The Equation of State of QCD up to the Electro-Weak scale

(part 1)

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PLAN OF THE TALK

- Introduction
- QCD EoS: two approaches with shifted b.c.
- The numerical results
- Conclusions

Introduction

Budapest-Wuppertal Collab., 2010, 2014
A. Bazavov, P. Petreczky, J. Weber, PRD 2018
HOT-QCD Collab., PRD 2014
WHOT-QCD Collab., PRD 2017

- QCD EoS: up to T~ 1-2 GeV using staggered quarks

  up to T~ 500 MeV using Wilson quarks

QCD in high-T regime:
unexplored

algorithmic challenges:
integral method

G. Boyd et al.
NPB 469 (1996) 419

↓

QCD in a moving
reference frame

L. Giusti and H. Meyer,
PRL 2011, JHEP 2011 and 2013

technical issues:
Lines of Constant Physics

↓

(L. Giusti 's talk)

use $g_R^2(\mu)$ to determine $a \leftrightarrow g_0$
at temperature $T \leftrightarrow \mu$

Alpha Coll. 2016, 2017, 2018
M. Bruno et al. PRD 2017

- Equation of State of SU(3) YM with high accuracy in the range 0 – 230 T_c

L. Giusti and M. Pepe, PRL 2014,
PRD 2015, PLB 2017

- Computation of mesonic and baryonic screening masses in QCD for T in 1-160 GeV

(D. Laudicina 's talk)

M. Dalla Brida, et al. JHEP 04 (2022) 034
L. Giusti, et al. JHEP 06 (2024) 205, PLB 855 (2024) 138799

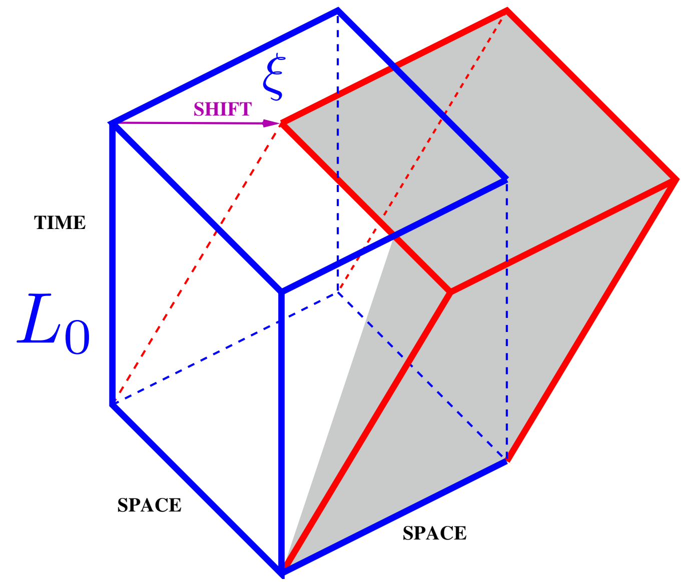
Thermal QCD in a moving frame

- Thermal theory in a moving reference frame corresponds to introducing a shift ξ when closing the b.c. along the temporal direction

$$U_\mu(x_0 + L_0, \mathbf{x}) = U_\mu(x_0, \mathbf{x} - L_0\xi)$$

$$\psi(x_0 + L_0, \mathbf{x}) = -\psi(x_0, \mathbf{x} - L_0\xi)$$

$$\bar{\psi}(x_0 + L_0, \mathbf{x}) = -\bar{\psi}(x_0, \mathbf{x} - L_0\xi)$$



By Lorentz invariance the free-energy is given by

$$f\left(L_0\sqrt{1+\xi^2}\right) = -\lim_{V\to\infty} \frac{1}{L_0 V} \log Z(L_0, V, \vec{\xi})$$

temperature of the system $\quad T = \dfrac{1}{L_0\sqrt{1+\xi^2}}$

- QCD EoS: matrix elements of the energy-momentum tensor $T_{\mu\nu}$

- Entropy density is the primary observable

$$\frac{s}{T^3} = -\frac{L_0^4(1+\xi^2)^3}{\xi_k}\langle T_{0k}\rangle_\xi$$

Thermal QCD in a moving frame on the lattice

- Lattice breaks continuum translation invariance

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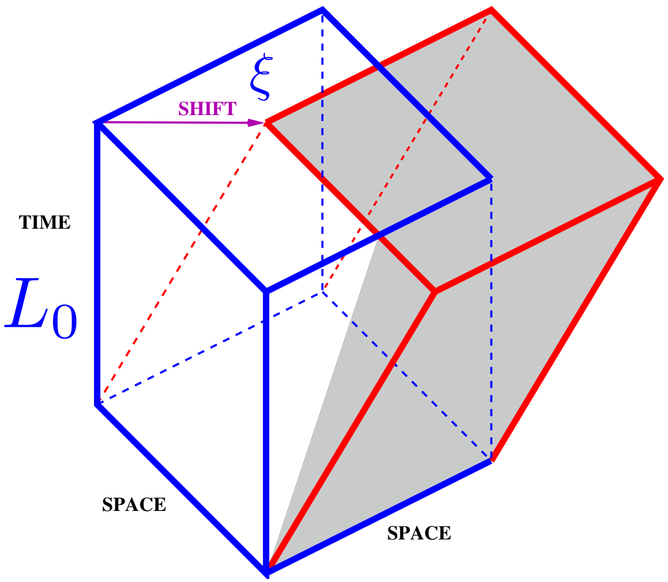


