UNIVERSITÄT BIELEFELD
Fakultät für Physik

CRC-TR 211

Quarkonia spectral functions from (2+1)-flavor QCD using non-perturbative thermal potential

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(HotQCD collaboration)

Lattice 2024, Liverpool, 31.07.2024

Motivation

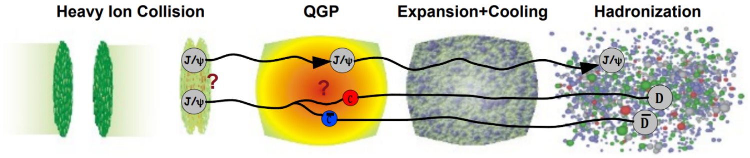
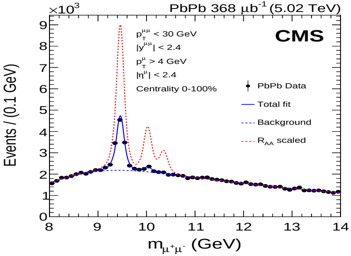


Figure: Steffen Bass

- Experimentally this QGP phase is recreated at RHIC and LHC in heavy ion collisions.
- QGP causes suppression of Quarkonia (bound states of heavy $q\bar{q}$), an important probe to study properties of QGP.



CMS Collaboration, PLB 790 (2019) 270

Correlators and spectral functions

- Heavy $q\bar{q}$: a thermometer of QGP in heavy ion collisions
- The spectral functions $\rho_H(\omega)$ contains information about the in-medium hadron properties

$$\sum_{\vec{x}} \left\langle \bar{\psi}\Gamma_H\psi(\tau,\vec{x})(\bar{\psi}\Gamma_H\psi(0,\vec{0}))^{\dagger}\right\rangle \equiv G_H(\tau) = \int_0^{\infty} \frac{d\omega}{\pi}\rho_H(\omega)\frac{\cosh(\omega(\tau-\frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

Strategy:

- $G_H(\tau)$ on the lattice
- Extract spectral function
- Estimate in-medium hadronic properties
- In addition transport coefficients, like heavy quark diffusion coefficients, are encoded in the vector meson spectral function

Spectral function in NRQCD

$$\rho_{PS}(\omega) \propto \lim_{r\to 0, r'\to 0} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\, C_>(t;\vec{r},\vec{r'})$$

$$C_>(t;\vec{r},\vec{r}') = \int d^3\vec{x} \langle \bar{\psi}(t,x+\frac{\vec{r}}{2})\gamma_5\, U\, \psi(t,x-\frac{\vec{r}}{2})\bar{\psi}(0,-\frac{\vec{r}'}{2})\gamma_5\, U\, \psi(0,-\frac{\vec{r}'}{2})\rangle_T$$

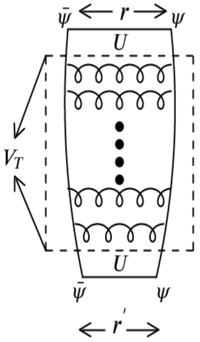
In the presence of Interaction,

$$\left\{ i\partial_t - \left[2M + V_T(r) - \frac{\nabla^2_{\vec{r}}}{M} \right] \right\} C_>(t;\vec{r},\vec{r'}) = 0$$

where V_T is defined in static limit,

$$V_T(r) = i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t} = V_{re}(r) - iV_{im}(r)$$

with $C_>(0;\vec{r},\vec{r'}) = \delta^3(\vec{r}-\vec{r}')$



M.Laine et al, JHEP 0703:054,2007

Wilson line correlator

- Non-perturbative formulation,
  A. Rothkopf et al., PRL. 108 (2012) 162001

$$W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, \rho(\omega, T) \exp(-\omega\tau)$$

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, \rho(\omega, T) \exp(-i\omega t)$$

