UNIVERSITÄT BIELEFELD

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Shear viscosity from quenched to full lattice QCD

Pavan

Olaf Kaczmarek, Guy D. Moore and Christian Schmidt

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Introduction

The quark-gluon-plasma produced in heavy-ion collisions are described by transport coefficients such as shear and bulk viscosity, and various conserved-number diffusion coefficients.

These transport coefficients are key input to hydrodynamical models

$$T_{ij} - T_{ij}^{eq} = -\eta\left(\nabla_i u_j + \nabla_j u_i - \frac{2}{3}\delta_{ij}\nabla_k u_k\right) - \zeta\,\delta_{ij}\nabla_k u_k$$

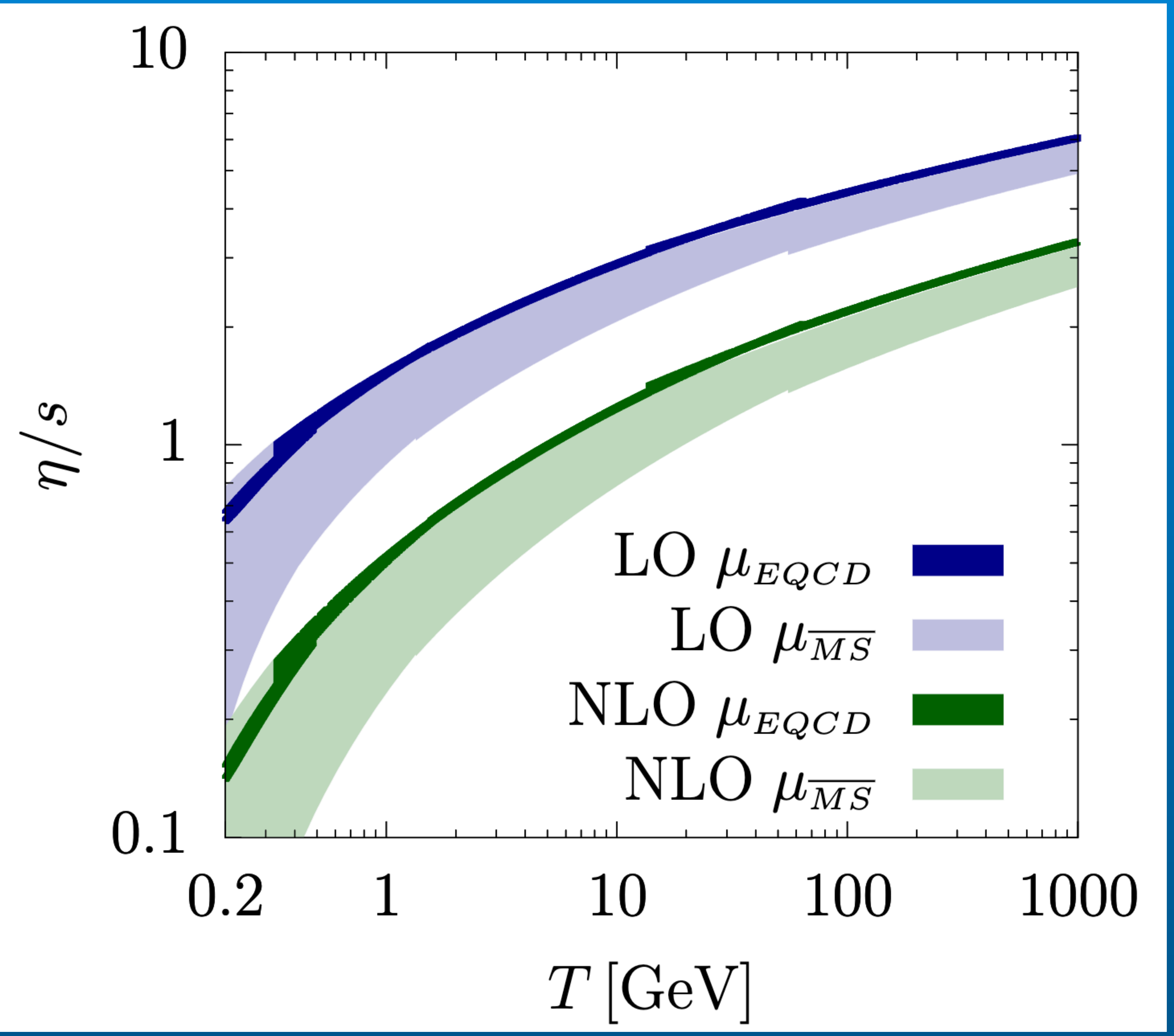
Here η and ζ are shear and bulk viscosities respectively.

Kubo relation for shear viscosity (η)

$$\eta = i\partial_\omega \int d^3x \int_0^\infty dt\, e^{i\omega t}\langle[T^{xy}(x,t), T^{xy}(0,0)]\rangle\,|_{\omega=0}$$

Perturbation theory doesn't converge; even at $T \sim 100$ GeV the shear viscosity to entropy ratio (η/s) for leading order (LO) is twice that of next-to-leading order (NLO).

The shear viscosity Kubo relation is written in real time but lattice QCD deals with Euclidean time correlation function.



Jacopo Ghiglieri *et al*., JHEP03 (2018) 179

Shear viscosity on lattice

The shear viscosity on lattice is accessible through analytical continuation

$$\eta(T) = \lim_{\omega \to 0} \frac{\rho_{shear}(\omega, T)}{\omega}$$

$$G(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{\cosh[\omega(1/2T - \tau)]}{\sinh(\omega/2T)} \rho(\omega, T) \qquad G(\tau) \equiv \langle T^{xy}(x, i\tau), T^{xy}(0,0) \rangle$$

This is known as ill-conditioned inversion problem that requires determining $\rho(\omega)$ from limited and incomplete information.

$$G_{shear}(\tau) \propto \int \mathrm{d}^3x \left\langle \pi_{ij}(0,\vec{0}), \pi_{ij}(\tau, \vec{x}) \right\rangle \qquad \pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij} T_{kk}$$

- The lattice doesn’t have continuous symmetry so no obvious choice for EMT.
- The EMT on lattice requires renormalisation constants.

Recent method for determining
Shear Viscosity in Quenched QCD

Harvey B. Meyer, Phys.Rev.D 76 (2007) 101701

Earlier pioneering work

F. Karsch and H. W. Wyld, Phys. Rev. D 35, 2518

Renormalisation

We will use gradient flow to write the EMT on lattice.

Define the density operator and the traceless tensor operator

$$E(x,\tau_F) = \frac{1}{4} F^a_{\rho\sigma}(x,\tau_F)\, F^a_{\rho\sigma}(x,\tau_F)$$

τ_F : Flow time

$$U_{\mu\nu}(x,\tau_F) = F^a_{\mu\rho}(x,\tau_F)\, F^a_{\nu\rho}(x,\tau_F) - \frac{1}{4}\delta_{\mu\nu}\, F^a_{\rho\sigma}(x,\tau_F)\, F^a_{\rho\sigma}(x,\tau_F)$$

