Pseudo-scalar meson spectral properties from spatial hadron correlators

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with

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Motivation

- Spectral representation of the temporal correlator:

$$\tilde{C}_\Gamma\left(\tau,\vec{p}\right) = \int_0^\infty \frac{d\omega}{2\pi}\rho_\Gamma\left(\omega,\vec{p}\right)K\left(\omega,\tau\right)$$

with $K\left(\omega,\tau\right) = \frac{\cosh\left[\omega\left(\frac{1}{2T}-\tau\right)\right]}{\sinh\left(\frac{\omega}{2T}\right)}$.

- Spectral function $\rho_\Gamma\left(\omega,\vec{p}\right)$ contains all information about all possible spectral excitations.

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- Spectral representation of the temporal correlator:

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- Spectral function $\rho_\Gamma(\omega,\vec{p})$ contains all information about all possible spectral excitations.
- To obtain $\rho_\Gamma(\omega,\vec{p})$ from $\tilde{C}_\Gamma(\tau,\vec{p})$ one has to solve an ill-posed problem.
- The temporal extent on the lattice is limited by the temperature $T = \frac{1}{aN_\tau}$.

$\Rightarrow$ Reconstruction of spectral function is difficult especially for small N_τ.

- Does there exist an alternative way to obtain spectral functions?

Spatial correlator

$$C_\Gamma(n_z) = \frac{1}{N_\sigma^2 N_\tau} \sum_{\tilde{\vec{n}}} \langle O_\Gamma\left(\tilde{\vec{n}}, n_z\right) \bar{O}_\Gamma\left(\tilde{\vec{0}}, 0\right)\rangle$$
$$= \sum_{k=0} A_k e^{-M_k n_z}$$

with $\tilde{\vec{n}} = \left(n_x, n_y, n_\tau\right)$ and $M_k < M_{k+1}$.

- Ground state is called screening mass $M_0 = m_0^{scr}$ and is dominant at large distances.

$\Rightarrow$ The spatial extent is not limited by the temperature.

Spectral function

- Spectral representation of the spatial correlator:

$$C_\Gamma\left(z\right) = \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} e^{ip_z z} \int_0^{\infty} \frac{d\omega}{\pi\omega} \rho_\Gamma\left(\omega, p_x = p_y = 0, p_z\right)$$

- $C_\Gamma\left(z\right)$ and $\tilde{C}_\Gamma\left(\tau, \vec{p}\right)$ share the same ρ_Γ.

Spectral function

- Using causality ($\left[\phi\left(x\right),\phi\left(y\right)\right]=0$, for $\left(x-y\right)^2<0$) as constraint we can write a real scalar spectral function as [Bros and Buchholz 1992]:

$$\rho(\omega,\vec{p})=\int_0^\infty ds\int\frac{d^3\vec{u}}{(2\pi)^2}\;\epsilon(\omega)\,\delta\Big(\omega^2-(\vec{p}-\vec{u})^2-s\Big)\,\widetilde{D}_T(\vec{u},s)$$

- $\epsilon(\omega)$ is the sign function to ensure Lorentz-invariance.
- Thermal spectral density $\widetilde{D}_T(\vec{u},s)$ contains all dynamical and temperature-dependent effects.
- Källén-Lehmann representation:

$$\widetilde{D}_T(\vec{u},s)\xrightarrow{T\to 0}(2\pi)^3\delta^3(\vec{u})\,\rho(s),$$

with zero temperature spectral density $\rho(s)$.

Thermoparticle

- Thermal medium may contain particle like excitations $D_{m,T}(\vec{x})$ named thermoparticles which are formed from the same discrete $\delta(s-m^2)$ component as mass m vacuum states [J. Bros and D. Buchholz, Zeitschrift für Physik C 55, 509-513].
- The contribution to the position-spaced thermal density is given by [J. Bros and D. Buchholz, Nucl. Phys. B 627, 289–310]

$$D_T\left(\vec{x}, s\right) = D_{m,T}\left(\vec{x}\right)\delta\left(s-m^2\right) + D_{c,T}\left(\vec{x}, s\right)$$

where $D_{c,T}\left(\vec{x}, s\right)$ contains all other contributions.

