# Baryon electric charge correlation as a magnetometer of QCD
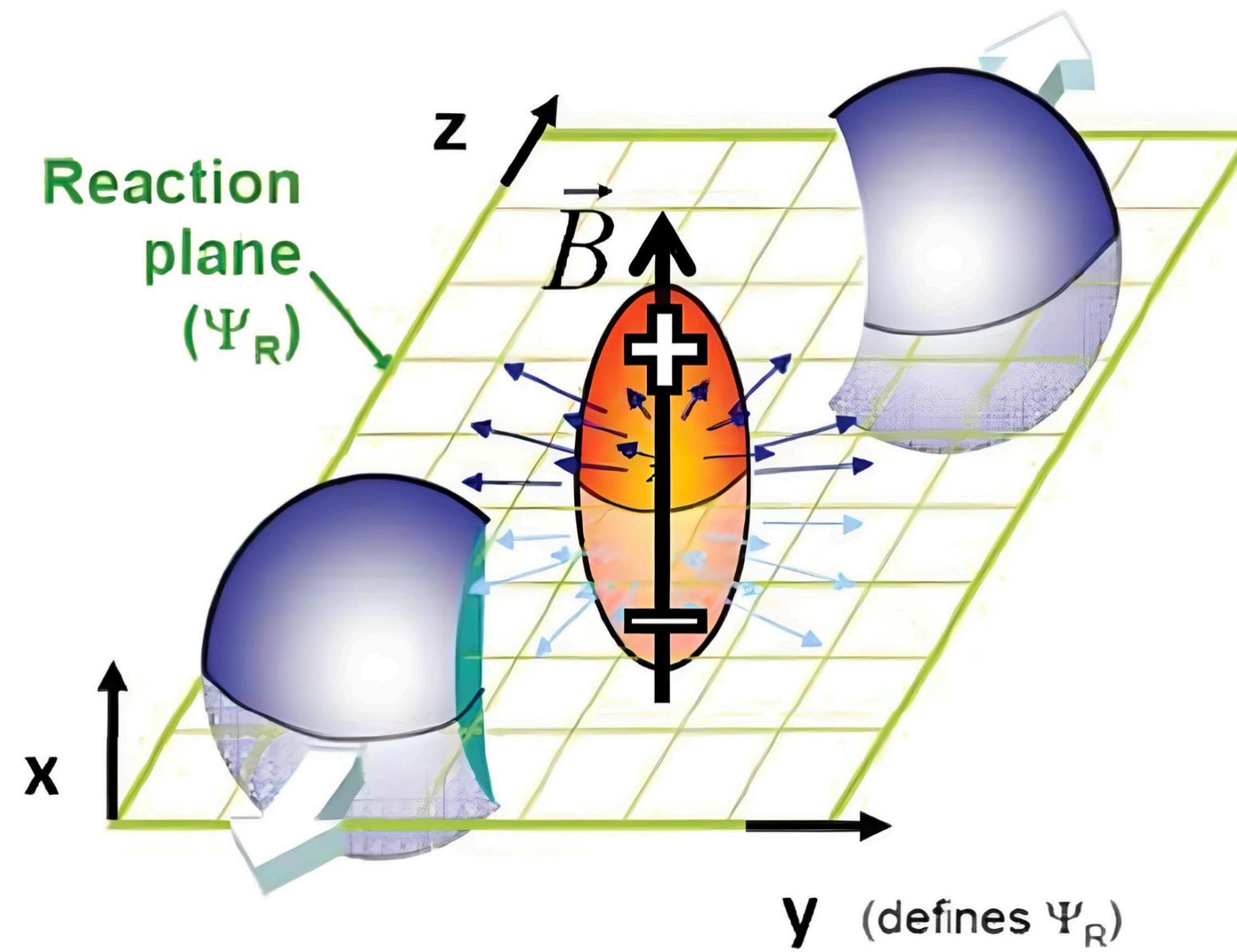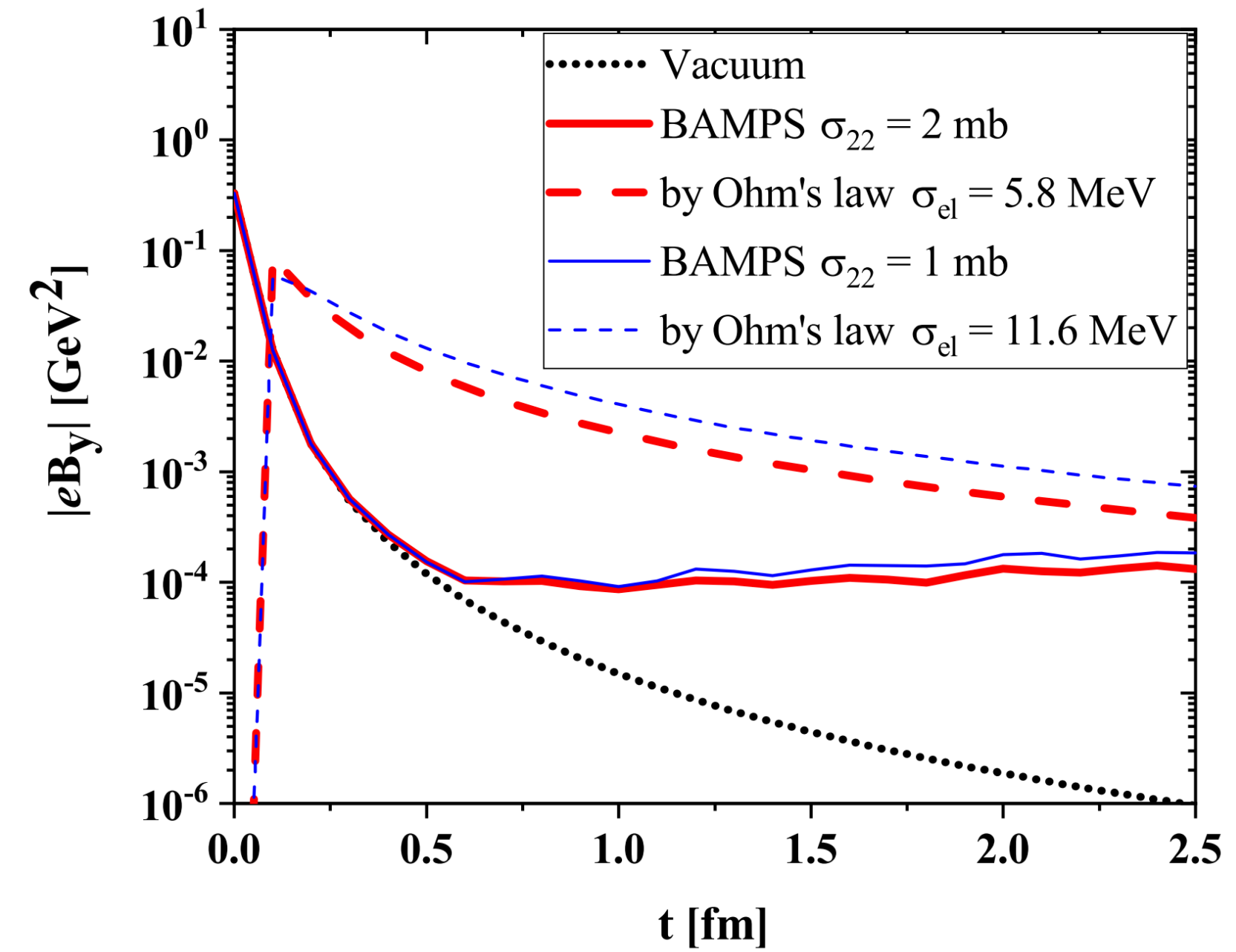
## Jin-Biao Gu

### Central China Normal University

Based on Phys. Rev. Lett. **132**, 201903 (2024)

In collaboration with H.-T. Ding, A. Kumar, S.-T. Li and J.-H. Liu

Nuclear Science
Computing Center at CCNU

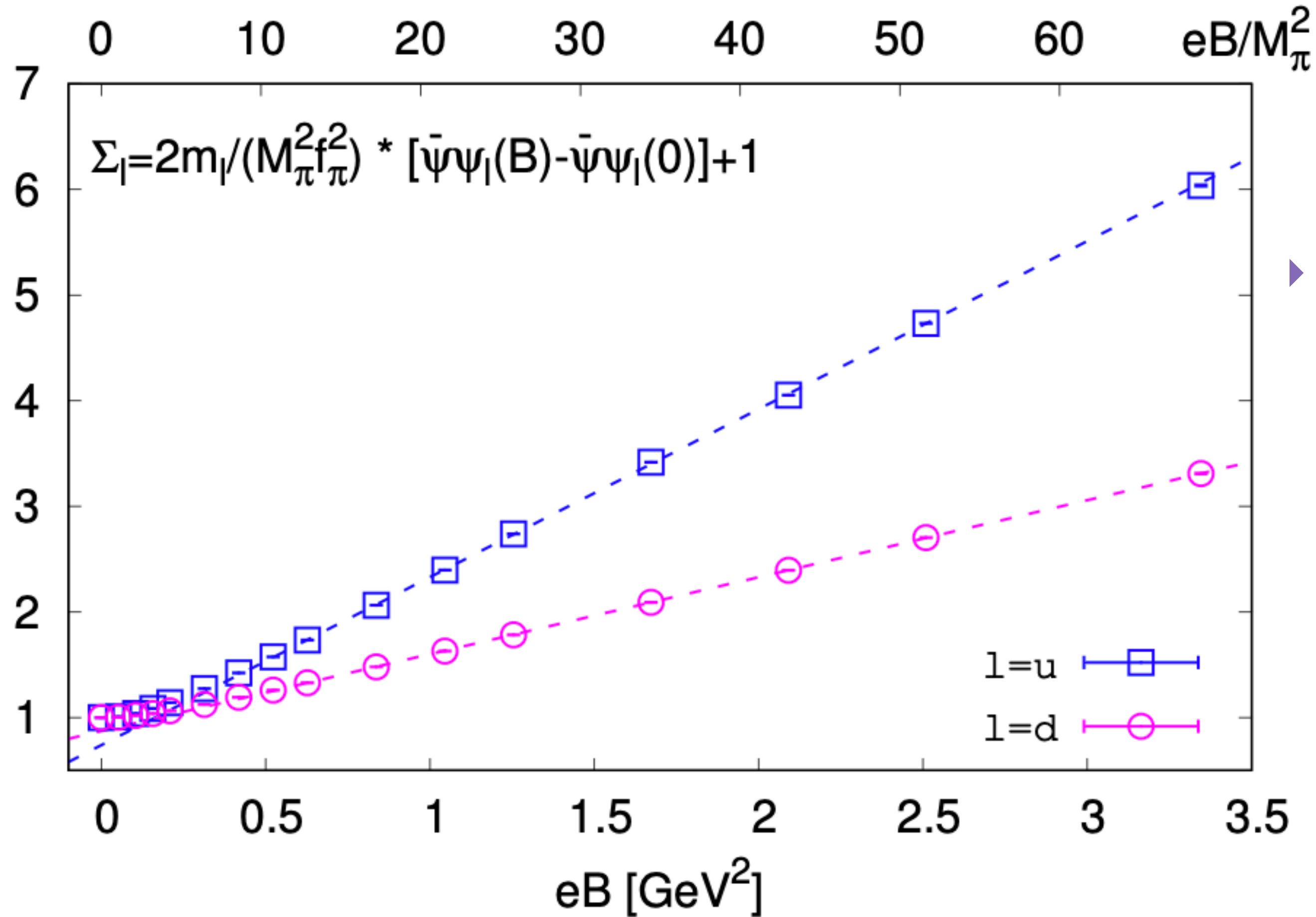*J. Zhao and F. Wang, Prog.Part.Nucl.Phys. 107 (2019) 200-236*



