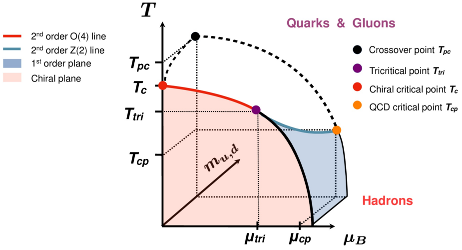
Towards a parameter-free determination of QCD critical exponents and chiral phase transition temperature

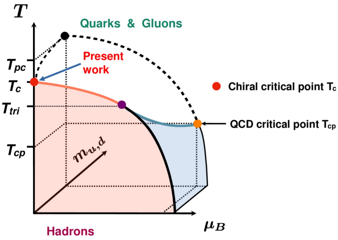
Sabarnya Mitra

Faculty of Physics, Universität Bielefeld

In collab. with Frithjof Karsch and Sipaz Sharma

Motivation : QCD phase diagram





- Aim : Locate **QCD critical point** ← **constraining** along T
- 3-dim. model analysis : $T_{cp} < T_{tri} < T_c < T_{pc}$.

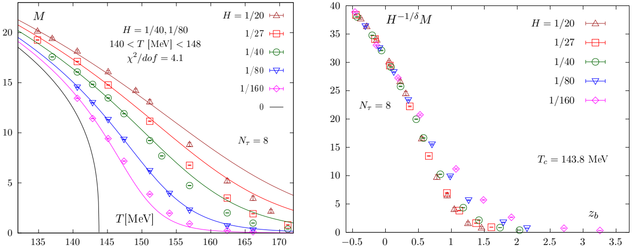
From Lattice QCD, $T_{pc} = 156.5 \pm 1.5$ MeV [HotQCD, 1812.08235],

$= 158.0 \pm 0.6$ MeV [Borsanyi et. al. 2002.02821]

$T_c = 132^{+3}_{-6}$ MeV [HotQCD, 1903.04801],

$= 134^{+6}_{-4}$ MeV [Kotov et. al. 2105.09842]

Previous work



In a previous work [Ding et. al. 2403.09390] ← We

- used critical exponents, scaling functions of $O(2)$ universality class
- adjusted fit range to $\boldsymbol{T_c \pm 3\,\%}$ → obtain fits based **only** on scaling ansatz (no regular terms)
- determined $\boldsymbol{T_c}$, non-universal parameters $\boldsymbol{t_0, h_0}$ in the scaling ansatz **from fits** ⇒ good scaling behavior **only within** fit range. So ⋯

This work: Aim

In this present work (topic of this talk),

- we try to find T_c in a way free of fits. This is done by estimating T_c:

1. **without using at all**, **any** $O(2)$ or $O(4)$ universality class **fitting parameter** or **other properties** of this class, and
2. with **no knowledge** of **critical exponents** β, δ at **input level**

- Thus $\rightarrow$ a **fit-independent**, "**parameter-free**" approach
- For this, work with an **improved** order parameter $\rightarrow$ try to **control**, reduce **regular** contribution of the **unimproved** one

A quick review $\cdots$

Universality class, critical exponents and scaling

- Chiral Ph. tr. $\rightarrow$ $\mathbf{2^{nd}}$ *order* [Pisarski, Wilczek, PRD 29, 338(R)]

  $\rightarrow SU(2)_L \times SU(2)_R\ [\textbf{QGP}] \xrightarrow[\textbf{breaking}]{\textbf{spontaneous}} SU(2)_V\ [\textbf{hadrons}]$

- Order parameter $\rightarrow$ light quark chiral condensate $\langle\bar{\psi}\psi\rangle_\ell$

  source/symmetry-breaking field $\rightarrow$ light quark mass m_ℓ, where

- This ph. tr. $\in$ $\boldsymbol{O(4)}$ univ. class $\xleftarrow[\text{by}]{\text{indicated}}$ $O(4)$ crit. expts.

