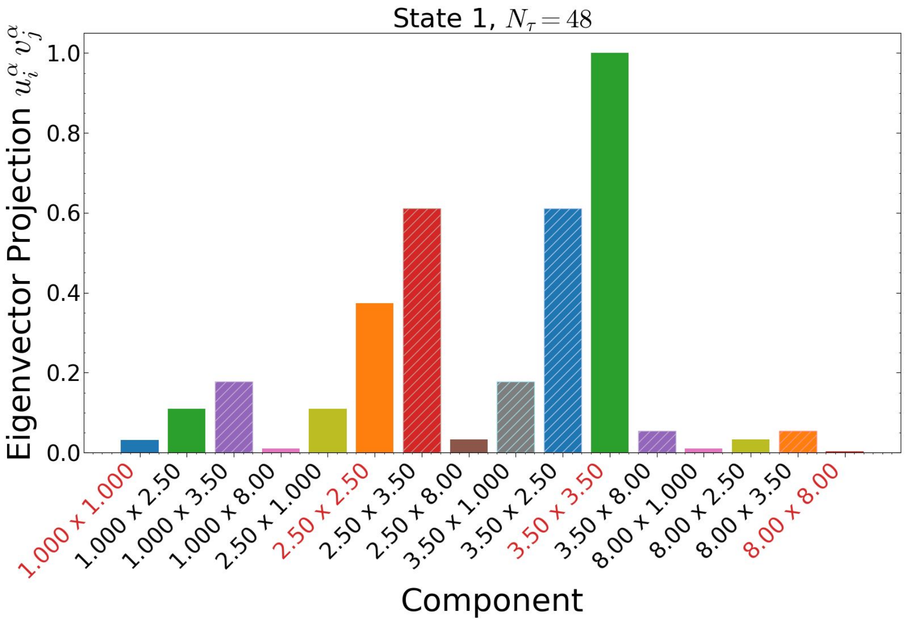

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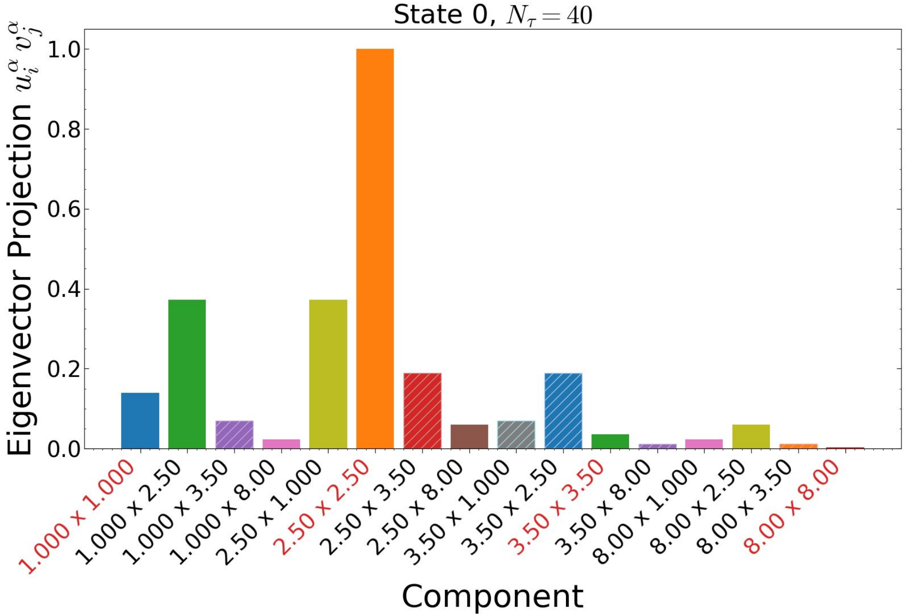

