Computation of the latent heat of the deconfinement phase transition of SU(3) Yang-Mills theory

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Outline

Motivation

- SU(3) Yang-Mills theory undergoes a first order deconfining phase transition, associated with the spontaneous breaking of $\mathbb{Z}_3$ center symmetry.
- Convenient framework: thermal field theory with shifted boundary conditions allows us to carry out Monte Carlo simulations exactly at the critical point T_c to evaluate L_H.

Outline

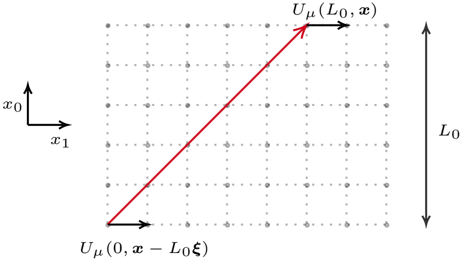
Thermal theory with shifted boundary conditions

Consider SU(3) Yang-Mills on a $L^3 \times L_0$ lattice, described by the Wilson action:

$$S[U] = -\frac{\beta}{6} \sum_{x,\mu\nu} \mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(x) \;\; \text{with: } \beta = 1/g_0^2$$

with shifted boundary conditions for gauge links: [L.Giusti, 14:30]

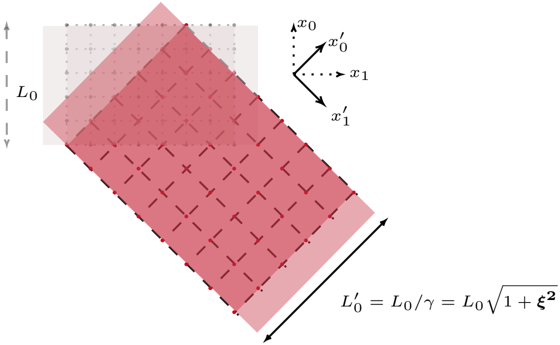
$$U_\mu(L_0, \boldsymbol{x}) = U_\mu(0, \boldsymbol{x} - L_0 \boldsymbol{\xi})$$



Thermal theory with shifted boundary conditions

In the thermodynamic limit, the continuum theory is invariant under Poincaré transformations: [Giusti and Meyer, 2011-13]

$$f(L_0, \boldsymbol{\xi}) = f\left(L_0\sqrt{1+\boldsymbol{\xi}^2}, \mathbf{0}\right)$$



Latent heat definition

The previous relation implies interesting Ward Identities, valid also on the lattice up to discretization errors. For instance for the entropy density [Giusti and Pepe, 2017]:

$$\frac{s(T)}{T^3} = -\frac{(1+\boldsymbol{\xi}^2)}{\xi_k} \frac{\langle T_{0k} \rangle_{\boldsymbol{\xi}}}{T^4} Z_T(g_0^2)$$

hence the latent heat is given by:

$$L_H = \frac{\Delta \varepsilon}{T_c^4} = \frac{\Delta s}{T_c^3} = -\frac{(1+\boldsymbol{\xi}^2)}{\xi_k} \frac{\Delta \langle T_{0k} \rangle_{\boldsymbol{\xi}}}{T_c^4} Z_T({g_0}^2_{\text{crit}})$$

where:

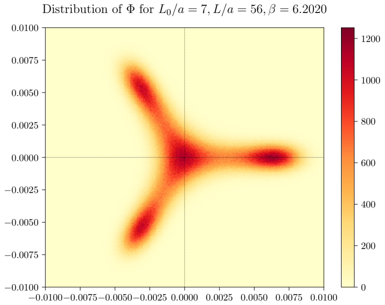
$$\Delta s = s_d(T_c) - s_c(T_c)$$
$$\Delta \langle T_{0k} \rangle_{\boldsymbol{\xi}} = \langle T_{0k} \rangle_{\boldsymbol{\xi},d} - \langle T_{0k} \rangle_{\boldsymbol{\xi},c}$$

Outline

Location of the transition

The transition point T_c can be determined in different ways, which are equivalent in the thermodynamic and continuum limit.

- Phase coexistence: at the transition point the probability of finding the system in either phase is the same. [Francis et al., 2015]
- The Polyakov Loop Φ is an order parameter for the breaking of the Z_3 symmetry.

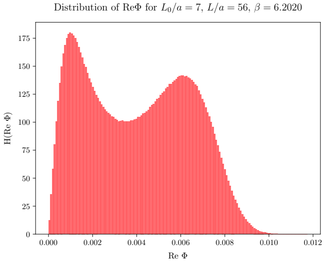


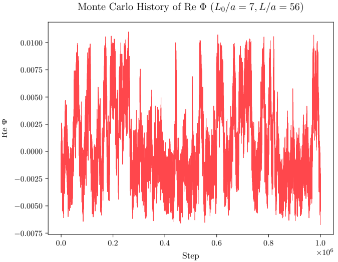
Location of the transition

The probability of finding the system in one phase is given by:

$$w_c(\beta, L) = \langle \theta(\Phi_c - \Phi) \rangle$$
$$w_d(\beta, L) = \langle \theta(\Phi - \Phi_c) \rangle$$
$$\beta = \beta_c \Leftrightarrow 3w_c = w_d$$

where Φ_c is the local minimum between the two peaks, and in the infinite volume limit it helps us define β_c unambiguously.