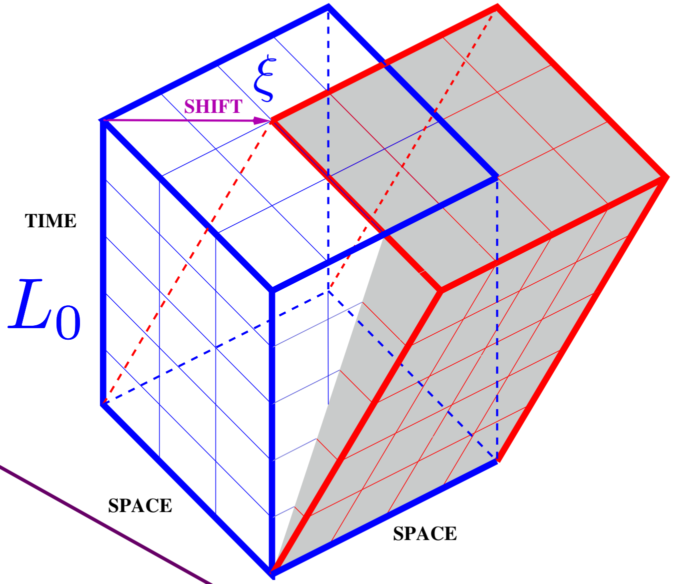
Thermal QCD in a moving frame on the lattice

- Lattice breaks continuum translation invariance

$$\frac{s}{T^3} = -\frac{L_0^4(1+\xi^2)^3}{\xi_k}\langle T^R_{0k}\rangle_\xi$$

$$T^R_{0k} = Z_G(g_0^2)\, T^G_{0k} + Z_F(g_0^2)\, T^F_{0k}$$

$$\langle T^R_{0k}\rangle_\xi = -\frac{\partial}{\partial \xi_k} f(\xi) = \frac{1}{L_0 V}\frac{\partial}{\partial \xi_k}\log \mathcal{Z}(L_0,\xi)$$



two possible strategies

M. Dalla Brida, L. Giusti, M. Pepe, JHEP 2020

- compute the renormalization constants: Z_G and Z_F (UV feature, not very large V)

- compute the expectation values of T^G_{0k} and T^F_{0k}

- compute directly the expectation value of T^R_{0k} as

Thermal QCD in a moving frame on the lattice

- Lattice breaks continuum translation invariance

$$\frac{s}{T^3} = -\frac{L_0^4(1+\xi^2)^3}{\xi_k}\langle T_{0k}^R\rangle_\xi$$

$$T_{0k}^R = Z_G(g_0^2)\, T_{0k}^G + Z_F(g_0^2)\, T_{0k}^F$$

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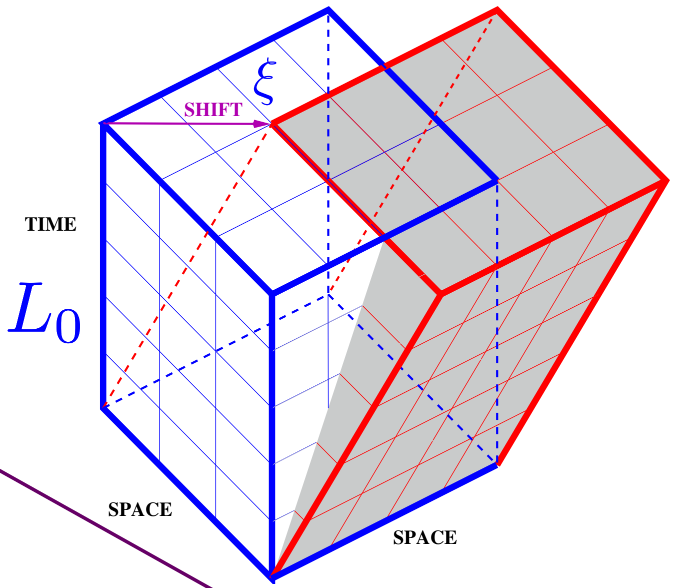

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M. Pepe, JHEP 2020

- compute the renormalization constants: Z_G and Z_F (UV feature, not very large V)

- compute the expectation values of T^G_{0k} and T^F_{0k}

we followed this way for SU(3) Yang-Mills theory

- compute directly the expectation value of T^R_{0k} as

- computing the renormalization constants in QCD requires quite some computational resources

The derivative of the free-energy density

$$\frac{\partial}{\partial \xi_k} f(\xi) = -\frac{1}{L_0 V} \frac{\partial}{\partial \xi_k} \log \mathcal{Z}(L_0, \xi) \to -\frac{1}{L_0 V} \frac{\log \mathcal{Z}(\xi_+) - \log \mathcal{Z}(\xi_-)}{\xi_+ - \xi_-} = -\frac{1}{L_0 V \Delta\xi} \log \frac{\mathcal{Z}(\xi_+)}{\mathcal{Z}(\xi_-)}$$

