The Equation of State of QCD up to the Electro-Weak scale

part 2

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Overview

1. **Thermal QCD in a moving frame**
2. **Strategy**
3. **Numerical results**
4. **Conclusions & outlook**

Thermal QCD in a moving frame

- QCD in a box $L_0 \times L^3$ in a moving frame:

$$Z(L_0, L, \boldsymbol{\xi}) = \mathrm{Tr}\left\{ e^{-L_0\left(\hat{H} - i\boldsymbol{\xi}\cdot\hat{\mathbf{P}}\right)} \right\}$$

  $\boldsymbol{\xi} \rightarrow$ euclidean boost parameter, 3d vector [Giusti, Meyer, JHEP 01 (2013), 140]

- Shifted boundary conditions

$$A_\mu(x_0 + L_0, \boldsymbol{x}) = A_\mu(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})$$

$$\psi(x_0 + L_0, \boldsymbol{x}) = -\psi(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})$$

$$\bar{\psi}(x_0 + L_0, \boldsymbol{x}) = -\bar{\psi}(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})$$

- Temperature range $1 - 100$ GeV: Scale setting through a NP-renormalized coupling $\bar{g}(\mu)$ [Leonardo Giusti's talk]

Strategy

Definition of free-energy density:

$$f(L_0, L, \boldsymbol{\xi}) = -\frac{1}{L_0 L^3} \ln Z(L_0, L, \boldsymbol{\xi})$$

Computation of the entropy density:

$$\frac{s}{T^3} = \frac{(1+\boldsymbol{\xi}^2)}{\xi_k} \frac{1}{T^4} \frac{\partial f_{\boldsymbol{\xi}}}{\partial \xi_k}\ , \qquad k = 1, 2, 3$$

[Dalla Brida, Giusti, Pepe, JHEP **04** (2020), 043]

Split in two contributions:

$$\frac{s}{T^3} = \frac{(1+\boldsymbol{\xi}^2)}{\xi_k} \frac{1}{T^4} \left\{ \frac{\partial}{\partial \xi_k} \left(f_{\boldsymbol{\xi}} - f_{\boldsymbol{\xi}}^{\infty} \right) + \frac{\partial}{\partial \xi_k} f_{\boldsymbol{\xi}}^{\infty} \right\}$$

where $f_{\boldsymbol{\xi}}^{\infty}$ free-energy of shifted QCD with quarks at infinite mass

Mass integral

$$\frac{s}{T^3} = \frac{(1+\boldsymbol{\xi}^2)}{\xi_k}\frac{1}{T^4}\left\{\frac{\partial}{\partial \xi_k}\left(f_{\boldsymbol{\xi}} - f_{\boldsymbol{\xi}}^{\infty}\right) + \frac{\partial}{\partial \xi_k} f_{\boldsymbol{\xi}}^{\infty}\right\}$$

Write the difference as an integral in subtracted mass $m_q = m_0 - m_{cr}$:

$$\frac{\partial}{\partial \xi_k}\left(f_{\boldsymbol{\xi}} - f_{\boldsymbol{\xi}}^{\infty}\right) = -\frac{\partial}{\partial \xi_k}\int_0^{\infty} dm_q\, \frac{\partial f_{\boldsymbol{\xi}}}{\partial m_q} = -\int_0^{\infty} dm_q\, \frac{\partial}{\partial \xi_k}\langle\bar{\psi}\psi\rangle_{\boldsymbol{\xi}}$$

On the lattice at given L_0/a and g_0:

$$\frac{\Delta}{\Delta \xi_k}\left(f_{\boldsymbol{\xi}} - f_{\boldsymbol{\xi}}^{\infty}\right) = -\int_0^{\infty} dm_q\, \frac{1}{\xi_+ - \xi_-}\left[\langle\bar{\psi}\psi\rangle_{\boldsymbol{\xi}_+} - \langle\bar{\psi}\psi\rangle_{\boldsymbol{\xi}_-}\right]$$