- $\rho(\omega, T)$ should have a form which is consistent with potential, $\lim_{t\to\infty} i\frac{\partial \log W(r,t)}{\partial t}$ should exist
- Gaussian spectral function doesn't have this limit (PRD 109, 074504)
- Simple Lorentzian has this limit but results depend on the lower cut-off (PRD 105, 054513)
- Bayesian analysis has a higher systematic error (PRL 114,082001)

Wilson line correlator and the potential

$$\log(W(r,\tau)) = -V_{re}(r)\tau - \int_{-\infty}^{\infty} du\, \sigma(r,u)\left[\exp(u\,\tau) + \exp(u\,(\beta-\tau))\right] + \dots$$

HTL like τ dependence.

- $\lim_{t\to\infty} i\frac{\partial \log W(r,t)}{\partial t} = \text{finite} \implies \lim_{u\to 0}\sigma(r,u) \sim \frac{1}{u^2}$
- Following HTL PT, $\sigma(r,u) = n_B(u)\left[\frac{V_{im}}{u} + c_1\,u + c_3\,u^3 + \dots\right]$
- Parametrization

$$W(r,\tau) = A\exp[-V_{re}(r)\tau - \frac{\beta\, V_{im}(r)}{\pi}\log(\sin(\frac{\pi\tau}{\beta})) + \dots]$$

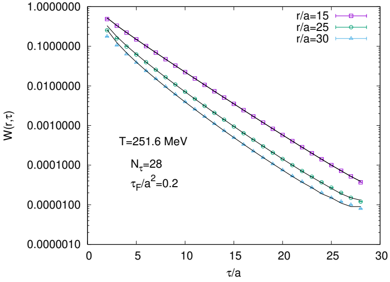
D. Bala et al, PRD 101, 034507

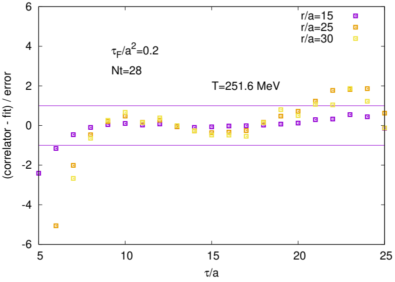
D. Bala et al, PRD 103, 014512

D. Bala et al, PRD 105, 054513

Wilson line correlator and potential

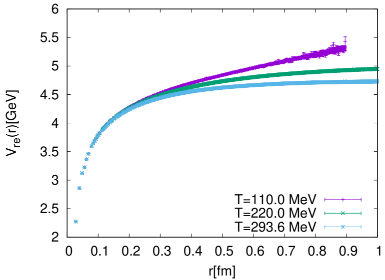
- Measure Wilson line correlator at finite flow time (τ_F)
- Three parameters fit ($\chi^2/dof \sim 1$) of Wilson line correlator for different distances.

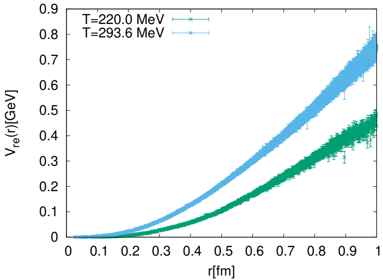




Color screening supported by the lattice data

β	a[fm]	m_l	N_σ	N_τ	T[MeV]
8.249	0.028	$m_s/5$	64	64	110.0
			96	32	220.0
			96	24	293.6

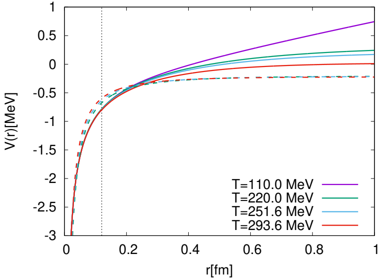


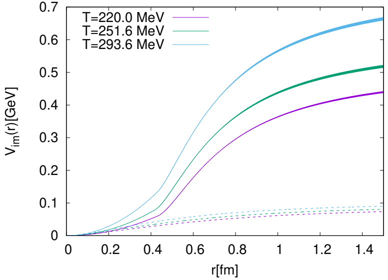


Functional form of the potential

$$V_{re}(r) = \frac{\sigma}{m_d}(1-\exp(-m_d r)) - \frac{\alpha}{r}\exp(-m_d r) + c$$

$$V_{im}(r) = \begin{cases} \frac{1}{2}br^2 & \text{for } r < r_0 \\ a_0 - \frac{a_1}{2r^2} - \frac{a_2}{4r^4} & \text{for } r \geq r_0 \end{cases}$$

