UNIVERSITÄT BIELEFELD

CRC-TR 211

Thermal photon production rate from lattice QCD

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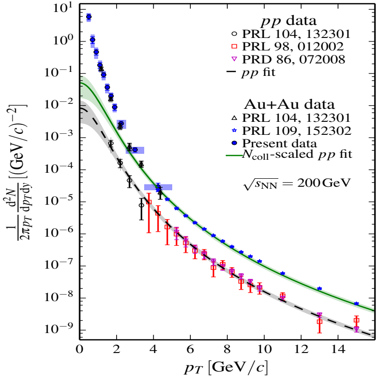
and
HotQCD Collaboration

arXiv: 2403.11647

- Photon and Di-lepton produced from QGP is an important probe to study Quark-Gluon-Plasma.
  - RHIC and LHC clearly shows excess of photon yield at low p_T region.
  - In addition it shows a large azimuthal anisotropy (Direct-photon puzzle).
  - The photon production rate (R_γ) from a thermalized QGP can be calculated in-terms of spectral function L.D. McLerran et al, PRD 31, 545



A. Adare et al, PRC 91,064904

J. Paquet et al, PRC 93, 044906

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)}{2\pi^2 k} g^{\mu\nu} \rho_{\mu\nu}(\omega = |\vec{k}|, \vec{k})$$

- $\rho_{\mu\nu}$ is defined in terms of Electromagnetic current $J_\mu(X) = \bar{\psi}(X)\gamma_\mu\psi(X)$,

$$\rho_{\mu\nu}(K = (\omega, \vec{k})) = \int d^4X \exp\{iK.X\}\langle[J_\mu(X), J_\nu(0)]\rangle_T$$

- On the lattice, we calculate the correlation function in Euclidean time. $G^E_{\mu\nu}(\tau, \vec{k}) = \int d^3\vec{x} \exp\left(i\vec{k}.\vec{x}\right)\langle J_\mu(\vec{x}, \tau)J_\nu(\vec{0}, 0)\rangle$
- Relation with spectral function

$$G^E_{\mu\nu}(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_{\mu\nu}(\omega, \vec{k})\frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

Numerically ill-conditioned problem.

1) Difference in the number of degrees of freedom.

2) Small error in G^E become very large error in ρ.

- $\rho_{\mu\nu}$ can be decomposed,

$$\rho_{\mu\nu}(\omega, \vec{k}) = P^T_{\mu\nu}\rho_T(\omega, \vec{k}) + P^L_{\mu\nu}\rho_L(\omega, \vec{k})$$

$$\rho_V(\omega, \vec{k}) = \rho^\mu_\mu(\omega, \vec{k}) = 2\rho_T(\omega, \vec{k}) + \rho_L(\omega, \vec{k})$$

- At the photon point $\rho_L(|\vec{k}|, \vec{k}) = 0$.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\rho_T(|\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\left(\rho_T(|\vec{k}|, \vec{k}) - \rho_L(|\vec{k}|, \vec{k})\right)$$

- At $T = 0$, $\rho_{\mu\nu} = (k_\mu k_\nu - g_{\mu\nu} k^2)\rho(k^2) \Rightarrow \rho_T = \rho_L$
- At finite T, $\rho_H = 2\,(\rho_T - \rho_L)$ displays pure thermal contribution.
- ρ_H is UV suppressed,

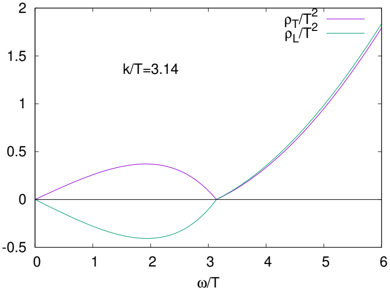
$$\rho_H \sim \frac{k^2\, O_4}{\omega^4}$$

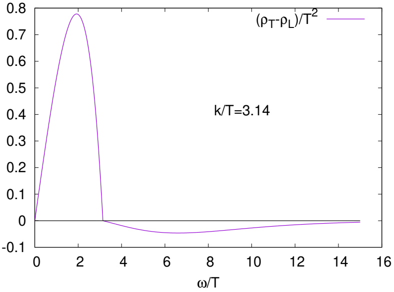
- Sum rule,

$$\int_0^\infty d\omega\, \omega\, \rho_H(\omega, |\vec{k}|) = 0$$

M. Ce et al, PRD 102, 091501

- Free result for ρ_T and ρ_L, G. Aarts and J.M. Resco,Nucl. Phys. B 726 (2005) 93