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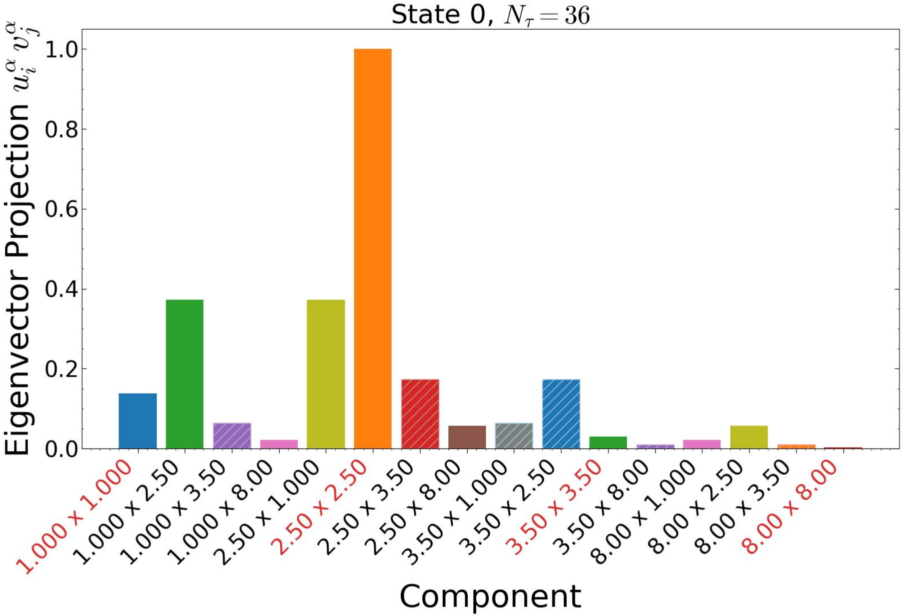

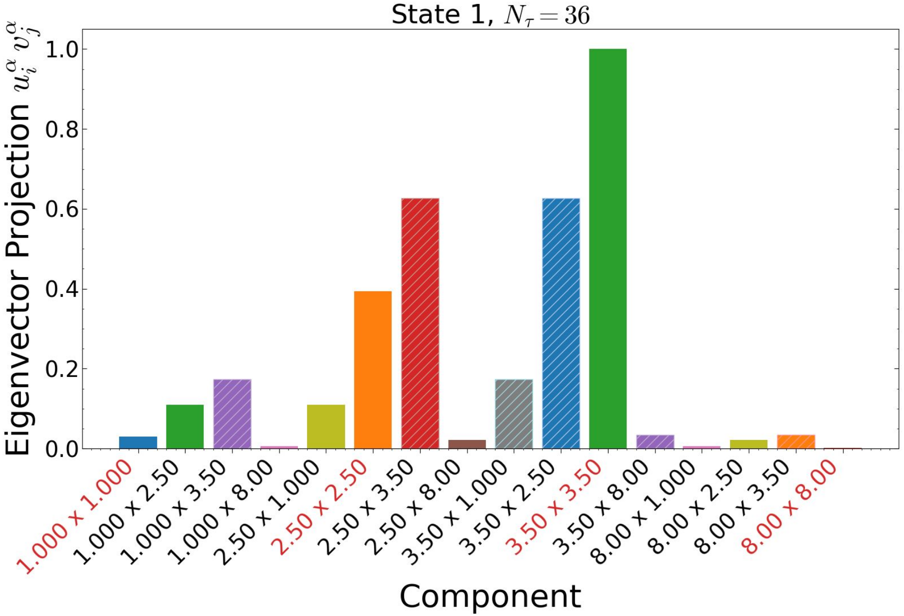

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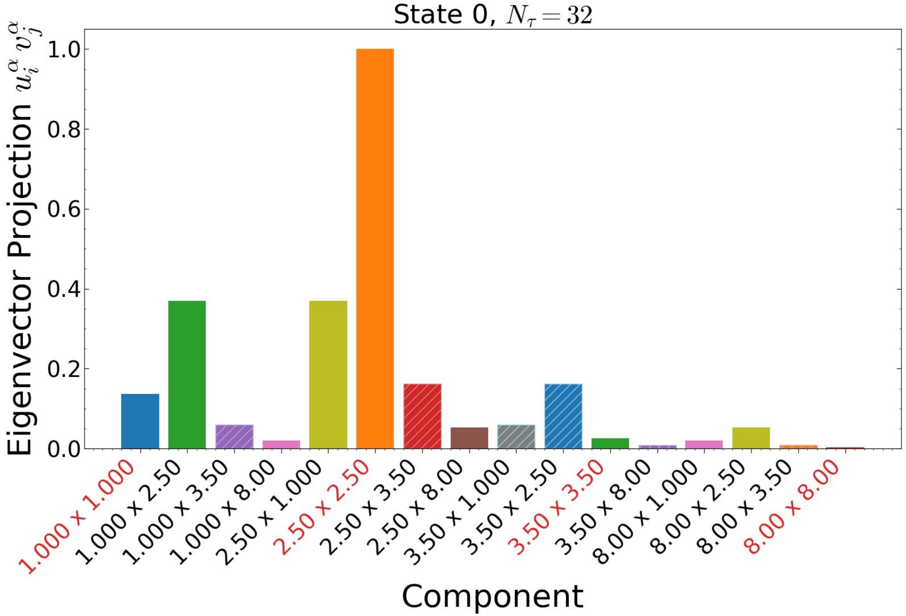