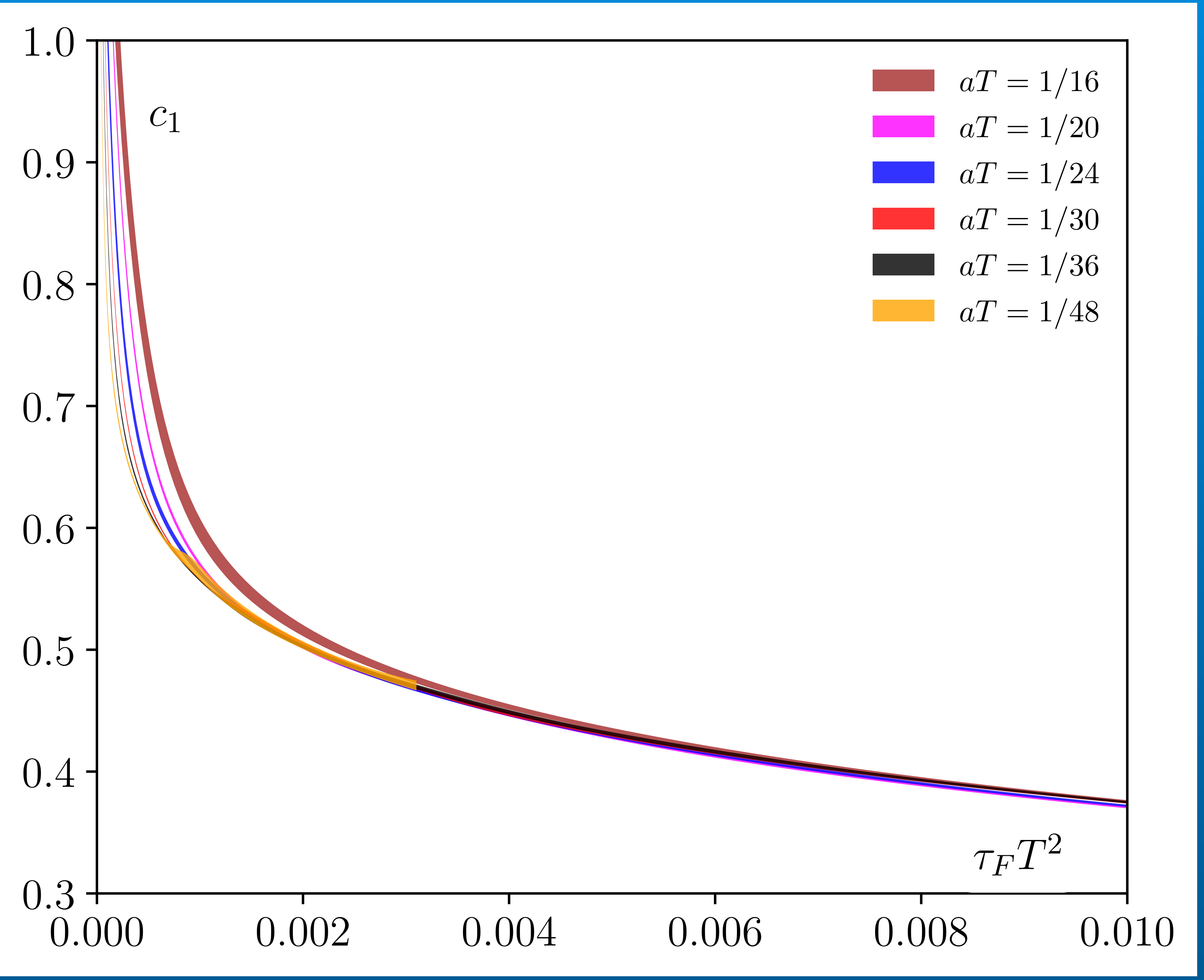
$$T_{\mu\nu}(x,\tau_F) = c_1(\tau_F)\, U_{\mu\nu}(x,\tau_F) + 4c_2(\tau_F)\, \delta_{\mu\nu}\, E(x,\tau_F)$$

Here c_1 and c_2 are the coefficients of the traceless and pure-trace parts of the EMT, respectively.

We determine the constant c_1 using a method inspired by the work of Giusti and Pepe.

$$\langle \epsilon + P \rangle_{\tau_F} = c_1(\tau_F) \left\langle \frac{1}{3} U_{ii}(\tau_F) - U_{00}(\tau_F) \right\rangle$$

[L. Giusti and M. Pepe, Phys. Rev. D 91, 114504 (2015)

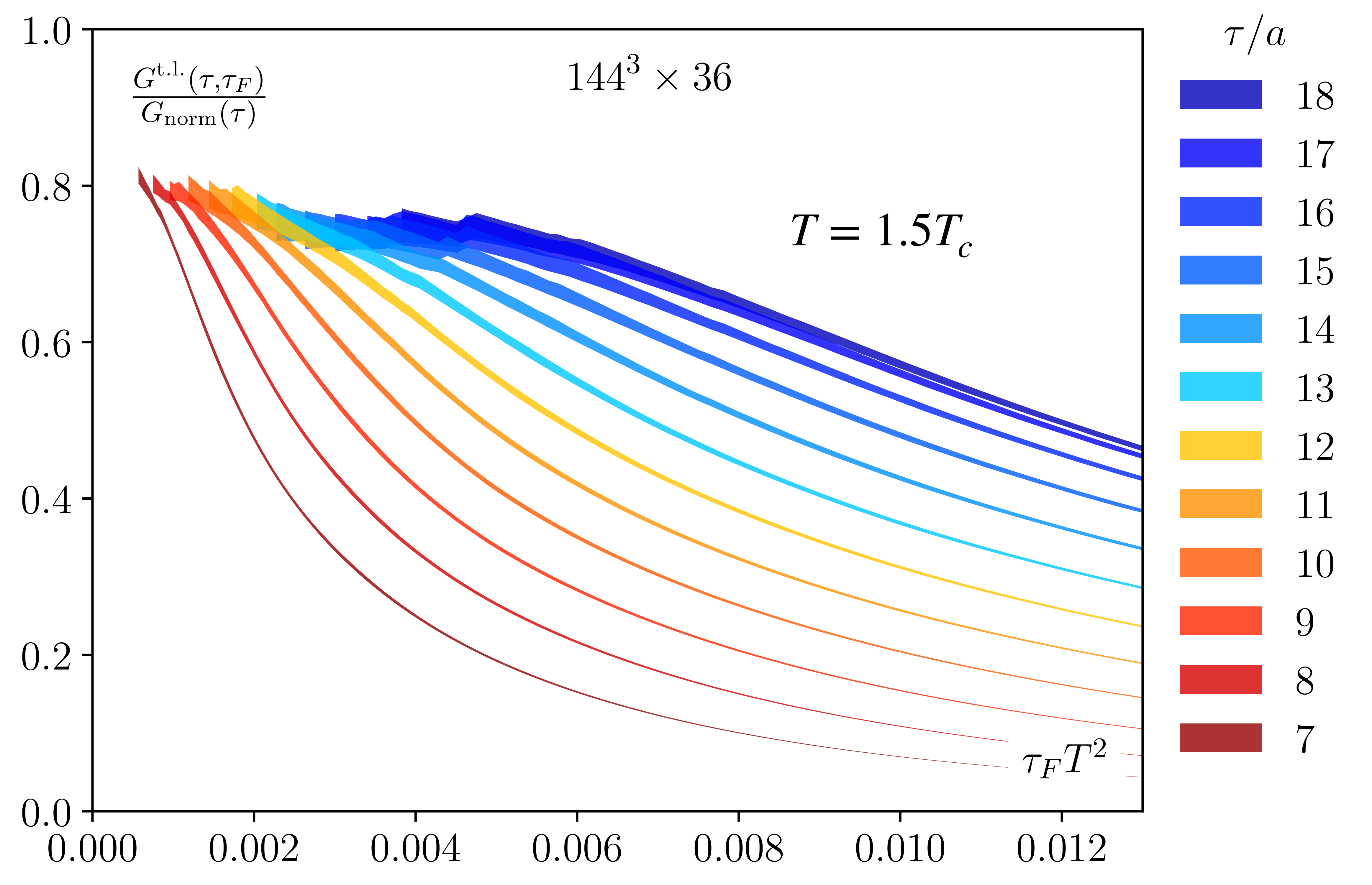


Combined c_1 at different lattice spacing
Luis Altenkort *et al.*, Phys. Rev. D 108, 014503

