Baryonic screening masses at very high temperatures

JHEP06(2024)205 & Phys.Lett.B 855 (2024)

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August 2, 2024

LATTICE 2024 LIVERPOOL

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- Interpolating operator with nucleon quantum numbers

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- Screening mass characterizes its exponential decay

$$m_{N^\pm} = - \lim_{x_3 \to \infty} \frac{d}{dx_3} \ln \left[C_{N^\pm}(x_3) \right] \qquad m_{N^\pm} = 3\pi T \text{ in the free theory}$$

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- **Probes of chiral symmetry restoration**
  In a chirally symmetric regime the positive and negative parity screening masses becomes degenerate

Lattice setup

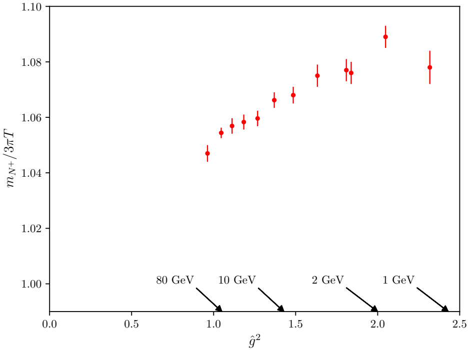
- 12 values of the temperature in the range **1.167-164.6 GeV**
- $N_f = 3$ $O(a)$-improved Wilson fermions
- Shifted boundary conditions with $\boldsymbol{\xi} = (1, 0, 0)$ [Giusti, Meyer (2011-13)]
- Lines of constant physics fixed with a non-perturbative definition of the running coupling [L.Giusti's talk]

	$\bar{g}^2(\mu)$	T (GeV)
T_0	-	164.6(5.6)
T_1	1.11000	82.3(2.8)
T_2	1.18446	51.4(1.7)
T_3	1.26569	32.8(1.0)
T_4	1.3627	20.63(63)
T_5	1.4808	12.77(37)
T_6	1.6173	8.03(22)
T_7	1.7943	4.91(13)
T_8	2.0120	3.040(78)
T_9	2.7359	2.833(68)
T_{10}	3.2029	1.821(39)
T_{11}	3.8643	1.167(23)

Lattice results

- Temperature dependence parameterized with

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\rm MS}}} + \frac{4}{9\pi^2} \ln \left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\rm MS}}} \right)$$



Lattice results

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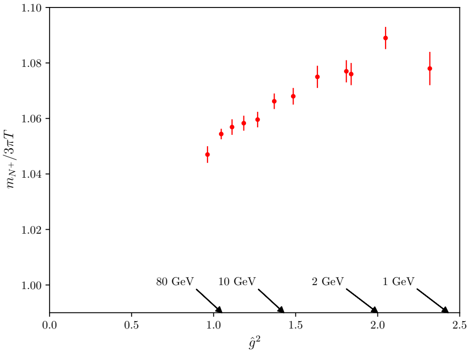

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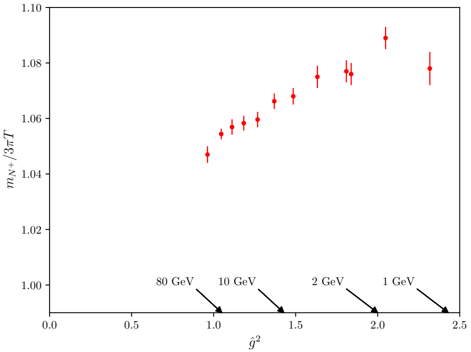
- Chiral symmetry: positive and negative parity masses are degenerate within statistical precision



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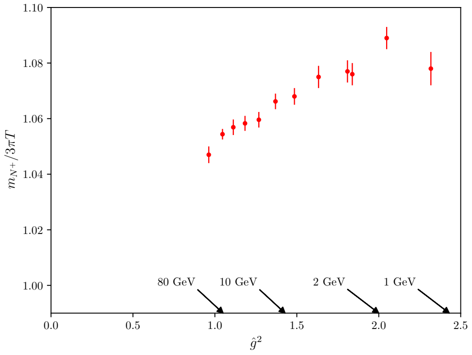

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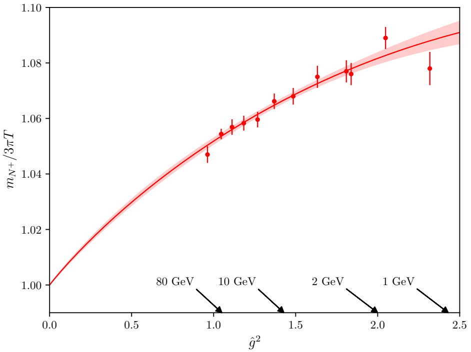