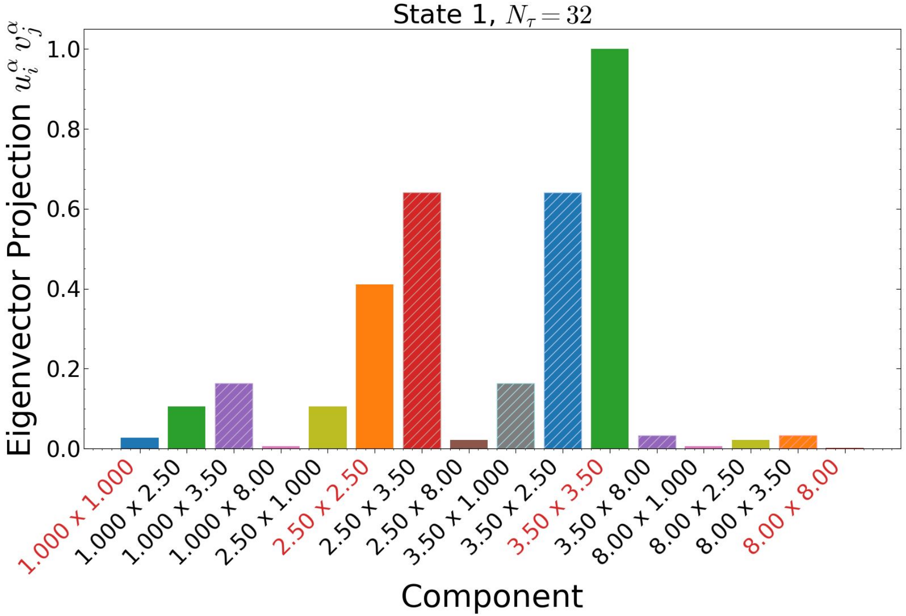

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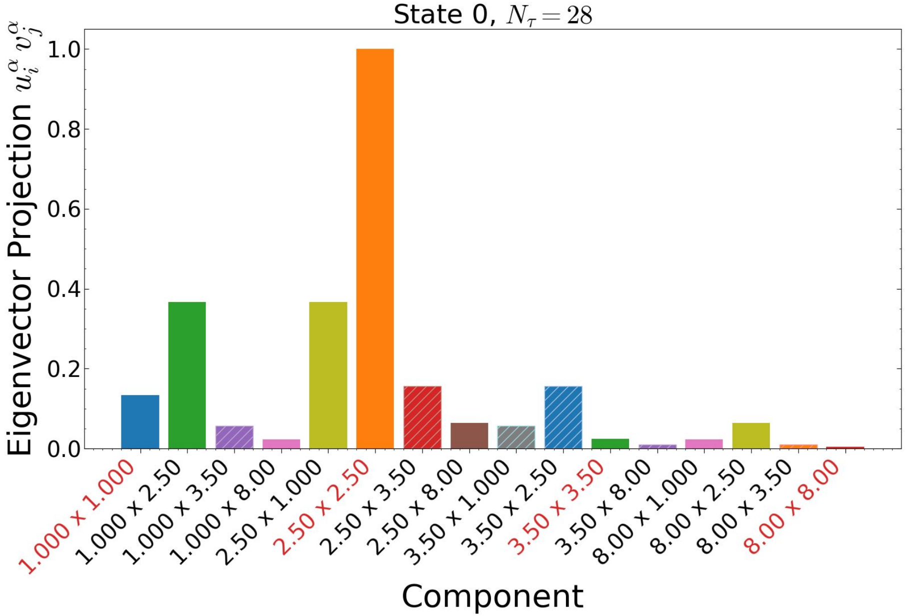