Large distance behaviour

- If $D_{c,T}(\vec{x}, s)$ is negligible we find:

$$C(z) \approx \frac{1}{2} \int_{|z|}^{\infty} dR e^{-m^{scr} R} D_{m,T}\left(|\vec{x}| = R\right)$$

⇒ Determine $D_{m,T}(R)$ from large distance behaviour of spatial correlator according to:

$$D_{m,T}\left(|\vec{x}| = z\right) \sim -2e^{m^{scr} z} \frac{dC(z)}{dz}$$

Pseudo-scalar channel ($\Gamma = \gamma_5$)

$$C_{\gamma_5}(z) \approx \sum_{i=0}^{N} A_i e^{-m_i^{scr} z}$$

$$\Rightarrow \; D^{(i)}_{m_i,T}(\vec{x}) = \alpha_i e^{-\gamma_i |\vec{x}|}, \quad \alpha_i = 2 A_i m_i^{scr}, \; \gamma_i = m_i^{scr} - m_i$$

$$\rho^{(i)}_{\mathsf{PS}}(\omega, \vec{p}) = \epsilon(\omega)\theta(\omega^2 - m_i^2) \cdot \frac{4\,\alpha_i \gamma_i \sqrt{\omega^2 - m_i^2}}{(|\vec{p}|^2 + m_i^2 - \omega^2)^2 + 2(|\vec{p}|^2 - m_i^2 + \omega^2)\gamma_i^2 + \gamma_i^4}$$

$$\rho_{\mathsf{PS}}(\omega, \vec{p}) = \sum_{i=0}^{N} \rho^{(i)}_{\mathsf{PS}}(\omega, \vec{p})$$

- With ρ_{PS} the spatial correlator $C(z)$ becomes a pure exponential.

Test the approach

1. Extract screening mass and first exited state at zero and finite temperature.
2. Compute ρ_{PS} with screening masses.
3. Predict $\tilde{C}(n_\tau)$ with ρ_{PS}.
4. Compare result with lattice data.

Lattice setup

CRC-TR 211

Table: HISQ configurations

β	N_F	$N_\sigma^3 \times N_\tau$	$m_{l,HISQ}$	$m_{s,HISQ}$	T [MeV]	a [fm]	Conf. #
7.010	$2+1$	$64^3 \times 64$	0.00132	0.0357	36.398	0.0847	227
		$64^3 \times 16$	0.00132	0.0357	145.591	0.0847	399
7.188	$2+1$	$64^3 \times 64$	0.00113	0.0306	43.072	0.0716	232
		$64^3 \times 16$	0.00113	0.0306	172.287	0.0716	395

- Mixed action approach: Möbius domain wall fermions (MDWF) on $(2+1)$ HISQ configurations
- Physical light and strange quark masses
- Analysis done for light-strange (K^0) and strange-strange ($\eta_{s\bar{s}}$) meson correlator.[D. Bala et al, JHEP 05, 332]
- Procedure was done for the light-light (π) meson correlator in [P. Lowdon and O. Philipsen, JHEP 10, 161].

Results

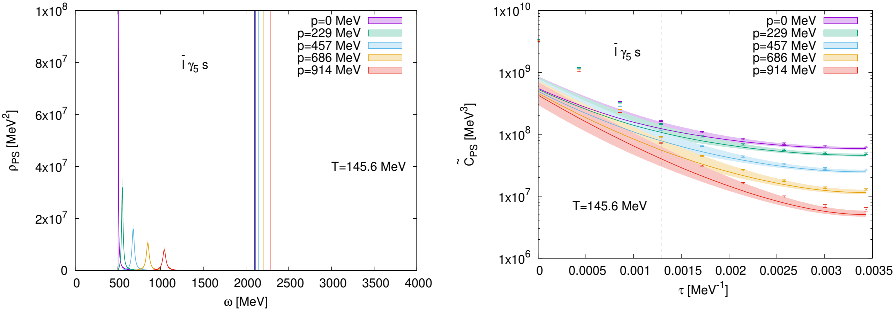


Figure: Spectral function (left) and comparison between predicted and lattice correlator (right) below T_c [D. Bala et al, JHEP 05, 332].

- Prediction is consistent with lattice data above the dashed line.
- Below dashed line contributions of higher exited states are necessary.

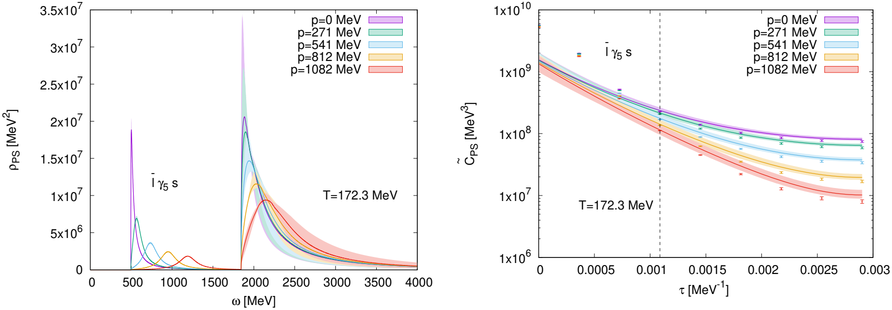


Figure: Spectral function (left) and comparison between predicted and lattice correlator (right) above T_c [D. Bala et al, JHEP 05, 332].