- ratio of partition functions is numerically challenging: poor overlap

$$\log \frac{\mathcal{Z}(\xi_+)}{\mathcal{Z}(\xi_-)} = \int_0^{g_0^2} \frac{\partial}{\partial \bar{g}_0^2} \log \frac{\mathcal{Z}(\bar{g}_0^2, \xi_+)}{\mathcal{Z}(\bar{g}_0^2, \xi_-)} \, d\bar{g}_0^2 \quad + \text{SB}$$

action

$S = S_G + S_F$

$$\int_0^{g_0^2} \frac{\partial}{\partial \bar{g}_0^2} \log \mathcal{Z}(\bar{g}_0^2, \xi_+) \, d\bar{g}_0^2 - \int_0^{g_0^2} \frac{\partial}{\partial \bar{g}_0^2} \log \mathcal{Z}(\bar{g}_0^2, \xi_-) \, d\bar{g}_0^2 = -\int_0^{g_0^2} \left[\Delta_\xi \langle \frac{\partial S_G}{\partial \bar{g}_0^2} \rangle + \Delta_\xi \langle \frac{\partial S_F}{\partial \bar{g}_0^2} \rangle \right] d\bar{g}_0^2$$

- we need large statistics: difficult in QCD

$$\log \frac{\mathcal{Z}(\xi_+)}{\mathcal{Z}(\xi_-)} = \log \frac{\mathcal{Z}(m_q, \xi_+)}{\mathcal{Z}(\infty, \xi_+)} - \log \frac{\mathcal{Z}(m_q, \xi_-)}{\mathcal{Z}(\infty, \xi_-)} + \log \frac{\mathcal{Z}(\infty, \xi_+)}{\mathcal{Z}(\infty, \xi_-)} =$$

$$= \int_\infty^{m_q} \frac{\partial}{\partial \bar{m}_q} \log \mathcal{Z}(\bar{m}_q, \xi_+) \, d\bar{m}_q - \int_\infty^{m_q} \frac{\partial}{\partial \bar{m}_q} \log \mathcal{Z}(\bar{m}_q, \xi_-) \, d\bar{m}_q - \int_0^{g_0^2} \left[\Delta_\xi \langle \frac{\partial S_G}{\partial \bar{g}_0^2} \rangle \right] d\bar{g}_0^2 \quad + \text{SB}$$