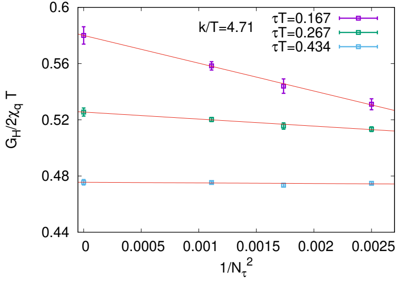
- $\rho_V = 2\rho_T + \rho_L$ has large UV part. G_V^E has large UV contribution.
- $\rho_H = 2(\rho_T - \rho_L)$ has small UV part. G_H^E has less UV contribution.

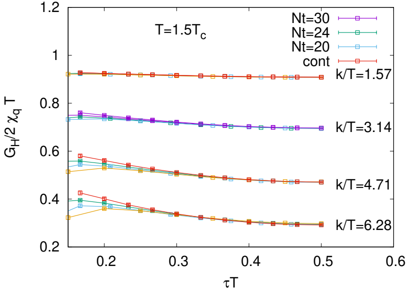
- We calculated the $T-L$ correlator in pure gluonic theory at $T=470$ MeV and in $N_f=2+1$ flavor QCD ($m_\pi=320$ MeV) at $T=220$ MeV.
- **Lattice Size:**
  - **Gluonic theory :** $120^3\times 30$, $96^3\times 24$, $80^3\times 20$, $L\sim 1.7$ fm
  - **Full QCD (HISQ):** $96^3\times 32$, $L\sim 2.7$ fm
- The available momentum for gluonic theory is $\frac{k}{T}=\frac{\pi n}{2}$ and for full QCD is $\frac{k}{T}=\frac{2\pi n}{3}$.
- We use clover improved Wilson fermion for the calculation of these correlation functions. $T\gg m_q$
- **Correlation Function:**

$$G_H^E(\tau,\vec{k})=\int_0^\infty \frac{d\omega}{\pi}2(\rho_T(\omega,\vec{k})-\rho_L(\omega,\vec{k}))\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Lattice data has cut-off effects and need continuum extrapolation.

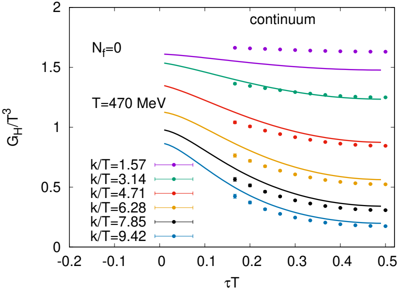
$$\frac{G_H}{2\chi_q T}(a) = \frac{G_H}{2\chi_q T}(a=0) + \frac{b}{N_\tau^2}$$

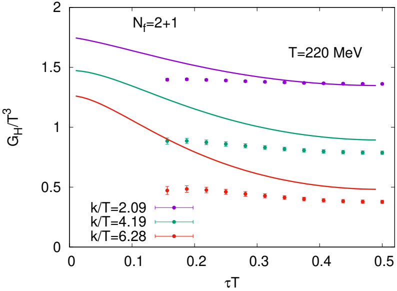




- Smaller cut-off dependence for G_H.
- Dominant contribution to G_H comes from the infrared part.

- $\chi_q = 0.897\,T^2$ for $N_f = 0$ (using non-perturbative parametrization)
  H-T Ding, O. Kaczmarek, and F. Meyer, PRD 94, 034504
  $\chi_q = 0.842\,T^2$ for $N_f = 3$ (order $g^6 log(g)$)
  A. Vuorinen, PRD 67, 074032





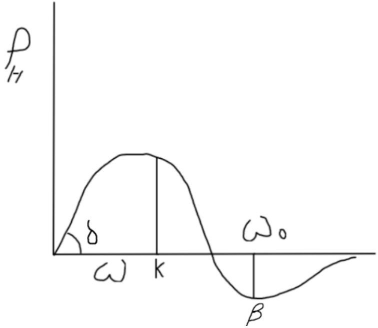
- Non-pertubative effects are important.