Continuum Extrapolation

Tree-level-improved EMT correlator

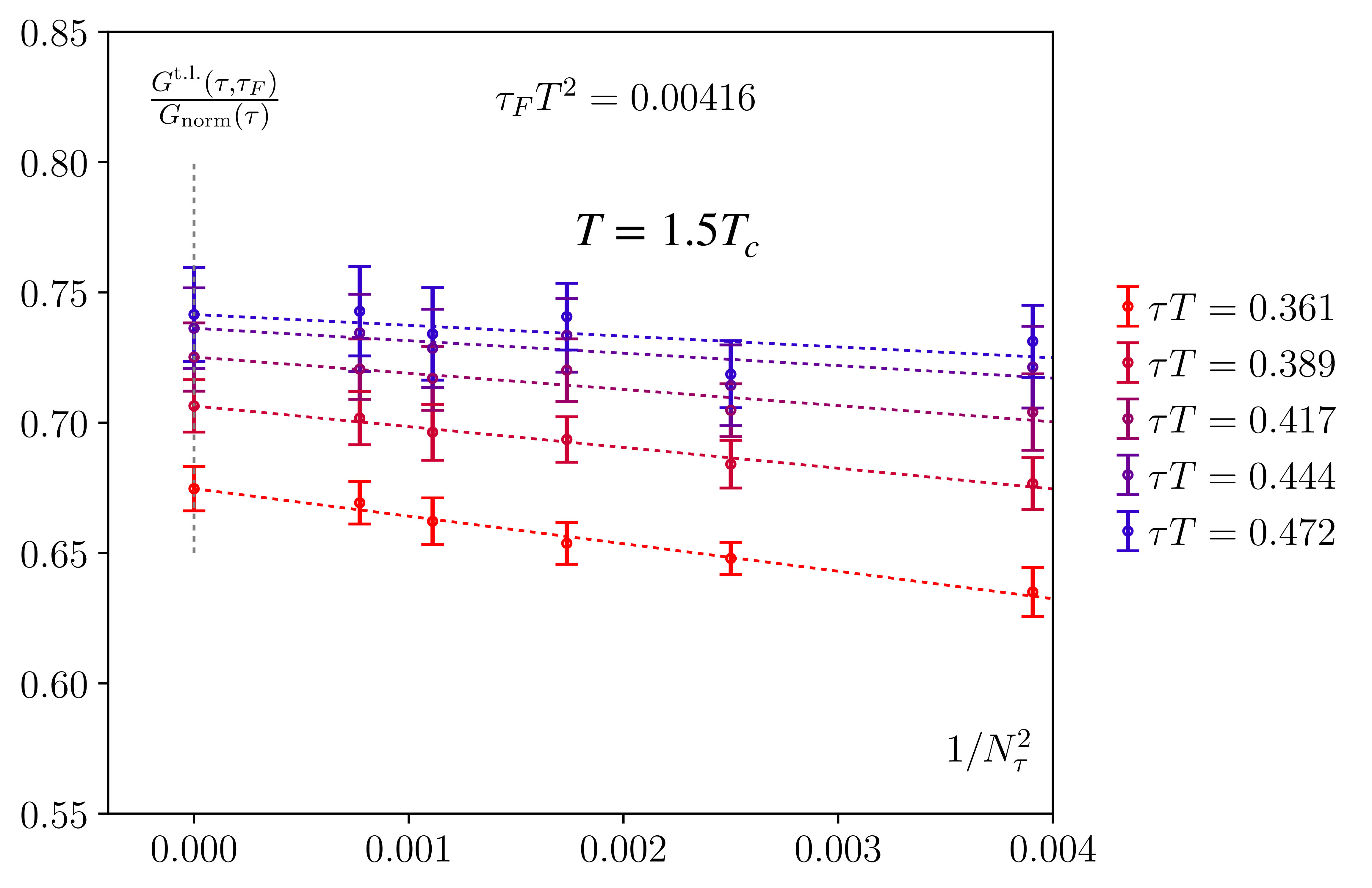
$$G^{\text{t.l.}}(\tau T) = G_{\text{lat}}(\tau T)\,\frac{G^{\text{LO}}_{\text{cont}}(\tau T)}{G^{\text{LO}}_{\text{lat}}(\tau T)}$$



Tree-level-improved EMT correlators in the shear channel

We perform the continuum extrapolation $a \to 0$

$$\frac{G^{\text{t.l.}}(N_\tau)}{G_{\text{norm}}(N_\tau)} = m \cdot N_\tau^{-2} + b$$