*Z. Wang et al., Phys.Rev.C 105 (2022) L041901*

$$eB_{\tau=0} \sim 5\, M_\pi^2 \text{ in RHIC}, \quad eB_{\tau=0} \sim 70\, M_\pi^2 \text{ in LHC}$$

## Does this strongly decaying magnetic field manifest itself in the final stage of HIC?

# Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



$\Sigma_l = 2m_l/(M_\pi^2 f_\pi^2) * [\bar{\psi}\psi_l(B) - \bar{\psi}\psi_l(0)] + 1$

- $l=u$
- $l=d$

*H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505*

▸ Sign of isospin symmetry breaking:

Non-degeneracy of *u* and *d* condensates at strong magnetic fields

$$\Sigma_l = 2m_l/(M_\pi^2 f_\pi^2) * [\bar{\psi}\psi_l(B) - \bar{\psi}\psi_l(0)] + 1$$

l=u
l=d
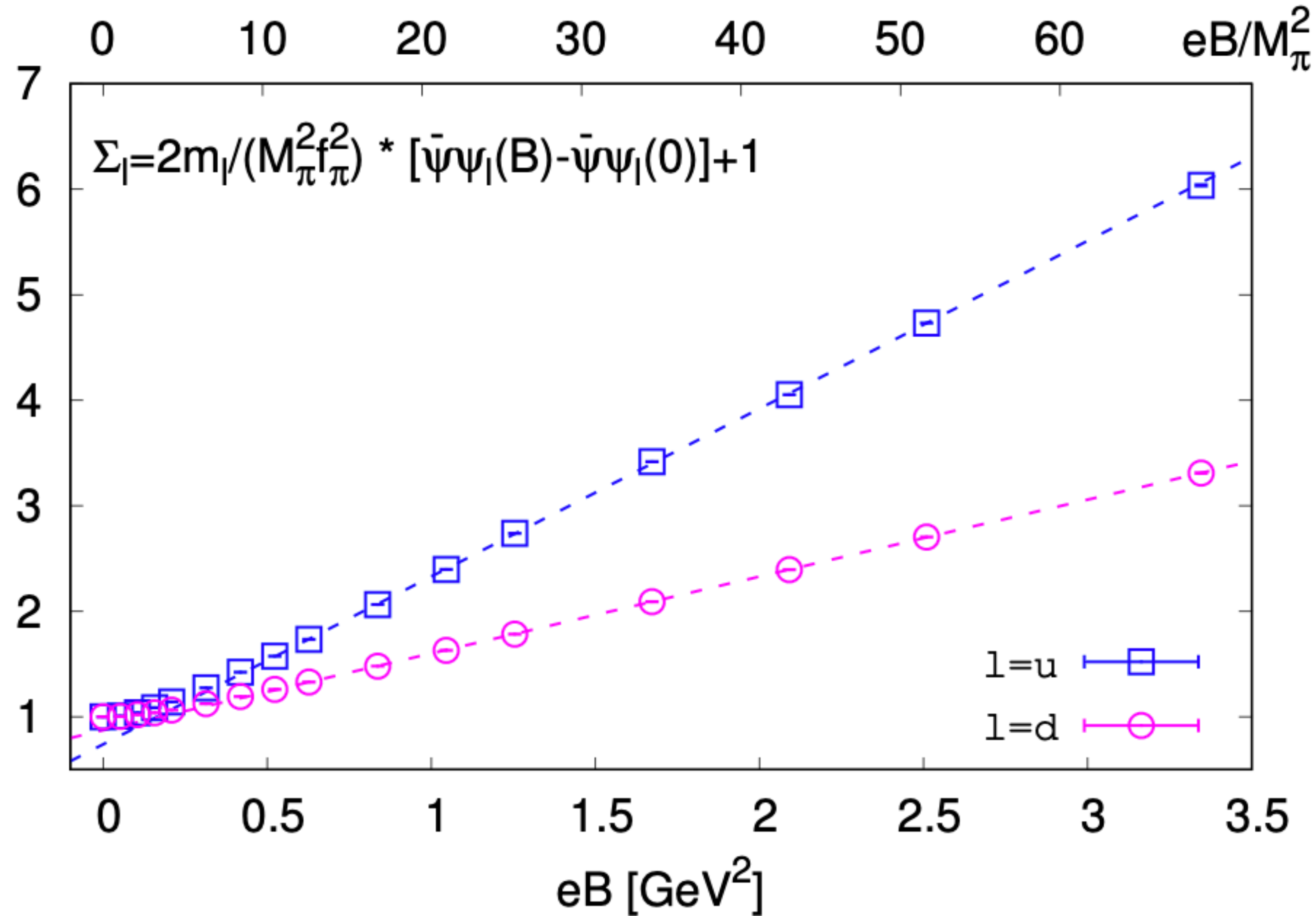
A clear effect but Not accessible in HIC experiments!

H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

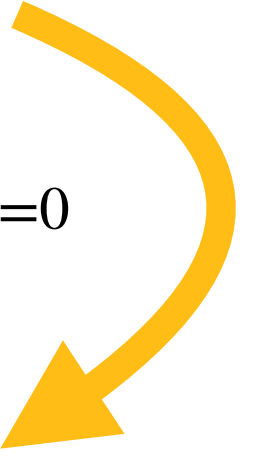# Fluctuations of net baryon number, electric charge and strangeness

Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathscr{Z}\left(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s\right) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{\text{BQS}}}{i!\,j!\,k!} \left(\frac{\mu_{\text{B}}}{T}\right)^i \left(\frac{\mu_{\text{Q}}}{T}\right)^j \left(\frac{\mu_{\text{S}}}{T}\right)^k$$

*C. Allton et al., Phys.Rev. D 66 (2002) 074507*

Taylor expansion coefficients at $\mu = 0$ are computable in LQCD

$$\chi_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\right|_{\mu_{u,d,s}=0}$$

$$\chi_{ijk}^{\text{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_{\text{B}}/T\right)^i \partial\left(\mu_{\text{Q}}/T\right)^j \partial\left(\mu_{\text{S}}/T\right)^k}\right|_{\mu_{\text{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_{\text{B}} + \frac{2}{3}\mu_{\text{Q}}$$

$$\mu_d = \frac{1}{3}\mu_{\text{B}} - \frac{1}{3}\mu_{\text{Q}}$$

$$\mu_s = \frac{1}{3}\mu_{\text{B}} - \frac{1}{3}\mu_{\text{Q}} - \mu_{\text{S}}$$

*LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007*

*Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112*

At $eB \neq 0$ a lot more need to be explored

**HRG:** *G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., Phys. Rev. D 98 (2018) 034003, Fukushima and Hidaka, Phys. Rev. Lett. 117 (2016)102301, Bhattacharyya et al., EPL 115(2016)62003*

**PNJL:** *W.-J. Fu, Phys. Rev. D 88 (2013) 014009*

✦ Highly improved staggered fermions and a tree-level improved Symanzik gauge action

✦ $N_f = 2 + 1$

✦ Lattice sizes : $32^3 \times 8$, $48^3 \times 12$; $64^3 \times 16$

✦ $m_s^{\text{phy}}/m_l = 27$, $M_\pi(eB = 0) \approx 135$ MeV

✦ $T$ window : $(144 \text{ MeV}, 166 \text{ MeV})$, i.e. $(0.9T_{pc}, 1.1T_{pc})$