- For $\omega \leq \omega_0$

$$\rho_H(\omega) = \frac{\beta\omega^3}{2\omega_0^3}\left(5 - 3\frac{\omega^2}{\omega_0^2}\right) - \frac{\gamma\omega^3}{2\omega_0^2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) + \delta_0\left(\frac{\omega}{\omega_0}\right)\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2$$

J. Ghiglieri, O. Kaczmarek, M. Laine, and F. Meyer, Phys. Rev. D 94, 016005.

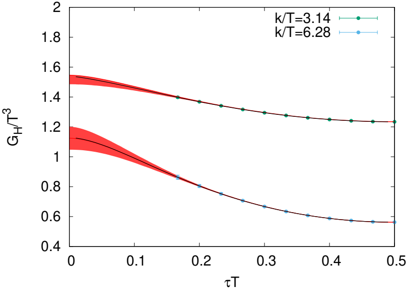
- $\beta = \rho_H(\omega_0)$ and $\gamma = \rho'_H(\omega_0)$
- For $\omega > \omega_0$ $\rho(\omega) = \sum_{i=0} \frac{A_i}{\omega^{4+2i}}$
- $\int_0^\infty d\omega\, \omega\, \rho_H(\omega, \beta, \gamma, \delta, A_i, \omega_0) = 0$
- $\omega_0 = \sqrt{k^2 + \nu(\pi T)^2}$

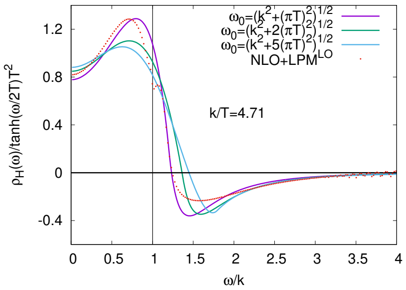


- Constrained fit with $\delta_0 \geq 0$,$\rho_H(k, \vec{k}) \geq 0$ and $\frac{\partial G_H}{\partial \tau} \leq 0$

Fitting of Mock Data

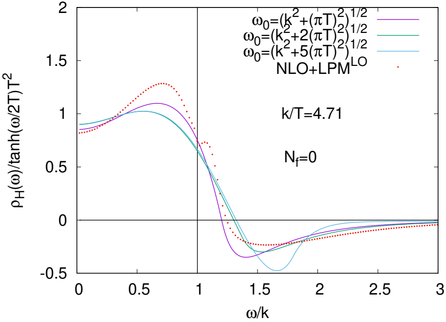
- Ten perturbative data points between 0.1875 to 0.5 in τT.
- Realistic error introduced similar to lattice correlator.

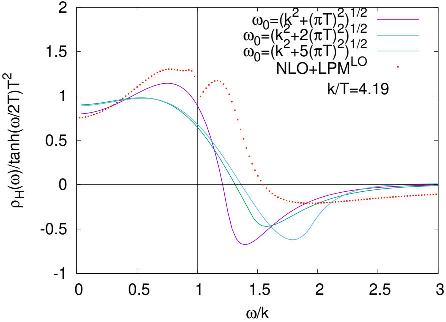




- The exact spectral function can be approximately caputured by the systematic uncertainty between $\omega_0 = \sqrt{k^2 + \pi^2 T^2}$ and $\omega_0 = \sqrt{k^2 + 5\pi^2 T^2}$