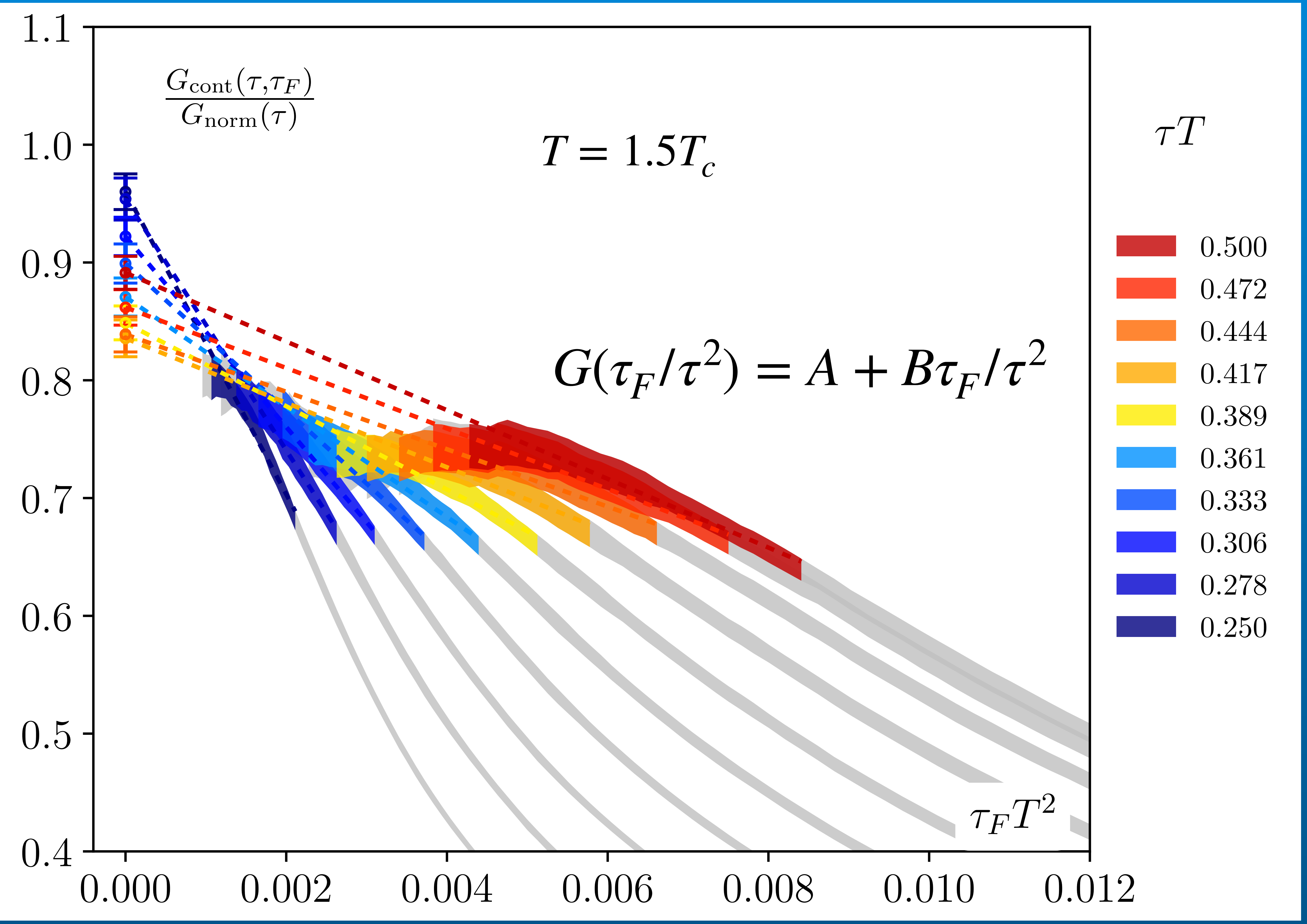


The continuum extrapolation of EMT correlators in shear channel

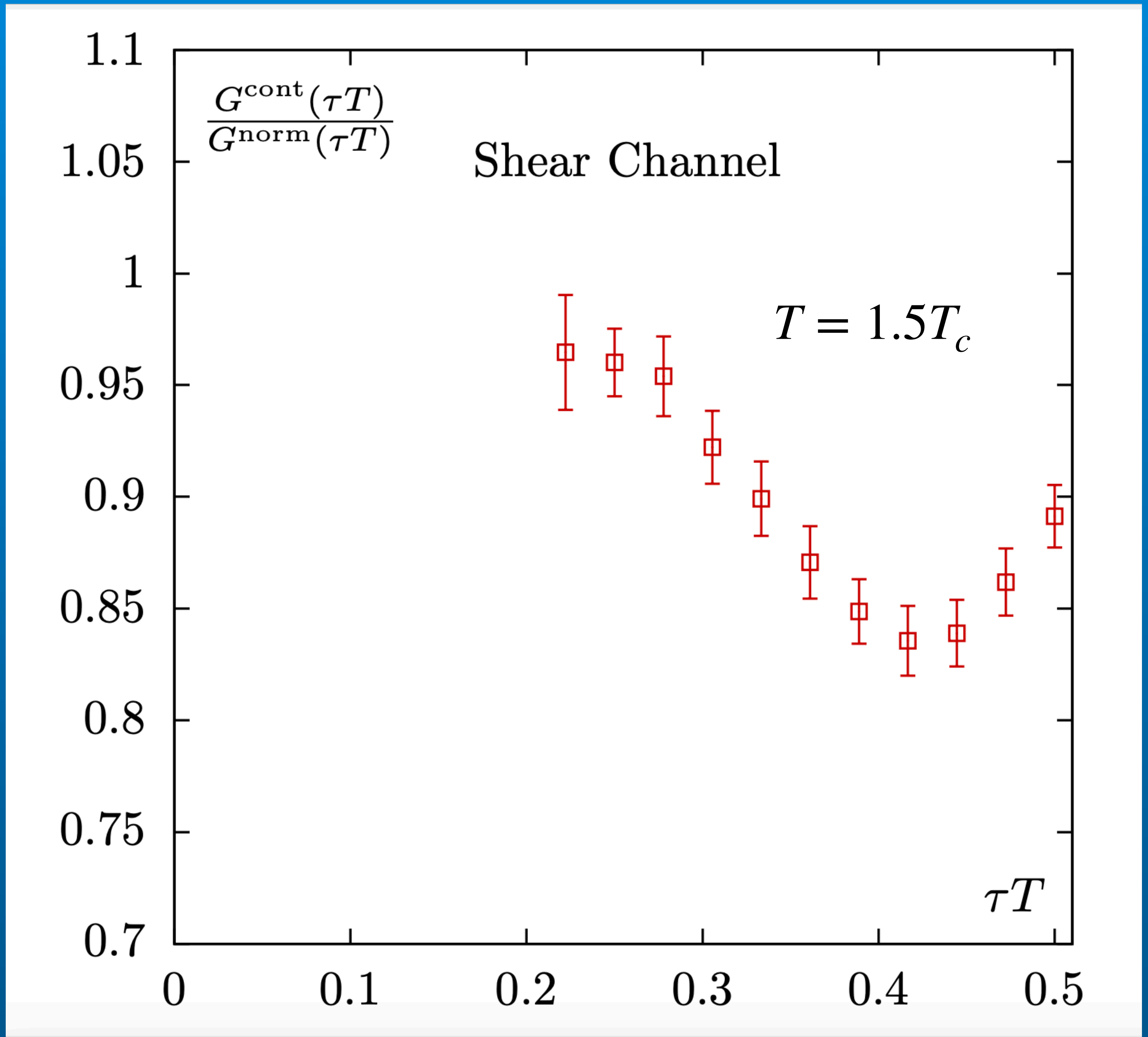
Luis Altenkort *et al.*, Phys. Rev. D 108, 014503

Zero-flow-time Extrapolation

The order of extrapolation is important because the continuum extrapolation eliminates terms of form a^2/τ_F , so that the τ_F extrapolation will consist only of positive powers.



The $\tau_F \to 0$ extrapolation of EMT correlators in shear channel



Extrapolated correlators in shear channel

Luis Altenkort *et al.*, Phys. Rev. D 108, 014503

Model Spectral Reconstruction

We first construct the spectral function using χ^2-fits with models based on perturbative calculations and then determine the viscosities …

$$\rho^{LO}_{shear}(\omega) = \frac{d_A \omega^4}{10\pi} \coth\left(\frac{\omega}{4T}\right)$$

d_A: dimension of adjoint representation

$$\rho^{NLO}_{shear}(\omega) = \rho^{LO}_{shear}(\omega) - 4 d_A \omega^4 \coth\left(\frac{\omega}{4T}\right) \frac{g^2(\bar{\mu}) N_c}{(4\pi)^3} \times \left[\frac{2}{9} + \phi^{\eta}_{T}(\omega)\right]$$

The infrared behaviour of the spectral function is not known a priori, and must be modelled.

M1: $$\frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^3} + B \frac{\rho_{\text{pert}}(\omega)}{\omega T^3}$$

M3: $$\frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^3} \frac{C^2}{C^2 + (\omega/T)^2} + B \frac{\rho_{\text{pert}}(\omega)}{\omega T^3}$$