✦ $eB$ window: $0 \leq eB < 9M_\pi^2$

$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}, \quad N_b = 0, 1, 2, 3, 4, 6$$
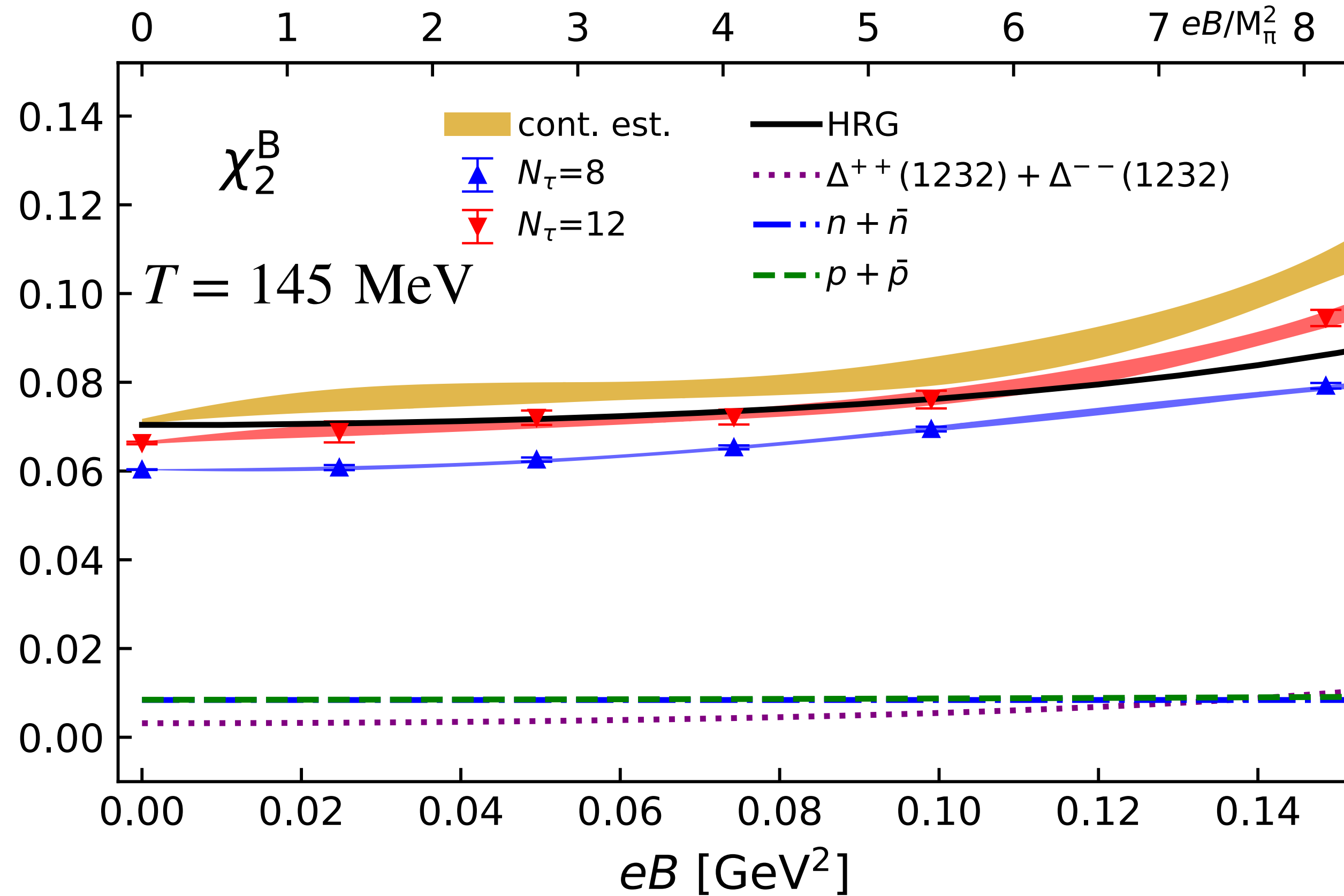
# Continuum estimate and extrapolation
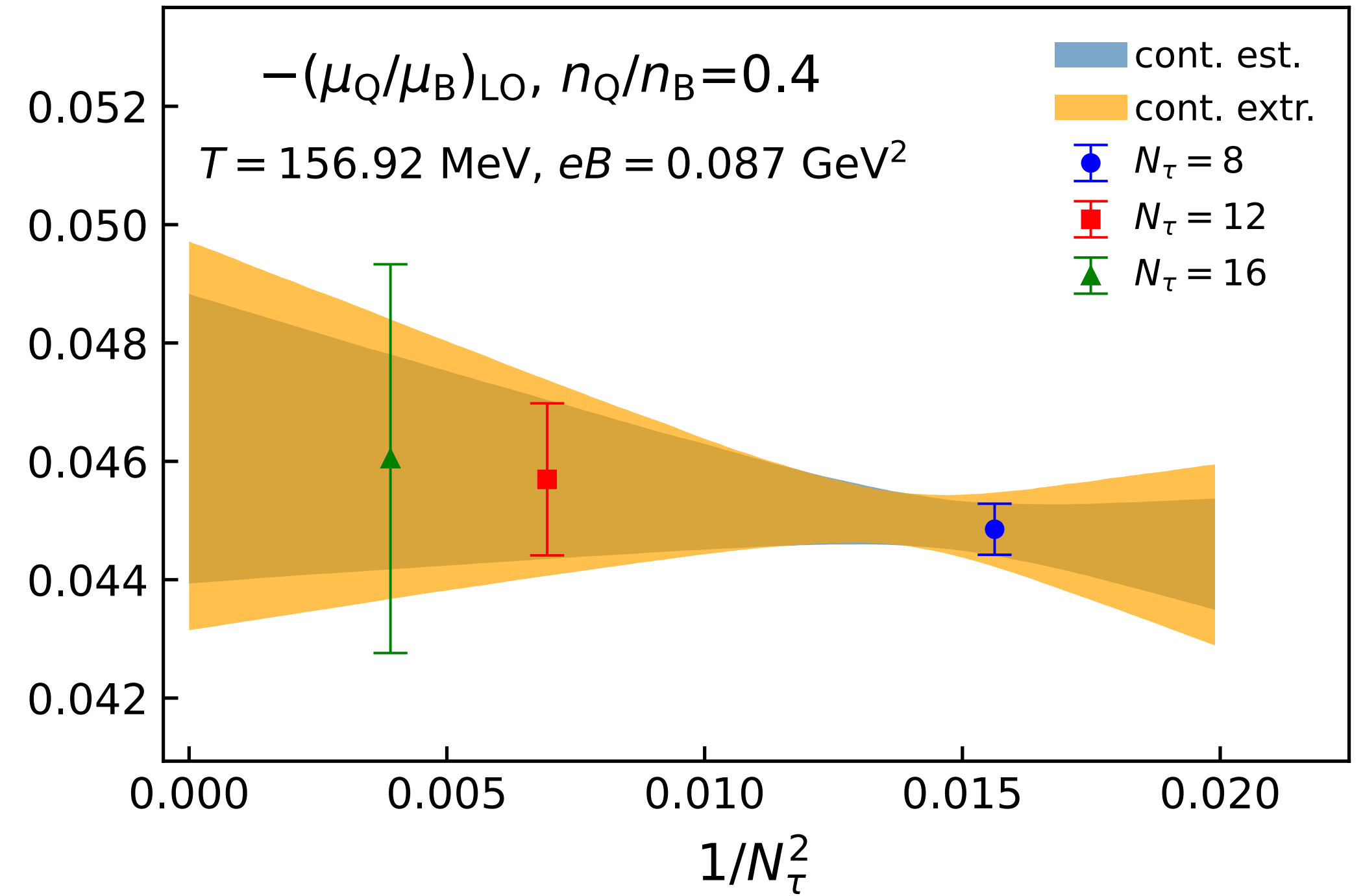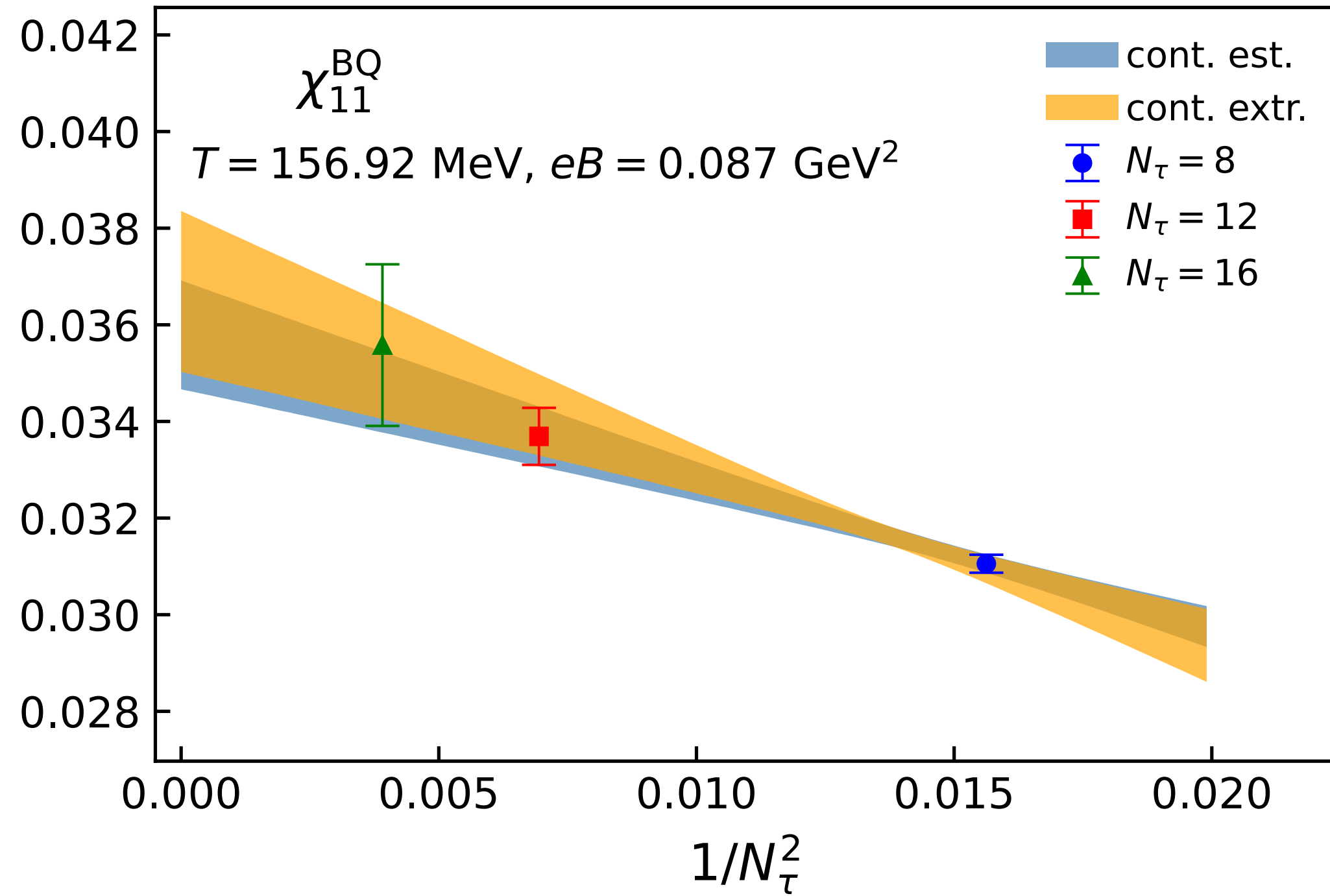


*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

$$\mathscr{O}\left(T, eB, N_\tau\right) = \mathscr{O}(T, eB) + \frac{c}{N_\tau^2}$$

Continuum estimate and continuum extrapolation are consistent within uncertainty

$$\chi_2^B$$

$T = 145$ MeV

*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

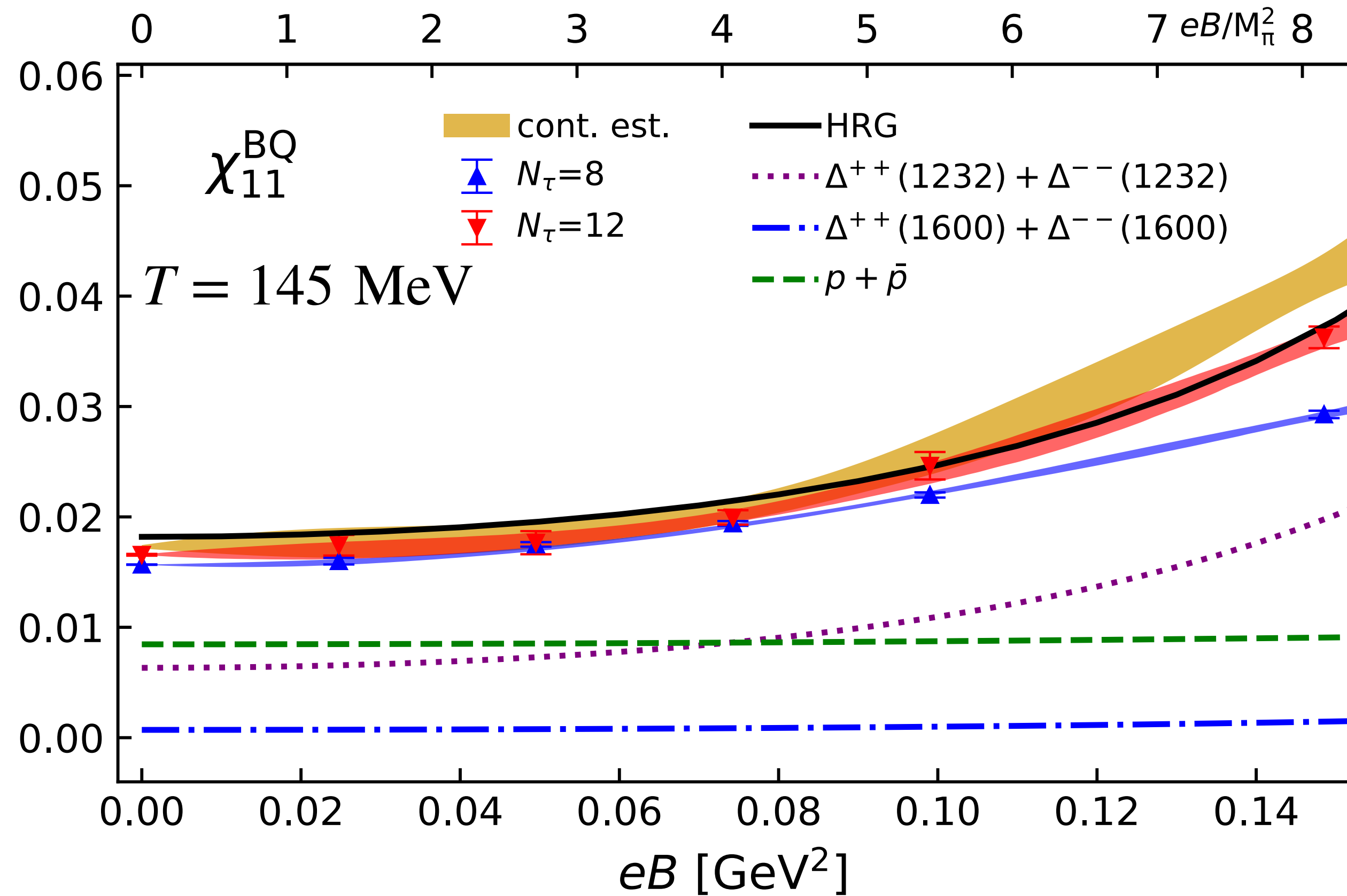❖ $\chi_2^B$ increases ~ 45% at $eB \sim 8M_\pi^2$

❖ Hadron Resonance Gas model (HRG):
Pressure arising from charged hadrons
$(eB \neq 0)$:

$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i| \, B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} \, K_1\left(\frac{n\varepsilon_0}{T}\right)$$

where $\varepsilon_0 = \sqrt{m_i^2 + 2\,|q_i|\,B\left(l + 1/2 - s_z\right)}$,

$K_1$ is the first-order modified Bessel function
of the second kind

*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

◆ $\chi_{11}^{\mathrm{BQ}}$ : **Magnetometer of QCD**

◆ The results of HRG model are consistent with LQCD up to $eB \sim 6M_\pi^2$

◆ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ give most of the contributions of magnetic field dependence of $\chi_{11}^{\mathrm{BQ}}$

◆ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ are not measurable in HIC experiments

$$\Delta^{++}(1232) \rightarrow p + \pi^+ \text{ : branching ratio almost } \mathbf{100\%} \text{ !}$$

HRG: Fluctuations expressed in terms of stable hadronic states:

$$\chi_{ijk}^{\mathrm{BQS}}\left(T, \hat{\mu}_{\mathrm{B}}, \hat{\mu}_{\mathrm{Q}}, \hat{\mu}_{\mathrm{S}}\right) = \sum_R \mathrm{B}_R^i \mathrm{Q}_R^j \mathrm{S}_R^k \frac{\partial^l p_R / T^4}{\partial \hat{\mu}_R^l}$$

net- B $: \tilde{p} + \tilde{n} + \tilde{\Lambda} + \tilde{\Sigma}^+ + \tilde{\Sigma}^- + \tilde{\Xi}^0 + \tilde{\Xi}^- + \tilde{\Omega}^-$

net- Q $: \tilde{\pi}^+ + \tilde{K}^+ + \tilde{p} + \tilde{\Sigma}^+ - \tilde{\Sigma}^- - \tilde{\Xi}^- - \tilde{\Omega}^-$

net- S $: \tilde{K}^+ + \tilde{K}^0 - \tilde{\Lambda} - \tilde{\Sigma}^+ - \tilde{\Sigma}^- - 2\tilde{\Xi}^0 - 2\tilde{\Xi}^- - 3\tilde{\Omega}^-$

$B_R, Q_R, S_R$ are the baryon number, electric charge and strangeness of the species $R$

*R. Bellwied et al., Phys. Rev. D 101, 034506 (2020)*

In HIC, fluctuations are related to the variance or covariance of net-multiplicity for Identified $\pi, K, p$

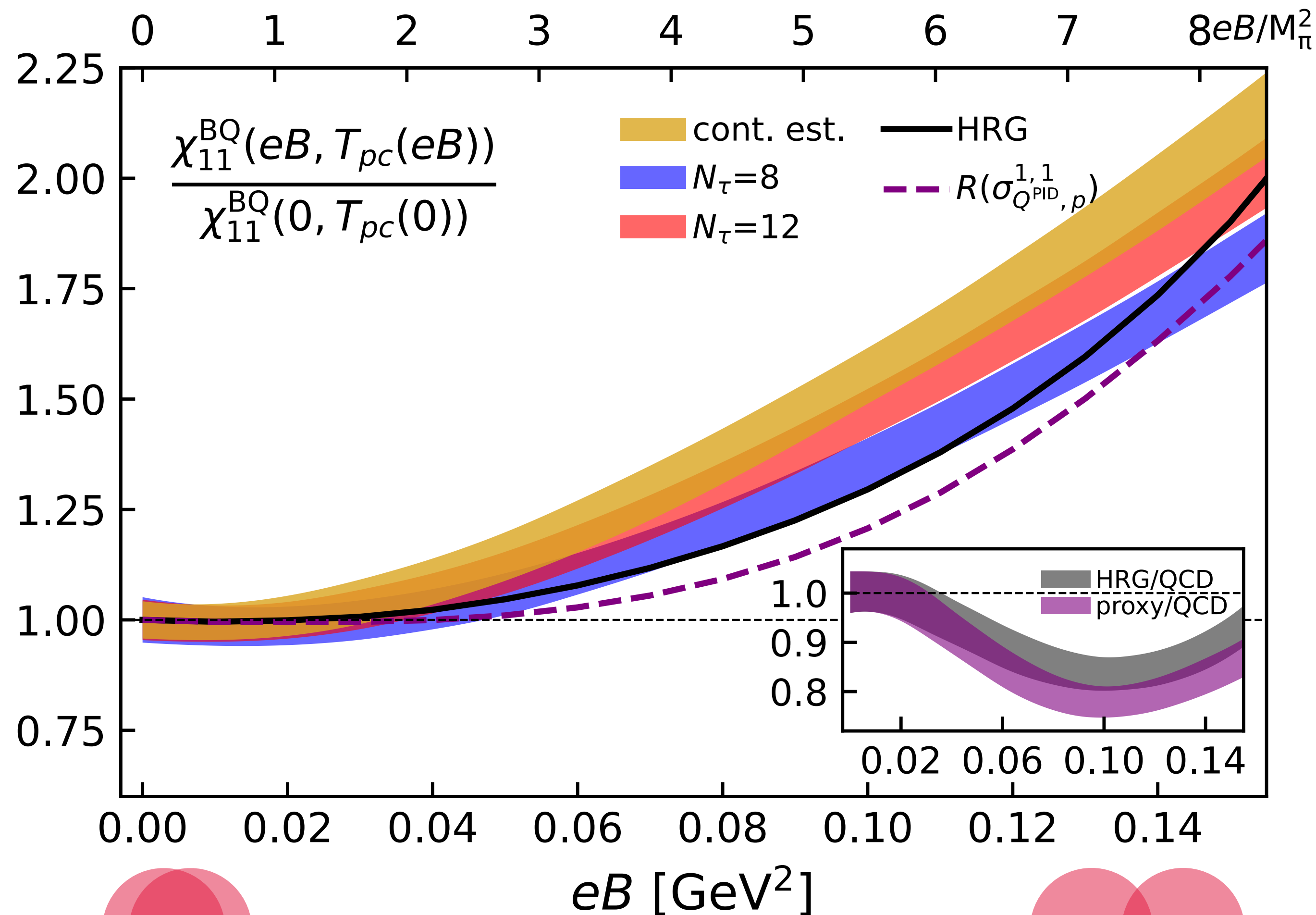*STAR, Phys.Rev.C 100 (2019) 1, 014902 ; STAR, Phys.Rev.C 105(2019) 2, 029901*

$\sigma_{Q^{\mathrm{PID}}, p}^{1,1}$ as proxy for $\chi_{11}^{\mathrm{BQ}}$:

$$\sigma_{Q^{\mathrm{PID}}, p}^{1,1} = \sum_R \left(P_{R \rightarrow \tilde{p}}\right)\left(P_{R \rightarrow Q^{\mathrm{PID}}}\right) \frac{\partial^2 p_R / T^4}{\partial \hat{\mu}_R^2} + \frac{\partial^2 p_{\tilde{p}} / T^4}{\partial \hat{\mu}_{\tilde{p}}^2}$$

where $P_{R \rightarrow i}$ represents number of particle $i$ produced by particle $R$ after the **entire decay chain**, $Q^{\mathrm{PID}} : \tilde{\pi}^+, \tilde{K}^+, \tilde{p}$

In proxy, contributions from all resonance decays are considered!

$$\frac{\chi_{11}^{BQ}(eB, T_{pc}(eB))}{\chi_{11}^{BQ}(0, T_{pc}(0))}$$

Legend:
- cont. est.
- $N_\tau = 8$
- $N_\tau = 12$
- HRG
- $R(\sigma_{Q^{PID},p}^{1,1})$

Inset legend:
- HRG/QCD
- proxy/QCD

Top axis: $eB/M_\pi^2$

Bottom axis: $eB$ [GeV$^2$]

Central Collisions ← → Peripheral Collisions
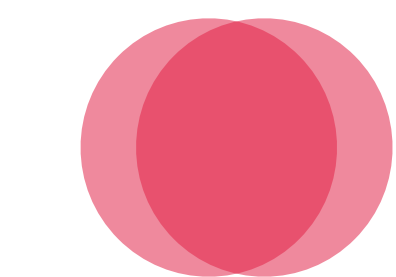
◆ At $eB \simeq 8M_\pi^2$, ratio of $\chi_{11}^{BQ} \sim 2.1$
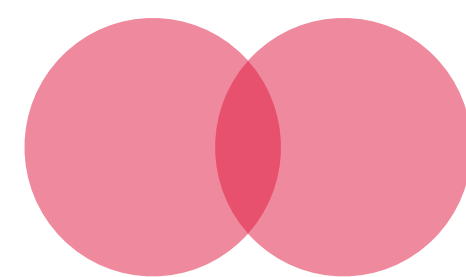
$$R(\sigma_{Q^{PID},p}^{1,1}) = \sigma_{Q^{PID},p}^{1,1}(eB)/\sigma_{Q^{PID},p}^{1,1}(eB = 0)$$

◆ The proxy $R(\sigma_{Q^{PID},p}^{1,1})$ can represent **80~85%** of the LQCD results