  (2 independent crit. expts. : $\boldsymbol{\beta}, \boldsymbol{\delta}$)

- Here, estimate T_c and δ $\left[\right.$ scl. rel. : $\boldsymbol{\langle\bar{\psi}\psi\rangle_\ell \sim m_\ell^{1/\delta}}$ at $T = T_c$ $\left.\right]$

- Our working S_{QCD} $\rightarrow$ **HISQ** action preserves $\boldsymbol{O(2)} \subset O(4)$

- Small **1%** diff. $\left[\delta_{(O(4))} \approx 1.01\, \delta_{(O(2))}\right]$ [Karsch et. al. 2304.01710]

In scaling region

- Close to T_c in scaling region (SR), we can write

$$\langle\bar{\psi}\psi\rangle_\ell = H^{\frac{1}{\delta}} f_G(z), \qquad \chi_\ell = \frac{\partial\langle\bar{\psi}\psi\rangle_\ell}{\partial m_\ell} = \frac{1}{\delta} H^{\frac{1}{\delta}-1} f_\chi(z) \tag{1}$$

- Here, $\langle\bar{\psi}\psi\rangle_\ell, \chi_\ell, m_\ell \rightarrow$ **dimensionless**, $H = m_\ell/m_s$
- $f_G(z), f_\chi(z) \rightarrow$ **scaling functions**. To some scaling parameters,
- Scaling variable $z = t\, H^{-1/\beta\,\delta}$, Reduced temp. $t = \frac{T}{T_c} - 1$
- Eqn.(1) $\rightarrow$ **only singular** cont. as *singular* $\gg$ *regular* in SR
- However, order parameter $\langle\bar{\psi}\psi\rangle_\ell$ has **divergences** (problems).

  So $\cdots$

Improved order parameter

We work with **improved** order parameter M, where

$$M = M_\ell - H\,\chi_\ell \quad \text{where} \tag{2}$$

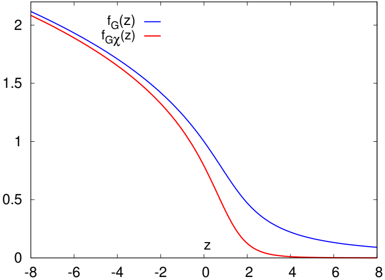
$$M_\ell = \frac{m_s}{f_K^4}\,\langle\bar{\psi}\psi\rangle_\ell\,, \qquad \chi_\ell = m_s\,\frac{\partial M_\ell}{\partial m_\ell} \quad \text{which makes}$$

$$M = H^{\frac{1}{\delta}}\boldsymbol{f_{G\chi}(z)} \quad \text{with} \qquad f_{G\chi}(z) = f_G(z) - f_\chi(z) \tag{3}$$

$f_K \rightarrow$ kaon decay constant. "Improved" features of this M:

- **No** additive divergences $\mathcal{O}(a^{-2}) \rightarrow$ **well-def.** in **continuum limit**
- **Mult. renorm.** by $m_s \rightarrow$ **no** log div. $\rightarrow$ **well-def.** in **chiral limit**
- **No** $\mathcal{O}(H)$ regular terms in $M \Rightarrow$ **reduced** $(M_{regular} < M_{\ell,regular})$
- Also **directly related** to scl. func. $\boldsymbol{f_{G\chi}(z)}$. Advantages :

Plot of the scaling function



- Inflection point of $f_{G_\chi}(z)$ **closer** to $\underline{z = 0}$ than $f_G(z)$
- $z_{t,G_\chi} = 0.629\,(10),\ z_{t,G} = 0.7991\,(96)$ [Ding. et. al., 2403.09390]
- So, **weaker** $\underline{m_\ell \text{ dependence of } T_c}$ while working with $f_{G_\chi}(z) \Rightarrow$
- **Improved** ord. par. $M \xleftarrow[\textbf{than}]{\textbf{better}}$ **unimproved** M_ℓ, for finding T_c. Using this,

The new observable

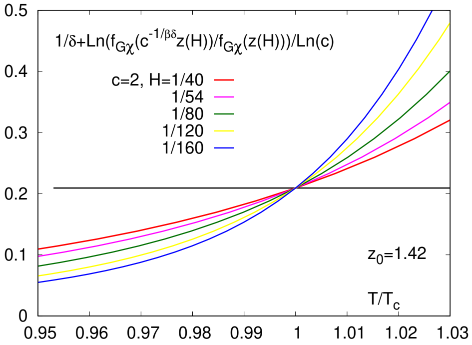
Propose a new observable $\boldsymbol{B(T, H_1, H_2)}$ in this work :

$$B(T, H_1, H_2) = \frac{\ln\left[\frac{M_\ell^{im}(H_2)}{M_\ell^{im}(H_1)}\right]}{\ln\left[\frac{H_2}{H_1}\right]} \qquad (\text{with } \mathrm{H}_2 = c\,\mathrm{H}_1) \tag{4}$$

$$= \frac{1}{\delta} + \frac{\ln\left[f_{G\chi}(z(H_2))/f_{G\chi}(z(H_1))\right]}{\ln(c)}, \quad \text{in scaling region} \tag{5}$$

