NRQCD Bottomonium at non-zero Temperature using Time-derivative Moments

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Motivation

- Heavy quarkonia states serve as probes for QGP as their masses are larger than other energy scales
- Heavy quarkonia states can be used as a thermometer for relativistic heavy-ion collisions
- NRQCD - b-quark mass larger than other mass scales and can be approximated to a non-relativistic particle

Euclidean meson correlator is given by

$$G(\tau; T) = \int_{\omega_{min}}^{\omega_{max}} \frac{\mathrm{d}\omega}{2\pi} K(\tau, \omega) \rho(\omega; T) \,,$$

NRQCD correlator,

$$G(\tau; T) = \int_{\omega_{min}}^{\omega_{max}} \frac{\mathrm{d}\omega}{2\pi} e^{-\omega\tau} \rho(\omega; T)$$

Ensembles

Gen	N_f	ξ	a_s (fm)	a_τ^{-1}(Gev)	m_π(MeV)	N_s
2	$2+1$	3.45	0.121	5.63	390	$24/32$
2L	$2+1$	3.45	0.112	6.08	240	32
3	$2+1$	7	*0.11	*11.66	*390	32

Lattice parameters for FASTSUM ensembles Generation 2, 2L and 3.

$T_c \sim 168$ MeV (Gen2L)

$T_c \sim 180$ MeV (Gen3)

Mass

To calculate the mass we approximate the spectral function to a Gaussian,

$$\rho(\omega; T) \propto \sum_{i=0}^{\infty} e^{-\frac{(\omega - m_i)^2}{2\Gamma_i^2}},$$

and then take the $\lim_{\Gamma^2 \to 0}$, so we have

$$\rho(\omega; T) \propto \sum_{i=0}^{\infty} \delta(\omega - m_i)$$

The correlator then becomes,

$$G(\tau; T) \propto \sum_{i=0}^{\infty} \int \frac{d\omega}{2\pi} e^{-\omega\tau} \delta(\omega - m_i)$$

Mass

Taking the time derivative

$$\frac{\partial G(\tau;T)}{\partial \tau} \propto \int -\frac{\mathrm{d}\omega}{2\pi}\omega e^{-\omega\tau}\delta_{\omega,m}$$

Divide by the correlator and evaluate the integral,

$$\frac{G'(\tau;T)}{G(\tau;T)} = \frac{\int -\frac{\mathrm{d}\omega}{2\pi}\omega e^{-\omega\tau}\delta_{\omega,m}}{\int \frac{\mathrm{d}\omega}{2\pi}e^{-\omega\tau}\delta_{\omega,m}}$$

$$\frac{G'(\tau;T)}{G(\tau;T)} = -m$$

We will use,

$$\frac{\partial(\log(G(\tau;T)))}{\partial\tau} = \frac{G'(\tau;T)}{G(\tau;T)}$$

We use a 4th order symmetric derivative.

Mass

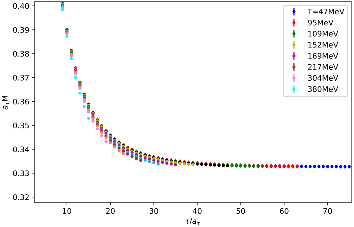
$$\rho(\omega) \propto \sum_{i=0}^{\infty} e^{-\frac{(\omega - m_i)^2}{2\Gamma_i^2}}$$

$$G(\tau) = \sum_{i=0}^{\infty} A_i e^{-m_i\tau + \Gamma_i^2\tau^2/2}$$

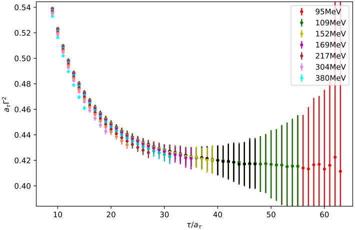
$$G(\tau) = A_0 e^{-m_0\tau + \Gamma_0^2\tau^2/2} \left(1 + \sum_{i=1}^{\infty} \frac{A_i}{A_0} e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}\right)$$

$$\frac{\partial \log(G(\tau))}{\partial\tau} = (-m_0 + \Gamma_0^2\tau) + \sum_{i=1}^{\infty} \frac{A_i}{A_0} e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}$$