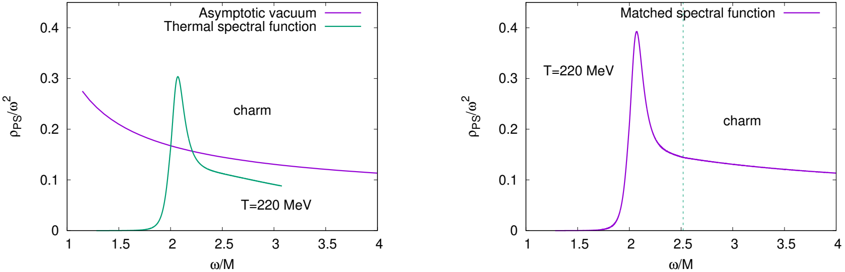




- Renormalon subtracted perturbative potential
- Non-perturbative thermal potential ≠ perturbative potential

Matching of the thermal and vacuum parts

$$\rho_{PS}^{mod}(\omega) = A_0\,\rho_{PS}^{T}(\omega)\theta(\omega_0-\omega) + \rho_{PS}^{T=0}(\omega)\theta(\omega-\omega_0)$$



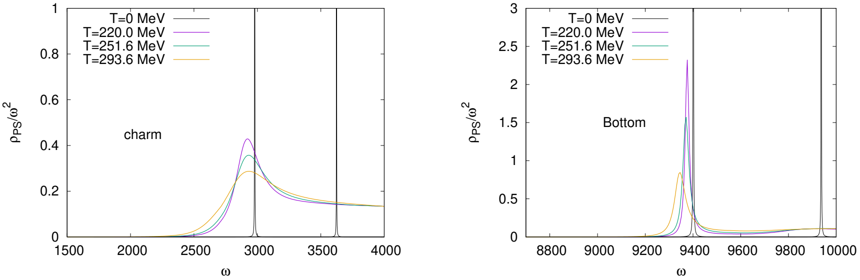
- $A_0 \sim 0.88\,M - 1.2\,M$
- $\omega_0 \sim 2\,M - 3\,M$

Similar spectral function using perturbative potential.

$N_f = 0$ M. Laine et al, JHEP11 (2017) 206

$N_f = 2+1$ Sajid Ali et al, Few-Body Syst 64, 52 (2023)

Spectral functions

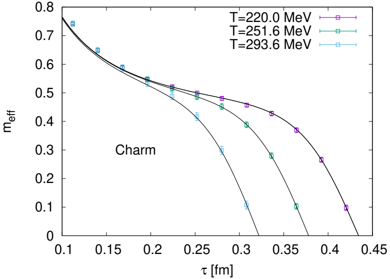


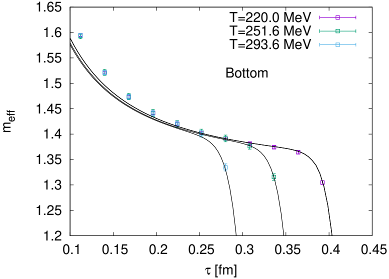
- (1S) state for bottom melts much after T_c ($T_c = 180 MeV$)
- Significant thermal effects on charmonium state
- Spectral function is not Gaussian around the peak

Consistency check with lattice correlator

$$G^E_{PS}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{PS}(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

$$m_{eff}(\tau_i) = \log\left(\frac{G^E_{PS}(\tau_i)}{G^E_{PS}(\tau_{i+1})}\right)$$