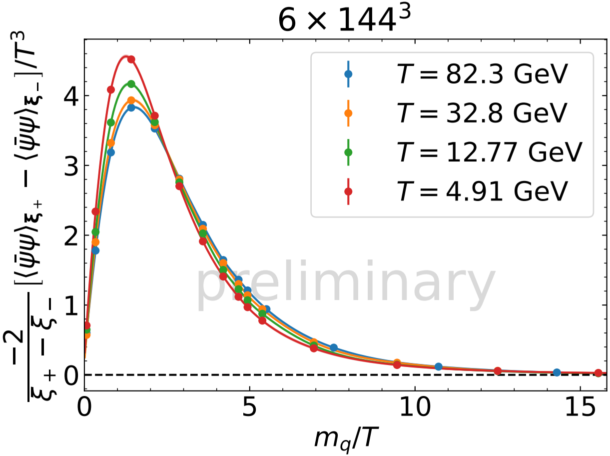
We choose $\boldsymbol{\xi} = (1,0,0)$, and $\boldsymbol{\xi}_{\pm} = (1 \pm \frac{a}{L_0}, 0, 0)$ [Giusti, Pepe, Phys. Rev. D **91** (2015), 114504]

Mass integral

- $N_f = 3$
  $\mathcal{O}(a)$-improved Wilson fermions
- Gauss quadratures:

interval	points
$0 \leq m_q/T \leq 5$	10
$5 \leq m_q/T \leq 20$	7
$m_q/T \geq 20$	3 (in κ)

- Relative error on integral $\sim 0.5\%$



g_0^2 integral

$$\frac{s}{T^3} = \frac{(1+\boldsymbol{\xi}^2)}{\xi_k}\frac{1}{T^4}\left\{\frac{\partial}{\partial \xi_k}\left(f_{\boldsymbol{\xi}} - f_{\boldsymbol{\xi}}^{\infty}\right) + \frac{\partial}{\partial \xi_k} f_{\boldsymbol{\xi}}^{\infty}\right\}$$

where $f_{\boldsymbol{\xi}}^{\infty} = f_{\boldsymbol{\xi}}^{\mathrm{YM}}$. We sample at many bare couplings

$$\frac{d}{dg_0^2}\left(\frac{\partial}{\partial \xi_k} f_{\boldsymbol{\xi}}^{\mathrm{YM}}\right) = -\frac{1}{g_0^2}\frac{\partial}{\partial \xi_k}\langle S^G\rangle_{\boldsymbol{\xi}}$$

and then integrate up to the desired value of g_0^2:

$$\frac{\partial}{\partial \xi_k} f_{\boldsymbol{\xi}}^{\infty} = \left(\frac{\partial}{\partial \xi_k} f_{\boldsymbol{\xi}}^{\mathrm{YM}}\right)_{\mathrm{free}} - \int_0^{g_0^2} du\,\frac{1}{u}\,\frac{\partial\langle S^G\rangle_{\boldsymbol{\xi}}}{\partial \xi_k}\bigg|_u$$

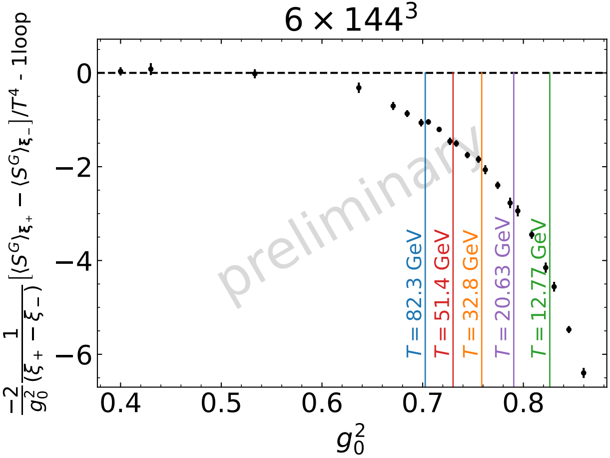
[Giusti, Pepe, Phys. Rev. D **91** (2015), 114504]

g_0^2 integral

- Integration rules:

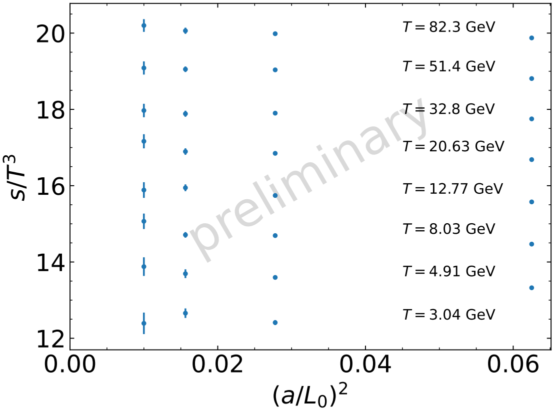
interval	points
$0 \leq g_0^2 \leq 0.4$	1 (trapz)
$0.4 \leq g_0^2 \leq 0.67$	3 (Gauss)
$T_i \to T_{i+1}$	3 (Gauss)