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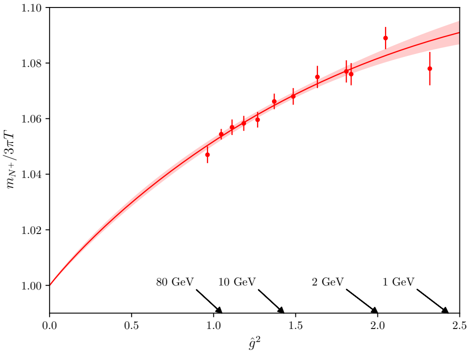
- Free theory value plus 4-8% positive deviation due to interactions
- Data fitted with a polynomial in $\hat{g}$
- Single $\sim \hat{g}^2$ correction is not enough to explain the temperature dependence down to 1 GeV



Three dimensional effective theory

- At high temperature QCD effectively behaves as a three dimesional effective theory with action [Linde (1980), Laine et al. (2005)]

$$S_{\mathrm{EQCD}} = \int d^3x \left\{ \frac{1}{2} \mathrm{Tr}\left[F_{ij}F_{ij}\right] + \mathrm{Tr}\left[(D_j A_0)(D_j A_0)\right] + m_{\mathrm{E}}^2 \mathrm{Tr}\left[A_0^2\right] \right\} + \ldots$$

with $F_{ij} = i[D_i, D_j]/g_{\mathrm{E}}$ and $D_i = \partial_i - i g_{\mathrm{E}} A_i$

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- Three dimensional gauge field A_i coupled to a massive scalar field A_0
- Low energy constant $m_{\rm E}^2$ and $g_{\rm E}^2$ matched to QCD at several orders in perturbation theory. At leading order [Kapusta (1979), Laine and Schroder (2005)]

$$m_{\rm E}^2 = g^2T^2\left(1 + \frac{N_f}{6}\right), \qquad g_{\rm E}^2 = g^2T$$

Three dimensional non-relativistic QCD

- At high temperature quarks are heavy fields with mass $\sim \pi T$. In the lowest Matsubara sector the dynamics is described by [Huang et al. (1996)]

$$S_{\mathrm{NRQCD}} = i \sum_{f=u,d,s} \int d^3x \Big\{ \bar{\chi}_f(x) \left[M - g_{\mathrm{E}} A_0 + D_3 - \frac{\nabla_\perp^2}{2\pi T} \right] \chi_f(x)$$

$$- \bar{\phi}_f(x) \left[M + g_{\mathrm{E}} A_0 + D_3 - \frac{\nabla_\perp^2}{2\pi T} \right] \phi_f(x) \Big\} + O\left(\frac{g_{\mathrm{E}}^2}{\pi T} \right)$$

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- χ and ϕ are three dimensional Weyl spinors related to the four dimensional fermion field ψ by

$$\psi_f(x_0, x) = \sqrt{T} e^{i\pi T x_0} \begin{pmatrix} \chi_f(x) \\ \phi_f(x) \end{pmatrix}$$

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- Matching coefficient M computed at next-to-leading order in perturbation theory [Laine et al. (2004)]

$$M = \pi T \left(1 + \frac{g^2}{6\pi^2} \right)$$

Equations of motion

- From the NRQCD action it is straighforward to see that the propagator for the χ field satisfies

$$\left\langle \left[M + \partial_3 - \frac{\nabla_\perp^2}{2\pi T} \right] S_\chi(x) \right\rangle = g_{\mathrm{E}} \left\langle \left[iA_3(x) + A_0(x) \right] S_\chi(x) \right\rangle - i\mathbb{1}\delta^{(3)}(x)$$

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- In perturbation theory, at next-to-leading order in g_E we write

$$S_\chi(\mathbf{r}, x_3) = S_\chi^{(0)}(\mathbf{r}, x_3) + g_\mathrm{E} S_\chi^{(1)}(\mathbf{r}, x_3) + O(g_\mathrm{E}^2) \,, \qquad \mathbf{r} = (x_1, x_2)$$

  where

$$S_\chi^{(0)}(\mathbf{r}, x_3) = -i\theta(x_3)\mathbb{1} \int \frac{d^2\mathbf{p}}{(2\pi)^2} e^{i\mathbf{p}\cdot\mathbf{r}} e^{-x_2\left(M + \frac{\mathbf{p^2}}{2\pi T}\right)}$$

$$S_\chi^{(1)}(\mathbf{r}, x_3) \simeq \int_0^{x_3} dz_3 \left[iA_3 + A_0 \right] \left(\frac{z_3}{x_3}\mathbf{r}, z_3 \right) S_\chi^{(0)}(\mathbf{r}, x_3)$$