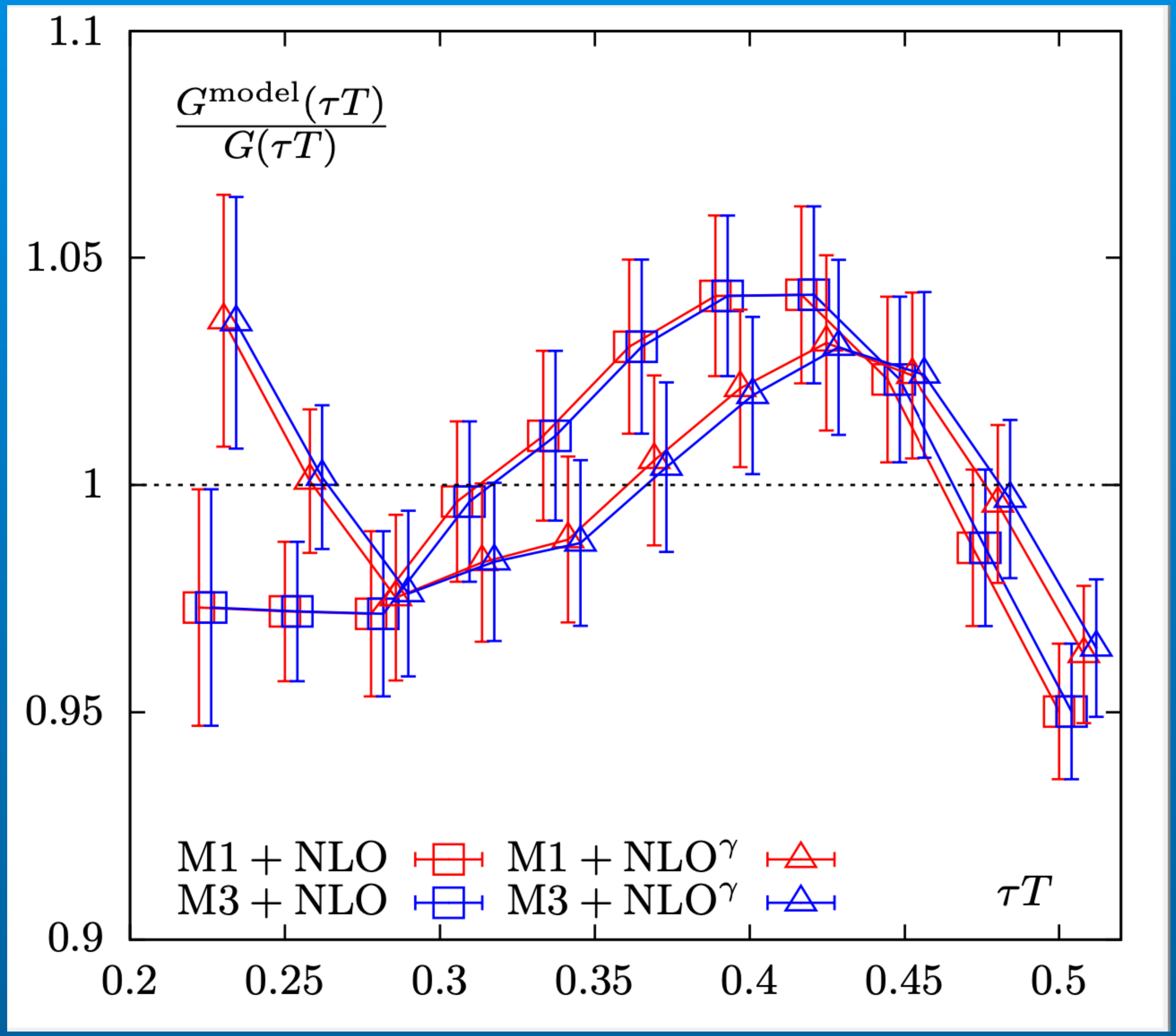
Here B is a coefficient allowing for a rescaling of the perturbative result, and A is the size of the IR contribution, which determines the transport coefficient of interest.

Now we integrate the spectral function with the kernel function to get the model correlators.

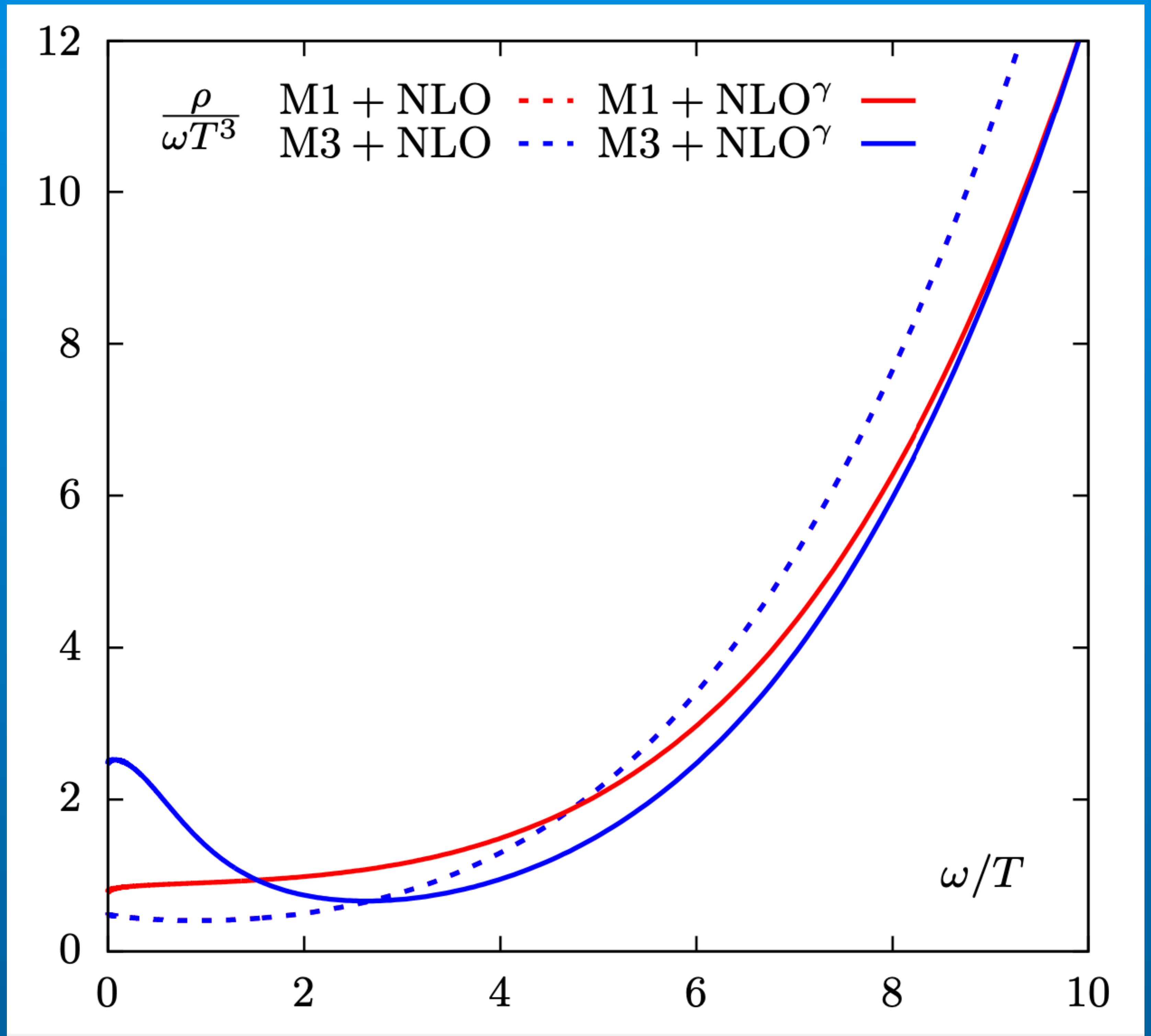
$$G(\tau) = \int_0^{\infty} \frac{\mathrm{d}\omega}{\pi} \frac{\cosh[\omega(1/2T - \tau)]}{\sinh(\omega/2T)} \rho(\omega, T)$$

Results

We have also tried to capture possible missing structure in the UV part of spectral function with anomalous dimension by replacing ω^4 with $\omega^{4+\gamma}$.



The comparison of fit and lattice correlators



Spectral fit function in the shear channel

Luis Altenkort *et al.*, Phys. Rev. D 108, 014503

$$\eta/s = 0.15 - 0.48, \quad T = 1.5T_c$$

Towards calculating
Shear Viscosity in Full QCD

Shear viscosity in Full QCD

We extend the methodology developed in quenched QCD for shear viscosity to full QCD

The inclusion of fermions adds significant complexity to the problem, both in terms of technical and computational aspects. The first challenge is determining the renormalisation constants.