Monte Carlo simulations in QCD

Monte Carlo simulations in Yang-Mills

- the computation can be performed with high precision with moderate effort

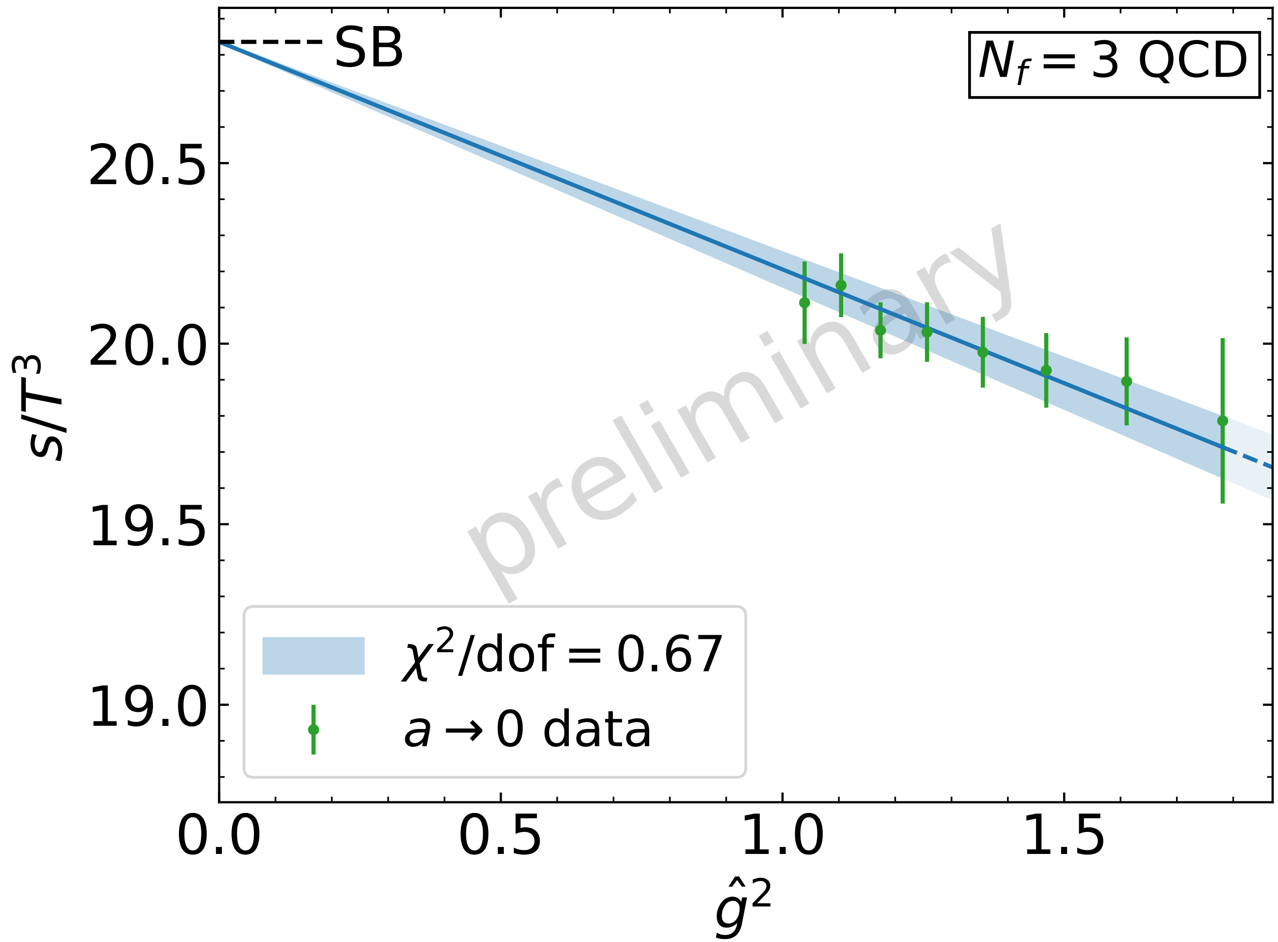
The numerical study

- QCD on the lattice with $N_f = 3$ quarks in the chiral limit
- reduced lattice artifacts: $O(a)$ - improved Wilson fermions
- continuum limit extrapolation: $L_0/a = 4, 6, 8, 10$
- large spatial volumes to have finite volume effects under control (checked explicitly): $LT \sim 10 - 25$
- shifted boundary conditions $\xi = (1, 0, 0)$
- 8 values of the temperature in the range $3 - 82$ GeV

T	T(GeV)
T_1	82.3(2.8)
T_2	51.4(1.7)
T_3	32.8(1.0)
T_4	20.63(63)
T_5	12.77(37)
T_6	8.03(22)
T_7	4.91(13)
T_8	3.040(78)

- we are adding one (two) more temperature: 165 (and 123) GeV

The entropy density vs T



$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} + \frac{4}{9\pi^2} \ln \left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} \right)$$

$$\Lambda_{\overline{\text{MS}}} = 341 \text{ MeV}$$

- Non-perturbative view: $\hat{g}^2(T)$ is just a function of the temperature chosen to simplify the comparison with perturbation theory (we use definition at 5 loops)

- The points result from the continuum limit extrapolation of 4 values of the lattice spacing for each physical temperature **(M. Bresciani 's talk)**

- We compute the EoS with a final accuracy $\lesssim 1\%$ covering a range of about 2 orders of magnitude in the temperature

Comparison with Perturbation Theory

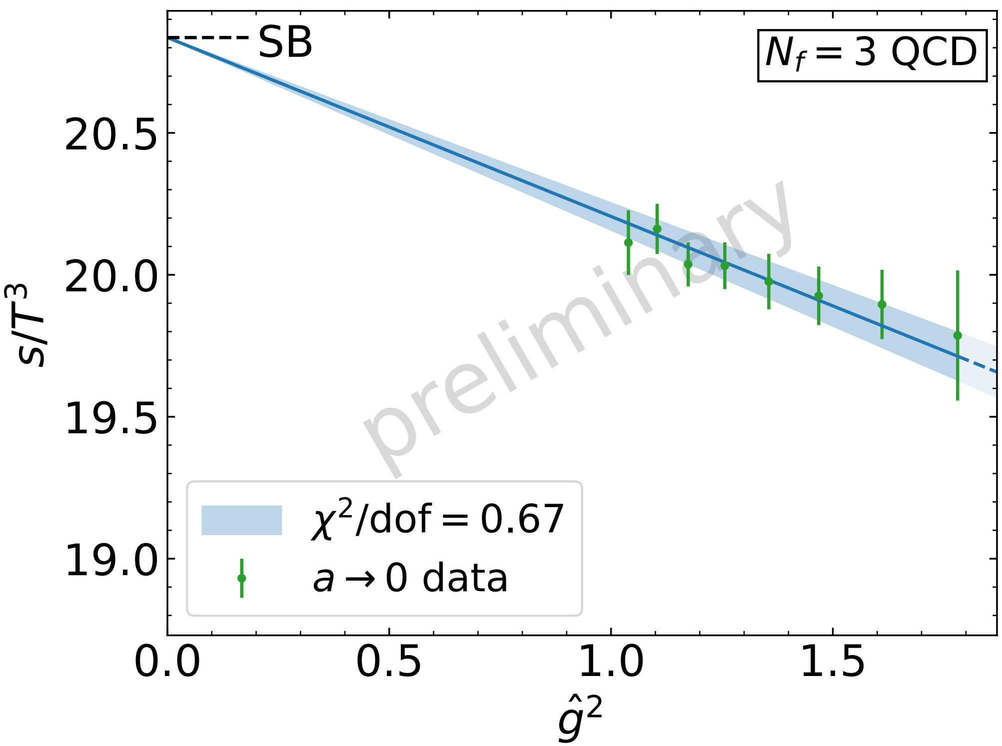
- When the temperature of the QCD plasma is high, T is the leading energy scale
  asymptotic freedom: $g^2(T)$ is small ➡ perturbative approach