Mass



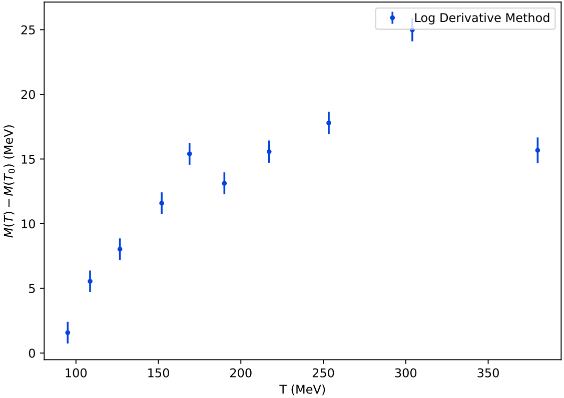
Mass in the Υ channel for all temperatures.



Mass in the χ_{b_1} channel for all temperatures.

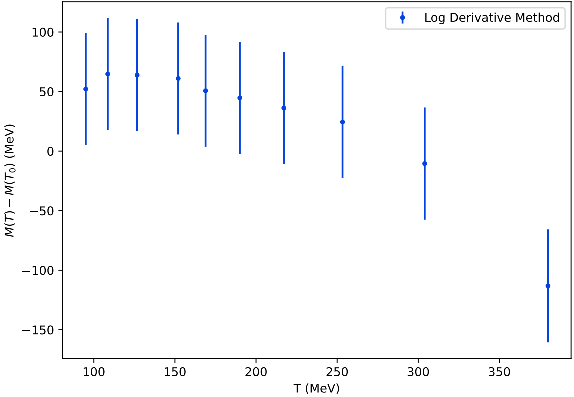
Υ - Mass

- fit to $M + ae^{-b\tau}$
- increase with temperature in agreement with other methods
- $M(T_0) =$ 9455(10)MeV, $M_{exp} =$ 9460MeV



χ_{b_1} - Mass

- fit to $M + ae^{-b\tau}$
- consider Γ in fit?
- $M(T_0) =$ 9965(79)MeV, $M_{exp} =$ 9892MeV



Width

To calculate the thermal width, Γ, we approximate the spectral function to a Gaussian function, centered at the mass and with a width of Γ,

$$\rho(\omega) \propto e^{-\frac{(\omega-m)^2}{2\Gamma^2}}$$

$$G(\tau;T) \propto \int \frac{d\omega}{2\pi} e^{-\omega\tau} e^{-\frac{(\omega-m)^2}{2\Gamma^2}}$$

Width

Since the spectral function $\rho(\omega)$ is a Gaussian then $\Gamma^2 = Var \langle \omega \rangle$.

$$\Gamma^2 = \left\langle \omega^2 \right\rangle - \langle \omega \rangle^2$$

We take a weighted average of ω and ω^2

$$\Gamma^2 = \frac{\int \frac{d\omega}{2\pi} \omega^2 e^{-\omega\tau} e^{-\frac{(\omega-m)^2}{2\Gamma^2}}}{\int \frac{d\omega}{2\pi} e^{-\omega\tau} e^{-\frac{(\omega-m)^2}{2\Gamma^2}}} - \left(\frac{\int \frac{d\omega}{2\pi} \omega e^{-\omega\tau} e^{-\frac{(\omega-m)^2}{2\Gamma^2}}}{\int \frac{d\omega}{2\pi} e^{-\omega\tau} e^{-\frac{(\omega-m)^2}{2\Gamma^2}}} \right)^2$$

and we can see that this is equal to,

$$\Gamma^2 = \frac{G''(\tau; T)}{G(\tau; T)} - \left(\frac{G'(\tau; T)}{G(\tau; T)} \right)^2$$

Width

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and we can see that this is equal to,

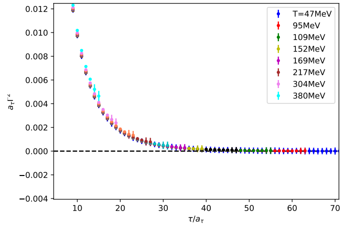
$$\Gamma^2 = \frac{G''(\tau; T)}{G(\tau; T)} - (M)^2$$