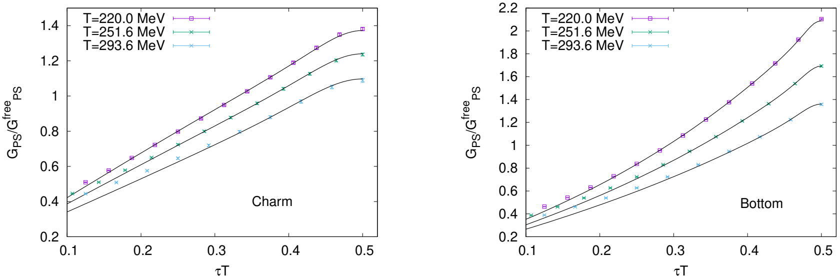


- Consistent with lattice data

Consistency check with lattice correlator

$$\rho_{PS}^{mod}(\omega, A) = A\rho_{PS}(\omega)$$

$$G_{PS}^{E}(\tau, A) = \int_0^\infty \frac{d\omega}{\pi} \rho_{PS}^{mod}(\omega, A) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$



- These spectral functions indeed describe the lattice correlator .

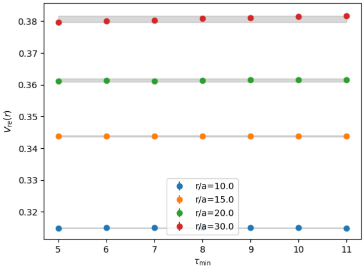
Conclusion and Outlook

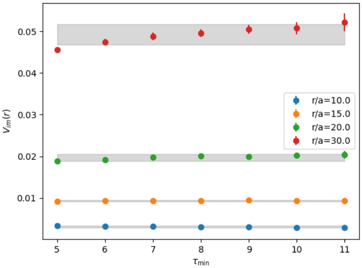
- Lattice data supports color screening of the non-perturbative thermal potential
- We observed a small thermal mass shift for the in-medium $\eta_b(1S)$ and $\eta_c(1S)$ channels and a large thermal width $(\Gamma_c(1S) \gg \Gamma_b(1S))$

Conclusion and Outlook

- Lattice data supports color screening of the non-perturbative thermal potential
- We observed a small thermal mass shift for the in-medium $\eta_b(1S)$ and $\eta_c(1S)$ channels and a large thermal width $(\Gamma_c(1S) \gg \Gamma_b(1S))$
- In contrast to Quenched QCD we see a bound state like structure of charmonium
- Study light quark mass effects by comparing $m_l = m_s/5$ and $m_l = m_s/27$
- Study cut-off effects and perform continuum extrapolation
- Estimate in-medium hadronic and transport properties (Kubo relation)

Thank you for your attention !


0.38
0.37
0.36
0.35
0.34
0.33
0.32
$V_{re}(r)$
5
6
7
8
9
10
11
$\tau_{\min}$
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0

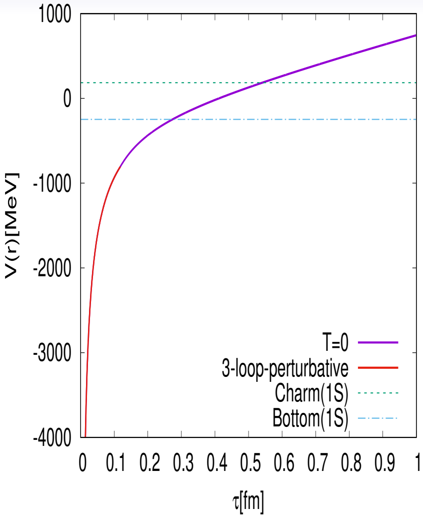

0.05
0.04
0.03
0.02
0.01
$V_{im}(r)$
5
6
7
8
9
10
11
$\tau_{\min}$
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0

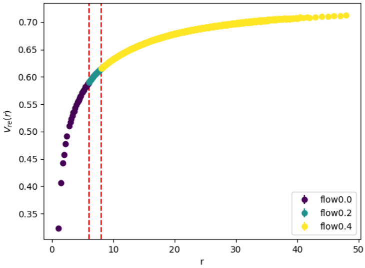
- Cornell fit of $T = 0$ lattice potential.
- Short distance matched renormalon subtracted peruturbative potential.