Baryonic correlators in the effective theory

- Nucleon interpolating operator in the effective theory, by displacing the fundamental fields in the transverse directions

$$N(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; x_3) \to \epsilon^{abc} \left[\chi_u^{aT}(\mathbf{r}_1, x_3)\sigma_2 \phi_d^b(\mathbf{r}_2, x_3) + \phi_u^{aT}(\mathbf{r}_1, x_3)\sigma_2 \chi_d^b(\mathbf{r}_2, x_3) \right] \chi_d^c(\mathbf{r}_3, x_3)$$

- The corresponding two-point correlation function

$$\begin{aligned} C_{N^\pm}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; x_3) &\equiv \frac{1}{T} \operatorname{Tr} \left\langle N(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; x_3) \overline{N}(0) P_\pm \right\rangle \\ &= \mp T^2 \left\langle 2W(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; x_3) + 3W(\mathbf{r}_2, \mathbf{r}_1, \mathbf{r}_3; x_3) \right\rangle \end{aligned}$$

where the Wick contraction is

$$W(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; x_3) \equiv -i\epsilon^{abc}\epsilon^{gfe} S_\chi^{ag}(\mathbf{r}_1, x_3) S_\phi^{bf}(\mathbf{r}_2, x_3) S_\chi^{ce}(\mathbf{r}_3, x_3)$$

- $C_{N^\pm}$ is a sum of two Wick contractions which propagate independently

Equation of motion for baryonic correlators

Combine

- Equations of motion for the fundamental fields propagators $S_\chi(\mathbf{r}, x_3)$ and $S_\phi(\mathbf{r}, x_3)$ at next-to-leading order
- Large x_3 limit to extract the screening mass

To obtain the equation of motion for W. It reads at $O(g_{\mathrm{E}}^2)$

$$\left[\partial_3 - \sum_{i=1}^{3} \frac{\nabla^2_{\mathbf{r}_i}}{2\pi T} + V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\right] \langle W(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; x_3)\rangle = 0$$

which is a Schrödinger equation with potential

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = 3M + \frac{1}{2}\left[V^-(r_{12}) + V^+(r_{13}) + V^-(r_{23})\right] , \qquad r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$$

and $V^\pm(r)$ defined as [Brandt et al. (2014)]

$$V^\pm(r) \equiv \frac{4}{3}\frac{g_{\mathrm{E}}^2}{2\pi}\left[\ln\left(\frac{m_{\mathrm{E}} r}{2}\right) + \gamma_{\mathrm{E}} \pm K_0(m_{\mathrm{E}} r)\right]$$

Schrödinger equation

- The equation of motion for a two-point correlation function with nucleon interpolating operators implies the two dimensional eigenvalue problem

$$\left[-\frac{\nabla_{\mathbf{r}_1}+\nabla_{\mathbf{r}_2}+\nabla_{\mathbf{r}_3}}{2\pi T}+V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\right]\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=E\,\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$$

- The baryonic screening mass is the energy eigenvalue corresponding to the ground state, i.e. $m_{N^\pm}=\min(E)$

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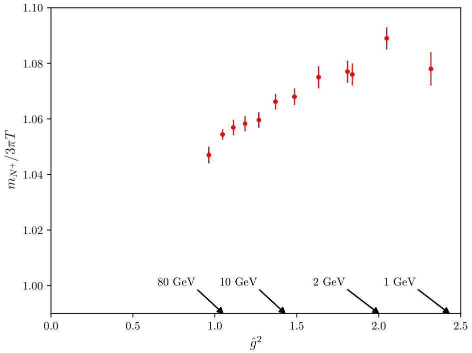
- Numerical solution found with
  - Two-dimensional hyperspherical harmonics method
  - Finite difference method

  Both providing

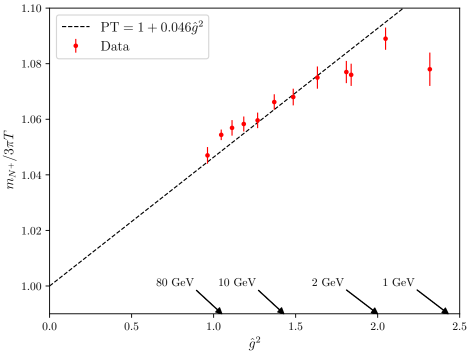
$$\boxed{E=3\pi T\left[1+0.046g^2+O(g^3)\right]}$$