- Relative error on integral $\sim 0.5\%$



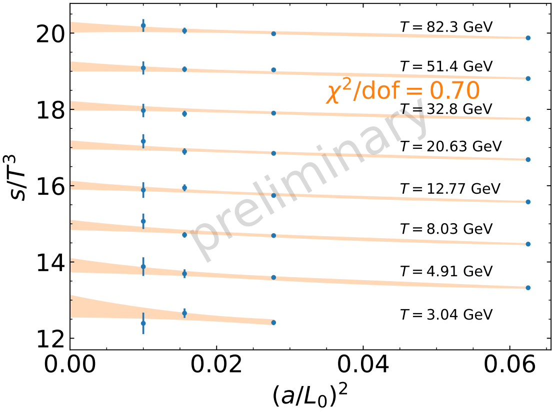
Continuum limit

- $L_0/a = 4, 6, 8, 10$ and $L/a = 144$
- 1loop improvement



Continuum limit

- $L_0/a = 4, 6, 8, 10$ and $L/a = 144$
- 1loop improvement
- Global fit of cutoff effects

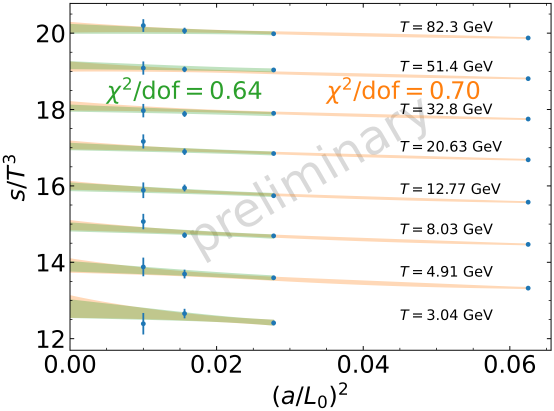


$$s(g_i, a/L_0)/T^3 = s_i + \left(\frac{a}{L_0}\right)^2 \left(s_{23}\, g_i^3 + s_{24}\, g_i^4\right) + \left(\frac{a}{L_0}\right)^3 \left(s_{33}\, g_i^3 + s_{34}\, g_i^4\right)$$

where $g_i = g(T_i)$

Continuum limit

- $L_0/a = 4, 6, 8, 10$ and $L/a = 144$
- 1loop improvement
- Global fit of cutoff effects

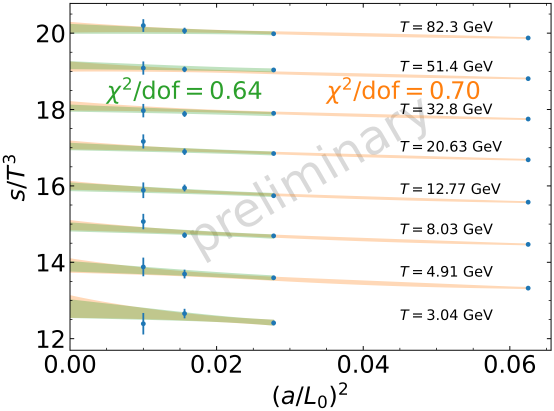


$$s(g_i, a/L_0)/T^3 = s_i + \left(\frac{a}{L_0}\right)^2 \left(s_{23}\, g_i^3 + s_{24}\, g_i^4 + s_{25}\, g_i^5\right)$$

where $g_i = g(T_i)$

Continuum limit

- $L_0/a = 4, 6, 8, 10$ and $L/a = 144$
- 1loop improvement
- Global fit of cutoff effects
- No impact from $\gamma \in [-1, 1]$
- Final accuracy $\lesssim 1\%$



$$s(g_i, a/L_0)/T^3 = s_i + [\alpha(a^{-1})]^\gamma \left(\frac{a}{L_0}\right)^2 \left(s_{23}\, g_i^3 + s_{24}\, g_i^4 + s_{25}\, g_i^5\right)$$