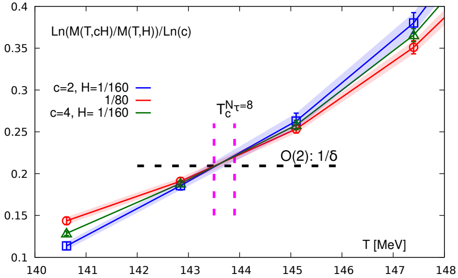
- This gives $B(\boldsymbol{T_c}, H_1, H_2) = 1/\boldsymbol{\delta}$, which ensures
- A **unique** **intersection** point for **different** H_1, H_2 lines. Whose
- $\boldsymbol{x}$-coordinate $\rightarrow \boldsymbol{T_c}$, $\boldsymbol{y}$-coordinate $\rightarrow 1/\boldsymbol{\delta}$ (in B vs T plane)
- How we expect it to look like:

Plot of B vs T



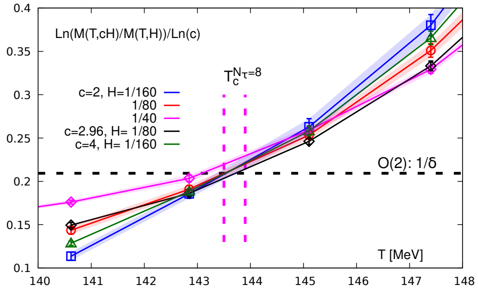
- **All** lines cross **through** a **unique point at** $T = T_c$
- **Ordering** of lines **reverses exactly** on **either sides** of $T = T_c$
- These → "**parameter-free**" features (holds also for other z_0, β, δ)
- Great, but how do LQCD results look like?

Direct Lattice QCD results



- With $N_\tau = \mathbf{8}$ lattices, working $H^{-1} = \mathbf{20}, \mathbf{27}, \mathbf{40}, \mathbf{80}, \mathbf{160}$ (used before)
- We find for $\boldsymbol{H^{-1}} \geq \mathbf{40}$, intersection pt. **within** T_c range [HotQCD, 1903.04801] and $\underline{y\text{-coordinate} \approx O(2) : 1/\delta}$
- And $\mathcal{O}(\boldsymbol{H^3})$ regular cont. **negligible** while **close to $\boldsymbol{T_c}$** for these masses. However, $\cdots$

Larger masses



- Lines involving $H^{-1} = 20$ and **27** cut lines of **lighter** m_ℓ **not** in a **unique** point. For these $m_\ell = m_s/20, m_s/27$,
- **Systematics not clear** at present $\Rightarrow$ WORK GOING ON; generate **more statistics**, add data at **additional** T-values $\rightarrow$ control effect of regular terms in the **vicinity** of T_c

Conclusions

- A new divergence-free estimator of $\underline{T_c \text{ and } \delta}$, well-defined for continuum and chiral limits with reduced regular contributions
- Within scaling region, one obtains a unique intersection point for lighter quark masses, also from lattice QCD
- This point lies within predicted T_c with results consistent with δ of $O(2)$ univ. class.

Outlook:

- Understand systematics for higher quark masses
- Check finite volume effects in future
- Find observable to resolve $O(2)$ and $O(4)$ critical exponents
- Proceed towards continuum limit.

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Outlook:

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- Find observable to resolve $O(2)$ and $O(4)$ critical exponents
- Proceed towards continuum limit. **THANK YOU ALL!**

Backup slides

$$t = \frac{T - T_c}{t_0\, T_c} = \frac{\tau}{t_0} \qquad h = \frac{m_\ell}{h_0\, m_s} = \frac{H}{h_0} \qquad \ell = \frac{\ell_0}{L}$$

where t_0, h_0, ℓ_0 are non-universal scaling parameters.

$$z = z_0\, z_b \;, \qquad z_L = z_{L,0}\, z_{L,b} \qquad \text{where}$$

$$z_0 = \frac{h_0^{1/\beta\delta}}{t_0} \;, \qquad z_{L,0} = h_0^{\nu/\beta\delta}\, \ell_0 \qquad \text{(fitting param.)}$$

$$z_b = \tau\, H^{-1/\beta\delta} \;, \qquad z_{L,b} = \frac{1}{L}\, H^{-\nu/\beta\delta}$$

Backup slides continued

$$M_\ell = h^{\frac{1}{\delta}}\, f_G(z) + f_{sub}\,(T, H)$$
$$\chi_\ell = h_0^{-1}\, h^{\frac{1}{\delta}-1}\, f_\chi(z) + g_{sub}\,(T, H)$$
$$M = h^{\frac{1}{\delta}}\, f_{G\chi}(z) + p_{sub}\,(T, H)$$