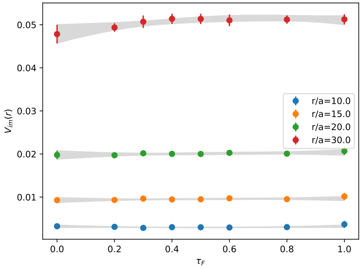
$$\left[-\frac{\nabla^2}{M} + V(r)\right]\psi_n(r) = E_n\psi_n(r)$$

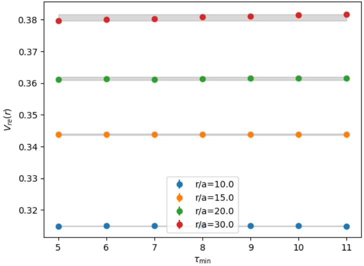
$$M^{1S} = 2M + E_0$$

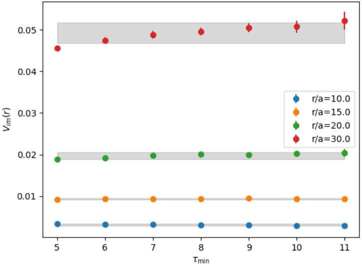
- $M^b = 4.78$ GeV
- $M^c = 1.35$ GeV

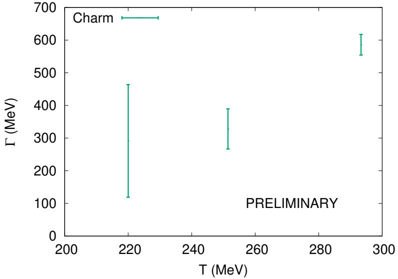


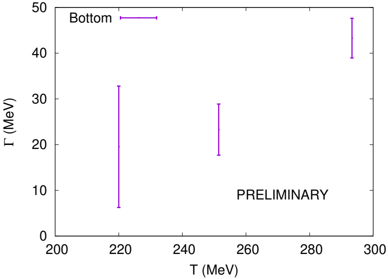

0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
$V_{re}(r)$
0
10
20
30
40
50
r
flow0.0
flow0.2
flow0.4


0.05
0.04
0.03
0.02
0.01
$V_{im}(r)$
0.0
0.2
0.4
0.6
0.8
1.0
τ_F
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0

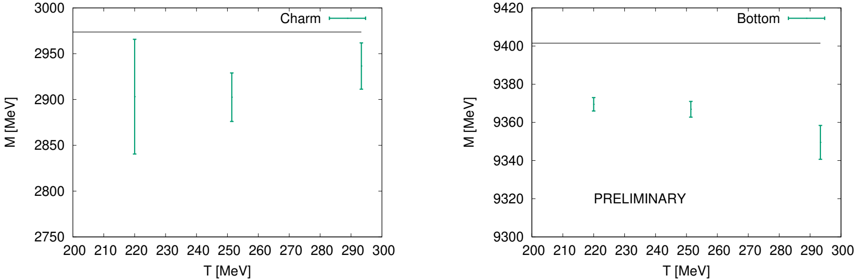

0.38
0.37
0.36
0.35
0.34
0.33
0.32
$V_{re}(r)$
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0
5 6 7 8 9 10 11
τ_{min}


0.05
0.04
0.03
0.02
0.01
$V_{im}(r)$
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0
5 6 7 8 9 10 11
τ_{min}





- We performed skewed Lorentzian fit near the peak.
- $\Gamma_c(1S) \gg \Gamma_b(1S)$



- Mass is identified with peak position of the spectral function.
- Finite mass shift is observed