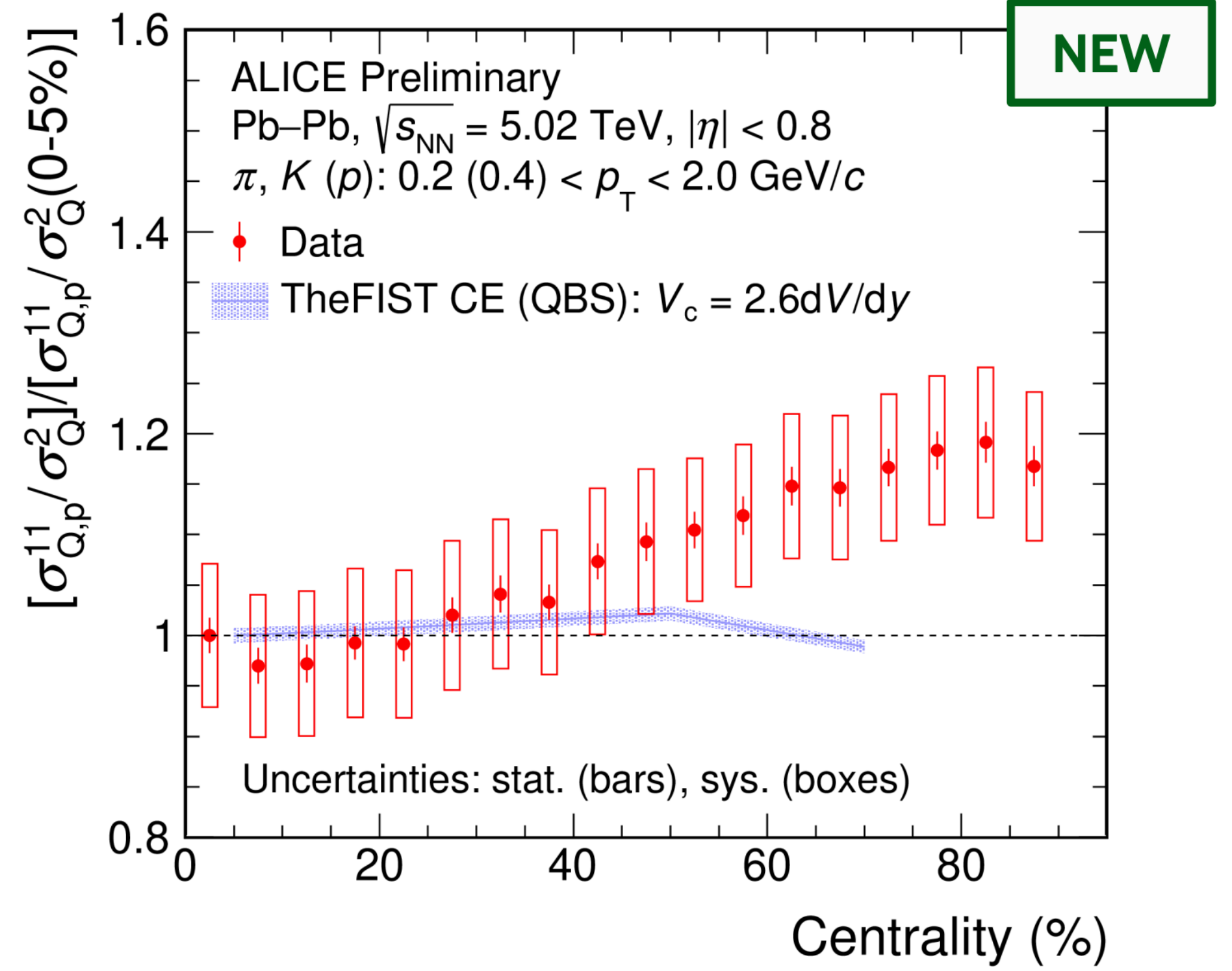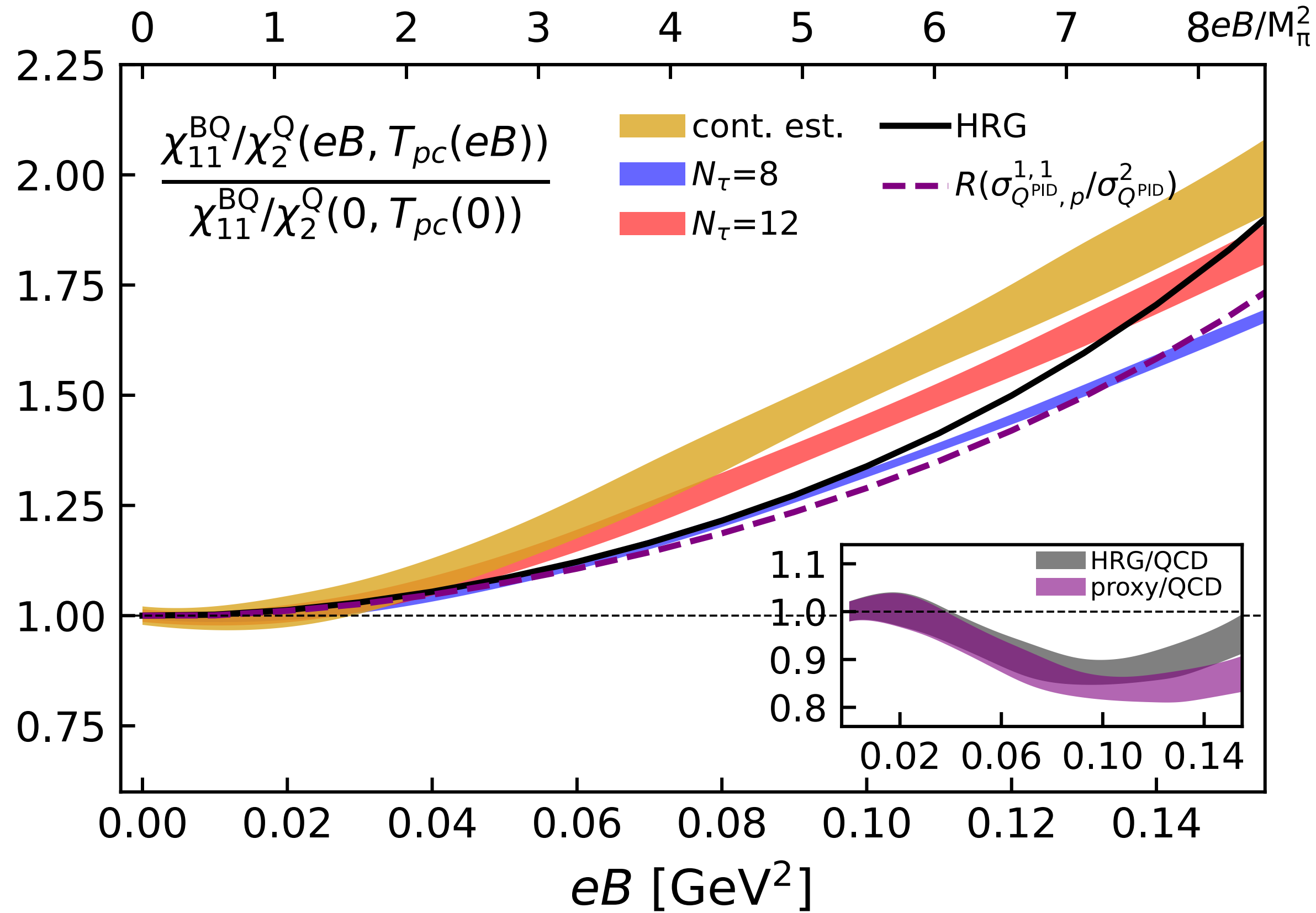◆ $R(\sigma_{Q^{PID},p}^{1,1})$ is a reasonable proxy for $\chi_{11}^{BQ}$

*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

$$\frac{\chi_{11}^{BQ}/\chi_2^Q(eB, T_{pc}(eB))}{\chi_{11}^{BQ}/\chi_2^Q(0, T_{pc}(0))}$$

- cont. est.
- HRG
- $N_\tau=8$
- $N_\tau=12$
- $R(\sigma_{Q^{PID},p}^{1,1}/\sigma_{Q^{PID}}^2)$

HRG/QCD
proxy/QCD

$8eB/M_\pi^2$

$eB$ [GeV$^2$]

ALICE Preliminary
Pb–Pb, $\sqrt{s_{NN}}$ = 5.02 TeV, $|\eta| < 0.8$
$\pi$, $K$ $(p)$: 0.2 (0.4) $< p_T < 2.0$ GeV/$c$

- Data
- TheFIST CE (QBS): $V_c = 2.6\mathrm{d}V/\mathrm{d}y$

$[\sigma_{Q,p}^{11}/\sigma_Q^2]/[\sigma_{Q,p}^{11}/\sigma_Q^2(0\text{-}5\%)]$

Uncertainties: stat. (bars), sys. (boxes)

Centrality (%)

NEW

ALI-PREL-573205

*S. Saha for the ALICE collaboration @ SQM 2024*

Central Collisions

Peripheral Collisions
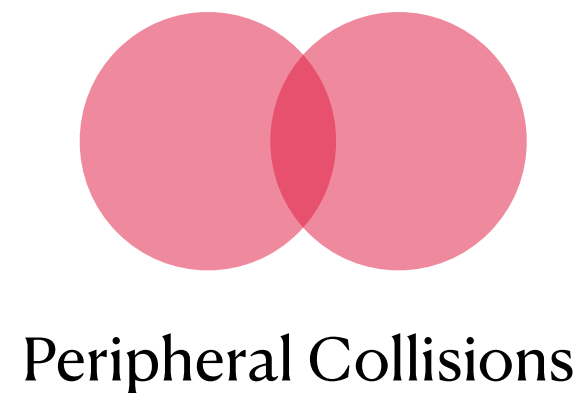
*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

# Electric charged chemical potential over baryon chemical potential
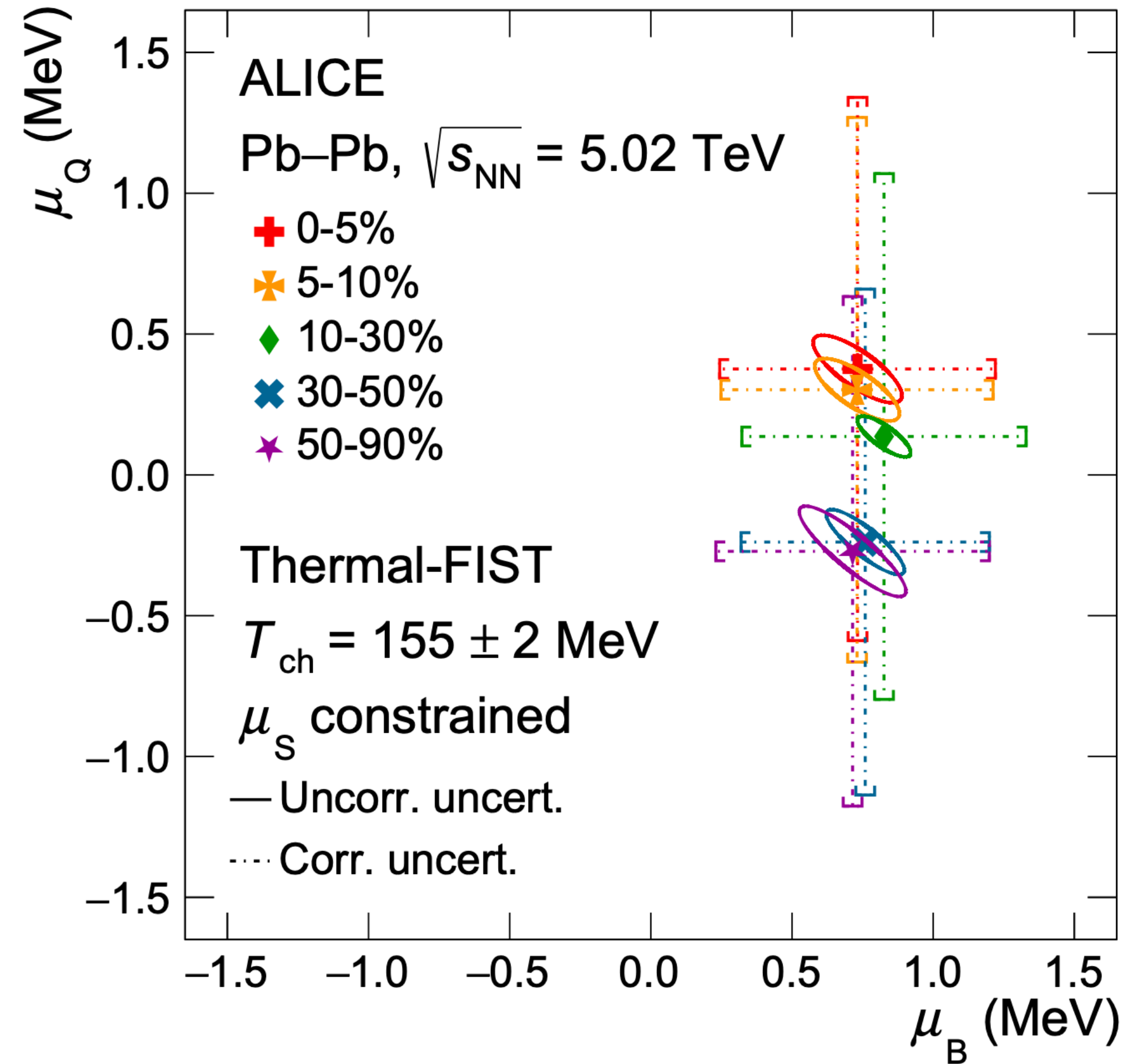


ALICE, arXiv:2311.13332

In the figure:

ALICE

Pb–Pb, $\sqrt{s_{NN}}$ = 5.02 TeV

- ✚ 0-5%
- ✳ 5-10%
- ◆ 10-30%
- ✖ 30-50%
- ★ 50-90%

Thermal-FIST

$T_{ch}$ = 155 ± 2 MeV

$\mu_S$ constrained

— Uncorr. uncert.

···· Corr. uncert.