In order to find $\beta_c(L)$:

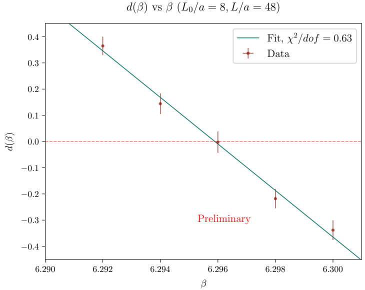
$$d(\beta, L) = \frac{3w_c - w_d}{3w_c + w_d}$$

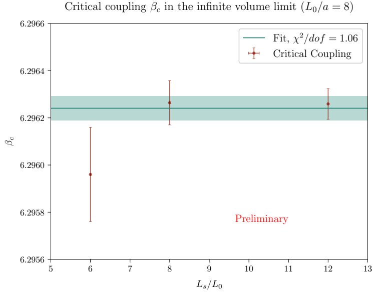
$$d(\beta, L) = 0 \Leftrightarrow \beta = \beta_c(L)$$

then the critical coupling is:

$$\beta_c = \lim_{L\to\infty} \beta_c(L)$$

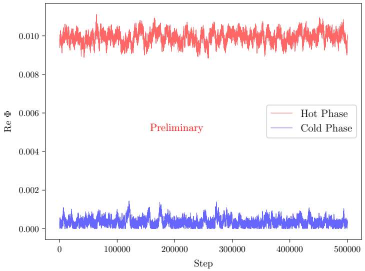
We investigated lattices with $(L_0/a) = 5, 6, 7$ and 8.



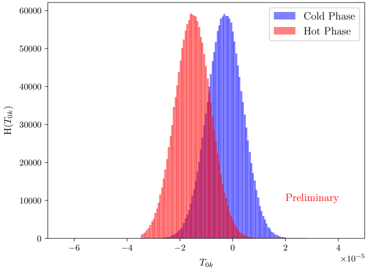


- Simulations are run at $T = T_c$ on large spatial volumes ($L/L_0 \sim 30 - 50$) in each phase, tunneling events are suppressed.
- $\Delta\langle T_{0k}\rangle$ has a $2 - 3‰$ precision, but the final uncertainty on s/T^3 is also affected by the error on the renormalization constant ($4 - 7‰$).

Monte Carlo History of Re Φ ($L_0/a = 6, L/a = 288$)



Estimation of $\langle T_{0k}\rangle_{cold}$ and $\langle T_{0k}\rangle_{hot}$ ($L_0/a = 6$, $\beta = \beta_c$)



Preliminary results for s/T^3 and L_H show a linear behaviour in $(a/L_0)^2$, and in the continuum limit:

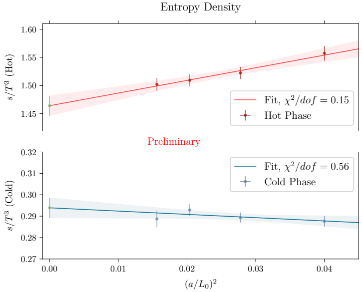
$$\frac{s_{hot}}{T^3} = 1.464(18)$$

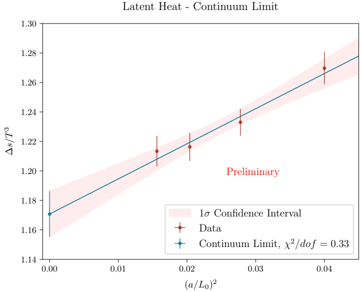
$$\frac{s_{cold}}{T^3} = 0.294(5)$$

while for the latent heat we quote the preliminary result:

$$L_H = 1.171(16)$$

which has a $\sim 3\sigma$ discrepancy with the latest result in literature. [Borsanyi et al., 2022]





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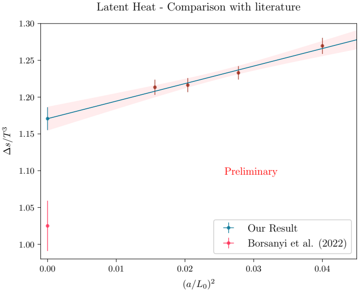

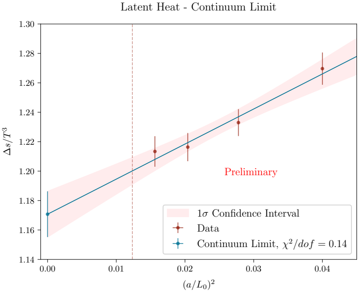

Outline

Conclusions

- Our estimate of $L_H = 1.171(16)$ has a $\sim 1.3\%$ precision, and this result currently produces a $\sim 3\sigma$ tension with other values in literature.
- With complete simulations and an additional point for $(L_0/a) = 9$ the accuracy will further increase, and the aforementioned tension may be clarified.
- Thermal field theories with SBC have been proven to be a convenient numerical setup for the computation of the latent heat of the deconfinement phase transition in SU(3) Yang-Mills.

thank you for your attention