EMT renormalisation in Full QCD

The EMT contains the following terms in $SO(4)$ symmetry

$$T^1_{\mu\nu}(x,\tau_F) = Z_1(\tau_F)\left[F^a_{\mu\rho}(x,\tau_F)F^a_{\nu\rho}(x,\tau_F) - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}(x,\tau_F)F^a_{\rho\sigma}(x,\tau_F)\right]$$

$$T^2_{\mu\nu}(x,\tau_F) = Z_2(\tau_F)\,\delta_{\mu\nu}F^a_{\rho\sigma}(x,\tau_F)F^a_{\rho\sigma}(x,\tau_F)$$

$$T^3_{\mu\nu}(x,\tau_F) = Z_3(\tau_F)\,\bar{\psi}(x)\left[\gamma_\mu\overleftrightarrow{D}_\nu + \gamma_\nu\overleftrightarrow{D}_\mu - \frac{1}{2}\delta_{\mu\nu}\gamma_\alpha\overleftrightarrow{D}_\alpha\right]\psi(x)$$

$$T^4_{\mu\nu}(x,\tau_F) \equiv Z_4(\tau_F)\,\delta_{\mu\nu}\bar{\psi}(x)\gamma_\alpha\overleftrightarrow{D}_\alpha\psi(x)$$

$$T^5_{\mu\nu}(x,\tau_F) \equiv Z_5(\tau_F)\,\delta_{\mu\nu}m_0\bar{\psi}(x)\psi(x)$$

Hiroki Makino, Hiroshi Suzuki PTEP 2014, 063B02

Shear viscosity: only $off-diagonal$ components contribute

One linear equation $\epsilon + p = \langle T_{xx} - T_{tt}\rangle$ but two unknowns Z_1 and Z_3

$\Rightarrow$ requires two independent ensembles to determine renormalisation factors

- Our idea is to vary the imaginary isospin chemical potential ($\mu = i\mu_I$)

Free Theory Motivation

The pressure contribution per fermionic degree of freedom

$$P(\theta) = -\frac{T^4}{2\pi^2} \sum_{n \in \mathcal{Z}-\{0\}} \frac{e^{in\theta}}{n^4}$$

Here θ is angle from Polyakov loop vacuum

For usual case $\theta = \pi$ $\quad P_0 = \frac{7}{8}\frac{\pi^2 T^4}{90}$

Roberge-Weiss phase transition occurs at $\mu_I/T = \pi/3$ for all the flavours

$$\frac{P}{P_0} = \frac{P_u(\pi - \pi/3) + P_d(\pi - \pi/3) + P_s(\pi - \pi/3)}{P_u(\pi) + P_d(\pi) + P_s(\pi)} = 0.55026455\ldots$$

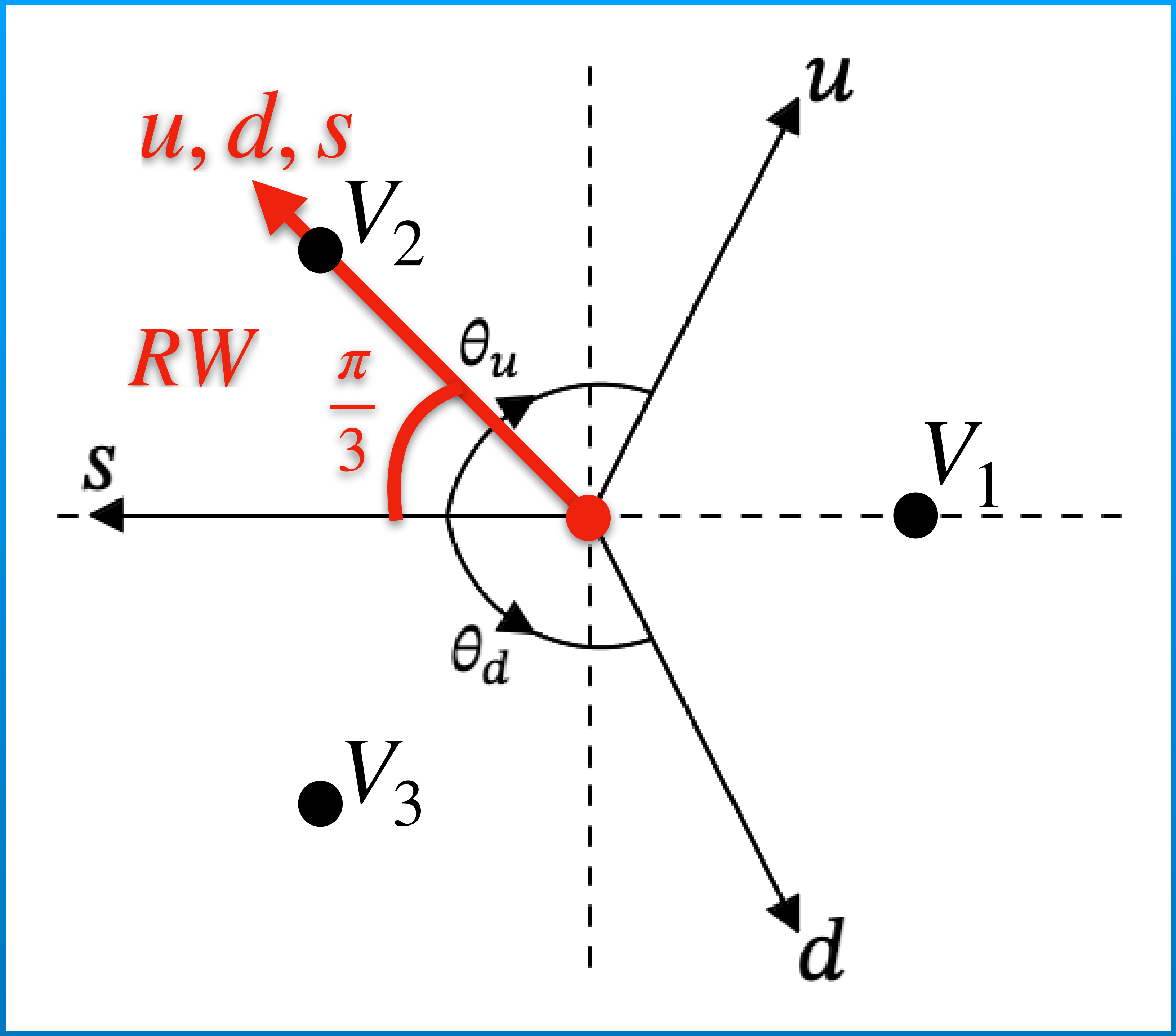
Change μ_I/T in following manner: $\mu_I^u/T = -\mu_I^d/T$ and $\mu_I^s/T = 0$