- Ground and exited state is significantly broadened.
- Prediction consistent with data for large τ and small values of momentum p.

- What happens if we use a non thermoparticle spectral function? E.g. the Breit-Wigner spectral function

$$\rho_{\mathsf{BW}}(\omega, \vec{p}) = \frac{4\omega\Gamma}{(\omega^2 - |\vec{p}|^2 - m^2 - \Gamma^2)^2 + 4\omega^2\Gamma^2}$$

- Gives rise to exponential spatial correlator contribution

$$C_{\mathsf{BW}}(z) = \frac{e^{-\sqrt{m^2+\Gamma^2}|z|}}{2\sqrt{m^2 + \Gamma^2}}$$

Results

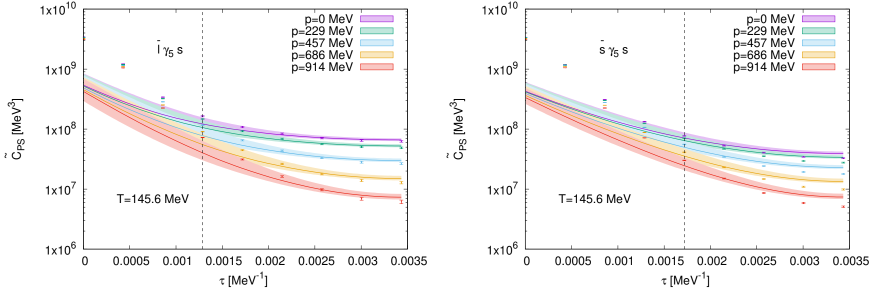


Figure: Comparison between temporal correlator prediction using a BW-spectral function and the corresponding lattice correlator at $145.6\mathrm{MeV}$[D. Bala et al, JHEP 05, 332].

- Prediction overshoots lattice data for large τ.

Conclusion and Outlook

Conclusion:

- We successfully used spatial hadron correlators to obtain pseudo-scalar spectral properties.
- The thermoparticle hypothesis provides a consistent description of how vacuum states undergo collisional broadening at finite temperature.

Outlook:

- Test thermoparticle hypothesis for charmonium and bottomonium.
- Extend approach to other meson channels.

Thank you for your attention !

Appendix

Meson operator

$$O_\Gamma(\vec{n}, n_\tau) = \bar{\psi}_{f_1}(\vec{n}, n_\tau)\,\Gamma\psi_{f_2}(\vec{n}, n_\tau)$$
$$\bar{O}_\Gamma(\vec{n}, n_\tau) = \bar{\psi}_{f_2}(\vec{n}, n_\tau)\,\Gamma\psi_{f_1}(\vec{n}, n_\tau)$$

State	J^{PC}	Γ	Particles
Scalar	0^{++}	$\mathbb{1}$, γ_4	f_0, a_0, K_0^*, ...
Pseudo-scalar	0^{-+}	γ_5 , $\gamma_4\gamma_5$	$\pi^\pm$, π^0, η, ...
Vector	1^{--}	γ_i , $\gamma_4\gamma_i$	$\rho^\pm$, ρ^0, ω, ...
Axial Vector	1^{++}	$\gamma_i\gamma_5$	f_1, a_1, ...

Table: [Gattringer and Lang 2010]

Meson correlators

- Temporal correlator:

$$\tilde{C}_\Gamma\left(n_\tau, \vec{p}\right) = \frac{1}{N_\sigma^3} \sum_{\vec{n}} \exp\left(-i\vec{n} \cdot \vec{p}\right) \langle O_\Gamma\left(\vec{n}, n_\tau\right) \bar{O}_\Gamma\left(\vec{0}, 0\right)\rangle$$

- Correlator behaves as:

$$\tilde{C}_\Gamma\left(n_\tau\right) = \sum_{k=0} A_k e^{-E_k n_\tau}$$

with $E_k < E_{k+1}$.

$\Rightarrow$ Ground state mass $m_0 = E_0$ is dominant at large distances

Meson correlators

- Spatial correlator:

$$C_\Gamma\left(n_z,\tilde{\vec{p}}\right)=\frac{1}{N_\sigma^2 N_\tau}\sum_{\tilde{\vec{n}}}\exp\left(-i\tilde{\vec{n}}\cdot\tilde{\vec{p}}\right)\left\langle O_\Gamma\left(\tilde{\vec{n}},n_z\right)\bar{O}_\Gamma\left(\tilde{\vec{0}},0\right)\right\rangle$$

with $\tilde{\vec{n}}=\left(n_x,n_y,n_\tau\right)$ and $\tilde{\vec{p}}=\left(p_x,p_y,p_\tau\right)$

- Correlator behaves as:

$$C_\Gamma\left(n_z\right)=\sum_{k=0}A_k e^{-M_k n_z}$$

with $M_k < M_{k+1}$.