Axes: $\mu_Q$ (MeV) vs $\mu_B$ (MeV)

- ⊙ In HIC experiments:

  $\mu_Q/\mu_B$ can be obtained from the thermal statistics fits to particle yields

- ⊙ Theoretically:

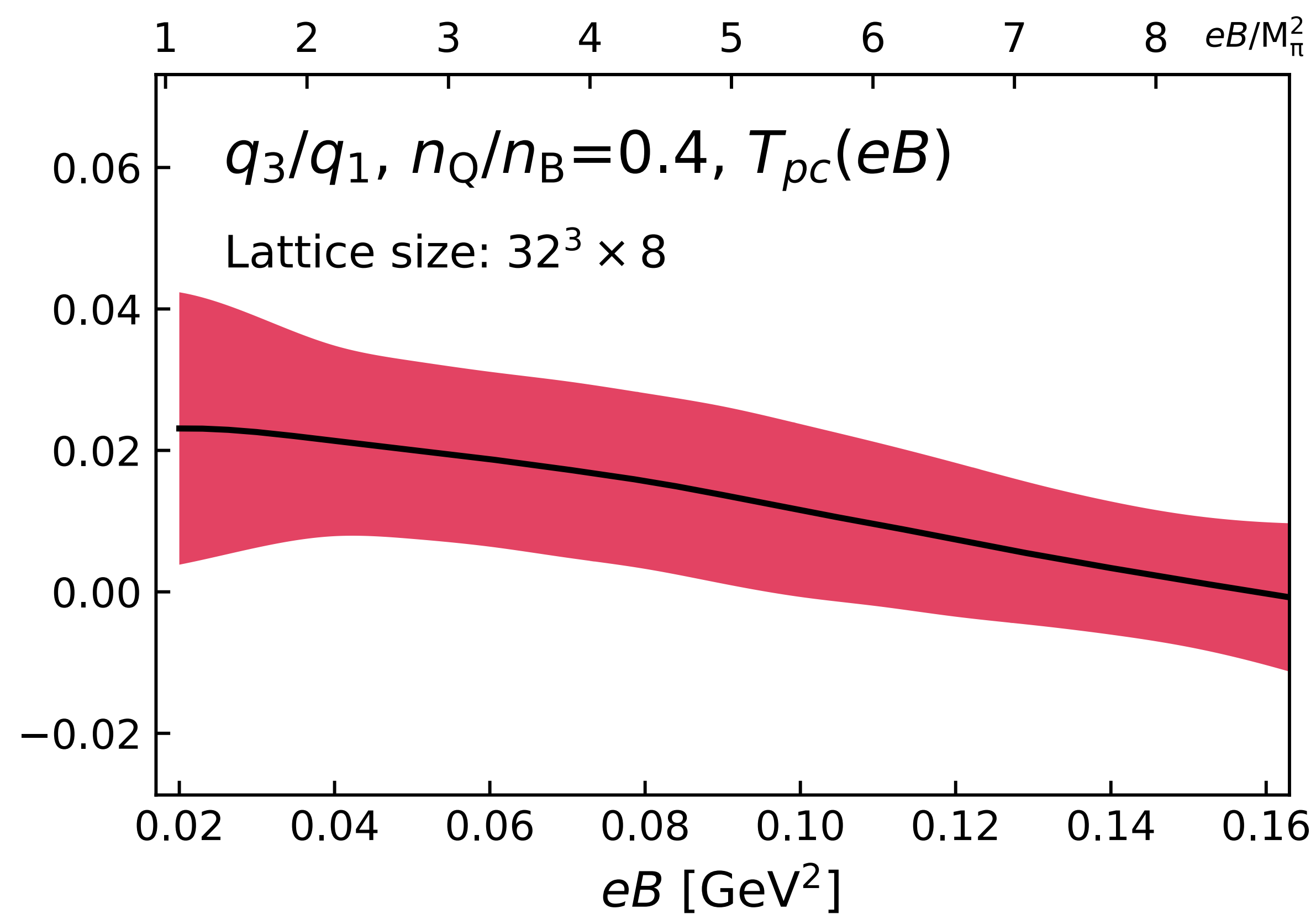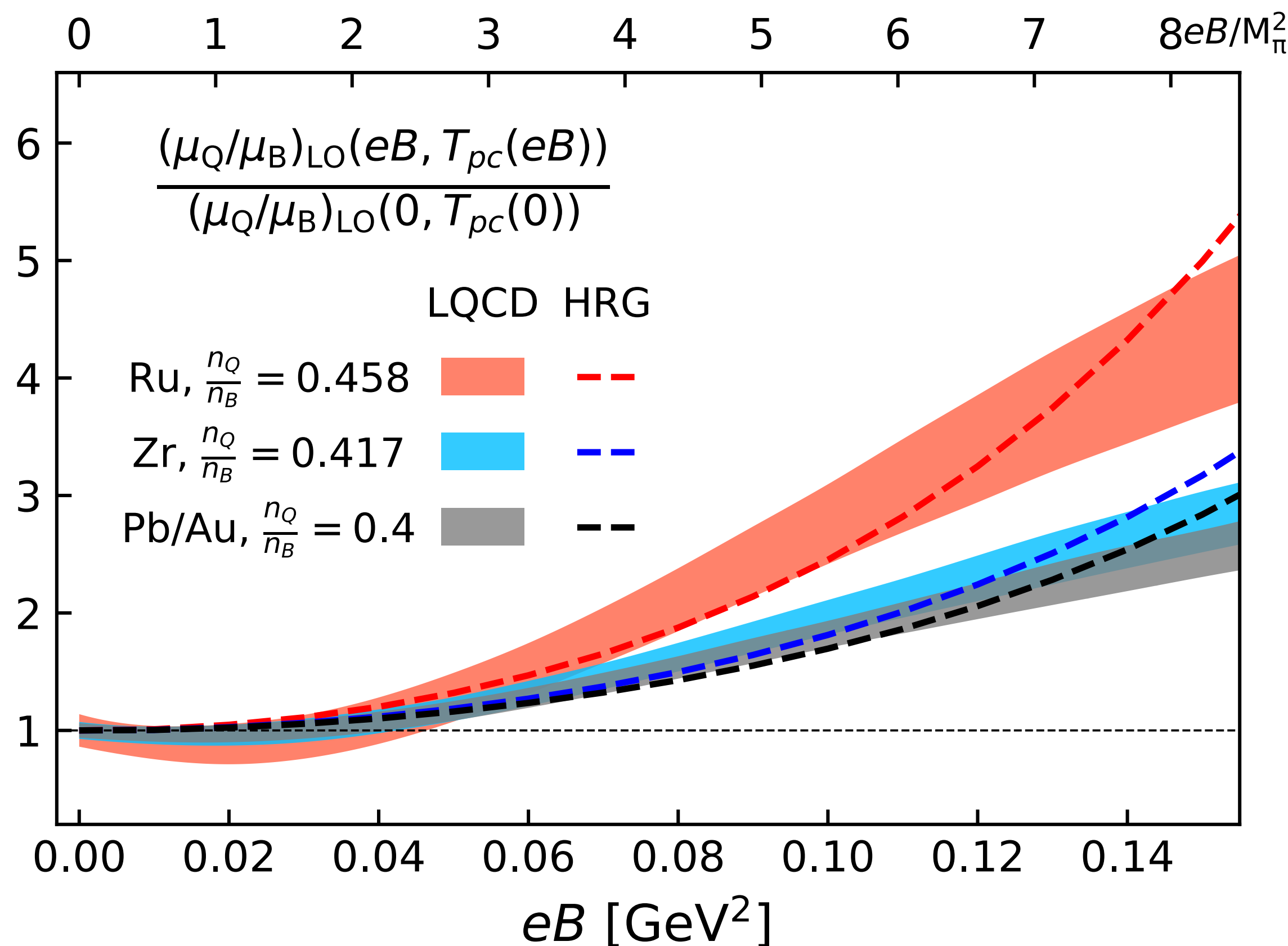$$\mu_Q/\mu_B = \boxed{q_1} + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$\boxed{q_1} = \frac{r\left(\chi_2^B \chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q \chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

with constraints: $r = n_Q/n_B$, $n_S = 0$

# Dependence of $\mu_Q/\mu_B$ on the magnetic field

$$\mu_Q/\mu_B = q_1 + q_3\ \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$



*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

# Summary

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges based on LQCD computation on $N_\tau = 8$ and $12$ lattices

- $\chi_{11}^{\mathrm{BQ}}$ is strongly affected by $eB$, and a reasonable proxy is provided for measurement in HIC

- The $\mu_{\mathrm{Q}}/\mu_{\mathrm{B}}$ show a significant dependence on the magnetic field and is sensitive to the initial $n_{\mathrm{Q}}/n_{\mathrm{B}}$
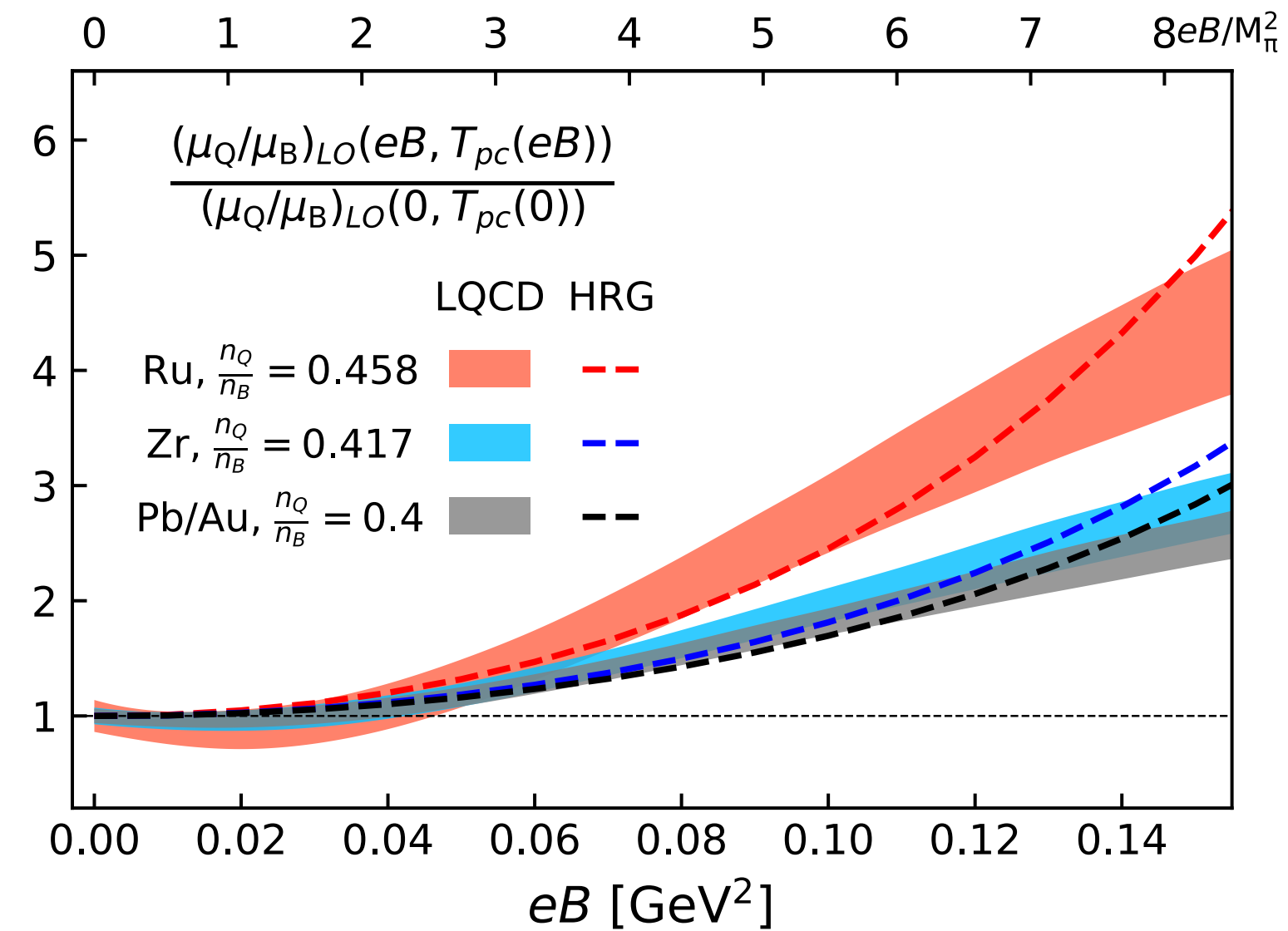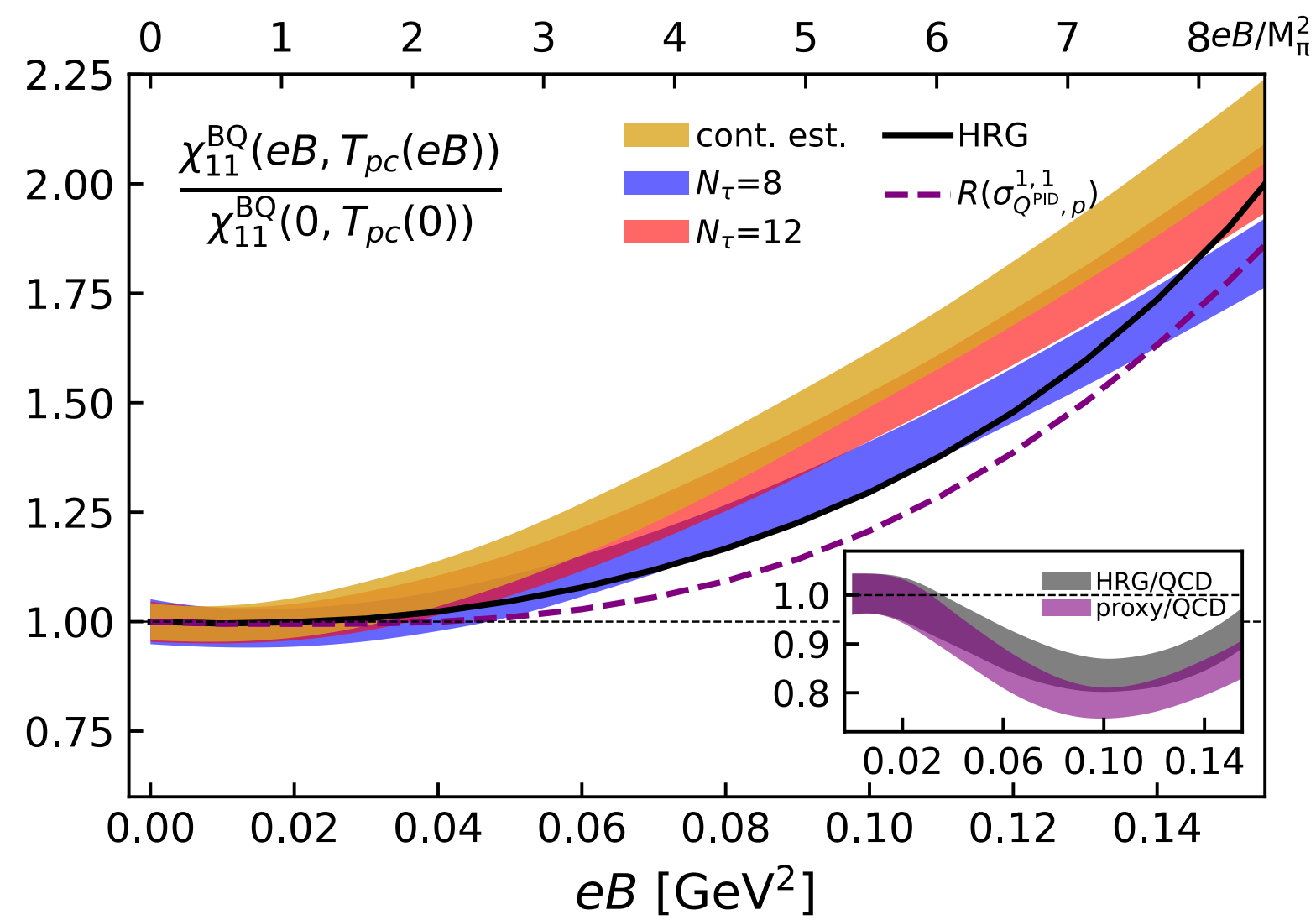