Comparison with the lattice



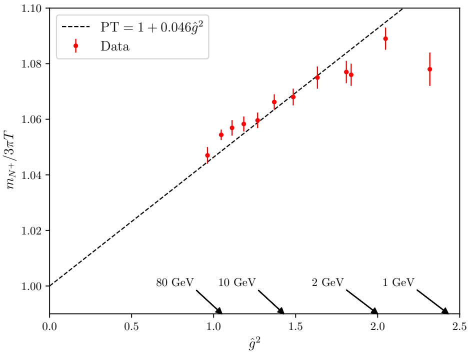
Comparison with the lattice

- Down to $T \sim 5$ GeV the difference between the perturbative expression and the non-perturbative data is within half a percent



Comparison with the lattice

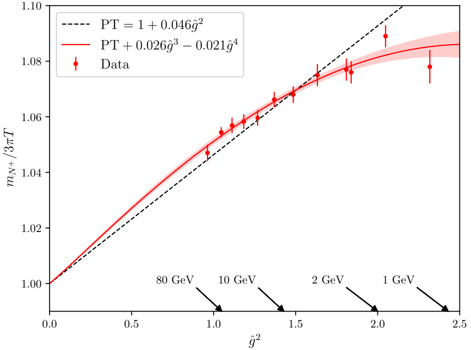
- Down to $T \sim 5$ GeV the difference between the perturbative expression and the non-perturbative data is within half a percent
- However a fast convergence of the perturbative series cannot be assumed, since the single $\hat{g}^2$ correction cannot parameterize the negative curvature of the lattice data



Final parameterization

- $m_{N^\pm}$ parameterized with a quartic polynomial in $\hat{g}$

$$\frac{m_{N^\pm}}{3\pi T} = b_0 + b_2\hat{g}^2 + b_3\hat{g}^3 + b_4\hat{g}^4$$



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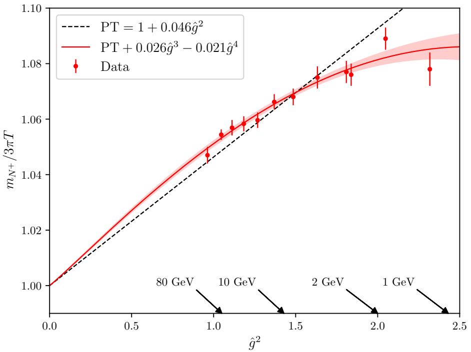

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- b_3 and b_4 with opposite signs and comparable in magnitude

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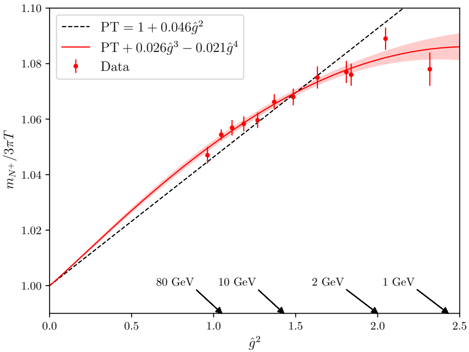

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- b_0 and b_2 compatible with the tree-level and next-to-leading order analytical values
- b_3 and b_4 with opposite signs and comparable in magnitude
- Other possible parameterizations lead to disagreement between b_2 and the next-to-leading prediction

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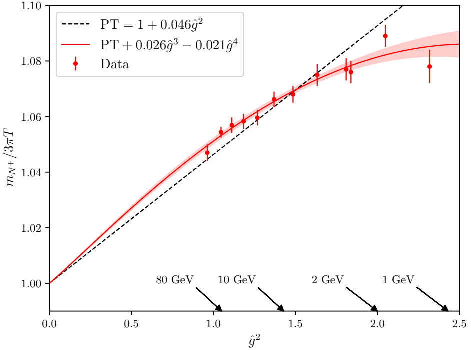

Conclusions & outlook

- We carried out the first detailed investigation of the baryonic screening masses with nucleon quantum numbers in the high temperature regime of QCD both on the lattice and in the effective theory
- If one assumes the perturbative series to be convergent the Coulomb interaction accounts for $\sim 90\%$ of the difference between the non-perturbative data and the free theory value $3\pi T$ down to $T \sim 5$ GeV
- However a single $O(g^2)$ correction is not sufficient to explain the temperature dependence of the non-perturbative data down to $T \sim 1$ GeV

- The study of spin-3/2 baryonic screening masses on the lattice is currently in progress