⇒ Ground state is called screening mass $M_0 = m_0^{scr}$ and dominant at large distances.

- $\left(m_0^{scr}\right)^{-1}$ describes a length at which the corresponding particle is effectively screened.

Ground state extraction

1. Compute $\tilde{C}\left(n_\tau, \vec{p}\right)$
2. Fit meson correlator using

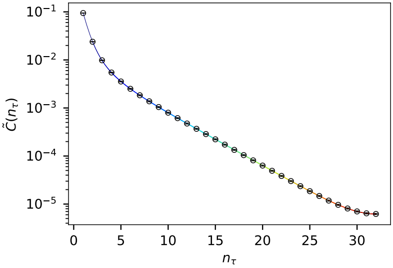
$$\tilde{C}_{\Gamma,lat}\left(n_\tau\right) = \sum_{k=0}^{N} A_k \left(e^{-E_k n_\tau} + e^{-E_k (N_\tau - n_\tau)}\right)$$
$$= \sum_{k=0}^{N} \tilde{A}_k \cosh\left(\frac{E_k}{2}\left(N_\tau - n_\tau\right)\right)$$

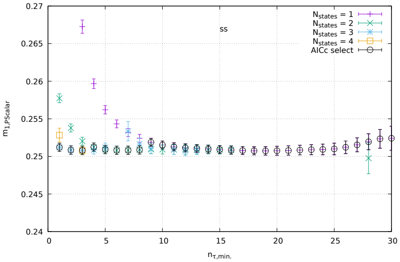
   for several fit intervals $\left[n_{\tau,min}, N_\tau/2\right]$ considering different numbers of states N.

3. Select best fit parameters for each interval using e.g. the Akaike information criterion (AIC) or χ^2/dof.
4. Plot all masses and average over values inside plateau area.

- Analogous for $C\left(n_z, \tilde{\vec{p}}\right)$

Ground state extraction





- Slope of temporal correlator $\rightarrow$ meson mass
- Slope of spatial correlator $\rightarrow$ screening mass

Spectral function and temporal correlator for $s\gamma_5\bar{s}$

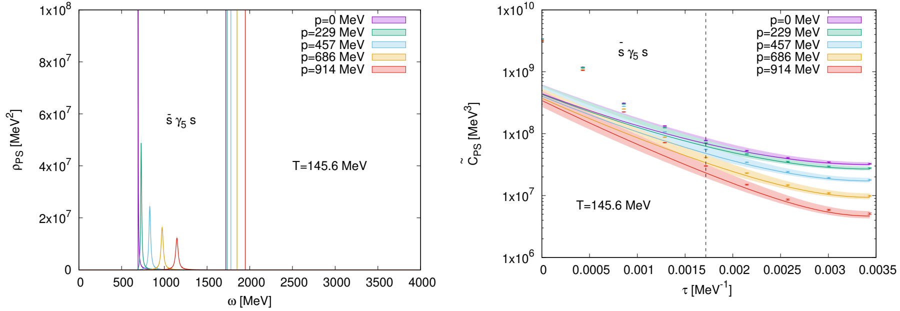


Figure: Spectral function (left) and comparison between predicted and lattice correlator (right) below T_c [D. Bala et al, JHEP 05, 332]

BW vs. thermoparticle

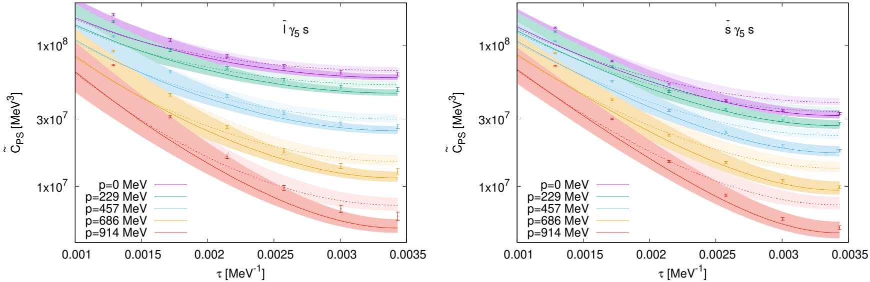


Figure: Comparison between temporal correlator prediction using thermoparticle and BW-spectral function at $145.6\mathrm{MeV}$[D. Bala et al, JHEP 05, 332]