# Thank you!

# Backup

$B$ pointing along the $z$ direction

$$u_x\left(n_x, n_y, n_z, n_\tau\right) = \begin{cases} \exp\left[-iqa^2 B N_x n_y\right] & (n_x = N_x - 1) \\ 1 & \text{(otherwise)} \end{cases}$$

$$u_y\left(n_x, n_y, n_z, n_\tau\right) = \exp\left[iqa^2 B n_x\right]$$

$$u_z\left(n_x, n_y, n_z, n_\tau\right) = u_t\left(n_x, n_y, n_z, n_\tau\right) = 1$$

## No sign problem !

Quantization of the magnetic field

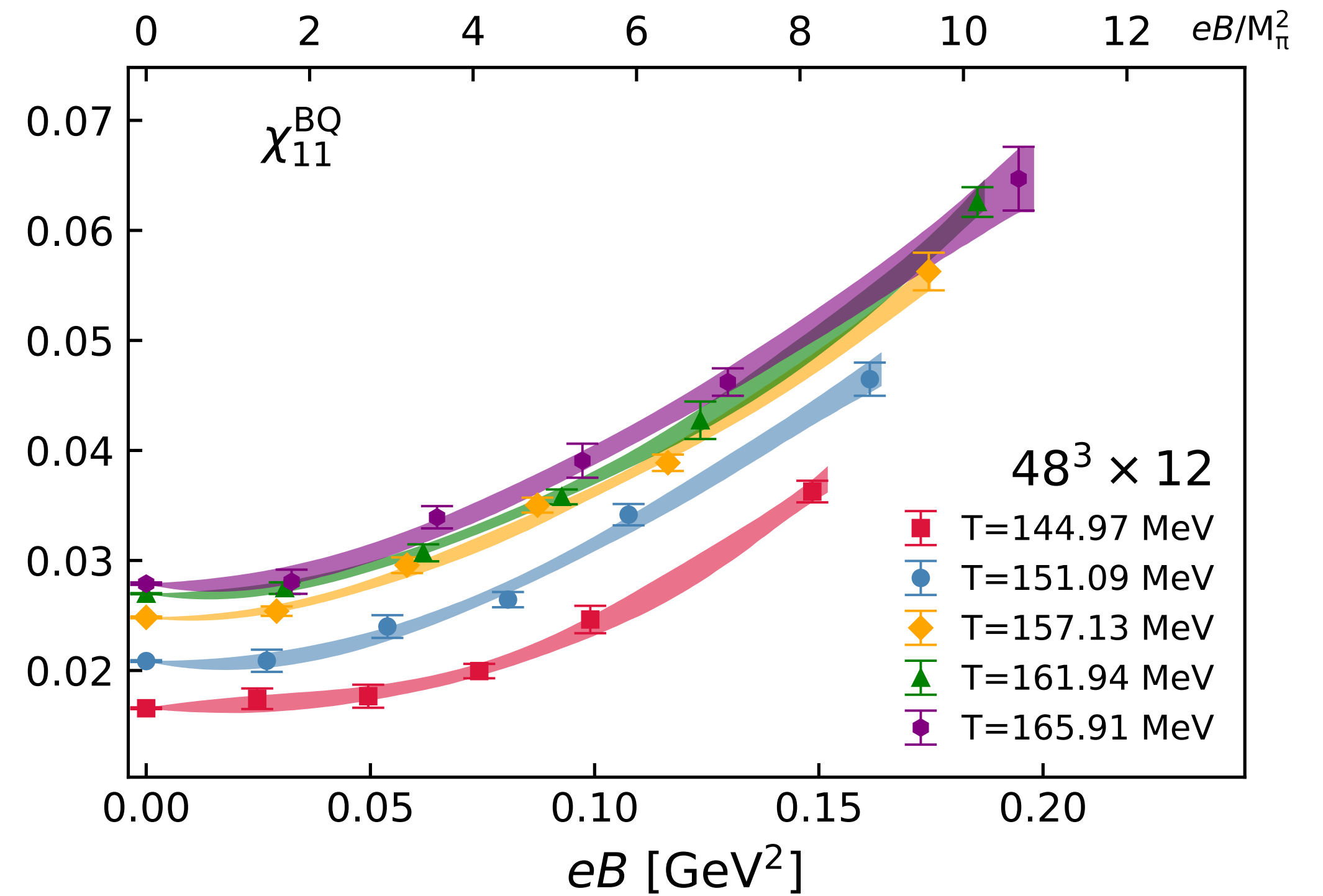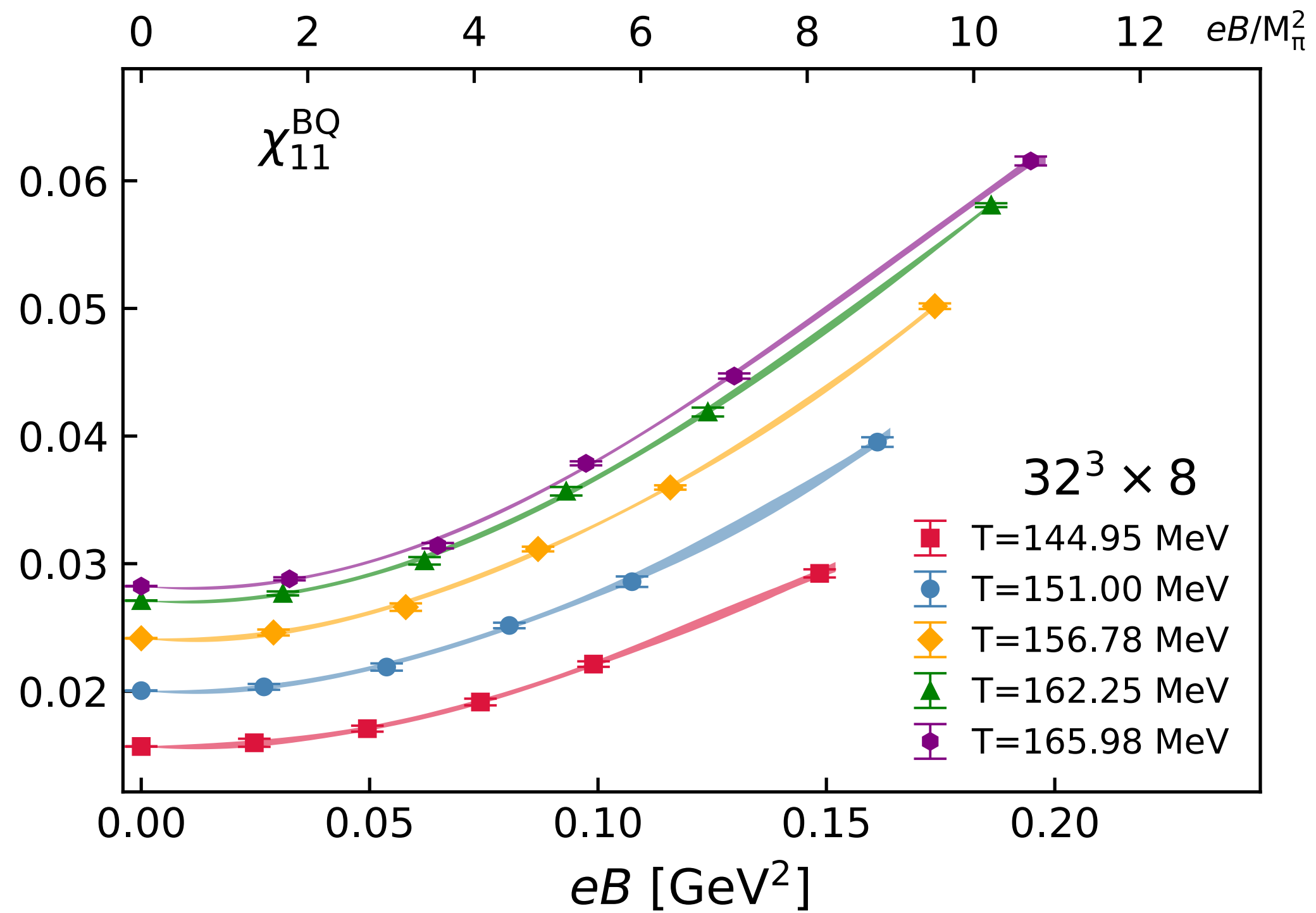$$q_u = 2/3 \ e$$
$$q_d = -1/3 \ e$$
$$q_s = -1/3 \ e$$