  E. Braaten, L. Yaffe, L. McLerran, R. Pisarski, S. Huang, M. Lissia, J._P. Blaziot, E. Iancu, M. Laine, …

- The PT relys on the hierarchy of 3 energy scales: $\pi T \gg g(T)\,T \gg g^2(T)\,T$ (hard, soft, ultrasoft)
  observables expressed as series expansions in $\frac{g(T)}{\pi}$
- Integrating out the hard modes, the system is static ➡ effective 3d theory;

! 3d Yang-Mills theory: confining theory, IR problems

A. Linde, RPP 1979, PLB 1980



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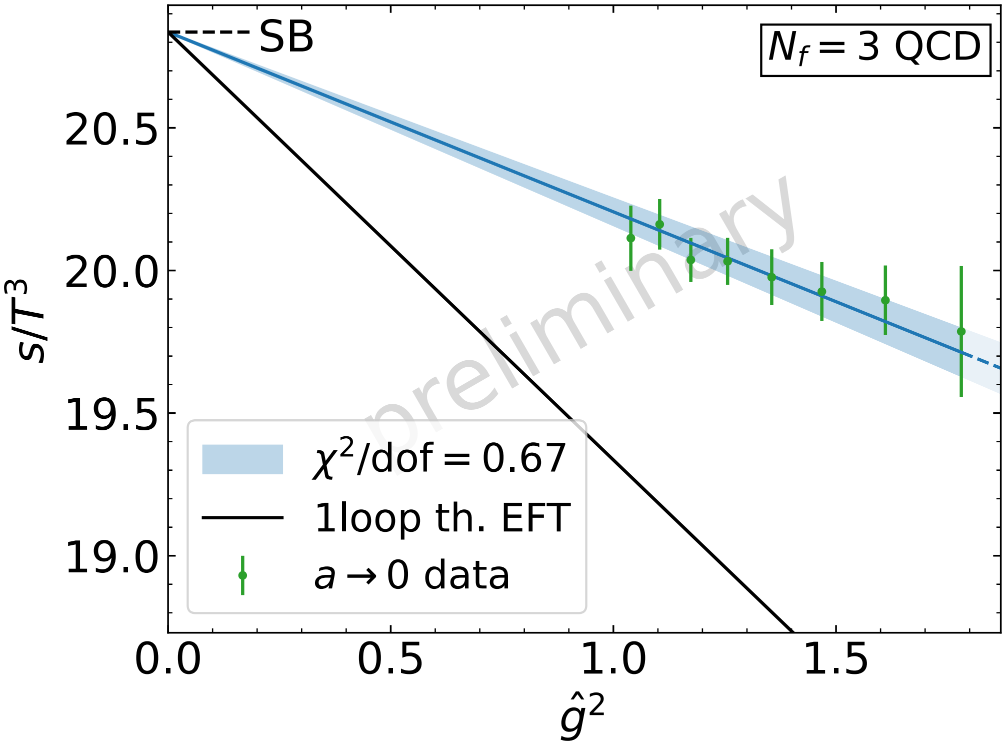