Fitting lattice data

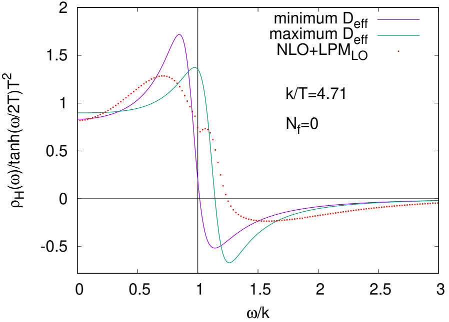


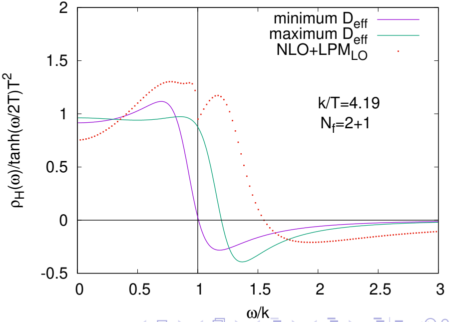


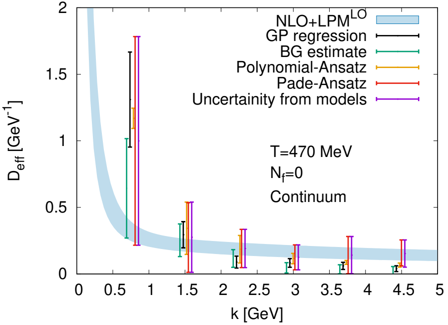
Pade Ansatz

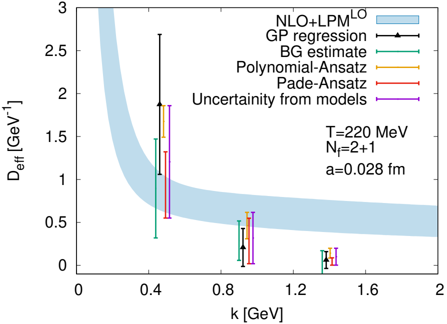
- $$\rho_H^{PADE}(\omega, \vec{k}) = A \frac{\tanh(\omega/2T)\ (1 + B\omega^2)}{(a^2 + \omega^2)((\omega - \omega_0)^2 + b^2)((\omega + \omega_0)^2 + b^2)}$$

  M. Ce et al, , PRD 102, 091501(R)
- The sum rule relates B with a, ω_0 and b.
- The fit has been performed on A, a, ω_0 and b.









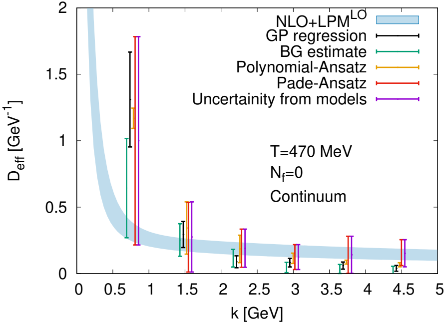
- Photon production rate,

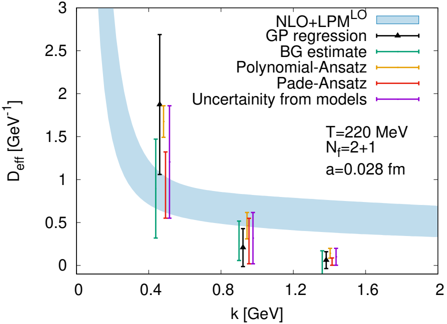
$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)\chi_q}{\pi^2} Q_i^2 D_{eff}(k)$$

- Effective diffusion coefficient,

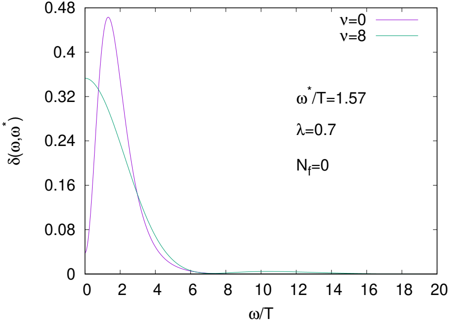
$$D_{eff}(k) = \frac{\rho_H(|\vec{k}|,\vec{k})}{2\chi_q|\vec{k}|}$$

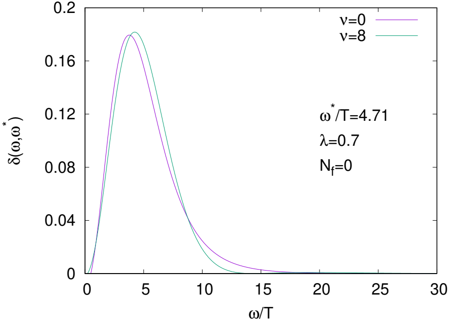
$$\lim_{k\to 0} D_{eff}(k) = D$$





- We calculated T-L correlator in Quenched and Full QCD.
- We obtained photon production rate using 4-different methods.
- We use OPE information, at large ω and sum rules to constrain the spectral reconstruction.