$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

$a$ is changed to get the targeted $T$, $T = \dfrac{1}{aN_\tau}$

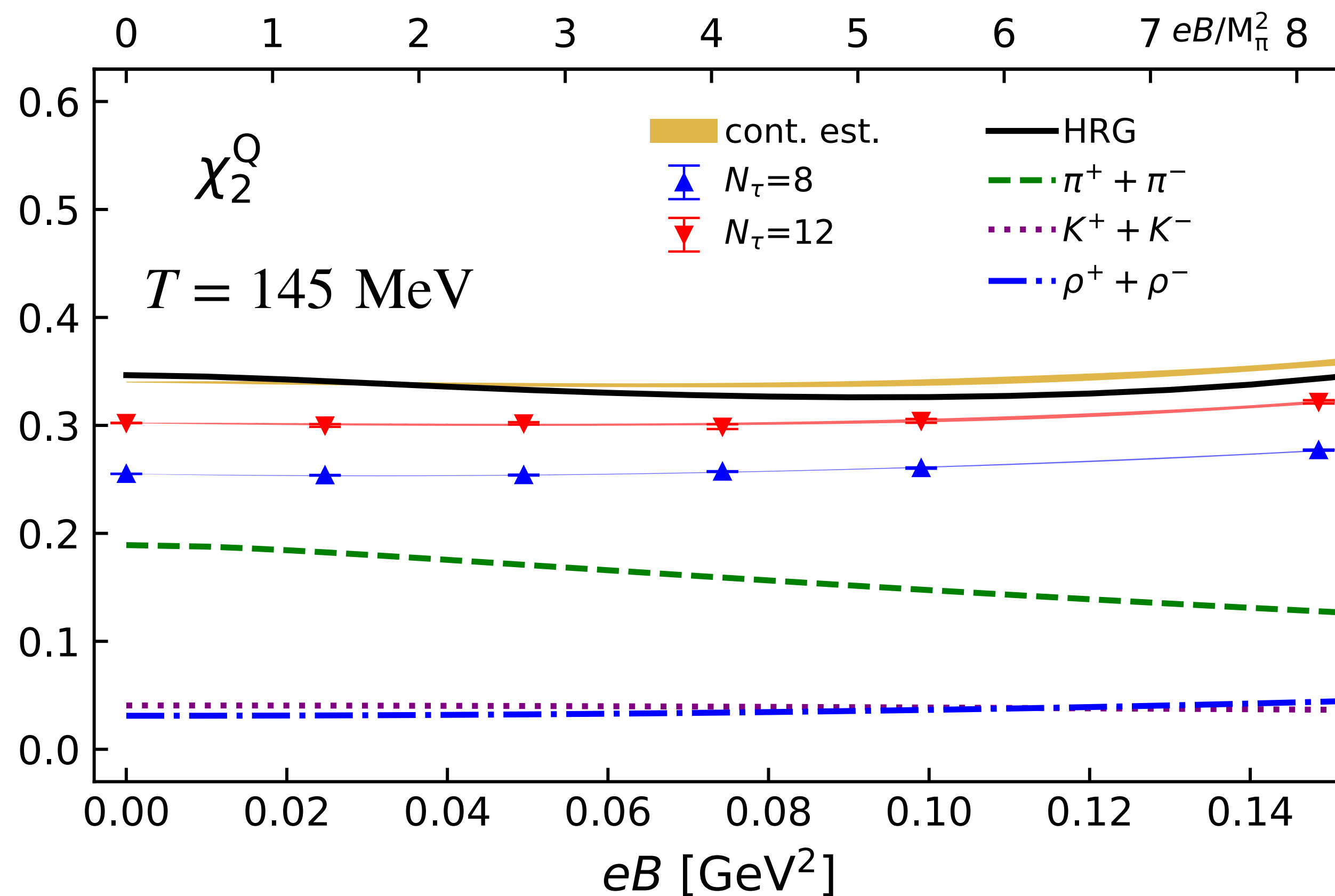✦ Statistics($eB \neq 0$): $N_\tau$=8: ~40000 ($N_{\text{rv}}$ : 603)

$N_\tau$=12: ~5000 ($N_{\text{rv}}$ : 102 ~ 705)

$N_\tau$=16: ~3000 ($N_{\text{rv}}$ : 603)

Landau gauge
G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

$\chi_2^Q$
$T = 145$ MeV

cont. est. — HRG
$N_\tau=8$ — $\pi^+ + \pi^-$
$N_\tau=12$ — $K^+ + K^-$
— $\rho^+ + \rho^-$

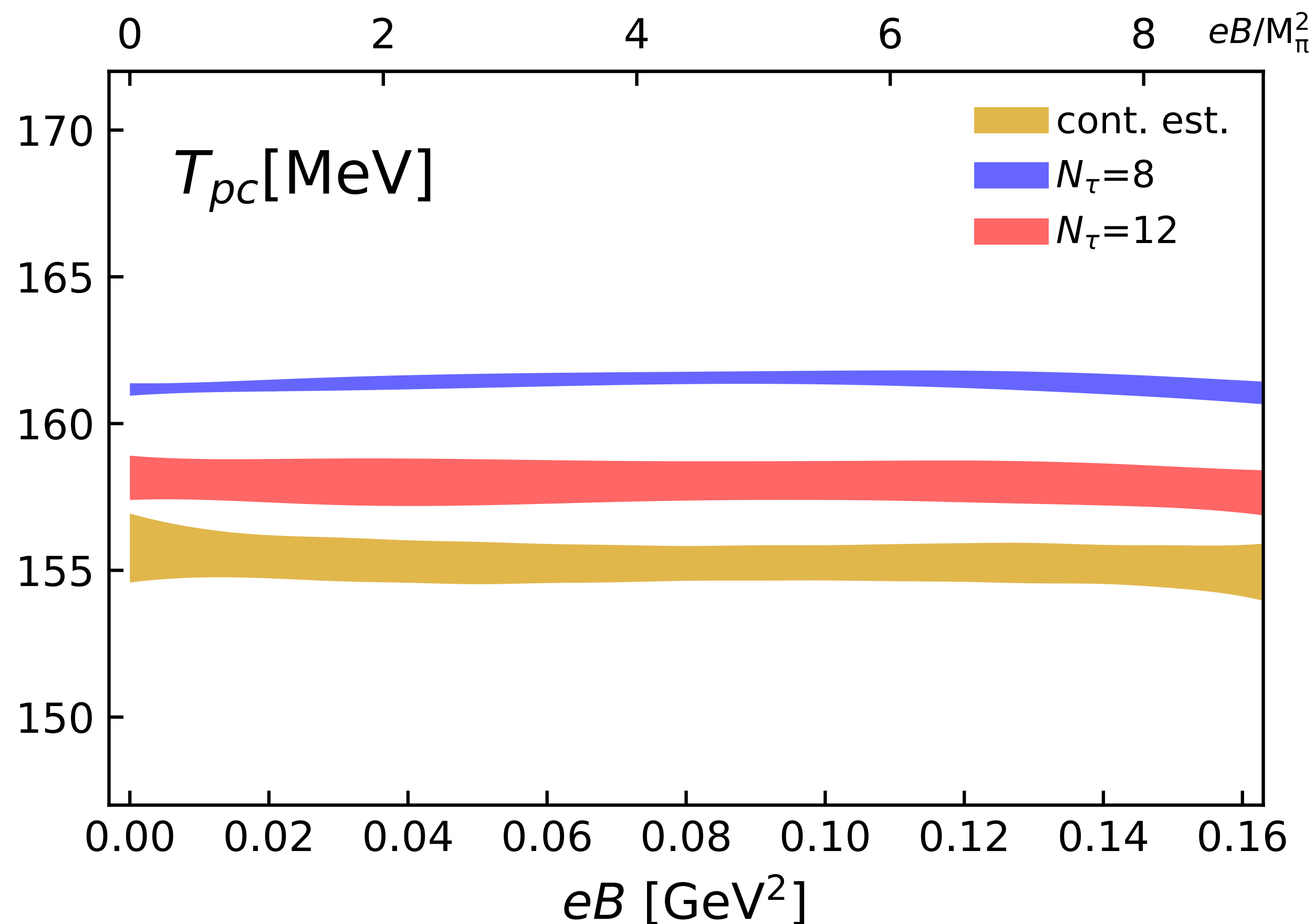*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

❖ $\chi_2^Q$ almost independent on $eB$

❖ Hadron Resonance Gas model (HRG):
Pressure arising from charged hadrons
$(eB \neq 0)$:

$$\frac{p_c^{M/B}}{T^4} = \frac{\left|q_i\right| B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1\left(\frac{n\varepsilon_0}{T}\right)$$

where $\varepsilon_0 = \sqrt{m_i^2 + 2\left|q_i\right| B\left(l + 1/2 - s_z\right)}$,

$K_1$ is the first-order modified Bessel function

# Transition line on $T - eB$ plane and $T_{\text{ch}}$ in experiment



$$M = \frac{1}{f_K^4} \left[ m_s \left( \langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d \right) - (m_u + m_d) \langle \bar{\psi}\psi \rangle_s \right]$$

$$\chi_M(eB) = \frac{m_s}{f_K^4} \left[ m_s \chi_l(eB) - 2\langle \bar{\psi}\psi \rangle_s (eB = 0) - 4m_l \chi_{su}(eB = 0) \right]$$

Finding the peak location of $\chi_M$ at each $eB$ value
to determine $T_{pc}(eB)$

*H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)*

# Proxy in experiment

✦ Conserved charges susceptibilities in experiment:

$$\chi_\alpha^2 = \frac{1}{VT^3}\kappa_\alpha^2, \quad \chi_{\alpha,\beta}^{1,1} = \frac{1}{VT^3}\kappa_{\alpha,\beta}^{1,1}$$

the second-order cumulants($\kappa$) are the variance or covariance($\sigma$) of the net-multiplicity $N$:

$$\kappa_\alpha^2 = \sigma_\alpha^2 = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)^2\rangle$$

$$\kappa_{\alpha,\beta}^{1,1} = \sigma_{\alpha,\beta}^{1,1} = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)(\delta N_\beta - \langle\delta N_\beta\rangle)\rangle$$

with $\delta N_\alpha = N_{\alpha^+} - N_{\alpha^-}$ and $\alpha, \beta = p, Q^{PID}, k$

- $p$ : a proxy for the net-baryon
- $k$ : a proxy for the net-strangeness
- $Q^{\text{PID}}$: identified $\pi$, $k$ and $p$

*STAR, Phys.Rev.C 100 (2019) 1, 014902*

$$\sigma_{Q^{\text{PID}},p}^{1,1} = \sigma_p^2 + \sigma_{p,\pi}^{1,1} + \sigma_{p,K}^{1,1}$$

$$\sigma_p^2 = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{p}}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}_R^2}$$

$$\sigma_{p,\pi}^{1,1} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{\pi}^+}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}_R^2}$$

$$\sigma_{p,K}^{1,1} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{K}^+}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}_R^2}$$

where $P_{R\to i} = \sum_\alpha N_{R\to i}^\alpha n_{i,\alpha}^R$

$n_{i,\alpha}^R$: numbers of $i$ produced by $R$ in decay channel $\alpha$

$N_{R\to i}^\alpha$: Branching ratio of channel $\alpha$