- 1-loop PT: unsatisfactory
- PT: consider higher orders

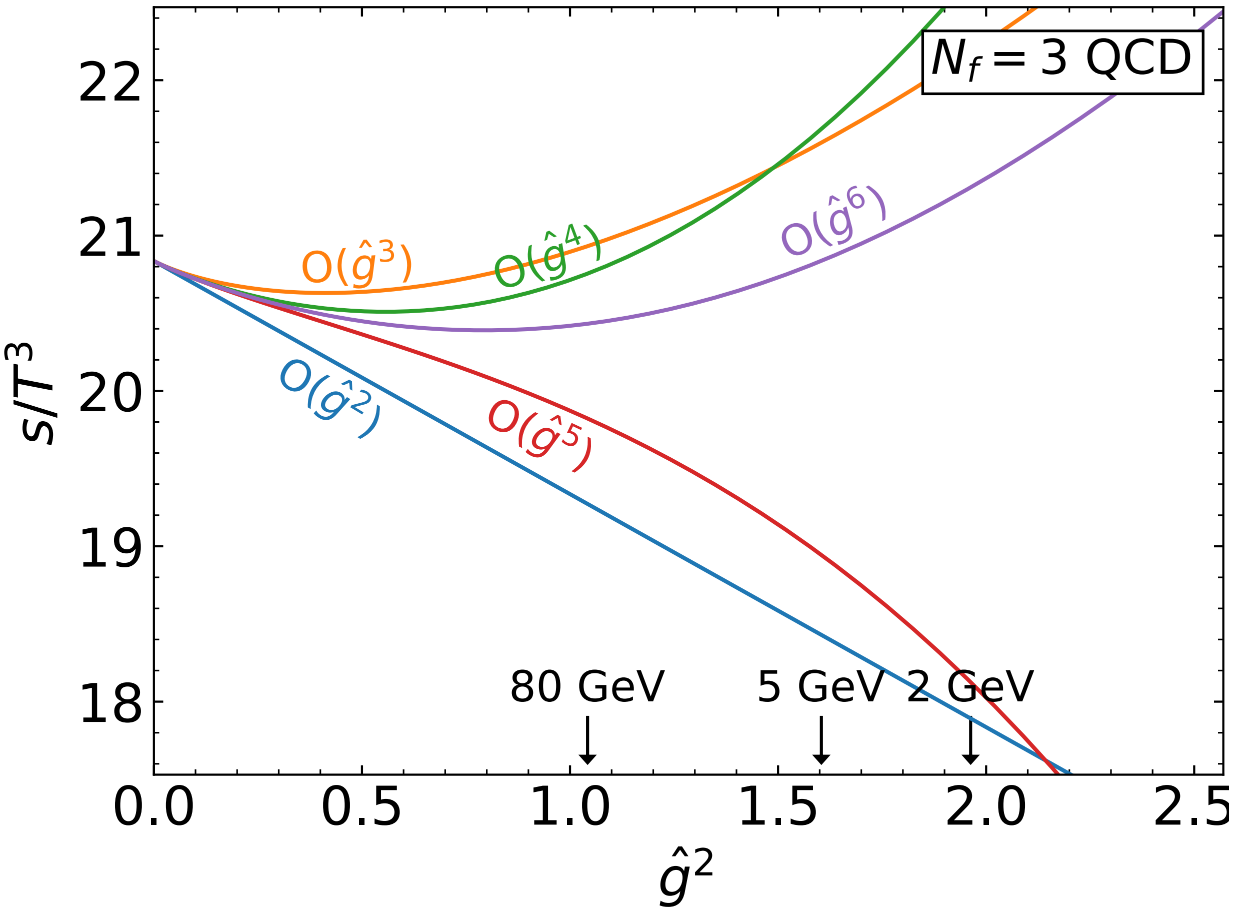
Comparison with Perturbation Theory

- PT computation up to $\hat{g}^6(T)$ order

E. Shuryak (1978), J. Kapusta (1979), T.Toimela (1983), P. Arnold and C.Zhai (1995), C. Zhai and B.Kastening (1995), K. Kajantie et al. (2003)

$$\frac{s(T)}{T^3} = \frac{32\pi^2}{45}\left[s_0 + s_2\hat{g}^2 + s_3\hat{g}^3 + s_4\hat{g}^2 + s_5\hat{g}^5 + \left(s_6 + \frac{q_c}{(2\pi)^6}\right)\hat{g}^6\right] + o(\hat{g}^6)$$

! unknown parameter related to the non-perturbative confining scale $g^2(T)T$



- Very poor convergence even at the Electro-Weak scale

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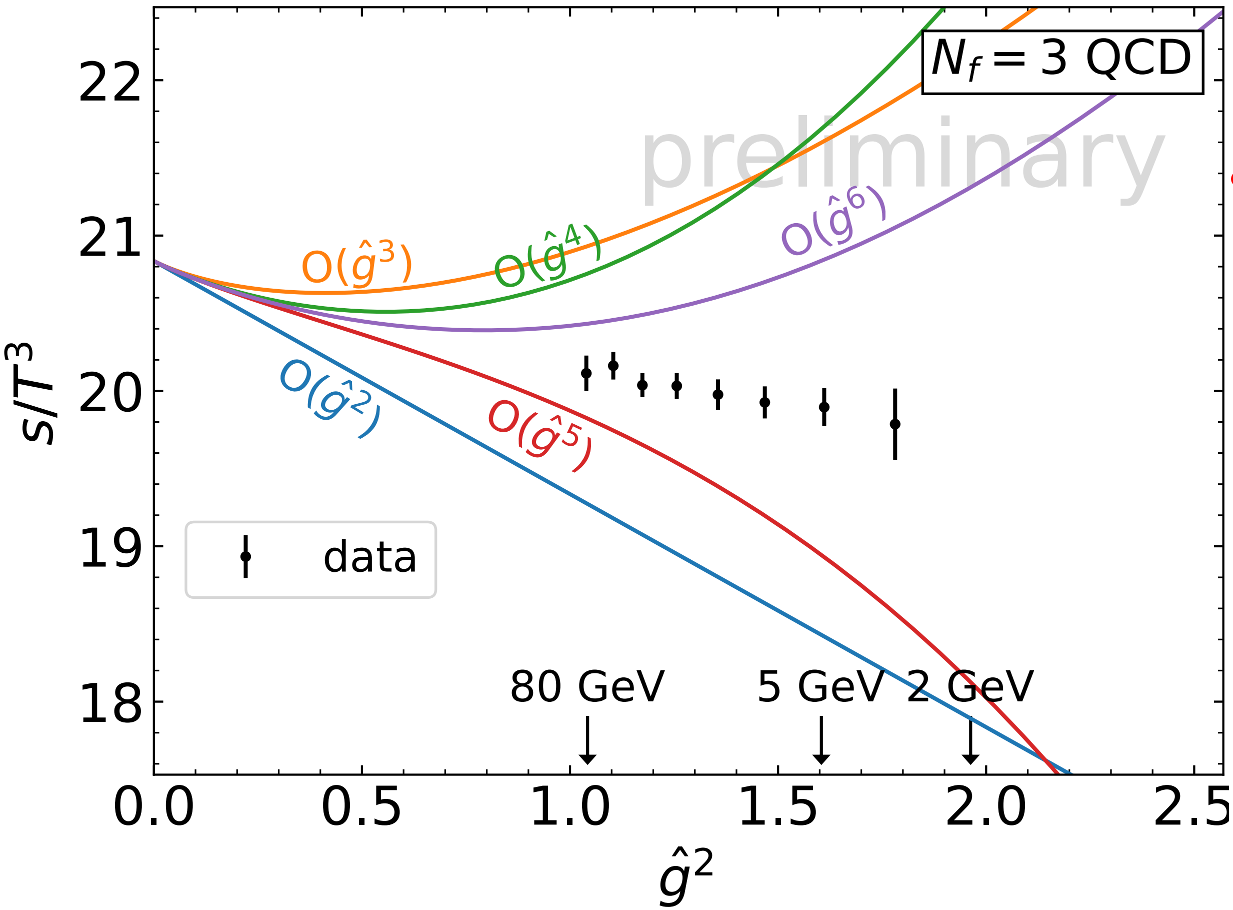