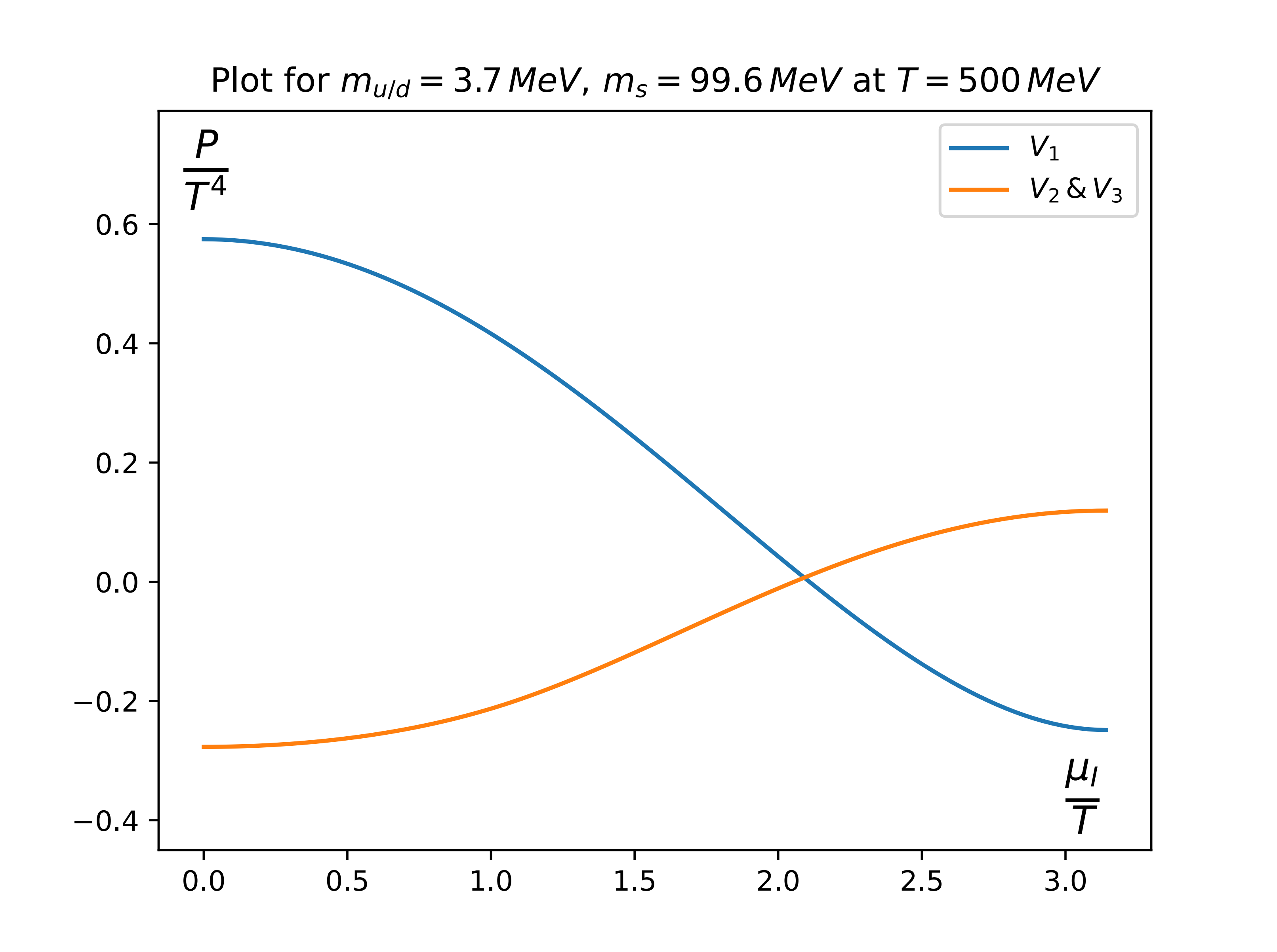
$$P_{V_1} = P(\pi) + P(\pi - \mu_I/T) + P(\pi + \mu_I/T)$$

$$P_{V_2/V_3} = P(\pi \mp 2\pi/3) + P(\pi \mp 2\pi/3 - \mu_I/T) + P(\pi \mp 2\pi/3 + \mu_I/T)$$

Phase transition at $\mu_I^u/T = -\mu_I^d/T = 2\pi/3$ and $\mu_I^s/T = 0$

$$\frac{P}{P_0} = \frac{P_u(\pi - 2\pi/3) + P_d(\pi + 2\pi/3) + P_s(\pi)}{P_u(\pi) + P_d(\pi) + P_s(\pi)} = \frac{1}{81}$$





V_1, V_2 and V_3 are Polyakov loop vacuums

So we will do our analysis for $\epsilon + p = \langle T_{xx} - T_{tt} \rangle$ at following ensembles,

Ensemble 1: $\mu_I^u/T = \mu_I^d/T = \mu_I^s/T = 0\ (\equiv \mu_i)$

Ensemble 2: $\mu_I^u/T = -\mu_I^d/T = 2\pi/3$ and $\mu_I^s/T = 0\ (\equiv \mu_f)$

$\epsilon + p$ measurement

$$\epsilon + p = \epsilon - 3p + 4p = I + 4p$$

Interaction measure I on these two ensemble

$$\frac{I(T)}{T^4}\frac{dT}{T} = N_t^4 \left[d\beta \langle -s_g \rangle_R + \sum_q dm_q \langle \bar{\psi}_q \psi_q \rangle_R \right]$$

We calculate pressure $p(\mu_f)$ as following

$$n = \frac{N}{V} = \frac{\partial p}{\partial \mu}\bigg|_T \qquad \Rightarrow \qquad p(\mu_f) = p(\mu_I = 0) + \int_0^{\mu_f} \frac{\partial p}{\partial \mu_I} d\mu_I$$

$\langle T_{xx} - T_{tt} \rangle$ measurement

$$T_{xx}^G - T_{tt}^G = Z_1(\tau_F)\big(G_{x\alpha}(x,\tau_F)G_{x\alpha}(x,\tau_F) - G_{t\alpha}(x,\tau_F)G_{t\alpha}(x,\tau_F)\big)$$

$$T_{xx}^F - T_{tt}^F = Z_3(\tau_F)\,\bar{\psi}(x)\big(\gamma_x \overleftrightarrow{D}_x - \gamma_t \overleftrightarrow{D}_t\big)\psi(x)$$