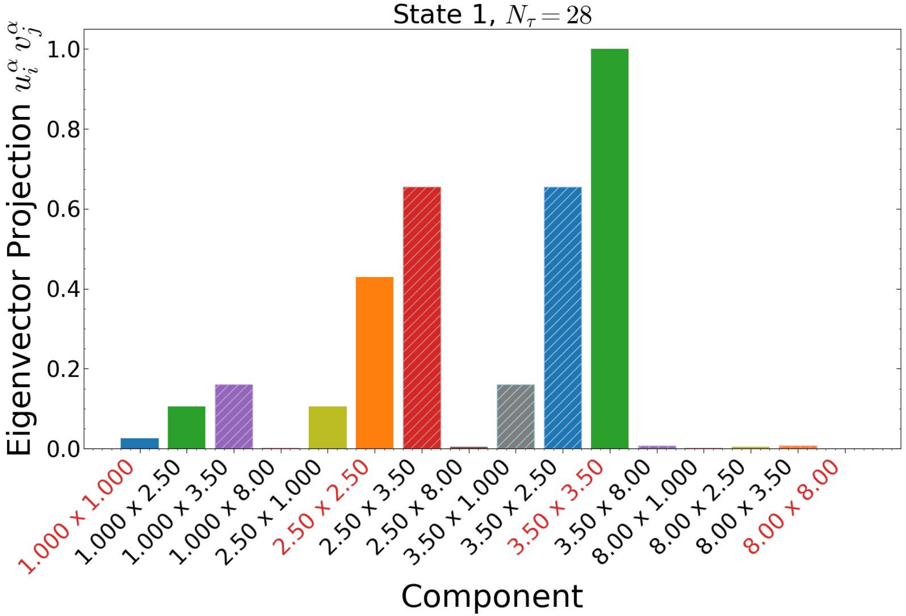

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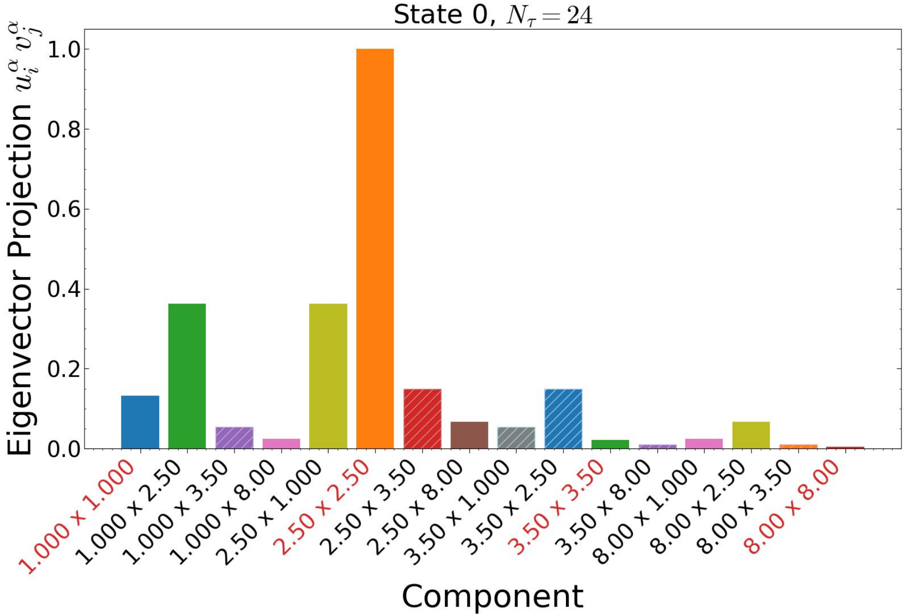

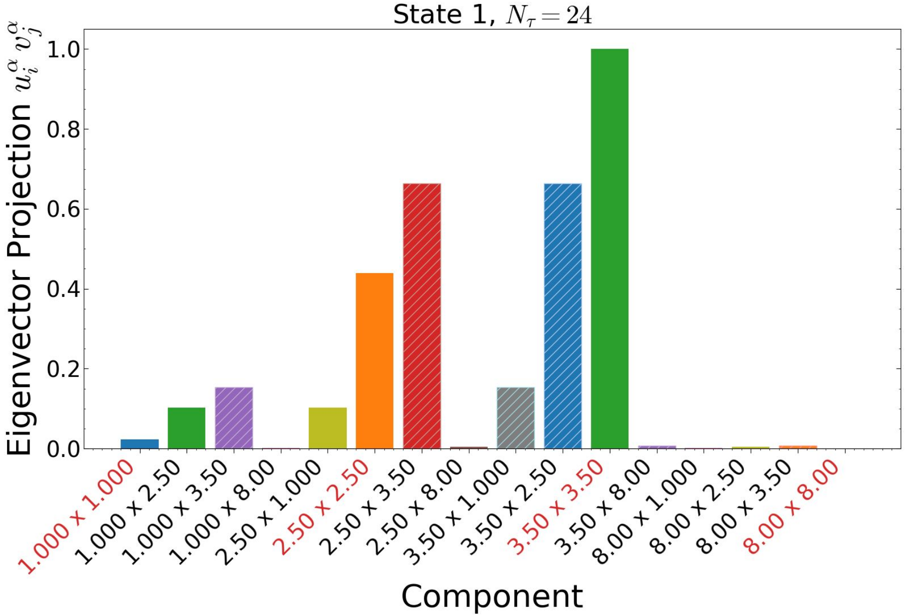

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