- Very poor convergence even at the Electro-Weak scale
- Data are not described by PT

  No surprise:

  $\hat{g}^2(80\ \text{GeV}) \simeq 1.05$ ➡ $\frac{\hat{g}}{\pi} \simeq 0.33$

  $\hat{g}^2(2\ \text{GeV}) \simeq 1.95$ ➡ $\frac{\hat{g}}{\pi} \simeq 0.44$

- Not matched the PT ansatz

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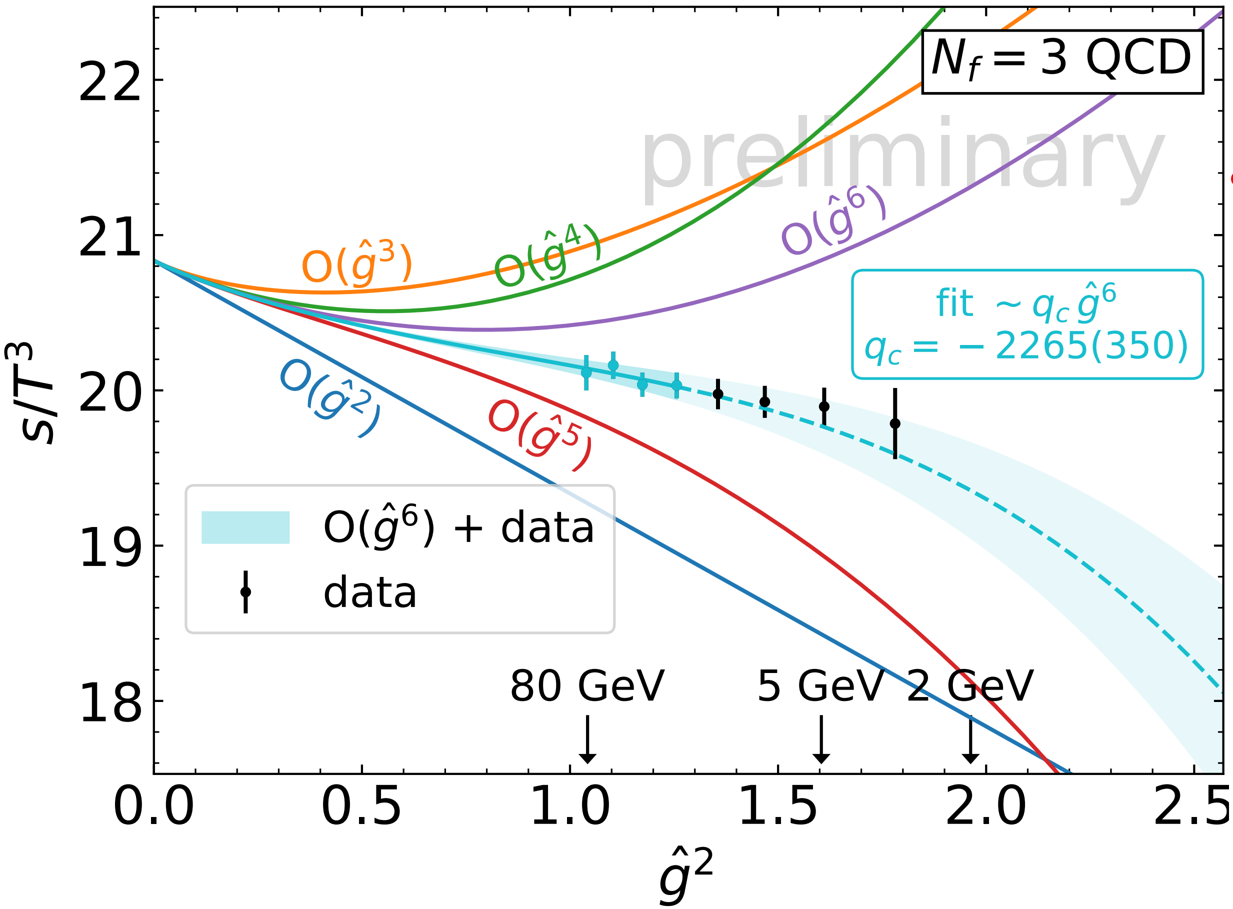