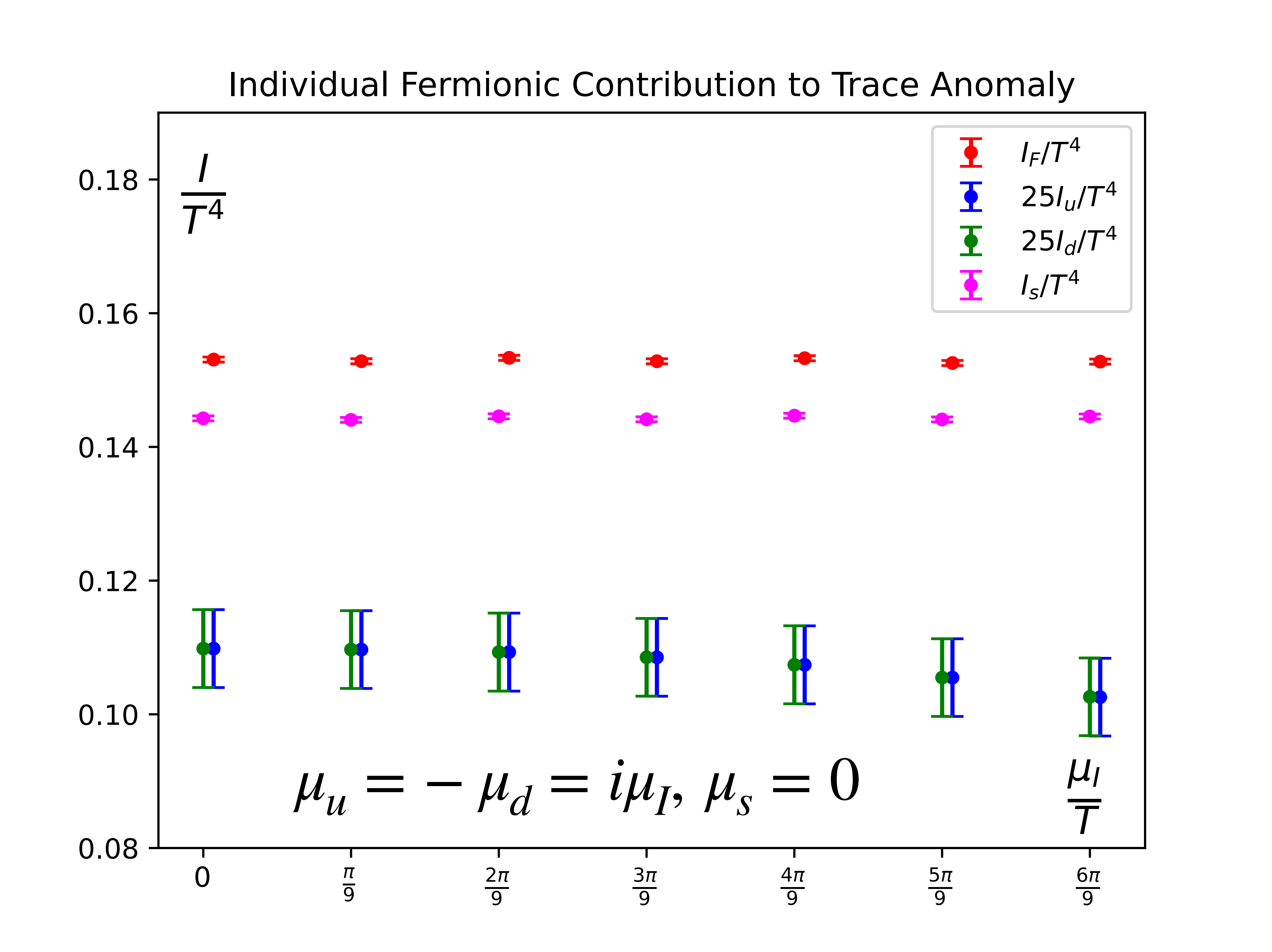
Trace Anomaly

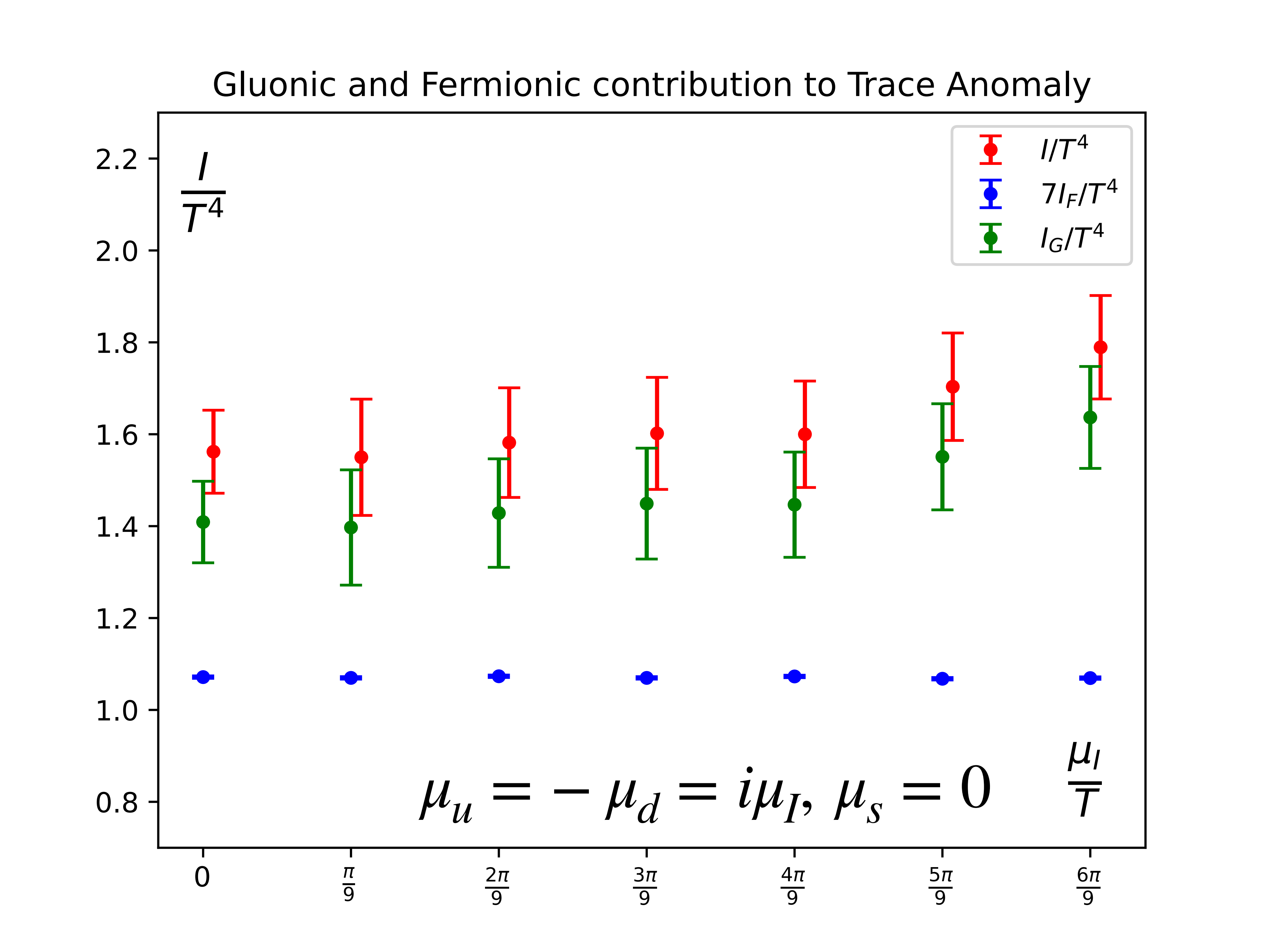
$$I = \delta_{\mu\nu}\{T_{\mu\nu}\}_R(x) = -\frac{\beta}{2g^3}\{F_{\mu\nu}F_{\mu\nu}\}_R(x) - (1+\gamma_m)m\{\bar{\psi}\psi\}_R(x)$$

due to running coupling

due to mass scale

The *zero* temperature values are taken from "The equation of state in (2+1)-flavor QCD" A. Bazavov, *et.al.*, Phys. Rev. D 90, 094503





The trace anomaly calculated with HISQ on $40^3 \times 8$ lattice ($T = 409.7$ MeV)

Pressure Measurement

Number density:

$$n = \frac{1}{4}\frac{T}{V}\left\langle \mathrm{Tr}\left[M(\mu)^{-1}\frac{\partial M(\mu)}{\partial \mu}\right]\right\rangle$$

Number density (n) measurement is in progress

We expect n to be smooth function of μ_I at high temperature

Integrating it will give us pressure at $p(\mu_f)$

$$p(\mu_f) = p(\mu_I = 0) + \int_0^{\mu_f} \frac{\partial p}{\partial \mu_I} d\mu_I$$

$\mu_I^u/T = -\mu_I^d/T = 2\pi/3$ and $\mu_I^s/T = 0 \; (\equiv \mu_f)$

Conclusion and Outlook

Quenched QCD

Constructed renormalised EMT using gradient flow

Continuum and zero-flow limit for renormalised correlators

Extracted the shear viscosity by modelling the spectral function

$$\eta/s = 0.15 - 0.48, \qquad T = 1.5T_c$$

Ample room for improving spectral modelling in $\omega \sim [1-5]T$ to get better fit results

Full QCD

Renormalised EMT for full QCD

Developed the idea to find renormalisation constants

We have trace anomaly in hand and are working on calculating the pressure.

Once the renormalisation constant is determined, we apply the method that we developed in quenched QCD.