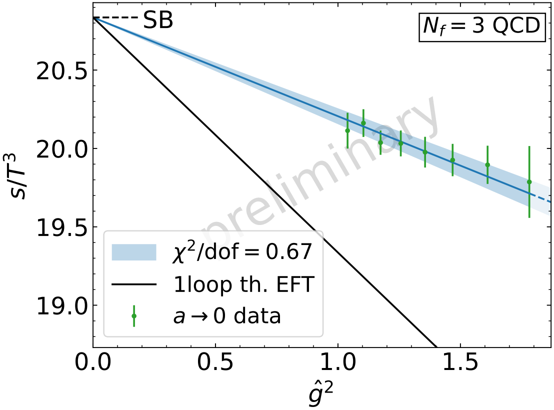
where $g_i = g(T_i)$

$N_f = 3$ QCD entropy density at high temperature

- $\hat{g}(T) \equiv g_{\overline{MS}}^{5\text{loop}}(2\pi T)$
- Fit of continuum values

$$s/T^3 = s_0 + s_1\,\hat{g}^2$$

- s_0 compatible with SB
- Fit with $s_0 = s_{SB}/T^3$



Conclusions & outlook

- QGP thermodynamics at high temperature made accessible by thermal QCD in a moving frame
- First computation of the entropy density of $N_f = 3$ QCD in the temperature range $1 - 100$ GeV
- Continuum extrapolations with 4 lattice spacings, final accuracy $\sim 1\%$

$\sim \circ \sim$

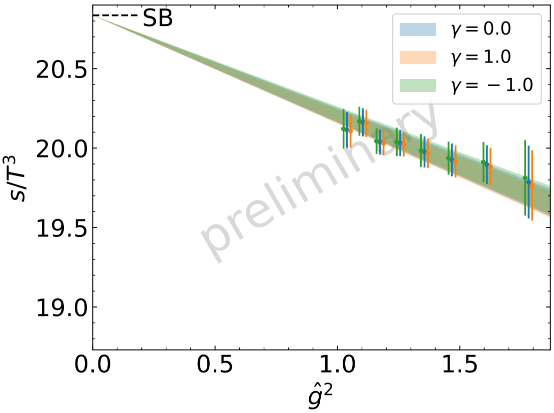
Non-perturbative renormalization of lattice QCD Energy-Momentum tensor

$$s/T^3 = -\frac{1+\boldsymbol{\xi}^2}{\xi_k} \langle T_{0k}^{R,\{6\}} \rangle_{\boldsymbol{\xi}} / T^4$$

$$\langle T_{\mu\nu}^{R,\{i\}} \rangle_{\boldsymbol{\xi}} = Z_G^{\{i\}} \langle T_{\mu\nu}^{G,\{i\}} \rangle_{\boldsymbol{\xi}} + Z_F^{\{i\}} \langle T_{\mu\nu}^{F,\{i\}} \rangle_{\boldsymbol{\xi}} \quad i = 3, 6$$

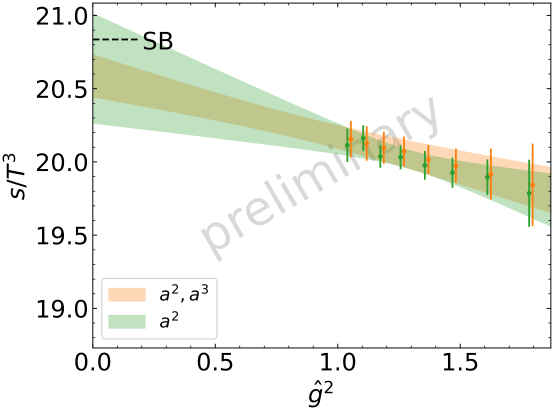
[Dalla Brida, Giusti, Pepe, JHEP **04** (2020), 043]

Log corrections to continuum limit



$$s(g_i, a/L_0)/T^3 = s_i + [\alpha(a^{-1})]^{\gamma} \left(\frac{a}{L_0}\right)^2 \left(s_{23}\, g_i^3 + s_{24}\, g_i^4 + s_{25}\, g_i^5\right)$$

Entropy vs $\hat{g}^2$



Fit with s_0, s_1 free parameters

$$s(T)/T^3 = s_0 + s_1 \hat{g}^2$$