#### Motivation LH in the thermal theory with SBC Computation of the LH Conclusions

Computation of the latent heat of the deconfinement phase transition of  $SU(3)$ Yang-Mills theory

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## Motivation

Definition of the latent heat in the thermal theory

- Computation of the latent heat
- Conclusions

### Outline

#### **1** Motivation

2 Definition of the latent heat in the thermal theory

- $\bigodot$  Computation of the latent heat
- 4 Conclusions

#### Motivation

- SU(3) Yang-Mills theory undergoes a first order deconfining phase transition, associated with the spontaneous breaking of  $\mathbb{Z}_3$  center symmetry.
- *•* Convenient framework: thermal field theory with shifted boundary conditions allows us to carry out Monte Carlo simulations exactly at the critical point  $T_c$  to evaluate  $L_H$ .

# Outline **1** Motivation 2 Definition of the latent heat in the thermal theory

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### Thermal theory with shifted boundary conditions

Consider SU(3) Yang-Mills on a  $L^3 \times L_0$  lattice, described by the Wilson action:

 $\begin{tabular}{lllllllll} Motivation \quad & LH\text{ in the thermal theory with SBC} \quad & \text{Computation of the LH} \quad & \text{Con} \quad$ 

$$
S[U] = -\frac{\beta}{6} \sum_{x,\mu\nu} \text{Re Tr} U_{\mu\nu}(x) \text{ with: } \beta = 1/g_0^2
$$

with shifted boundary conditions for gauge links: [L.Giusti, 14:30]



## Thermal theory with shifted boundary conditions

In the thermodynamic limit, the continuum theory is invariant under Poincaré transformations: [Giusti and Meyer, 2011-13]

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$$
f(L_0,\xi) = f\left(L_0\sqrt{1+\xi^2},\mathbf{0}\right)
$$

$$
L_0
$$
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$$
L_0
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L_1
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\n
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L_2
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L_3
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L_4
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\n
$$
L_5 = L_0 / \gamma = L_0 \sqrt{1 + \xi^2}
$$

#### Latent heat definition

The previous relation implies interesting Ward Identities, valid also on the lattice up to discretization errors. For instance for the entropy density [Giusti and Pepe, 2017]:

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$$
\frac{s(T)}{T^3}=-\frac{(1+\pmb{\xi}^2)}{\xi_k}\frac{\langle T_{0k}\rangle_{\pmb{\xi}}}{T^4}Z_T(g_0^2)
$$

hence the latent heat is given by:

$$
L_H = \frac{\Delta \varepsilon}{T_c^4} = \frac{\Delta s}{T_c^3} = -\frac{(1+\xi^2)}{\xi_k} \frac{\Delta \langle T_{0k} \rangle_{\xi}}{T_c^4} Z_T(g_{0\text{crit}}^2)
$$

where:

$$
\Delta s = s_d(T_c) - s_c(T_c)
$$

$$
\Delta \langle T_{0k} \rangle_{\xi} = \langle T_{0k} \rangle_{\xi,d} - \langle T_{0k} \rangle_{\xi,c}
$$

Outline



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### 3 Computation of the latent heat

4 Conclusions

#### Location of the transition

The transition point  $T_c$  can be determined in different ways, which are equivalent in the thermodynamic and continuum limit.

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- *•* Phase coexistence: at the transition point the probability of finding the system in either phase is the same. [Francis et al., 2015]
- *•* The Polyakov Loop Φ is an order parameter for the breaking of the *Z*<sup>3</sup> symmetry. <sup>−</sup>0.0100−0.0075−0.0050−0.<sup>0025</sup> <sup>0</sup>.<sup>0000</sup> <sup>0</sup>.<sup>0025</sup> <sup>0</sup>.<sup>0050</sup> <sup>0</sup>.<sup>0075</sup> <sup>0</sup>.<sup>0100</sup> <sup>−</sup>0.<sup>0100</sup>



#### Location of the transition

The probability of finding the system in one phase is given by:

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$$
w_c(\beta, L) = \langle \theta(\Phi_c - \Phi) \rangle
$$
  

$$
w_d(\beta, L) = \langle \theta(\Phi - \Phi_c) \rangle
$$
  

$$
\beta = \beta_c \Leftrightarrow 3w_c = w_d
$$

where  $\Phi_c$  is the local minimum between the two peaks, and in the infinite volume limit it helps us define  $\beta_c$  unambiguously.



## $0.4 -$

In order to find  $\beta_c(L)$  :

$$
d(\beta, L) = \frac{3w_c - w_d}{3w_c + w_d}
$$
  

$$
d(\beta, L) = 0 \Leftrightarrow \beta = \beta_c(L)
$$

then the critical coupling is:

$$
\beta_c = \lim_{L \to \infty} \beta_c(L)
$$

We investigated lattices with  $(L_0/a) = 5, 6, 7$  and 8.



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### Motivation LH in the thermal theory with SBC **Computation of the LH** Conco coo $\circ$

- *•* Simulations are run at  $T = T_c$  on large spatial volumes (*L*/*L*<sup>0</sup> *∼* 30 *−* 50) in each phase, tunneling events are suppressed.
- *•* ∆*⟨T*0*k⟩* has a 2 *−* 3‰ precision, but the final uncertainty on  $s/T^3$  is also affected by the error on the renormalization constant  $(4 - 7\%)$ .



−6 −4 −2 0 2 4  $T_{\rm cr}$ 

 $_{0}$   $\pm$ 10000 20000

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 $\times10<sup>-5</sup>$ 

Preliminary

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Preliminary results for *s*/*T* 3 and *L<sup>H</sup>* show a linear behaviour in  $(a/L_0)^2$ , and in the continuum limit:

$$
\frac{s_{hot}}{T^3} = 1.464(18)
$$

$$
\frac{s_{cold}}{T^3} = 0.294(5)
$$

while for the latent heat we quote the preliminary result:

$$
L_H=1.171(16)
$$

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which has a *∼* 3*σ* discrepancy with the latest result in literature. [Borsanyi et al., 2022]

Entropy Density 1.60 s/T 3 (Hot) 1.55 1.50 Fit,  $\chi^2/dof = 0.15$ <br>Hot Phase 1.45 Preliminary  $0.32 -$ Fit,  $\chi^2/dof = 0.56$ <br>Cold Phase 0.31 s/T 3 (Cold)  $0.30 +$  $0.29 _{0.2}$ 0.27 0.00 0.01 0.02 0.03 0.04  $(a/L_0)^2$ Latent Heat - Continuum Limit 1.30 1.28 1.26 1.24 r<br>a<br>∃ 1.22 1.20 Preliminary 1.18 1σ Confidence Interval<br>Data<br>Continuum Limit,  $\chi^2/d$  $rac{1}{1}$ 1.16 iit,  $\chi^2/dof = 0.3$ 1.14 0.00 0.01 0.02 0.03 0.04  $(a/L_0)^2$ 

Motivation LH in the thermal theory with SBC  $\begin{array}{cc} \text{Computation of the LH} \qquad \text{Con} \qquad \text{co} \, \$ 

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#### **Conclusions**

- 
- *•* Our estimate of *L<sup>H</sup>* = 1*.*171(16) has a *∼* 1*.*3% precision, and this result currently produces a  $\sim 3\sigma$  tension with other values in literature.

- *•* With complete simulations and an additional point for  $(L_0/a) = 9$  the accuracy will further increase, and the aforementioned tension may be clarified.
- *•* Thermal field theories with SBC have been proven to be a convenient numerical setup for the computation of the latent heat of the deconfinement phase transition in SU(3) Yang-Mills.

# thank you for your attention