Width

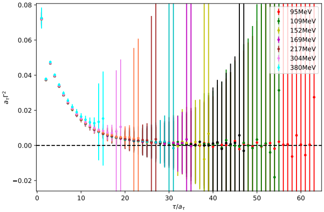
$$\frac{\partial^2(\log(G(\tau;T)))}{\partial\tau^2} = \frac{G''(\tau;T)G(\tau;T) - G'(\tau;T)^2}{G(\tau;T)^2}$$
$$= \frac{G''(\tau;T)}{G(\tau;T)} - \left(\frac{G'(\tau;T)}{G(\tau;T)}\right)^2$$

$$\frac{\partial\log(G(\tau))}{\partial\tau} = (-m_0 + \Gamma_0^2\tau) + \sum_{i=1}^{\infty}\frac{A_i}{A_0}e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}$$
$$\frac{\partial^2\log(G(\tau))}{\partial\tau^2} = (\Gamma_0^2) + \sum_{i=1}^{\infty}B_i e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}$$

Width



Width in the Υ channel for all temperatures.

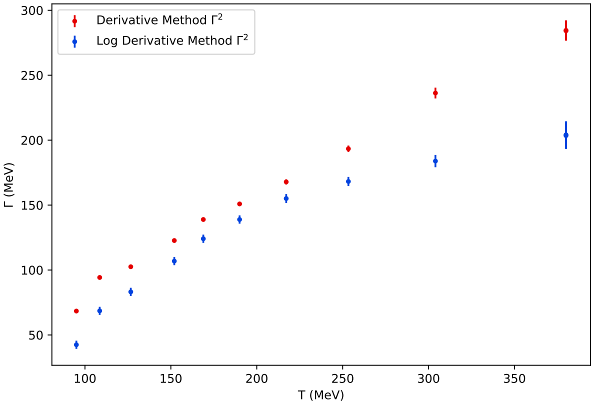


Width in the χ_{b_1} channel for all temperatures.

Υ - Width

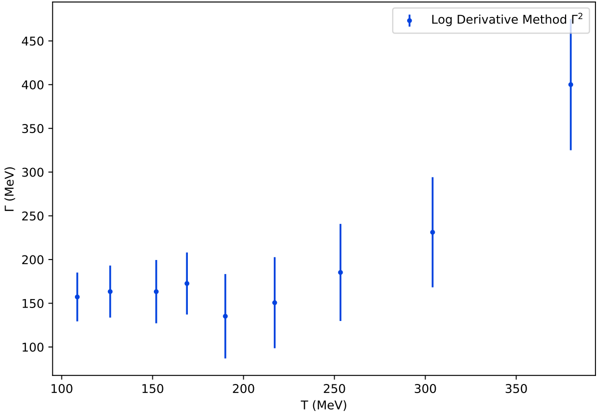
- fit to $\Gamma^2 + ae^{-b\tau}$
- increase with temperature in agreement with other methods

$$\frac{\partial^2(\log(G(\tau;T)))}{\partial\tau^2} = \frac{G''(\tau;T)}{G(\tau;T)} - \left(\frac{G'(\tau;T)}{G(\tau;T)}\right.$$



χ_{b_1} - Width

- fit to $\Gamma^2 + ae^{-b\tau}$
- increase with temperature in agreement with other methods



Conclusions

Mass

- Increase in mass with increasing temperature, in agreement with other methods
- Particles with a larger width need further study

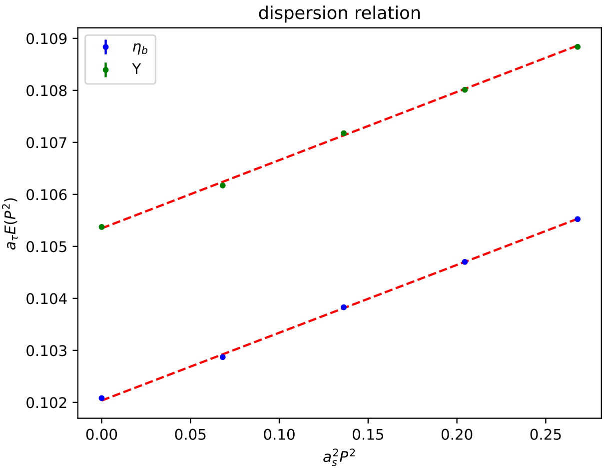
Width

- Increase in width with increasing temperature, in agreement with other methods

Outlook

- Gen 3
- Mass of other particles with significant width

Gen 3 Tuning



- Zero temperature dispersion relations used to determine 1S spin average kinetic mass, $M_2(\overline{1S})$.
- Tune the heavy quark mass by requiring $M_2(\overline{1S})$ to equal its experimental value.