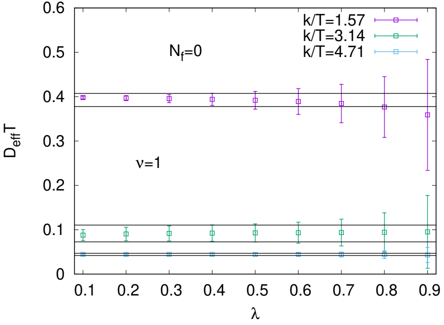
$$\frac{\rho^{BG}(\omega)}{f(\omega)} = \sum_i q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega}\delta(\omega,\bar{\omega})\frac{\rho(\bar{\omega})}{f(\bar{\omega})}.$$

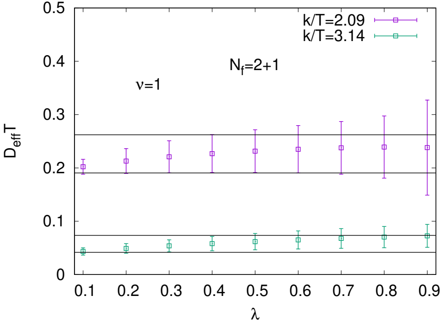
$\delta(\omega,\bar{\omega}) = \sum_i q_i(\omega)\, K(\bar{\omega},\tau_i) f(\bar{\omega})$.

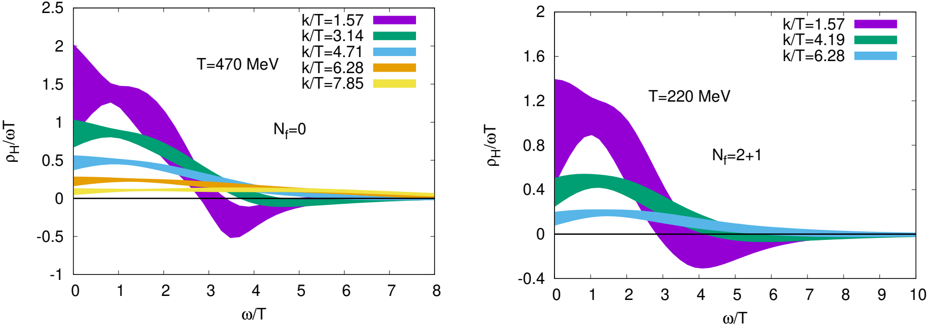
- Minimize $F(\omega) = \lambda\, Width[\delta(\omega,\bar{\omega})] + (1-\lambda)\, var[\rho_{BG}(\omega)]$

$$f(\omega) = \frac{\tanh^5(\omega/\omega_0)}{(\omega/\omega_0)^4}$$

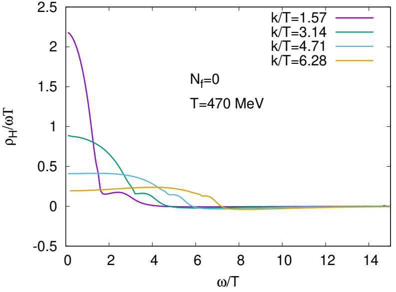
where, $\omega_0 = \sqrt{k^2 + \nu\pi^2 T^2}$.

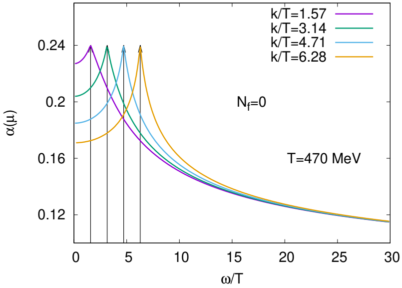

k/T=1.57
k/T=3.14
k/T=4.71
N_f=0
ν=1
D_eff T
λ


k/T=2.09
k/T=3.14
N_f=2+1
ν=1
D_eff T
λ


2.5
2
1.5
1
0.5
0
-0.5
-1
0 1 2 3 4 5 6 7 8
ω/T
ρH/ωT
T=470 MeV
Nf=0
k/T=1.57
k/T=3.14
k/T=4.71
k/T=6.28
k/T=7.85
2
1.6
1.2
0.8
0.4
0
-0.4
0 1 2 3 4 5 6 7 8 9 10
ω/T
ρH/ωT
T=220 MeV
Nf=2+1
k/T=1.57
k/T=4.19
k/T=6.28

- Away from the light cone, naive perturbation theory works well.
- Near the light cone, one has to perform LPM resummation (coherent scattering by gluons).
- Renormalization scale $\mu = \sqrt{|\omega^2 - k^2| + (2\pi T\zeta)^2}$





Coupling is maximum at the light cone.

S. Caron-Huot, PRD 79, 065039

- Scale setting:

  $T_c/\Lambda_{\overline{MS}} = 1.24$ for $N_f = 0$

  $T_c/\Lambda_{\overline{MS}} = 0.521$ for $N_f = 3$