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- Not matched the PT ansatz

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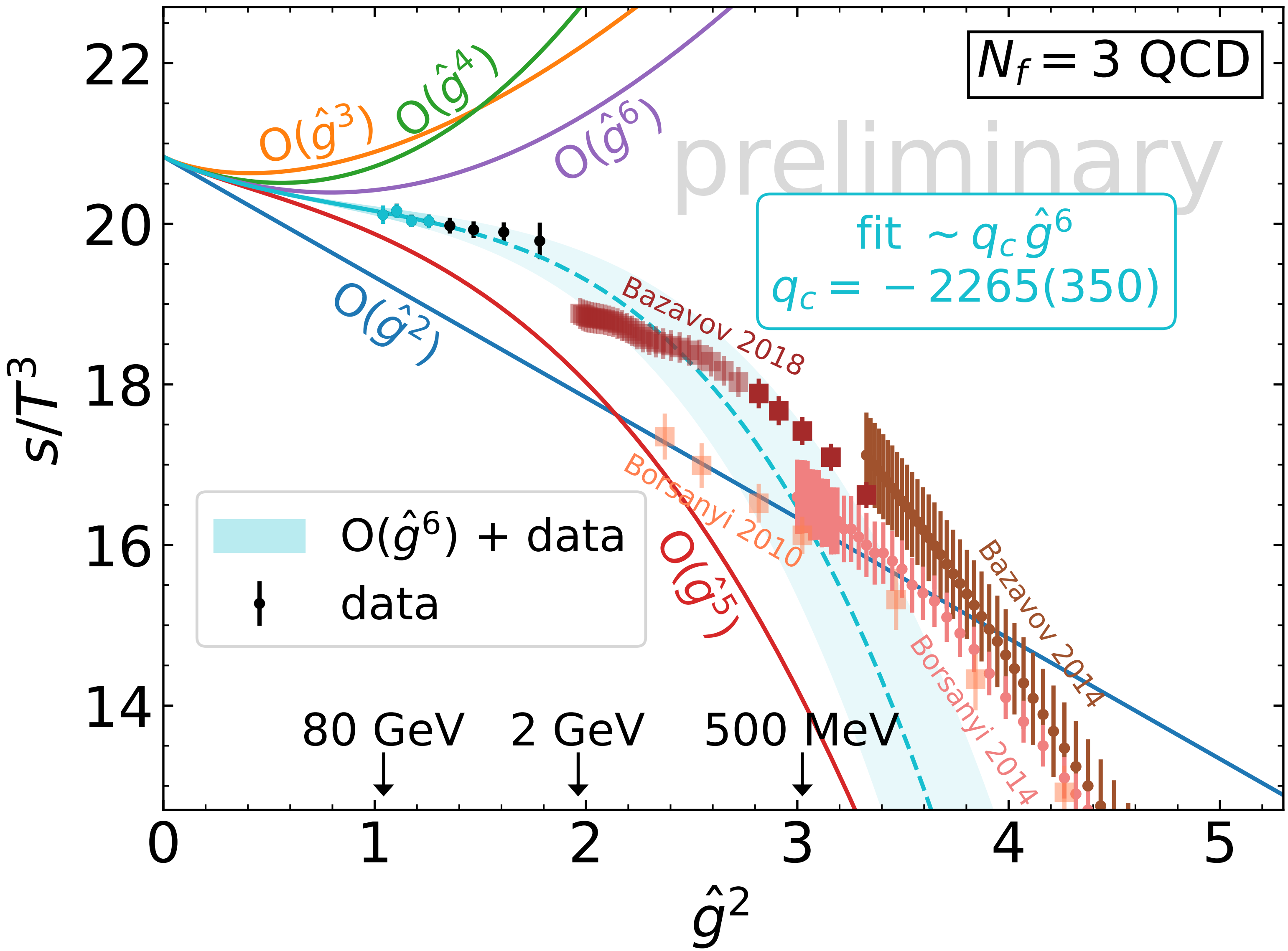
! Non-perturbative and higher order effects fit by q_c represent 45% at the Electro-Weak scale

Connecting to low-temperature results



- PT + q_c do not connect well to low-temperature data available in the literature

Conclusions

- We have computed the EoS of QCD N_f=3 covering a range of about 2 orders of magnitude in temperature in the unexplored regime between 3 and 82 GeV. Simulations are in progress at T = 165 GeV and, maybe, 123 GeV.

- The framework of shifted boundary conditions is a simple, robust and efficient method to investigate with precision $\lesssim$ 1% the thermal features of a QFT.

- The perturbative prediction shows a very poor convergence rate up to the Electro-Weak scale where $\frac{\hat{g}}{\pi} \simeq 0.33$ is quite large and the separation of the hard, soft and ultrasoft scales does not hold.

- Assuming that PT holds, then the non-perturbative physics contributes to $\frac{s(T)}{T^3}$ at order $g^6(T)$: fitting this term to our data, it represents 45% of the total deviation from the Stefan-Boltzmann limit at the Electro-Weak scale

- QCD dynamics remains strongly non-perturbative up to the Electro-Weak scale, and this regime can only be reliably investigated through lattice simulations.